Cooperation and Asymmetries in Common-Pool Resources

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Preliminary

Abstract

This paper examines common-pool resource, such as an irrigation system, that is asymmetric in the sense that the headender has a first mover advantage in appropriating water. The headender could fill their fields with water, leaving little for the tailender. However, irrigation system needs maintenance and the headender may leave more water for the tailender to incentivize him to participate in maintenance. I show that first best symmetric cooperation, where the two parties share the water equally and make equal investments in maintenance, is self-enforcing when the headender's appropriation capacity is moderate and maintenance is costly, which are typical conditions in traditional irrigation systems with long track record of well-functioning cooperation. If symmetric cooperation is not sustainable, the headender's incentives can be improved by allocating her more than equal share of the water.

1 Introduction

This paper examines a common-pool resource, such as an irrigation system, that is asymmetric in the sense that the headender has a first mover advantage in appropriating water. The headender could fill their fields with water, leaving little for the tailender. However, irrigation system needs maintenance and the headender may leave more water for the tailender to incentivize him to participate in maintenance. Ostrom and Gardner (1993) analyzed such an asymmetric setup as a static bargaining problem where irrigators decide on appropriation and provision rules in an annual meeting. This paper takes the relational contracts approach and examines self-enforcing appropriation and provision rules, taking particular interest in when symmetric cooperation is incentive compatible.

The paper starts by examining symmetric cooperation where the two parties share the water equally – despite their asymmetric positions – and contribute equally to maintenance. Symmetric cooperation is common in traditional irrigation systems where water is shared on rotation basis and households have equal maintenance obligations. I show that symmetric first best cooperation is self-enforcing when, in addition to large enough discount factor, the headender's appropriation capacity is moderate and the maintenance of the irrigation system is costly. Both factors reduce the headender's temptation to deviate from the relational contract. Both factors are also typical characteristics of traditional irrigation systems.

If symmetric first best cooperation is not self-enforcing, the headender's incentives can be improved by allocating her more than half of the water in the relational contract. However, this will weaken the tailender's incentives to participate in maintenance. I solve for the largest share of water that can go to the headender without violating the tailender's incentive compatibility constraint. The share is adjusted to headender's appropriation capacity, so that first-best cooperation is incentive compatible as long as maintenance is sufficiently costly.

Table 1 summarizes the results. FB refers to first best investments and SB to second best investments.

	High-cost $c(y) = y^3$	Low-cost $c(y) = y^{\frac{3}{2}}$
Patient $\delta = 0.7$	Symmetric and asymmetric FB	Symmetric SB, Asymmetric FB
Impatient $\delta = 0.45$	Asymmetric FB	Asymmetric SB

Table 1

Briefly noting some related literature, Halonen-Akatwijuka (2024) examines incentives to cooperate in maintenance of a public good. Here I introduce appropriation of common-pool resource and how it interacts with maintenance incentives. Harstad et al. (2019) analyzes the interaction of technology and incentives to cooperate in the context of environmental treaties. Ramey and Watson (1997) and Halac (2015) examine the effect of one-off specific investment on the incentives to cooperate in an employment relationship. This paper analyzes repeated investments.

2 The model

Two agents $i = 1, 2$ share a common-pool resource. In the *investment phase* each agent makes an investment y_i in the common-pool resource, e.g. maintains an irrigation system. The value of the common-pool resource is $\theta(y_1 + y_2)$, where $\theta > 0$. Investment costs are given by $c(y_i) = (y_i)^{\frac{1}{\alpha}}$, where $\alpha \in (0, 1)$. Agent i's investment is observable to the other agent[1], but the investment and the value of the common-pool resource are not verifiable.[2]

In the *appropriation phase* the agents move sequentially. For example, agent 1 is the headender of the irrigation system and appropriates share s_1 of the water, leaving share $s_2 = 1 - s_1$ to agent 2, the tailender.[3] The main focus of this paper is to examine how such asymmetry in appropriation affects the incentives to cooperate.

[1]The communities typically gather to maintain the common-pool resource in specific days and attendance is observable.

[2]Alternatively, the common-pool resource is in a developing country with weak institutions, so that contracts contingent on the investment or the value of the resource cannot be enforced by court.

[3]Similar situation can arise e.g. in a hierarchy where the boss decides on the resource allocation, while the value of resources available depends on both the boss's and the employee's effort.

The agents utilities are

$$u_i = s_i\theta(y_1 + y_2) - c(y_i). \tag{1}$$

The maximum share agent 1 can appropriate is κ_1. For example, agent 1 has the capacity to capture κ_1 of the flowing water in the irrigation system.

Assumption 1. $\kappa_1 \in (\frac{\sqrt{5}-1}{2}, 1)$.

Assumption 1 simplifies the analysis. Its role will become clear in the course of the paper. Note that $\frac{\sqrt{5}-1}{2} \approx 0.62$.

Joint surplus, S, equals

$$S = \theta(y_1 + y_2) - c(y_1) - c(y_2). \tag{2}$$

Thus, the first-best investments, $y_1^* = y_2^* \equiv y^*$, are given by

$$\theta - c'(y^*) = 0. \tag{3}$$

2.1 One-shot game

We can solve the one-shot game by backward induction, starting from the appropriation phase. Agent 1 maximizes her utility by appropriating the maximal κ_1 of the common-pool resource, leaving the remaining $\kappa_2 = 1 - \kappa_1$ to agent 2. Thus, in the investment phase the agents maximize

$$u_i(y_1, y_2) = \kappa_i\theta(y_1 + y_2) - c(y_i), \tag{4}$$

resulting in investments y_i^N given by

$$\kappa_i\theta - c'(y_i^N) = 0. \tag{5}$$

According to (3) and (5), the agents underinvest relative to first best, $y_i^N < y^*$, due to ignoring the positive externality on the value of the common-pool resource appropriated by the other agent. In other words, the agents freeride in maintenance.

