The Economics of Power Consolidation*

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Abstract

Power and resources have been concentrating within major nations around the world, echoing a century-old prediction in political economy that remains a longstanding empirical puzzle with renewed urgency. I develop an economic model of how a society's distribution of power and resources evolves over time. Multiple lineages of players compete by accumulating power, which is modeled as an asset that increases one's probability of winning conflicts over resources. This model provides sharp equilibrium predictions for how a society's distribution of power evolves and whether it approaches equality, oligarchy, or dictatorship in the long run. My main result shows that power and resources *generically* fall into the hands of a few when political competition is left unchecked in large societies, suggesting that today's trends are no anomaly and will not self-correct.

JEL Codes: C73, D02, D63, D72, D74, E22, O40, P00

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1 Introduction

As inequality continues to rise in the United States, so have concerns among economists and other social scientists[1] that it may be drifting towards *oligarchy*, a concern that has been shared by American voters and presidents across history and the political spectrum.[2] However, this is not an exceptionally American trend: rising political and economic inequality has also been observed in the United Kingdom, in several other European and OECD nations, and in other large societies such as Russia, China, and India, alongside global trends of spreading authoritarianism and mounting anxieties about the health of democracies.[3] This has made the following age-old question as relevant as ever: what allows – or indeed prevents – power and resources from falling into the hands of a few in a society? And today, why are so many nations – differing widely in their institutions, norms, geography, etc. – growing more unequal in ways that no longer feel temporary?

Concentrating power and wealth is considered a top global emergency by over 600 leading economists (Stiglitz et al., 2025; Equals, 2025) but the forces driving these trends remain murky. A rising number of leading economists have stressed that without a more systematic understanding of power dynamics, our understanding of economic inequality, economic growth, and many other important economic phenomena will remain incomplete.[4]

This paper develops a framework that precisely predicts how a society's distribution of power evolves over time using standard economic modeling tools from dynamic contests. The analysis allows these complex dynamics to be simply understood in terms of familiar marginal analysis. My results show that today's authoritarian and oligarchic trends *generically* emerge from standard power accumulation incentives, and will not self-correct. This turns out to address puzzles in political economy that have echoed over the past century through today. Before turning to the precise details of my contribution and approach, it is worth briefly taking stock of what is known – and what is puzzling – about the dynamics of political inequality.

There is extensive research explaining how these dynamics directly result from particular qualitative features of a nation (culture, geography, population size, democratic norms like voter turnout, etc.) or its de jure institutions (democratic/authoritarian or inclusive/extractive in nature, bureaucratic or hierarchical in structure, or other official, explicit details about its constitution, laws, legislative procedures, checks and balances, organization, etc.). These factors are relevant, but since de facto power consolidation can take place regardless of de jure institutions or norms (Acemoğlu and Robinson, 2008; Winters, 2011; Francois et al., 2015) it is prudent to "avoid the comfortable idea that the mere form of government can of itself safeguard a nation against despotism" (Encyclopædia Britannica, 1946). More generally, otherwise similar/different countries can end

[1] Dal Bó et al. (2009); Winters and Page (2009); Winters (2011); Gilens and Page (2014); Krugman (2014, 2025); Piketty (2014); Solow (2014a,b); Saez and Zucman (2019); Galbraith and Choi (2020); Acemoğlu (2024b); Fung and Lessig (2025); Khan (2025).

[2] For bipartisan concern among U.S. voters, see University of Chicago AP-NORC Center (2018) and Pew Research Center (2023). For U.S. presidents, this includes Adams (Mayville, 2018), Jefferson (1820), Jackson (1837), Roosevelt (1901), Roosevelt (1938), Eisenhower (1961), Carter (2015), and Biden (2025).

[3] For the United Kingdom, see Ansell and Gingrich (2024); for European nations, see Lupu and Pontusson (2023); for OECD nations, see OECD (2008, 2011, 2012, 2015, 2021, 2025); for Russia, China, and India, see Chancel, Piketty, Saez and Zucman (2022) and Brookings Institution (2023); for spreading authoritarianism, see Freedom House (2022), United Nations (2023), Reuters (2023), V-Dem (2025), and Stiglitz et al. (2025); for democratic trends, see Hyde (2020), Freedom House (2020, 2024), Deutsche Welle (2024), Economist Intelligence Unit (2020, 2024, 2025), and Riedl et al. (2024).

[4] Acemoğlu (2012, 2024a); Acemoğlu and Khan (2024); Aghion et al. (2021); Ansell and Gingrich (2024); Beramendi et al. (2024); Blanchard and Rodrik (2023); Callander et al. (2022); Deaton (2024); Gilens (2012); Gilens and Page (2014); Houle (2018); Krugman (2020, 2025); Lagunoff (2024); Lupu and Pontusson (2023); Piketty (2014 p. 20, 35; 2015); Rau and Stokes (2025); Rausser et al. (2011); Remington (2023); Schäfer and Schwander (2019); Stiglitz (2016, 2011); World Bank (2006, 2017); Winters and Page (2009); Winters (2011); United Nations (2020).

up at vastly different/similar regimes (Acemoğlu and Robinson, 2019). No unified framework can yet explain how oligarchy, dictatorship, and equality each emerge, but a growing body of research has pointed to a central thread: political competition over de facto power. It has been identified as the key driver of how oligarchies emerge (Winters and Page, 2009; Winters, 2011; Gilens, 2012; Gilens and Page, 2014). It also lies at the heart of Acemoğlu and Robinson (2023) where equality emerges within a *narrow corridor* through which lockstep political competition incentivizes elites and non-elites to keep power in balance, while outside this corridor dictatorship emerges as one side dominates the other. The narrow corridor framework's well-founded modeling approach (built using tools from dynamic contests and asset accumulation) is evident in its ability to explain the emergence of equality and dictatorship across countries and history (Acemoğlu and Robinson, 2019). However, today's global trends then become all the more puzzling: why has power been consolidating within countries lying in the narrow corridor as well as those outside it? This introduces a much deeper puzzle: over a century ago, Michels (1911) predicted that in large enough groups of people, power and resources inevitably fall into the hands of a few, stating this as *The Iron Law of Oligarchy*. Despite the uncanny prescience of the Iron Law's prediction for today's global trends, its underlying mechanism has remained largely elusive (Leach, 2005, 2015; Acemoğlu and Robinson, 2012, p. 361; Winters, 2011, p. 2; Diefenbach, 2019; Van den Bosch, 2025, p. 574).

Contributions My initial results (Propositions 1 and 2) provide a unifying framework that sharply predicts how a society's distribution of power endogenously evolves over time and whether it will approach equality, dictatorship, or one of the various degrees of oligarchy in between. Built using standard economic tools, this framework shows that the same competitive forces underlie the emergence of these vastly different regimes, and it also reveals a deeper connection across the research frontier: consistent with Acemoğlu and Robinson (2019, 2023) competitive pressure propels societies through a narrow corridor to equality via the Red Queen effect[5] while the discouragement effect[6] pulls them towards dictatorship, whereas oligarchies emerge through a novel combination of both effects in a competitive process that reflects empirical observations by Winters and Page (2009), Winters (2011), Gilens (2012), and Gilens and Page (2014).

My main results (Propositions 3 through 5) show that power and resources must fall into the hands of a few in sufficiently large societies: egalitarianism always destabilizes past some population size, and only strong dictatorships and sufficiently concentrated oligarchies remain stable. This addresses the long-standing open question surrounding Michels' (1911) Iron Law, showing that it generically emerges under standard economic assumptions on power accumulation incentives. The underlying mechanism is, disquietingly, the gradual collapse of the narrow corridor: as countries become more populous or fragmented, the same competitive pressure that once kept power in check intensifies and turns pernicious, causing this corridor to narrow further until it vanishes completely, and equality destabilizes.

Modeling Approach These novel results rely only on standard economic modeling tools. This paper constructs an economic framework for how a society's distribution of de facto political power endogenously evolves due to intergenerational competition over resources. Society is populated by multiple lineages/dynasties of players, who can be interpreted literally as individuals or as

[5]Red Queen effect: competitive pressure between closely matched opponents induces each to accumulate power, but in relative terms they cannot outrun one another (positive feedback loop).

[6]Discouragement effect: the further ahead or behind one is from the rest, the weaker their incentive to accumulate power to close or widen the gap (negative feedback loop).

representative agents of different socioeconomic groups aligned by interest, nominal status, etc.[7] Power is modeled as an asset that increases one's chances of winning distributional conflicts over consumable resources/rents such as public funds, natural resources, favorable policies, etc.[8] Players compete over time by accumulating and passing along stocks of power to increase their chances of winning conflicts, which drives how their distribution of power evolves. Concretely, this notion of power captures forms of de facto political influence that can be accumulated through lobbying, bribery, campaign contributions, dynastic entrenchment, social and political connections, organizational capacity, or hard power like weapons and armies. These forms of power behave like endowable assets (costly to accumulate, depreciate without upkeep) and benefit holders by increasing their chances of exacting favorable conflict outcomes.[9] Only two standard, substantive assumptions are made: convex accumulation costs and victory probabilities that depend on power differences.[10]

This modeling approach is known as a dynamic contest. As discussed above, evidence increasingly points to political competition over de facto power as the central driver of how a society's distribution of power evolves across national and institutional contexts. Contests are a well-established game-theoretic modeling approach to this exact kind of political competition,[11] but they have not been used to study how an entire society's distribution of power evolves over time (Dixit, 2021). The present work fills this gap by providing a portable benchmark framework for understanding these dynamics using a well grounded economic modeling approach motivated by empirical observation.

The rest of this paper is organized as follows. The extensive background details of the modeling approach and related literature are discussed below. Section 2 constructs the model. Section 3 provides equilibrium predictions for how a society's distribution of power and resources evolves, and how dictatorial, oligarchic, and egalitarian regimes each emerge in the long run. Section 4 contains the main results of this paper which show how large societies inevitably become unequal in the absence of intervention. Section 5 presents the conclusions. The Appendix contains the proofs of the results in the main text of this paper (Section A), auxiliary technical results (Section B), and extensions of the results to the case where societies' resource endowments vary with population size (Section C).

[7]Individual players can model the rise of political dynasties or individual oligarchs (reflecting empirical work by Dal Bó et al. (2009) and Winters (2011), respectively) or more generally how a society's distribution of power can also approach dictatorship or equality. Representative agents can model competition between different interest groups (Becker, 1983, 1985; Grossman and Helpman, 1994) or different socioeconomic groups such as the nominal rulers/elites and civilians/non-elites (Acemoğlu and Robinson, 2019, 2023), different parties or factions (Prato and Invernizzi, 2024), different ethnic groups (Francois et al., 2015; Laurent-Lucchetti et al., 2023), etc.

[8]For distributional conflicts over public funds and material gain, see Mohtadi and Roe (2003); Winters (2011); for natural resources, see Hirshleifer (1989); for favorable policies, see Becker (1983, 1985), Grossman and Helpman (1994); Alesina and Rodrik (1994); for more general discussions of distributional conflicts see Konrad (2009); Piketty (2015); Beviá and Corchón (2024).

[9]For material power (e.g., lobbying, bribery, donations) see Becker (1983, 1985), Winters and Page (2009), and Winters (2011); for campaign spending see Acharya et al. (2024) and Prato and Invernizzi (2024); for entrenchment see Dal Bó et al. (2009); for hard power see Hirshleifer (1989), Konrad (2009), Ewerhart (2021), and Beviá and Corchón (2024); for organizational power and social/political connections see Ehrlich and Lui (1999), Acemoğlu and Robinson (2008), and Akciğit et al. (2023).

[10]For convex accumulation costs see Dixit and Pindyck (1994), Acemoğlu (2009), Fang, Noe and Strack (2020), and Prato and Invernizzi (2024). For victory probabilities, this paper assumes the difference-form contest success function introduced by Hirshleifer (1989) and axiomatized by Skaperdas (1996), which is also standard and natural for the present context, as discussed in the Related Literature and Remark 1 in the main text.

[11]Dixit (1987); Hirshleifer (1989); Skaperdas (1996); Konrad (2009); Dechenaux et al. (2015); Francois et al. (2015); Levine and Mattozzi (2022); Acemoğlu and Robinson (2023); Beviá and Corchón (2024); Acharya et al. (2024)

Related Literature This paper joins Acemoğlu and Robinson (2023) in a nascent, promising research direction (Dixit, 2021, p. 1362-63, 1369-70, 1375) towards systematically understanding the dynamics of political inequality using well-established tools from dynamic contests.[12] Acemoğlu and Robinson (2023) – abbreviated AR hereafter – analyze a two player difference-form dynamic contest between "the state"/"elites" and "civil society"/"non-elites" where power accumulation costs are convex and sufficiently powerful players have an additional linear cost advantage.[13] This paper advances this direction in several significant ways by (i) generalizing and reframing the analysis to societies of any finite population; (ii) allowing general convex costs with or without power-based cost advantages; (iii) relaxing several technical assumptions made in AR (parts 1 through 4 of their Assumption 2; parts 2 and 3 of their Assumption 3). In addition to the general interest contributions above, this paper reveals two surprising inversions of the main results in AR, along with other qualitatively new insights.

First, one of the main insights from AR is that competitive pressure drives societies through a narrow corridor towards equality; my analysis shows how this mechanism unexpectedly inverts in larger societies. That is, the same competitive pressure that sustains the narrow corridor to equality in small societies also causes it to gradually collapse in larger populations, eventually destabilizing egalitarianism and triggering the dynamics behind Michels' (1911) Iron Law of Oligarchy. Second, I show how power consolidation (in the form of dictatorship or concentrated oligarchy) can take place with or without power-based cost advantages, which were naturally thought to play a key role in the emergence of dictatorship and in the framework's dynamics more broadly (AR, p. 412-414; Dixit, 2021, p. 1364). Third, another central result in AR is that the strongest states/elites emerge through sustained competition with (rather than domination of) civil society/non-elites, since dictators always hold less power than they would under egalitarianism (due to insufficient competitive pressure). In contrast, the framework in this paper admits both weak and strong dictatorships; while only the former is possible in AR, the latter form of dictatorship overwhelmingly dominates as societies become large. Fourth, this paper allows for a spectrum of oligarchic regimes to emerge through a novel combination of the forces that push societies toward equality and dictatorship.

Developing this approach speaks to calls by Dixit (2021) to build on AR and by Piketty (2015) to put "distribution back at the center of economics." More broadly, it contributes to ongoing efforts towards solving the political inequality puzzle outlined at the start of this paper. This is a highly multifaceted challenge, and scholars have been advancing along multiple fronts such as ideology and information (Piketty, 2019; Bowen et al., 2022; Levy et al., 2022; Acemoğlu et al., 2025), trade (Tabellini and Magistretti, 2024), education (Acemoğlu et al., 2015; Gethin et al., 2022), moral hazard (Prato and Invernizzi, 2024), bargaining protocols (Bernheim et al., 2006; Bowen and Zahran, 2012; Jeon and Hwang, 2022; Ali et al., 2018), economic aspirations (Genicot and Ray, 2017), scalar stress (Perret et al., 2020), social norms (Acemoğlu et al., 2025), technology (Piketty, 2014; Acemoğlu and Johnson, 2023; Acemoğlu and Restrepo, 2025), policy decay (Callander and Martin, 2025), and many others.[14] This paper advances the front focused on de facto political power accumulation and distributive conflicts, which have been observed to play key roles in the emergence of dictatorial,

[12] Acemoğlu and Robinson (2019) provide an extensive view of history through the lens of their model formalized in Acemoğlu and Robinson (2023), which is microfounded in Acemoğlu (2005). Acemoğlu, Egorov and Sonin (2008a) axiomatically study coalition formation in a related framework.

[13] Specifically, the authors assume that $C(I, x) = c(I) + \max\{\gamma - x, 0\}I$ is the cost of accumulating power at rate I given current power level x, where c is smooth, convex, and increasing and $\gamma \geq 0$ is the threshold below which players incur the extra linear accumulation cost.

[14] See Gavrilets and Currie (2023), Lupu and Pontusson (2023), Meng et al. (2023), Elkjær and Klitgaard (2024), Lagunoff (2024), and Egorov and Sonin (2025) for helpful and thorough surveys. See Acemoğlu and Robinson (2019, 2023) and Winters (2011) for detailed discussions of previous approaches focused on structural factors (e.g. culture, geography, etc.) and de jure institutions and norms (e.g. voting, turnout, etc.), respectively.

egalitarian, and oligarchic distributions of power across institutional and national settings (Winters and Page, 2009; Winters, 2011; Gilens and Page, 2014; Acemoğlu and Robinson, 2019).

Dynamic contests model situations where players compete over time by making costly investments that increase their chances of winning conflicts over a prize.[15] In the case of the power accumulation contest seen here, this approach models Weber's (1925) widely-adopted definition of de facto power as "the probability that one actor within a social relationship will be in a position to carry out his own will despite resistance, regardless of the basis on which this probability rests," and the fact that it must be *accumulated* by, for instance, "expenditures of time and money on campaign contributions, political advertising, and ... other ways that exert political pressure" (Becker, 1983).

Convex accumulation costs are standard (Dixit and Pindyck, 1994; Acemoğlu, 2009; Fang, Noe and Strack, 2020). The cost and marginal cost of power accumulation are also assumed weakly diminishing in one's current amount of power to flexibly permit both (1) the standard case where costs do not depend on current power and (2) the intuitive case where power yields a cost advantage. The results of this model do not rely on whether costs are in/dependent of current power.[16]

A Contest Success Function (CSF) gives each player's likelihood of winning conflicts given their power relative to their opponents. This paper uses a standard[17] difference-form CSF, which was introduced by Hirshleifer (1989, 1991) to relax certain overly idealized aspects of the ratio-form CSF (Tullock, 1980); both forms of CSF are axiomized in Skaperdas (1996).[18] The substantive properties of this CSF are that victory probabilities (1) only depend on power differences, (2) each player's marginal benefit of accumulating power is increasing in how closely matched they are with their opponents and (3) powerless players have a small but non-zero chance of winning conflicts. For other papers that employ the difference-form CSF to study political competition and conflict, see Garfinkel and Skaperdas (2007), Gartzke and Rohner (2011), Francois et al. (2015), Acemoğlu and Robinson (2023), and Prato and Invernizzi (2024). Outside of contest theory, the difference-form CSF is known as the *softargmax* function, since it provides a noisy analogue to the argmax function in a parsimonious (Maximum Entropy) sense (Jaynes, 2003, p. 343-371; Goodfellow et al., 2016, p. 183; Franke and Degen, 2023, p. 16-18).

As mentioned earlier, dictatorship emerges through the *discouragement effect*, egalitarianism through the *Red Queen effect*, and oligarchy through a novel combination of both effects. The discouragement effect is observed in a variety of dynamic contests. See Konrad (2012) for a survey of dynamic contests focused on the discouragement effect. The Red Queen effect mirrors the same phenomenon from evolutionary biology (Van Valen, 1973) which explains how, for instance, foxes and rabbits induced one another to become similarly fast. The name derives from the Red Queen's Race in *Through the Looking-Glass*, Lewis Carroll's (1871) sequel to *Alice in Wonderland*, where progress is purely relative ("it takes all the running you can do, to keep in the same place... to get somewhere else, you must run at least twice as fast as that!"). Analogously, along paths towards equality or oligarchy, closely matched players induce each other to accumulate power, but none of them can outrun the rest, so in relative terms they each stand still. For more discussion of the Red

[15]For detailed and helpful references on contest theory, see Konrad (2009) and Beviá and Corchón (2024); for surveys, see Corchón (2007), Corchón and Serena (2018), and Fu and Wu (2019); Dechenaux et al. (2015) provide a survey of the experimental literature. See Jia and Skaperdas (2012) and Jia et al. (2013) for discussions focused on empirical estimation.

