

War, Politics, and the Rise of Nations

How Superpowers Choose Allies

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Abstract: This paper offers a framework for understanding the rise of nations via war and alliance politics. The economic rationale of alliance decisions are summarized by two pairs of inequalities comparing the relative levels of wealth and military strengths of nations. These inequalities suggest that a superpower may fight alone or ally with strong but poor nations to rob wealthy but weak nations. Other nations ally with or against the superpower depending on its wealth and strength. At a tipping point, a marginal change in economic conditions shocks alliance relationships. The theory has profound empirical and policy implications.

Keywords: War, Ally, Politics, Wealth, Superpower, Rise of Nations, World Order.

JEL code: A10, D74, F50, H00, L00, N00, P00.

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1. Introduction

This paper offers a novel framework for understanding the rise of nations via wars and alliance politics in a superpower-centric world.

In the millennia of civilized history, some nations achieved superpower status, such as the Persian, Macedonian, Roman, Arab, and Mongol Empires in ancient times, and the United Kingdom and the United States in modern times. With their supreme military, political, and economic powers, superpowers shape the world order in which other nations find their places. When a superpower is toppled, the world order is changed with power and wealth redistributed across nations.

Superpowers have diverse institutional – cultural, economic, political, etc. – and geographical orientations. However, there is a prominent commonality among them: the road to superpower status is paved by military victories. This fact suggests the compelling need to include military competition in the study of the rise nations. The question is: how exactly do nations compete militarily? A related question is: how is military competition intertwined with political and economic competition? A particularly puzzling question is: how can a previously small, divided, and weak nation – that is, an underdog, such as the Greeks in the fourth century BC and that of the Mongols in the thirteenth centuries reviewed later in this paper – suddenly rise to topple a seemingly invincible incumbent superpower? The theory of alliance politics offered in this paper addresses these questions.

For a quick grasp of the essence of the theory, imagine a world of three nations (parties): *A*, *B*, and *S*. Each party is endowed with a certain amount of wealth, military

resources, and a military technology. S is the superpower, which has more military resources and a superior military technology than A and B . Culturally, A and B are close, while S is alien.

Each party makes two sequential decisions: how much of its military resource to invest in the capacity for war and whom to ally with or fight against in war. In making the second decision, S decides whether to fight alone against both A and B or ally with one of them against the other. Meanwhile, A and B decide whether to ally with or against S . These decisions affect the organization of war resources and war outcomes.

The alliance decisions are not arbitrary but have their underlying economic logic, which is summarized in two pairs of mathematical inequalities presented later in the paper. In plain words, having allies makes a superpower stronger and better able to preserve the power and wealth that it currently holds. However, the opportunity cost is that it has fewer nations and less wealth left to rob. This trade-off between preserving and grabbing suggests that a superpower does not need allies when it is strong but not so wealthy. For a superpower, weak and wealthy nations are good targets rather than helpful allies for preservative purposes. In terms of the model developed in this paper, S would fight against both A and B if they were wealthy and weak, but S would ally with one of them if S itself were wealthy but not so strong. Meanwhile, A and B are motivated to team up against S when S is sufficiently wealthy. The motivation is stronger when S is weaker, not surprisingly.

Between the regimes of alliance relationships shaped by the two pairs of

inequalities, there are two tipping points at which a marginal change in the parameters can cause drastic changes in the world order: the superpower may abandon its allies and the allies may abandon the superpower.

One of the theory's rich empirical implications is that the world may have two or three classes of nations. When a superpower fights alone, the world is polarized, with the mighty and wealthy superpower on one side and everybody else on the other. When the parameters are such that the superpower has allies, the world has three classes of nations: the superpower, its allies, and its enemies.¹

The theory suggests that a superpower's alliance policy is likely to change with its relative level of wealth. A superpower much wealthier than other nations would willingly subsidize its allies. But the superpower would become resentful of and terminate the subsidies as it becomes poorer. Beyond a certain point, the superpower would abandon the allies and start to prey on them.

The theory suggests that a change in the world order would be an abrupt and swift process after a relatively long period of stability.² The reason for this pattern of change lies in the existence of tipping points at which, but not before, a marginal change in the parameters can send shocks to the existing alliance relationships. The right leader at the right time, such as Alexander the Great or Genghis Khan, may well be such a critical marginal change.

¹ The relationships of the European colonists with the indigenous peoples in the Americas seem to be examples of a polarized world. The current world has three classes of nations as no nation has the military capacity to overwhelm all other nations.

² The rise of the Greeks and that of the Mongols are examples. In modern times, world order often changes in a process of which "...almost everybody is surprised" (Dalio, 2021, p. 8).

The theory also suggests that along with military victories, nations that join an alliance against the incumbent superpower can gain large amounts of wealth.

According to the theory, a leading indicator of the rise of a new superpower is that culturally close parties stop infighting and form an alliance against the incumbent superpower, as the Greek poleis had done before they toppled the Persian Empire and the Mongol tribes had done before they toppled the Jin Dynasty.

In addition to explaining historical experiences and generating testable empirical predictions, the theory sheds light on important policies shaping the current world order. Examples include the Marshall Plan, which was adopted after World War II. The theory suggests that the Trump administration's current policies toward allies of the United States reflect the poorer Americans' rational calls (further discussed later in the paper).

Scholars have studied the rise of nations, focusing on economic efficiency and the wealth of nations. Influential works include Smith (1776), Weber (1904), Olson (1965, 1982, 2000), North (1991), Weingast (1995), Greif (2008), Besley and Persson (2011), Acemoglu and Robinson (2006, 2012), Gorodnichenko and Roland (2017), McCleary and Barro (2019), Aghion et al. (2021), and Morris (2022). The framework offered here can accommodate existing theories by incorporating economic efficiency into the model as a cause of changes in the economic conditions affecting alliance politics.

However, the pure economic focus also limits the explanatory power of existing theories. As sustained gains in productivity and economic growth are a modern

phenomenon, existing theories can better explain the rise of nations in recent centuries than that in the millennia of history. Neither can the pure economic focus help explain the rich experiences of nations, such as the sudden and swift rise of nations with diverse institutional and geographical conditions. The framework offered here complements existing theories by moving beyond the pure economic focus to study multidimensional – military, political, and economic – competition among nations for fresh insights on the topic.³

An academic predecessor of this paper is Mearsheimer’s realist theory of international politics, which focuses on security competition among great powers (Mearsheimer and Rosato, 2023). In a broader picture, foreign policy and alliance politics have been debated in the United States since the very beginning of the nation’s history. With many volumes of works devoted to it, the debate has involved realist, ideological, and emotional factors.⁴ This paper contributes to the debate a first-of-its-kind rigorous realist framework in which ideology can find a proper place. It provides an income-based rational explanation of the emotional side of the debate.