2.2 Repeated game

The stage game of the repeated game includes a *transfer phase* before the investment and appropriation phases. In the transfer phase the agents can make voluntary monetary transfers to each other. In particular, if agent i has deviated in the previous period, he can pay a fine F_i to agent j and renegotiate back to cooperation.

The stage game is repeated infinitely and the agents have have a common discount factor of $\delta \in (0, 1)$. Note that the investments depreciate fully before the beginning of the subsequent period, e.g. the maintenance investments are washed away in monsoon rains.

This paper examines the nature of cooperation in the repeated game. Cooperation is *symmetric* if the agents share the common-pool resource equally, $s_1^c = s_2^c = \frac{1}{2}$, and make equal investments, $y_1^c = y_2^c$, where superscript c refers to cooperation. Alternatively, cooperation can be *asymmetric* so that either the resource is not shared equally, $s_1^c \neq s_2^c$, or the agents do not make equal investments $y_1^c \neq y_2^c$.

3 Relational contract

The agents form a relational contract, agreeing to make cooperative investments (y_1^c, y_2^c) and to share the common-pool resource according to (s_1^c, s_2^c). In essence, agent 1 promises to take less than κ_1 if the common-pool resource, $s_1^c < \kappa_1$, and both agents promise not to freeride in maintenance, $y_i^c > y_i^N$. Furthermore, fines (F_1, F_2) are determined for any deviation from cooperative investments or sharing.[4] The assumption this paper makes about disagreement – i.e. if agent i deviates, but does not pay F_i in the transfer phase of the subsequent period – is that agent j

[4] Many local commons punish absence from maintenance activities or overappropriation by fines paid to the community. Note that these fines are not externally enforced.

punishes i by choosing zero investment, $y^p = 0$, and appropriating the maximal κ_j of the resource (if $j = 1$) in that period. Agent i, on the other hand, continues to behave noncooperatively for the rest of the period, investing y_i^N and appropriating κ_i of the resource (if $i = 1$).

The optimal relational contract maximizes the agents' discounted joint surplus,

$$(1-\delta)\sum_{t=1}^{\infty}\delta^{t-1}\big[\theta(y_1+y_2)-c(y_1)-c(y_2)\big]. \tag{6}$$

The payoffs are normalized by $(1-\delta)$ as is standard. The maximization problem is subject to the following incentive compatibility constraints (IC).

1. IC for y_i^c and s_i^c to be self-enforcing, denoted by IC-C.
2. IC for F_i to be self-enforcing, denoted by IC-F.
3. IC for punishment under disagreement to be self-enforcing, denoted by IC-P.

Before defining these ICs formally, let us describe the agents' strategies. The strategy depends on whether the agent is the first mover – and can choose how much of the common-pool resource to appropriate – or not.

Phase 1

(i) $y_i = y_i^c$ (and $s_i = s_i^c$ if $i = 1$) in $t = 1$. $y_i = y_i^c$ (and $s_i = s_i^c$ if $i = 1$) in $t > 1$ if $y_j = y_j^c$ (and $s_j = s_j^c$ if $j = 1$) in $t - 1$.

(ii) Phase 2 starts against agent j in t if $y_j \neq y_j^c$ (or $s_j \neq s_j^c$ if $j = 1$) in $t - 1$.

Phase 2

(iii) If agent j pays F_j to agent i in the transfer phase of t, $y_i = y_i^c$ (and $s_i = s_i^c$ if $i = 1$) in t and Phase 1 starts in $t + 1$.

(iv) If agent j does not pay F_j to agent i, $y_i = 0$ (and $s_i = \kappa_i$ if $i = 1$) in t and Phase 2 re-starts in $t + 1$.

(v) If agent i deviates in Phase 2 and does not choose $y_i = y_i^c$ (or $s_i = s_i^c$ if $i = 1$) as prescribed in (iii), or $y_i = 0$ (or $s_i = \kappa_i$ if $i = 1$) as prescribed in (iv), Phase 2 starts against agent i in $t + 1$.

We are now ready to define the IC's. Agent i cooperates by investing y_i^c (and appropriating s_i^c of the resource if $i = 1$) if the following IC-C is satisfied.

$$C_i \geq (1 - \delta)D_i + \delta[C_i - (1 - \delta)F_i] \tag{7}$$

where C_i is agent i's payoff from cooperation,

$$C_i = s_i^c \theta(y_1^c + y_2^c) - c(y_i^c), \tag{8}$$

and D_i is the payoff from deviation,

$$D_i = \kappa_i \theta(y_i^N + y_j^c) - c(y_i^N). \tag{9}$$

Note that IC-C is symmetric despite the agents' asymmetric roles. Consider first agent 1. If 1 cooperates, she chooses investment y_1^c and appropriates s_1^c of the resource, earning continuation value C_i. If she deviates, she invests y_1^N and appropriates κ_1, but can renegotiate back to cooperation by paying fine F_1 in the transfer phase of the subsequent period. For agent 2 cooperation involves choosing investment y_2^c and receiving share s_2^c if also 1 cooperates by appropriating s_1^c. Agent 2 deviates by investing y_2^N, which agent 1 observes and responds in the appropriation phase of same period by taking κ_1 of the resource, leaving κ_2 to agent 1. This explains why IC-C is symmetric for the agents. However, note that $\kappa_2 < \frac{1}{2} < \kappa_1$.

Equation (7) simplifies to

$$\delta F_i \geq D_i - C_i \equiv T_i, \tag{10}$$

where T_i is the temptation to deviate. Agent i will cooperate if the discounted value of the fine exceeds the temptation to deviate.

