

Conflict, Social Order, and Complementarities in Production

Ekrem T. Başer* & Tanguy Le Fur[†]

December 17, 2024

Work-in-progress. We have extensive simulations in support of all our propositions and have analytical results for special cases, but our proofs are yet incomplete.

Abstract

This paper studies the role of complementarities in collective production in the sustainability of inclusive institutions. We investigate how initial endowments and the elasticity of substitution in the aggregate production function jointly determine the distribution of both economic and coercive power among social groups, and their respective incentives to consent or not to a binding social contract. Drawing on a formal model of conflict, we show that i) a relatively balanced distribution of economic and coercive power is necessary for inclusive institutions to emerge and be sustainable, ii) there is a non-monotonic relationship between the degree of complementarities and the sustainability of inclusive institutions, and iii) complementarities determine which group favors conflict over social order. We argue that our theoretical framework has both normative and positive applications and can be used to design policies aimed at ensuring the stability of inclusive institutions and to shed light on a number of historical cases in which such institutions either appeared or broke down.

*Bilkent University, International Relations, ekrem.baser@bilkent.edu.tr

[†]LEM, University of Lille, tanguy.le-fur@univ-lille.fr

1 Introduction

Well-functioning economic and political institutions are often considered prerequisites for peace, prosperity, and development. Further, there is widespread agreement that inequality is inimical for social outcomes, and that a relatively balanced distribution of power is required for good institutions to emerge and be sustainable. Scholars, however, have focused on different types of power. Some, from David Ricardo and Karl Marx to Piketty (2017), have emphasized inequality pertaining to economic power and argued that the distribution of resources can bend institutions. Others, from Max Weber to Acemoglu and Robinson (2012, 2022), stressing the role of inequality in coercive power—social groups’ ability to prey on one another—, have focused instead on the role of violence and conflict in shaping institutions. However, economic and coercive power are often considered separately, which leads us to overlook the potential relationship between the two.

In this paper, we try to understand how the distribution of both coercive and economic power matter for the emergence and sustainability of good institutions. To do so, we consider a formal model of conflict in which two groups pool resources to produce, but can also engage in coercive activities. The novelty is the possibility for both groups to sign a binding social contract and thereby adopt a bundle of inclusive institutions. For the sake of simplicity, we adopt a reduced-form view of institutions: inclusive political institutions prevent groups from engaging in coercion, and inclusive economic institutions ensures the functioning of a market economy with perfect competition that values productive inputs according to their marginal productivity. In a state of nature without such institutions, the shares of aggregate (collective) production that accrue to each group depend on their relative coercive power. Under inclusive institutions, both groups relinquish the possibility of coercion and those shares are instead determined by their relative economic power—or how the market values their respective productive inputs and turns them into wealth. We show that a social order characterized by a set of inclusive political and economic institutions can emerge and be sustainable if and only if the distribution of both coercive and economic power across social groups is relatively balanced, such that neither group has incentives to break the social contract.

Both coercive and economic power are endogenous, and are determined by (i) the initial distribution of resources, and (ii) the elasticity of substitution between groups’ productive inputs in the

aggregate production function. The group with a greater share of initial resources is always able to muster greater coercive power in equilibrium. However, whether having a greater endowment of resources implies greater economic power depends on the elasticity of substitution—the degree of complementarity (or substitutability) between groups’ productive inputs. When both inputs are substitutes, the market values abundant inputs and having more resources also translates into greater economic power in addition to coercive power. However, when both inputs are complements such that the market values scarce inputs, being endowed with more resources results in lower productivity and therefore less economic power. Coercive and economic power can therefore be either positively or negatively correlated, and complementarities in aggregate production matter for the distributions of both types of power.

The degree to which distributions of coercive and economic power overlap among social groups across countries and over time exhibit significant variation. In some instances, coercive and economic power are indeed negatively correlated: both Marx’s description of class conflict as well as theories of franchise extension by an elite threatened by disenfranchised masses (Acemoglu and Robinson 2000) suggest that capitalists benefited from substantial economic power, while workers wielded greater coercive power during the Industrial Revolution. Looking at other cleavages, such as across racial, ethnic, or religious lines, also provides examples of coercive and economic power being negatively correlated. Minority groups, such as Jews in early 20th-century Germany or Greeks in Ottoman Turkey, lacked coercive power but benefited from significant economic power. In other majority-minority settings, however, coercive and economic power can be positively correlated: it is the case in the United States, where Whites still have significantly greater coercive and economic power relative to African-Americans today, but also in China between the Han Chinese and the Uighurs, or historically in the British Empire between the Irish and English populations.

What then determines the incentives of various social groups to abide by a social contract and adopt inclusive institutions? Because coercion is inefficient in the sense that it does not produce value, the opportunity cost of coercive investments is central to the trade-off between social order and conflict. A group’s dilemma between order and conflict is akin to choosing between a larger share of a smaller pie or a smaller share of a larger pie. Formally, a group is better off with conflict, and thus resists the adoption of inclusive institutions, if its relative coercive power is suf-

ficiently greater than its relative economic power to compensate for the loss of output that is due to the wasteful nature of conflict. The initial distribution of resources as well as the elasticity of substitution between groups' inputs in aggregate production are therefore going to be key for the emergence and sustainability of inclusive institutions. Importantly, the degree of complementarities in production is also going to determine which group is better off with conflict and prevents the adoption of inclusive institutions.

When both groups' inputs are substitutes in aggregate production, the group with the smaller share of initial resources is weaker in terms of both coercive and economic power but may nonetheless get a larger share of the pie in conflict compared to social order. This is because the group with greater resources—while stronger in both dimensions of power—also has greater opportunity costs of coercive investments. When groups' productive inputs are complements instead, the picture is different: as the market values scarce inputs, the group with the smaller share of initial resources has relatively more economic power and thus faces a greater opportunity cost of coercion. The group with a larger share of initial resources can not only wield greater coercive power, its productive inputs are less valued by the market such that it can increase its share of aggregate output in conflict relative to order if it is sufficiently weak in terms of economic power.

Both the initial distribution of resources and the elasticity of substitution in aggregate production determines social groups' relative coercive and economic power, and therefore their incentives to consent or not to the adoption of inclusive institutions. We show that the distribution of resources must be sufficiently equitable for both groups to forego coercive activities, but also that the threshold level of initial inequality that allows inclusive institutions to be sustainable depends on the degree of complementarities between groups' contribution to aggregate production. Interestingly, we find that there is a non-monotonic relationship, displayed in Figure 1, between the elasticity of substitution and the maximum level of inequality that can prevent conflict to prevail. Furthermore, we also show that social order is harder to sustain when inputs are complements rather than substitutes as it demands a more equal initial distribution of resources for both groups not to favor conflict over order.

This paper contributes to various strands of literature. Inequality is often considered detrimental to social order and the adoption of inclusive institutions requires a balanced distribution of power: Acemoglu and Robinson 2022 argue that there is a *Narrow Corridor* to liberty and pros-

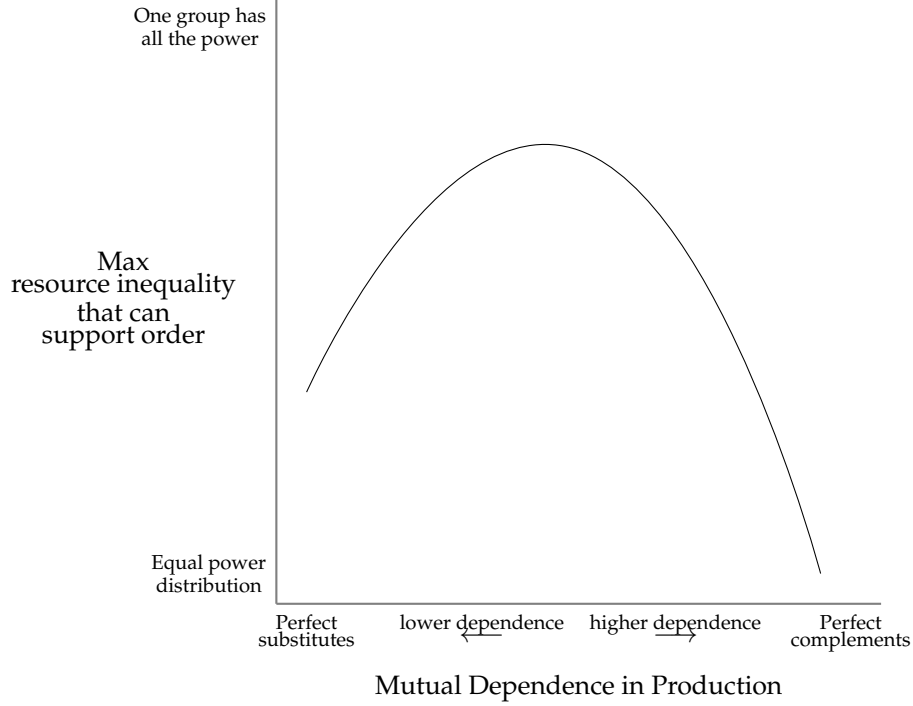


Figure 1: Maximum inequality in resource distribution that can sustain social order as a function of the mutual dependence of societal groups for producing shared outcomes.

perity. We show that the size of that corridor depends crucially on the degree of complementarities between groups' economic activities. Our results therefore also speak to a relatively recent literature on complementarities and social order. While complementarities have been associated with peaceful outcomes (Becker and Pascali 2019; Jha 2013, 2018; Silve 2018), we argue instead that, all else equal, complementarities in aggregate production makes social order less likely to be sustainable as it requires a more equal distribution of initial resources. However, because the relationship between the elasticity of substitution and order is non-monotonic, increasing complementarity can lead to more peaceful outcomes at low to medium levels, but it can also lead to less peaceful outcomes at medium to high levels.

