

Mechanism Design by a Politician*

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Abstract

A set of agents has to make a decision about the provision of a public good and its financing. Agents have heterogeneous values for the public good and each agent's value is private information. An agenda-setter has the right to make a proposal about a public-good level and a vector of contributions. For the proposal to be approved, only the favourable votes of a subset of agents are needed. If the proposal is not approved, a type-dependent outside option is implemented. I characterize the optimal public-good provision and the coalition-formation for any outside option in dominant strategies. Optimal public-good provision might be a non-monotonic function of the outside option public-good level. Moreover, the optimal coalition might be a non-convex set of types.

Keywords: legislative coalitions, public goods, mechanism design, reservation utility

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1 Introduction

This paper revisits a classic mechanism design problem, the provision of a public good, by considering a political economy environment. To this end, I aim at unifying two different views on political economy.

From a macroeconomics perspective, political economy developed at the intersection of Economics and Political Science as an attempt to incorporate political dynamics into the analysis of economic policy. In the words of [Alesina and Perotti \(1994\)](#):

“Political-economy models begin with the assertion that economic policy choices are not made by social planners, who live only in academic papers. Rather, economic policy is the result of political struggle within an institutional structure.”

On the other hand, drawing on the developments in the theory of screening¹, an alternative view of political economy has been put forth by the work of Jean-Jacques Laffont and coauthors.² The Constitution-politician agency problem is reminiscent of the principal-agent problem in the theory of contracts. As it was made clear in [Laffont \(2000\)](#),

“[p]olitical economy is the recognition of the incompleteness of the Constitution [...]. Because the Constitution is an incomplete contract, politicians have a lot of discretion, that they use to further interest groups.”

In particular, the incompleteness of the Constitution entails the absence of adequate institutional tools to cope with asymmetric information. In fact, according to [Laffont \(2001\)](#):

“... a major inefficiency of political conflicts follows from the inefficiency of redistributive instruments due to asymmetric information. [...] Politics is then a game of redistribution of information rents within the realm of discretion left by the Constitution.”

In this paper, I consider a public-good provision model with privately-informed, heterogeneous agents where the mechanism designer is one of the agents. In particular, this agent, whom I refer to as the ‘politician’, is the agenda-setter in a collective choice problem with three main features.

First, the collective choice entails the provision of a nonrival, nonexcludable public good and its financing. Second, the contributions borne by different agents are allowed to be different. Third, in order for the proposal to be implemented, the agenda-setter only needs the support of a subset of the agents (e.g., simple or supermajority).

If the proposal is not approved, a type-dependent outside option is implemented. Drawing from insights in the theory of screening, I characterize optimal public-good provision

¹For a recent and thorough overview, see [Börger et al. \(2015\)](#).

²For an extensive treatment, see [Laffont \(2001\)](#).

and coalition formation for any outside option. I focus on dominant-strategy implementation.

The main idea is a fundamental connection between monopolistic screening and agenda-setting. When there is a single agent, the two problems are identical. Like a monopolist, the agenda-setter has to elicit from the agent his/her value for the public-good and offer him/her a contract involving a quantity (in this case, a public-good level) in exchange for a transfer. When there are multiple agents, the two problems are still mathematically equivalent *ex ante*. However, the *ex post* constraints are fundamentally different.

In fact, whereas in monopolistic screening the quantity of the good traded is consumed privately by each agent, the public good is nonrival and nonexcludable; hence, once a public-good level has been chosen, all agents will consume it, regardless of the individual contributions to its financing.

Building on key contributions in monopolistic screening with type-dependent outside options (Lewis and Sappington (1989), Maggi and Rodriguez-Clare A. (1995), Jullien (2000)), I show that the shape of the reservation utility profile, namely the utility profile yielded by the outside option, crucially determines the optimal contract, and hence public-good provision and coalition formation.

Intuitively, for a given outside option, once a public-good level has been provided, some agents will have the *ex post* incentive to understate their preference for the public good, whereas others will have the *ex post* incentive to overstate it. If the agenda-setter can provide a public-good level which is consistent with all these constraints, then the public-good provision will differ from the outside option. Otherwise, the outside option will be implemented.

When the agenda-setter only needs a subset of favourable votes for the proposal to be implemented, some agents can be excluded from the winning coalition and therefore their participation constraints can be violated. If the incentive compatibility constraints are required to hold only for the members of the coalition, then it is optimal to randomly choose the agents and propose the mechanism to $q - 1$ of them, while taxing the others to some upper bound $\bar{\tau}$.

If the incentive compatibility constraints have to hold for all agents, including those left out of the winning coalition, then also this set of excluded agents will affect the optimal public-good provision, since those agents still have to be given the incentive to truthfully reveal their value for the public good.

Interestingly, the winning coalition varies significantly with the outside option and the shape of the reservation utility profile. When the outside option public-good level is sufficiently low (high), it is optimal to choose the agents with the ‘highest’ (‘lowest’) value for the public good, *regardless* of the shape of the reservation utility profile.

On the other hand, for intermediate values of the outside option public-good level, the winning coalition crucially depends on the concavity or convexity of the reservation

utility profile. When it is concave, the winning coalition can be a *non-convex* set of agents, comprising agents with high and low values for the public good, while excluding other agents with intermediate values. When the reservation utility profile is convex, the winning coalition is a convex set of agents, excluding either the agents with the highest, or lowest, value for the public good.

Optimal public-good provision is indeed the result of a ‘game of redistribution of information rents’, as pointed out by [Laffont \(2001\)](#).

The rest of the paper is structured as follows. [Section 2](#) reviews the literature. [Section 3](#) introduces the model. [Section 4](#) and [Section 5](#) lay out the analysis. [Section 6](#) foreshadows the main results, which are then presented in [Section 7](#). [Section 8](#) characterizes the implications of the main results. [Section 9](#) discusses the relaxations of the assumptions and [Section 10](#) concludes. All proofs are in [Appendix A](#).

2 Related Literature

This paper contributes to the literature on public-good provision in political economy and mechanism design. In political economy, an established literature develops a positive theory of public-good provision in a legislative bargaining framework, as in [Baron and Ferejohn \(1989\)](#), [Battaglini and Coate \(2007\)](#), [Battaglini and Coate \(2008\)](#) and [Bowen et al. \(2014\)](#). These contributions study the bargaining between heterogeneous players in a complete-information setting. In contrast to them, I consider a setting in which agents are privately informed about their valuations for the public good.

Within mechanism design, the contribution of this paper is twofold. First, following the pioneering contribution of [Green and Laffont \(1977\)](#), a branch of the literature has studied the design of optimal mechanisms for the provision of a public good in dominant strategies.³ The optimality of the mechanisms in these models hinges on the standpoint of a benevolent social planner seeking to maximize aggregate utilitarian welfare. Moreover, these models concern the provision of a fixed-size public project.

In this paper, the mechanism designer is one of the agents; hence, s/he is privately informed and seeks to maximize his/her utility, subject to the institutional constraints. In addition to this, the size of the project (the public-good level) is endogenous to the coalition-formation problem, and not fixed *a priori*.

There is also a relatively small literature on monopolistic screening with agents having type-dependent outside options ([Lewis and Sappington \(1989\)](#), [Baron and Myerson \(1982\)](#), [Maggi and Rodriguez-Clare A. \(1995\)](#), [Jullien \(2000\)](#), [Figuerola and Skreta \(2009\)](#)). These models consider the optimal contract of a monopolist to a privately-informed agent who has an outside option, which also depends on his/her type.

³For a comprehensive treatment, see [Green and Laffont \(1979\)](#). For recent contributions, see [Kuzmics and Steg \(2017\)](#) and the references therein.

As an interesting application of the same framework, [Teichgräber et al. \(2022\)](#) consider the optimal design of short-time work schemes when firms have private information about their productivity and type-dependent outside options.

The present paper adopts a similar viewpoint, whereby the agenda-setter needs to propose a mechanism to the agents in order to maximize his/her utility. However, the mechanism designer is providing the agents with a nonrival, non-excludable public good, and not with a private good. Therefore, the *ex post* constraints differ from the ones in those models.

At the intersection of political economy and mechanism design, [Bierbrauer and Hellwig \(2016\)](#) consider coalition-incentive compatible mechanisms and show that they are voting mechanisms. [Scheuer and Wolitzky \(2016\)](#) study the optimal dynamic capital tax policy for a government with limited commitment power due to political coalition formation. In order for a policy to be credible, the government needs the support of a large enough share of the electorate, hence leading to an endogenous coalition formation. To the best of my knowledge, this is the first paper studying a coalition-formation problem in a mechanism design setting.

The closest contributions to the present paper are [Gersbach et al. \(2023\)](#) and [Dragu and Laver \(2017\)](#).

The former studies the coalition-formation problem of an agenda-setter in an environment where agents have heterogeneous preferences for the public good and their utilities are logarithmic. Importantly, however, the authors consider a complete information setting. Instead the focus of the present paper is on a private information setting; moreover, a more general utility function is considered. A thorough comparison between the two contributions is deferred to [Section 9.6](#).

[Dragu and Laver \(2017\)](#) adopts a mechanism design approach to study which legislative majority coalitions will be formed in an environment where politicians are privately informed about their policy preferences. It is shown that the equilibrium policy coalitions will be either center left or center right, depending on whether the median party has a policy position to the left or right of the outside option.

The present paper departs from this analysis in two fundamental regards. First, the agenda-setter is one of the agents and s/he is able to control the resource constraint; i.e., in addition to a public-good level (or a specific policy), s/he can propose a vector of net contributions which can differ across agents, reflecting the information rents that have to be paid for them to report their type truthfully.

Second, whereas [Dragu and Laver \(2017\)](#) normalize the outside option to zero, I characterize the solution for any outside option. I show that, depending on the shape of the reservation utility profile and the outside option public-good level, very different coalitions can arise in equilibrium.

3 Model

There is a set $N = \{1, \dots, n\}$ of agents, with $n \geq 2$. They have to make a collective choice about the provision of a nonrival, nonexcludable public good and its financing. Agents are heterogeneous with respect to their value for the public good. For each agent $i \in N$, the value for the public good (or the ‘type’) θ_i is private information. In particular, $\theta_i \in \Theta_i \equiv [\underline{\theta}, \bar{\theta}]$ has a cumulative distribution function F_i with strictly positive density $f_i(\theta_i) > 0$. Denote by θ the generic vector $(\theta_1, \theta_2, \dots, \theta_n)$, where $\theta \in \Theta \equiv [\underline{\theta}, \bar{\theta}]^N$. Moreover, denote $f(\theta) = \prod_{i \in N} f_i(\theta_i)$.

The agenda-setter, $a \in N$, makes a proposal consisting of a public-good level and a vector of contributions. Formally, the proposal is a $n + 1$ -tuple $(g, t_i)_{i \in N}$, where $g \in \mathbb{R}_+$ is the public-good level and $t_i \in \mathbb{R}$ is the contribution of agent i . The cost of producing the public good is linear: $c(g) = g$. The proposal has to satisfy a resource constraint

$$g \leq \sum_{i \in N} t_i.$$

In other words, the total contributions by the agents need to be at least as high as the cost of providing the public good. Given a policy proposal, agent i ’s utility is given by

$$u_i(\theta_i, g, t_i) = \theta_i \phi(g) - t_i$$

with $\phi : \mathbb{R}_+ \longrightarrow \mathbb{R}_+$ being a twice-differentiable, non-decreasing and concave function. Moreover, it satisfies $\phi(0) = 0$.

The agenda-setter needs the support of $q \leq n$ agents to have his/her policy implemented. If the proposal is not approved, then an outside option public-good level is implemented. This yields utility $\bar{v}_i(\theta_i, g^\circ)$, where $\bar{v}_i$ is non-decreasing in g° , twice-differentiable and it satisfies $\bar{v}_i(\cdot, 0) = 0$. Importantly, $\bar{v}_i(\theta_i, g^\circ)$ is type-dependent. Moreover, the utility levels from the agenda-setter’s proposal may differ from those under the outside option. This generality is meant to capture some potential irreversibilities entrenched in the institutional framework.

For example, a set of local governments might want to provide a nation-wide public-good provision. In this case, the provision of the public good at the national level generates utility levels which are different under the outside option. This might be due to the presence of fixed costs or the impossibility of providing the same public-good level at the same cost with the local technology.⁴

Alternatively, the outside option might be linked to the marginal benefit of the policy proposal in a nonlinear way. For example, some agents might benefit from the policy proposal the most because they would be the worst-off under the outside option.

⁴Caveat: different technologies might imply different utility function $\bar{v}_i$.

Therefore, no restriction is placed on the shape of the reservation utility profile.

4 First-Best Analysis

What is the best the agenda-setter can do? By the Revelation Principle (Myerson (1979)), we can restrict our attention to direct mechanisms without loss of generality.

Definition 1. A direct mechanism is a set of functions g and t_i , $\forall i \in N$

$$\begin{aligned} g : \Theta &\longrightarrow \mathbb{R}_+, \\ t_i : \Theta &\longrightarrow \mathbb{R}. \end{aligned}$$

4.1 VCG Mechanisms

We now consider the class of VCG mechanisms and see whether they can implement the ‘efficient’ (utilitarian first-best) public-good level.⁵ Is this allocation implementable? We first need two definitions.

Definition 2. The public-good level g is *efficient* if it solves

$$\sum_N \theta_i \phi'(g) = 1. \quad (1)$$

Definition 3. A direct mechanism $(g, t_i)_{i \in N}$ is called a ‘Vickrey-Clarke-Groves’ (VCG) mechanism if g is efficient and if for every $i \in N$ there is a function

$$\tau_i : \Theta_i \longrightarrow \mathbb{R}$$

such that

$$t_i(\theta) = - \sum_{j \neq i} u_j(\theta, g(\theta)) + \tau_i(\theta_{-i}).$$

Proposition 1 (Krishna and Maenner (2001)). *Suppose that for every $i \in N$ the set Θ_i is a convex subset of a finite-dimensional Euclidean space. Moreover, assume that for every $i \in N$ the function $u_i(\theta_i, g(\theta))$ is a convex function of θ_i . Suppose that $(g, t_i)_{i \in N}$ is a dominant strategy incentive-compatible mechanism. Then, a direct mechanism with the same $(g(\theta), t'_i)_{i \in N}$ is dominant strategy incentive-compatible if and only if for every $i \in N$ and every θ_{-i} there is a number $\tau_i(\theta_{-i}) \in \mathbb{R}$ such that*

$$t'_i(\theta) = t_i(\theta) + \tau_i(\theta_{-i}) \quad \forall \theta \in \Theta.$$

⁵The discussion in this section follows Börgers et al. (2015).