Paying the fine has also to be self-enforcing. IC-F is given by

$$C_i - (1-\delta)F_i \geq (1-\delta)P_i + \delta[C_i - (1-\delta)F_i], \tag{11}$$

where P_i is the punishment payoff,

$$P_i = \kappa_i \theta y_i^N - c(y_i^N). \tag{12}$$

If agent i pays the fine in the transfer phase, cooperation is restored for the investment and appropriation phases of the same period. Thus, agent i earns continuation value C_i from this period. If i does not pay the fine, agent j makes zero punishment investment and i receives κ_i of the resource. By the one shot deviation principle cooperation resumes in the subsequent period.

(11) gives an upperbound for the fine:

$$F_i \leq \frac{C_i - P_i}{1-\delta} = \frac{V_i}{1-\delta}, \tag{13}$$

where $V_i \equiv C_i - P_i$ is the value of the relationship. Since higher fine relaxes IC-C, the maximal fine, $F_i = \frac{V_i}{1-\delta}$, is optimal. Substituting the maximal fine in (10), IC-C simplifies to

$$\delta \geq \frac{T_i}{T_i + V_i} \equiv \underline{\delta}_i. \tag{14}$$

The final incentive compatibility constraint, IC-P, is for punishment. Agent j has to sacrifice current period profits by choosing zero punishment investment, but then receives F_i from agent i in the subsequent period. If j does not choose zero investment, punishment starts against j in the following period. IC-P is thus given by

$$(1-\delta)\hat{P}_j + \delta[(1-\delta)F_i + C_j] \geq (1-\delta)\hat{D}_j + \delta[C_j - (1-\delta)F_j], \tag{15}$$

where the punisher's payoff $\hat{P}_j$ is

$$\hat{P}_j = \kappa_j \theta y_i^N, \tag{16}$$

while deviation from the punishment gives payoff $\hat{D}_j$,

$$\hat{D}_j = \kappa_j \theta(y_1^N + y_2^N) - c(y_j^N). \tag{17}$$

(15) simplifies to

$$\delta(F_1 + F_2) \geq (\hat{D}_j - \hat{P}_j) \equiv T_j^p, \tag{18}$$

where T_j^p is the temptation to deviate from the punishment investment. Substituting the maximal fines in (18) obtains

$$\delta \geq \frac{T_j^p}{T_j^p + V_1 + V_2} \equiv \underline{\delta}_j^p. \tag{19}$$

Lemma 1 summarizes the above analysis.

Lemma 1 *Optimal relational contract solves*

$$\max_{\{y_1^c, y_2^c, s_1^c, s_2^c\}} (1-\delta) \sum_{t=1}^{\infty} \delta^{t-1} \left[\theta(y_1^c + y_2^c) - c(y_1^c) - c(y_2^c)\right]$$

subject to $\delta \geq max[\underline{\delta}_1, \underline{\delta}_2, \underline{\delta}_1^p, \underline{\delta}_2^p]$.

The optimal relational contract limits agent 1's appropriation to $s_1^c < \kappa_1$ and selects the cooperative investments (y_1^c, y_2^c) to maximize joint surplus subject to IC-C and IC-P.

4 Incentives to cooperate

To solve for optimal relational contract, let us first work out the explicit form of $\underline{\delta}_i$. It follows from (8) and (9) that the temptation to deviate is

$$\begin{aligned} T_i = D_i - C_i &= [\kappa_i \theta(y_i^N + y_j^c) - c(y_i^N)] - [s_i^c \theta(y_1^c + y_2^c) - c(y_i^c)] \\ &= (\kappa_i - s_i^c)\theta y_j^c + [\kappa_i \theta y_i^N - c(y_i^N)] - [s_i^c \theta y_i^c - c(y_i^c)]. \end{aligned} \tag{20}$$

From (8) and (12), the value of the relationship is

$$V_i = C_i - P_i = [s_i^c \theta(y_1^c + y_2^c) - c(y_i^c)] - [\kappa_i \theta y_i^N - c(y_i^N)]. \tag{21}$$

Thus, it follows from (20) and (21) that

$$T_i + V_i = \kappa_i \theta y_j^c. \tag{22}$$

Finally, it is convenient to define cooperative investment by

$$\eta_i \theta - c'(y_i^c) = 0, \tag{23}$$

where $\eta_i \in (\kappa_i, 1]$ is endogenous degree of cooperation.

It is proved in the Appendix that

$$\underline{\delta}_i = \frac{T_i}{T_i + V_i} = \frac{(\kappa_i - s_i^c)}{\kappa_i} + (1-\alpha)\left(\frac{\kappa_i}{\eta_j}\right)^{\frac{\alpha}{1-\alpha}} - \frac{(s_i^c - \eta_i \alpha)}{\kappa_i}\left(\frac{\eta_i}{\eta_j}\right)^{\frac{\alpha}{1-\alpha}}. \tag{24}$$

(24) shows how $\underline{\delta}_i$ depends on exogenous absorption capacity, κ_i, and the productivity of the investment, α, in addition to endogenous degrees of cooperation, η_i, η_j, and cooperative share, s_i^c. Note that the first term is positive for agent 1 and negative for agent 2. By deviating, 1 takes a larger share of 2's cooperative investment, $\kappa_1 > s_1^c$, while 2 receives a smaller share of 1's cooperative investment by deviating, $\kappa_2 < s_2^c$, as 1 responds to 2's freeriding by appropriating κ_1 of the resource. Note also that θ cancels out in (24) due to its proportional effect on both T_i and V_i.