Another academic predecessor of this paper is the conceptual framework of Hirshleifer (1994, 1995) and North et al. (2009) for studying violence, which can trace its origins to Heraclitus of ancient Greece, who stated that “war is the father of all things” (Hirshleifer, 1994, p. 9), and Hobbes (1651), who considered a “war of all against all” as the default social order. This paper is an effort to operationalize their

³ A more general framework would discuss the relative importance of taking and making – that is, acquiring wealth by force and by being productive.

⁴ See Kissinger (1994), Zoellick (2020), and Mearsheimer and Rosato (2023) for reviews.

conceptual framework to study an important economic and political issue.

The rest of the paper is organized as follows. Section 2 lays out the model.

Sections 3 and 4 study how alliances are formed and their implications for the world order, respectively. Section 5 discusses historical, empirical, and current policy issues. Section 6 concludes the paper.

2. The model⁵

2.1. Players and moves

There are three nations or parties in the world: A , B , and S . A and B are each endowed with wealth $w_A = w_B = w_i \geq 0$, a war resource $0 < I_A = I_B = I_i \leq w_i$, and a war technology $\tau_A = \tau_B = \tau_i > 0$.⁶ S is endowed with wealth $w_S > 0$, a war technology $\tau_S > \tau_i$, and a war resource I_S , $w_S \geq I_S > I_i > 0$. A and B share the same culture and, for that reason, are referred to as fellow parties, while S is culturally distant from A and B .⁷

Wealth may be redistributed via war.⁸ Before launching a war, at $t = 0$, a party needs to invest $I_k > 0$ to build its capacity for war $F_k(I_k, \tau_k)$, where $k = A, B$, or S . The variable τ_k determines how effectively I_k is transformed into F_k . For simplicity without the loss of generality, assume that $F(I_k, \tau_k)$ has the explicit form of $F(I_k, \tau_k) = \tau_k I_k$. A

⁵ The methodology of this study is of game theory. See Fudenberg and Tirole (1991) and Gibbons (1992) on game theory.

⁶ The variable I_k could be the number of men in a society where all men were soldiers. τ_k is different when people fight with swords or guns, or under different leaders. In the model, w_A , w_B , and w_i are assumed equal and used interchangeably.

⁷ One might think of A and B as two Greek poleis and S as their centuries-old enemy Persia.

⁸ Redistribution of wealth by force is a starting point in political economists' discussion of property rights. See, for example, Barzel (1997).

party investing $I_k = 0$ resulting in $F_k(0) = 0$ is incapable of war and it will surely lose its wealth.⁹

After investing I_k , a party at $t = 1$ makes the alliance decision. Alliance A-B means that a joint force $F_{AB}(I_A, I_B)$ is built. Alliance A-S means that a joint force $\beta F_{AS}(I_A, I_S)$ is built, where $\beta > 0$ stands for the cross-cultural effect on Alliance A-S' capacity for war. Under these assumptions, the military capacity of the joint force is $F_{AB}(I_A, I_B) = \tau_i(I_A + I_B)$ for Alliance A-B and $\beta F_{AS}(I_A, I_S) = \beta(\tau_i I_A + \tau_s I_S)$ for Alliance A-S.¹⁰

The parties are risk-neutral wealth maximizers. Investing I_A and allying with B to war against S , A 's expected wealth is

$$E_{AB}(\pi_A) = p_{AB}[(w_A - I_A) + .5(w_S - I_S)] - (1 - p_{AB})(w_A - I_A) \quad (1)$$

Allying with S to war against B , A 's expected wealth is

$$E_{AS}(\pi_A) = p_{AS}[(w_A - I_A) + .5(w_B - I_B)] - (1 - p_{AS})(w_A - I_A) \quad (2)$$

In these equations, $0 \leq p_{AS}, p_{AB} \leq 1$ are the odds of winning; 0.5 is the assumed sharing rule; $[(w_A - I_A) + .5(w_S - I_S)]$ and $[(w_A - I_A) + .5(w_B - I_B)]$ are A 's respective wealth after winning the war against S or B , respectively; and $(w_A - I_A)$ is the amount of A 's wealth lost after losing the war.

S also has an alliance decision to make. If, after investing $I_s > 0$, it fights alone to rob both A and B , its expected wealth is

$$E_S(\pi_s) = p_s[(w_S - I_s) + 2(w_i - I_i)] - (1 - p_s)(w_S - I_s) \quad (3)$$

⁹ F_k is written as F when doing so does not cause confusion. See Bates et al. (2002), Smith et al. (2014), and Hirshleifer (1991) on military investments and technologies.

¹⁰ Alliance A-B or Alliance A-S here is a question of whether A and B are a unified or separated nations. Bolton and Roland (1997) study the topic from a different angle.

In this equation, $0 \leq p_s \leq 1$ is the odds that S can win over Alliance A - B , and the other terms are similar to their counterparts in equations 1 and 2.

If S allies with say A to fight against B , its expected wealth is

$$E_{SA}(\pi_S) = p_{AS}[(w_S - I_S) + 0.5(w_B - I_B)] - (1 - p_{AS})(w_S - I_S) \quad (4)$$

Following the convention in the literature of contest theory and with F in the form assumed above, the odds of winning a war can be defined as follows.¹¹

$$p_{AB} \equiv F_{AB}(I_A, I_B) / [F_{AB}(I_A, I_B) + F_S(I_S)] \quad (5a)$$

$$= \tau(I_A + I_B) / [\tau(I_A + I_B) + \tau_S I_S]$$

$$p_{AS} \equiv \beta F_{AS}(I_A, I_S) / [\beta F_{AS}(I_A, I_S) + F_B(I_B)] \quad (5b)$$

$$= \beta(\tau I_A + \tau_S I_S) / [\beta(\tau I_A + \tau_S I_S) + \tau I_B]$$

$$p_S \equiv F_S(I_S) / [F_{AB}(I_A, I_B) + F_S(I_S)] \quad (5c)$$

$$= \tau_S I_S / [\tau(I_A + I_B) + \tau_S I_S]$$

$$= 1 - p_{AB}$$

As is commonly recognized, the outcomes of wars are never certain. The specification of $E_{AB}(\pi_A)$, $E_{AS}(\pi_A)$, and $E_S(\pi_S)$ reflect the uncertainty.

The two-step game described above is summarized in Figure 1 in Appendix 1.