Corollary 1 (Börger et al. (2015)). Suppose that for every $i \in N$ the set Θ_i is a convex subset of a finite-dimensional Euclidean space. Moreover, assume that for every $i \in N$ the function $u_i(\theta, g(\theta))$ is a convex function of θ_i . Suppose that $(g, t_i)_{i \in N}$ is a dominant strategy incentive-compatible mechanism, and suppose that g is efficient. Then, $(g, t_i)_{i \in N}$ is a VCG mechanism.

In our setting, the two convexity conditions are met. Hence, VCG mechanisms are the only mechanisms that make efficient public-good level decision rules dominant strategy incentive-compatible.

However, we have the following

Theorem 1. No VCG mechanism is implementable.

Proof. For simplicity, assume $\phi(g) = \ln(1+g)$. Consider the public good g^* and the transfers $t_i^*(\theta, g^*)_{i \in N \setminus \{a\}}$ of a VCG mechanism. By standard arguments (Green and Laffont (1979)), transfers do not cover the cost of the public good and external funds are needed to balance the budget. Now, pick the agent with the lowest type, and denote it θ_j . Consider agent θ_j 's variation in utility brought about by a slight decrease in g^* offset by a decrease in t_j^* . Formally

$$\begin{aligned} \theta_j \ln(1+g^*-\delta) - \theta_j \ln(1+g^*) &= \underbrace{t_j^*(\theta, g^*-\delta) - t_j^*(\theta, g^*)}_{\epsilon} \\ \theta_j \ln\left(\sum_{i \in N} \theta_i - \delta\right) - \theta_j \ln\left(\sum_{i \in N} \theta_i\right) &= \epsilon \\ \delta &= \sum_{i \in N} \theta_i (e^{\frac{\epsilon}{\theta_j}} - 1) \end{aligned}$$

By construction, agent j 's utility is unchanged. All other agents θ_k , with $k \in N \setminus \{a, j\}$, have a utility loss equal to $\epsilon - \frac{\theta_k}{\theta_j} \epsilon$. Therefore, for the transfers $t_i^*(\theta, g^*-\delta)_{i \in N \setminus \{a\}}$ to be still incentive-compatible, the proposer needs to compensate them. Now, consider the variation in the proposer's utility.

$$\begin{aligned} \theta_a \ln(1+g^*-\delta) - \theta_a \ln(1+g^*) - \sum_{i \in N \setminus \{a\}} \left(\frac{\theta_i}{\theta_j} - 1 \right) \epsilon - [(g^*-\delta) - g^*] \\ = \sum_{i \in N} \theta_i (e^{\frac{\epsilon}{\theta_j}} - 1) - \sum_{i \in N} \frac{\theta_i}{\theta_j} \epsilon \\ = O(\epsilon) > 0, \end{aligned}$$

where the last equality follows from the Taylor expansion of e^x around $x = 0$. This argument generalizes to arbitrary functions $\phi(g)$. Hence, g^* is not implementable. $\square$

Theorem 1 has a simple interpretation. Suppose that the efficient public-good level is known and it is set in the Constitution. Moreover, suppose that there is availability of external funds for the set of agents to implement the efficient public-good provision. Even

in that case, it would not be possible to find a mechanism to implement that public-good level in dominant strategies.

5 Second-best Mechanisms

We now consider second-best mechanisms. Formally, there is the following sequence of events.

1. An agenda-setter $a \in N$ has the right to make a proposal.
2. Each agent $i \in N$ is privately informed about his/her own value for the public-good θ_i .
3. The agenda-setter asks each agent $i \in N$ to reveal his/her value for the public good and simultaneously makes a take-it-or-leave-it policy proposal consisting of a public-good level $g(\theta)$ and a vector of transfers to finance it, $t_i(\theta)$.
4. The voting stage occurs secretly. If at least $q \leq n$ agents vote in favor of the policy proposal, then it is implemented. If not, an outside option public-good level is implemented, yielding utilities $\bar{v}_i(\theta_i, g^\circ)$ to each agent $i \in N$.

Denote

$$\mathcal{Q} \equiv \{Q \subseteq N : a \in Q \text{ and } |Q| = q\}$$

as the set of all minimal winning coalitions that include a . The agenda-setter solves the following problem

$$\begin{aligned} & \max_{g(\theta), t(\theta), Q} \int \theta_a \phi(g(\theta)) - t_a(\theta) dF(\theta) \\ & \text{subject to} \quad g(\theta) \leq t_a(\theta) + \sum_{i \in N \setminus \{a\}} t_i(\theta), \\ & \quad \text{and for some } Q \in \mathcal{Q}, \\ & \quad \theta_i \phi(g(\theta)) - t_i(\theta) \geq \theta_i \phi(g(\theta'_i, \theta_{-i})) - t_i(\theta'_i, \theta_{-i}) \quad \forall \theta_i, \theta'_i \in \Theta_i, \theta_{-i} \in \Theta_{-i} \text{ and } i \in N \setminus \{a\}, \\ & \quad \theta_j \phi(g(\theta)) - t_j(\theta) \geq \bar{v}_j(\theta_j, g^\circ) \quad \forall j \in Q. \end{aligned}$$

The first constraint is the *resource constraint*: the net contributions of the agents have to be enough to cover the public-good expenditure.

The second is the *incentive compatibility constraint*: every agent $i \in N \setminus \{a\}$ needs to have an incentive to reveal his/her true type truthfully to the agenda-setter.

The last constraint is the *participation constraint*: the utility yielded by the proposed policy to the agents in the coalition needs to be at least as high as the one yielded by the outside option.

The participation constraint needs to be satisfied only for those $q - 1$ agents included in the coalition; the remaining agents can be forced to participate. However, they have to be given the incentive to reveal their type truthfully; hence, the incentive compatibility constraint has to be satisfied for all agents.

This is consistent with alternative interpretations of the model. First, there might be political economy constraints preventing the agenda-setter to extract resources from agents who do not participate in the mechanisms.⁶ Second, the agenda-setter might be thought of as a government placing any set of welfare weights to the agents. For example, an egalitarian government might place a unit weight on the agent(s) with the lowest type. In order to identify the type(s) of interest, the types of all the agents have to be elicited.⁷ To begin with, a few assumptions are in order. They will be relaxed in [Section 9](#).

Assumption 1. *The agenda-setter has full commitment power.*

This assumption rules out the possibility that the agenda-setter might modify the allocation rule after it has been announced and the preferences of the other agents have been revealed.

Assumption 2. *The density functions $f_i(\theta_i)$, for every $i \in N$, are log-concave.*

This assumption is standard in the literature and it ensures that the inverse of the hazard rate is increasing in types.

Assumption 3. *The reservation utility profile is implementable.*

This assumption is referred to as *homogeneity* in [Jullien \(2000\)](#). It requires that the reservation utility profile be implementable without excluding any agent. It implies that $\bar{v}_i$ is non-decreasing in types.

Standard results in mechanism design theory ([Maggi and Rodriguez-Clare A. \(1995\)](#)) yield the following lemma.

Lemma 1. *A direct mechanism $(g, t_i)_{i \in N}$ is dominant-strategy incentive compatible if and only if for every $\theta_i \in \Theta_i$ and $\theta_{-i} \in \Theta_{-i}$:*

(i) $g(\theta_i, \theta_{-i})$ is increasing in θ_i ;

(ii) For every $\theta_i \in [\underline{\theta}, \bar{\theta}]$ we have

$$dU_i/d\theta_i = \phi(g(\theta)) - \bar{v}'_i(\theta_i, g^\circ), \quad (2)$$

⁶By this, I do not mean the agents who participate but whose participation constraints are violated. Rather, they are agents who are not even asked by the agenda-setter to participate in the mechanism; e.g., this would be the case if the agenda-setter chose randomly $q - 1$ agents and proposed them a direct mechanism.

⁷However, when the agenda-setter is indeed one of the agents seeking to maximize his/her expected utility, it might be reasonable to assume that the incentive compatibility constraint has to hold only for the members of the winning coalition. In [Section 9](#), it is shown that the agents in the winning coalition Q will be chosen randomly.

where $U_i(\theta_i) = \theta_i \phi(g(\theta)) - \bar{v}_i(\theta_i, g^\circ)$.

The first condition is the standard *monotonicity constraint*, a well-known necessary condition for implementability of the allocation rule.

Condition (2) is an envelope condition which shows the two ‘countervailing incentives’⁸ at play. On the one hand, agents have the incentive to understate their value for the public good: since it is nonrival and nonexcludable, all agents will benefit from its consumption, independently of who contributes to its financing.

On the other hand, agents have the incentive to overstate their types so as to overstate their reservation utility, the utility yielded by the outside option. Depending on which effects dominates, agents will have the overall incentive to understate or overstate their value for the public good.

Condition (2) implies that, over the set of types for whom the dominating incentive is to understate, information rents will be *increasing* in types. Analogously, when the dominating incentive is to overstate the types, information rents will be *decreasing* over this set of types.

This implication of dominant-strategy incentive compatibility will play a major role in the subsequent analysis.

6 Linear Reservation Utility Profile

In order to foreshadow the main results, we consider a particular functional form for the reservation utility profiles, namely the case in which $\bar{v}_i(\cdot)$ is linear in types:

$$\bar{v}_i(\theta_i, g^\circ) = \theta_i \phi(g^\circ) - \frac{g^\circ}{n}, \quad \forall i \in N.$$

The outside option is given by a outside option public-good level g° , and a uniform array of transfers to finance it.⁹ Importantly, by condition (2), the incentive to understate or overstate the type is given by

$$dU_i/d\theta_i = \phi(g(\theta)) - \phi(g^\circ). \quad (3)$$

Hence, the sign of (3) is type-independent. The analysis is thus greatly simplified.

⁸To the best of my knowledge, the term was coined by [Lewis and Sappington \(1989\)](#) and later used by [Maggi and Rodriguez-Clare A. \(1995\)](#).

⁹This is a common assumption in the political economy literature; see e.g., [Battaglini and Coate \(2008\)](#) and [Gersbach et al. \(2023\)](#).

6.1 Unanimity

Suppose first that the institutional constraints require that the proposal be approved by unanimity; in other words, $q = n$. The agenda-setter solves the following problem

$$\begin{aligned}
& \max_{g(\theta), t(\theta)} \int \theta_a \phi(g(\theta)) - t_a(\theta) dF(\theta) \\
& \text{subject to} \quad g(\theta) \leq t_a(\theta) + \sum_{i \in N \setminus \{a\}} t_i(\theta), \\
& \quad \frac{dg(\theta)}{d\theta} \geq 0, \\
& \quad dU_i/d\theta_i = \phi(g(\theta)) - \phi(g^\circ), \\
& \quad \theta_i \phi(g(\theta)) - t_i(\theta) \geq \theta_i \phi(g^\circ) - \frac{g^\circ}{n} \quad \forall i \in N \setminus \{a\}.
\end{aligned} \tag{LU}$$

Denote $g^*(\theta)$ as the solution to LU.

Lemma 2. *If $g^*(\theta) \neq g^\circ$, then the participation constraint can bind at most at one type.*

Proof. Suppose not. Then, there exist two agents, θ_k and θ_l , with $\theta_k > \theta_l$, whose participation constraints are binding. However, by condition (3) the incentive to understate or overstate the type is type-independent.

But then, if $g^*(\theta) > g^\circ$, all agents have *ex post* the incentive to understate their type. Analogously, if $g^*(\theta) < g^\circ$, all agents have *ex post* the incentive to overstate their type. Therefore, in the first case θ_k could pay a lower contribution by reporting θ_l ; in the second case, θ_l could pay a lower contribution by reporting θ_k . This contradicts the optimality of the participation constraints of both agents being binding. $\square$

Then, from Lemma 1 and Lemma 2 we immediately have the following result.

Proposition 2. *There exist two threshold values, g_U^L and g_U^H , such that:*

$$g^*(\theta) = \begin{cases} g_U^L & \text{if } g^\circ < g_U^L, \\ g^\circ & \text{if } g_U^L \leq g^\circ \leq g_U^H, \\ g_U^H & \text{if } g^\circ > g_U^H. \end{cases}$$

where g_U^L and g_U^H solve, respectively,

$$\begin{aligned}
& \left[\theta_a + \sum_{i \in N \setminus \{a\}} \left(\theta_i - \frac{1 - F_i(\theta_i)}{f_i(\theta_i)} \right) \right] \phi'(g(\theta)) = 1 \\
& \left[\theta_a + \sum_{i \in N \setminus \{a\}} \left(\theta_i + \frac{F_i(\theta_i)}{f_i(\theta_i)} \right) \right] \phi'(g(\theta)) = 1.
\end{aligned}$$

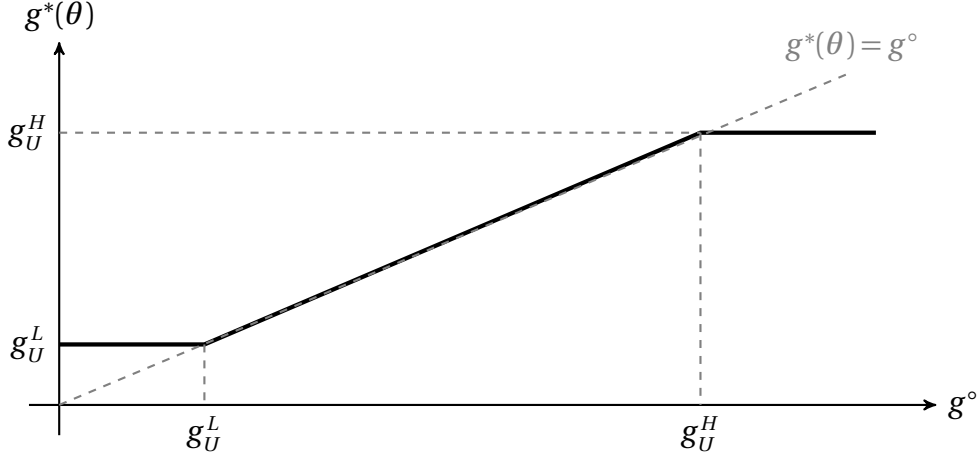


Figure 1: $g^*(\theta)$ in the linear case with unanimity.