Our research also sheds light on the conditions under which the group with greater coercive power might prey on the weak and vice versa. On the one hand, common wisdom suggests that the weak has to settle with worse deals, and thus the powerful would have greater incentives to prey on the weak (Fearon 1995; Powell 1999). On the other hand, the literature also highlights a *Paradox of Power*: the fact that the weaker side may fight harder, or may even be better off with conflict (Hirshleifer 1991; Sechser 2010). In our model, we observe both the strong preying on

the weak and the weak preying on the strong in equilibrium, depending on the degree of complementarities in aggregate production. When groups' inputs are substitutes, the group with low coercive power may favor conflict over order if it also has relatively low economic power. When groups' inputs are complements instead, the group with greater coercive power does indeed prey on the other. By allowing economic and coercive power to interact with each other depending on the elasticity of substitution, we therefore nest both phenomena in a unified framework. This is related to recent work that emphasizes the importance of considering different dimensions of power to explain the occurrence of conflict (Herrera, Morelli, and Nunnari 2022; Morelli, Ogliari, and Hong 2023).

The closest works to this paper are Skaperdas (1992), Skaperdas and Syropoulos (1998), and Silve (2018), all of which study contest models with production complementarities. Skaperdas (1992) and Skaperdas and Syropoulos (1998) focus on interactions between two agents under anarchy and do not consider the implications for sustaining inclusive institutions, which is our focus here. Our conflict state of the world is similar to these papers, and we rely on their method to prove uniqueness. Silve (2018) is similar to our paper in the sense that it considers complementarities on the intensity of conflict and on the social order that can emerge. Unlike Silve (2018), our model features a shared pool for economic and coercive activities, enabling agents to fully internalize the opportunity costs of coercive investments. This distinction is critical for our results and absent in Silve (2018), where the pool of resources used for production and coercion are different.

Moving forward, we introduce the "state of nature," which captures the conditions under which societal groups compete for their share of aggregate production via coercive activities. We then endow social groups with the ability to choose between this conflict state and a social order characterized by inclusive institutions.

2 The State of Nature

To begin our analysis, we will paint a picture of life in the state of nature, a world without a social contract, property rights, or institutions to enforce agreements and punish wrongdoers. Consider a society consisting of two groups, which we take as unitary actors, indexed by $i = 1, 2$. These groups could represent the nobility and farmers, elites and non-elites, different ethnic or religious

groups, or capitalists and workers. Each group possesses resources e_i , which it can allocate to either production y_i or coercion p_i , such that $e_i = y_i + p_i$. Production generates economic value that can be used for consumption, while coercive effort is useful only for forcefully appropriating what others produce.

This state of nature is a classic scenario of anarchy, well-known to scholars of conflict (Garfinkel and Skaperdas 2007; Hirshleifer 1991; Waltz 1979). Although we refer to this scenario as the state of nature, this type of setup applies—and has long been used—to broadly capture situations where agents determine their respective outcomes via unproductive influence activities (e.g., lobbying), as opposed to relying on contracts or property rights (Tullock 1967).

2.1 The Economy

We model economic activity in a reduced-form manner, where an aggregate production function governs economic output and describes the shape of the economy. Each group turns its resources devoted to production y_i into an intermediate input Y_i . In turn, groups' intermediate inputs Y_i determine the aggregate production Y . Aggregate production is governed by the following generalized Constant Elasticity of Substitution (CES) aggregate production function (Arrow et al. 1961), a staple in macroeconomics:

$$Y = \left(\alpha_1 Y_1^{\frac{\sigma-1}{\sigma}} + \alpha_2 Y_2^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} \quad (1)$$

where α_i are share parameters that determine the relative importance of intermediate inputs in production, and they sum to one ($\alpha_1 + \alpha_2 = 1$). The elasticity of substitution between intermediate inputs is captured by the parameter $\sigma \in (0, \infty)$. To simplify the analysis, we assume that the intermediate production technologies are linear, such that the supply of each intermediate input equals the amount of resources allocated to production by the corresponding agent: $Y_i = y_i$.

This simple economy can capture a wide range of situations in which groups depend on each other to create value. As $\sigma \rightarrow 0$, the groups' inputs become perfect complements, which means that increasing production is only possible by raising inputs from both groups.¹ In this case, a group cannot increase production simply by increasing its own inputs. Thus, this scenario repre-

1. As $\sigma \rightarrow 0$, equation 1 converges to a Leontieff production function.

sents an extreme form of mutual dependence.

In contrast, as $\sigma \rightarrow \infty$, the groups' inputs become perfect substitutes, which means that one group's input is as good as the other's.² In this case, a group does not require the other's input to raise production. Therefore, this situation represents an extreme form of mutual independence.

More generally, if $\sigma > 1$, inputs are gross substitutes, and groups are more independent of each other in creating value. If $\sigma < 1$, inputs are gross complements, and groups are more dependent on each other than independent.³

2.2 Conflict

In the absence of market institutions and property rights, it is unclear how aggregate production, the total pie, should be divided between the groups. Conflict becomes relevant because each group can forcefully seize a share of the production by allocating resources towards coercion. We model this conflict as a contest following existing literature (Garfinkel and Skaperdas 2007; Hirshleifer 1991). The share of production an agent i receives through conflict is given by a standard Tullock contest success function:⁴

$$\delta_i^c = \begin{cases} \frac{p_i}{p_i + p_{-i}} & \text{iff } p_i + p_{-i} \neq 0 \\ 1/2 & \text{otherwise} \end{cases} \quad (2)$$

Equation 2 implies that if one group invests in coercion while the other does not, the group with coercive investments seizes all production. Consequently, as is common in models featuring Tullock contest success functions, there is no equilibrium where no one engages in coercion.

Coercive investments are useful for seizing production forcefully and defending against predation, but they are otherwise wasteful since they do not contribute to production. The more groups invest in coercion, the fewer resources they have to invest in production. We denote the aggregate production in the presence of conflict as Y^c . We can express Y^c to reflect the resource constraint of each group $e_i = y_i + p_i$ as follows:

2. As $\sigma \rightarrow \infty$, equation 1 converges to a linear production function.

3. If $\sigma = 1$, the production function is equivalent to the Cobb-Douglas function.

4. While a Tullock contest success function usually gives a probability of seizing the entirety of a prize, here we interpret the outcome as the share of the pie—aggregate output—that accrues to each group.

$$Y^c = \left(\alpha_1 (e_1 - p_1)^{\frac{\sigma-1}{\sigma}} + \alpha_2 (e_2 - p_2)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} \quad (3)$$

Trivially, aggregate output strictly decreases with the predation efforts of each player. Production would be greater in the absence of conflict. Conflict is indeed wasteful.

2.3 Equilibrium

Groups allocate their resources between production and coercion, and this decision jointly determines both the total output produced and the share each group receives. They do so to maximize their income $I_i = \delta_i Y^c$ subject to their resource constraint. Substituting for the constraint, the maximization problem amounts to choosing the level of predation effort p_i^* . Formally:

$$p_i^* = \operatorname{argmax} \delta_i(p_i, p_{-i}) \cdot Y^c(p_i, p_{-i}) \quad (4)$$

The first order condition depicts the fundamental trade-off facing group i :

$$\underbrace{\frac{\partial \delta^i}{\partial p_i} \cdot Y^c}_{>0} + \underbrace{\delta^i \cdot \frac{\partial Y^c}{\partial p_i}}_{<0} = 0 \quad (5)$$

The first term describes the benefits of coercion which increases the share of production that group i seizes. The second term characterizes the negative effect of predation efforts on aggregate output.

Throughout the paper, we focus on pure-strategy Nash equilibria. Our first result is that in the state of nature, there is a unique equilibrium where both groups invest in coercion. This means there is no peace in the state of nature. The proof of the following proposition can be found in the appendix.