2.2. Equilibrium

The subgame perfect Nash equilibrium of the game is $\{I_k, A-i\}$, at which the expected wealth is maximized for all parties, where $I_k > 0$ or $I_k = 0$ and $A-i$ is Alliance

¹¹ This lotus probability form with the property of being homogeneous of degree zero is from the contest theory. See Corchón and Serena (2018) for an insightful review of the literature.

$A-B$ or Alliance $A-S$.¹² With $I_k > 0$ as the obvious dominant strategy at $t = 0$, the equilibrium of the game is one of the following two situations.

$$\{I_A, I_B, I_S; A-B\} \text{ or } \{I_A, I_B, I_S; A-S\}$$

Two pairs of inequalities determine the equilibrium of the game. A chooses Alliance $A-B$ if

$$E_{AB}(\pi_A) > E_{AS}(\pi_A) \quad (6)$$

A chooses Alliance $A-S$ when the inequality is reversed to $E_{AB}(\pi_A) < E_{AS}(\pi_A)$.

Similarly, S chooses to fight alone if

$$E_S(\pi_S) > E_{SA}(\pi_S) \quad (7)$$

When the inequality is reversed to $E_S(\pi_S) < E_{SA}(\pi_S)$, S wants to ally with another party, again say A .

In terms of the alliance relationships, S fighting alone and Alliance $A-B$ are equivalent concepts, which suggests $E_S(\pi_k) = E_{AB}(\pi_k)$. $E_{AS}(\pi_k)$ and $E_{SA}(\pi_k)$ are also equivalent concepts, meaning that $E_{AS}(\pi_k) = E_{SA}(\pi_k)$.

Ignoring the switching cost, $E_{SA}(\pi_S) = E_S(\pi_S)$ and $E_{AB}(\pi_A) = E_{AS}(\pi_A)$ are the tipping points at which the existing alliance relationships shift with a marginal change in the parameters.

Alliance $A-S$ is the equilibrium when it is in the best interests of both A and S , that is, when $E_{AS}(\pi_A) > E_{AB}(\pi_A)$ and $E_{AS}(\pi_S) > E_{AB}(\pi_S)$ both hold. Otherwise, Alliance $A-B$ is the equilibrium.

¹² An alternative modeling strategy is to assume $I_k = \alpha w_k$, $\alpha \leq 1$, and let I_k be a continuous variable. This alternative strategy complicates the study without adding much value to it.

3. A stable world order

3.1. The concept of a stable world order

A superpower-centric world has a stable order when A and B have more to gain from siding with S than fighting against S . Such a world emerges when

$$E_{AB}(\pi_i) < E_{AS}(\pi_i) \quad i = A, B \quad (8a)$$

Inequality $E_{AB}(\pi_i) < E_{AS}(\pi_i)$ means that a party is better off allying with S than with the other party. In such a world, A and B join Alliance $S-i$ whenever possible although they may be forced into an alliance against S .

After some mathematical operations, inequality 8a is reduced to the explicit form of 8b as follows.

$$p_{AB}/p_{AS} < 2.5/(2 + .5K) \quad (8b)$$

where $K = (w_S - I_S)/(w_i - I_i)$. Inequality 8b holds when the parameters are in the ranges stated in Proposition 1.

Proposition 1. The world order is stable when inequality 8b is satisfied, which is true when I_i , τ_i , and w_S are small or I_S , τ_S , w_i , and β are large.¹³

Inequality 8b is intuitively easy to understand. The fraction on its left-hand side is the comparative odds of winning for the two alliances, which reflect the comparative military capacities of the alliances. A prefers Alliance $A-S$ when the odds of Alliance $A-B$ winning are small relative to those of Alliance $A-S$, which is true when τ_k , I_k , and β have the stated values, where $k = A, B, S$.

¹³ All proofs are in Appendix 2.

On the right-hand side of inequality 8b, K measures the comparative levels of wealth of A and S . The condition suggests that the wealthier A is relative to S , the less likely A wants to fight S and challenge the existing world order.¹⁴ As a mind exercise, imagine an arbitrarily large $(w_i - I_i)$ or an arbitrarily small $(w_s - I_s)$ so that K is arbitrarily small. The right-hand side will be arbitrarily close to its maximal value of 1.25, making inequality 8b easily satisfied. When $(w_s - I_s) = 0$, A would not want to ally with B and challenge S even when Alliance $A-B$ has a better chance of winning than Alliance $A-S$.

It is worth repeating that a stable world does not rule out the possibility of Alliance $A-B$ as $E_{AB}(\pi_A) < E_{AS}(\pi_A)$ represents only A 's intention rather than actual Alliance $A-S$. For Alliance $A-S$ to actualize, it also needs to be the choice of S as explained in the next two subsections.

3.2. A stable world with two classes of nations

In a stable world, S may want to fight alone rather than ally with another party. This alliance policy divides the world into two classes of nations: S as the mighty and wealthy superpower and everybody else left weak and impoverished. With condition 8a for a stable world satisfied, a two-class world emerges when

$$E_s(\pi_s) > E_{sA}(\pi_s) \quad (9a)$$

By equations 3 and 4, $E_s(\pi_s) > E_{sA}(\pi_s)$ means that

$$p_s[(w_s - I_s) + 2(w_i - I_i)] - (1 - p_s)(w_s - I_s)$$

¹⁴ A conventional wisdom is that the rich support an existing social order. This result confirms that the wisdom applies to the world order.

$$> p_{AS}[(w_S - I_S) + .5(w_B - I_B)] - (1 - p_{AS})(w_S - I_S)$$

After some mathematical operations, this inequality is reduced to the explicit form of

$$p_S/p_{AS} > (2K + .5)/(2K + 2) \quad (9b)$$

Proposition 2 specifies the conditions under which this inequality is satisfied.

Proposition 2. With inequalities 8b and 9b both satisfied, the world is stable with two classes of nations: the superpower and everybody else. Inequalities 8b and 9b are both satisfied when β is of intermediate size; τ_S , I_S , and w_i are large; and τ_i , I_i , and w_s are small.

The fraction on the left-hand side of inequality 9b is, again, the relative war capacities of the parties, whereas the relative levels of wealth are on the right-hand side. Inequality 9b says that with the comparative levels of wealth given, S is more likely to fight alone when it is stronger compared to Alliance A - S . With the relative capacities for war given, S is more likely to fight alone when K is smaller, meaning that S is not so wealthy relative to A and B . Inequalities 8b and 9b suggest that β cannot be so small that A never wants to ally with S nor so large that S always wants to ally with A . At the same time, a large w_i suggests that A and B are ideal prey for S , and S would bear a high cost if it shared the wealth obtained from war with an ally. Finally, a small w_s makes S more willing to take the risk of fighting without an ally.