Figure 1 illustrates the solution.

The intuition behind the result is very simple. The incentive to understate or overstate the type is *ex post* type-independent: once a public-good level $g^*(\theta)$ has been provided, all agents will have the *ex post* incentive to either understate or overstate their type, depending on the sign of (3).

If the outside option public-good level is sufficiently low ($g^o \leq g_U^L$), all agents will have the incentive to understate their type. If the outside option public-good level is sufficiently high ($g^o \geq g_U^H$), all agents will have the incentive to overstate their type.

Finally, if the outside option public-good level assumes intermediate values ($g_U^L \leq g^o \leq g_U^H$), then some agents will have the incentive to understate, whereas some other agents will have the incentive to overstate. The two conflicting incentives cannot be consistent *ex post* with (3). Hence, the agenda-setter can only propose the outside-option public-good level.

6.2 Majority

Suppose now that $q \leq n$. The agenda-setter solves

$$\begin{aligned}
 & \max_{g(\theta), t(\theta), Q} \int \theta_a \phi(g(\theta)) - t_a(\theta) dF(\theta) \\
 & \text{subject to} \quad g(\theta) \leq t_a(\theta) + \sum_{i \in N \setminus \{a\}} t_i(\theta), \\
 & \quad \quad \quad \frac{dg(\theta)}{d\theta} \geq 0, \tag{LM} \\
 & \quad \quad \quad dU_i/d\theta_i = \phi(g(\theta)) - \bar{v}'_i(\theta_i, g^o) \quad \forall i \in N \setminus \{a\}, \\
 & \quad \quad \quad \text{and for some } Q \in \mathcal{Q}, \\
 & \quad \quad \quad \theta_j \phi(g(\theta)) - t_j(\theta) \geq \theta_j \phi(g^o) - \frac{g^o}{n} \quad \forall j \in Q.
 \end{aligned}$$

The first three constraints are the same as in the unanimity problem. However, in addition to $g(\theta)$ and $\mathbf{t}(\theta)$, the agenda-setter now also needs to choose a coalition $Q \in \mathcal{Q}$.

For any $Q \in \mathcal{Q}$, let $\Xi(Q) = \sum_{i \in Q} \theta_i$ be Q 's value for the public good. Define

$$\underline{Q} \equiv \arg \min_{Q \in \mathcal{Q}} \Xi(Q) \quad \text{and} \quad \overline{Q} \equiv \arg \max_{Q \in \mathcal{Q}} \Xi(Q),$$

to be, respectively, the 'lowest' and 'highest' coalitions as measured by their value for the public good. Along the same lines as in the unanimity case, we obtain the following

Proposition 3. *There exist a set of agents I , with $|I| \leq n - q$, two cutoff types, $\tilde{\theta}, \bar{\theta} \in [\underline{\theta}, \bar{\theta}]$ and two threshold public-good values, $g_M^L, g_M^H \in \mathbb{R}_+$, such that, if $g_M^L < g_M^H$,*

$$g^*(\theta) = \begin{cases} g_M^L & \text{if } g^\circ < g_M^L, \\ g^\circ & \text{if } g_M^L \leq g^\circ \leq g_M^H, \\ g_M^H & \text{if } g^\circ > g_M^H, \end{cases}$$

where g_M^L and g_M^H solve, respectively,

$$\begin{aligned} \left[\theta_a + |I| \cdot \tilde{\theta} + \sum_{i \in \overline{Q} \setminus \{a\}} \left(\theta_i - \frac{1 - F_i(\theta_i)}{f_i(\theta_i)} \right) \right] \phi'(g(\theta)) &= 1, \\ \left[\theta_a + |I| \cdot \bar{\theta} + \sum_{i \in \underline{Q} \setminus \{a\}} \left(\theta_i + \frac{F_i(\theta_i)}{f_i(\theta_i)} \right) \right] \phi'(g(\theta)) &= 1. \end{aligned}$$

Moreover,

$$Q^* = \begin{cases} \overline{Q} & \text{if } g^\circ < g_M^L, \\ \text{any } Q \in \mathcal{Q} & \text{if } g_M^L \leq g^\circ \leq g_M^H, \\ \underline{Q} & \text{if } g^\circ > g_M^H. \end{cases}$$

If $g_M^L > g_M^H$ for some $q \leq n$, then

$$g^*(\theta) = \begin{cases} g_M^L & \text{if } g^\circ < g_M^L, \\ g_M^H & \text{if } g^\circ > g_M^L. \end{cases}$$

Moreover,

$$Q^* = \begin{cases} \overline{Q} & \text{if } g^\circ < g_M^L, \\ \underline{Q} & \text{if } g^\circ > g_M^L. \end{cases}$$

Figure 2 and Figure 3 illustrate the solution.

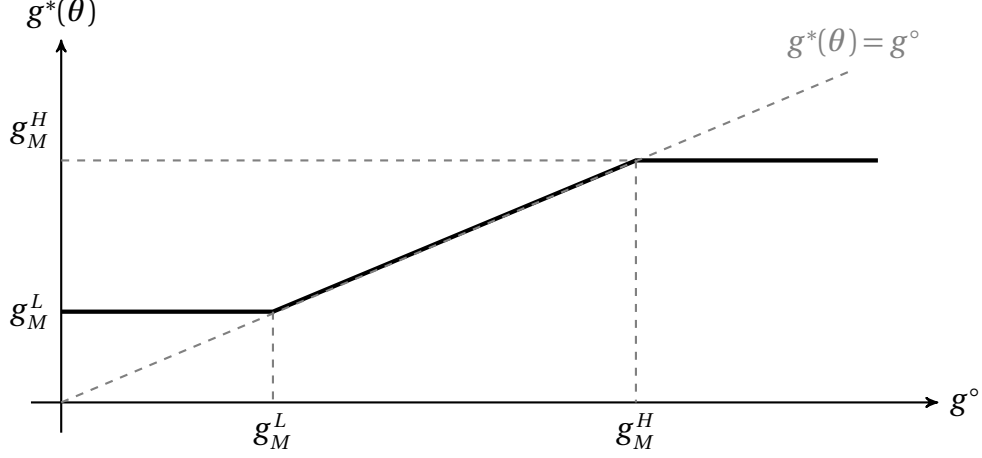


Figure 2: $g^*(\theta)$ in the linear case with majority ($g_M^L < g_M^H$).

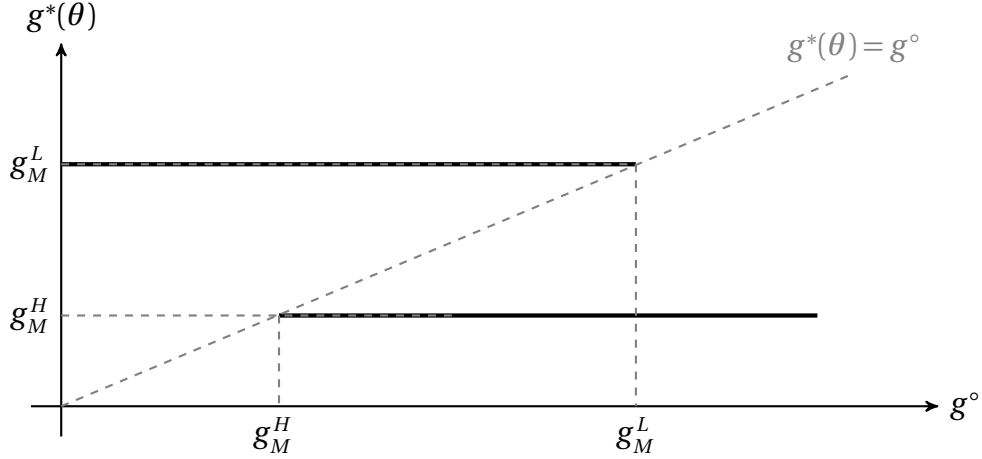


Figure 3: $g^*(\theta)$ in the linear case with majority ($g_M^L > g_M^H$).

The reasoning is similar to the unanimity case. Notice, however, that the thresholds g_M^L and g_M^H are different.

In fact, unlike the unanimity case, the agenda-setter can now exclude some agents from the winning coalition; i.e., some agents can be forced to participate in the collective choice. However, they have to be given the incentive to truthfully report their types. Therefore, they will also affect public-good provision. The intuition behind [Proposition 3](#) is again related to (3).

If $g^*(\theta) > g^o$, then all agents have the *ex post* incentive to understate their types. Moreover, information rents have to be increasing in types.

Intuitively, when the outside option is low, incentive constraints bind downwards and the participation constraint binds on some type $\tilde{\theta}$. All agents with higher types must earn information rents; however, the types below can be bunched at the same allocation of $\tilde{\theta}$,

thus violating their participation constraints. It is then optimal to choose $\tilde{\theta}$ in such a way that the highest $q-1$ agents earn information rents. An analogous reasoning goes through for the case where g° is high.

Interestingly, when for some q it is the case that $g_M^L > g_M^H$, public-good provision becomes non-monotonic in the outside option public-good level. This is the same result obtained in [Gersbach et al. \(2023\)](#).

The results in this section come as no surprise. When the reservation utility profile is linear in types, condition (3) states that the agents' incentive to misreport their types is type-independent. Hence, the agenda-setter is able to implement a collective choice which is different from the outside option only when the incentives are aligned for all agents. This is the case when the outside option public-good level is sufficiently low or sufficiently high.

On the other hand, when the outside option public-good level assumes intermediate values, the only collective choice which can be implemented in dominant strategies is proposing the outside option.

The same reasoning applies when the agenda-setter only needs the support of a subset of agents. Dominant strategy incentive compatibility severely restricts the ability of the agenda-setter to 'exploit' the agents excluded from the winning coalition.

7 Main Results

In this section, I consider a general reservation utility profile $\bar{v}_i$. Before stating the two main results of this paper, some preliminary definitions and results are in order.

A key insight from [Jullien \(2000\)](#) is that the shadow value on the participation constraints at the optimal allocation can be represented by $\gamma^*(\theta_i)$, which is a cumulative distribution function over the set of types. The agenda-setter surplus can be rewritten so as to account for the information rents brought about by incentive compatibility.

Lemma 3. *The agenda-setter's objective function can be rewritten as*

$$\sigma(\theta, \gamma(\theta), g) \equiv \int_{\Theta} \left[\sum_{i \in N} \theta_i \phi(g(\theta)) - \sum_{i \in N \setminus \{a\}} \left(\frac{\gamma^*(\theta_i) - F_i(\theta_i)}{f_i(\theta_i)} \right) \phi(g(\theta)) - g(\theta) \right] f(\theta) d\theta. \quad (4)$$

Since (4) is strictly quasi-concave in the public-good level, the objective function will have a unique maximizer.

Definition 4.

$$\xi(\gamma(\theta), \theta) = \arg \max_g \sigma(\theta, \gamma(\theta), g).$$

One last piece of notation. It is useful to define the partition of $N \setminus \{a\}$ depending on whether agents have the *ex post* incentive to understate or overstate their types.

Definition 5.

$$N \setminus \{a\} = K(g^\circ) \cup L(g^\circ) \cup M(g^\circ),$$

where

$$\begin{aligned} K(g^\circ) &= \{i \in N \setminus \{a\} \mid dU_i/d\theta_i < 0\} \\ L(g^\circ) &= \{i \in N \setminus \{a\} \mid dU_i/d\theta_i = 0\} \\ M(g^\circ) &= \{i \in N \setminus \{a\} \mid dU_i/d\theta_i > 0\}. \end{aligned}$$

7.1 Unanimity

The agenda-setter solves

$$\begin{aligned} & \max_{g(\theta), t(\theta)} \int \theta_a \phi(g(\theta)) - t_a(\theta) dF(\theta) \\ \text{subject to} \quad & g(\theta) \leq t_a(\theta) + \sum_{i \in N \setminus \{a\}} t_i(\theta), \\ & \frac{dg(\theta)}{d\theta} \geq 0, \\ & dU_i/d\theta_i = \phi(g(\theta)) - \bar{v}_i'(\theta_i, g^\circ), \\ & U_i(\theta_i, g(\theta)) \geq \bar{v}_i(\theta_i, g^\circ) \quad \forall i \in N \setminus \{a\}. \end{aligned} \tag{U}$$

Theorem 2. *Under Assumptions (1)-(3), there exists a unique optimal solution to U. An implementable allocation $g^*(\theta)$ is optimal if and only if there exists a cumulative distribution function $\gamma^*(\theta_i)$ such that*

$$\begin{cases} (\mathcal{A}) & d\gamma^*(\theta_i) > 0 \quad \text{for } \theta_i \in L(g^\circ), \\ (\mathcal{B}) & g^*(\theta) = \xi(\gamma^*(\theta_i), \theta). \end{cases}$$

The optimal allocation partitions the set of agents $N \setminus \{a\}$ into $K(g^\circ)$, $L(g^\circ)$ and $M(g^\circ)$. Otherwise, $g^(\theta) = g^\circ$.*

Theorem 2 is closely related to Theorem 1 in Jullien (2000). Condition $\mathcal{A}$ identifies which participation constraints bind and, hence, which types earn no information rents. It states that the participation constraint can bind on a single interior type and at the extremes: the lowest and highest types.