Proposition 1. *There is a unique Nash equilibrium in the state of nature, where both agents invest in conflict, $p_i^* > 0$.*

The intuition is straightforward: if one group does not invest any resources in conflict, the other group can seize all production with an infinitesimally small investment in coercion. There-

fore, in equilibrium, both groups invest resources in coercion. This result aligns with existing literature on the political economy of conflict (e.g., Hirshleifer 1991). In the state of nature, the distribution of output reflects the relative *coercive power* of each group. The relative coercive power of each group in turn depends on its initial endowment relative to the other group, denoted by $\epsilon_i = e_i/e_{-i}$.

3 Social contract and conflict

According to neoclassical theories of markets, social welfare is maximized under ideal institutional conditions where groups can engage in economic production and competition unburdened by contractual problems and protected property rights (Arrow and Debreu 1954). According to the Acemoglu-Robinson framework, such "inclusive" economic institutions emerge and are sustained in the presence of "inclusive" political institutions, where the state is strong enough to enforce property rights and does not prey on its citizens (Acemoglu and Robinson 2019). Together, these inclusive institutions allow societies to tap into the benefits of free markets, use resources efficiently, and accumulate wealth.

In this section, we aim to (i) capture the key features of the social contract under inclusive institutions and (ii) allow groups to choose between this social contract and conflict. Our objective is to examine the conditions under which such a social contract can be supported in equilibrium. We are particularly interested in how two societal factors—resource inequality and the degree of elasticity of substitution in groups' productive efforts—affect the emergence of social order.

We modify the game as follows: the groups now have the ability to choose between the state of nature and a social contract characterized by inclusive economic and political institutions, which we describe below. Group i favors the social order over the state of nature if and only if its total income is higher in the former. If at least one group is better off in the state of nature however, conflict prevails. Therefore, for the social order to emerge, both groups must consent to be bound by it. Once both groups consent to the social contract, they relinquish their ability to invest in coercion. We therefore consider the endogenous existence of the social order and show that it is determined by the structure of economic production and the initial distribution of resources.

3.1 Social order with inclusive institutions

We capture the social order implied by the inclusive political and economic institutions in a reduced-form manner. On the political side, we assume that the state does not engage in predation over citizens but only prevents them to engage in any kind of coercive activity, essentially protecting property rights. As no group bound by the social contract can engage in predation, both allocate all their resources to productive activities: $y_i = e_i$. Production is therefore maximized, and so is social welfare. We denote by Y^o aggregate output under social order:

$$Y^o = \left(\alpha_1 e_1^{\frac{\sigma-1}{\sigma}} + \alpha_2 e_2^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}$$

On the economic side, we assume that markets function properly and the forces of perfect competition are at play. As in the vast majority of macroeconomic models, both groups are therefore paid according to their marginal contribution to aggregate production. In such a setting, it is economic rather than coercive power that determines the distribution of output. Formally, each group's total income under social order is $I_i^o \equiv \frac{\partial Y}{\partial y_i} \cdot y_i = \frac{\partial Y}{\partial e_i} \cdot e_i$. We denote by δ_i^o its share of total output under social order:

$$\begin{aligned} \delta_i^o \equiv \frac{I_i^o}{Y^o} &= \frac{(\partial Y / \partial e_i) \cdot e_i}{Y} \\ &= \frac{\alpha_i e_i^{\frac{\sigma-1}{\sigma}}}{\alpha_i e_i^{\frac{\sigma-1}{\sigma}} + \alpha_{-i} e_{-i}^{\frac{\sigma-1}{\sigma}}} \end{aligned} \quad (6)$$

The share of output that accrues to each group under social order does not reflect its relative *coercive power* but rather its relative *economic power*. The relative economic power of each group is also determined by the distribution of endowments. However, the mapping from endowments to economic power under social order is not the same as the mapping from endowments to coercive power in the state of nature. Let $\epsilon_i = e_i / e_{-i}$ be the relative endowment of group i again. Its relative *economic power*, measured by its share of total output distributed by the market, is:

$$\delta_i^o(\epsilon_i) = \frac{\alpha_i \epsilon_i^{\frac{\sigma-1}{\sigma}}}{\alpha_i \epsilon_i^{\frac{\sigma-1}{\sigma}} + \alpha_{-i}} \quad (7)$$

The elasticity of substitution between both groups' inputs in aggregate production σ plays a key role in determining how relative endowments translate into relative economic power with inclusive economic institutions. To see this, let us take the derivative of the share of group i with respect to its relative endowments:

$$\frac{\partial \delta_i^o(\epsilon_i)}{\partial \epsilon_i} = \frac{\sigma - 1}{\sigma} \cdot \frac{\alpha_i \epsilon_i^{-\frac{1}{\sigma}} \alpha_{-i}}{\left(\alpha_i \epsilon_i^{\frac{\sigma-1}{\sigma}} + \alpha_{-i} \right)^2} \left\{ \begin{array}{ll} > 0 & \Leftrightarrow \sigma > 1 \\ = 0 & \Leftrightarrow \sigma = 1 \\ < 0 & \Leftrightarrow \sigma < 1 \end{array} \right. \quad (8)$$

The relationship between relative endowments and the share of production that accrues to each group under such economic institutions depends on how productive inputs are valued by the market. A baseline case is when the elasticity of substitution σ is equal to one. The aggregate production function is Cobb-Douglas and the shares of both groups are constant and equal to $\delta_i^o = \alpha_i$. In this case, the relative economic power of each group does not depend on relative endowments.

However, the distribution of endowments matters when the elasticity of substitution is different than one. When $\sigma > 1$, both groups' inputs are gross substitutes in the production process. Since both groups' productive efforts can (imperfectly) substitute for each other, an increase in the relative supply of a group's inputs decreases the market demand for the other group's. As a result, a group's share of total output increases with its relative supply of productive inputs. In other words, an increase in endowments relative to the other group translates into greater economic power.

Conversely, if $\sigma < 1$, the groups' activities are gross complements in aggregate production. The productivity of one group's inputs depends on the supply of the other group's. An increase in the relative supply of a group's productive input increases the market demand for the other's, increasing the share of the total output of the latter. In this case, an increase in relative endowments, therefore, translates into a lower economic power.

In the next section, we explore the conditions under which groups choose to be bound by this social order rather than engage in conflict.

3.2 The choice between conflict and inclusive institutions

In this section, we are interested in the incentives of each group to consent to give up on coercive activities and sign a social contract that allows both inclusive political and economic institutions to emerge. We study, in particular, how those incentives vary with relative endowments. In order not to conflate distribution effects from income effects, we consider that the total supply of initial resources is fixed and equal to E , such that $e_1 + e_2 = E$. We denote by γ_i the share of resources endowed to group i . Therefore, $e_i = \gamma_i E$ and $\gamma_i + \gamma_{-i} = 1$. The relative endowment of group i is thus $\epsilon_i = \gamma_i / (1 - \gamma_i)$.

Each group favors the institutions (or lack thereof) that maximize its total income I_i . The choice, therefore, amounts to comparing total income in the state of nature I_i^c and under social order I_i^o . Group i is indifferent between conflict and order when $I_i^c = I_i^o$, is better off in the state of nature when $I_i^c > I_i^o$ and favors inclusive institutions when $I_i^o > I_i^c$. Using the definition of total income in the state of nature and under social order, group i is better off in the state of nature if and only if:

$$\frac{\delta_i^c}{\delta_i^o} > \frac{Y^o}{Y^c} \quad (9)$$

The above inequality says that a group will resist the adoption of inclusive institutions when its relative coercive power is sufficiently greater than its relative economic power to compensate for the loss of aggregate output due to the wasteful nature of conflict. Conversely, for a group to favor social order, with competitive markets, over the state of nature, its economic power needs to be substantial relative to the coercive power it relinquishes by signing the social contract.

As we saw in the previous subsections, both types of power are determined by the relative endowment of each group. While coercive power always increases with the relative endowment of a group in the state of nature, the mapping from relative endowments to economic power in a market economy is contingent on the elasticity of substitution between the two groups' productive inputs. Looking at equation 8, we see that when the elasticity of substitution between both groups' inputs is lower than one—that is, they are gross complements—an increase in the relative endowment of a group raises its coercive power but decreases its economic power as the market would value the scarcer productive inputs of the other group more. All else equal, this increases the likelihood that the better-endowed group would be better off in the state of nature

and thus refuse to adopt inclusive institutions. On the other hand, when the elasticity of substitution is greater than one, and both groups' inputs are gross substitutes in economic production, an increase in the relative endowment of a group results in greater economic power. The effect on the ratio of relative coercive and economic power is therefore ambiguous.