As for temptation to deviate from the punishment investment, it follows from (17) and (16) that

$$T_i^p = \hat{D}_i - \hat{P}_i = [\kappa_i \theta(y_1^N + y_2^N) - c(y_i^N)] - \kappa_i \theta y_j^N = \kappa_i \theta y_i^N - c(y_i^N). \tag{25}$$

In the Appendix it is shown that

$$\underline{\delta}_i^p = \frac{T_i^p}{T_i^p + V_1 + V_2} = \frac{\kappa_i(1-\alpha)(\kappa_i)^{\frac{\alpha}{1-\alpha}}}{\sum_{j=1}^2 (1-\alpha\eta_j)(\eta_j)^{\frac{\alpha}{1-\alpha}} - \kappa_j(1-\alpha)(\kappa_j)^{\frac{\alpha}{1-\alpha}}}. \tag{26}$$

4.1 Symmetric cooperation

Let us start by analyzing symmetric cooperation. That is, the agents choose equal investments and share the resource equally. Substituting $\eta_1 = \eta_2 \equiv \eta$ and $s_i^c = \frac{1}{2}$ in (24) obtains

$$\underline{\delta}_i = 1 - \frac{(1-\eta\alpha)}{\kappa_i} + (1-\alpha)\Big(\frac{\kappa_i}{\eta}\Big)^{\frac{\alpha}{1-\alpha}}. \tag{27}$$

It is straightforward that $\frac{\partial \underline{\delta}_i}{\partial \kappa_i} > 0$, implying that $\underline{\delta}_1 > \underline{\delta}_2$. Thus, IC-C for symmetric cooperation is not binding for agent 2. Lemma 2 provides some useful properties of agent 1's critical discount factor $\underline{\delta}_1$.

Lemma 2 *Suppose $\eta_1 = \eta_2 = \eta$ and $s_i^c = \frac{1}{2}$.*
(i) $lim \underline{\delta}_{1\alpha\to 0} = 2 - \frac{1}{\kappa_1} > 0$.
(ii) $lim \underline{\delta}_{1\alpha\to 1} = 1 - \frac{(1-\eta)}{\kappa_1} \leq 1$.
(iii) $\frac{\partial \underline{\delta}_1}{\partial \alpha} > 0$.
(iv) $\frac{\partial \underline{\delta}_1}{\partial \eta} > 0$ if and only if $\eta > \kappa_1$.

All the proofs are in the Appendix.

According to Lemma 2, the minimum value of $\underline{\delta}_1$ is obtained for the lowest productivity $\alpha \to 0$. If δ is less than this minimum value, $\delta < 2 - \frac{1}{\kappa_1}$, IC-C for symmetric cooperation cannot be satisfied. This is the case when κ_1 is relatively large as it is too tempting for agent 1 to expropriate most of the resource instead of sharing it equally.[5] While lower degree of cooperation, η, generally relaxes IC-C (Lemma 2(*iv*)), it has no effect in the limit at $\alpha \to 0$. Thus, a necessary condition for symmetric cooperation of any degree is that κ_1 is sufficiently low, so that $\delta > 2 - \frac{1}{\kappa_1}$.

Cooperation incentives depend also on productivity. By Lemma 2(*ii*) symmetric first best cooperation, $\eta = 1$, is not self-enforcing for the highest productivity since $lim \underline{\delta}_{1\alpha\to 1} = 1$, while IC-C can be relaxed by lower degree of cooperation by Lemma 2(*iv*). Furthermore, lower productivity reduces $\underline{\delta}_1$ according to Lemma 2(*iii*).

[5] Large κ_1 also reduces the value of the relationship because the fallback payoff is larger, also weakening the incentives to cooperate.

Let us examine why higher productivity weakens cooperation incentives, starting from the value of the relationship. Total differentiation of (21) gives

$$\frac{dV_1}{d\alpha} = \Big[\frac{\partial c(y_1^N)}{\partial \alpha} - \frac{\partial c(y_1^c)}{\partial \alpha}\Big] + \Big[s_1^c\theta - c'(y_1^c)\Big]\frac{\partial y_1^c}{\partial \alpha} + s_1^c\theta\frac{\partial y_2^c}{\partial \alpha} - \Big[\kappa_1\theta - c'(y_1^N)\Big]\frac{\partial y_1^N}{\partial \alpha} \quad (28)$$

By (5) the last term in the square brackets equals zero and by (23) $c'(y_1^c) = \theta\eta_1$. Furthermore, taking into account symmetric cooperation, $y_1^c = y_2^c \equiv y^c$ and $s_1^c = \frac{1}{2}$, (28) is equivalent to

$$\frac{dV_1}{d\alpha} = \Big[\frac{\partial c(y_1^N)}{\partial \alpha} - \frac{\partial c(y^c)}{\partial \alpha}\Big] + \theta(1-\eta)\frac{\partial y^c}{\partial \alpha} > 0. \quad (29)$$

The term in the square brackets in positive because higher productivity reduces the cost of higher cooperative investment more than the cost of Nash investment. The second term is non-negative because higher productivity increases the cooperative investment. Therefore, the value of the relationship is increasing in α, *strengthening* the incentives to cooperate. Weaker incentives must then be driven by greater temptation. Total differentiation of (20) gives

$$\begin{aligned}\frac{dT_1}{d\alpha} &= -\Big[\frac{\partial c(y_1^N)}{\partial \alpha} - \frac{\partial c(y_1^c)}{\partial \alpha}\Big] + \theta(\kappa_1 - s_1^c)\frac{\partial y_2^c}{\partial \alpha} + \Big[\kappa_1\theta - c'(y_1^N)\Big]\frac{\partial y_1^N}{\partial \alpha} - \Big[s_1^c\theta - c'(y_1^c)\Big]\frac{\partial y_1^c}{\partial \alpha} \\ &= -\Big[\frac{\partial c(y_1^N)}{\partial \alpha} - \frac{\partial c(y^c)}{\partial \alpha}\Big] + \theta(\kappa_1 + \eta - 1)\frac{\partial y^c}{\partial \alpha} > 0 \quad (30)\end{aligned}$$

The first term in (30) is negative and the second term is positive since by Lemma 2(*iv*) $\eta \geq \kappa_1 > \frac{1}{2}$. The second term is the dominant effect. Greater productivity increases the cooperative investment, increasing agent 1's temptation both because she can expropriate from 2's larger investment and because she obtains a greater payoff increase from freeriding.