[16]Note that more powerful players can incur higher costs as an *equilibrium effect*; on certain equilibrium paths competitive pressure may induce players to accumulate power more rapidly (thus incurring higher costs) as they become more powerful.

[17]Skaperdas (1996), Baik (1998), Francois et al. (2015), Cubel and Sanchez-Pages (2016); Ewerhart (2021); Ewerhart and Sun (2024); Beviá and Corchón (2024, §2.2).

[18]See also the axiomizations provided by Clark and Riis (1996) and Cubel and Sanchez-Pages (2016); see Baik (1998) and Che and Gale (2000) for generalizations and variants of the difference-form CSF.

Queen effect in economics, see Baumol (2004).

This paper has a number of interesting parallels with Fang, Noe and Strack (2020), who also study an all-pay contest with risk-neutral players and convex costs, but they largely consider a static setting with homogeneous players and also focus on optimal (effort-maximizing) contest design. The authors observe a discouraging effect from competitive pressure due to a large number of players, which partially corresponds to some of this paper's main results. This is analogous to the destabilization of the egalitarian regime in large societies observed in my paper, but does not coincide with my results that dictatorial and sufficiently concentrated oligarchies remain stable.[19] Indirectly related to this work are Berry (1993), Clark and Riis (1996), and Chowdhury and Kim (2014) who study multi-winner contests. While the model herein employs a conventional single-winner contest mechanism, one may interpret there being multiple effective winners in oligarchic and egalitarian power structures.

Bowen et al. (2022) study a qualitatively distinct form of power known as *personal power*. Their notion is similar in that it increases the probability of actualizing one's ideal outcome by asserting one's will. However, it is qualitatively different in that personal power derives from one's personal characteristics, and its effectiveness must be learned by others. This paper instead focuses on the forms of power that are accumulated and inherited and whose effectiveness is common knowledge; studying the case where one's stock of power is private information would be a fruitful direction for future research.

The emergence of oligarchies in large groups[20] of people was viewed as an inevitability by Michels (1911) when he first articulated his Iron Law of Oligarchy. His main thesis was that larger groups eventually establish hierarchical, bureaucratic institutions in order to relieve "scalar stress" (i.e. difficulty coordinating collective actions), and this naturally causes power and resources to fall into the hands of a few. Leach (2005) notes that "[d]espite almost a century of scholarly debate … there is still no consensus about whether and under what conditions Michels's claim holds true." This assessment appears to be supported by the thorough reviews of the modern literature provided by Leach (2005, 2015), Tolbert (2010), Diefenbach (2019) and Van den Bosch (2025).

This paper joins a burgeoning literature studying the Iron Law of Oligarchy rigorously. My contribution is a rigorous analytical characterization of how the Iron Law emerges in sufficiently large groups in a way that is portable across de jure institutional environments (bureaucratic or otherwise). My results show how it naturally emerges from standard properties of power accumulation incentives. Perret et al. (2020) use simulations to investigate the hypothesized role of scalar stress in the formation of hierarchies in large societies. Turchin et al. (2022) empirically investigate this (and related) hypotheses, and identify military and agricultural technological progress as key causal factors. Group size plays no role in remaining rigorous work related to the Iron Law focus on other aspects. Prato and Invernizzi (2024) considers the counterweights to the Iron Law in a party politics setting, studied through the lens of a moral hazard and contest design problem. They model a one-shot game between a party conference (who designs a reward scheme to maximize the party's chances of winning against an opposing party with exogenous strength) and the party's factions (who each exert costly effort that increases the party's collective chances of winning the interparty contest and the faction's individual chances of winning the intraparty

[19]In section S.4 of Fang, Noe and Strack's (2020) Online Appendix, the authors consider an extension with heterogeneous agents, but in a significantly qualitatively different way than herein. Proposition S-2 therein appears loosely related to how oligarchies remain stable in this paper.

[20]While Michels's (1911) discussion usually focuses on political parties, he also posited that the Iron Law applies more broadly to large organizations and groups in general (p. 70). This conjecture has been maintained by later scholars (Diefenbach, 2019, p. 545; Tolbert, 2010; Winters, 2011, p. 2; Van den Bosch, 2025, p. 574) and is reflected in the trends discussed at the start of this paper and confirmed by the results of the present work.

contest that follows). Their main results show that parties may find internal power sharing more efficient than power consolidation under certain conditions. Acemoğlu and Robinson (2006) study a game between nominal elites and non-elites, and their analysis focuses on the interaction between de jure and de facto power rather than group size. Their main results show how elites can use de facto power to circumvent de jure constraints. Although, Prato and Invernizzi (2024), Acemoğlu and Robinson (2006), and the present work study very different aspects of the Iron Law, their modeling frameworks are quite compatible. A combined framework may be a potentially fruitful approach to studying how Iron Law can be counteracted in a way that is dynamically robust to being unraveled by elites over time.

Jeon and Hwang (2022) consider a dynamic bargaining framework where dictatorial or diarchic (two oligarchs) regimes can emerge at any group size. In their model, dictatorial power structures are unstable given that agents are sufficiently forward-looking. Interestingly, the results of the framework here hold if players are short lived, and assuming longer-lived agents does not provide substantive additional insight.[21]

The emergence of egalitarian, oligarchic, and dictatorial regimes has historically been treated as disparate phenomena, but modern empirical work seems to increasingly point towards a key, common factor: competition with respect to de facto power accumulation. Winters (2011) provides an extensive study of how oligarchies emerge and persist in a variety of societies around the world, where he also notes that a "consistent pattern in human history is for very small minorities to amass great wealth and power" (p. 25). Rather than focusing on how particular institutional structures allow or preclude the formation of oligarchies, he instead argues that wealth defense and the accumulation of material power are far more important factors in the formation of oligarchies. Moreover, he emphasizes how oligarchies can emerge even in the presence of democratic norms and institutions, thus becoming democracies only in name. This is also stressed in Winters and Page's (2009) empirical study on the distribution of material power in the United States. Gilens (2012) and Gilens and Page (2014) observe that competition with respect to de facto power accumulation is a key driver of how political inequality evolves in the U.S. In parallel fashion, the observations in Acemoğlu and Robinson (2019) strongly suggest that this same driver has also been at the heart of how equality and dictatorship emerge, in a general pattern that has been stable across history and different structural factors (culture, geography, etc.).

Although this paper focuses on the role of de facto power accumulation, investigating the interaction between de jure and de facto power is critical for getting full, detailed understanding of how the distribution of power evolves. Bai and Lagunoff (2011) study the Faustian dynamics between enacting one's preferred policies and retaining political power. Buchheim and Ulbricht (2020) study the interaction between regime change, revolutionary pressure from disenfranchised citizens, and preemptive policy reforms. Subsection 4.1 of the present paper provides complementary results on the effect of dictatorial power by characterizing how it induces higher political power. Lagunoff (2024) provides a groundbreaking analysis of the interaction between targeted punishments/rewards by authoritarian regimes and the dynamics of wealth inequality. The analysis of endogenously evolving political institutions (Aghion et al., 2004; Lagunoff, 2009; Wolitzky, 2013) is another fruitful line of research studying the interaction between de facto and de jure power.

There exist several other related papers that study how political competition drives distributions/balances of power in a variety of settings. Francois et al. (2015), De Luca et al. (2018), and Laurent-Lucchetti et al. (2023) empirically and theoretically study power sharing between ethnic

[21]The results of this paper remain qualitatively unchanged when players are forward looking and not exceedingly patient; when players are patient enough, the results do qualitatively change, but not for an insightful reason. The reasoning and details are summarized in Remark 3, as this is not among the significant differences with AR discussed earlier.

groups. Myerson (1993), Lizzeri and Persico (2005), and Acharya et al. (2024) focus on electoral campaigns, while Acemoğlu, Egorov and Sonin (2008b), Perisco (2023), Polburn and Polburn (2025) focus on non-democratic settings. Myerson (1993) and Lizzeri and Persico (2005) observe a drawback of electoral competition when the number of parties grow large, showing that unequal and inefficient outcomes emerge due to increased pressure to appeal to narrow interests. The main results of my paper reveal a generalized drawback from competitive pressure due to a crowding out of incentives as the number of competitors grows, which causes power consolidation to eventually become the generic stable outcome.

Acemoğlu et al. (2019) observe that as societies become large, social conflicts become more likely due to increased competitive pressure. This paper confirms this relationship between population size and competitive pressure, but also demonstrates its capacity to cause all but the most powerful to capitulate. Acemoğlu et al. (2025) observe a positive feedback loop in sufficiently successful democracies arising from a strengthening of democratic norms among citizens. This suggests one channel that may endogenously keep the narrow corridor more stable.

This paper is also related to Bowen and Zahran (2012). In the present model, dictatorial and oligarchic power structures are reached by trajectories that originate sufficiently nearby. These classes respectively have qualitative similarities to the compromise and no-compromise classes in Bowen and Zahran (2012), which are reached in an analogous fashion. The dynamics in this paper also have parallels to those seen in Genicot and Ray (2017), who analyze how inequality and aspirations evolve and interact over time. Namely, the magnitude of agents' aspirations in their model has a similar encouraging/discouraging effect as small/large power differences do in my model. Another related paper is Li et al. (2017), who study power dynamics in organizations using a principal-agent framework.

2 Model

This section constructs the framework of this paper. Several variations of this model yield the same core insights; the details are discussed in Footnotes 22-27 and Remarks 1-7 as they become relevant.

2.1 Setup

Time $t \in \{0, \Delta, 2\Delta, ...\}$ has an infinite horizon and is initially taken to be discrete with period length, $\Delta > 0$. Period length is later made small when attention is brought to model dynamics, which are more tractably characterized in continuous time.

There are $N \geq 2$ lineages of players that are replaced each period.[22] Lineage $i \in \{1, ..., N\}$ is formally defined as $\mathbb{i} \equiv \{i_0, i_\Delta, i_{2\Delta}, ...\}$, where i_t denotes the player from lineage i born in generation t. Players compete by accumulating and passing along stocks of *power*, an asset that increases one's chances of winning conflicts over resources (in a way made precise below). The amount of power held by the lineage-i player at time t is given by $x_{it} \in [0, \chi]$, for some fixed $\chi > 0$.[23]

Each lineage $i \in \{1, ..., N\}$ is initially endowed with x_{i0} units of power. This is held by player i_0, who remains inactive and simply serves to initialize the game. Play then proceeds as follows: at the beginning of each period $t \in \{\Delta, 2\Delta, ...\}$, player i_t inherits their predecessor's power $x_{i,t-\Delta}$ which linearly[24] depreciates at rate, $\delta > 0$ (hence by amount $\delta\Delta$ each period). Players then simultaneously

[22]Considering forward-looking players yields no new insights. The reason is discussed in Remark 3, as soon as the necessary context is established.

[23]Assuming that power takes values in $[0, \chi]$ is standard (Skaperdas, 1996, p. 284) and vastly simplifies exposition while leaving the results qualitatively unchanged. This artificial constraint is fully relaxed in section 4. Assuming $x_\cdot \in \mathbb{R}_+$ ex ante yields identical results but forces the discussion to become more cumbersome and technically complicated.

choose how much to invest in their own power. Formally, player i_t commits to accumulating power at rate, $I_{it} \geq 0$ throughout the period, which adds $I_{it}\Delta$ newly-created units of power to i_t's stock by the end of the period. The instantaneous flow cost of investing at rate I_{it} given inherited power $x_{i,t-\Delta}$ is given by $C(I_{it}, x_{i,t-\Delta})$. The marginal cost of investment is denoted by $C_I(I_{it}, x_{i,t-\Delta}) \equiv \frac{\partial}{\partial I_{it}} C(I_{it}, x_{i,t-\Delta})$.

After players accumulate power, society endows a lump-sum unit of consumable resources.[25] Players compete over these resources through a conflict whose victor is chosen according to the conditional probability distribution,[26]

$$H(x_{it}, \boldsymbol{x}_{\text{-}i,t}; N) \equiv \text{Prob}\{\text{Player } i_t \text{ wins the conflict} \,|\, \text{Players powers are } \boldsymbol{x}_t\}. \tag{1}$$

That is, each players' victory probability depends not only on how much power they hold (x_{it}), but also that held by others ($\boldsymbol{x}_{\text{-}i,t} \equiv (x_{jt})_{j \neq i}$).[27] At the end of period t, player i_t earns an expected payoff of $H(x_{it}, \boldsymbol{x}_{\text{-}i,t}; N) - \Delta \cdot C(I_{it}, x_{i,t-\Delta})$, where $C(\cdot,\cdot)$ is weighted by period length – while $H(\cdot,\cdot;\cdot)$ is not – since the former is a *flow* cost while the latter is a *lump sum* benefit. For more details about this conventional modeling approach in asset accumulation and dynamic contests, useful resources include Fudenberg and Tirole (1991, p. 121), Dixit and Pindyck (1994, p. 357-393), and Acemoğlu and Robinson (2023, p. 412).

Two standard assumptions are made in this paper: one on the cost C of power accumulation (Assumption 1) and one on its benefit H (Assumption 2).

Assumption 1. Power accumulation costs $C : \mathbb{R}_+^2 \to \mathbb{R}_+$ satisfy the following:

1. $C(I_\cdot, x_\cdot)$ and $C_I(I_\cdot, x_\cdot)$ are strictly increasing in $I_\cdot$;
2. $C(I_\cdot, x_\cdot)$ and $C_I(I_\cdot, x_\cdot)$ are weakly decreasing in $x_\cdot$;
3. C_I is continuously differentiable in its first argument and continuous in its second.

The first part assumes convex power accumulation costs, which is standard (Dixit and Pindyck, 1994; Acemoğlu, 2009; Fang, Noe and Strack, 2020). The second part flexibly captures both the intuitive case where power yields a cost advantage (Dixit, 2021; Acemoğlu and Robinson, 2023) and standard case where accumulation cost $C(I_{it}, x_{i,t-\Delta})$ only depends on I_{it}. The third part is a mild smoothness assumption that permits $C_I(I_\cdot, \cdot)$ to have "kinks."

For players' benefit H of holding power, I assume the standard (Francois et al., 2015; Cubel and Sanchez-Pages, 2016; Ewerhart, 2021; Ewerhart and Sun, 2024; Beviá and Corchón, 2024) difference-form contest function whose properties were notably studied by Hirshleifer (1989) and Skaperdas (1996).[28]

[24] This paper assumes linear depreciation to facilitate comparison with Acemoğlu and Robinson (2023); assuming geometric depreciation yields similar results but is analytically intractable.

[25] Section C in the Appendix relaxes the normalization to unity and allows the size of the resource endowment $y_N \geq 0$ to arbitrarily vary with the number of players N. To provide a benchmark model for how societies' distributions of power evolve over time, it is appropriate to concentrate on an endowment economy to focus attention on players' power accumulation choices. The case where players can also accumulate productive capital is currently being investigated in a follow-up work to this paper.

[26] The results of this paper do not depend on the assumptions that players necessarily engage in conflict or that there is a single winner. Another version of this model can allow players to peacefully split resources through bargaining: players simultaneously, publicly announce their desired share of resources and then their acceptance or rejection of the proposed split; if players agree to a feasible split they enact it, and otherwise conflict ensues. This bargaining stage does not affect results, so it is omitted for parsimony.

[27] A model with heterogeneous costs or heterogeneous valuations over the endowment is possible, but this generalization does not provide substantive insight and complicates the statement of the results of this paper.

[28] In addition to being a standard modeling choice in Contest Theory, the difference-form contest function is more broadly known as the softargmax function, a smooth, noisy analogue of $\arg\max\{x_{1\cdot}, \ldots, x_{N\cdot}\}$ that introduces noise in a tractable, parsimonious way (Jaynes, 2003, p. 343-371; Goodfellow et al., 2016, p. 183; Franke and Degen, 2023, p. 16-18).

Assumption 2. Given $\boldsymbol{x}_{\cdot} = (x_{1\cdot}, \dots, x_{N\cdot})$, player $i_{\cdot}$ wins conflicts with probability

$$H(x_{i\cdot}, \boldsymbol{x}_{-i\cdot}; N) \equiv \frac{e^{\lambda x_{i\cdot}}}{\sum_{j=1}^{N} e^{\lambda x_{j\cdot}}} = \frac{1}{1 + \sum_{j \neq i} e^{-\lambda(x_{i\cdot} - x_{j\cdot})}}, \quad (\lambda > 0). \tag{2}$$

This standard assumption is equivalent to assuming that H takes no particular functional form and instead satisfies a collection of natural properties. This follows from Skaperdas (1996, Theorem 3), which is discussed in the remark below for readers' convenience.

Remark 1. By Skaperdas (1996, Theorem 3), Assumption 2 is equivalent to only assuming that victory probabilities (i) are continuous (ii) directly depend only on the power differences between participating players, (iii) are increasing in one's own power and decreasing in opponents' powers, and (iv) yield consistent conditional victory probabilities across group sizes.[29] The non-trivial property is in (iv), which is "a basic consistency property, implying that the contests among smaller numbers of players are qualitatively similar to those among a larger number of them" (Skaperdas, 1996, p. 286). This actively works against my main results (Propositions 3 through 5) which show how as N becomes large, a qualitative inversion of incentives always takes place. The difference-form contest function thus provides a useful benchmark in addition to an intuitive, concrete discussion of results. That said, my results do not depend on the difference-form contest function and each result holds under mild conditions; these technical details are discussed in Appendix A (p. 31).

Player $i_{\cdot}$'s marginal benefit of power accumulation ("contest incentive") corresponds to how their victory probability increases with their power $x_{i\cdot}$, holding others' powers $\boldsymbol{x}_{-i\cdot}$ fixed:

$$h(x_{i\cdot}, \boldsymbol{x}_{-i\cdot}; N) \equiv \frac{\partial}{\partial x_{i\cdot}} H(x_{i\cdot}, \boldsymbol{x}_{-i\cdot}; N) = \frac{\lambda \sum_{j \neq i} e^{-\lambda(x_{i\cdot} - x_{j\cdot})}}{\left[1 + \sum_{j \neq i} e^{-\lambda(x_{i\cdot} - x_{j\cdot})}\right]^2}. \tag{3}$$

Parameter λ corresponds to how effectively power influences conflict outcomes due to de jure institutional constraints, social norms, or inherent noise (Hirshleifer, 1989). Larger λ correspond to conflicts that are less noisy in that their outcome depends more heavily on players' relative powers. To illustrate, when $\lambda = 0$, victors are chosen uniformly at random regardless of power but as $\lambda \to \infty$, only the strongest have a chance of winning, like in an all-pay auction.[30] Note that the *effectivity* of player $i_{\cdot}$'s power in influencing conflict outcomes is given by $e^{\lambda x_{i\cdot}}$ (Corchón and Dahm, 2010, p. 83), which is monotonically increasing in λ.