Under social order, both groups put all of their resources in production. Given that $\epsilon_i = e_i/e_{-i} = \epsilon_i(\gamma_i)$, the income of each group is therefore:

$$I_i^o = \delta_i^o Y^o = \frac{\alpha_i \epsilon_i(\gamma_i)^{\frac{\sigma-1}{\sigma}}}{\alpha_i \epsilon_i(\gamma_i)^{\frac{\sigma-1}{\sigma}} + \alpha_{-i}} \left(\alpha_i (\gamma_i E)^{\frac{\sigma-1}{\sigma}} + \alpha_{-i} ((1 - \gamma_i) E)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} \quad (10)$$

In the state of nature, each group invests $p_i^* > 0$ amount of resources into coercive activities and $e_i - p_i^*$ into production. The optimal amount of resources invested in coercive activities at equilibrium is a function of the exogenous parameters of the model, including γ_i . The income of each group under conflict is thus:

$$I_i^c = \delta_i^c Y^c = \frac{p_i^*(\gamma_i)}{p_i^*(\gamma_i) + p_{-i}^*(\gamma_i)} \left(\alpha_i (\gamma E - p_i^*(\gamma_i))^{\frac{\sigma-1}{\sigma}} + \alpha_{-i} ((1 - \gamma_i) E - p_{-i}^*(\gamma_i))^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} \quad (11)$$

Both the distribution of resources, given by γ_i , and the elasticity of substitution σ play a key role in determining income under both the state of nature and social order and, therefore, the incentives of each group to adopt inclusive institutions. Lemma 1 provides a detailed characterization of each group's decision:

Lemma 1. Suppose $\alpha_i = \alpha_{-i} = 0.5$. For player i , there exists a relative endowment level $\tilde{\gamma}_i$, such that if $\gamma_i = \tilde{\gamma}_i$, player i is indifferent between order and conflict. If $\sigma > 1$, player i strictly prefers conflict over order if and only if $\gamma_i < \tilde{\gamma}_i$. If $\sigma < 1$ player i strictly prefers conflict over order if and only if $\gamma_i > \tilde{\gamma}_i$.

A few things are worth noting before going further. First, relative endowments determine whether a given group prefers the state of nature or social order. Second, and most importantly, for a given distribution of resources, the elasticity of substitution dictates which group has incentives to resist the adoption of inclusive institutions. Recall that relative endowments determine both coercive and economic power. Intuitively, it is always the group that has the less economic power, that is, the group that gets a lower share of output under social order, that can benefit from conflict.

When $\sigma > 1$ and both groups' inputs are gross substitutes in aggregate production, the less-endowed group has low economic power relative to the better-endowed group. Despite also having low coercive power, it can be better off in the state of nature as the powerful group finds it costly to invest productive resources into coercive activities as it reduces aggregate production by a substantial margin. This is reminiscent of the *paradox of power* (Hirshleifer 1991) in which the less powerful group improves its relative position through conflict.

When $\sigma < 1$ and both groups inputs are gross complements in aggregate production, it is the better-endowed group that has low relative economic power. The market indeed values the scarce productive input more. In this case, the better endowed-group can muster its relatively strong coercive power in the state of nature to compensate for its low economic power under social order. There is no paradox anymore: the powerful group in terms of coercion preys on the weaker group.

We can then study how the threshold $\tilde{\gamma}_i$ varies with the elasticity of substitution σ . The results are described in Lemma 2:

Lemma 2. *$\tilde{\gamma}_i$ is the relative endowment level that makes group i indifferent between order and conflict. Suppose $\alpha_i = \alpha_{-i} = 0.5$, then:*

$$\text{For } \sigma \neq 1, \quad \frac{\partial \tilde{\gamma}_i}{\partial \sigma} > 0 \quad \& \quad \lim_{\sigma \rightarrow \infty} \tilde{\gamma}_i < \lim_{\sigma \rightarrow 0} \tilde{\gamma}_i = 0.5 \quad \& \quad 1 > \lim_{\sigma \rightarrow 1^-} \tilde{\gamma}_i > \lim_{\sigma \rightarrow 1^+} \tilde{\gamma}_i > 0$$

The main results of the paper laid out in the next subsection follow from both Lemma 1 and 2. Figure 2 provides a useful visual summary of both, focusing on a single group's incentives to consent or not to social order:

3.3 The adoption of inclusive institutions

We assume that for social order to be sustainable, both groups must consent to it. If one of the two groups is better off in the state of nature, then no social contract is signed and inclusive institutions are not adopted. Let $\bar{\gamma} = \max\{\tilde{\gamma}_1, \tilde{\gamma}_2\}$ and $\underline{\gamma} = \min\{\tilde{\gamma}_1, \tilde{\gamma}_2\}$. We assume that group i opts for order when indifferent. Proposition 2 characterizes the conditions under which social order can be supported in equilibrium:

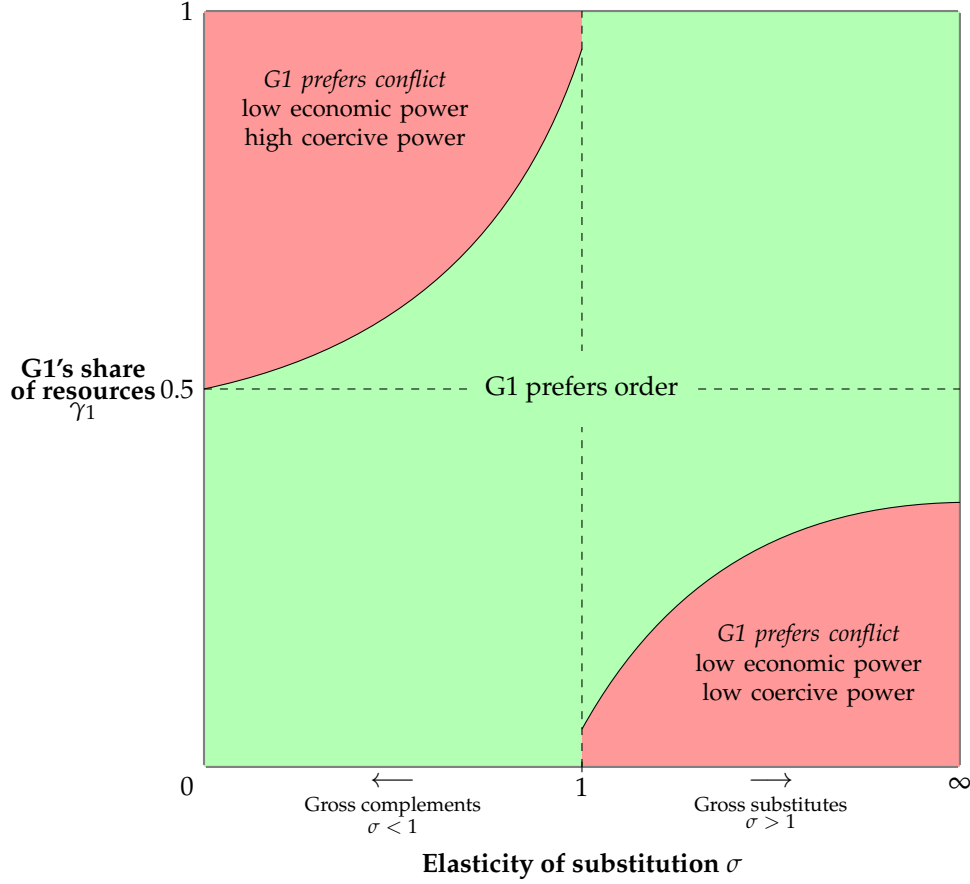


Figure 2: A group's choice between order and conflict. On the x-axis is the elasticity of substitution of groups' productive efforts σ and on the y-axis is the group's share of total resources. The solid black lines are the inequality at which the group is indifferent $\tilde{\gamma}_1$. We set $\alpha_i = \alpha_{-i} = 0.5$.

Proposition 2. *In equilibrium, social order prevails if and only if the distribution of resources in the society is sufficiently balanced: $\gamma \in [\underline{\gamma}, \bar{\gamma}]$.*

The result follows directly from Lemma 1. Proposition 2 underlines that inequality in both coercive and economic power can directly undermine social order. Inequality makes different groups favor the state of nature over social order depending on the elasticity of substitution. As social order requires both groups to benefit from inclusive institutions, the distribution of resources has to be sufficiently balanced. Figure 3 displays the incentives of both groups to consent or not to social order and, therefore, shows the region in which inclusive institutions are actually sustainable at equilibrium. Conflict prevails in equilibrium in the blue- and red-shaded regions, whereas social order prevails in the green-shaded center region.