Finally, as for punishment incentives, substituting $\eta_1 = \eta_2 \equiv \eta$ in (26) we obtain

$$\underline{\delta}_i^p = \frac{\kappa_i(1-\alpha)(\kappa_i)^{\frac{\alpha}{1-\alpha}}}{2(1-\alpha\eta)(\eta)^{\frac{\alpha}{1-\alpha}} - \kappa_j(1-\alpha)(\kappa_j)^{\frac{\alpha}{1-\alpha}}}. \quad (31)$$

Lemma 3 explores the properties of $\underline{\delta}_i^p$.

Lemma 3 *(i)* $\underline{lim\delta}^p_{i\,\alpha\to 0} = \frac{\kappa_i}{1+\kappa_i}$.

(ii) $\underline{lim\delta}^p_{i\,\alpha\to 1} = 0$.

According to Lemma 3, $\underline{\delta}_1^p$ obtains its maximal value when $\alpha \to 0$. If this is less than the minimal value of $\underline{\delta}_1$, i.e. $\frac{\kappa_1}{1+\kappa_1} < 2 - \frac{1}{\kappa_1}$, agent 1's IC-P is not binding. This is the case if $\kappa_1 \geq \frac{\sqrt{5}-1}{2} \approx 0.62$, which is satisfied by Assumption 1.[6] This implies that if IC-C is satisfied for agent 1, all the other IC's are satisfied. In other words, symmetric cooperation is self-enforcing if and only $\delta \geq \underline{\delta}_1$.

By Lemma 2 $\underline{\delta}_1$ is monotonically increasing and ranges from $2 - \frac{1}{\kappa_1}$ to 1 if $\eta = 1$. Assuming $\delta > 2 - \frac{1}{\kappa_1}$, there exists a unique $\hat{\alpha} \in (0,1)$ such that $\delta = \underline{\delta}_1$. Therefore, symmetric first best cooperation is self-enforcing if and only if $\alpha \in (0, \hat{\alpha}]$. While for $\alpha \in (\hat{\alpha}, 1)$, symmetric first best cooperation is not sustainable and the degree of cooperation has to be reduced so that IC-C just binds for agent 1. Proposition 4 spells out these conditions.

Proposition 4 *(i) For a given δ, symmetric first best cooperation, $y_1^c = y_2^c = y^*$ and $s_1^c = s_2^c = \frac{1}{2}$, is supported in SPE if and only if $\kappa_1 < \frac{1}{2-\delta}$ and $\alpha \in (0, \hat{\alpha}]$, where $\hat{\alpha} \in (0,1)$.*

(ii) Suppose $\kappa_1 < \frac{1}{2-\delta}$. For $\alpha \in (\hat{\alpha}, 1)$ the highest self-enforcing symmetric degree of cooperation, $\hat{\eta}$, with $s_1^c = s_2^c = \frac{1}{2}$ is given by

$$1 - \frac{(1-\hat{\eta}\alpha)}{\kappa_1} + (1-\alpha)\Big(\frac{\kappa_1}{\hat{\eta}}\Big)^{\frac{\alpha}{1-\alpha}} = \delta,$$

where $\hat{\eta} \in [\kappa_1, 1)$.

According to Proposition 4, symmetric first best cooperation is self-enforcing if productivity is relatively low and agent 1's appropriation capacity is not too high,

[6] Similarly, if the maximal value of $\underline{\delta}_2^p$ is less than the minimal value of $\underline{\delta}_1$, agent 2's IC-P is not binding either. This is the case if $\frac{\kappa_2}{1+\kappa_2} = \frac{1-\kappa_1}{2-\kappa_1} < 2 - \frac{1}{\kappa_1}$. The inequality is satisfied if and only if $\kappa_1 \geq 2 - \sqrt{2} \approx 0.59$, which holds by Assumption 1.

both limiting 1's temptation to deviate. These are typical conditions in the traditional irrigation systems e.g. in Nepal and Philippines with long track record of well-functioning cooperation. Traditional irrigation systems are very costly to maintain and the headender's absorption capacity is limited. Water is typically shared on rotation basis and households have equal maintenance obligation.

Proposition 4 also shows that if productivity is high, 1's incentives can be improved by reducing the degree of cooperation. However, a more efficient way to improve incentives is to allocate a larger share of the common-pool resource to agent 1 while maintaining the investments at first best level. It is to this asymmetric cooperation that we turn our attention in Section 4.2.

Example. Suppose $\kappa_1 = \frac{3}{4}$; $\alpha = \frac{1}{3}$ or $\frac{2}{3}$; $\delta = 0.7$ or 0.45.

Substituting $\eta = 1$ and $\kappa_1 = \frac{3}{4}$ in (27) obtains $\underline{\delta}_1 = 0.688$ for $\alpha = \frac{1}{3}$ and $\underline{\delta}_1 = 0.743$ for $\alpha = \frac{2}{3}$. Thus, if the agents are relatively patient, $\delta = 0.7$, symmetric first best cooperation is self-enforcing for low productivity, $\alpha = \frac{1}{3}$. Symmetric cooperation is also sustainable for high productivity, $\alpha = \frac{2}{3}$, if the degree of cooperation is reduced to $\eta = 0.9$. However, if the agents are relatively impatient, $\delta = 0.45$, symmetric cooperation of any degree is not possible since the necessary condition requires $\delta > \frac{2}{3}$.