Equilibrium Concept I focus on Markov perfect equilibrium (Maskin and Tirole, 2001). The state variable in period t is $\boldsymbol{x}_{t-\Delta} \in [0, \chi]^N$, the previous period's power structure; the initial power structure $\boldsymbol{x}_0 \in [0, \chi]^N$ is exogenously fixed. Given $\boldsymbol{x}_{t-\Delta}$, player i_t's action set is $X_{it}(\boldsymbol{x}_{t-\Delta}) \equiv [\max\{x_{i,t-\Delta} - \delta\Delta, 0\}, \chi]$.[31] A *strategy* $x_{it} : [0, \chi]^N \to X_{it}$ for player i_t maps each state $\boldsymbol{x}_{t-\Delta}$ to an action x_{it} in $X_{it}(\boldsymbol{x}_{t-\Delta})$. The sequence $\{(x^*_{1t}, ..., x^*_{Nt})\}_{t \in \{\Delta, 2\Delta, ...\}}$ is a (*Markov perfect*) *equilibrium* – henceforth simply referred to as "equilibrium" – if at each t, $x^*_{it}(\boldsymbol{x}^*_{t-\Delta})$ is a best response to $\boldsymbol{x}^*_{-i,t}(\boldsymbol{x}^*_{t-\Delta})$ $\forall i \in \{1, ..., N\}$.

[29] More specifically, the property in (iv) states that the probability of winning an N-player conflict is consistent with the conditional probability of winning a larger $\hat{N}$-player conflict, given that the victor is a player from $\{1, ..., N\}$. Formally, $H(x_i, \boldsymbol{x}_{-i}; N) = H(x_i, \hat{\boldsymbol{x}}_{-i}; \hat{N}) / \sum_{j=1}^{N} H(x_j, \hat{\boldsymbol{x}}_{-j}; \hat{N})$ holds for all $i \leq N < \hat{N}$, $x = (x_1, ..., x_N)$, and $\hat{\boldsymbol{x}} = (x_1, ..., x_N, \hat{x}_{N+1}, ..., \hat{x}_{\hat{N}})$.

[30] More formally, $\lim_{\lambda \to 0^+} H(x_i, \boldsymbol{x}_{-i}; N) = 1/N$ and $\lim_{\lambda \to \infty} H(x_i, \boldsymbol{x}_{-i}; N) = \mathbb{1}\{i \in \operatorname{argmax}\{x_j\}_{j=1}^N\} / \#\operatorname{argmax}\{x_j\}_{j=1}^N$.

[31] Notice that since x_{it} and I_{it} "pin down" one another, the number of choice variables can be reduced to one. For the purposes of defining strategies and equilibria, x_{it} is considered the only choice variable of player i_t.

2.2 Discussion of Modeling Approach

For readers' convenience, this section briefly summarizes the extensive theoretical and empirical background of the dynamic contest modeling approach to political competition employed in this paper. The full details are provided in the Introduction and Related Literature.

Players can be interpreted either literally as individuals from different lineages/dynasties (Dal Bó et al., 2009; Winters, 2011) or as representative agents of different socioeconomic groups aligned by de jure elite/non-elite status (Acemoğlu and Robinson, 2019, 2023), ethnicity (Francois et al., 2015), interest (Becker, 1983, 1985), etc. Power captures the various forms of de facto political power one can accumulate such as lobbying capacity, campaign contributions (Becker, 1983, 1985), political entrenchment (Dal Bó et al., 2009), social capital/connections (Ehrlich and Lui, 1999), organizational capacity, or plain hard power (Acemoğlu and Robinson, 2008). This modeling approach operationalizes the core features of de facto power discussed by Weber (1925), Becker (1983), and Acemoğlu and Robinson (2008): it is an asset that is accumulated through costly investment, decays without upkeep, and increases an agent's likelihood of exacting favorable political outcomes. That is, it increases one's chances of winning political conflicts to extract rents such as public funds (Mohtadi and Roe, 2003), favorable policies/regulations (Becker, 1983; Grossman and Helpman, 1994), control over natural resources or territory (Beviá and Corchón, 2024), etc.

The reduced form, stylized structure and focus on de facto power are both deliberate. This allows this framework to focus on the economic forces that drive how a society's distribution of power evolves, study the incentives that allow – or indeed prevent – the emergence of egalitarian, oligarchic, and dictatorial regimes. Moreover, recent empirical (Dal Bó et al., 2009; Winters and Page, 2009; Winters, 2011; Gilens and Page, 2014) and analytical (Acemoğlu and Robinson, 2008) work on power consolidation point to competition over de facto political power as a first order concern, particularly for understanding the parallel oligarchic trends taking place around the world today.

3 Initial Results: Equilibrium Power Dynamics

This section solves the above game to provide unique equilibrium predictions of how a society's distribution of power endogenously evolves over time (Proposition 1) and whether it will approach equality, oligarchy, or dictatorship in the long run (Proposition 2). The results in this section are purposefully natural, and lay a solid foundation for the main results in the next section.

3.1 How the Distribution of Power Endogenously Evolves in Equilibrium

The optimization problem faced by the lineage-i player in period $t \in \{\Delta, 2\Delta, ...\}$ is given by

$$\begin{cases} \max_{x_{it}, I_{it}} & H(x_{it}, \boldsymbol{x}_{-i,t}) - \Delta \cdot C(I_{it}, x_{i,t-\Delta}) \\ \text{s.t.} & x_{it} = I_{it}\Delta + x_{i,t-\Delta} - \delta\Delta \\ & 0 \leq x_{it} \leq \chi \\ & I_{it} \geq 0 \end{cases} \tag{4}$$

Proposition 1 solves the above game using (4) and makes period length Δ arbitrarily small so that the unique equilibrium dynamics of $\boldsymbol{x}_t^*$ can be studied in continuous time, which is far more tractable for analysis.

Proposition 1. *As $\Delta \to 0$, the equilibrium is unique and power x^*_{it} obeys the following law of motion*

$$\dot{x}^*_{it} \equiv \lim_{\Delta \to 0} \frac{x^*_{it} - x^*_{i,t-\Delta}}{\Delta} = \begin{cases} -\delta \mathbb{1}_{(0,\chi]}(x^*_{it}), & \text{if } h(x^*_{it}, \boldsymbol{x}^*_{-i,t}) < C_I(0, x^*_{it}) \\ 0, & \text{if } h(x^*_{it}, \boldsymbol{x}^*_{-i,t}) > C_I(\delta, x^*_{it}) \text{ and } x^*_{it} = \chi \\ C_I^{-1}\big(h(x^*_{it}, \boldsymbol{x}^*_{-i,t}), x^*_{it}\big) - \delta \mathbb{1}_{(0,\chi]}(x^*_{it}), & \text{otherwise,} \end{cases} \tag{5}$$

for each player $i \in \{1, \ldots, N\}$ at each time $t \in \mathbb{R}_+$, where C_I^{-1} denotes the inverse function of C_I with respect to its first argument, keeping its second argument fixed.

Proof. Found in appendix subsection A.1. ■

The first two parts of (5) correspond to the corner solutions of (4) while the third corresponds to the interior solution. The first part states that when the marginal benefit $h(x^*_{it}, \boldsymbol{x}^*_{-i,t})$ of power accumulation is less than the marginal cost $C_I(I_{it}, x^*_{it})$ of accumulating power at any $I_{it} \geq 0$, then it is optimal for player i_t to not add any power to their stock so that it depreciates unabated ($\dot{x}^*_{it} = -\delta$) or remains at zero. The second equation implies that player i_t optimally maintains the maximum level of power ($x^*_{it} = \chi$) when the net marginal gain ($h(\chi, \boldsymbol{x}^*_{-i,t}) - C_I(\delta, \chi)$) of doing so is positive. Otherwise the third equation applies, and the optimal $\dot{x}^*_{it}$ equalizes the marginal benefit and marginal cost of power accumulation:

$$\dot{x}^*_{it} = C_I^{-1}\big(h(x^*_{it}, \boldsymbol{x}^*_{-i,t}), x^*_{it}\big) - \delta \mathbb{1}_{(0,\chi]}(x^*_{it}) \Leftrightarrow \underbrace{h(x^*_{it}, \boldsymbol{x}^*_{-i,t})}_{\text{marginal benefit}} = \underbrace{C_I(\dot{x}^*_{it} + \delta \mathbb{1}_{(0,\chi]}(x^*_{it}), x^*_{it})}_{\text{marginal cost}}. \tag{6}$$

This characterization of players' equilibrium power accumulation behavior provides a natural, intuitive foundation for the main results of this paper, based solely on the cost and benefit of power accumulation. These two economic forces underlie players' equilibrium power accumulation decisions, and thus how the society's distribution of power $\boldsymbol{x}^*_t$ evolves over time. Intriguingly, depending on how unequal the society currently is, each force can either reinforce inequality or push toward greater balance. This will be illustrated explicitly in Section 3.3, below.

Remark 2. Notice that the law of motion in (5) is *time-invariant*: how a group's power structure evolves in equilibrium depends only on its current power structure ($\boldsymbol{x}^*_t = \boldsymbol{x}^*_{t'} \Leftrightarrow \dot{x}^*_{it} = \dot{x}^*_{it'}\ \forall i, t, t'$). Moving forward, notation will often be simplified by suppressing time subscripts and asterisks.

3.2 The Emergence of Egalitarian, Oligarchic, and Dictatorial Regimes

Now that the equilibrium dynamics of $\boldsymbol{x}_t$ have been fully characterized, attention is turned to the stable power structures that can arise in the long run. Intuitively, a power structure $\bar{\boldsymbol{x}}$ is stable if it attracts a region ("basin of attraction") of sufficiently nearby $\boldsymbol{x}_t$ which all converge to $\bar{\boldsymbol{x}}$ and stay at rest thereafter. This is formally defined below.

Definition 1. A power structure $\bar{\boldsymbol{x}} \in [0, \chi]^N$ is *stable* if both of the following hold:

1. $\dot{x}_{it} = 0\ \forall\, i$ when $\boldsymbol{x}_t = \bar{\boldsymbol{x}}$. ("steady state")
2. $\forall \varepsilon > 0, \exists \rho > 0$ such that if $\|\boldsymbol{x}_0 - \bar{\boldsymbol{x}}\|_2 < \rho$, then $\|\boldsymbol{x}_t - \bar{\boldsymbol{x}}\|_2 < \varepsilon\ \forall\, t \geq 0$ and $\lim_{t\to\infty} \|\boldsymbol{x}_t - \bar{\boldsymbol{x}}\|_2 = 0$.

As will be shown below, the stable power structures that can emerge in the long run always take one of the following forms:

1. *Egalitarian*, where players are all powerless or all hold χ units power. That is,

$$\bar{\boldsymbol{x}} \in \left\{(0, \ldots, 0), (\chi, \ldots, \chi)\right\} =: \mathscr{E}.$$

   I will often respectively refer to $(0,\ldots,0)$ and $(\chi,\ldots,\chi)$ as *de-escalated* and *escalated* egalitarian.

2. *Oligarchic*, where $k \in \{2, \ldots, N-1\}$ players ("the oligarchs") hold χ units of power, and the remaining $N-k$ players are powerless. That is,

$$\bar{\boldsymbol{x}} \in \left\{\boldsymbol{x} \in \{0, \chi\}^N : \textstyle\sum_{i=1}^N x_i = k\chi\right\} =: \mathscr{O}_k.$$

   I refer to $\mathscr{O}_k$ as the set of *k-archic* power structures. Their union $\bigcup_{k=2}^{N-1} \mathscr{O}_k$ is the set of all oligarchic power structures.

3. *Dictatorial*, where only one player ("the dictator") holds a strictly positive amount $d \in (0, \chi]$ of power. That is,

$$\bar{\boldsymbol{x}} \in \left\{(d, 0, \ldots, 0), \ldots, (0, \ldots, 0, d)\right\} =: \mathscr{D}_d.$$

   I refer to $\bar{\boldsymbol{x}}$ as *strong dictatorial* if $d = \chi$ and *weak dictatorial* otherwise.

With this terminology in hand, the following proposition characterizes the necessary and sufficient conditions under which stable egalitarian, oligarchic, and dictatorial power structures emerge (parts 1-5) and shows that nothing else can be stable (part 6).

Proposition 2. *$\bar{\boldsymbol{x}}$ is stable if and only if it is egalitarian, oligarchic, or dictatorial. More specifically:*

1. *The escalated egalitarian power structure $\bar{\boldsymbol{x}} = (\chi,\ldots,\chi)$ is stable if and only if*
$$h\big(\chi, (\chi,\ldots,\chi)\big) > C_I(\delta, \chi) \tag{I}$$

2. *The de-escalated egalitarian power structure $\bar{\boldsymbol{x}} = (0,\ldots,0)$ is stable if and only if*
$$h\big(0, (0,\ldots,0)\big) \leq C_I(0, 0) \tag{II}$$

3. *Let $k \in \{2,\ldots,N-1\}$. Each k-archic power structure $\bar{\boldsymbol{x}} \in \mathscr{O}_k$ is stable if and only if*
$$h\big(\chi, (\overbrace{\chi,\ldots,\chi}^{k-1}, \overbrace{0,\ldots,0}^{N-k})\big) > C_I(\delta, \chi) \text{ and } h\big(0, (\overbrace{\chi,\ldots,\chi}^{k}, \overbrace{0,\ldots,0}^{N-k-1})\big) \leq C_I(0, 0). \tag{III}$$

4. *Let $d \in (0, \chi)$. Each weak dictatorial power structure $\bar{\boldsymbol{x}} \in \mathscr{D}_d$ is stable if and only if*
$$h\big(\cdot, (0,\ldots,0)\big) \text{ intersects } C_I(\delta, \cdot) \text{ from above at } d \text{ and } h\big(0, (d, 0,\ldots,0)\big) \leq C_I(0, 0) \tag{IV}$$

5. *Each strong dictatorial power structure $\bar{\boldsymbol{x}} \in \mathscr{D}_\chi$ is stable if and only if*
$$h\big(\chi, (0,\ldots,0)\big) > C_I(\delta, \chi) \text{ and } h\big(0, (\chi, 0,\ldots,0)\big) \leq C_I(0, 0). \tag{V}$$

6. *No other stable power structures exist.*

Proof. Found in Appendix subsection A.2. ■

This result provides a unified characterization of how equality, oligarchy, and dictatorship emerge, each under natural conditions: the powerful (if any) have strong enough incentives to maintain their position (Condition I, and the first parts of Conditions III-V), and the powerless (if any) lack sufficient incentive to challenge it (Condition II, and the second parts of Conditions

III-V).[32] Dictatorships are qualitatively distinct because dictators potentially settle at an interior level of power due to a lack of closely matched opponents. When weak dictatorships emerge, dictators settle at the level of power where their marginal benefit matches the marginal cost of offsetting depreciation, as illustrated in Figure 1. As this figure also shows, whether a strong or weak dictatorship emerges depends on the dictators' power level at the moment their opponents' powers depreciate to zero.

Competitive forces generically push $\boldsymbol{x}_t$ towards an egalitarian, oligarchic, or dictatorial regime over time. The intuition behind this result is quite natural. If players are sufficiently closely matched, they all either accumulate power (or let their powers decay) at similar rates.[33] Otherwise, stronger players try to outrun weaker players. If two players hold different interior power levels, the stronger player has stronger power accumulation incentives, so it is not possible for $\boldsymbol{x}_t$ to say at rest since the stronger player profitably deviates by pulling ahead.[34]

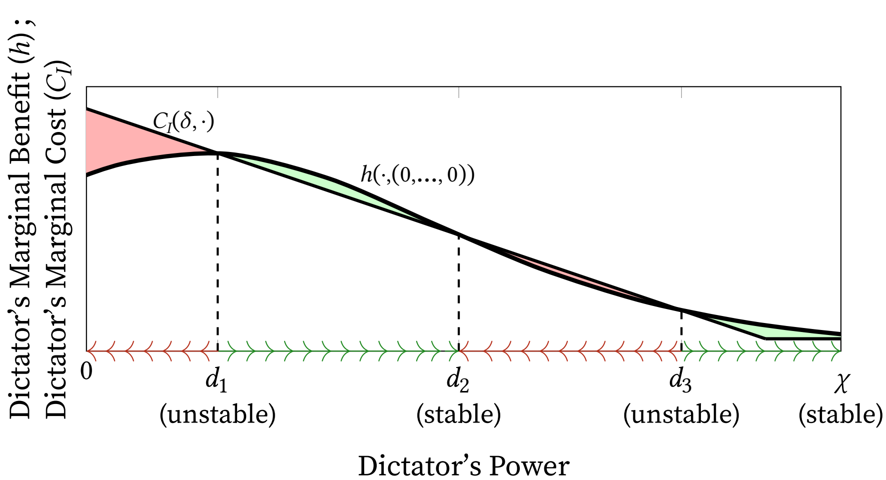


Figure 1: Power level $d \in (0, \chi)$ is sustained in a stable *weak* dictatorship only if the dictator's marginal benefit $h\big(\cdot, (0, \ldots, 0)\big)$ of power intersects marginal cost $C_I(\delta, \cdot)$ at d from above. Power level $d = \chi$ is sustained in a stable *strong* dictatorship only if a dictator's marginal benefit $h(\chi, (0, \ldots, 0)$ of maintaining χ units of power strictly outweighs the marginal cost $C_I(\delta, \chi)$ of maintaining that power level. Notice that this intuition still holds if costs do not depend on inherited power (e.g. when $C_I(\delta, x_.) = \gamma \, \forall \boldsymbol{x}_.$ for some $\gamma \in (h(d_1, (0, \ldots, 0)), h(\chi, (0, \ldots, 0)))$.

Remark 3. The result in Proposition 2 remains unchanged when players are forward-looking but sufficiently impatient. If players are exceedingly patient, the escalated egalitarian power structure

[32] Condition II and the second parts of Conditions III-V can be relaxed from "$h(0, \cdot) \leq C_I(0, 0)$" to "$h(0, \cdot) < C_I(\delta, 0)$" without qualitatively altering the result in Proposition 2. These relaxed conditions are necessary and sufficient under a slightly weaker (but visually indistinguishable) notion of stability (a "Filipov solution") that allows *Zeno behavior* (loosely put, $\boldsymbol{x}_t$ can infinitesimally oscillate instead of being perfectly at rest). Intuitively, if $h(0, \cdot) \in (C_I(0, 0), C_I(\delta, 0))$, powerless players will repeatedly try to "rise up" (accumulate power) before instantaneously giving up (letting their powers depreciate to zero). Remark 6 in the Appendix provides additional discussion.

[33] It is possible to have a steady state where multiple players have the same interior power level, but it cannot be stable. This is because giving these interior players an equal, arbitrarily small windfall of power induces each to accumulate in unison, driving the power structure away from the original point. The reasoning is similar in the case where there is only one player with interior power, and at least one other player at the power cap.

[34] If players i and j hold different interior power levels $0 < x_i < x_j < \chi$, the stronger player has a higher net marginal gain of power accumulation ($h(x_j, \boldsymbol{x}_{-j}) > h(x_i, \boldsymbol{x}_{-i})$ and $C_I(\cdot, x_j) \leq C_I(\cdot, x_i)$), so if the weaker player wishes to rest ($\dot{x}_j = 0 \Leftrightarrow h(x_i, \boldsymbol{x}_{-i}) = C_I(\delta, x_i)$), then the stronger player must find it optimal to accumulate power ($h(x_j, \boldsymbol{x}_{-j}) > h(x_j, \boldsymbol{x}_{-i}) = C_I(\delta, x_i) \geq C_I(\delta, x_j) \Rightarrow \dot{x}_i > 0$).

becomes the only stable power structure, although the reason does not provide much insight: with enough patience, the cost of accumulating the maximum level of power χ (which is endured over a *finite* number of periods) is outweighed by the high benefit of holding χ units of power (which is enjoyed over an *infinite* time horizon). This forward-looking extension is not among the several significant differences between this paper and Acemoğlu and Robinson (2023), which were discussed in the Related Literature, and it is indeed proven in virtually the same way as Proposition 3 in Acemoğlu and Robinson (2023), with only trivial differences.