As hinted in the previous section, the types of groups that opt for conflict differ qualitatively

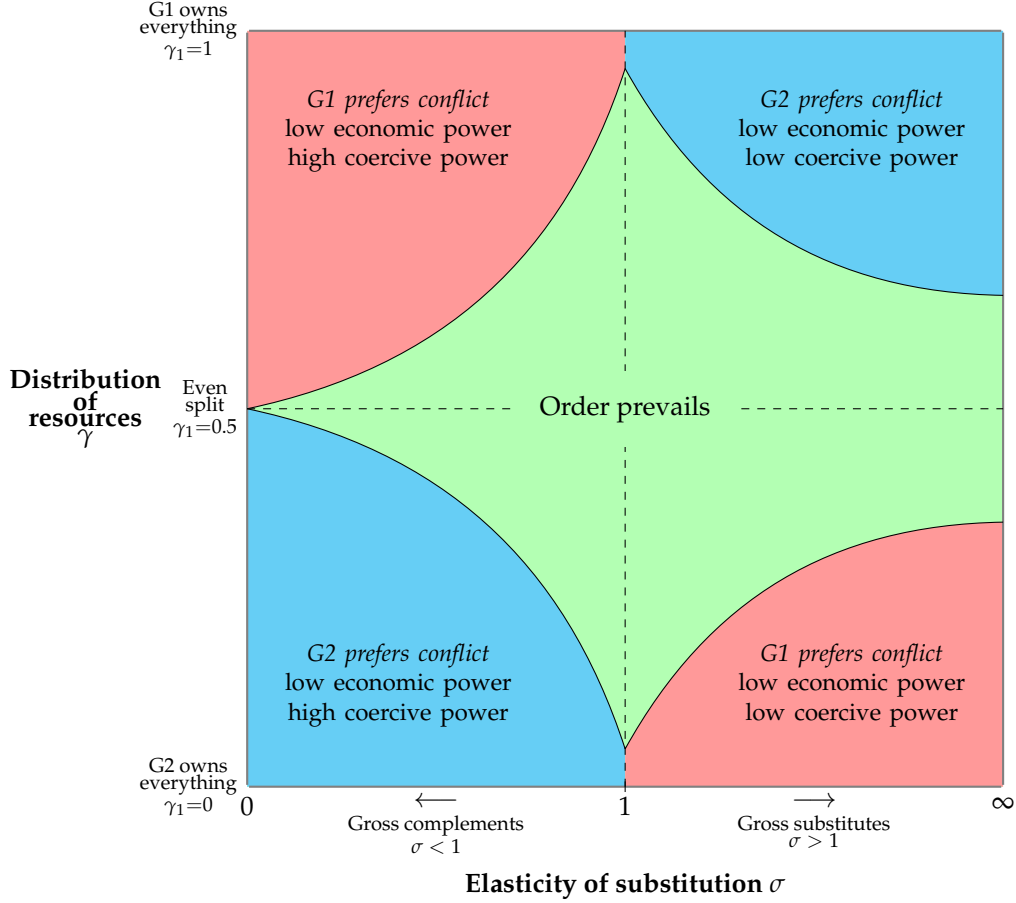


Figure 3: Equilibrium regions by the the elasticity of substitution of groups' productive efforts σ (x-axis) and the distribution of resources γ (y-axis). We set $\alpha_i = \alpha_{-i} = 0.5$.

depending on the elasticity of substitution. When it is high ($\sigma > 1$) such that both groups' inputs are gross complements in production, the group that prefers conflict over social order has relatively low coercive power. When both groups' inputs are gross complements ($\sigma < 1$), the group that refuses social order has greater coercive power. However, the group that favors conflict, whether it has low or high coercive power, is always the group that has the lowest economic power. In other words, a group favors conflict over social order when inclusive economic institutions do not reward its productive efforts enough.

Whether economic and coercive power are positively or negatively correlated depends on the elasticity of substitution. Two different scenarios arise. On the one hand, when $\sigma > 1$, the group that benefits from conflict has both low economic and coercive power. Although it remains poorer than the other group, it is able to improve its relative position through conflict. The well-known

paradox of power thus exists as an equilibrium outcome in our framework when inequality is sufficiently high and productive inputs are gross substitutes. On the other hand, when $\sigma < 1$, the group that triggers conflict has low economic but high coercive power. It is not only able to improve its position relative to the group with high economic power, but also to seize the greater share of the pie through conflict. In other words, when productive inputs are gross complements, the breakdown of social order sees a reversal of fortunes: the group that would have been the poorest actually becomes the richest by wielding its coercive power.

As can be seen in Figure 3, the shape of the region in which social order is sustainable in equilibrium is not the same for σ greater or lower than one. Comparing the right half of the figure where $\sigma > 1$ and the left half where $\sigma < 1$, we see conflict as an equilibrium outcome more frequently in the left half. Social order is thus easier to sustain in an economy where groups' productive efforts are gross substitutes or gross complements. Our next result formalizes this. To do so, we define $\Delta(\sigma) = \bar{\gamma}(\sigma) - \underline{\gamma}(\sigma)$, which measures the amount of resource inequality that can be consistent with social order in equilibrium for a given elasticity of substitution σ . Then, $\int_1^\infty \frac{\Delta(\sigma)}{\sigma} d\sigma$ measures the prevalence of social order equilibrium for $\sigma > 1$, while $\int_0^1 \frac{\Delta(\sigma)}{\sigma} d\sigma$ measures the prevalence of social order equilibrium for $\sigma < 1$.

Proposition 3. $\int_1^\infty \frac{\Delta(\sigma)}{\sigma} d\sigma > \int_0^1 \frac{\Delta(\sigma)}{\sigma} d\sigma$ and $\lim_{\sigma \rightarrow \infty} \Delta(\sigma) > \lim_{\sigma \rightarrow 0} \Delta(\sigma)$. Therefore, social order can be sustained under greater resource inequality when groups' production efforts are gross substitutes $\sigma > 1$ as opposed to gross complements $\sigma < 1$.

Proposition 3 shows that social order can be supported in equilibrium under greater resource inequality when groups' productive efforts are gross substitutes rather than gross complements. Conflict is indeed more likely to prevail, for a given level of inequality, when $\sigma < 1$. Therefore, another important difference between the gross complements and substitutes cases is the degree of inequality required for social order to be sustainable. We show that, overall, when groups' productive efforts are complementary, social order requires a more equitable resource distribution than when they are substitutable. Conflict is wasteful in both cases, but the opportunity cost of investing in coercion is different. When groups' inputs are gross complements, the better-endowed group has excess resources that have little use in production and are not valued by the market. When inputs are gross substitutes, the better-endowed group is the most productive one.

Therefore, diverting resources to coercion shrinks the pie more when inputs are gross substitutes rather than gross complements.

Note that since conflict is a contest, the most severe damage to the economy occurs under the most equitable resource distributions because in those conditions both sides invest the most in coercion. The discontented group, the group with relatively low economic power, faces a trade-off: stay under order and receive a lower share of a larger pie or opt for conflict and receive a higher share of a smaller pie. Therefore, in the presence of complementarities, the group with low economic power finds getting the "higher share of a smaller pie" more tempting under more equitable resource distributions.

Finally, note how the size of the green-shaded region in which inclusive institutions are sustainable first increases then decreases with σ . Our final result in this section characterizes this non-monotonic relationship between the elasticity of substitution and equilibrium outcomes.

Proposition 4. *Suppose $\gamma \notin [\lim_{\sigma \rightarrow \infty} \underline{\gamma}(\sigma), \lim_{\sigma \rightarrow \infty} \bar{\gamma}(\sigma)]$ and $\gamma \in [\underline{\gamma}(1), \bar{\gamma}(1)]$. Then, conflict prevails in equilibrium for sufficiently low and sufficiently high levels of σ , while social order prevails in between.*

The relationship between σ and conflict can be non-monotonic. First, in the very top and bottom regions of Figure 3, conflict is the only equilibrium outcome. Here, changing σ does not change the equilibrium outcome. Second, in the center region where order prevails even when groups' efforts are perfect substitutes ($\sigma \rightarrow \infty$), increasing σ (thus reducing mutual dependence) can transition the equilibrium outcome from conflict to order. Thus, here too, the relationship between σ and conflict is monotonic. Finally, in the upper middle and lower middle regions, conflict can occur in equilibrium for both high and low levels of σ but not for intermediate values. Hence, here, the relationship between σ and conflict is nonmonotonic, as stated in Proposition 4 and depicted in Figure 1. Depending on the initial degree of mutual dependence, increasing mutual dependence can lead to order or conflict.

4 Discussion: Complementarities, conflict, and social order

The theoretical framework that we lay out and analyze in the previous sections is arguably stylized. It nonetheless allows for an interesting investigation of the role complementarities between

social groups in the collective production process play in the emergence of inclusive institutions and the sustainability of social order. In this section, we discuss our results at greater length and highlight how they might contribute to various strands of literature.

The conditions for inclusive institutions. In the framework developed by Daron Acemoglu and James Robinson over the years (Acemoglu and Robinson 2006, 2012, 2019, 2022), the emergence and resilience of inclusive institutions require a balanced distribution of power across social groups. They argue that it is only in this "narrow corridor" in which power is rather evenly distributed between elites and non-elites (or the state and civil society) that the institutions required for prosperity are sustainable. In our model, both coercive and economic power stems from endowments and Proposition 2 shows that a relatively balanced distribution of initial resources is indeed required for social groups to consent to sign a binding social contract.