4.2 Asymmetric cooperation

If symmetric first best cooperation is not sustainable, agent 1's incentives can be improved by a sharing rule that gives her more than half of the common-pool resource, $s_1^c > \frac{1}{2}$. However, this will weaken agent 2's incentives. Increasing s_1^c improves incentives for cooperate to the point where $\underline{\delta}_1 = \underline{\delta}_2$. That is,

$$\frac{(\kappa_1 - s^*)}{\kappa_1} + (1-\alpha)(\kappa_1)^{\frac{\alpha}{1-\alpha}} - \frac{(s^* - \alpha)}{\kappa_1} = \frac{(\kappa_2 - 1 + s^*)}{\kappa_2} + (1-\alpha)(\kappa_2)^{\frac{\alpha}{1-\alpha}} - \frac{(1 - s^* - \alpha)}{\kappa_2} \tag{32}$$

Solving for s^* from (32),

$$s^* = \frac{1}{2}\Big[(2-\alpha)\kappa_1 + \alpha\kappa_2 + (1-\alpha)\kappa_1\kappa_2\Big[(\kappa_1)^{\frac{\alpha}{1-\alpha}} - (\kappa_2)^{\frac{\alpha}{1-\alpha}}\Big]\Big]. \tag{33}$$

Substituting (33) back to (32), we can solve for the lowest $\underline{\delta}_1 = \underline{\delta}_2 \equiv \underline{\delta}$ for first best investments that can be obtained by asymmetric sharing.

$$\underline{\delta} = (2\alpha - 1) + (1 - \alpha)\left[\kappa_1(\kappa_1)^{\frac{\alpha}{1-\alpha}} + \kappa_2(\kappa_2)^{\frac{\alpha}{1-\alpha}}\right] \tag{34}$$

Lemma 5 explores the properties of $\underline{\delta}$.

Lemma 5 *(i)* $lim\underline{\delta}_{\alpha\to 0} = 0.$
(ii) $lim\underline{\delta}_{\alpha\to 1} = 1.$

Lemma 5 shows that for any given δ there exists $\tilde{\alpha} \in (0, 1)$ such that IC-C for first best investments is satisfied for both agents if and only if $\alpha \in (0, \tilde{\alpha}]$. This is the case for any κ_1. With equal sharing, as shown in Proposition 4(i), large κ_1 limits the sustainability of first best investments. When we allow agent 1's share to be adjusted to κ_1, IC-C can be satisfied even if 1's appropriation capacity is high as long as productivity is not too high.

However, we need to keep track of IC-P as well. (31) shows that $\underline{\delta}_i^p$ does not depend on s_i^c. Accordingly, Lemma 3 is relevant also for asymmetric sharing. To guarantee that IC-P is satisfied, it is now assumed that $\delta \geq \frac{\kappa_1}{1+\kappa_1}$.[7]

Proposition 6 *Suppose* $\delta \geq \frac{\kappa_1}{1+\kappa_1}$. *With asymmetric sharing first best investments,* $y_1 = y_2 = y^*$, *can be supported in SPE if and only if* $\alpha \in (0, \tilde{\alpha}]$, *where* $\tilde{\alpha} \in (0, 1)$.

Propositions 4 and 6 show that first best investments in common-pool resource are self-enforcing if productivity is relatively low, with some additional constraints for the headender's absorption capacity (stricter conditions for symmetric cooperation). Halonen (2024) analyzes pure public goods and shows that also in that framework first best investments are sustainable if productivity is relatively low.

Example. Let us continue with the previous example, which showed that symmetric first best cooperation cannot be sustained if agents are impatient, $\delta = 0.45$,

[7] Note that this is not a necessary condition.

or productivity is high, $\alpha = \frac{2}{3}$.[8] Substituting $\kappa_1 = \frac{3}{4}$ and $\alpha = \frac{1}{3}$ in (33), the maximum share the relational contract can award to agent 1 is $s^* = 0.69$, reducing the critical discount factor to $\underline{\delta} = 0.18$ according to (34). While if $\alpha = \frac{2}{3}$, we obtain $s^* = 0.60$ and $\underline{\delta}_1 = 0.48$. Therefore, with asymmetric sharing first best cooperation is self-enforcing even when the agents are impatient, $\delta = 0.45$, as long as productivity is low. Similarily, first best cooperation can be sustained even when productivity is high, as long as the agents are patient, $\delta = 0.7$.

[8] Note that assumption $\delta \geq \frac{\kappa_1}{1+\kappa_1} = \frac{3}{7}$ is satisfied.

Appendix

Given the functional forms, closed-form solutions for the investments can be obtained from (3), (5) and (23).

$$y^* = (\theta\alpha)^{\frac{\alpha}{1-\alpha}} \tag{35}$$

$$y_i^N = (\kappa_i\theta\alpha)^{\frac{\alpha}{1-\alpha}} \tag{36}$$

$$y_i^c = (\eta_i\theta\alpha)^{\frac{\alpha}{1-\alpha}} \tag{37}$$

First, let us derive $\underline{\delta}_i$. From (20),

$$T_i = (\kappa_i - s_i^c)\theta y_j^c + [\kappa_i\theta y_i^N - c(y_i^N)] - [s_i^c\theta y_i^c - c(y_i^c)]$$

$$= (\kappa_i - s_i^c)\theta(\eta_j\theta\alpha)^{\frac{\alpha}{1-\alpha}} + [\kappa_i\theta(\kappa_i\theta\alpha)^{\frac{\alpha}{1-\alpha}} - (\kappa_i\theta\alpha)^{\frac{1}{1-\alpha}}] - [s_i^c\theta(\eta_i\theta\alpha)^{\frac{\alpha}{1-\alpha}} - (\eta_i\theta\alpha)^{\frac{1}{1-\alpha}}]$$

$$= (\theta)^{\frac{1}{1-\alpha}}(\alpha)^{\frac{\alpha}{1-\alpha}}\Big[(\kappa_i - s_i^c)(\eta_j)^{\frac{\alpha}{1-\alpha}} + \kappa_i(\kappa_i)^{\frac{\alpha}{1-\alpha}}(1-\alpha) - (\eta_i)^{\frac{\alpha}{1-\alpha}}(s_i^c - \eta_i\alpha)\Big]. \tag{38}$$