As mentioned in Footnote 1, probably the best examples of a two-class world are the European colonial experiences in the Americas. With large amounts of wealth found there – precious metals, land, and other highly valued goods – and very limited

military capacities, the colonies offered ideal conditions for a two-class world.¹⁵

3.3. A stable world with three classes of nations

Under certain conditions, a stable world may have three classes of nations: the superpower, its allies, and its enemies. In terms of the model in this paper, such a world emerges when the parameters are such that two inequalities are satisfied:

$E_{AS}(\pi_A) > E_{AB}(\pi_A)$ and $E_{AS}(\pi_S) > E_S(\pi_S)$, which means that S is better off allying with A and at the same time A is better off allying with S .

By equations 3 and 4, $E_{SA}(\pi_S) > E_S(\pi_S)$ means that

$$\begin{aligned} p_S[(w_S - I_S) + 2(w_i - I_i)] - (1 - p_S)(w_S - I_S) \\ < p_{AS}[(w_S - I_S) + .5(w_B - I_B)] - (1 - p_{AS})(w_S - I_S) \end{aligned} \quad (10a)$$

With some mathematical operations, inequality 10a can be reduced to the explicit form of

$$p_S/p_{AS} < (2K + .5)/(2K + 2) \quad (10b)$$

Proposition 3. When inequalities 8b and 10b are both satisfied, a stable world with three classes of nations emerges: S as the superpower, A as its ally, and B as its enemy. Inequalities 8b and 10b are both satisfied when β is large and the differences between w_S and w_i , I_S and I_i , and τ_S and τ_i are not too large.

It is apparent that inequality 10b is likely satisfied when β is large making the left-hand side of the inequality smaller. The difference between I_S and I_i or that between τ_S and τ_i cannot be so large as to make the left-hand side of 10b very large.

Inequality 10b is likely satisfied when K is large, making the right-hand side of

¹⁵ Adam Smith (1776) discussed how Europeans benefited from their American colonies.

the inequality larger. However, if K is too large, the condition for a stable world – i.e., inequality 8b – may be violated, so the difference between w_s and w_i should be large but not too large.

Examples of a world with three classes of nations are easy to find. It is a common state of the world for the reason that an incumbent superpower often does not have the military capacity to overwhelm all other nations.

4. Changes in the world order¹⁶

4.1. Two types of changes

An existing world order may change mildly or fundamentally depending on how the economic parameters change.

In a relatively mild change in the world order, the superpower shifts from having an ally to not having one, or does the reverse. Initiating such a change, S is better off from the change, but A worse off depending on if it gains or loses ally status.

A fundamental alteration of the world order occurs when A jumps from Alliance $A-S$ to Alliance $A-B$. When this happens, S wants to maintain its superpower status by dividing A and B to conquer one of them. However, the fellow parties A and B find it more rewarding to team up to topple the superpower. Such a change makes S worse off and the fellow parties better off.

These two types of changes are discussed in the next two subsections.

¹⁶ Dalio (2021) offers an insightful conceptual framework for studying the changing world order and backs it with detailed empirical information.

4.2. A mild change in the world order

The world would go through a relatively mild change when the loyalty of A to Alliance A - S remains unchanged but S wants to abandon or have A as its ally. To be precise, this means that inequality 8a, $E_{AB}(\pi_i) < E_{AS}(\pi_i)$, remains satisfied but $E_S(\pi_S) < E_{SA}(\pi_S)$ shifts to $E_S(\pi_S) > E_{SA}(\pi_S)$ or the reverse happens. The conditions under which such a shift occurs are explained in Proposition 4.

Proposition 4. There are parameters at which $E_S(\pi_S) = E_{SA}(\pi_S)$ while $E_{AS}(\pi_i) > E_{AB}(\pi_i)$.

At this point, the world may shift from an order of three classes of nations to an order of two classes of nations with a marginal increase in w_i , I_S , or τ_S , or a marginal decrease in w_S , β , I_i , or τ_i . The shift makes S and B better off at the cost of A . An opposite shift occurs when these parameters change in the opposite directions.

The stated changes in β , I_i , τ_i , I_S , and τ_S imply an increased military capacity of S relative to that of A and B , resulting in a larger p_S/p_{AS} ratio. At the same time, a smaller w_S or a larger w_i makes S weigh more on robbing than preserving its own wealth and less willing to share newly acquired wealth with an ally, resulting in its decision to abandon A as its ally.

4.3. A fundamental change in the world order

In a stable world, Alliance A - B may exist not because A and B want to ally against S but because S pushes them together. This happens when inequalities 8b and 9b both hold. With its own interests better aligned with S than with B – that is, with $E_{AB}(\pi_A) < E_{AS}(\pi_A)$, A abandons Alliance A - B whenever it has a chance to join Alliance A - S . Such

a chance emerges when $E_S(\pi_S) > E_{SA}(\pi_S)$ is reversed to $E_S(\pi_S) < E_{SA}(\pi_S)$.

The world becomes fundamentally unstable when the inequalities guiding A 's alliance policy change – that is, when $E_{AB}(\pi_A) < E_{AS}(\pi_A)$ is reversed to $E_{AB}(\pi_A) > E_{AS}(\pi_A)$ or, equivalently, when $p_{AB}/p_{AS} < 2.5/(2 + .5K)$ is replaced by

$$p_{AB}/p_{AS} > 2.5/(2 + .5K) \quad (11)$$

The important thing about inequality 11 is that it makes A and B whole-heartedly determined to team up in fighting against S , resulting in a solid Alliance A - B that cannot be broken by S with a change in its alliance policy.

Proposition 5 explains when and how this may happen.

Proposition 5. There are parameters at which $E_{AB}(\pi_i) = E_{AS}(\pi_i)$ and $E_{SA}(\pi_S) > E_S(\pi_S)$. At this point, A abandons Alliance A - S for Alliance A - B if K , I_i , or τ_i becomes marginally larger or β , I_S , or τ_S becomes marginally smaller. Such a change makes A and B better off at the cost of S .

A numerical example is provided in Appendix 2, after the Proof of Proposition 5, in which the parameters are such that $E_{AB}(\pi_i) = E_{AS}(\pi_i)$ and $E_{SA}(\pi_S) > E_S(\pi_S)$ both hold, meaning that S prefers Alliance A - S while A is indifferent. At this point, a marginal increase in K would make A jump from Alliance A - S to Alliance A - B while S remains loyal to Alliance A - S .

The conditions under which various world orders would emerge, as stated in Propositions 1 through 5, are summarized in Table 1 in Appendix 1.