Moreover, we show that in a framework in which social groups have to pool resources together to produce, the size of the "narrow corridor" is shaped by the elasticity of substitution between their respective productive inputs, σ . Propositions 3 and 4 demonstrate that the degree of initial inequality that can sustain social order in equilibrium changes significantly based on the degree of complementarities between groups' inputs in collective production. We find that both very high substitutability (low σ) and very high complementarity (high σ) generate a greater need for resource equality to maintain social order. The relationship between the degree of initial inequality and the likelihood that order prevails over conflict is non-monotonic in the elasticity of substitution. Furthermore, social order can be sustained under greater levels of inequality under high substitutability than high complementarity, making the latter a more difficult case for inclusive institutions.

The technology with which social groups collectively produce therefore matters a lot for the sustainability of inclusive institutions. While the role of the resources endowments in shaping institutions has long been recognized, we show the degree of complementarities is crucial in determining the distribution of resources that allows inclusive institutions to emerge and be sustainable. As such, while *ex ante* redistribution appears as a natural lever to promote social order, we argue that it needs to be contingent on the elasticity of substitution in the aggregate production function.

The winners and losers of inclusive institutions. By considering both coercive power (the ability to use resources to appropriate a larger share of production) and economic power (how resources are valued by the market under inclusive institutions), we are able to characterize which social group might have incentives to break the social contract. While a group with relatively low economic power is more likely to be worse off under social order, it needs not have greater coercive power to favor conflict over social order. In our framework, the relationship between coercive and economic power depends on the value of the elasticity of substitution in the aggregate production function. The degree of complementarities thus determines which group might resist the emergence of inclusive institutions or challenge them, or in other words: the winners and losers of inclusive institutions.⁵

Who benefits from conflict has been the subject of an extensive literature in economics, political science, and international relations. While a greater coercive power suggests that a powerful group might be better off with conflict (Fearon 1995; Powell 1999), it may be the case that a group with lower coercive power can also have incentives to engage in conflict (Hirshleifer 1991; Sechser 2010). Hirshleifer (1991) labels the latter case in which a relatively less powerful group favors conflict over order the "Paradox of Power". Our framework allows to nest those two polar cases: whether a group with greater or lower relative coercive power might be better off with conflict than under social order is contingent on the value of the elasticity of substitution as shapes the distribution of economic power as well.

The elasticity of substitution indeed determines the relationship between initial endowments and both coercive and economic power. While more resources always translate into greater coercive power, whether groups' productive inputs are substitutable or complimentary determines how they will be valued by the market according to their respective supply. When σ is lower than one and inputs are gross complements, the market values the scarce input more such that the group with a greater initial endowment of resources might favor conflict as it has greater coercive power but lower economic power. However, when σ is greater than one and inputs are gross substitutes, the market values more inputs in abundant supply. The group with a lower relative endowment of resources might be worse off under inclusive institutions and thus favor conflict

5. While the relationship between coercive and economic power has been examined by scholars focusing on social interactions under anarchy (Hirshleifer 1991; Skaperdas 1992; Skaperdas and Syropoulos 1997), they do not investigate how this correspondence matters for sustaining good institutions.

over order, despite having lower coercive power.

The relationship between complementarities and social order. Finally, our findings have implications for understanding the relationship between complementarities among groups' economic activities and desirable outcomes such as peace. Prevailing arguments in the literature suggest that stronger complementarities reduce incentives for conflict and increase the likelihood of peace (Becker and Pascali 2019; Jha 2013, 2018; Silve 2018). However, our results suggest that not only the relationship between complementarities and the probability of a peaceful outcome is non-monotonic, social order is actually harder to sustain under gross complementarity than gross substitutability as it requires a more equal distribution of initial resources. This divergence is due to existing research either not considering the opportunity costs of conflict investments in production or assuming that the group with the scarce resource has a stronger outside option.

In Silve (2018), for example, the resource pools used for coercion and production are separated. Therefore, when a group invests in coercion, it does not need to consider the opportunity costs in terms of production as in our model. Under gross complementarity, the group with the greater share of resources (which is also the poorer group under order) has "excess" resources that yield little in production and are thus better used in conflict. The groups' internalization of the opportunity cost of their coercive investments is the main channel through which order can break down under strong complementarities. Separating the resource pools used for conflict and production assumes away the downside of complementarities, leaving only the upside.

Second, scholars arguing that strong complementarities are associated with peace rely on a theoretical framework where the group with the scarce resource has a stronger outside option than the other group (Becker and Pascali 2019; Jha 2013, 2018). In Jha (2013), which lays down the foundation of this theoretical framework, the dominant group cannot abuse the minority under complementarities because the members of the minority group can simply leave the economy. This outside option circumscribes the majority group's ability to prey on the minority for fear of severely shrinking the pie. In our model, giving an outside option to the group with low coercive power would be akin to making the coercive activities of the powerful group ineffective, thereby increasing the maximum degree of resource inequality that can support order in equilibrium. While the exit option assumption is reasonable in many specific cases—perhaps for Muslims

living in port towns in medieval India (Jha 2018)—, we argue that in the absence of an exit options, complementarities might actually foster conflict. To nuance our claim however, note that because the relationship between the degree of complementarities and the likelihood of conflict is non-monotonic, it might be the case as show in Proposition 4 that for a range of resource distributions, starting from perfect substitutes ($\sigma = 0$), increasing complementarities first changes equilibrium from conflict to order, and then back to conflict.

5 Historical case studies

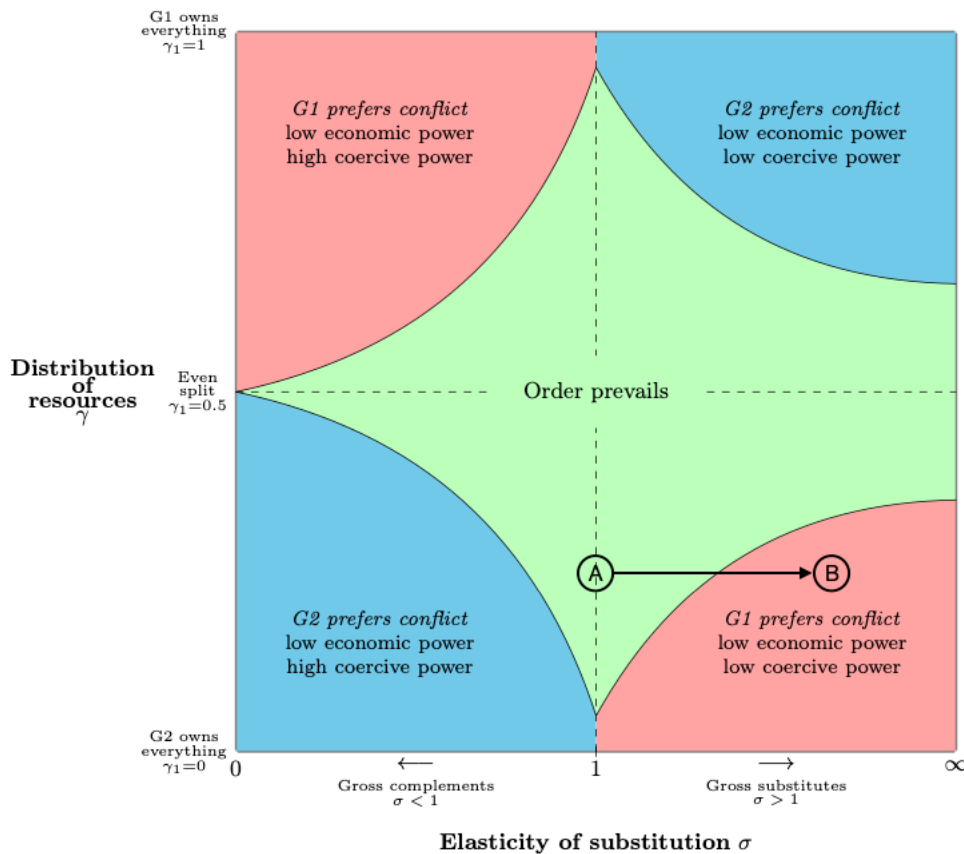
Our very stylized framework can be used to interpret a variety of historical episodes of different natures as it sheds light on the conditions required for social order to either hold or break down. Two parameters lie at the heart of the analysis: the initial distribution of endowments γ and the elasticity of substitution σ . Shifts in either of those can trigger a regime switch, from social order to conflict or vice versa. Inclusive institutions can thus emerge or disappear. In this section, we consider a series of historical case studies that correspond to several potential situations in which a change in some parameter make various social groups willing to sign a social contract or instead renege it.