From (39),

$$T_i + V_i = \kappa_i\theta y_j^c = \kappa_i\theta(\eta_j\theta\alpha)^{\frac{\alpha}{1-\alpha}}. \tag{39}$$

Therefore,

$$\underline{\delta}_i = \frac{T_i}{T_i + V_i} = \frac{(\kappa_i - s_i^c)(\eta_j)^{\frac{\alpha}{1-\alpha}} + \kappa_i(\kappa_i)^{\frac{\alpha}{1-\alpha}}(1-\alpha) - (\eta_i)^{\frac{\alpha}{1-\alpha}}(s_i^c - \eta_i\alpha)}{\kappa_i(\eta_j)^{\frac{\alpha}{1-\alpha}}}$$

$$= \frac{(\kappa_i - s_i^c)}{\kappa_i} + (1-\alpha)\Big(\frac{\kappa_i}{\eta_j}\Big)^{\frac{\alpha}{1-\alpha}} - \frac{(s_i^c - \eta_i\alpha)}{\kappa_i}\Big(\frac{\eta_i}{\eta_j}\Big)^{\frac{\alpha}{1-\alpha}}. \tag{40}$$

Second, to derive $\underline{\delta}_i^p$, T_i^p from (18) is

$$T_i^p = \kappa_i\theta y_i^N - c(y_i^N) = \kappa_i\theta(\kappa_i\theta\alpha)^{\frac{\alpha}{1-\alpha}} - (\kappa_i\theta\alpha)^{\frac{1}{1-\alpha}} = \kappa_i\theta(\kappa_i\theta\alpha)^{\frac{\alpha}{1-\alpha}}(1-\alpha). \tag{41}$$

From (39),

$$T_i^p + V_1 + V_2 = [\kappa_i\theta y_i^N - c(y_i^N)] + [s_1^c\theta(y_1 + y_2) - c(y_1^c)] - [\kappa_1\theta y_1^N - c(y_1^N)]$$

$$+[s_2^c\theta(y_1+y_2)-c(y_2^c)]-[\kappa_2\theta y_2^N-c(y_2^N)]$$

$$=[\theta(y_1+y_2)-c(y_1^c)-c(y_2^c)]-[\kappa_j\theta y_j^N-c(y_j^N)]$$

$$=[\theta(\eta_1\theta\alpha)^{\frac{\alpha}{1-\alpha}}+\theta(\eta_2\theta\alpha)^{\frac{\alpha}{1-\alpha}}-(\eta_1\theta\alpha)^{\frac{1}{1-\alpha}}-(\eta_2\theta\alpha)^{\frac{1}{1-\alpha}}]-[\kappa_j\theta(\kappa_j\theta\alpha)^{\frac{\alpha}{1-\alpha}}-(\kappa_j\theta\alpha)^{\frac{1}{1-\alpha}}]$$

$$=\theta^{\frac{1}{1-\alpha}}\alpha^{\frac{\alpha}{1-\alpha}}[(1-\eta_1\alpha)(\eta_1)^{\frac{\alpha}{1-\alpha}}+(1-\eta_2\alpha)(\eta_2)^{\frac{\alpha}{1-\alpha}}-\kappa_j(\kappa_j)^{\frac{\alpha}{1-\alpha}}(1-\alpha)]. \tag{42}$$

Therefore,

$$\underline{\delta}_i^p=\frac{T_i^p}{T_i^p+V_1+V_2}=\frac{\kappa_i(1-\alpha)(\kappa_i)^{\frac{\alpha}{1-\alpha}}}{\sum_{j=1}^2(1-\alpha\eta_j)(\eta_j)^{\frac{\alpha}{1-\alpha}}-\kappa_j(1-\alpha)(\kappa_j)^{\frac{\alpha}{1-\alpha}}}. \tag{43}$$

Proof of Lemma 2.

(i)-(ii) Proof is straightforward from (27).

(iii) Differentiating (27) with respect to α,

$$\frac{\partial\underline{\delta}_1}{\partial\alpha}=\frac{\eta}{\kappa_1}+\Big[\frac{ln(\frac{\kappa_1}{\eta})}{(1-\alpha)}-1\Big]\Big(\frac{\kappa_1}{\eta}\Big)^{\frac{\alpha}{1-\alpha}}. \tag{44}$$

Evaluating (44) at $\alpha=0$,

$$\frac{\partial\underline{\delta}_1}{\partial\alpha}=\frac{\eta}{\kappa_1}-1+ln(\frac{\kappa_1}{\eta})=\frac{\eta}{\kappa_1}-1-ln(\frac{\eta}{\kappa_1})>0 \tag{45}$$

for any $\eta\neq\kappa_1$.

$$\frac{\partial^2\underline{\delta}_1}{\partial\alpha^2}=-\frac{ln(\frac{\kappa_1}{\eta})}{(1-\alpha)^2}\Big(\frac{\kappa_1}{\eta}\Big)^{\frac{\alpha}{1-\alpha}}+\frac{ln(\frac{\kappa_1}{\eta})}{(1-\alpha)^2}\Big(\frac{\kappa_1}{\eta}\Big)^{\frac{\alpha}{1-\alpha}}+\frac{\Big[ln(\frac{\kappa_1}{\eta})\Big]^2}{(1-\alpha)^3}\Big(\frac{\kappa_1}{\eta}\Big)^{\frac{\alpha}{1-\alpha}}$$

$$=\frac{\Big[ln(\frac{\kappa_1}{\eta})\Big]^2}{(1-\alpha)^3}\Big(\frac{\kappa_1}{\eta}\Big)^{\frac{\alpha}{1-\alpha}}>0 \tag{46}$$

Since $\underline{\delta}_1$ is increasing in α at $\alpha=0$ and is convex in α, it follows that $\frac{\partial\underline{\delta}_1}{\partial\alpha}>0$ for all $\alpha\in(0,1)$.