Condition $\mathcal{B}$ characterizes the optimal public-good provision as the public-good level which maximizes the agenda-setter's utility adjusted to take into account the information rents induced by incentive compatibility. Depending on the outside option public-good level g° , some agents will have the incentive to understate their type, whereas others will have the incentive to overstate it. In particular, if $\gamma^*(\theta_i) > F_i(\theta_i)$, the dominating incentive

is to understate the type; if $\gamma^*(\theta_i) < F_i(\theta_i)$, the dominating incentive is to overstate the type.

The main idea behind [Theorem 2](#) is very simple: if there exists a cdf $\gamma^*(\theta_i)$ such that the collective choice, the public-good provision $g^*(\theta)$, is consistent with the agents' *ex post* incentives to misreport their types, then such collective choice will be implemented. If not, the agenda-setter can do no better than proposing the outside option.

7.2 Majority

We consider the following problem faced by the agenda-setter

$$\begin{aligned}
& \max_{g(\theta), t(\theta), Q} \int \theta_a \phi(g(\theta)) - t_a(\theta) dF(\theta) \\
& \text{subject to} \quad g(\theta) \leq t_a(\theta) + \sum_{i \in N \setminus \{a\}} t_i(\theta), \\
& \quad \quad \quad \frac{dg(\theta)}{d\theta} \geq 0 \\
& \quad \quad \quad \text{and for some } Q \in \mathcal{Q}, \\
& \quad \quad \quad dU_i/d\theta_i = \phi(g(\theta)) - \bar{v}'_i(\theta_i, g^\circ) \quad \forall i \in N \setminus \{a\}, \\
& \quad \quad \quad U_j(\theta_j, g(\theta)) \geq \bar{v}_j(\theta_j, g^\circ) \quad \forall j \in Q \setminus \{a\}.
\end{aligned} \tag{M}$$

We now state the second main result.

Theorem 3. *There exists a unique optimal solution to $\mathcal{M}$. An implementable allocation $g^*(\theta)$ is optimal if and only if there exists a cumulative distribution function $\gamma^*(\theta_i)$ such that*

$$\begin{cases}
(\mathcal{A}) & d\gamma^*(\theta_i) > 0 \quad \text{for } \theta_i \in L(g^\circ), \\
(\mathcal{B}) & g^*(\theta) = \xi(\gamma^*(\theta_i), \theta).
\end{cases}$$

The optimal allocation partitions the set of agents $N \setminus \{a\}$ into $K(g^\circ)$, $L(g^\circ)$ and $M(g^\circ)$. Moreover, there exists a set of agents

$$I(g^\circ) = \{i \in N \mid U_i(\theta_i) < 0\},$$

with $|I(g^\circ)| \leq n - q$. Otherwise, $g^(\theta) = g^\circ$.*

The interpretation of [Theorem 3](#) is very similar. Conditions $\mathcal{A}$ and $\mathcal{B}$ still apply. Moreover, the solution will induce a partition of agents into $K(g^\circ)$, $L(g^\circ)$ and $M(g^\circ)$. However, the agenda-setter needs to choose a coalition Q , for whose members the participation constraints need to be satisfied. There is a set $I(g^\circ)$ for whom the participation constraints is violated.

Notice that this type of exclusion is different from the exclusion of some agents in [Jullien \(2000\)](#). The reason is that, in the present framework, agents can be forced to participate

in the mechanism, and they will still consume the public good once it has been provided. However, they still need to be given the incentives to reveal their types truthfully *ex post*.

The idea is again very simple. $K(g^\circ)$ and $M(g^\circ)$ are two convex sets.¹⁰ This is because, by condition (2), information rents have to be increasing (decreasing) over the set of agents who have the incentive to understate (overstate) their types. The agenda-setter has to identify a set of agents whose participation constraints can be violated, but whose incentives to report truthfully are still required to hold. The choice of this set crucially depends on the shape of the reservation utility profile.

Moreover, depending on how far public-good provision is from the efficient level, a subset of $I(g^\circ)$ will be bunched at the allocation of the agent(s) whose participation constraints is (are) binding.

Therefore, Theorem 2 and Theorem 3 imply that the optimal public-good provision is either equal to - or a step function of - the outside option public-good level.

8 Implications

Theorem 2 and Theorem 3 show that optimal public-good provision crucially depends on the shape of the reservation utility profile $\bar{v}_i$.

When the outside option public-good level g° is low, all agents have the incentive to understate their types. By familiar arguments, it is then optimal to underprovide the public good by distorting the allocations downwards. The participation constraint binds for the lowest type; all others earn information rents. By Condition (2), rents are increasing in types.

Analogously, when the outside option public-good level is sufficiently high, all agents have the incentive to overstate their types. The optimal public-good provision now entails distorting the allocation upwards. The participation constraints binds only for the highest type. All other agents earn information rents, which are now decreasing in types by Condition (2).

When g° assumes intermediate values, however, agents have countervailing incentives: some agents will have the incentive to understate their types, whereas others will have the incentive to overstate their types. Therefore, the optimal public-good provision depends on the concavity or convexity of the reservation utility profile.

Section 6 considered the linear case. We now consider the concave and convex cases.

¹⁰Hence, for any three types $\theta_1 < \theta_2 < \theta_3$, if θ_1 and θ_3 are included in either set, then also θ_2 is included in the same set.

8.1 $\bar{v}_i$ Concave

Lemma 4. Suppose $\bar{v}_i'' \leq 0$. Then, for some $k, l \in \{0, 1, \dots, r\}$ with $k \leq l$,

$$K(g^\circ) = \{\theta_1, \dots, \theta_k\},$$

$$L(g^\circ) = \{\theta_{k+1}, \dots, \theta_l\},$$

$$M(g^\circ) = \{\theta_{l+1}, \dots, \theta_r\}.$$

Moreover, the indices k and l are increasing in g° . If values are all distinct, then $L(g^\circ)$ has at most one agent.

Lemma 4 orders the partition of the agents, depending on whether they have the incentive to understate or overstate their types, or they are indifferent between the two.

When values are all distinct, there will be a single type $\tilde{\theta}$ whose participation constraint binds. When g° is very low, then $\tilde{\theta} = \theta_1$. As the outside option public-good level increases, the cutoff type $\tilde{\theta}$ will be an interior type. By Lemma 4, all types below (above) $\tilde{\theta}$ will have the incentive to overstate (understate) their types. Figure 4 displays the information rents as a function of the types in this case.

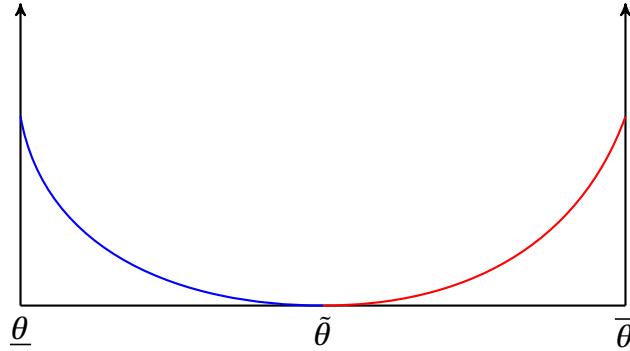


Figure 4: $\bar{v}_i$ concave: unanimity

In the majority case, the same argument applies. However, Theorem 3 states that a subset of agents I might be bunched at the allocation of the type whose participation constraint binds. Depending on g° , the participation constraint binds on a different type and hence also the set of agents excluded from the coalition differs. Figure 5 displays the solution when the cutoff type is interior.

Interestingly, the winning coalition can be a *non-convex* set: there can be three types $\theta_i < \theta_k < \theta_j$ such that types θ_i and θ_j might be included in Q , whereas θ_k is excluded.

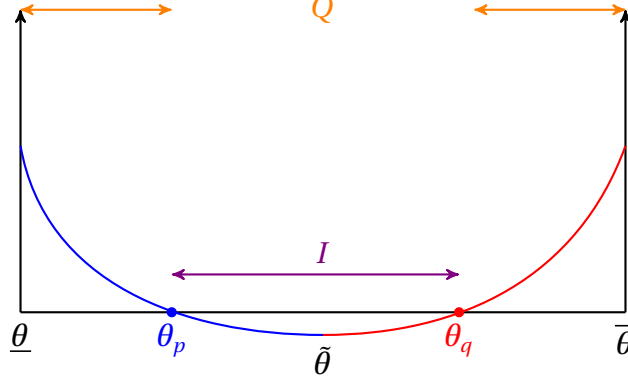


Figure 5: $\bar{v}_i$ concave with exclusion

8.2 $\bar{v}_i$ Convex

Lemma 5. Suppose $\bar{v}_i'' \geq 0$. Then, for some $k, l \in \{0, 1, \dots, r\}$ with $k \leq l$,

$$K(g^\circ) = \{\theta_{r-1}, \dots, \theta_l\},$$

$$L(g^\circ) = \{\theta_1, \theta_r\},$$

$$M(g^\circ) = \{\theta_k, \dots, \theta_2\},$$

Moreover, the indices k and l are decreasing in g° .

Analogously to Lemma 4, Lemma 5 orders the partition of the agents. However, the transition between sets is more involved than in the concave case. The reason is again Condition (2).

As g° increases, the first agent whose incentive switches from understating to overstating is the agent with the highest type. However, the information rents have to be decreasing for the set of agents having the incentive to overstate. The only way to preserve incentive compatibility is then to have the participation constraint binding at both extremes of the type space. Interestingly, there will be a single interior type where the two countervailing incentives cross out and the agent will be just indifferent between understating or overstating.¹¹ This cutoff type is decreasing in g° : it coincides with θ_n when g° is very low and it coincides with lower types as g° increases. Figure 6 illustrates this property. In the majority case, there will be a set of agents I whose participation constraint is violated. These agents might be bunched at the allocation of the type(s) with binding participation constraint(s). Contrary to the concave case, the winning coalition is now a convex set. Figure 7 illustrates this property.

¹¹See Maggi and Rodriguez-Clare A. (1995).

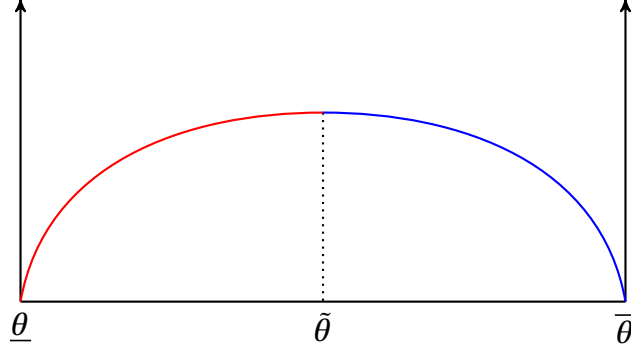


Figure 6: Information rents when $\bar{v}_i$ is convex

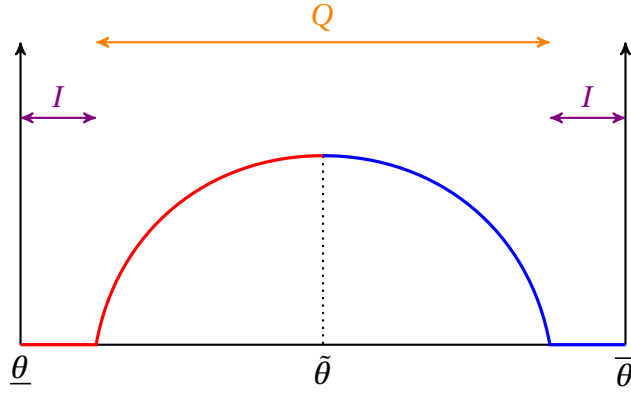


Figure 7: $\bar{v}_i$ convex with exclusion

9 Extensions

9.1 Limited Commitment

Suppose that the agenda-setter cannot fully commit to reporting her true parameter θ_a after collecting the preferences of the other agents.

Recall the efficient public-good level in (1) and denote it g^{FB} . Then, we have the following result.

Proposition 4. *The optimal report $\hat{\theta}_a$ is given by*

$$\hat{\theta}_a = \begin{cases} 0 & \text{if } g^*(\theta) \geq g^{FB}, \\ \min\{g^{FB} - g^*(\theta), \bar{\theta}\} & \text{if } g^*(\theta) < g^{FB}, \end{cases}$$

if $g^*(\theta) \neq g^\circ$. Otherwise, $\hat{\theta}_a = \theta$, for any $\theta \in [\underline{\theta}, \bar{\theta}]$.

Proof. The agenda-setter's objective function is concave and maximized at g^{FB} . Therefore, any reduction in the public-good level by reporting a lower type has a first-order effect on the objective function only when public-good provision is higher than the efficient level. $\square$

9.2 Bunching

9.2.1 Relaxing (Assumption 2)-(Assumption 3)

9.2.2 Negatively-sloped Reservation Utility Profile

Suppose $\bar{v}'_i(\theta_i, g^\circ) < 0$, for every $i \in N$. Then, condition (2) yields

$$dU_i/d\theta_i = \phi(g(\theta)) - \bar{v}'_i(\theta_i, g^\circ) > 0$$

for all $\theta_i \in \Theta_i$. The dominating incentive is then always to understate the type. The equilibrium public-good provision is then the same as in Proposition 2 and Proposition 3 when $\bar{v}_i(\theta, g^\circ) \equiv 0$.

9.3 Common Values

9.4 Stochastic Mechanisms

Mechanisms can be stochastic in two ways: for each agent, the inclusion in the winning coalition Q (*stochastic coalitions*); or the public-good provision g (*stochastic public-good provision*).