5. History, empirics, and policies

5.1. History

The theory of alliance politics presented here draws heavily from historical experiences such as those of the Ancient Greeks and the Mongols. This subsection reviews these experiences.

The Ancient Greeks.¹⁷ The Ancient Greeks shined in history with many contributions to human civilization in politics, philosophy, and other aspects. In the fourth century BC, the Greeks defeated and replaced the Persians as the new superpower. The Hellenistic Empire established by them covered vast territories in Asia, Africa, and Europe. An aspect of the Greeks' experience that was quite striking is that they toppled the Persian Achaemenid Dynasty, ending its 220 years of history, via a war that lasted only four years from 334 to 330 BC.

Equally striking is how divided the Greeks were in the centuries before then. The Greek history started in 1200 BC and, by the 8th century BC, the Balkan Peninsula had become crowded with hundreds of poleis as the basic political units. The poleis fought each other for political, economic, and various other reasons. The infighting stopped from 490 to 479 BC during the war against the invading superpower Persia, and resumed shortly after the war. The infighting eventually escalated to two all-out wars fought from 460 to 445 BC and from 431 to 404 BC. Those wars were between two leagues that involved nearly all the poleis, the Delian League led by Athens and the Peloponnesian League led by Sparta. The wars ended with a defeated and ruined

¹⁷ This review draws heavily from Pavlac (2011), Furtado (2013), and Shiono (2022).

Athens, a weakened Sparta, and impoverished other poleis. Persia encouraged the infighting with financial and military support to some poleis.

The infighting among the Greeks largely ended in 337 BC when Philip II, King of Macedon, founded the Corinthian League, with all the poleis (except Sparta) joining the League and committing to peace among them. In 336 BC, Alexander, the son and successor of King Philip II, made all the member poleis re-confirm their commitment to the League. A final round of infighting occurred in 335 BC when Alexander led the League to crush the rebellious poleis Thebes.

The Greeks had previously established leagues and signed contracts to stop their infighting. All these efforts failed until this time with Persia identified as the common and utmost enemy of all Greeks and an empire to be eradicated from Earth.

With the League behind him, Alexander launched the war against Persia in 334 BC and ended its more 200-year history in four years.

The war against Persia made Alexander the wealthiest man on Earth. Before the war, he was essentially bankrupt, with 70 talents of gold in his treasury and a debt of 1300 talents. Yet, he needed 300 talents annually to maintain his military forces. For a quick comparison, Athens in its peak years had 6,000 talents of gold in reserve and an annual inflow of 1,000 talents.

The war against Persia solved Alexander's financial problems. He seized 3,000 talents of gold in Issus, 18,000 talents in Susa, and many more in other cities. He shared the newly acquired wealth generously with his soldiers from different poleis.

His generosity made it easy for him to recruit new soldiers when he rotated out the veterans who became rich and wanted to return home.

The Mongol Empire.¹⁸ More than one and a half millennia after Alexander's eastward march to establish the huge Hellenistic Empire, in the thirteenth century, the Mongols repeated the experience of starting in a corner of the world and becoming a superpower in a short period of time. The corner in which they started was at the other end of the world – in the cold and barren Northeastern Asia – and their march was westward.

Before Genghis Khan officially started his dynasty in 1206, the Mongols had been ruled by two successive regional superpowers for about three centuries, the Liao Dynasty (907–1125) and the Jin Dynasty (1115–1234). In addition to taxing and randomly robbing the Mongols, the Jin also regularly killed Mongol men and boys to control the size of their male population, which was one of the most important military resources of that time. The Jin had some Mongol tribes as their allies and others as their targets.

After he became the ruler of the Mongols in 1206, Genghis Khan spent a few years to crush sporadic rebellions here and there. In 1211, he led an army that was about one tenth of the size of the Jin army to start the war against the Jin. By 1214, the Mongols had made Jin army an insignificant, no-threat existence and taken vast territories from the Jin, including its capital, Beijing. The Jin Dynasty survived for two more decades as the Mongols turned their formidable force to other conquests. In

¹⁸ This review draws heavily from Christian (1998) and Sun (2009).

1219 and the decades followed, the Mongols marched westward through Central and Western Asia deep into Europe to establish the largest empire ever in human history.

The Mongols killed along the way as they did not have enough manpower to secure all the cities and lands that they conquered. In the centuries that followed, they ruled much of the world with power and wealth that no other nation could match.

Comments. The Greek and Mongol experiences are not isolated. The Persia Empire, the Jin Dynasty, and other superpowers went through a similar process in their own histories. They had been divided and oppressed before a genius leader emerged to unite and lead the people to fight and topple an oppressive external power.

It would be difficult to explain the Greek or Mongol experience by changes in their centuries-old cultural, political, and economic institutions or geographical conditions. However, the model in this paper suggests that the world order can be pushed to the tipping point and changed drastically and swiftly by a marginal change in the military technology in favor of the oppressed nation or a marginal change in wealth in favor of the incumbent superpower. In the Greek case, King Philip II's effort to improve the phalanx and the weaponry and King Alexander's new Cavalry tactics may well have been among the marginal changes in favor of the Greeks. At the same time, with the wealth accumulated by the Persians increasing over time, the Greeks would eventually realize that the best way to enrich themselves was to team up to rob the Persians. This belief was proven true.

In the case of the Mongols, the theory developed in this paper suggests looking

into how the military tactics introduced by Genghis Khan changed the balance on the battlefields in favor of the Mongols, and how the wealth that the Jin accumulated over time motivated the Mongols for a war against the Jin. Again, the theory in this paper suggests that with these changes accumulating and eventually passing the tipping point, it could be expected that someday the Mongols would stop their infighting and team up to rob the Jin.

5.2. Empirical predictions

The theory of the world order developed in this paper posits the following empirical predictions.

1. When a superpower is overwhelmingly strong or not so wealthy, it does not want allies. Thus the world has two classes of nations: the superpower and everybody else. Otherwise, the superpower would have allies, resulting in a world of three classes of nations: the superpower, its allies, and its enemies.
2. When a superpower is sufficiently wealthy but not sufficiently strong, other nations are likely to team up against and prey on the superpower.
3. Whatever change is occurring in the world order – whether the superpower is abandoning its allies or the allies are abandoning the superpower – it is likely a swift and abrupt process following a period of stability. The reason for this is that at a tipping point, but not before, a marginal change in economic conditions can cause sweeping changes in alliance relationships.
4. A leading indicator of a toppling world order is that other nations, first and

foremost those that are culturally close, stop fighting each other and join in an alliance against the incumbent superpower.