5.1 Substituting capital for labor: the Luddites rage against the machines

At the onset of the industrial revolution in England, manufacturing production was essentially conducted by highly skilled craftsmen and artisans who progressively disappeared in the face of machine competition. The division of labor, mechanization, and the rise of the factory system indeed rendered them useless as physical capital coupled to unskilled labor could substitute for their skills. Far from the cliché of naive opponents to technological progress, the Luddites were in fact those skilled workers who fought for their employment opportunities.

In light of our model, what pushed the Luddites to break the social contract and engage in rioting was a series of technological innovations that can be viewed as shifting the elasticity of substitution between skilled labor and physical capital. Consider that group 1 and group 2 are skilled laborers and capitalists respectively, and suppose an initial situation, depicted by point A in the figure below, in which we conservatively assume a unitary elasticity of substitution be-

tween physical capital and high skilled labor and a relatively low supply of the latter. Social order prevailed, but industrial technological progress progressively allowed the replacement of such workers with machines. As the market income of the soon-to-be Luddites declined, they were less and less satisfied with the prevailing economic institutions as we move to point B and eventually broke the social contract and the machines that replaced them.

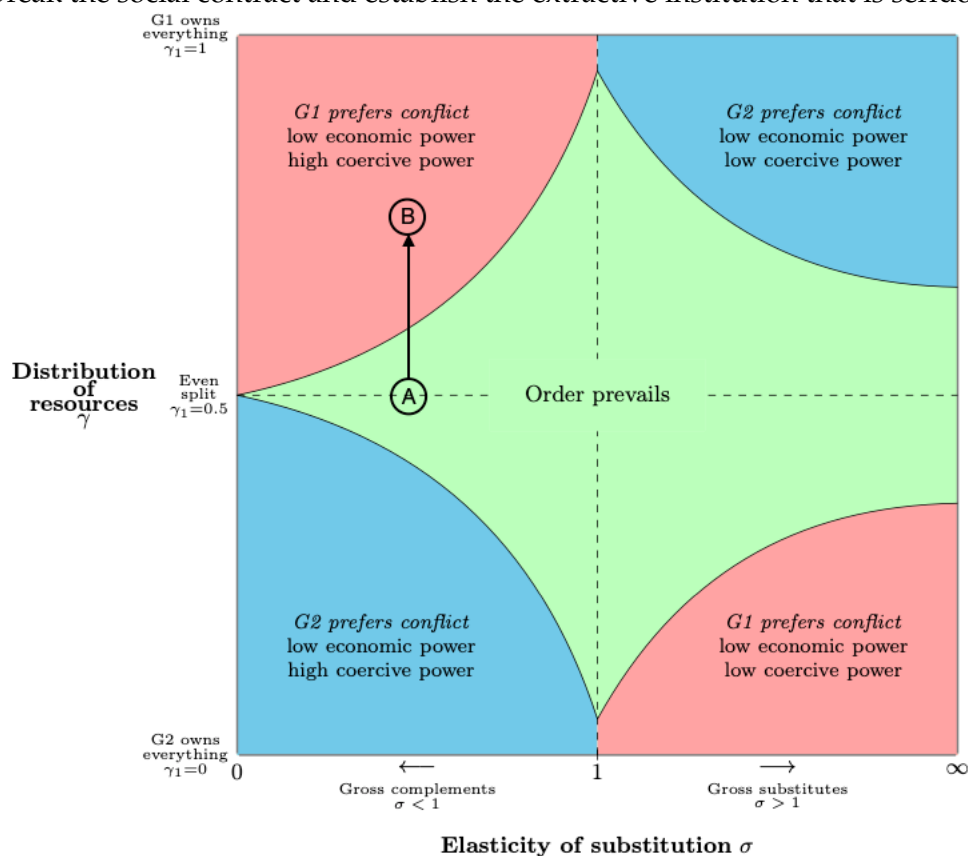


5.2 Changes in the land-labor ratio: Domar's serfdom hypothesis

Rather than a shift in the elasticity of substitution, conflict can also arise from a change in the initial distribution of resources—or factor endowments—depicted here by γ . Factor endowments have indeed been considered crucial in the establishment and persistence of serfdom, an extractive institution ubiquitous in medieval Europe, since the hypothesis laid out by Evsey Domar in the 1970s (Domar 1970) that the origin of serfdom lied in a high land to labor ratio. Domar indeed conjectured that the substantial increase in the land to labor ratio in Eastern Europe following a series of conquests in the 16th century and the ensuing depopulation due to peasant migration to the new territories force led to the establishment of coercive institutions by powerful landowners

willing to retain scarce labor in the central area of Russia.

Domar's serfdom hypothesis can be easily rationalized with our framework as increase in relative factor endowments, or a shift in the distribution of initial resources γ . Let us consider a situation in which group 1 and group 2 represents peasants and landowners respectively. Suppose a relatively equal distribution of endowments between both, and also that the corresponding factors of production (labor and land) are complements such that the elasticity of substitution is lower than one. This initial situation is depicted by point A in the figure below. A rise in the land-to-labor ratio (a decrease in γ) as depicted by point B raises the marginal productivity of peasant labor and the associated market income. Landowners on the other hand see the share of aggregate output they would be entitled too in a functioning market economy decline, which pushes them to break the social contract and establish the extractive institution that is serfdom.



6 Conclusion

This paper is about the conditions required for inclusive institutions to emerge and be sustainable. We argue that the degree of complementarities between social groups productive inputs in collective production—or how they rely on each other to create value—determines whether an initial distribution of resources is compatible with social order or yields conflict instead. More specifically, we show that i) a relatively balanced distribution of economic and coercive power is necessary for inclusive institutions to emerge and be sustainable, ii) there is a non-monotonic relationship between the degree of complementarities and the sustainability of inclusive institutions, and iii) complementarities determine which group favors conflict over social order.

We therefore argue that, in addition to distribution of power within society, the shape of the technology with which social groups produce collectively, characterized here by the elasticity of substitution in the aggregate production function, matters for the type of institutions that eventually prevail. Furthermore, by disentangling coercive and economic power and showing how both depend on the initial distribution of resources, we are able to characterize the winners and losers of inclusive institutions and show that social groups not rewarded enough by the market might challenge such institutions irrespective of their relative coercive power.

We believe our framework, albeit very stylized, has both positive and normative applications. On the one hand, it can be used to shed light on a number of historical case studies where inclusive institutions emerged or social order broke down following a shift in either factor endowments or the elasticity of substitution. The model provides a simple yet useful way to think about the Luddites movement in early 18th century England or the rise of serfdom in medieval Eastern Europe. On the other hand, our analysis suggests to policy makers that an *ex ante* redistribution of resources might align the incentives of various groups with social order. However, we show that the magnitude of the redistribution needed depends on the value of the elasticity of substitution between those groups' inputs in the aggregate production function.

References

Acemoglu, Daron, and James Robinson. 2000. "Why Did the West Extend the Franchise? Democracy, Inequality, and Growth in Historical Perspective." *The Quarterly Journal of Economics* 115 (4): 1167–1199.

- Acemoglu, Daron, and James Robinson. 2006. *Economic Origins of Dictatorship and Democracy*. New York: Cambridge University Press.
- . 2012. *Why Nations Fail*. New York: Crown.
- . 2019. *The Narrow Corridor: States, Societies and the Fate of Liberty*. New York: Penguin.
- . 2022. “Weak, Despotic, or Inclusive? How State Type Emerges from State versus Civil Society Competition.” *American Political Science Review*, 1–14.
- Arrow, Kenneth J., Hollis B. Chenery, Bagicha S. Minhas, and Robert M. Solow. 1961. “Capital-Labor Substitution and Economic Efficiency.” *The Review of Economics and Statistics* 43 (3): 225–250.
- Arrow, Kenneth J., and Gerard Debreu. 1954. “Existence of an Equilibrium for a Competitive Economy.” *Econometrica* 22 (3): 265.
- Becker, Sascha O., and Luigi Pascali. 2019. “Religion, Division of Labor, and Conflict.” *American Economic Review* 109 (5): 1764–1804.
- Domar, Evsey D. 1970. “The Causes of Slavery or Serfdom: A Hypothesis.” *The Journal of Economic History* 30 (1): 18–32. Accessed December 13, 2024.
- Fearon, James D. 1995. “Rationalist Explanations for War.” *International Organization* 49 (3): 379–414.
- Garfinkel, Michelle R., and Stergios Skaperdas. 2007. “Economics of Conflict: An Overview.” In *Handbook of Defense Economics*, edited by Todd Sandler and Keith Hartley, 2:649–709. Amsterdam: Elsevier.
- Herrera, Helios, Massimo Morelli, and Salvatore Nunnari. 2022. “A Theory of Power Wars.” *Quarterly Journal of Political Science* 17 (1): 1–30.
- Hirshleifer, Jack. 1991. “The Paradox of Power.” *Economics and Politics* 3 (3): 177–200.
- Jha, Saumitra. 2013. “Trade, Institutions, and Ethnic Tolerance: Evidence from South Asia.” *American Political Science Review* 107 (4): 806–832.
- . 2018. “Trading for peace.” *Economic Policy* 33 (95): 485–526.
- Morelli, Massimo, Laura Ogliari, and Long Hong. 2023. “Power Mismatch and Civil Conflict: An Empirical Investigation.” *Working Paper*.
- Piketty, Thomas. 2017. *Capital in the Twenty-First Century*. Cambridge: Harvard University Press.
- Powell, Robert. 1999. *In the Shadow of Power: States and Strategies in International Politics*. Princeton: Princeton University Press.
- Sechser, Todd S. 2010. “Goliath’s Curse: Coercive Threats and Asymmetric Power.” *International Organization* 64 (4): 627–660.
- Silve, Arthur. 2018. “Asset Complementarity, Resource Shocks, and the Political Economy of Property Rights.” *Journal of Conflict Resolution* 62 (7): 1489–1516.
- Skaperdas, Stergios. 1992. “Cooperation, Conflict, and Power in the Absence of Property Rights.” *American Economic Review* 82 (4): 720–739.