9.4.1 Stochastic Coalitions

Suppose now that the incentive compatibility constraint has to be satisfied only for the members of the coalition $Q \in \mathcal{Q}$. We consider the following problem faced by the agenda-setter

$$\begin{aligned} \max_{g(\theta), t(\theta), Q} \quad & \int \theta_a \phi(g(\theta)) - t(\theta_a) dF(\theta) - \bar{v}_a(\theta_a, g^\circ) \\ \text{subject to} \quad & g(\theta) \leq t(\theta_a) + \sum_{i \in N \setminus \{a\}} t(\theta_i), \\ & \frac{dg(\theta)}{d\theta} \geq 0, \\ & \text{and for some } Q \in \mathcal{Q}, \\ & dU_j/d\theta_j = \phi(g(\theta)) - \bar{v}'_j(\theta_j, g^\circ), \\ & U_j(\theta_j, g(\theta)) \geq \bar{v}_j(\theta_j, g^\circ) \quad \forall j \in R. \end{aligned} \tag{S}$$

Problem $\mathcal{S}$ differs from Problem $\mathcal{M}$ only insofar as the individual rationality constraints of the agents excluded from the coalition are concerned. The resource constraint is relaxed, since the agents in $N \setminus Q$ can now be taxed to some upper bound $\bar{\tau}$.

Proposition 5. *The coalition Q is randomly chosen. Within this set of agents, the solution from Problem $\mathcal{U}$ applies.*

9.4.2 Stochastic Public-Good Provision

Since the utility function is linear in types and quasilinear in the transfers, the agents are risk-neutral. Moreover, the agenda-setter's objective function is concave in the public good. Therefore, it is straightforward to show that there is no gain in proposing a stochastic public-good provision.

Proposition 6. *Deterministic public-good provision is optimal.*

9.5 Dynamics

Consider now the simplest possible dynamic setting. The agenda-setter is the same in each period $\tau = 1, \dots, T$ and the agents' values for the public good do not evolve over time. Denote $\delta \in [0, 1)$ the common discount factor of the agents. Moreover, let g_τ and t_τ be the public-good provision and transfer in period τ , respectively. Agent i 's total utility is then given by

$$U_i^T(\theta_i) = \sum_{\tau=1}^T \delta^{\tau-1} (\theta_i \phi(g_\tau) - t_\tau).$$

Analogously to [Definition 1](#), the notion of direct mechanism can be extended to this simple dynamic setting.

Definition 6. A dynamic direct mechanism is a set of functions $g = (g_1, \dots, g_T)$ and $t_i = (t_{i,1}, \dots, t_{i,T})$, $\forall i \in N$

$$\begin{aligned} g &: \Theta \longrightarrow \mathbb{R}_+^T, \\ t_i &: \Theta \longrightarrow \mathbb{R}^T. \end{aligned}$$

Moreover, as in [Section 5](#), the notions of incentive compatible and individually rational (static) mechanisms can be naturally extended to dynamic mechanisms.

Definition 7. A dynamic direct mechanism is incentive compatible if, $\forall i \in N$,

$$U_i^T(\theta) = \sum_{\tau=1}^T \delta^{\tau-1} (\theta_i \phi(g_\tau(\theta)) - t_\tau(\theta)) \geq \sum_{\tau=1}^T \delta^{\tau-1} (\theta_i \phi(g_\tau(\theta'_i, \theta_{-i})) - t_\tau(\theta'_i)) \quad \forall \theta_i, \theta'_i \in [\underline{\theta}, \bar{\theta}].$$

Definition 8. A dynamic direct mechanism is individually rational if, $\forall i \in N$,

$$U_i^T(\theta) \geq \bar{v}_i(\theta_i, g^\circ) \beta \quad \forall \theta_i \in [\underline{\theta}, \bar{\theta}],$$

where $\beta = \frac{1-\delta^T}{1-\delta}$.

In this simple dynamic setting, the agenda-setter has full commitment power and s/he proposes an intertemporal profile of public-good levels. If the agents vote in favour, then

the public-good provision g_τ will be implemented in each period τ . If not, the outside option $\bar{v}_i(\theta_i, g^\circ)$ will be implemented in the current and future periods, without the possibility to renegotiate the public-good provision.

In the screening literature, the seller cannot benefit from ‘deviating’ from the (static) optimal mechanism: posting the same price p^* is the best s/he can do.¹² Then, a natural conjecture in the present setting is that optimal dynamic public-good provision will be the same as if agents were ‘myopic’ in the sense of [Gersbach et al. \(2023\)](#); i.e., at each period τ , they take into account only the payoff in period τ . If the participation constraint in [Definition 8](#) has to hold only for a coalition $Q \in \mathcal{Q}$, then public-good provision might assume more complicated intertemporal dynamics. A more thorough analysis is then left for future research.

9.6 Comparison with [Gersbach et al. \(2023\)](#)

As noted in [Section 2](#), [Gersbach et al. \(2023\)](#) consider a very similar model in a complete information setting. The presence of private information implies several significantly different results.

First, the optimal choice of the winning coalition depends on the shape of the reservation utility profile and the outside option public-good level. In [Gersbach et al. \(2023\)](#), it is optimal to choose the highest or lowest coalition, depending on whether the outside option public-good level is lower or higher than a cutoff value.

As shown in [Section 8](#), extreme coalitions are optimal when the outside option public-good level is sufficiently high or low, regardless of the shape of $\bar{v}_i$. When it assumes intermediate values, however, optimal coalitions might be a non-convex set of types and exclude the highest or lowest types.

Second, in [Gersbach et al. \(2023\)](#) public-good provision is a non-monotonic function of the outside option public-good level; moreover, it is never optimal to propose the outside option. As shown [Section 6](#), however, even when the reservation utility profile is the same as in [Gersbach et al. \(2023\)](#), the latter result is reversed.

Third, lowering q never decreases public-good provision. This is the exact opposite of [Gersbach et al. \(2023\)](#), whereby *increasing* q can never decrease public-good provision.

10 Conclusions

In this paper, I embedded a mechanism design problem about the provision of a public good into a political economy framework. In line with the previous contributions from the screening literature, the shape of the outside option crucially determines the optimal

¹²See Chapter 11 in [Börger et al. \(2015\)](#).

public-good provision and coalition formation. Several directions for future research remain to be pursued.

First, dominant-strategy implementation circumvents the issue of the ‘informed principal’ in the sense of [Myerson \(1983\)](#). An analysis in Bayesian implementation would raise significant complications, as there is no guarantee that the revelation principle holds.¹³ However, relaxing the requirement of dominant-strategy implementation constitutes a promising direction for future research.

Second, the present paper only considered a static problem. Exploring public-good provision and coalition formation in dynamic setting and limited commitment, along the lines of [Doval and Skreta \(2022\)](#), would also constitute an interesting exercise.

Finally, the paper considered a single dimension of private information. Multidimensional screening problems are well-known to pose severe difficulties (e.g., [Rochet and Choné \(1998\)](#)). Nonetheless, members of organizations are likely to be heterogeneous along different dimensions of private information. Making progress on extending the model to multiple dimensions would provide additional insights into the coalition formation problem.

¹³See [Mylovanov and Tröger \(2014\)](#) and the references therein.

A Proofs

A.1 Proof of Lemma 1

Proof. Consider $\theta_i > \theta_j$. By incentive compatibility, we have that

$$U_i(\theta_i) \geq \theta_i \phi(g(\theta_j, \theta_{-i})) - t(\theta_j) - \bar{v}_i(\theta_i, g^\circ) = U_j(\theta_j) + (\theta_i - \theta_j) \phi(g(\theta_j, \theta_{-i})) - (\bar{v}_i(\theta_i, g^\circ) - \bar{v}_j(\theta_j, g^\circ)) \quad (5)$$

and

$$U_j(\theta_j) \geq \theta_j \phi(g(\theta_i, \theta_{-j})) - t(\theta_i) - \bar{v}_j(\theta_j, g^\circ) = U_i(\theta_i) + (\theta_j - \theta_i) \phi(g(\theta_i, \theta_{-j})) - (\bar{v}_j(\theta_j, g^\circ) - \bar{v}_i(\theta_i, g^\circ)) \quad (6)$$

which imply

$$\phi(g(\theta_i, \theta_{-i})) - \frac{\bar{v}_i(\theta_i, g^\circ) - \bar{v}_j(\theta_j, g^\circ)}{\theta_i - \theta_j} \geq \frac{U_i(\theta_i) - U_j(\theta_j)}{\theta_i - \theta_j} \geq \phi(g(\theta_j, \theta_{-j})) - \frac{\bar{v}_j(\theta_j, g^\circ) - \bar{v}_i(\theta_i, g^\circ)}{\theta_i - \theta_j}.$$

As $\theta_j \rightarrow \theta_i$, the conditions in Lemma 1 are implied.

To establish the converse, assume the above mentioned conditions hold. Then,

$$\begin{aligned} U_i(\theta_i) - U_j(\theta_j) &= \int_{\theta_j}^{\theta_i} \phi(g(x, \theta_{-j})) - \bar{v}'_j(x, g^\circ) dx \\ &\geq \int_{\theta_j}^{\theta_i} \phi(g(\theta_j, \theta_{-j})) - \bar{v}'_j(\theta_j, g^\circ) dx \\ &= (\theta_i - \theta_j) \phi(g(\theta_j, \theta_{-j})) - (\bar{v}_i(\theta_i, g^\circ) - \bar{v}_j(\theta_j, g^\circ)). \end{aligned}$$

This implies (5). An analogous argument proves (6). □

A.2 Proof of Proposition 2

Proof.

Case I $dU_i/d\theta_i > 0, \forall i \in N$.

The proposer's problem Equation ($\mathcal{L}\mathcal{U}$) becomes

$$\begin{aligned}
& \max_{g(\theta), t(\theta)} \int \theta_a \phi(g(\theta)) - t(\theta_a) dF(\theta) \\
& \text{subject to } g(\theta) \leq t(\theta_a) + \sum_{i \in N \setminus \{a\}} t(\theta_i), \\
& \quad \frac{dg(\theta)}{d\theta} \geq 0, \\
& t(\theta_i) = \theta_i \phi(g(\theta_i, \theta_{-i})) - \int_{\underline{\theta}}^{\theta_i} \phi(g(x, \theta_{-i})) dx, \\
& t(\theta_i) \leq \theta_i \phi(g(\theta)) \quad \forall i \in N \setminus \{a\}.
\end{aligned}$$

Consider the proposer's expected utility. Since all constraints are binding, it is equal to

$$\begin{aligned}
& \int_{\Theta} \left[\theta_a \phi(g(\theta)) - \sum_{i \in N \setminus \{a\}} t(\theta_i) - g(\theta) \right] f(\theta) d\theta \\
&= \int_{\Theta} \left[\theta_a \phi(g(\theta)) + \sum_{i \in N \setminus \{a\}} \left[\theta_i \phi(g(\theta)) - \int_{\underline{\theta}}^{\theta_i} \phi(g(x, \theta_{-i})) dx \right] - g(\theta) \right] f(\theta) d\theta \\
&= \int_{\Theta} \left[\sum_{i \in N} \theta_i \phi(g(\theta)) - \sum_{i \in N \setminus \{a\}} \left(\int_{\underline{\theta}}^{\theta_i} \phi(g(x, \theta_{-i})) dx \right) - g(\theta) \right] f(\theta) d\theta \\
&= \int_{\Theta} \left[\sum_{i \in N} \theta_i \phi(g(\theta)) - \sum_{i \in N \setminus \{a\}} \left(\frac{1 - F_i(\theta_i)}{f_i(\theta_i)} \right) \phi(g(\theta)) - g(\theta) \right] f(\theta) d\theta
\end{aligned}$$

where the last equality follows from Fubini's theorem. Pointwise maximization with respect to $g(\theta)$ yields

$$\left[\theta_a + \sum_{i \in N \setminus \{a\}} \left(\theta_i - \frac{1 - F_i(\theta_i)}{f_i(\theta_i)} \right) \right] \phi'(g(\theta)) = 1. \quad (7)$$

[Assumption 2](#) ensures the monotonicity constraint is satisfied.

The same goes through for any $g^\circ \leq g^L$.

Case II $dU_i/d\theta_i < 0, \forall i \in N$.

The proposer's problem Equation (LW) becomes

$$\begin{aligned}
& \max_{g(\theta), t(\theta)} \int \theta_a \phi(g(\theta)) - t(\theta_a) dF(\theta) \\
& \text{subject to } g(\theta) \leq t(\theta_a) + \sum_{i \in N \setminus \{a\}} t(\theta_i), \\
& \quad \frac{dg(\theta)}{d\theta} \geq 0, \\
& \quad t(\theta_i) = \theta_i \phi(g(\theta_i, \theta_{-i})) - \int_{\theta_i}^{\bar{\theta}} \bar{v}_i(x, g^\circ) - \phi(g(x, \theta_{-i})) dx, \\
& \quad \theta_i \phi(g(\theta)) - t(\theta_i) \geq \theta_i \phi(g^\circ) - \frac{g^\circ}{n} \quad \forall i \in N \setminus \{a\}.
\end{aligned}$$

Consider the proposer's expected utility. Since all constraints are binding, it is equal to

$$\begin{aligned}
& \int_{\Theta} \left[\theta_a \phi(g(\theta)) - \sum_{i \in N \setminus \{a\}} t(\theta_i) - g(\theta) \right] f(\theta) d\theta \\
&= \int_{\Theta} \left[\theta_a \phi(g(\theta)) + \sum_{i \in N \setminus \{a\}} \left[\theta_i \phi(g(\theta)) - \int_{\theta_i}^{\bar{\theta}} \bar{v}_i(x, g^\circ) - \phi(g(x, \theta_{-i})) dx \right] - g(\theta) \right] f(\theta) d\theta \\
&= \int_{\Theta} \left[\sum_{i \in N} \theta_i \phi(g(\theta)) - \sum_{i \in N \setminus \{a\}} \left(\int_{\theta_i}^{\bar{\theta}} \bar{v}_i(x, g^\circ) - \phi(g(x, \theta_{-i})) dx \right) - g(\theta) \right] f(\theta) d\theta \\
&= \int_{\Theta} \left[\sum_{i \in N} \theta_i \phi(g(\theta)) + \sum_{i \in N \setminus \{a\}} \left(\frac{F_i(\theta_i)}{f_i(\theta_i)} \right) \phi(g(\theta)) - \sum_{i \in N \setminus \{a\}} \left(\int_{\theta_i}^{\bar{\theta}} \bar{v}_i(x, g^\circ) \right) - g(\theta) \right] f(\theta) d\theta.
\end{aligned}$$

Pointwise maximization with respect to $g(\theta)$ yields

$$\left[\theta_a + \sum_{i \in N \setminus \{a\}} \left(\theta_i + \frac{F_i(\theta_i)}{f_i(\theta_i)} \right) \right] \phi'(g(\theta)) = 1. \quad (8)$$

Again, by Assumption 2, the monotonicity constraint is satisfied. Moreover, the same goes through for any $g^\circ \geq g^H$.