5. There are three direct causes of war that changes the world order: culture, strength, and wealth.

Culture-induced wars occur when cross-cultural cooperation becomes very difficult, meaning that in terms of the model developed in this paper, β for some reason becomes very close to zero. In this type of war, culturally close nations form alliances against a culturally distant power. This observation suggests that the rise of a powerful religion or ideology is a leading indicator of an unstable world order.¹⁹

Strength-induced wars occur when a change in I_i , τ_i , I_s , or τ_s results a change in nations' relative military capacities. Proposition 5 suggests that, at the critical point, a marginal change in these parameters in favor of A and B may encourage them to use their newly gained military strength to topple the existing world order. This observation suggests that breakthroughs in military technologies are a leading indicator of an unstable world.

Greed-induced wars occur when wealth becomes so unevenly distributed that poor nations would be motivated to launch a war against a wealthy nation. This is the reason for S to fight both A and B when w_i is large and w_s is small. It is also the reason for A and B to fight S even though the odds of winning may be small, as a sufficiently large prize makes the risk worth taking. It is an empirical question where the threshold is above which wealth concentration encourages other nations to destabilize

¹⁹ Huntington (1993), his supporters and critics extensively discussed this topic.

the existing world order.

5.3. Policy issues

The findings of this study have direct policy implications, in light of which some policies can be reviewed.

Policies toward challengers. The study suggests the need for the superpower to control a challenger's access to military resources and technologies to ensure its own military competitiveness and the stability of the existing world order. In ancient times when men were one of the most important war resource, many nations had the policy of directly controlling the number of men of an enemy nation.

In modern times, controlling a challenger's access to war resources and technologies remains a common policy practice. A key difference, however, is that economic efficiency plays a far more important role in determining a nation's wealth and military strength. Economic efficiency is also a much more decisive variable for a superpower to maintain its military dominance in the existing world order.

How to choose allies. The study suggests that an incumbent superpower does not always need allies when it is militarily very strong but economically not so wealthy compared to other nations. Under these conditions, the superpower is better off acting alone and ignoring the interests and moves of the allies.

However, when a superpower is not so strong and at the same time wealthy relative to other nations, it is better off having allies. When a superpower has allies, it

has more to gain by allying with strong and poor nations to prey on weak and wealthy nations.

Policies to preserve a stable world order. At the critical point $E_{AB}(\pi_A) = E_{AS}(\pi_A)$ or, equivalently, $p_{AB}/p_{AS} = 2.5/(2 + .5K)$, a marginal increase in w_s resulting in a marginal increase in K would destabilize the world order. At the same time, a marginal decrease in w_s resulting in a marginal decrease in K would restore the stability of the world order. This finding has the policy implication that a way to stabilize an existing world order is for S to transfer some of its wealth to its allies, as the United States did through the Marshall Plan in the years after World War II.

Complicating international politics among allies, however, is the sentiment of domestic politics. In the decades after World War II, both the United States and the Soviet Union voluntarily adopted many “unfair” policies that favored their respective allies at the cost of their own peoples, such as financial transfers, unequal market access, and unequal shares of military spending. As their allies recovered from the ruins of the war and seem to have reached high living standards, the people of the Soviet Union, especially the Russians, eventually became very unhappy with and resentful of policies that were favorable to their allies. This popular sentiment contributed to the eventual collapse of the Soviet Union by the end of the last century. Such “unfair” policies lasted longer between the United States and its allies, and added to the inequality problem associated with economic globalization discussed by Maskin (2015) and others. Over time and for the same reason, such policies have also

become a source of popular resentment. The Trump administration's efforts to reverse them seem to reflect this popular sentiment, although they may be destabilizing to the existing world order.

6. Conclusion

Alliance politics has been central in foreign policy debates in the United States since the founding of the nation. Ongoing changes in the world order are adding new fuel to the centuries-old debate. This paper contributes to the debate by offering a rigorous framework for understanding the economic rationale of alliance decisions.

The value of the theory developed in this paper can be appreciated by recognizing the fact that, in much of the millennia of civilized history, nations existed in a superpower-centric world. In such a world, alliance relationships, particularly those with or against a superpower, are critical for determining the wealth and even the survival of nations. To understand the rise of nations in such a world, it is necessary to study how nations form alliances, how alliances change, and how such changes affect the world order.

The framework developed in this paper aimed to address these questions. The modeled game starts with the recognition that nations want wealth and are ready to use force to acquire it. It reveals that, two sets of variables are critical for understanding how war and alliance politics determine the rise or decline of nations: the relative military capacities and the relative levels of wealth, which determine the

odds of winning and the expected gains via war, respectively.

This mechanism of how the war and wealth variables impact alliance decisions is summarized by two pairs of inequalities. Between the alliance regimes given by these inequalities, two equations describe the tipping points at which a marginal change in the parameters causes a drastic change in the world order, with alliance relationships changed and an incumbent superpower possibly toppled.

The theory offered here move beyond the focus on pure economic efficiency to study multidimensional competition among nations and the resulting world order. It explains the rise of nations with diverse institutional and geographical orientations throughout history. The theory is an effort to operationalize the conceptual framework of Hirshleifer (1994, 1995) and North et al. (2009) to study violence.²⁰ It sheds light on important empirical and policy issues.

The study can be extended for a deeper and more comprehensive understanding of the rise of nations. On the empirical front, although the paper provided reviews and discussion of historical experiences and ongoing events to support the theory, they are not substitutes for rigorous empirical study.

The theoretic construct should also be extended and enriched. Future studies should consider the trade-off between taking wealth, as emphasized in this paper, and creating wealth, as emphasized in efficiency-oriented theories. This requires recognizing that a party can acquire wealth by robbing others or by being productive itself. In modern times, being productive through economic cooperation is far more

²⁰ See Wang (2012, 2017, 2022, 2024) for more works to operationalize the framework.

important than it was in ancient times. Thus, forming alliance is a decision that affects not only the static military capacities, but also the dynamics of how these capacities change over time. Although wealthy nations may be better grabbing targets from which a superpower may obtain immediate gains in wealth, they are also likely more productive partners in economic cooperation for long-term gains. That said, it is important to recognize that taking – that is, acquiring wealth by force – is not an antique but still a common and significant phenomenon in the present world. In ancient times and modern times alike, it can be implemented in many forms, such as outright war, negotiated arrangements under military threats, or institutionalized rules in favor of some parties at the cost of other parties.