3.3 Three Player Illustration

This section illustrates the global equilibrium dynamics in the case with $N = 3$ players. In the interest of clearly visualizing these results, I focus on the case where the set of stable power structures is

$$\Big\{ \underbrace{(\chi, \chi, \chi)}_{\substack{\text{escalated}\\ \text{egalitarian}}}, \underbrace{(d, 0, 0), (0, d, 0), (0, 0, d)}_{\text{dictatorial}}, \underbrace{(\chi, \chi, 0), (\chi, 0, \chi), (0, \chi, \chi)}_{\substack{\text{oligarchic}\\ \text{(2-archic)}}} \Big\},$$

where $d \in (0, \chi]$. Figure 2 visualizes the equilibrium dynamics in this case.

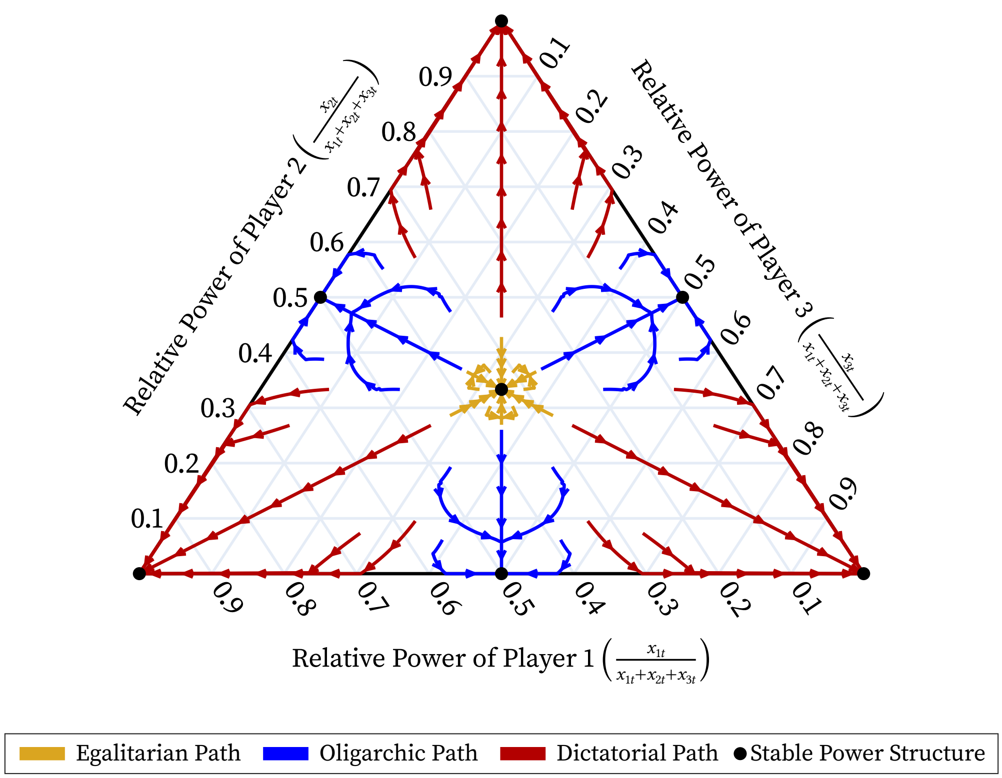


Figure 2: Simplex plot representation of equilibrium paths produced using power accumulation cost function $C(I_\cdot, x_\cdot) = I_\cdot^2 + (1 - x_\cdot)I_\cdot$, institutional constraint/inverse noise parameter $\lambda = 5.5$, depreciation $\delta = 0.1$, and power cap $\chi = 1$.

When players' powers are initially close to one another, the *(escalated) egalitarian* power structure is reached through competition. Notice that players' power accumulation incentives are strongest when they are evenly matched with one another. Thus, when players begin sufficiently closely matched, they each begin with similarly strong power accumulation incentives. Moreover, convex accumulation costs prevent each player from outrunning the rest, since rapid power accumulation is increasingly expensive. Consequently, players begin accumulating power at similar rates in equilibrium, resulting in each becoming more powerful in *absolute* terms while their *relative* powers remain similar. This cycle repeats until each player reaches χ units of power.

Dictatorial power structures are reached through a qualitatively different process. When one player – say, player 1 – begins significantly more powerful than the rest, that player will eventually be the only one holding power (the "dictator"). Players' power accumulation incentives are weaker than before due to this power disparity, but by construction, player 1's incentives are strong enough for power accumulation to be optimal. Other players may also find it initially optimal to accumulate power, but not so fast as to close the gap with player 1, since convex investment costs make rapid accumulation relatively expensive. Consequently, this power disparity widens, further discouraging players 2 and 3 from catching up with player 1. This process continues until weaker players 2 and 3 find power accumulation sub-optimal, allowing their respective powers to depreciate to zero, at which point player 1 has fully consolidated power. Notice that this process did not rely on player 1 having a cost advantage (i.e., this intuition holds even when $C(I_\cdot, x_\cdot)$ is constant in $x_\cdot$).

Finally, *oligarchic* (here, 2-archic) power structures are reached through a combination of the above two processes. These are reached when two players (say, players 1 and 2) begin closely matched to each other but outmatch the rest (here, player 3). Players 1 and 2 compete with one another, each driving the other player's power up in the same way that the escalated egalitarian power structure is reached. This causes these players to "outrun" (in terms of power accumulation) player 3, who eventually allows their power to fully depreciate, like in the dictatorial case.

Remark 4. Stochastic shocks can be systematically incorporated into this model, but this makes analytical results intractable while leaving the core insight intact: the deeper $\boldsymbol{x}_t$ is within the basin of attraction for one regime (e.g. oligarchy), the larger the shock required to divert it towards another type of regime (e.g. egalitarianism). For example, including shocks to the power structure $\boldsymbol{x}_t$, such as outcome-dependent gains (resp. losses) in power to players that win (resp. lose) conflicts or outcome-independent shocks (e.g. distributed i.i.d. across lineages and time). The paths taken by $\{\boldsymbol{x}_t\}_{t\in\mathbb{R}_+}$ remain qualitatively similar apart from being noisy (e.g. in Figure 2, the paths would now have random fluctuations) but can cause $\boldsymbol{x}_t$ to jump over basin boundaries. Note that shocks that depend on conflict outcomes mechanically make unequal regimes more likely. Another example of shocks can be to model primitives (such as λ, δ, etc.), which would cause the basins of attraction boundaries to shift.

4 Main Results: How Large Societies Become Unequal

I now turn to the main result of this paper: in the following proposition, I show that there always exists a finite group size past which the escalated egalitarian power structure ceases to be stable.

Proposition 3. *The escalated egalitarian power structure $(\chi, \ldots, \chi)$ is unstable in groups larger than*

$$\bar{N}_\chi^{\mathscr{E}} \equiv \left\lceil \frac{\lambda + \sqrt{(\lambda - 4C_I(\delta,\chi))\lambda}}{2C_I(\delta,\chi)} \right\rceil \cdot \mathbb{1}_{\left[4C_I(\delta,\chi),\infty\right)}(\lambda) \tag{7}$$

Proof. Found in Appendix subsection A.3. ■

This result implies that when political competition is left unchecked,[35] sufficiently large societies *inevitably* become unequal; Proposition 4 (below) strengthens this result by showing that only dictatorships and concentrated oligarchies will remain stable. This provides a rigorous microfoundation for the Iron Law of Oligarchy, showing how it holds under standard economic conditions. This addresses a century-old puzzle with renewed urgency amid rising inequality in the US and other large countries; this result suggests that these concerning trends will not self-correct.

The economic intuition from this result reveals a surprising reversal of incentives that is especially unexpected given the central insight from Acemoğlu and Robinson (2019, 2023): competitive pressure between closely matched competitors is what drives nations through a *narrow corridor*[36] towards equality. My analysis confirms that this mechanism holds for small populations, but that as societies grow, this corridor to equality grows ever narrower until it eventually *vanishes* past size $N_\chi^{\mathscr{E}}$. This results, ironically, from high competitive pressure among closely matched competitors.

To see how competitive pressure eventually destabilizes the escalated egalitarian power structure in sufficiently large groups, first recall that by Proposition 2.1, the escalated egalitarian power structure is stable if and only if Condition I holds:

$$(N-1)\lambda/N^2 = h(\chi, (\chi, \dots, \chi); N) > C_I(\delta, \chi),$$

which is used to derive the formula for $N_\chi^{\mathscr{E}}$ in (7).[37] Notice that the strength of players' power accumulation incentives h are strictly increasing in how closely matched they are with their aggregate competition.[38] At the escalated egalitarian power structure, every player faces $N-1$ opponents that are each as strong as they are. This is why each player has strong power accumulation incentives when N is small; in fact they are as strong as possible when there are just $N = 2$ players. However, as

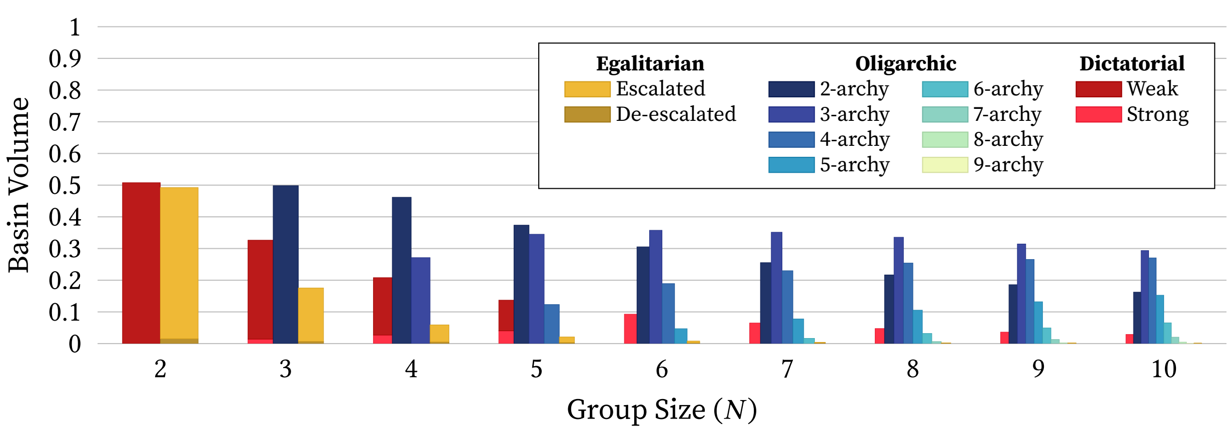


Figure 3: Numerical simulation of the N-volume (Lebesgue measure) of the basins of attraction of each type of stable power structure for group sizes $N \in \{2, \dots, 10\}$ assuming $\lambda = 4$, $\delta = 0.1$, $\chi = 1$, and cost function $C(I, x) = 1.5I^2 + 1.5\max\{0.5 - x, 0\}I$.

[35]By "unchecked political competition," I precisely mean that the game modeled in this paper proceeds without intervention originating externally to the game.

[36]The eponymous "narrow corridor" of Acemoğlu and Robinson (2019) refers to the basin of attraction of the escalated inclusive regime, which broadly corresponds to players being sufficiently closely matched.

[37]The presence of the indicator function in equation (7) reflects the fact that Condition I fails to hold at any $N \geq 2$ when $\lambda/4 < C_I(\delta, \chi)$.

[38]To see why this is true for general N, first note that $e^{\lambda x_i}$ measures the *effectivity* of player i's power (Corchón and Dahm, 2010); that is, how effectively their power influences their victory probability. Then, notice that $h(x_i, \boldsymbol{x}_{-i})$ can be rewritten as a function of the relative effectivity of player i's power $e^{\lambda x_i} / \sum_{j \neq i} e^{\lambda x_j}$, which is unimodal at unity.

N grows large, players become overwhelmed by their aggregate competition at $\boldsymbol{x} = (\chi, \ldots, \chi)$. Thus, the very competitive pressure that allows equality to emerge in small societies is what eventually causes equality to destabilize in large ones.

I now analyze the institutional features that make the escalated egalitarian power structure more (or less) robust to population size in the following Corollary. Even though this result naturally follows from Condition I/equation (7), its implications are quite counter-intuitive at first glance.

Corollary 1. *$\bar{N}^{\mathscr{E}}_{\chi}$ is increasing in λ and decreasing in $C_I(\delta, \chi)$.*

Proof. Found in Appendix section A.4. ■

On its face, this result seems obvious: making power more effective in influencing conflict outcomes (increasing λ) or cheaper to maintain[39] each increase the net marginal gain in Condition I, so the escalated inclusive power structure remains stable at larger population sizes (i.e. $\bar{N}^{\mathscr{E}}_{\chi}$ increases). However, its implications seem far less obvious since increasing λ and decreasing $C(\delta, \chi)$ intuitively seem more likely to cause equality to destabilize faster.

First notice that raising λ increases competitive pressure (the force just seen to destabilize equality) by making winning conflicts more dependent on holding the most power. Further notice that the escalated inclusive power structure can be made fully robust to population size if $\lambda \to \infty$, which effectively turns conflicts into an *all-pay auction* where only the strongest have a chance of winning. While perhaps paradoxical at first, it is actually quite intuitive: since only the strongest can win, everyone has an extreme incentive to *exactly* match one another, and no incentive to surpass one another. That being said, this can only address a "last-mile problem" for societies already sufficiently close to equality, but in unequal societies it accelerates power consolidation by incentivizing sufficiently stronger player(s) to outrun the rest faster.

The hard upper bound χ on power is a similarly interesting institutional constraint. Increasing χ may raise the egalitarian stability threshold $\bar{N}^{\mathscr{E}}_{\chi}$ if this reduces maintenance cost $C_I(\delta, \chi)$, but has no effect otherwise. This implies that equality may be kept stable in large, competitive societies by letting $\chi \to \infty$, which effectively removes formal restrictions on how much political influence x. players can amass. This is intriguing, given the stylized resemblance to of the U.S. Supreme Court's decision in *Citizens United v. FEC (2010)*, which eliminated caps on independent political expenditures. The potentially pro-equality effect of $\chi \to \infty$ becomes more intuitive upon closer inspection. First, this only addresses the same last-mile problem mentioned above. Second, even in sufficiently equal societies, letting $\chi \to \infty$ is at most a stopgap solution against population size. As I now show in Proposition 4.1, if one makes χ arbitrarily large, $\bar{N}^{\mathscr{E}}_{\chi}$ remains finite in all but a knife-edge case where the marginal cost of maintaining an arbitrarily large amount of power tends towards *exactly* zero. The remainder of this proposition characterizes how dictatorships and concentrated oligarchies are far more robust to population size.

Proposition 4. *Suppose that $\lim_{\chi \to \infty} C_I(\delta, \chi) = \gamma > 0$. If χ is made arbitrarily large, then*

1. *The group size past which the escalated egalitarian power structure is unstable remains finite.*
2. *Apart from the trivial case when $\lambda/4 \leq \gamma$,[40] dictatorial power structures remain stable at arbitrarily large group sizes.*
3. *k-archies remain stable at arbitrarily large group sizes if k is no greater than*

$$\bar{k} \equiv \left\lfloor \frac{\lambda + \sqrt{(\lambda - \gamma)\lambda}}{\gamma} \right\rfloor \cdot \mathbb{1}_{(\gamma, \infty)}(\lambda), \tag{8}$$

[39] By changing C, decreasing δ, or increasing χ (if $C_I(\delta, \cdot)$ is not constant).

[40] When $\lambda/4 \leq \gamma$, dictatorships are trivially unstable $\forall N$ due to insufficient contest incentives.

and k-archies with $k \in \{\bar{k}+1, \bar{k}+2, \dots\}$ are not stable at any group size.

Proof. Found in Appendix section A.5. ■

This result completes the characterization of the Iron Law of Oligarchy: in large enough societies, only dictatorships and sufficiently concentrated oligarchies – those with at most $\bar{k}$ oligarchs – remain stable. The reasoning for oligarchies mirrors the earlier case. When there are less than $\bar{k}$ oligarchs, each faces more than one – but not too many – closely matched opponents, which ensures strong competition incentives for each oligarch even as N becomes large. Otherwise, the oligarchs become overwhelmed like the players in the egalitarian regime, which is why more diffuse oligarchies eventually destabilize.

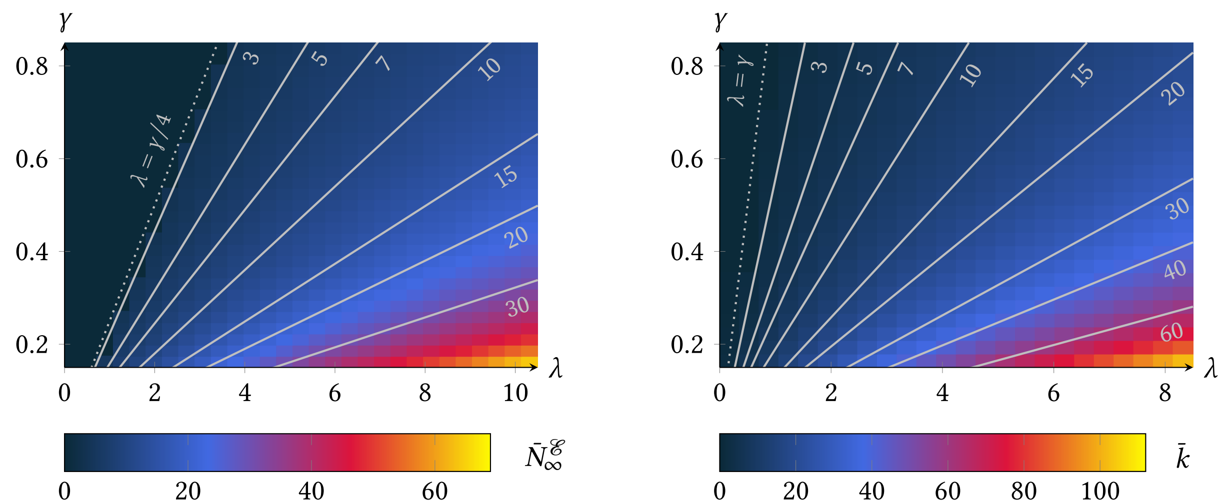


Figure 4: How $\bar{N}_\infty^{\mathscr{E}} \equiv \lim_{\chi\to\infty} \bar{N}_\chi^{\mathscr{E}}$ and $\bar{k}$ vary with λ and $\gamma \equiv \lim_{\chi\to\infty} C_I(\delta, \chi)$.

It is more curious why dictatorships remain stable at arbitrarily large group sizes: Since dictators have no closely matched competitors, shouldn't their power accumulation incentives be weak? This is indeed the case when N is small, but as N grows large the dictator's contest incentives rise due to mounting pressure from powerless players. This is investigated in further detail in the next section.

Remark 5. For simplicity, society's resource endowment is normalized to one. Appendix C extends the model to variable endowments, and shows that more egalitarian regimes can remain stable in large societies if and only if endowments grow fast enough – potentially at an arbitrarily steep rate – as N increases.