Case III $g^L < g^\circ < g^H$.

Consider a simple three-agent example, with $N = \{\theta_a, \theta_k, \theta_l\}$. Suppose $\theta_a = 0$ and $\theta_k > \theta_l \neq 0$. This is a classic screening problem as in Maggi and Rodriguez-Clare A. (1995). Figure 8 illustrates the problem faced by the agenda-setter.

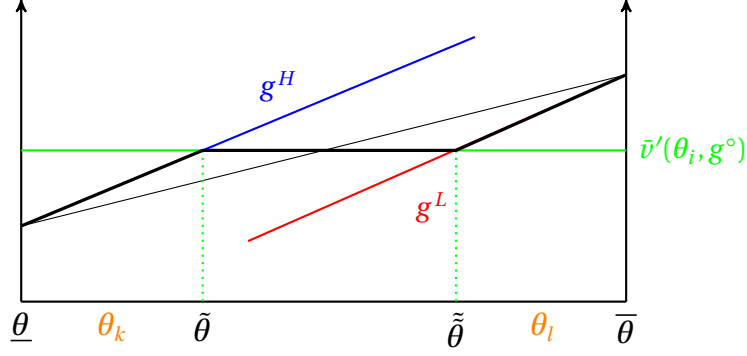


Figure 8: Two-agent screening example

Suppose $\theta_k \in (\underline{\theta}, \tilde{\theta})$ and $\theta_l \in (\tilde{\bar{\theta}}, \bar{\theta})$ and recall the IC condition (3). For both,

$$dU_{k,l}/d\theta_{k,l} = \phi(g(\theta)) - \phi(g^\circ).$$

On the one hand, if $g(\theta) > g^\circ$, then θ_k has *ex post* the incentive to overstate. On the other hand, if $g(\theta) < g^\circ$, then θ_l has *ex post* the incentive to understate. Hence, a can do no better than choosing the outside option. $\square$

A.3 Proof of Proposition 3

Proof. We begin with the following two lemmata.

Lemma 6. *The optimal choice of coalition is*

$$Q^* = \begin{cases} \bar{Q} & \text{if } g^\circ < g_M^L, \\ \text{any } Q \in \mathcal{Q} & \text{if } g_M^L \leq g^\circ \leq g_M^H, \\ \underline{Q} & \text{if } g^\circ > g_M^H, \end{cases}$$

if $g_M^L < g_M^H$. Otherwise

$$Q^* = \begin{cases} \bar{Q} & \text{if } g^\circ < g_M^L, \\ \underline{Q} & \text{if } g^\circ > g_M^L. \end{cases}$$

Proof. Consider first the case $g^\circ < g_M^L$. Obviously, the solution from the analogous case in Proposition 2 is still feasible. However, it is possible for the solution to the majority case to violate some individual rationality constraints, since they are required to hold only for $q-1$ agents. Denote this set of agents I , with $|I| \leq n-q$. Condition (3) requires that information rents be increasing over the whole set of types. Moreover, incentive constraints are pointing in the direction of the lowest type. Also, if the participation constraint is satisfied for type θ_1 , then it is also satisfied for $\theta_2 > \theta_1$. Therefore, utility maximization requires that

the cutoff agent $\tilde{\theta}$ be the agent with the lowest type still included in the coalition:

$$\theta_1, \theta_2, \dots, \underbrace{\tilde{\theta}, \dots, \theta_{n-1}, \theta_n}_{\text{at most } q-1}.$$

An analogous reasoning applies to the case $g^\circ > g_M^L$, with the corresponding cutoff type $\tilde{\tilde{\theta}}$:

$$\underbrace{\theta_1, \theta_2, \dots, \tilde{\tilde{\theta}}}_{\text{at most } q-1}, \dots, \theta_{n-1}, \theta_n.$$

□

The number of agents whose participation constraint is violated depends on the value of q .

Lemma 7. *The cardinality of the set I , $|I|$, is inversely related to q .*

Proof. Recall (7). If agents with the lowest types are bunched at the same allocation of a cutoff type $\tilde{\theta}$, the analogous condition is

$$\left[\theta_a + |I| \cdot \tilde{\theta} + \sum_{i \in Q \setminus \{a\}} \left(\theta_i - \frac{1 - F_i(\theta_i)}{f_i(\theta_i)} \right) \right] \phi'(g(\theta)) = 1. \quad (9)$$

Thus, the optimal public-good provision $g^*(\theta)$ is higher when $I \neq \emptyset$. However, as q decreases further, by concavity of the utility functions the efficient ('first-best') level (1) is reached for some $\tilde{q}$. For any $q < \tilde{q}$, it is optimal to maintain the same cutoff type $\tilde{\theta}$. □

With the help of Lemma 6, the analogous cases as in Proposition 2 now follow.

Case I $dU_i/d\theta_i > 0, \forall i \in N$.

The agenda-setter's problem Equation ($\mathcal{LM}$) becomes

$$\begin{aligned} & \max_{g(\theta), t(\theta)} \int \theta_a \phi(g(\theta)) - t(\theta_a) dF(\theta) \\ & \text{subject to } g(\theta) \leq t(\theta_a) + \sum_{i \in N \setminus \{a\}} t(\theta_i), \\ & \quad \frac{dg(\theta)}{d\theta} \geq 0, \\ & \quad t(\theta_i) = \theta_i \phi(g(\theta_i, \theta_{-i})) - \int_{\tilde{\theta}}^{\theta} \phi(g(x, \theta_{-i})) dx \quad \forall i \in N \setminus \{a\}, \\ & \quad t(\theta_i) \leq \theta_i \phi(g(\theta)) \quad \forall i \in Q. \end{aligned}$$

By Lemma 6, it is then optimal to make the participation constraint of $\tilde{\theta}$ binding.

Claim 1. For all $\theta_i \leq \tilde{\theta}$, the participation constraint is violated.

Proof. We have that

$$\begin{aligned} t(\theta_i) &= \theta_i \phi(g(\theta)) - \int_{\tilde{\theta}}^{\theta_i} \phi(g(x, \theta_{-i})) dx \\ &= \theta_i \phi(g(\theta)) + \int_{\theta_i}^{\tilde{\theta}} \phi(g(x, \theta_{-i})) dx \\ &\geq \theta_i \phi(g(\theta)), \end{aligned}$$

where the last inequality follows from the monotonicity of the allocation rule. Notice that, in order to preserve incentive compatibility, it must be the case that $t(\theta_i) = t(\tilde{\theta})$, for all $\theta_i \leq \tilde{\theta}$. $\square$

Consider now the expected transfer of types $\theta_i \geq \tilde{\theta}$. It is equal to

$$\int_{\underline{\theta}}^{\tilde{\theta}} t(\theta_i) f_i(\theta_i) d(\theta_i) = \int_{\underline{\theta}}^{\tilde{\theta}} \theta_i \phi(g(\theta_i, \theta_{-i})) - \int_{\underline{\theta}}^{\tilde{\theta}} \int_{\tilde{\theta}}^{\theta} \phi(g(x, \theta_{-i})) dx f_i(\theta_i) d(\theta_i).$$

Consider the second term.

$$\begin{aligned} &\int_{\underline{\theta}}^{\tilde{\theta}} \int_{\tilde{\theta}}^{\theta} \phi(g(x, \theta_{-i})) dx f_i(\theta_i) d(\theta_i) \\ &= \int_{\underline{\theta}}^{\tilde{\theta}} \int_{\underline{\theta}}^{\theta} \phi(g(x, \theta_{-i})) dx f_i(\theta_i) d(\theta_i) - \int_{\underline{\theta}}^{\tilde{\theta}} \int_{\underline{\theta}}^{\tilde{\theta}} \phi(g(x, \theta_{-i})) dx f_i(\theta_i) d(\theta_i). \end{aligned}$$

Simplifying the first term is a standard procedure in the literature ([Börkers et al. \(2015\)](#)). A very similar procedure can be applied to simplify also the second term. A graphical representation helps to illustrate how the standard procedure has to be modified in order to take into account the excluded types.¹⁴ For completeness, I include the derivations for both terms.

¹⁴The steps follow closely Chapter 2 in [Börkers et al. \(2015\)](#).

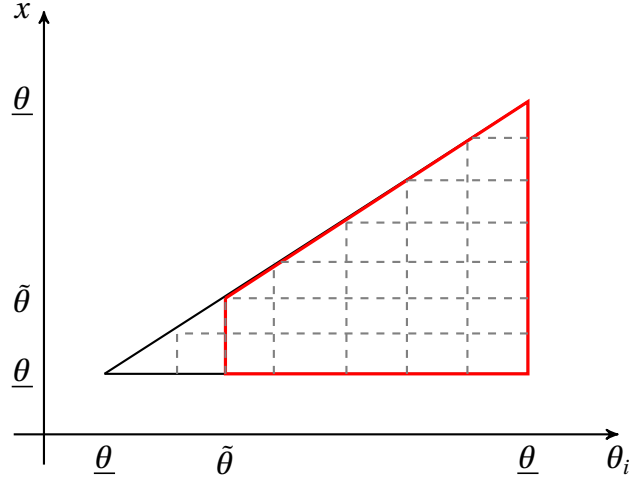


Figure 9: Change in the order of integration.

The first term simplifies as follows:

$$\begin{aligned}
\int_{\underline{\theta}}^{\bar{\theta}} \int_{\underline{\theta}}^{\theta} \phi(g(x, \theta_{-i})) dx f_i(\theta_i) d(\theta_i) &= \int_{\underline{\theta}}^{\bar{\theta}} \int_{\underline{\theta}}^{\theta} \phi(g(x, \theta_{-i})) f_i(\theta_i) dx d(\theta_i) \\
&= \int_{\underline{\theta}}^{\bar{\theta}} \int_{\underline{\theta}}^{\theta} \phi(g(x, \theta_{-i})) f_i(\theta_i) \mathbb{1}_{x \leq \theta} d(\theta_i) dx \\
&= \int_{\underline{\theta}}^{\bar{\theta}} \phi(g(x, \theta_{-i})) \int_{\underline{\theta}}^{\theta} f_i(\theta_i) \mathbb{1}_{x \leq \theta} d(\theta_i) dx \\
&= \int_{\underline{\theta}}^{\bar{\theta}} \phi(g(x, \theta_{-i})) \int_x^{\bar{\theta}} f_i(\theta_i) d(\theta_i) dx \\
&= \int_{\underline{\theta}}^{\bar{\theta}} \phi(g(x, \theta_{-i})) [1 - F(x)] dx \\
&= \int_{\underline{\theta}}^{\bar{\theta}} \phi(g(\theta_i)) [1 - F_i(\theta_i)] d\theta_i.
\end{aligned}$$

When applying Fubini's theorem in the second equality, the whole black triangle is considered. On the other hand, for the second term, only the red polygon has to be taken into

account.

$$\begin{aligned}
& \int_{\underline{\theta}}^{\bar{\theta}} \int_{\underline{\theta}}^{\bar{\theta}} \phi(g(x, \theta_{-i})) dx f_i(\theta_i) d(\theta_i) = \int_{\underline{\theta}}^{\bar{\theta}} \int_{\underline{\theta}}^{\bar{\theta}} \phi(g(x, \theta_{-i})) f_i(\theta_i) \mathbb{1}_{\theta \leq \bar{\theta}} d(\theta_i) dx \\
& = \int_{\underline{\theta}}^{\bar{\theta}} \phi(g(x, \theta_{-i})) \int_x^{\bar{\theta}} f_i(\theta_i) d(\theta_i) dx \\
& = \int_{\underline{\theta}}^{\bar{\theta}} \phi(g(x, \theta_{-i})) \int_{\underline{\theta}}^{\bar{\theta}} f_i(\theta_i) d(\theta_i) dx - \int_{\underline{\theta}}^{\bar{\theta}} \phi(g(x, \theta_{-i})) \int_{\underline{\theta}}^x f_i(\theta_i) d(\theta_i) dx \\
& = \int_{\underline{\theta}}^{\bar{\theta}} \phi(g(x, \theta_{-i})) F(\bar{\theta}) dx - \int_{\underline{\theta}}^{\bar{\theta}} \phi(g(x, \theta_{-i})) F(x) dx \\
& = \int_{\underline{\theta}}^{\bar{\theta}} \phi(g(x, \theta_{-i})) [F(\bar{\theta}) - F(x)] \mathbb{1}_{x \leq \bar{\theta}} dx \\
& = \int_{\underline{\theta}}^{\bar{\theta}} \phi(g(\theta)) [F(\bar{\theta}) - F_i(\theta_i)] \mathbb{1}_{\theta_i \leq \bar{\theta}} d\theta_i.
\end{aligned}$$

Then, the expected transfer of the excluded types are given by

$$\begin{aligned}
& \int_{\underline{\theta}}^{\bar{\theta}} t(\theta_i) f_i(\theta_i) d(\theta_i) = \int_{\underline{\theta}}^{\bar{\theta}} \theta_i \phi(g(\theta_i, \theta_{-i})) - \int_{\bar{\theta}}^{\theta} \phi(g(x, \theta_{-i})) dx f_i(\theta_i) d(\theta_i) \\
& = \int_{\underline{\theta}}^{\bar{\theta}} \bar{\theta} \phi(g(\theta)) d\theta_i - \left[\int_{\underline{\theta}}^{\bar{\theta}} \phi(g(\theta)) [1 - F_i(\theta_i)] d\theta_i - \int_{\underline{\theta}}^{\bar{\theta}} \phi(g(\theta)) [F(\bar{\theta}) - F_i(\theta_i)] \mathbb{1}_{\theta_i \leq \bar{\theta}} d\theta_i \right] \\
& = \int_{\underline{\theta}}^{\bar{\theta}} \left[\bar{\theta} - \frac{1 - F(\bar{\theta})}{f_i(\theta_i)} \right] \phi(g(\theta)) d\theta_i
\end{aligned}$$

In other words, types $\theta_i \leq \bar{\theta}$ are bunched at the same allocation of the cutoff type $\bar{\theta}$, whose participation constraint is binding. Putting everything together, and going through the same derivations as for [Proposition 2](#), yields the following result.