Future studies should also recognize that the modern world order is built on complex webs of financial, scientific and technological, trade, human, education, and military relationships among nations as observed by Li and Maskin (2021) and Dalio (2021). These complex webs are far more important for both economic efficiency and military capacities in modern times than they were in ancient times. They need to be carefully and systematically studied for a better understanding of the modern world order. While Dalio's (2021) conceptual framework and quantitative analyses are an extremely valuable step, more rigorous analytical frameworks can add momentum to research along this line.

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Appendix 1

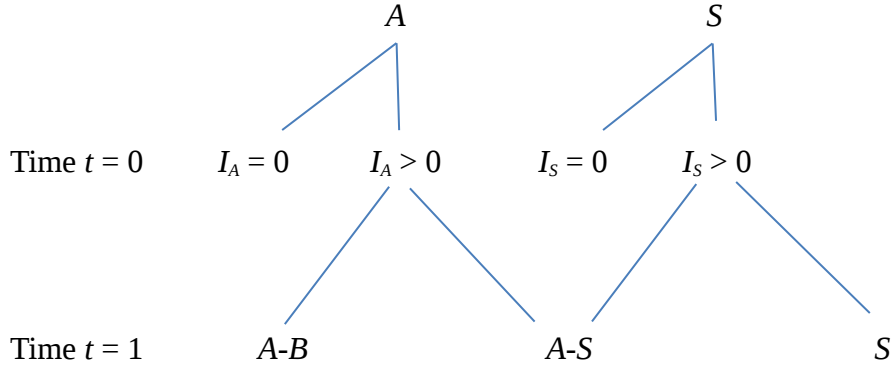


Figure 1: The moves of A and S.

Table 1: Alliance decisions by S and A and resulting world orders

S: Ally or not? A: S or B?	$E_S(\pi_S) > E_{SA}(\pi_S)$ $p_S/p_{AS} > (2K+.5)/(2K+2)$ Superpower without allies	$E_S(\pi_S) = E_{SA}(\pi_S)$ $p_S/p_{AS} = (2K+.5)/(2K+2)$ Tipping point	$E_S(\pi_S) < E_{SA}(\pi_S)$ $p_S/p_{AS} < (2K+.5)/(2K+2)$ Superpower wants an ally
$E_{AB}(\pi_i) < E_{AS}(\pi_i)$ $p_{AB}/p_{AS} < 2.5/(2 + .5K)$ A wants Alliance A-S	Stable world with two classes of nations.	Switching between two classes & three classes of nations.	Stable world with three classes of nations.
$E_{AB}(\pi_i) = E_{AS}(\pi_i)$ $p_{AB}/p_{AS} = 2.5/(2 + .5K)$ Tipping point	Switching between a stable & an unstable world order.	X	Switching between a stable & an unstable world order.
$E_{AB}(\pi_i) > E_{AS}(\pi_i)$ $p_{AB}/p_{AS} > 2.5/(2 + .5K)$ A wants Alliance A-B	Unstable world order.	Unstable world order.	Unstable world order.

Appendix 2: Proofs of the Propositions

Proof of Proposition 1. The world is stable when $E_{AS}(\pi_A) > E_{AB}(\pi_A)$, that is, when

$$\begin{aligned} p_{AS}[(w_A - I_A) + .5(w_B - I_B)] - (1 - p_{AS})(w_A - I_A) \\ > p_{AB}[(w_A - I_A) + .5(w_S - I_S)] - (1 - p_{AB})(w_A - I_A) \end{aligned}$$

Let $K = (w_S - I_S)/(w_i - I_i)$, $0 < K < \infty$. Dividing both sides of the inequality and

rearranging to obtain

$$\begin{aligned} p_{AS}(1 + .5) - (1 - p_{AS}) &> p_{AB}(1 + .5K) - (1 - p_{AB}) \\ 2.5p_{AS} &> p_{AB}(2 + .5K) \\ p_{AB}/p_{AS} &< 2.5/(2 + .5K) \end{aligned}$$

It is straightforward to confirm that the inequality holds when I_i , τ_i , and w_S are small or when, I_S , τ_S , w_S , and β are large. Q.E.D.

Proof of Proposition 2. A two-class world prevails when $E_S(\pi_A) > E_{AS}(\pi_A)$, that is, when

$$\begin{aligned} p_S[(w_S - I_S) + 2(w_i - I_i)] - (1 - p_S)(w_S - I_S) \\ > p_{AS}[(w_S - I_S) + .5(w_B - I_B)] - (1 - p_{AS})(w_S - I_S) \end{aligned}$$

Again, let $K = (w_S - I_S)/(w_i - I_i)$, $0 < K < \infty$. Dividing both sides of the inequality and

rearranging to obtain

$$\begin{aligned} p_S(K + 2) - (1 - p_S)K &> p_{AS}(K + .5) - (1 - p_{AS})K \\ p_S/p_{AS} &> (2K + .5)/(2K + 2) \end{aligned}$$

A final check is that this condition can be satisfied in the stable world, which requires $p_{AB}/p_{AS} < 2.5/(2 + .5K)$. Both inequalities are indeed satisfied when β is of

intermediate size, τ_S large, and τ_A small so that p_S/p_{AS} is large while p_{AB}/p_{AS} is small. For example, when $K = 0$, a two-class world emerges with $p_S/p_{AS} > 0.25$ and $p_{AB}/p_{AS} < 1.25$.

Proof of Proposition 3. A three-class world emerges when $E_{AS}(\pi_A) > E_{AB}(\pi_A)$ and $E_{SA}(\pi_S) > E_S(\pi_S)$.

$E_{AS}(\pi_A) > E_{AB}(\pi_A)$ means

$$\begin{aligned} p_{AS}[(w_A - I_A) + .5(w_B - I_B)] - (1 - p_{AS})(w_A - I_A) \\ > p_{AB}[(w_A - I_A) + .5(w_S - I_S)] - (1 - p_{AB})(w_A - I_A) \end{aligned}$$

Dividing both sides by $(w_i - I_i)$ to obtain

$$p_{AS}(1 + .5) - (1 - p_{AS}) > p_{AB}(1 + .5K) - (1 - p_{AB})$$

The inequality after rearranging can be written as

$$2.5p_{AS} > p_{AB}(2 + .5K)$$

$$p_{AB}/p_{AS} < 2.5/(2 + .5K)$$

$E_{SA}(\pi_S) > E_S(\pi_S)$ means

$$\begin{aligned} p_S[(w_S - I_S) + 2(w_i - I_i)] - (1 - p_S)(w_S - I_S) \\ < p_{AS}[(w_S - I_S) + .5(w_B - I_B)] - (1 - p_{AS})(w_S - I_S) \end{aligned}$$

Dividing both sides by $(w_i - I_i)$ to obtain

$$p_S(K + 2) - (1 - p_S)K < p_{AS}(K + .5) - (1 - p_{AS})K$$

which after rearranging can be written as

$$p_S(2K + 2) < p_{AS}(2K + .5)$$

$$p_S/p_{AS} < (2K + .5)/(2K + 2)$$

The two inequalities, $p_{AB}/p_{AS} < 2.5/(2 + .5K)$ and $p_S/p_{AS} < (2K + .5)/(2K + 2)$, are both satisfied when β is sufficiently large as stated in the proposition.