4.1 How Strong Dictatorships Emerge in Large Societies

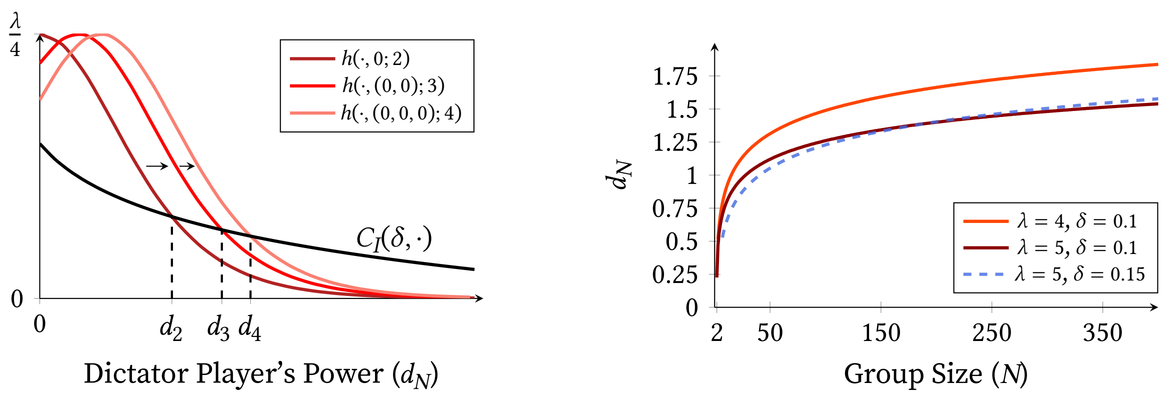
This section characterizes the comparative statics of stable dictatorial power, the amount of power held by the strongest player ("the dictator") in a stable dictatorship. Recall that Proposition 2 established that weak dictatorships (with dictatorial power $d \in (0, \chi)$) are stable if and only if Condition IV holds and strong dictatorships (with dictatorial power $d = \chi$) are stable if and only if Condition V holds. Depending on model primitives, it is possible for no dictatorships to be stable or for *multiple* levels of power to be sustained in stable dictatorships as in Figure 1 in subsection

3.2.[41] For ease of exposition, assume throughout this subsection that *exactly one* level of power d is sustained in a stable dictatorship.[42] Analogous results hold when multiple types of dictatorship are stable, but are substantially more cumbersome to state, and offer insubstantial additional insight.

Proposition 5. *The amount of power held in stable dictatorships increases in group size N.*

Proof. Found in Appendix subsection A.6. ■

Larger group sizes lead to stronger dictatorships because powerless players have a small but non-zero probability of victory, reflecting the inherent noisiness of real-world conflicts (Hirshleifer, 1989). As a result, powerless players collectively exert competitive pressure on the dictator player. This pressure grows with the number of powerless players, thereby inducing the dictator to hold an increasingly high level of power. Geometrically, this is because increasing group size N translates the dictator's marginal benefit $h(\cdot,(0,\dots,0);N)$ rightward. This is illustrated in Figure 5A, below.



(A) Larger N induce higher d_N because it shifts the dictator's marginal benefit of power accumulation $h(\cdot,(0,\dots,0);N)$ rightward.

(B) Simulated relationship between N and d_N, assuming cost function $C(I_i, x_i) = 3.25I_i^2 + \max\{0.5 - x_i, 0\}I_i$.

Figure 5: How larger group size N induces higher levels of power d_N held by the strongest player ("dictator") in stable dictatorships.

As N becomes large, dictators' power accumulation incentives – and thus their equilibrium behavior – increasingly resemble those of oligarchs. The way in which dictators optimally respond to increases in group size is what ultimately causes their contest incentives to grow with N and hence allows dictatorships to be robust to group size in Proposition 4. Strong dictators emerge when (the rest of) society is *collectively* strong. While this resembles Acemoğlu and Robinson's (2023) main result, there is an added twist: non-dictator players are individually powerless, having only collective strength in numbers. Other comparative statics properties of the amount of power d held in stable dictatorships are given in the result below.

Proposition 6.

1. *Uniformly increasing the marginal cost of investment $C_I(\cdot,\cdot)$ decreases d.*
2. *d is decreasing in δ.*

[41]Recall that the equilibrium dynamics in (5) are always unique, even when there exist multiple stable levels of dictatorial power.

[42]Formally put: assume that either (1) Condition IV holds for exactly one $d \in (0,\chi)$ and Condition V fails or (2) IV fails at all $d \in (0,\chi)$ and Condition V holds.

3. *d increasing in λ if and only if* $\frac{\lambda d-1}{\lambda d+1}e^{\lambda d} < N-1$.

Proof. Found in Appendix subsection A.7. ■

The first two parts of this proposition consider the negative relationship between the amount of power d held by dictators in stable dictatorships and the marginal cost $C_I(\delta, d)$ of maintaining d units of power. If the latter value were to increase – say, due to an unexpected shock or a policy intervention that makes accumulating power more costly – the dictator's marginal cost of maintaining d units of power would outweigh its marginal benefit $h(d, (0, \ldots, 0))$. The dictator consequently lets their power depreciate until stabilizing at a new, lower level of power.

Institutional constraint parameter λ has a less straightforward relationship with stable dictatorial power d. Increasing λ induces an increase in d if and only if they are both sufficiently small, a requirement that becomes less stringent as N increases. As λ becomes large, simply surpassing the other players – rather than the amount by which one surpasses – becomes the dominant influencing factor in winning conflicts. The role played by group size is also natural: larger N correspond to more powerless players, who always have a strictly positive probability of winning conflicts when λ is finite. Thus, dictators in larger groups face more pressure to maintain higher levels of power in parallel fashion to Proposition 5.

5 Concluding Remarks

The power gap continues to widen in many nations around the world – in democracies and non-democracies alike – showing no signs of stopping. Without rigorously understanding the incentives that underlie these parallel trends, attempts at policy intervention risk being ineffective.

With this motivation in hand, this paper developed an economic framework of how a society's distribution of power and resources endogenously evolves over time due to competition. It provides sharp equilibrium predictions for how political inequality evolves, and whether equality, dictatorship, or oligarchy emerge in the long run. This analysis demonstrates that these regimes do not arise from fundamentally different forces, but rather from the same power accumulation incentives. Most critically, my main results show that once societies become large enough, power *inevitably* concentrates into the hands of a few in the absence of external intervention. This provides a solid economic foundation for the prediction of Michels' (1911) Iron Law of Oligarchy whose mechanism has remained elusive, even as it seems to be playing out in real time today around the world. The analysis shows how the Iron Law endogenously emerges from standard economic assumptions on power accumulation incentives; specifically, the same competitive pressures that sustain equality in small societies eventually lead to its collapse as societies grow.

Michels (1911, p. 406) asserted that "[h]istorical evolution mocks all the prophylactic measures that have been adopted for the prevention of oligarchy." Given the opacity of the Iron Law's mechanism for over a century, it's not hard to see why Michels was so pessimistic. Although my main results confirm the Iron Law's prediction, Michels's fatalistic conclusion is not the right takeaway. Over 600 leading economists and inequality experts recently contended that "inequality is not inevitable; it is a policy choice" (Equals, 2025). This is very likely true. However, understanding the incentives that drive the Iron Law is necessary for designing mechanisms and policies that effectively counteract it. This paper provides significant contributions towards this end and a useful foundation for future research, but of course does not close the book on the rapidly developing issue of rising political inequality. The results of this paper suggest that these trends will not self-correct. Much work remains to be done, and we are racing against the clock.

There are several fruitful avenues for future research. Currently, two lines of follow-up research are underway by the author. First is to develop a model that allows players to accumulate productive capital alongside power. This studies an interesting tradeoff (between productive investment and power accumulation) and may also reveal when/how societies can escape the Iron Law of Oligarchy through sufficiently rapid economic growth, a requirement suggested by the results in Appendix C. There may be cause for optimism towards this end. My results show that overwhelmingly high competitive pressure under more egalitarian regimes is the key catalyst of the Iron Law while Besley, Persson and Sturm (2010) and Acemoglu et al. (2019) respectively observe how political competition and democratic institutions encourage economic growth. Second, allowing agents to form coalitions may reveal how richer hierarchies emerge, and the potential for coalition-formation to counteract the Iron Law. Related research in this direction includes Acemoğlu et al. (2008a), Baron and Bowen (2015), and Polburn and Polburn (2025).

Exploring tradeoffs between various forms of accumulable de facto power (lobbying capacity, social influence, arms, etc.) would be very fruitful, especially for putting data to this model. Given that this framework is constructed using conventional economic tools, it can be easily embedded into other models. This would allow for further investigation of how power accumulation interacts with other considerations, such as ideology (Piketty, 2019; Gethin et al., 2022; Levy et al., 2022; Acemoğlu et al., 2025; Ba et al., 2025), uncertain personal power (Bowen et al., 2022), legislative bargaining protocols (Bernheim et al., 2006; Bowen and Zahran, 2012; Ali et al., 2018), sentiment over democratic performance (Acemoğlu et al., 2025), economic integration with democratic nations (Tabellini and Magistretti, 2024), endogenous de jure institutions (Aghion et al., 2004; Acemoğlu and Robinson, 2006; Lagunoff, 2009; Wolitzky, 2013), as well as the many other institutional and structural factors discussed in the introduction. For thorough and thoughtful discussions thereof, please see Gavrilets and Currie (2023), Lupu and Pontusson (2023), Elkjær and Klitgaard (2024), Lagunoff (2024), and Egorov and Sonin (2025).

Future work must also grapple with the design of effective policy interventions, which has two vexing challenges. First, the results of Acemoğlu and Robinson (2008) suggest that elites can effectively circumvent formal de jure institutional constraints using de facto power. Second, achieving external intervention is fundamentally challenging since whoever intervenes has, by doing so, entered the strategic environment themselves. Designing a mechanism that can account for this remains as a pressing challenge.

Appendix A Proofs

This section contains the proofs to the results in the main text (Propositions 1-6). Note that in the main text, resources are normalized to unity, for simplicity. Section C of this Appendix considers an extension where the size of resources is $y_N \geq 0$, which can vary with population size $N \in \{2, 3, \ldots\}$. In preparation for this section, Propositions 1-2 are proven assuming the aforementioned extended model. This proves the results in the main text (since it is the special case where $y_N = 1\ \forall N$) and avoids redundancy in Section C.

A.1 Proof of Proposition 1

Arbitrarily fix $\boldsymbol{x}_{t-\Delta} \in [0,\chi]^N$. Notice that the optimization problem in (4) is equivalent to the optimization problem

$$\begin{cases} \max\limits_{x_{it}} & H(x_{it}, \boldsymbol{x}_{-i,t}) - \Delta \cdot C\left(\frac{x_{it}-x_{i,t-\Delta}}{\Delta} + \min\left\{\frac{x_{i,t-\Delta}}{\Delta}, \delta\right\}, x_{i,t-\Delta}\right) \\ \text{s.t.} & 0 \leq x_{it} \\ & x_{it} \leq \chi \\ & x_{i,t-\Delta} - \min\{x_{i,t-\Delta}, \delta\Delta\} \leq x_{it} \end{cases} \tag{4$'$}$$

wherein x_{it} is the only choice variable. This is achieved by rearranging the equality constraint of (4)

$$x_{it} = \Delta \cdot I_{it} + \max\{x_{i,t-\Delta} - \delta, 0\} \Leftrightarrow I_{it} = \frac{x_{it} - x_{i,t-\Delta}}{\Delta} + \min\left\{\frac{x_{i,t-\Delta}}{\Delta}, \delta\right\}$$

and substituting out I_{it}. Optimization problem (4′) is now solved. I form the Lagrangian

$$\begin{aligned} \mathcal{L} = \; & H(x_i, \boldsymbol{x}_{-i}) - \Delta \cdot C\left(\tfrac{x_{it}-x_{i,t-\Delta}}{\Delta} + \min\left\{\tfrac{x_{i,t-\Delta}}{\Delta}, \delta\right\}, x_{i,t-\Delta}\right) \\ & + x_{it}\mu_1 + (\chi - x_{it})\mu_2 + (x_{it} - x_{i,t-\Delta} + \min\{x_{i,t-\Delta}, \delta\Delta\})\mu_3 \end{aligned} \tag{9}$$

where μ_1, μ_2, and μ_3 respectively denote the Lagrange multiplier of the first, second, and third inequality constraints of (4′). The first order condition is given by

$$h(x_{it}, \boldsymbol{x}_{-i,t}) = C_I\left(\frac{x_{it} - x_{i,t-\Delta}}{\Delta} + \min\left\{\frac{x_{i,t-\Delta}}{\Delta}, \delta\right\}, x_{i,t-\Delta}\right) - \mu_1 + \mu_2 - \mu_3. \tag{FOC}$$

Recall that $h(x_{it}, \boldsymbol{x}_{-i,t}) \equiv \frac{\partial}{\partial x_{it}} H(x_{it}, \boldsymbol{x}_{-i,t})$ (equation (3)) and C_I denotes the partial derivative of C with respect to its first argument. The Karush Kuhn-Tucker (KKT) conditions are given by (CS_1), (CS_2), and (CS_3), which respectively correspond to the first, second, and third inequality constraints of (4′).

$$x_{it} \geq 0, \; \mu_1 \geq 0, \; x_{it}\mu_1 = 0 \tag{CS$_1$}$$

$$x_{it} \leq \chi, \; \mu_2 \geq 0, \; (\chi - x_{it})\mu_2 = 0 \tag{CS$_2$}$$

$$x_{i,t-\Delta} - \min\{x_{i,t-\Delta}, \delta\Delta\} \leq x_{it}, \; \mu_3 \geq 0, \; \left[\frac{x_{it} - x_{i,t-\Delta} + \min\{x_{i,t-\Delta}, \delta\Delta\}}{\Delta}\right]\mu_3 = 0 \tag{CS$_3$}$$

Case 1: Suppose the first two constraints of (4′) are both binding. This implies that $x_{it} = 0$ and $x_{it} = \chi$, which is a contradiction since $\chi > 0$. Therefore it is never possible for the first two constraints of (4′) to both be binding.

Case 2: Now suppose that only the first constraint of (4′) is slack. This implies that $x_{it} = \chi$ and $x_{it} = x_{i,t-\Delta} - \min\{x_{i,t-\Delta}, \delta\Delta\}$, which in turn jointly imply that $x_{i,t-\Delta} = \chi + \min\{x_{i,t-\Delta}, \delta\Delta\}$. This equation yields a contradiction when $x_{i,t-\Delta} > 0$ (since it implies that $x_{i,t-\Delta} = \chi + \delta\Delta > \chi$, which is impossible) and when $x_{i,t-\Delta} = 0$ (since it implies that $x_{i,t-\Delta} = \chi \neq 0$). Thus it is never possible for only the first constraint of (4′) to be slack.

Case 3: I now turn to the case where only the second constraint of (4′) is binding. In this case, $\mu_1 = \mu_3 = 0$, $\mu_2 \geq 0$, $x_{it} = \chi$, and $x_{it} > x_{i,t-\Delta} - \min\{x_{i,t-\Delta}, \delta\Delta\}$. In this case, it follows from (FOC) that

$$h(\chi, \boldsymbol{x}_{-i}) \geq C_I\left(\frac{\chi - x_{i,t-\Delta}}{\Delta} + \min\left\{\frac{x_{i,t-\Delta}}{\Delta}, \delta\right\}, x_{i,t-\Delta}\right) \tag{10}$$

Since h is a bounded function and C_I is strictly increasing in its first argument, it follows that there exists a sufficiently small $\tilde{\Delta}_1 > 0$ such that $\forall \Delta < \tilde{\Delta}_1$ the above inequality holds only at $x_{i,t-\Delta} = \chi$ and $\chi/\Delta > \delta$.

Case 4: Now consider the case where only the third constraint of (4′) is binding. In this case $\mu_1 = \mu_2 = 0$, $\mu_3 \geq 0$, $x_{it} \in (0, \chi)$, and $x_{it} = x_{i,t-\Delta} - \min\{x_{i,t-\Delta}, \delta\Delta\}$. If $x_{i,t-\Delta} \in [0, \delta\Delta]$, then we must have $x_{it} = x_{i,t-\Delta} - x_{i,t-\Delta} = 0$, which contradicts the fact that the first constraint of (4′) is slack in the present case. Therefore we must have $x_{i,t-\Delta} \in (\delta\Delta, \chi]$. In this case, it follows from (FOC) that

$$h(x_{it}, \boldsymbol{x}_{-i,t}) \leq C_I\left(\frac{x_{it} - x_{i,t-\Delta}}{\Delta} + \delta, x_{i,t-\Delta}\right) = C_I(0, x_{i,t-\Delta}) \tag{11}$$

where the equality follows from the fact that the third constraint of (4′) is binding. Therefore it is established that $(x_{it} - x_{i,t-\Delta})/\Delta = -\delta$ if both $h(x_{it}, \boldsymbol{x}_{-i,t}) \leq C_I(0, x_{i,t-\Delta})$ and $x_{i,t-\Delta} \in (\delta\Delta, \chi]$ are true.

Case 5: Now suppose that only the second constraint of (4′) is slack. In this case, we have $\mu_1 \geq 0$, $\mu_2 = 0$, $\mu_3 \geq 0$, $x_{it} = 0$, and $x_{it} = x_{i,t-\Delta} - \min\{x_{i,t-\Delta}, \delta\Delta\}$. In this case, we must have $x_{i,t-\Delta} \in [0, \delta\Delta]$. This is because when $x_{i,t-\Delta} \in (\delta\Delta, \chi]$, the binding first and third constraints of (4′) imply that $x_{i,t-\Delta} = \delta\Delta$, which is a contradiction. When $x_{i,t-\Delta} \in [0, \delta\Delta]$, these binding constraints imply that $x_{it} = x_{i,t-\Delta} - x_{i,t-\Delta} = 0$, which is true. It follows from this and from (FOC) that $x_{it} = 0$ if $h(0, \boldsymbol{x}_{-i,t}) \leq C_I(0, \delta\Delta)$ and $x_{i,t-\Delta} \in [0, \delta\Delta]$.

Case 6: Next, consider the case where only the first constraint of (4′) is binding. Here, we have $\mu_1 \geq 0 = \mu_2 = \mu_3$, $x_{it} = 0$, and $x_{it} > x_{i,t-\Delta} - \min\{x_{i,t-\Delta}, \delta\Delta\}$. The binding first constraint and slack third constraint of (4′) imply that $x_{i,t-\Delta} < \min\{x_{i,t-\Delta}, \delta\Delta\}$. This inequality yields a contradiction when $x_{i,t-\Delta} \leq \delta\Delta$ (since it implies that $x_{i,t-\Delta} < x_{i,t-\Delta}$) and when $x_{i,t-\Delta} > \delta\Delta$ (since it implies that $x_{i,t-\Delta} < \delta\Delta$). Thus it is impossible for only the first constraint of (4′) to bind.