Lemma 8. *Optimal public-good provision solves*

$$\left[\theta_a + |I| \cdot \bar{\theta} + \sum_{i \in \bar{Q} \setminus \{a\}} \left(\theta_i - \frac{1 - F_i(\theta_i)}{f_i(\theta_i)} \right) \right] \phi'(g(\theta)) = 1. \quad (10)$$

Case II $dU_i/d\theta_i < 0, \forall i \in N$.

Along the same lines as in the previous case, the following result holds.

Lemma 9. *Optimal public-good provision solves*

$$\left[\theta_a + |I| \cdot \tilde{\theta} + \sum_{i \in Q \setminus \{a\}} \left(\theta_i + \frac{F_i(\theta_i)}{f_i(\theta_i)} \right) \right] \phi'(g(\theta)) = 1. \quad (11)$$

The same goes through for any $g^\circ \geq g^H$.

Case III $g^L < g^\circ < g^H$.

The argument is the same as in Case III in [Proposition 2](#). □

A.4 Proof of Theorem 2

A.4.1 Proof of Lemma 3

Proof. Consider the expected transfer $\int t(\theta_i) f_i(\theta_i) d\theta_i$ from agent $i \in N$. As standard in the literature, we can rewrite it as

$$\int_{\underline{\theta}}^{\bar{\theta}} s_i(\theta_i, g(\theta)) - u_i(\theta_i) f_i(\theta_i) d\theta_i$$

where $s_i(\theta_i, g(\theta)) = \theta_i \phi(g(\theta))$. As in [Jullien \(2000\)](#), define $\gamma^*(\theta_i)$ to be the shadow value of a uniform reduction in the participation constraint for types between $\underline{\theta}$ and θ_i . Then, $d\gamma^*(\theta_i)$ is the shadow value on the individual rationality constraint for agent with type θ_i .

Step 1. $\gamma^*(\theta_i)$ is a cumulative distribution function.

Increasing θ_i extends the set of agents for whom the constraint is relaxed; hence, $\gamma^*(\theta_i)$ is positive and nondecreasing. Furthermore, relaxing the participation constraint for the whole set of agents is equivalent to a uniform increase in $u_i(\theta_i)$. Therefore, $\gamma^*(\bar{\theta}) = 1$.

Step 2. *From a single to many agents.*

The expected transfer is then

$$\int_{\underline{\theta}}^{\bar{\theta}} s_i(\theta_i, g(\theta)) - u_i(\theta_i) f_i(\theta_i) d\theta_i + \int_{\underline{\theta}}^{\bar{\theta}} u_i(\theta_i) d\gamma^*(\theta_i). \quad (12)$$

Integrating by parts (12) twice yields

$$\begin{aligned}
& s_i(\theta_i, g(\theta)) - u_i(\theta_i)F_i(\theta_i) \Big|_{\underline{\theta}}^{\bar{\theta}} - \int_{\underline{\theta}}^{\bar{\theta}} [s'_i(\theta_i, g(\theta)) - u'_i(\theta_i)]F_i(\theta_i) d\theta_i + u_i(\theta_i)\gamma(\theta_i) \Big|_{\underline{\theta}}^{\bar{\theta}} - \int_{\underline{\theta}}^{\bar{\theta}} u'_i(\theta_i)\gamma^*(\theta_i) \\
&= s_i(\bar{\theta}, g(\theta)) - \int_{\underline{\theta}}^{\bar{\theta}} [s'_i(\theta_i, g(\theta)) - u'_i(\theta_i)]F_i(\theta_i) d\theta_i - \int_{\underline{\theta}}^{\bar{\theta}} u'_i(\theta_i)\gamma^*(\theta_i) \\
&= s_i(\bar{\theta}, g(\theta)) - \left[s_i(\theta_i, g(\theta))F_i(\theta_i) \Big|_{\underline{\theta}}^{\bar{\theta}} - \int_{\underline{\theta}}^{\bar{\theta}} s_i(\theta_i, g(\theta))f_i(\theta_i) d\theta_i - \int_{\underline{\theta}}^{\bar{\theta}} u'_i(\theta_i)F_i(\theta_i) \right] - \int_{\underline{\theta}}^{\bar{\theta}} u'_i(\theta_i)\gamma^*(\theta_i) \\
&= \int_{\underline{\theta}}^{\bar{\theta}} s_i(\theta_i, g(\theta))f_i(\theta_i) d\theta_i + \int_{\underline{\theta}}^{\bar{\theta}} u'_i(\theta_i)F_i(\theta_i) - \int_{\underline{\theta}}^{\bar{\theta}} u'_i(\theta_i)\gamma^*(\theta_i) \\
&= \int_{\underline{\theta}}^{\bar{\theta}} \left[s_i(\theta_i, g(\theta)) - \frac{\gamma^*(\theta_i) - F_i(\theta_i)}{f_i(\theta_i)} u'_i(\theta_i) \right] f_i(\theta_i) d\theta_i
\end{aligned}$$

Summing over $i \in N \setminus \{a\}$ and substituting back the original utility functions into the budget constraint yield (4). $\square$

A.4.2 Defining the thresholds

Consider now the implicit function

$$G(\theta_i; g^\circ) \equiv \phi(g(\theta)) - \bar{v}'_i(\theta_i, g^\circ) = 0. \quad (13)$$

The following result ensures that, for given public-good provision $g(\theta)$ and outside option public-good level g° , agents are ordered according to their incentive to misreport their types. A crucial role is played by the shape of the reservation utility profile.

Lemma 10. *For $\theta_i \in [\underline{\theta}, \bar{\theta}]$ and $x \in \mathbb{R}_+$, there exists a unique $\bar{g}(\theta_i, x)$ such that*

$$G(\theta_i; \bar{g}(\theta_i, x)) = x.$$

In addition, if $\bar{v}_i$ is concave (convex), then $\bar{g}(\theta_i, x)$ is strictly increasing in x and strictly increasing (decreasing) in θ_i .

Proof of Lemma 10. It can be readily checked that $G(\theta_i; g^\circ)$ has the following properties:

- Suppose $\bar{v}$ is concave. Then, for $\theta_i < \theta_j$, $G(\theta_i; g^\circ) \leq G(\theta_j; g^\circ)$ on its domain. Equality holds if and only if $g^\circ = 0$.
- Suppose $\bar{v}$ is convex. Then, for $\theta_i < \theta_j$, $G(\theta_j; g^\circ) \leq G(\theta_i; g^\circ)$ on its domain. Equality holds if and only if $g^\circ = 0$.

Moreover, $G(\theta_i; g^\circ)$ is monotone increasing in g° , for all $g^\circ \in \mathbb{R}_+$. $\square$

As [Lemma 10](#) shows, the ordering of agents according to their types hinges on the concavity or convexity of the reservation utility profile. Therefore, each case is considered separately.

A.5 Concavity

I begin with the proof of [Lemma 4](#).

Proof. Consider $\theta_k \in K(g^\circ)$, $\theta_l \in L(g^\circ)$ and $\theta_m \in M(g^\circ)$, with $\theta_k \leq \theta_l \leq \theta_m$. We have that

$$\begin{aligned} dU_m/\theta_m &= \phi(g(\theta)) - \bar{v}'_m(\theta_m, g^\circ) \geq \phi(g(\theta)) - \bar{v}'_l(\theta_l, g^\circ) \\ &= dU_l/d\theta_l \\ &= 0 \\ &\geq \phi(g(\theta)) - \bar{v}'_k(\theta_k, g^\circ) \\ &= dU_k/d\theta_k. \end{aligned}$$

$\square$

I now define a partition of the support of g° according to the agents' incentive to understate or overstate their types. In particular, define $\bar{g}^{|K|, |M|}$ as the outside option public-good level solving (13) when $G(\cdot)$ solves the following condition

$$\left[\theta_a + \sum_{k \in K} \left(\theta_k + \frac{1 - F_k(\theta_k)}{f_k(\theta_k)} \right) + \sum_{m \in M} \left(\theta_m - \frac{F_m(\theta_m)}{f_m(\theta_m)} \right) \right] \phi'(g(\theta)) = 1. \quad (14)$$

In order to keep notation compact and consistent with [Section 6](#), define the following thresholds as

$$\begin{aligned} \bar{g}^L &\equiv \left[\theta_a + \sum_{i \in L, M} \left(\theta_i - \frac{1 - F_i(\theta_i)}{f_i(\theta_i)} \right) \right] \phi'(g(\theta)) = 1, \\ \bar{g}^{L-1} &\equiv \left[\theta_a + \left(\theta_1 + \frac{F_1(\theta_1)}{f_1(\theta_1)} \right) + \sum_{i \in L, M} \left(\theta_i - \frac{1 - F_i(\theta_i)}{f_i(\theta_i)} \right) \right] \phi'(g(\theta)) = 1, \\ \bar{g}^{H-1} &\equiv \left[\theta_a + \sum_{i \in K} \left(\theta_i + \frac{F_i(\theta_i)}{f_i(\theta_i)} \right) + \left(\theta_r - \frac{1 - F_r(\theta_r)}{f_r(\theta_r)} \right) \right] \phi'(g(\theta)) = 1, \\ \bar{g}^H &\equiv \left[\theta_a + \sum_{i \in L, K} \left(\theta_i + \frac{F_i(\theta_i)}{f_i(\theta_i)} \right) \right] \phi'(g(\theta)) = 1. \end{aligned}$$

Then, [Lemma 4](#) and [Lemma 10](#) imply the following proposition.

Proposition 7. Suppose $\bar{v}_i''(\theta_i, g^\circ) \leq 0$. The indices $k = k(g^\circ)$ and $l = l(g^\circ)$ are given by:

$$k(g^\circ) = \begin{cases} 0 & \text{if } 0 \leq g^\circ < \bar{g}^L \\ 1 & \text{if } \bar{g}^L \leq g^\circ < \bar{g}^{L-1} \\ \vdots & \text{if } \vdots \\ r-2 & \text{if } \bar{g}^{H-1} \leq g^\circ < \bar{g}^H \\ r-1 & \text{if } \bar{g}^H \leq g^\circ, \end{cases}$$

$$l(g^\circ) = \begin{cases} 1 & \text{if } 0 \leq g^\circ < \bar{g}^L \\ 2 & \text{if } \bar{g}^L \leq g^\circ < \bar{g}^{L-1} \\ \vdots & \text{if } \vdots \\ r-1 & \text{if } \bar{g}^{H-1} \leq g^\circ < \bar{g}^H \\ r & \text{if } \bar{g}^H \leq g^\circ. \end{cases}$$

Proof.

- $k = 0, l = 1$. Same as in [Proposition 2](#).
- $0 < k < l < r$. By [Lemma 4](#), the set of types is ordered. But then, rents are minimized if the participation constraint binds on a single interior type. Denote this type by θ_l . Formally, the proposer's problem becomes

$$\begin{aligned} & \max_{g(\theta), t(\theta)} \int \theta_a \phi(g(\theta)) - t(\theta_a) dF(\theta) \\ & \text{subject to } g(\theta) \leq t(\theta_a) + \sum_{i \in N \setminus \{a\}} t(\theta_i), \\ & \quad \frac{dg(\theta)}{d\theta} \geq 0, \\ & t(\theta_i) = \begin{cases} \theta_i \phi(g(\theta_i, \theta_{-i})) - \int_{\theta}^{\theta_i} \phi(g(x, \theta_{-i})) dx & \text{for } \theta_i \geq \theta_l, \\ \theta_i \phi(g(\theta_i, \theta_{-i})) - \int_{\theta_i}^{\bar{\theta}} \bar{v}_i(x, g^\circ) - \phi(g(x, \theta_{-i})) dx & \text{for } \theta_i < \theta_l, \end{cases} \\ & u_i(\theta_i, g(\theta)) \geq \bar{v}_i(\theta_i, g^\circ) \quad \forall i \in N \setminus \{a\}. \end{aligned}$$

Then, going through the same derivations as in [Proposition 2](#) yields the following condition for the optimal $g(\theta)$

$$\left[\theta_a + \sum_{i < l} \left(\theta_i + \frac{F_i(\theta_i)}{f_i(\theta_i)} \right) + \sum_{i \geq l} \left(\theta_i - \frac{1 - F_i(\theta_i)}{f_i(\theta_i)} \right) \right] \phi'(g(\theta)) = 1.$$

- $k = r - 1, l = r$. Same as in [Proposition 2](#).

□

A.6 Convexity

I begin with the proof of [Lemma 5](#).

Proof. Consider $\theta_k \in K(g^\circ)$, $\theta_l \in L(g^\circ)$ and $\theta_m \in M(g^\circ)$, with $\theta_m \leq \theta_l \leq \theta_k$. We have that

$$\begin{aligned} dU_m/\theta_m &= \phi(g(\theta)) - \bar{v}'_m(\theta_m, g^\circ) \geq \phi(g(\theta)) - \bar{v}'_l(\theta_l, g^\circ) \\ &= dU_l/d\theta_l \\ &= 0 \\ &\geq \phi(g(\theta)) - \bar{v}'_k(\theta_k, g^\circ) \\ &= dU_k/d\theta_k. \end{aligned}$$

However, in contrast to [Lemma 4](#), there cannot be a single interior type in $L(g^\circ)$.