- Skaperdas, Stergios, and Constantinos Syropoulos. 1997. "The Distribution of Income in the Presence of Appropriative Activities." *Economica* 64 (253): 101–107.
- . 1998. "Complementarity in contests." *European Journal of Political Economy* 14 (4): 667–684.
- Tullock, Gordon. 1967. "The welfare costs of tariffs, monopolies, and theft." *Economic Inquiry* 5 (3): 224–232.
- Waltz, Kenneth N. 1979. *Theory of International Politics*. New York: McGraw-Hill.

7 Appendix

7.1 Proof of existence and uniqueness of the Nash equilibrium

The proof follows Skaperdas and Syropoulos 1997. First, we know that if the payoff function I_i of each player is quasiconcave with respect to that player's strategy, then a pure strategy Nash equilibrium always exists. It is straightforward to show that the payoff function of each player is strictly concave, which ensures existence:

$$\frac{\partial^2 I_i}{\partial p_i^2} = \frac{\partial^2 \delta_i}{\partial p_i^2} \cdot Y + 2 \frac{\partial \delta_i}{\partial p_i} \frac{\partial Y}{\partial p_i} + \delta_i \frac{\partial^2 Y}{\partial p_i^2} < 0$$

Before going further, note the properties of CES production functions and write the derivatives of the production function with respect to production effort rather than predation effort instead:

$$\frac{\partial Y}{\partial p_i} = -\frac{\partial Y}{\partial y_i} < 0 \quad ; \quad \frac{\partial^2 Y}{\partial p_i^2} = \frac{\partial^2 Y}{\partial y_i^2} < 0 \quad ; \quad \frac{\partial^2 Y}{\partial p_i \partial p_{-i}} = \frac{\partial^2 Y}{\partial y_i \partial y_{-i}} > 0$$

Both players' best response functions are denoted by ϕ_i such that $p_i = \phi_i(p_{-i})$.

Then, as in Skaperdas and Syropoulos (1997), if at least one pure-strategy equilibrium is in the interior of the strategy space—which we assume—, then that equilibrium is unique if and only if:

$$0 < \phi'_1(p_2^*)\phi'_2(p_1^*) < 1$$

where p_1^* and p_2^* are players' coercive efforts at equilibrium. Taking the first order conditions of player i and using the implicit function theorem, we have:

$$\phi'_i(p_{-i}) = -\frac{\frac{\partial^2 I_i}{\partial p_i \partial p_{-i}}}{\frac{\partial^2 I_i}{\partial p_i^2}}$$

Let us start with the numerator. We start by rewriting the first derivative of the payoff function of player i as follow:

$$\begin{aligned}
\frac{\partial I_i}{\partial p_i} &= \frac{\partial \delta_i}{\partial p_i} Y + \delta_i \frac{\partial Y}{\partial p_i} \\
&= \frac{\delta_i \delta_{-i}}{p_i} Y - \delta_i \frac{\partial Y}{\partial y_i} \\
&= \frac{\delta_i}{p_i} \left(\delta_{-i} Y - p_i \frac{\partial Y}{\partial y_i} \right)
\end{aligned}$$

Taking the derivative with respect to the other player's strategy gives us the numerator of $\phi'_i(p_{-i})$.

Noting that $\frac{\partial \delta_i}{\partial p_{-i}} = -\frac{\delta_i}{p_i + p_{-i}}$ we get:

$$\begin{aligned}
\frac{\partial^2 I_i}{\partial p_i \partial p_{-i}} &= \frac{1}{p_i + p_{-i}} \underbrace{\frac{\delta_i}{p_i} \left(\delta_{-i} Y - p_i \frac{\partial Y}{\partial y_i} \right)}_{= \frac{\partial I_i}{\partial p_i}} + \frac{\delta_i}{p_i} \left(\underbrace{\frac{\partial \delta_{-i}}{\partial p_{-i}} Y - \delta_{-i} \frac{\partial Y}{\partial y_{-i}}}_{= \frac{\partial I_{-i}}{\partial p_{-i}}} + p_i \frac{\partial^2 Y}{\partial y_i \partial y_{-i}} \right)
\end{aligned}$$

At equilibrium $\frac{\partial I_i}{\partial p_i} = \frac{\partial I_{-i}}{\partial p_{-i}} = 0$, therefore:

$$\frac{\partial^2 I_i}{\partial p_i \partial p_{-i}} = \delta_i \frac{\partial^2 Y}{\partial y_i \partial y_{-i}} > 0$$

Turning to the denominator, let us rewrite it as follows:

$$\frac{\partial^2 I_i}{\partial p_i^2} = \frac{\partial^2 \delta_i}{\partial p_i^2} Y - 2 \frac{\partial \delta_i}{\partial p_i} \frac{\partial Y}{\partial y_i} + \delta_i \frac{\partial^2 Y}{\partial y_i^2} < 0$$

Before going further, note that as the numerator is strictly positive at equilibrium and the denominator is strictly negative, it follows that $\phi'_i(p_{-i}^*) > 0$.

Because $\frac{\partial^2 \delta_i}{\partial p_i^2} Y - 2 \frac{\partial \delta_i}{\partial p_i} \frac{\partial Y}{\partial y_i}$ is strictly negative, it is trivial to bound the denominator as follows:

$$\frac{\partial^2 I_i}{\partial p_i^2} < \delta_i \frac{\partial^2 Y}{\partial y_i^2}$$

Using the above inequality, we obtain:

$$\begin{aligned}
\frac{\partial^2 I_i}{\partial p_i^2} < \delta_i \frac{\partial^2 Y}{\partial y_i^2} &\Leftrightarrow -\frac{\partial^2 I_i}{\partial p_i^2} > -\delta_i \frac{\partial^2 Y}{\partial y_i^2} \\
&\Leftrightarrow -\frac{1}{\frac{\partial^2 I_i}{\partial p_i^2}} < -\frac{1}{\delta_i \frac{\partial^2 Y}{\partial y_i^2}} \\
&\Leftrightarrow -\frac{\frac{\partial^2 I_i}{\partial p_i \partial p_{-i}}}{\frac{\partial^2 I_i}{\partial p_i^2}} < -\frac{\frac{\partial^2 I_i}{\partial p_i \partial p_{-i}}}{\delta_i \frac{\partial^2 Y}{\partial y_i^2}} \\
&\Leftrightarrow \phi'_i(p_{-i}^*) < -\frac{\delta_i \frac{\partial^2 Y}{\partial y_i \partial y_{-i}}}{\delta_i \frac{\partial^2 Y}{\partial y_i^2}} \\
&\Leftrightarrow \phi'_i(p_{-i}^*) < -\frac{\frac{\partial^2 Y}{\partial y_i \partial y_{-i}}}{\frac{\partial^2 Y}{\partial y_i^2}} \\
&\Leftrightarrow \phi'_i(p_{-i}^*) \phi'_{-i}(p_i^*) < \frac{\left(\frac{\partial^2 Y}{\partial y_i \partial y_{-i}}\right)^2}{\frac{\partial^2 Y}{\partial y_i^2} \frac{\partial^2 Y}{\partial y_{-i}^2}}
\end{aligned}$$

Because the CES production function Y is linearly homogeneous, the following is true:

$$y_i \frac{\partial^2 Y}{\partial y_i^2} = -y_{-i} \frac{\partial^2 Y}{\partial y_i \partial y_{-i}}$$

Inserting this into the inequality above, we can show that the right-hand side is exactly equal to one such that:

$$0 < \phi'_i(p_{-i}^*) \phi'_{-i}(p_i^*) < 1$$

which ensures the uniqueness of the equilibrium.