Case 7 (Interior): Finally, attention is turned to the *interior* case where all constraints of (4′) are slack. Here, $\mu_i = 0 \ \forall i \in \{1, 2, 3\}$, $x_{it} \in (0, \chi)$, and $x_{it} > x_{i,t-\Delta} - \min\{x_{i,t-\Delta}, \delta\Delta\}$. Equation (FOC) then implies that x_{it} satisfies the following equality:

$$h(x_{it}, \boldsymbol{x}_{-i,t}) = C_I\left(\frac{x_{it} - x_{i,t-\Delta}}{\Delta} + \min\left\{\frac{x_{i,t-\Delta}}{\Delta}, \delta\right\}, x_{i,t-\Delta}\right). \tag{12}$$

Note that $h(\cdot, \boldsymbol{x}_{-i,t})$ and its partial derivative with respect to its first argument are both bounded given any $\boldsymbol{x}_{-i,t} \in [0, \chi]^{N-1}$ and recall that $C_I(\cdot, x_{i,t-\Delta})$ is strictly increasing given any $x_{i,t-\Delta} \in [0, \chi]$. It then follows that either the solution is never interior at any period length or there exists a sufficiently small $\tilde{\Delta}_2 > 0$ such that given any fixed $\Delta < \tilde{\Delta}_2$, a unique interior value of x_{it} satisfies (12) and that at said x_{it}

$$\frac{\partial}{\partial x_{it}}\left[h(x_{it}, \boldsymbol{x}_{-i,t}) - C_I\left(\frac{x_{it} - x_{i,t-\Delta}}{\Delta} + \min\left\{\frac{x_{i,t-\Delta}}{\Delta}, \delta\right\}, x_{i,t-\Delta}\right)\right] < 0. \tag{13}$$

Note that the "$\min\left\{\frac{x_{i,t-\Delta}}{\Delta}, \delta\right\}$" term in (12) is equal to δ when $x_{i,t-\Delta} \in [\delta\Delta, \chi]$ and equal to zero when $x_{i,t-\Delta} = 0$. When $x_{i,t-\Delta} \in (0, \delta\Delta)$, the right-hand side of (12) is $C_I(x_{it}/\Delta, x_{i,t-\Delta})$. Note that $(0, \delta\Delta)$ converges to the empty set as Δ is made arbitrarily small in the sense that for any sequence $\{\hat{\Delta}_k\}_{k\in\mathbb{N}}$ such that $\hat{\Delta}_k \to 0$ as $k \to \infty$, we have $\cap_{k\in\mathbb{N}}(0, \delta\hat{\Delta}_k) = \emptyset$.

Given the above, for all sufficiently small Δ the maximizer x_{it}^* of equation (4′) is unique and characterized as follows: $x_{it}^* = (x_{i,t-\Delta} - \delta\Delta)\mathbb{1}_{(\delta\Delta,\chi]}(x_{i,t-\Delta})$ if $h(x_{it}^*, \boldsymbol{x}_{-i,t}) \leq C_I(\delta, x_{i,t-\Delta})$, $x_{it}^* = \chi$ if $x_{i,t-\Delta} = \chi$ and $h(\boldsymbol{x}_{it}*, \boldsymbol{x}_{-i,t}) \geq C_I(\delta, x_{i,t-\Delta})$, and otherwise x_{it}^* satisfies (12). Making Δ arbitrarily small yields the autonomous system in (5). (Kumagai, 1980). ■

A.2 Proof of Proposition 2

Parts 1, 2, 3, 4, and 5, respectively establish the necessary and sufficient conditions under which escalated egalitarian, de-escalated egalitarian, oligarchic, weak dictatorial, and strong dictatorial power structures are stable. Part 6 then shows that all other power structures will fail to be stable. The following notation will be convenient: let $\mathbf{1}_k$ (resp. $\mathbf{0}_k$) denote a vector of k ones (resp. zeroes), let $\boldsymbol{e}_i \in \mathbb{R}^N$ denote the i^{th} standard basis vector, and let $B_\varepsilon(\boldsymbol{x})$ denote the ε-ball centered at $\boldsymbol{x} \in \mathbb{R}^N$.

To establish the stability of egalitarian and oligarchic power structures, I use Lyapunov's Direct/Second Method (Lyapunov, 1892, 1992). For readers' convenience, I summarize this method before proceeding with the proof.[43] This method shows that $\bar{\boldsymbol{x}} \in [0,\chi]^N$ is stable if there exists a differentiable $\Lambda : \mathbb{R}^N \times \mathbb{R}^N \to \mathbb{R}$ and an ε-ball of $\bar{\boldsymbol{x}}$, $B_\varepsilon(\bar{\boldsymbol{x}})$, such that (1) $\Lambda(\boldsymbol{x}_t = \bar{\boldsymbol{x}}; \bar{\boldsymbol{x}}) = 0$ and $\Lambda(\boldsymbol{x}_t; \bar{\boldsymbol{x}}) > 0$ $\forall \boldsymbol{x}_t \in B_\varepsilon(\bar{x}) \setminus \{\bar{\boldsymbol{x}}\}$ and (2) $\dot{\Lambda}(\boldsymbol{x}_t, \bar{\boldsymbol{x}}) := \frac{d}{dt}\Lambda(\boldsymbol{x}_t; \bar{\boldsymbol{x}}) < 0$ $\forall \boldsymbol{x} \in B_\varepsilon(\bar{x}) \setminus \{\bar{\boldsymbol{x}}\}$. Λ is often referred to as a *Lyapunov function* and thought of as an "energy function." Intuitively, Lyapunov's Direct Method amounts to showing that the energy of the system *strictly* decreases to zero along all trajectories starting sufficiently close to a steady state $\bar{\boldsymbol{x}}$. In this proof, I use a commonly-employed Lyapunov function, $\Lambda(\boldsymbol{x}_t; \bar{\boldsymbol{x}}) \equiv \frac{1}{2}\sum_{i=1}^N (\bar{x}_i - x_{it})^2$, whose time-derivative is $\dot{\Lambda}(\boldsymbol{x}_t; \bar{\boldsymbol{x}}) \equiv \frac{d}{dt}\Lambda(\boldsymbol{x}_t) = -\sum_{i=1}^N (\bar{x}_i - x_{it})\dot{x}_{it}$. Notice that by construction, given any $\bar{\boldsymbol{x}} \in [0,\chi]^N$, $\Lambda(\bar{\boldsymbol{x}}; \bar{\boldsymbol{x}}) = 0$ and $\Lambda(\boldsymbol{x}_t; \bar{\boldsymbol{x}}) > 0$ when $\boldsymbol{x}_t \neq \bar{\boldsymbol{x}}$. I now proceed with the proof of Proposition 2, proving each part in turn.

To economize space, parts 1-3 and 5 are first proven simultaneously as follows. Let $\boldsymbol{z}_k \equiv \mathbf{0}_N + \chi \sum_{j \le k} \boldsymbol{e}_k$ for each $k \in \{0, 1, .., N\}$ and define the following inequalities:

$$h(z_i, \boldsymbol{z}_{-i}) > C_I(\delta, z_i) \text{ if } i \le k, \tag{14}$$

$$h(z_i, \boldsymbol{z}_{-i}) \le C_I(0, z_i) \text{ if } i > k \tag{15}$$

Assume that $\bar{\boldsymbol{x}} = \boldsymbol{z}_k$. Suppose that (14) and (15) both hold when $k \in \{1, \dots, N\}$, and when $k = 0$ (resp. $k = N$) only the latter (resp. former) holds.[44] Then by continuity

$$h(x_i, \boldsymbol{x}_{-i}) > C_I(\delta, x_i) \;\forall i \le k; \;\; h(x_i, \boldsymbol{x}_{-i}) < C_I(\delta, x_i) \;\forall i > k$$

must hold $\forall \boldsymbol{x} \in B_\varepsilon(\bar{\boldsymbol{x}}) \cap [0,\chi]^N$ for any sufficiently small $\varepsilon > 0$. Then at every $\boldsymbol{x}$ in this ε-ball, we have for any $i \le k$ that $\dot{x}_i$ is positive (resp. zero) if x_i is less than (resp. equal to) χ, and for any $i > k$ we have $\dot{x}_i$ is negative (resp. zero) if x_i is greater than (resp. equal to) zero by Lemma 1. Therefore

$$\dot{\Lambda}(\boldsymbol{x}, \bar{\boldsymbol{x}}) = -\left[\sum_{i=1}^k (\bar{x}_i - x_i)\dot{x}_i + \sum_{i=k+1}^N (\bar{x}_i - x_i)\dot{x}_i\right] < 0 \;\forall \boldsymbol{x} \in (B_\varepsilon(\bar{\boldsymbol{x}}) \cap [0,\chi]^N) \setminus \{\bar{\boldsymbol{x}}\}$$

and $\dot{\Lambda}(\bar{\boldsymbol{x}}, \bar{\boldsymbol{x}}) = 0$. Therefore $\bar{\boldsymbol{x}}$ is stable. Now suppose that $k > 0$ and $h(\chi, (\chi\mathbf{1}_{k-1}, \mathbf{0}_{N-k})) < C_I(\delta, \chi)$. Then it follows from Lemma 1 that $\dot{x}_i < 0$ $\forall i \le k$ when $\boldsymbol{x} = \bar{\boldsymbol{x}}$, so it cannot be a steady state. If instead $h(\chi, (\chi\mathbf{1}_{k-1}, \mathbf{0}_{N-k})) = C_I(\delta, \chi)$, then $\bar{\boldsymbol{x}}$ cannot be stable. Notice that $\frac{\partial}{\partial w}h(w, w\mathbf{1}_k, \mathbf{0}_{N-k})$ is strictly positive when $0 < k < N$ and zero when $k = N$. Thus, for any sufficiently small $\varepsilon > 0$ and any $\rho \in (0, \varepsilon)$, if $\boldsymbol{x}_t = \bar{\boldsymbol{x}} - \rho\sum_{j \le k} \boldsymbol{e}_j$, it then follows from Lemma 1 that $\dot{x}_i = \dot{x}_j < 0$ $\forall i, j \le k$ and $\dot{x}_\ell = 0$ for any $\ell > k$, so we must have $x_{t'} \notin B_\varepsilon(\bar{\boldsymbol{x}})$ at some finite $t' \ge t$. Now suppose that $k < N$ and $h(\chi, (\chi\mathbf{1}_k, \mathbf{0}_{N-k-1})) \le C_I(0, 0)$. Then $\dot{x}_i > 0$ $\forall i > k$ when $\boldsymbol{x} = \bar{\boldsymbol{x}}$, so it cannot be a steady state. Thus (14) and (15) are necessary and sufficient for $\bar{\boldsymbol{x}} = \boldsymbol{z}_k$ to be stable. This completes the proof for parts 1,2,3, and 5.

Turning to the proof of part 4, let $\bar{\boldsymbol{x}} = (d, \mathbf{0}_{N-1})$ for some $d \in (0, \chi)$. Assume that Condition IV holds. Then in any sufficiently small ε ball of $\bar{\boldsymbol{x}}$ we must have $h(x_i, \boldsymbol{x}_{-i}) < C_I(0, x_i)$ $\forall i \ge 2$ and $h(x_1, \mathbf{0}_{N-1})$ strictly greater (resp. less) than $C_I(\delta, x_1)$ for all $x_1 \in (d - \varepsilon, d)$ (resp. $x_1 \in (d, d+\varepsilon)$). It then follows

[43] For more information on Lyapunov's Direct Method, please see LaSalle (1960), La Salle and Lefschetz (2012), and Chiang and Alberto (2015).

[44] When $k = 0$ (resp. $k = N$) then inequality (14) (resp. (15)) is vacuously true.

from Lemma 1 that $\dot{x}_i = -\delta\ \forall i \geq 2$ at all $\boldsymbol{x} \in B_\varepsilon(\bar{\boldsymbol{x}}) \cap [0,\chi]^N$, $\dot{x}_1 > 0\ \forall \boldsymbol{x} \in (d-\varepsilon, d) \times \{\mathbf{0}_{N-1}\}$, and $\dot{x}_1 < 0$ $\forall \boldsymbol{x} \in (d, d+\varepsilon) \times \{\mathbf{0}_{N-1}\}$. By the Implicit Function Theorem, there exists a continuous hypersurface $x_1 = g(\boldsymbol{x}_{-1})$ that bisects $B_\varepsilon(\bar{\boldsymbol{x}}) \cap [0,\chi]^N$ s.t. $\operatorname{sgn}(\dot{x}_1) = \operatorname{sgn}(g(\boldsymbol{x}_{-1}) - x_1)\ \forall \boldsymbol{x} \in B_\varepsilon(\bar{\boldsymbol{x}}) \cap [0,\chi]^N$. Therefore if $\boldsymbol{x}_0 \in B_\varepsilon(\bar{\boldsymbol{x}}) \cap [0,\chi]^N$ then $\|\boldsymbol{x}_t - \bar{\boldsymbol{x}}\|_2 \to 0$ as $t \to \infty$, and thus $\bar{\boldsymbol{x}}$ is stable. If $h(0,(d,\mathbf{0}_{N-2})) > C_I(0,0)$ then $\dot{x}_i > 0\ \forall i > 1$ so $\bar{\boldsymbol{x}}$ cannot be stable. This just leaves the case where only the first part of Condition IV fails to hold. If $h(d,\mathbf{0}_{N-1}) \neq C_I(\delta, d)$, then $\dot{x}_1 \neq 0$. If $h(\cdot,\mathbf{0}_{N-1})$ intersects $C_I(\delta,\cdot)$ at d, but not from above, then $\bar{\boldsymbol{x}}$ is a steady state but is not stable. Let $\hat{\varepsilon}$ be sufficiently small so that $h(x_i, \boldsymbol{x}_{-i}) < C_I(0 < x_i)\ \forall i \geq 2$ at every $\boldsymbol{x} \in B_{\hat{\varepsilon}}(\bar{\boldsymbol{x}}) \cap [0,\chi]^N$. Suppose that $h(x_1,\mathbf{0}_{N-1}) \leq C_1(\delta, x_1)\ \forall x_1 \in (d-\hat{\varepsilon}, d)$. Then for any $\rho \in (0,\hat{\varepsilon})$, if $\boldsymbol{x}_t = (d-\rho, \mathbf{0}_{N-1})$, then it follows from Lemma 1 that $\dot{x}_{1t} < 0$ and $\dot{x}_{it} = 0\ \forall i \geq 2$, so that $\boldsymbol{x}_t$ leaves the $\hat{\varepsilon}$ ball of $\bar{\boldsymbol{x}}$ after time finite time t' or does not converge to $\bar{\boldsymbol{x}}$ as $t \to \infty$. The reasoning is similar after supposing that $h(x_1,\mathbf{0}_{N-1}) \geq C_1(\delta, x_1)\ \forall x_1 \in (d, d+\hat{\varepsilon})$. Then for any $\rho \in (0,\hat{\varepsilon})$, if $\boldsymbol{x}_t = (d+\rho, \mathbf{0}_{N-1})$, then it follows from Lemma 1 that $\dot{x}_{1t} > 0$ and $\dot{x}_{it} = 0\ \forall i \geq 2$, so that $\boldsymbol{x}_t$ leaves the $\hat{\varepsilon}$ ball of $\bar{\boldsymbol{x}}$ after time finite time t' or does not converge to $\bar{\boldsymbol{x}}$ as $t \to \infty$. This completes the proof for Part 4.

I now turn to the proof of Part 6. First suppose that $\bar{\boldsymbol{x}}$ has two distinct interior components $0 < \bar{x}_i < \bar{x}_j < \chi$. Then $\bar{\boldsymbol{x}}$ is not a steady state since $h(x_j, \boldsymbol{x}_{-j}) > h(x_i, \boldsymbol{x}_{-i})$ and $C_I(\cdot, x_j) \leq C_I(\cdot, x_i)$, so $\dot{x}_j = C_I^{-1}(h(x_j, \boldsymbol{x}_{-j}), x_j) - \delta > C_I^{-1}(h(x_i, \boldsymbol{x}_{-i}), x_i) - \delta = \dot{x}_i$. Now suppose that there is a steady state $\bar{\boldsymbol{x}}$ whose first $m \in \{1, ..., N\}$ components have the same interior value $w \in (0,\chi)$ any remaining components are either 0 or χ. Notice that $\frac{\partial}{\partial w} h(w, (w\mathbf{1}_m, \cdot)) > 0$. Fix sufficiently small $\varepsilon > 0$ and let $\rho \in (0,1)$. If $\boldsymbol{x}_t$ has $x_{it} = w + \rho\varepsilon$ for all $i \leq m$ and $x_{it} = \bar{x}_{it}$ for any $i > m$, then by Lemma 1 it follows that $\dot{x}_i = \dot{x}_j > 0$ $\forall i,j \leq m$ and $\dot{x}_\ell = 0$ for any other $\ell \geq m$. Thus, $\boldsymbol{x}_\tau$ leaves the ε ball of $\bar{\boldsymbol{x}}$ by some $\tau \in (t,\infty)$. Thus such a $\bar{\boldsymbol{x}}$ cannot be stable. Now suppose that $\bar{\boldsymbol{x}}$ has at least one interior $\bar{x}_i \in (0,\chi)$ and at least one maximally powerful $\bar{x}_j = \chi$, and any other components are either 0 or χ. Notice that if $x_i \leq \max\{x_\ell\}_{\ell=1}^N$, then $\frac{\partial}{\partial x_i} h(x_i, \boldsymbol{x}_{-i}) > 0$. This case then proceeds like the previous one. Fix sufficiently small $\varepsilon > 0$ and let $\rho \in (0,1)$. If $\boldsymbol{x}_t$ has $x_{it} = w + \rho\varepsilon$ and $x_{\ell t} = \bar{x}_{it}\ \forall \ell \neq i$, then $\dot{x}_{it} > 0$ and $\dot{x}_{\ell t}\ \forall \ell \neq i$. Thus, $\boldsymbol{x}_\tau$ leaves the ε ball of $\bar{\boldsymbol{x}}$ by some $\tau \in (t,\infty)$, thus $\bar{\boldsymbol{x}}$ is not stable. ■

Remark 6. Oligarchic and dictatorial power structures remain stable in a technically weaker – but visually indistinguishable – Filippov (2013) sense if one relaxes the second parts of Conditions III, IV, and V by replacing "$\leq C_I(0,0)$" with "$< C_I(\delta,0)$." The same is true for the de-escalated egalitarian power structure if II is relaxed in the same way. If the left-hand side of each condition is strictly between $C_I(0,0)$ and $C_I(\delta,0)$, the corresponding power structure $\bar{x}$ is technically no longer a steady state (since part (a) of Definition 1 is violated) but still behaves very similarly to before (since part (b) of Definition 1 is still satisfied). In this case, x_t still approaches $\bar{\boldsymbol{x}}$ within finite time (and remains arbitrarily close forever afterwards). The only difference from before is that once $\boldsymbol{x}_t$ reaches $\bar{\boldsymbol{x}}$, it does not remain *exactly* there; it proceeds to *infinitesimally* oscillate around $\bar{\boldsymbol{x}}$. Formally, this is known as a type of *Zeno behavior*. In physical terms, the phenomenon just described is similar to a rubber ball that is arbitrarily close to being at rest (i.e. it is vibrating by an imperceptibly small amount).

The economic intuition is quite natural. This is illustrated in the case of a strong dictatorship $\bar{\boldsymbol{x}} = \chi\boldsymbol{e}_i$, where $i \in \{1,\dots,N\}$ is the dictator player; the intuition is the same in the cases of weak dictatorship, oligarchy, and the de-escalated egalitarian steady state. Suppose that $h(\chi,\mathbf{0}_{N-1}) > C_I(\delta,\chi)$, $h(0,(\chi,\mathbf{0}_{N-1})) \in (C_I(0,0), C_I(\delta,0))$, and that $\boldsymbol{x}_t = \chi\boldsymbol{e}_i$ at some fixed time $t \in \mathbb{R}_+$. The former inequality guarantees that it is optimal for the dictator player i to maintain χ units of power at time t. The latter inequality implies that powerless players $j \neq i$ will accumulate power at some positive level, since their marginal benefit of power accumulation $h(0,(\chi,\mathbf{0}_{N-1}))$ exceeds the marginal cost of investment $C_I(I,0)$ when $I = 0$ and because C is convex in I. However, these players will then proceed to let their powers depreciate, because the net marginal gain of maintaining any positive level of power $(h(x_j, x_{-j}) - C_I(\delta, x_j))$ is locally negative. Formally, this is because – under the aforementioned supposition – $h(0,(\chi,\mathbf{0}_{N-1})) < C_I(\delta,0)$, and because $h(\cdot,\cdot)$ and $C_I(\cdot,\cdot)$ are assumed continuous. The behavior of these weak players has a natural, concrete interpretation: they repeatedly try to "rise up" against the dictator by accumulating power before quickly acquiescing.