(iv) Differentiating (27) with respect to η,

$$\frac{\partial \underline{\delta}_1}{\partial \eta} = \frac{\alpha}{\kappa_1} - \alpha\Big(\frac{1}{\eta}\Big)\Big(\frac{\kappa_1}{\eta}\Big)^{\frac{\alpha}{1-\alpha}} = \frac{\alpha}{\kappa_1}\Big[1 - \Big(\frac{\kappa_1}{\eta}\Big)^{\frac{1}{1-\alpha}}\Big]. \tag{47}$$

It follows from (47) that $\frac{\partial \underline{\delta}_1}{\partial \eta} > 0$ if and only if $\eta > \kappa_1$.

Q.E.D.

Proof of Lemma 3.

(i) It follows from (31) and $(\kappa_1 + \kappa_2) = 1$ that

$$lim \underline{\delta_i^p}_{\alpha \to 0} = \frac{\kappa_i}{2 - \kappa_j} = \frac{\kappa_i}{1 + \kappa_i} \tag{48}$$

(ii) Taking limits from (31) obtains

$$lim \underline{\delta_i^p}_{\alpha \to 1} = \frac{0}{2(1-\eta)}. \tag{49}$$

Therefore, $lim \underline{\delta_i^p}_{\alpha \to 1} = 0$ if $\eta < 1$.

Proof of Proposition 4.

From (27)

$$\underline{\delta}_i = 1 - \frac{(1-\eta\alpha)}{\kappa_i} + (1-\alpha)\Big(\frac{\kappa_i}{\eta}\Big)^{\frac{\alpha}{1-\alpha}}. \tag{50}$$

Since $\frac{\partial \underline{\delta}_i}{\partial \kappa_i} > 0$, the binding IC-C is for agent 1. If the minimum value of $\underline{\delta}_1$ is greater than the maximum value of $\underline{\delta}_1^p$, IC-P is not binding for agent 1. This is the case according to Lemmas 2 and 3 if $2 - \frac{1}{\kappa_1} > \frac{\kappa_1}{1+\kappa_1}$, that is if $\kappa_1 \geq \frac{\sqrt{5}-1}{2}$. IC-P is not binding for agent 2 either if $2 - \frac{1}{\kappa_1} > \frac{\kappa_2}{1+\kappa_2} = \frac{1-\kappa_1}{2-\kappa_1}$, that is if $\kappa_1 \geq 2 - \sqrt{2}$, where $2 - \sqrt{2} < \frac{\sqrt{5}-1}{2}$. Accordingly, assuming $\kappa_1 \geq \frac{\sqrt{5}-1}{2}$ we only need to check that $\delta \geq \underline{\delta}_1$.

Assume $\kappa_1 \geq \frac{\sqrt{5}-1}{2}$ and $\delta > 2 - \frac{1}{\kappa_1}$.

(i) Suppose $\eta = 1$. According to Lemma 2 $lim \underline{\delta_1}_{\alpha \to 0} = 2 - \frac{1}{\kappa_1}$, $lim \underline{\delta_1}_{\alpha \to 1} = 1$ and $\frac{\partial \underline{\delta}_1}{\partial \alpha} > 0$. Therefore, there exists $\hat{\alpha} \in (0,1)$ such that $\delta \geq \underline{\delta}_1$ if and only if $\alpha \in (0, \hat{\alpha}]$.

(ii) For $\eta = 1$ agent 1's IC-C is not satisfied if $\alpha \in (\hat{\alpha}, 1)$. By Lemma 2 $\frac{\partial \underline{\delta}_1}{\partial \eta} > 0$ if and only if $\eta > \kappa_1$ and therefore η can be reduced so that $\underline{\delta}_1 = \delta$. Since $lim \underline{\delta_1}_{\alpha \to 1} =$

$1-\frac{(1-\eta)}{\kappa_1}$, symmetric cooperation is self-enforcing for even $\alpha\to 1$ if the degree of cooperation is reduced to $\underline{\eta}$ such that $1-\frac{(1-\underline{\eta})}{\kappa_1}=\delta$, implying $\underline{\eta}=1-\kappa_1(1-\delta)$. Therefore, for $\alpha\in(\hat{\alpha},1)$ the highest self-enforcing degree of cooperation, $\hat{\eta}$, is given by

$$1-\frac{(1-\hat{\eta}\alpha)}{\kappa_i}+(1-\alpha)\left(\frac{\kappa_i}{\hat{\eta}}\right)^{\frac{\alpha}{1-\alpha}}=\delta, \tag{51}$$

where $\hat{\eta}\geq\kappa_1$.

Q.E.D.

References

[1] Halac, Marina. 2015. "Investing in a Relationship." *Rand Journal of Economics* 46 (1):165-185.

[2] Halonen-Akatwijuka, Maija. 2024. "Climate Change and Incentives to Cooperate in Local Commons", mimeo.

[3] Harstad, Bård, Francesco Lancia and Alessia Russo. 2017. "Compliance Technology and Self-Enforcing Agreements." *Journal of the European Economic Association* 17 (1): 1-29.

[4] Ostrom, Elinor and Roy Gardner. 1993. “Coping with Asymmetries in the Commons: Self-Governing Irrigation Systems Can Work.” *Journal of Economic Perspectives* 7 (4): 93–112.

[5] Ramey, Garey and Joel Watson. 1997. "Contractual Fragility, Job Destruction, and Business Cycles." *Quarterly Journal of Economics* 112 (3):873–911.