It is difficult to satisfy both inequalities at the same time when K is too small or too large. When $K = 0$, the right-hand side of the second inequality $[(2K + .5)/(2K + 2)] = 0.25$, which is its lower limit and makes the inequality not as easy to satisfy as when K is at other values. When $K \rightarrow \infty$, the right-hand side of the first inequality $[2.5/(2 + .5K)] \rightarrow 0$, which is its lower limit and hard to satisfy. The fact that $2.5/(2 + .5K)$ is decreasing while $(2K + .5)/(2K + 2)$ is increasing in K implies that it is more difficult to satisfy both inequalities when K is extremely large or small. K is large or small when w_S and w_i are far apart.

Similarly, it is difficult to satisfy both inequalities when I_S and I_i or τ_S and τ_i are far apart resulting in extreme values of p_{AB}/p_{AS} and p_S/p_{AS} . Q.E.D.

Proof of Proposition 4. To confirm that there are parameter values at which $E_{SA}(\pi_S) = E_S(\pi_S)$, recall Propositions 2 and 3, which confirm that there are parameters at which $E_{SA}(\pi_S) > E_S(\pi_S)$ and also parameters at which $E_{SA}(\pi_S) < E_S(\pi_S)$. With all parameters being continuous, there must be parameter values at which the equal sign holds.

Also confirmed by Propositions 2 and 3 is the fact that there are parameter values at which $E_{AB}(\pi_i) < E_{AS}(\pi_i)$ when $E_{SA}(\pi_S) > E_S(\pi_S)$ or $E_{SA}(\pi_S) < E_S(\pi_S)$. It follows that there are parameter values at which $E_{SA}(\pi_S) = E_S(\pi_S)$ and $E_{AB}(\pi_i) < E_{AS}(\pi_i)$.

Proof of Proposition 5. It is easy to confirm that there are parameter values at which $E_{AB}(\pi_i) < E_{AS}(\pi_i)$ and also parameter values at which $E_{AB}(\pi_i) > E_{AS}(\pi_i)$. As all parameters are continuous, there must be parameter values at which $E_{AB}(\pi_i) = E_{AS}(\pi_i)$.

What needs to be confirmed next is that when the parameters are such that $E_{AB}(\pi_i) = E_{AS}(\pi_i)$ or $E_{AB}(\pi_A) > E_{AS}(\pi_A)$, inequality $E_{AS}(\pi_S) > E_S(\pi_S)$ holds so that Alliance A-B is not what S wishes to see.

Proposition 3 tells us that $E_{AB}(\pi_A) > E_{AS}(\pi_A)$ is equivalent to $p_{AB}/p_{AS} > 2.5/(2 + .5K)$ and $E_{AS}(\pi_S) > E_S(\pi_S)$ is equivalent to $p_S/p_{AS} < (2K + .5)/(2K + 2)$. In other words, when

$$p_{AB}/p_{AS} > 2.5/(2 + .5K)$$

$$p_S/p_{AS} < (2K + .5)/(2K + 2)$$

A and B would team up to topple the world order while S prefers Alliance A-S.

The right-hand side of the first inequality decreases in K while the right-hand side of the second inequality increases in K , suggesting that a sufficiently large K can make the two inequalities satisfied at the same time, provided that the other parameters – that is, I_i , τ_i , I_S , τ_S , and β – do not have extreme values.

For a concrete example, consider a case where $\tau = \tau_S$, $I_S = 2I_A$, and $\beta = 1$. These parameter values suggest that

$$\begin{aligned} p_{AB} &\equiv F_{AB}(I_A, I_B)/[F_{AB}(I_A, I_B) + F_S(I_S)] \\ &= \tau(I_A + I_B)/[\tau(I_A + I_B) + \tau_S I_S] \\ &= 2\tau I_A/(2\tau I_A + 2\tau I_A) \\ &= 1/2 \end{aligned} \tag{5a}$$

$$p_{AS} \equiv \beta F_{AS}(I_A, I_S) / [\beta F_{AS}(I_A, I_S) + F_B(I_B)] \quad (5b)$$

$$= \beta(\tau I_A + \tau_S I_S) / [\beta(\tau I_A + \tau_S I_S) + \tau I_B]$$

$$= 3\tau I_A / (3\tau I_A + \tau I_B)$$

$$= 3/4$$

$$p_S \equiv F_S(I_S) / [F_{AB}(I_A, I_B) + F_S(I_S)] \quad (5c)$$

$$= \tau_S I_S / [\tau(I_A + I_B) + \tau_S I_S]$$

$$= 1 - p_{AB}$$

$$= 1/2$$

This means that $p_{AB}/p_{AS} = p_S/p_{AS} = 2/3$. It follows that $E_{AB}(\pi_A) > E_{AS}(\pi_A)$ and $E_{AS}(\pi_S) >$

$E_S(\pi_S)$ – that is, $p_{AB}/p_{AS} > 2.5/(2 + .5K)$ and $p_S/p_{AS} < (2K + .5)/(2K + 2)$ become

$$2/3 > 2.5/(2 + .5K) \rightarrow 0, \text{ when } K \rightarrow \infty$$

$$2/3 < (2K + .5)/(2K + 2) \rightarrow 1, \text{ when } K \rightarrow \infty$$

Both inequalities are satisfied when $K = (w_S - I_S)/(w_B - I_B)$ is sufficiently large.

With $\tau = \tau_S$, $I_S = 2I_A$, and $\beta = 1$, $E_{AB}(\pi_A) = E_{AS}(\pi_A)$ or, equivalently, $2/3 = 2.5/(2 + .5K)$

when $K = 7.5 - 4 = 3.5$.

Verify that, when $K = 3.5$, $(2K + .5)/(2K + 2) = 7.5/9 > 6/9$, meaning that $E_{AS}(\pi_S) > E_S(\pi_S)$ holds. This implies that, at $K = 3.5$, S prefers Alliance A-S while A is indifferent. It means that the world is stable when $K < 3.5$ but not when $K > 3.5$ and, with all other parameters remaining constant, a marginal increase in K would shift A from Alliance A-S to Alliance A-B while S remains loyal to Alliance A-S.