A.3 Proof of Proposition 3

Recall that by part 1 of Proposition 2, the $(\chi,\ldots,\chi)$ is stable if and only if $h(\boldsymbol{x},(\chi,\ldots,\chi);N) > C_I(\delta,\chi)$ which is equivalent to $\frac{(N-1)\lambda}{N^2} > C_I(\delta,\chi)$. Rearranging this yields the quadratic inequality $0 > C_I(\delta,\chi)N^2 - \lambda N + \lambda$. When $\frac{\lambda}{4} \leq C_I(\delta,\chi)$, the escalated egalitarian power structure is not stable for any N; this quickly follows from Condition I.[45] Otherwise, solving the above quadratic inequality for N yields

$$\frac{\lambda - \sqrt{\big(\lambda - 4C_I(\delta,\chi)\big)\lambda}}{2C_I(\delta,\chi)} < N < \frac{\lambda + \sqrt{\big(\lambda - 4C_I(\delta,\chi)\big)\lambda}}{2C_I(\delta,\chi)}. \tag{16}$$

It is easily verified that the left-most term is always less than two:

$$\frac{\lambda - \sqrt{\big(\lambda - 4C_I(\delta,\chi)\big)\lambda}}{2C_I(\delta,\chi)} - 2 = \frac{\left[\sqrt{\lambda - 4C_I(\delta,\chi)} - \sqrt{\lambda}\right]\sqrt{\lambda - 4C_I(\delta,\chi)}}{2C_I(\delta,\chi)} \leq 0.$$

Noting that N is a natural number implies that no escalated egalitarian steady state exist for group sizes larger than $\bar{N}^I_\chi \equiv \left\lfloor \frac{\lambda + \sqrt{\big(\lambda - 4C_I(\delta,\chi)\big)\lambda}}{2C_I(\delta,\chi)} \right\rfloor$. ■

A.4 Proof of Corollary 1

Suppose that $C_I(\delta,\chi) = q$ for some $q > 0$ and that $\lambda \geq 4q$. Note that

$$\frac{\partial}{\partial\lambda}\left[\frac{\lambda + \sqrt{\lambda\cdot(\lambda - 4q)}}{2q}\right] = \frac{1}{2q}\left[\frac{(\lambda - 2q) + \sqrt{\lambda\cdot(\lambda - 4q)}}{\sqrt{\lambda\cdot(\lambda - 4q)}}\right], \tag{17}$$

and that

$$\frac{\partial}{\partial q}\left[\frac{\lambda + \sqrt{\lambda\cdot(\lambda - 4q)}}{2q}\right] = \frac{-\sqrt{\lambda}}{2q^2\sqrt{\lambda - 4q}}\left[(\lambda - 2q) + \sqrt{\lambda\cdot(\lambda - 4q)}\right]. \tag{18}$$

Observe that equations (17) and (18) are respectively positive and negative if $(\lambda - 2q) + \sqrt{\lambda\cdot(\lambda - 4q)}$ is positive, which is always the case under the aforementioned supposition: $(\lambda - 2q) + \sqrt{\lambda\cdot(\lambda - 4q)} > \underbrace{(\lambda - 4q)}_{\geq 0} + \underbrace{\sqrt{\lambda}}_{>0}\underbrace{\sqrt{\lambda - 4q}}_{\geq 0} \geq 0$. ■

[45] Notice that if $h(\chi,\chi;2) = \lambda/4 < C_I(\delta,\chi)$, then Condition I ($h(\boldsymbol{x},(\chi,\ldots,\chi);N) \geq C_I(\delta,\chi)$) fails at all $N \geq 2$ since $h(\boldsymbol{x},(\chi,\ldots,\chi);N) = \frac{(N-1)\lambda}{N^2}$ is decreasing in N on $\{2,3,\ldots\}$.

A.5 Proof of Proposition 4

Arbitrarily fix $\varepsilon > 0$; suppose that $C_I(\delta, \cdot)$ is bounded by ε from below. By equation (16) in Proposition 3, the escalated egalitarian power structure is stable only in group sizes smaller than

$$\bar{N}_\chi^I \equiv \left\lfloor \frac{\lambda + \sqrt{(\lambda - 4C_I(\delta, \chi))\lambda}}{2C_I(\delta, \chi)} \right\rfloor$$

when $\lambda > 4C_I(\delta, \chi)$, and is otherwise not stable in any group size permitted in this model ($N \in \{2, 3, \ldots\}$). If $\lambda \leq 4\varepsilon$, Proposition 4.1 trivially follows because of Assumption 1.2. Otherwise, $\bar{N}_\chi^I$ clearly remains finite as $\chi \to \infty$ given the aforementioned supposition.

Proposition 7 in section B of this Appendix showed in equation (26) that for each $k \in \{2, 3, \ldots\}$, k-archic power structures are stable only in groups smaller than

$$\bar{N}_\chi^{O_k} \equiv \left\lfloor k + e^{\lambda\chi} \left(\frac{\lambda + \sqrt{(\lambda - C_I(\delta, \chi))\lambda}}{C_I(\delta, \chi)} - k \right) \right\rfloor$$

when $\lambda > C_I(\delta, \chi)$ and not stable at any N otherwise. Similarly to before, if $\lambda < \varepsilon$ then Proposition 4.2 trivially follows because of Assumption 1.2. In the remaining case where $\exists z > 0$ s.t. $\lambda > C_I(\delta, \chi)$ $\forall \chi > z$, notice that the limit of $\bar{N}_\chi^{O_k}$ as $\chi \to \infty$ only depends on the sign of

$$\lim_{\chi \to \infty} \frac{\lambda + \sqrt{(\lambda - C_I(\delta, \chi))\lambda}}{C_I(\delta, \chi)} - k.$$

$\bar{N}_\chi^{O_k} \to \infty$ as $\chi \to \infty$ if the above is strictly negative and $\lim_{\chi \to \infty} \bar{N}_\chi^{O_k} \leq 0$ otherwise.

Proposition 8 in section B of this Appendix showed in equation (29) that dictatorships are stable only in groups smaller than

$$\bar{N}_\chi^D \equiv \left\lfloor e^{\lambda\chi} \cdot \left\lfloor \frac{\lambda - 2C_I(\delta, \chi) + \sqrt{\lambda}\sqrt{\lambda - 4C_I(\delta, \chi)}}{2C_I(\delta, \chi)} \right\rfloor \right\rfloor$$

which becomes arbitrarily large as $\chi \to \infty$ given the aforementioned supposition.

■

A.6 Proof of Proposition 5

As in the above proofs, let $\boldsymbol{e}_i$ denote the i^{th} standard basis vector, and for each $n \in \mathbb{N}$ let $\mathbf{0}_n \in \mathbb{R}^n$ denote the vector of zeros. Recall that by Proposition 2, a dictatorial power structure where the strongest player has $d \in (0, \chi)$ units of power is stable if and only if

$$\begin{aligned} &h\big(\cdot, (0, \ldots, 0); N\big) \text{ intersects } C_I(\delta, \cdot) \text{ from above at } d, \text{ and} \\ &h\big(0, (d, 0, \ldots, 0); N\big) < C_I(0, 0), \end{aligned} \qquad \text{(Condition IV)}$$

holds, and that strong dictatorial power structures are stable if and only if

$$h\big(\chi, (0, \ldots, 0); N\big) > C_I(\delta, \chi) \text{ and } h\big(0, (\chi, 0, \ldots, 0; N)\big) < C_I(0, 0). \qquad \text{(Condition V)}$$

holds. Arbitrarily fix $N \in \{2,3,...\}$ and all other model primitives (χ, δ, λ, and C) such that for each $M\{N, N+1\}$, either (1) Condition IV holds at exactly one $d_M \in (0,\chi)$ and condition V fails or (2) Condition IV fails at all $d \in (0,\chi)$ and Condition V holds. The result of the proof is immediate in case where Condition V holds when the group size is $N+1$.

Now suppose that for each $M \in \{N, N+1\}$, Condition IV holds at exactly one $d_M \in (0,\chi)$ and condition V fails. Note that for all $n \in \{2,3,...\}$ and $\ell \in \{0,1,2,...\}$,

$$h\left(x_i - \frac{1}{\lambda}\ln\left(\frac{n+\ell-1}{n-1}\right), \mathbf{0}_{n-1}; n\right) = h\left(x_i, \mathbf{0}_{n+\ell-1}; n+\ell\right). \tag{19}$$

That is, given an initial group size of n, adding ℓ more players is equivalent to translating $h(\cdot, \mathbf{0}_{n-1}; n)$ rightward by $\frac{1}{\lambda}\ln\left(\frac{n+\ell-1}{n-1}\right)$. Moreover, observe that $h(\cdot, \mathbf{0}_{N-1}; N)$ can only intersect $C_I(\delta,\cdot)$ from above after the former attains its global maximum at $x_i = \frac{\ln(N-1)}{\lambda}$ as it is assumed that both functions are continuous and $C_I(I,x)$ is weakly decreasing in its second argument. It then follows that $d_M \in \left(\frac{\ln(M-1)}{\lambda}, \chi\right)$ $\forall M \in \{N, N+1\}$. If $d_N \in \left(\frac{\ln(N-1)}{\lambda}, \frac{\ln(N)}{\lambda}\right)$, then $d_N < d_{N+1}$ follows from the fact that $d_{N+1} > \frac{ln(N)}{\lambda}$. If instead $d_N \in \left(\frac{\ln(N)}{\lambda}, \chi\right)$, note that $h(\cdot, \mathbf{0}_{N-1}; N)$ and $h(\cdot, \mathbf{0}_N; N+1)$ are strictly decreasing on $\left(\frac{\ln(N)}{\lambda}, \chi\right)$. Since the latter is a rightward translation of the former, and since $C_I(\delta,\cdot)$ is decreasing, it follows that $d_N < d_{N+1}$. Note that when restricting attention to the interval $\left(\frac{\ln(N)}{\lambda}, \chi\right]$, translating $h(\cdot, \mathbf{0}_{N-1}; N)$ rightward is equivalent to translating it upward; this immediately yields a contradiction upon supposing that Condition V holds for N and Condition IV holds for $N+1$ at exactly one $d_{N+1} \in (0,\chi)$. Noting that $h(0, (d, \mathbf{0}_{N-2}); N)$ is decreasing in N and d $\forall (N, d, \lambda) \in \{2,3,...\} \times (0,\infty)^2$, the proof is complete. ■

A.7 Proof of Proposition 6

I consider without loss of generality case where player 1 is a (weak) dictator: $\hat{\boldsymbol{x}} = (d, \mathbf{0}_N)$, where $d \in (0,\chi)$ is as in the first part of Condition IV, which is reproduced and discussed in the proof of Proposition 5, found immediately above. Assume that $C(\cdot,\cdot)$ complies with Assumption 1. Fix some small $\varepsilon > 0$ and choose some $\tilde{C}$ such that $\tilde{C}_I(\cdot,\cdot) = C_I(\cdot,\cdot) + \varepsilon$. Since $h(\cdot, \mathbf{0}_N)$ intersects $C_I(\delta,\cdot)$ from above at $d < \chi$, it follows from Assumptions 1 and 2 that $h(\cdot, \mathbf{0}_N) - C_I(\delta,\cdot)$ is locally decreasing around d. Hence, for sufficiently small but positive ε, $h(\cdot, \mathbf{0}_N)$ intersects $\tilde{C}_I(\delta,\cdot)$ at $\tilde{d} < d$. This completes the proof for part (i) of this proposition. Note that since C is assumed convex in its first argument, part (ii) immediately follows. Turning to part (iii), we consider the effect of an increase in λ on d. Note that

$$\frac{\partial}{\partial\lambda}h(x_i, \mathbf{0}_{N-1}) = \underbrace{-\frac{(N-1)e^{\lambda x_i}}{\left[(N-1)+e^{\lambda x_i}\right]^3}}_{<0\ \because N\geq 2}(1 - e^{\lambda x_i} - N + \lambda x + \lambda x e^{\lambda x_i} - \lambda N x) \tag{20}$$

Setting the second term to less than zero and rearranging yields the inequality in part (iii) of this proposition, thereby completing its proof. ■

Remark 7. The first proposition holds for any uniformly equicontinuous family $\{H(\cdot;N)\}_{N=2}^{\infty}$ of contest functions. Intuitively, this ensures marginal benefits $h(\cdot;N)$ to not explode to infinity. Mathematically, it guarantees that the solution to the dynamical system (5) in Proposition 1 exists and is unique (Picard–Lindelöf/Cauchy–Lipschitz theorem). No additional properties of H are needed to show that the stability conditions I-V are sufficient for the emergence of egalitarian, oligarchic, and dictatorial regimes in Proposition 2. The proof showing necessity involves an intuitive, mild property: if a player is not strictly stronger than the rest, then their marginal benefit is increasing (resp. decreasing) in how much power they (resp. their opponents) hold. This property also causes certain regimes with interior power player(s) that face at least one weakly stronger opponent to not be stable[46] A similarly mild property is involved in the proof that regimes with different interior power levels are unstable. In the standard case where $C_I(I, x)$ is constant in x, regimes with distinct interior power levels are unstable when players with different powers have different power accumulation incentives; in the less standard case where $C_I(I, x)$ is weakly diminishing in x, it is sufficient for stronger players have weakly stronger power accumulation incentives. The main results in Propositions 3 and 4 showing the Iron Law of Oligarchy hold under a very mild property: as $k \in \{2, ..., N\}$ players become equally, arbitrarily powerful (and any remaining players' powers remain finite), their marginal benefits converge to the case where they are the only players.[47] Finally, in Proposition 5 stable dictatorial power is increasing in group size because powerless players have a non-zero chance of winning conflicts; otherwise, dictators ruling over one powerless player versus a billion powerless players would face the same competitive pressure.

Appendix B Auxiliary Results

Lemma 1. *For each* $i \in \{1, \dots, N\}$, *sign* $(\dot{x}_i) =$ *sign* $(h(x_i, \boldsymbol{x}_{-i}) - C_I(\delta, x_i))\, \mathbb{1}_{(0,\chi)}(x_i)$ *at every* $\boldsymbol{x}$ *in* $\{\boldsymbol{x} \in [0, \chi]^N : x_i \in (0, \chi]\}$.

Proof. Assume throughout that $x_i \in (0, \chi]$. First suppose that $h(x_i, \boldsymbol{x}_{-i}) < C_I(0, x_i)$. This implies that $h(x_i, \boldsymbol{x}_{-i}) < C_I(\delta, x_i)$ since $C_I(\cdot, x_i)$ is assumed to be strictly increasing. Then $\dot{x}_i = -\delta$ by the first piece of (5). Assume henceforth that $h(x_i, \boldsymbol{x}_{-i}) \geq C_I(0, x_i)$. It then follows from (5) that $\dot{x}_i$ satisfies

$$h(x_i, \boldsymbol{x}_{-i}) = C_I(\dot{x}_i + \delta, x_i). \tag{21}$$

If $h(x_i, \boldsymbol{x}_{-i}) = C_I(\delta, x_i)$, then we must have $\dot{x}_i = 0$ in order for (21) to hold. If $h(x_i, \boldsymbol{x}_{-i})$ is strictly greater (resp. less) than $C_I(\delta, x_i)$, $\dot{x}_i$ must be strictly positive (resp. negative) in order for (21) to hold because $C_I(\cdot, x_i)$ is assumed to be strictly convex. However, recall that by the second part of (5), $\dot{x}_i = 0$ when $h(x_i, \boldsymbol{x}_{-i}) > C_I(\delta, x_i)$ and $x_i = \chi$. This is why there is an indicator function $\mathbb{1}_{(0,\chi)}(x_i)$ in the equation of this lemma. ■

Lemma 2. *Let* $i \in \{1, \dots, N\}$ *and arbitrarily fix* $\boldsymbol{x} \in \{\boldsymbol{x} \in [0, \chi]^N : x_i = 0\}$. *Then* $\dot{x}_i > 0$ *if* $h(x_i, \boldsymbol{x}_{-i}) > C_I(0, x_i)$ *and* $\dot{x}_i = 0$ *if* $h(x_i, \boldsymbol{x}_{-i}) = C_I(0, x_i)$.

Proof. Assume that $x_i = 0$ throughout. If $h(x_i, \boldsymbol{x}_{-i}) < C_I(0, x_i)$ then $\dot{x}_i = 0$ by the first piece of (5). Suppose that $h(x_i, \boldsymbol{x}_{-i}) = C_I(0, x_i)$. Since $x_i = 0$, (5) implies that $\dot{x}_i$ must satisfy

$$h(x_i, \boldsymbol{x}_{-i}) = C_I(\dot{x}_i, x_i). \tag{22}$$

In order for (22) to hold in the present case we must have $\dot{x}_i = 0$, since $C_I(\cdot, x_i)$ is strictly increasing. Finally, suppose that $h(x_i, \boldsymbol{x}_{-i}) > C_I(0, x_i)$. Then $\dot{x}_i$ must satisfy (22), which implies that $\dot{x}_i > 0$ since $C_I(\cdot, x_i)$ is strictly increasing. ■

[46] This is because a small windfall of power to interior power player(s) boosts their power accumulation incentives and causes $\boldsymbol{x}_t$ to not return to its original location before the perturbation.

[47] Notice that for any conditional probability distribution, the marginal benefit must vanish to zero if all players are equally powerful and the number of players converges to infinity. Thus there is always a group size past which only dictatorship an oligarchy are the only stable regimes.

Proposition 7. *Let $k \in \{2, \ldots, N-1\}$. k-archies are never stable past group size*

$$\bar{N}_{\chi}^{O_k} = \left\lceil k + e^{\lambda\chi}\left(\frac{\lambda + \sqrt{(\lambda - C_I(\delta,\chi))\lambda}}{C_I(\delta,\chi)} - k\right)\right\rceil \tag{23}$$

Proof. To simplify notation, let q_χ denote $C_I(\delta, \chi)$ Recall that in the proof of Part 3 of Proposition 2, it was established that

$$h(\chi, (\chi\mathbf{1}_{k-1}, \mathbf{0}_{N-k}); N) > q_\chi \text{ and } h(0, (\chi\mathbf{1}_k, \mathbf{0}_{N-k-1}; N) \leq C_I(0,0) \tag{24}$$

are the necessary and sufficient conditions for the stability of each element of

$$\left\{\boldsymbol{x} \in \{0, \chi\}^N : \sum_{i=1}^{N} x_i = k\chi\right\}.$$

Note that the first inequality in (24) is equivalent to

$$\frac{\lambda\left[k - 1 + (N-k)e^{-\lambda\chi}\right]}{\left[k + (N-k)e^{-\lambda\chi}\right]^2} > q_\chi, \tag{25}$$

which yields the following quadratic inequality in N. Letting $\alpha = e^{-\lambda\chi}$ and $\beta = \frac{\lambda}{q_\chi}$, this is as follows:

$$\alpha^2 N^2 + \alpha[2(1-\alpha)k - \beta]N + \left\{\left[(1-\alpha)k - \frac{\beta}{2}\right]^2 + \left(1 - \frac{\beta}{4}\right)\beta\right\} < 0$$

Note that the coefficient of N^2 is positive; by the formula for the vertex of a parabola, it follows that no N satisfies this inequality if $\beta \cdot \left(1 - \frac{\beta}{4}\right) > 0$ ($\Leftrightarrow \lambda < 4q_\chi$). Otherwise, solving the above quadratic inequality yields the following:

$$k + e^{\lambda\chi}\left[\frac{\lambda - \sqrt{(\lambda - q_\chi)\lambda}}{q_\chi} - k\right] < N < k + e^{\lambda\chi}\left[\frac{\lambda + \sqrt{(\lambda - q_\chi)\lambda}}{q_\chi} - k\right] \tag{26}$$

Therefore, k-archies are never stable if N is greater than or equal to

$$\bar{N}_{\chi}^{O_k} = \left\lceil k + e^{\lambda\chi}\left(\frac{\lambda + \sqrt{(\lambda - q_\chi)\lambda}}{q_\chi} - k\right)\right\rceil$$

Finally note that if the second inequality of (24) holds for some $N = \tilde{N} \in \{2, 3, \ldots\}$ then it holds for all $N \geq \tilde{N}$ since $\frac{\partial}{\partial N} h(0, (\chi\mathbf{1}_k, \mathbf{0}_{N-k-1}; N) < 0$. ■

Proposition 8. *Suppose Condition IV holds for some N and χ, then there exist finite $\underline{N}_{\chi}^{D_W}, \bar{N}_{\chi}^{D_W}, \bar{N}_{\chi}^{D_S}$ such that*

1. *Weak dictatorships are stable if $\underline{N}_{\chi}^{D_W} \leq N < \bar{N}_{\chi}^{D_W}$.*

2. *Only strong dictatorships are stable if* $\bar{N}_\chi^{D_W} \leq N \leq \bar{N}_\chi^{D_S}$

3. *Weak and strong dictatorships are unstable if* $N > \bar{N}_\chi^{D_S}$.

Proof. Recall that the marginal benefit of investment for player i when her power is $x_i \in [0, \chi]$ and all other players have zero power is given by

$$h(x_i, \mathbf{0}_{N-1}; N) \equiv \frac{\lambda(N-1)e^{-\lambda x_i}}{(1+(N-1)e^{-\lambda x_i})^2} \quad (\lambda > 0). \tag{27}$$

Recall that by (19) given an initial group size of N, adding K more players shifts marginal benefit $h(\cdot, \mathbf{0}_{N-1}; N)$ rightward.

Suppose that $d_N \boldsymbol{e}_i$ ($d_N \in (0,1)$) is stable when group size is $N \in \{2, 3, ...\}$. This is only possible if $h(\cdot, \mathbf{0}_{N-1}; N)$ intersects $C_I(\delta, \cdot)$ from above at d_N. I demonstrate this via proof by contrapositive. If $h(\cdot, \mathbf{0}_{N-1}; N)$ is strictly greater than (strictly less than) $C_I(\delta, \cdot)$ at d_N, then the player i's marginal benefit of investment is strictly greater than (strictly less than) her marginal cost when $\boldsymbol{x} = d_N \boldsymbol{e}_i$, hence $\dot{x}_i > 0$ ($\dot{x}_i < 0$) at this point. Therefore if $h(d_N, \mathbf{0}_{N-1}; N) \neq C_I(\delta, d_N)$ then $d_N \boldsymbol{e}_i$ is not a steady state. If the intersection is from below, then $d_N \boldsymbol{e}_i$ is not stable. Let $\varepsilon > 0$. Perturbing x_i to $d_N + \varepsilon$ ($d_N - \varepsilon$) causes the marginal benefit of investment to become strictly greater than (strictly less than) the marginal cost for player i, thereby inducing $\dot{x}_i > 0$ ($\dot{x}_i < 0$) at this perturbed point. $d_N \boldsymbol{e}_i$ is not stable if $h(\cdot, \mathbf{0}_{N-1}; N)$ is tangent to $C_I(\delta, \cdot)$ at d_N. This is shown through similar reasoning. Finally, we consider the case where $h(x_i, \mathbf{0}_{N-1}; N) = C_I(\delta, x_i)\ \forall\ x_i \in (d_N - \varepsilon, d_N + \varepsilon)$ for some $\varepsilon > 0$. (That is, $h(\cdot, \mathbf{0}_{N-1}; N)$ and $C_I(\delta, \cdot)$ overlap in some ε-neighborhood of $x_i = d_N$.) Note that if $d_N \boldsymbol{e}_i$ is a steady state, we must have that $h(0, (d_N, \mathbf{0}_{N-2}); N) < C_I(\delta, 0)$ Otherwise $\dot{x}_j > 0\ \forall j \neq i$ at this point. By the continuity of h and C_I, there must be some η-neighborhood of $d_N \boldsymbol{e}_i$ throughout which this strict inequality holds. Consider the perturbation to $\boldsymbol{x}' = (d_N + \nu)\boldsymbol{e}_i$, where $0 < \nu < \min\{\varepsilon, \eta\}$. By construction $h(d_N + \nu, \mathbf{0}_{N-1}; N) = C_I(\delta, d_N + \nu)$, so $\dot{x}_i = 0$ at this point. Similarly, $h(0, (d_N + \nu, \mathbf{0}_N); N) < C_I(\delta, 0)$, so $\dot{x}_j = 0\ \forall j \neq i$. Therefore a trajectory that begins at $\boldsymbol{x}_0 = \boldsymbol{x}'$ does *not* approach $d_N \boldsymbol{e}_i$ in the limit, thereby ruling out its stability. Note that $\max_{x_i \in \mathbb{R}} h(x_i, 0; 2) = \frac{\lambda}{4}$; this global maximum is attained at $x_i = 0$. Since (19) implies that $h(\cdot, \mathbf{0}_{N-1}; N)$ is a rightward horizontal translation of $h(\cdot, 0; 2)$ ($N = 2, 3, ...$), it follows that $\max_{x_i \in \mathbb{R}} h(x_i, \mathbf{0}_{N-1}; N) = \frac{\lambda}{4}$ for every such N. Recall that $h(x_i, \mathbf{0}_{N-1}; N)$ attains its global maximum (about which it is unimodal) at $x_i = \frac{\ln(N-1)}{\lambda}$. Notice that this is monotonically increasing in N when $N \geq 2$. Since $C_I(I, x)$ is weakly decreasing in its second argument, it follows that $\min_{x_i \in [0,\chi]} C_I(\delta, x_i) = C_I(\delta, \chi)$. Finally, notice that $\lim_{x_i \to \infty} h(x_i, \mathbf{0}_N; N) = 0$. The desired result is immediate.

If $\lambda < 4C_I(\delta, \chi)$ then $\bar{N} = 2$. Now assume $\lambda > 4C_I(\delta, \chi)$ throughout the remaining duration of this proof. Since $h(x, \mathbf{0}_{N-1}; N)$ is unimodal about $\frac{\ln(N-1)}{\lambda}$ it follows that there exist exactly two values of N that solve $h(\chi, \mathbf{0}_{N-1}; N) = C_I(\delta, \chi)$. These are

$$\widehat{N}_1 = 1 + \left(\frac{e^{\lambda \chi}}{2C_I(\delta, \chi)}\right)\left(\lambda - 2C_I(\delta, \chi) - \sqrt{\lambda}\sqrt{\lambda - 4C_I(\delta, \chi)}\right) \tag{28}$$

and

$$\widehat{N}_2 = 1 + \left(\frac{e^{\lambda \chi}}{2C_I(\delta, \chi)}\right)\left(\lambda - 2C_I(\delta, \chi) + \sqrt{\lambda}\sqrt{\lambda - 4C_I(\delta, \chi)}\right). \tag{29}$$

Let $\bar{N}_\chi^{D_W} = \lceil \widehat{N}_1 \rceil$. If $\widehat{N}_2 \in \mathbb{N}$, then let $\bar{N}_\chi^{D_S} = \widehat{N}_2 - 1$; otherwise let $\bar{N}_\chi^{D_S} = \lfloor \widehat{N}_2 \rfloor$. When $N = \bar{N}_\chi^{D_S}$, we know that $h(x_i, \mathbf{0}_{\bar{N}_\chi^{D_S}-1}; \bar{N}_\chi^{D_S}) > C_I(\delta, x_i)\ \forall\ x_i \in (\chi - \varepsilon, \chi]$ for some $\varepsilon > 0$ and the reverse inequality holds in

$[0, \chi - \varepsilon]$. Therefore the only stable dictatorships that exist are $\{\chi \boldsymbol{e}_i\}_1^{\bar{N}_\chi^{D_S}}$. It follows from (19) that for all $N > \bar{N}_\chi^{D_S}$, $h(x_i, \mathbf{0}_{N-1}; N) < C_I(\delta, x_i)\ \forall\ x_i \in [0, \chi]$. Therefore no dictatorial steady state can exist at any such N. By construction $h(x_i, \mathbf{0}_{\bar{N}_\chi^{D_W}-1}; \bar{N}_\chi^{D_W}) > C_I(\delta, x_i)\ \forall\ x_i \in \left(\frac{\ln\left(\bar{N}_\chi^{D_W}-1\right)}{\lambda}, \chi\right]$. It then follows that the only dictatorial steady states that exist are $\{\chi \boldsymbol{e}_i\}_1^M$. It follows from (19) that the same is true for all $N \in \{\bar{N}_\chi^{D_W}, ..., \bar{N}_\chi^{D_S}\}$. ■

Appendix C Population-Varying Resource Endowments

For simplicity, the baseline model in the main text assumes a fixed resource endowment size normalized to unity. This section explores the extension where the size of a society's resource endowment $y_N \geq 0$ can vary with its population size $N \in \{2, 3, ...\}$. As discussed in Footnote 25 in the main text, focusing on an endowment economy is appropriate given this paper's focus on understanding how societies' distributions of power evolves over time. The case with a production economy is currently planned follow-up work to this paper.

Remark 8. This section only assumes positive resource endowments $\{y_N\}_{N=2}^\infty \subseteq \mathbb{R}_{++}$ and imposes no restrictions on how they vary with population size N. As $N \to \infty$, if y_N grows roughly linearly in N, then there are no population *scale effects*, while asymptotically faster (resp. slower) than linear growth corresponds to persistent positive (resp. negative) population scale effects (Acemoğlu, 2009, p. 448).[48] Empirical evidence seems to largely suggest that the expected/benchmark case has no persistent population scale effects, i.e. N should not have a direct, systematically positive or negative effect on y_N/N as $N \to \infty$ (Lucas, 1993; Jones, 1995a,b; Junius, 1997; Young, 1998; Acemoğlu, 2009; Jones and Romer, 2010; Bond-Smith, 2019).

Mechanically, this extension generalizes one specific part of the setup: player i.'s benefit and marginal benefit from power (given power structure $\boldsymbol{x}$) are now respectively given by $y_N \cdot H(x_{i\cdot}, \boldsymbol{x}_{-i,\cdot})$ and $y_N \cdot h(x_{i\cdot}, \boldsymbol{x}_{-i,\cdot})$; the main text focused on the special case where $y_N = 1\ \forall N$. The effect of this extension is trivial before Proposition 3: everything[49] in the main text remains identical apart from replacing each instance of "$H(\cdot)$" (resp. "$h(\cdot)$") with "$y_N H(\cdot)$" (resp. "$y_N h(\cdot)$").

The more intriguing finding from this extension regards the main results in the main text (Propositions 3 and 4), which showed that in sufficiently large societies, escalated egalitarian regimes become unstable while dictatorships and concentrated oligarchies remain stable in arbitrarily large societies. It is reasonable to suspect that this result may be sensitive to this extension, since an increasingly large number of players compete over a fixed economic pie.[50] Interestingly, this turns out not to be the case. This is demonstrated by Proposition 9, below. This result also provides a tight analytical relationship between the stability of egalitarian, oligarchic, and dictatorial regimes and the growth rate of y_N.

Proposition 9. *As population size N grows arbitrarily large, the egalitarian power structure remains stable if and only if the size of the society's economy y_N grows sufficiently quickly in N; dictatorial and*

[48]Note that y_N is not assumed to necessarily always grow with N in this section; it it is permitted to be decreasing or non-monotonic in N.

[49]I.e. the model, exposition, results (i.e. Propositions 3 and 4), corresponding proofs, etc.

[50]To see why, recall why regimes with sufficiently many powerful players – that is, the escalated egalitarian regime and oligarchies with more than $\bar{k}$ oligarchs (characterized in (8)) – eventually became unstable past some finite population size N in the main text: as N grows large, the marginal benefit of maintaining high levels of power in such regimes shrinks, eventually falling below its corresponding marginal cost, rendering the regime unstable thereafter.

oligarchic regimes remain stable if and only if y_N grows sufficiently quickly, but not too quickly. More precisely:

a. *The egalitarian power structure $\boldsymbol{x} = \chi \mathbf{1}_N$ is stable at arbitrarily large N if and only if the following inequality holds at arbitrarily large N:*

$$y_N \geq \frac{C_I(\delta, \chi)}{\lambda} \frac{N^2}{N-1} =: \underline{y}_N^I. \tag{30}$$

b. *For any fixed $k \in \{2, 3, \ldots\}$, all k-archies $\boldsymbol{x} \in \mathcal{O}_k$ are stable at arbitrarily large N if and only if the following holds at arbitrarily large N:*

$$\underline{y}_N^{\mathcal{O}_k} := \frac{C_I(\delta, \chi)}{\lambda} \frac{[(1-e^{-\lambda\chi})k + e^{-\lambda\chi}N]^2}{(1-e^{-\lambda\chi})k - 1 + e^{-\lambda\chi}N} < y_N \leq \frac{C_I(0,0)}{\lambda} \frac{[(e^{\lambda\chi}-1)k + N]^2}{(e^{\lambda\chi}-1)k - 1 + N} =: \bar{y}_N^{\mathcal{O}_k}. \tag{31}$$

c. *All strong dictatorships $\boldsymbol{x} \in \mathcal{D}_\chi$ are stable at arbitrarily large N if and only if*

$$\underline{y}_N^D := \frac{C_I(\delta, \chi)}{\lambda} \frac{[1+(N-1)e^{-\lambda\chi}]^2}{(N-1)e^{-\lambda\chi}} < y_N \leq \frac{C_I(0,0)}{\lambda} \frac{[e^{\lambda\chi}+N-1]^2}{e^{\lambda\chi}+N-2} =: \bar{y}_N^D \tag{32}$$

holds for arbitrarily large N.

Proof. Power structure $\boldsymbol{x} = \chi \mathbf{1}_N$ is stable if and only if $y_N h(\chi, \mathbf{1}_N; N) > C_I(\delta, \chi)$, which is equivlent to $y_N > [h(\chi, \mathbf{1}_N; N)]^{-1} C_I(\delta, \chi)$; part a of the result then immediately follows from equation (3). For any fixed $k \in \{2, 3, \ldots\}$ and $N \in \{k+1, k+2, \ldots\}$, all k-archies $\boldsymbol{x} \in \mathcal{O}_k$ are stable if and only if inequalities $y_N h(\chi, (\chi \mathbf{1}_{k-1}, \mathbf{0}_{N-k}); N) > C_I(\delta, \chi)$ and $y_N h(0, (\chi \mathbf{1}_k, \mathbf{0}_{N-k-1}); N) \leq C_I(0,0)$ both hold; these inequalities are easily rearranged as $C_I(\delta, \chi)[h(\chi, (\chi \mathbf{1}_{k-1}, \mathbf{0}_{N-k}); N)]^{-1} < y_N \leq C_I(0,0)[h(0, (\chi \mathbf{1}_k, \mathbf{0}_{N-k-1}); N)]^{-1}$. Part b then immediately follows from equation (3). All strong dictatorships $\boldsymbol{x} \in \mathcal{D}_\chi$ are stable if and only if $y_N h(\chi, \mathbf{0}_{N-1}; N) > C_I(\delta, \chi)$ and $y_N h(0, (\chi, \mathbf{0}_{N-2}); N) \leq C_I(0,0)$ both hold. These inequalities are simply rearranged as $C_I(\delta, \chi)[h(\chi, \mathbf{0}_{N-1}; N)]^{-1} < y_N \leq C_I(0,0)[h(0, (\chi, \mathbf{0}_{N-2}); N)]^{-1}$. Part c immediately follows from equation (3). ∎

All parts of the above characterization are non-trivial, as there is no universal observed relationship between population size and the size of a society's economy in terms of curvature or even monotonicity.[51] Note that all upper and lower bounds ($\underline{y}_N$ and $\bar{y}_N$, respectively) in Proposition 9 are monotonically increasing and convex, and are asymptotically affine.

Part a of Proposition 9 shows that the escalated egalitarian power structure can only remain stable in large societies if and only if y_N persistently grows sufficiently rapidly in N; specifically, it must persistently outpace $\underline{y}_N^E$, whose slope can be arbitrarily large. Parts b and c show that the stability of oligarchic and dictatorial regimes rely on y_N growing sufficiently fast, but not too fast. Specifically, in order for k-archies (resp. strong dictatorships) to remain stable, y_N must persistently remain above $\underline{y}_N^{\mathcal{O}_k}$ and below $\bar{y}_N^{\mathcal{O}_k}$ (resp. above $\underline{y}_N^D$ and below $\bar{y}_N^D$). This is illustrated in Figure 6.

[51] Becker et al. (1999); Alesina et al. (2005); Acemoğlu (2009); Peterson (2017); Bucci and Prettner (2020); Jones (2022).

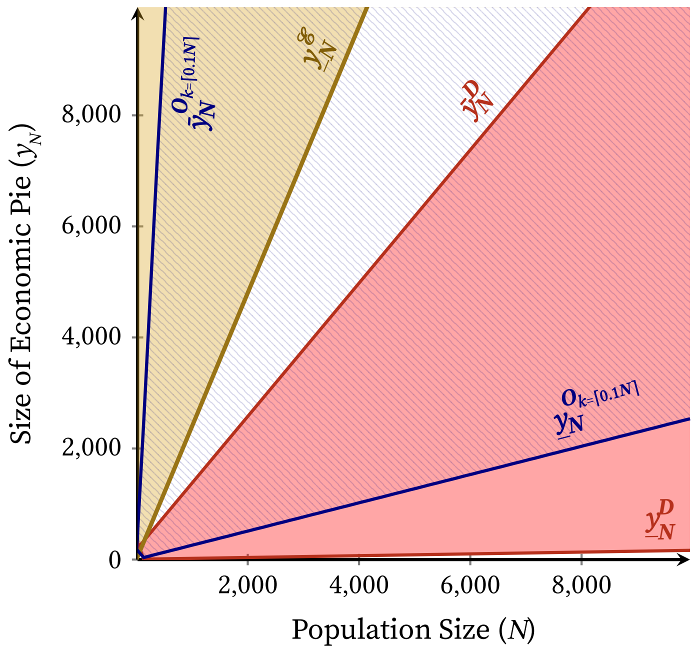


Figure 6: This figure assumes $\lambda = 5$, $C(I, z) = 6I + 6I^2$, $\delta = 0.5$, and $\chi = 1$.

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