Intuitively, as g° increases, the agents with the highest types transition into $K(g^\circ)$. By Condition (2), information rents have to be decreasing in $K(g^\circ)$ and increasing in $M(g^\circ)$. Since $\gamma^*(\theta_i)$ is nondecreasing, it must be constant for all types; i.e., $\gamma^*(\theta_i) = \gamma^*$, for all $i \in N \setminus \{a\}$. But then, information rents are minimized if the IR constraints of the agents with the highest and lowest types bind.

Formally, define $R(\gamma^*)$ to be the utility differential between the high type and the low type

$$R(\gamma^*) = U(\bar{\theta}) - U(\underline{\theta}) = \int_{\underline{\theta}}^{\bar{\theta}} \phi(\hat{g}(\theta, \gamma^*)) - \bar{v}'_i(\theta_i, g^\circ) d\theta,$$

where $\hat{g}$ solves (4). Clearly, $R(\gamma^*)$ is decreasing. To determine the optimal choice of γ^* , we have the following lemma.

Lemma 11 ([Maggi and Rodriguez-Clare A. \(1995\)](#)). *The optimal γ^* satisfies*

$$\gamma^* \equiv \begin{cases} 1 & \text{if } R(1) \geq 0 \\ R^{-1}(0) & \text{if } R(1) < 0 < R(0) \\ 0 & \text{if } R(0) \leq 0. \end{cases}$$

Proof. When $R(1) \geq 0$ or $R(0) \leq 0$, the same reasoning as in [Proposition 2](#) applies. Consider now the case $R(1) < 0 < R(0)$. Choosing γ^* ensures that $R(\gamma^*) = 0$ and incentive compatibility is preserved. Moreover, denote

$$J(\theta; g^\circ) \equiv \int_{\underline{\theta}}^{\theta} \phi(\hat{g}(x; \gamma^*)) - \bar{v}'_i(x, g^\circ) dx \quad (15)$$

where $\hat{g}$ solves (4). We have the following lemma.

Lemma 12. $J(\theta; g^\circ)$ is quasiconcave in θ .

Proof. $J'(\theta; g^\circ) = 0$ when $G(\theta_i; g^\circ) = 0$, which occurs for a single θ_i . By convexity of $\bar{v}_i$, $J'(\theta; g^\circ)$ is decreasing for $\theta' > \theta$. $\square$

Then, $U(\theta) = J(\theta; g^\circ) + U(\underline{\theta})$ and, by quasiconcavity, $U(\theta) \geq \min\{U(\underline{\theta}), U(\bar{\theta})\}$ for all $\theta_i \in \Theta_i$. Hence, if the individual rationality constraint is satisfied for the highest and lowest types, it is satisfied for all types. $\square$

This concludes the proof of [Lemma 5](#).

As in [Appendix A.5](#), it is useful to define the following thresholds of the support of g° recursively as follows:

$$\begin{aligned}\bar{g}^L &\equiv \left[\theta_a + \sum_{i \in L, M} \left(\theta_i - \frac{1 - F_i(\theta_i)}{f_i(\theta_i)} \right) \right] \phi'(g(\theta)) = 1 \\ \bar{g}^{L-1} &\equiv \left[\theta_a + \left(\theta_r - \frac{\gamma_{L-1}^* - F_r(\theta_r)}{f_r(\theta_r)} \right) + \sum_{i \in L, M} \left(\theta_i - \frac{\gamma_{L-1}^* - F_i(\theta_i)}{f_i(\theta_i)} \right) \right] \phi'(g(\theta)) = 1 \\ \bar{g}^{H-1} &\equiv \left[\theta_a + \sum_{i \in K} \left(\theta_i - \frac{\gamma_{H-1}^* - F_i(\theta_i)}{f_i(\theta_i)} \right) + \left(\theta_1 - \frac{\gamma_{H-1}^* - F_1(\theta_1)}{f_r(\theta_1)} \right) \right] \phi'(g(\theta)) = 1 \\ \bar{g}^H &\equiv \left[\theta_a + \sum_{i \in L, K} \left(\theta_i + \frac{F_i(\theta_i)}{f_i(\theta_i)} \right) \right] \phi'(g(\theta)) = 1.\end{aligned}$$

Notice that γ^* changes with the outside option public-good level, as shown in [Lemma 11](#). [Lemma 5](#) and [Lemma 10](#) imply the following proposition.

Proposition 8. Suppose $\bar{v}_i''(\theta_i, g^\circ) \geq 0$. The indices $k = k(g^\circ)$ and $l = l(g^\circ)$ are given by:

$$k(g^\circ) = \begin{cases} 0 & \text{if } 0 \leq g^\circ < \bar{g}^L \\ 1 & \text{if } \bar{g}^L \leq g^\circ < \bar{g}^{L-1} \\ \vdots & \text{if } \vdots \\ r-2 & \text{if } \bar{g}^{H-1} \leq g^\circ < \bar{g}^H \\ r-1 & \text{if } \bar{g}^H \leq g^\circ, \end{cases}$$

$$l(g^\circ) = \begin{cases} r & \text{if } 0 \leq g^\circ < \bar{g}^L \\ r-1 & \text{if } \bar{g}^L \leq g^\circ < \bar{g}^{L-1} \\ \vdots & \text{if } \vdots \\ 2 & \text{if } \bar{g}^{H-1} \leq g^\circ < \bar{g}^H \\ 1 & \text{if } \bar{g}^H \leq g^\circ. \end{cases}$$

Proof.

- $k = 0, l = 1$. Same as in [Proposition 2](#).
- $0 < k < l < r$. In order to preserve incentive compatibility, according to condition (2) information rents have to be increasing (decreasing) over the set of types who have the incentive to understate (overstate) their type. By [Lemma 5](#), the set of types is ordered. But then, rents are minimized if the participation constraints of the lowest and highest types bind. Formally, the proposer's problem becomes

$$\begin{aligned}
& \max_{g(\theta), t(\theta)} \int \theta_a \phi(g(\theta)) - t(\theta_a) dF(\theta) \\
& \text{subject to } g(\theta) \leq t(\theta_a) + \sum_{i \in N \setminus \{a\}} t(\theta_i), \\
& \quad \frac{dg(\theta)}{d\theta} \geq 0, \\
& t(\theta_i) = \begin{cases} \theta_i \phi(g(\theta_i, \theta_{-i})) - \int_{\theta}^{\theta_i} \phi(g(x, \theta_{-i})) dx & \text{for } \theta_i \leq \theta_l, \\ \theta_i \phi(g(\theta_i, \theta_{-i})) - \int_{\theta_i}^{\theta} \bar{v}_i(x, g^\circ) - \phi(g(x, \theta_{-i})) dx & \text{for } \theta_i > \theta_l, \end{cases} \\
& u_i(\theta_i, g(\theta)) \geq \bar{v}_i(\theta_i, g^\circ) \quad \forall i \in N \setminus \{a\}.
\end{aligned}$$

Then, going through the same derivations as in [Proposition 2](#) yields the following condition for the optimal $g(\theta)$

$$\left[\theta_a + \sum_{i>l} \left(\theta_i - \frac{\gamma^* - F_i(\theta_i)}{f_i(\theta_i)} \right) + \sum_{i \leq l} \left(\theta_i - \frac{\gamma^* - F_i(\theta_i)}{f_i(\theta_i)} \right) \right] \phi'(g(\theta)) = 1.$$

- $k = r - 1, l = r$. Same as in [Proposition 2](#).

□

This concludes the proof.

□

A.7 Proof of [Theorem 3](#)

Proof. From [Theorem 2](#), we immediately have the following corollaries.

Corollary 2. *For a given coalition $Q \in \mathcal{Q}$, the solution to the unanimity problem has to hold.*

Corollary 3. *The participation constraint holds for at least q agents.*

Corollary 4. *$K(g^\circ)$ and $M(g^\circ)$ are convex sets.*

What remains to be done is identifying the set of agents whose participation constraint is not satisfied.

Proposition 9. *Suppose $dU_i/d\theta_i$ has the same sign for all agents. Then, the participation constraint binds on a single type θ_l . There exists a set $I(g^\circ) = \{i \in N \mid U(\theta_i) < 0\}$, with $|I(g^\circ)| = n - q$, such that*

(i) *If $dU_i/d\theta_i < 0 \forall i \in N$, then*

$$I = \{i \in N \mid \theta_i \leq \theta_l\} ,$$

(ii) *If $dU_i/d\theta_i > 0 \forall i \in N$, then*

$$I = \{i \in N \mid \theta_i \geq \theta_l\} .$$

Moreover, a subset of agents in I is bunched at the allocation of θ_l .

[Proposition 9](#) applies when g° is sufficiently high or low. When agents have heterogeneous incentives to misreport their types - $dU_i/d\theta_i$ does not have the same sign for all agents - then, the choice of the set $I(g^\circ)$ hinges on the distribution of information rents induced by incentive compatibility; hence, it crucially depends on the shape of the reservation utility profile. The concave and convex cases are therefore dealt with separately.

A.7.1 Concavity

The construction needed is shown in [Figure 5](#) and can be derived with a very similar reasoning as in [Lemma 11](#). Intuitively, it is efficient for the agenda-setter to violate $n - q$ participation constraints. To preserve incentive compatibility, we need the following lemma. Formally, define $R(\gamma^*)$ to be the utility differential between θ_p and θ_q

$$R(\gamma^*) = U(\theta_q) - U(\theta_p) = \int_{\theta_p}^{\theta_q} \phi(\hat{g}(\theta, \gamma^*)) - \bar{v}'_i(\theta_i, g^\circ) d\theta ,$$

where $\hat{g}$ solves (4). Clearly, $R(\gamma^*)$ is decreasing.

Lemma 13. *The optimal γ^* satisfies*

$$\gamma^* \equiv \begin{cases} F_q(\theta_q) & \text{if } R(F_q(\theta_q)) \geq 0 \\ R^{-1}(0) & \text{if } R(F_q(\theta_q)) < 0 < R(F_p(\theta_p)) \\ F_p(\theta_p) & \text{if } R(F_p(\theta_p)) \leq 0 . \end{cases}$$

Proof. When $R(F_q(\theta_q)) \geq 0$ or $R(F_p(\theta_p)) \leq 0$, [Proposition 9](#) applies. Consider now the case $R(F_q(\theta_q)) < 0 < R(F_p(\theta_p))$. Choosing γ^* such that $R(\gamma^*) = 0$ ensures incentive compatibility is preserved.

As above, denote

$$J(\theta; g^\circ) \equiv \int_{\theta_p}^{\theta} \phi(\hat{g}(x; \gamma^*)) - \bar{v}_i'(x, g^\circ) dx \quad (16)$$

where $\hat{g}$ solves (4). We have the following lemma.

Lemma 14. *$J(\theta; g^\circ)$ is quasiconvex in θ .*

Proof. $J'(\theta; g^\circ) = 0$ when $G(\theta_i; g^\circ) = 0$, which occurs for a single θ_i . By concavity of $\bar{v}_i$, $J'(\theta; g^\circ)$ is increasing for $\theta' > \theta$. $\square$

Then, $U(\theta) = J(\theta; g^\circ) + U(\theta_p)$ and, by quasiconvexity, $U(\theta) \leq \max\{U(\theta_p), U(\theta_q)\}$ for all $\theta \in [\theta_p, \theta_q]$. Hence, if the participation constraint is satisfied for θ_p and θ_q , it is violated for all types in between. $\square$

Finally, the choice of the interval (θ_p, θ_q) can only be the largest interval comprising $n - q$ agents.

Lemma 15. *The choice of the interval (θ_p, θ_q) is such that:*

$$\theta_1, \theta_2, \dots, \underbrace{\theta_p, \theta_{p+1}, \dots, \theta_{q-1}, \theta_q}_{|I| \leq n-q}, \dots, \theta_{n-1}, \theta_n.$$

Proof. Suppose not. If (θ_p, θ_q) comprises more than $n - q$ agents, then the participation constraint is satisfied for fewer than $q - 1$ agents, and the proposal cannot be implemented. On the other hand, if (θ_p, θ_q) comprises fewer than $n - q$ agents, then extending it to include just $n - q$ agents does not affect incentive compatibility and decreases the rents paid out by the agenda-setter, a contradiction. $\square$

A.7.2 Convexity

When the reservation utility profile is convex, it was shown in [Appendix A.6](#) that the participation constraints bind on the highest and lowest types. Along a very similar reasoning as in [Proposition 9](#), we have the following proposition.

Proposition 10. *There exist two types θ_p and θ_q such that*

$$I(g^\circ) = \{i \in N \mid \theta_i \leq \theta_p \cup \theta_i \geq \theta_q\}.$$

Moreover, a subset of agents might be bunched at the same allocation of θ_p and θ_q .

Proof. When g° is sufficiently low or high, [Proposition 9](#) applies. For intermediate values of g° , in the unanimity case the participation constraints are binding for the lowest and highest types, as shown in [Appendix A.6](#). Incentive compatibility is preserved if transfers

are increased uniformly so as to induce the participation constraints of types $\theta_p > \underline{\theta}$ and $\theta_q < \bar{\theta}$ bind. Moreover, whenever $g(\theta)$ is lower than (1), agents $\theta_i < \theta_p$ can be bunched at the allocation of θ_p so as to *increase* public-good provision. Analogously, when $g(\theta)$ is higher than (1), agents $\theta_i > \theta_q$ can be bunched at the allocation of θ_q in order to *decrease* public-good provision. $\square$

$\square$

A.8 Proof of Proposition 6

As in Jullien (2000), define $g(\theta)$ as the public-good level such that

$$\phi(g(\theta)) = \mathbb{E}_\mu[\phi(g(\theta)) \mid \theta],$$

where $\mathbb{E}_\mu[\cdot \mid \theta]$ is the conditional expectation operator for the distribution $\mu(\theta, dg)$.

Proof. Recall condition (2) and consider two types, θ_1 and θ_2 . We have that

$$\begin{aligned} \int_{\theta_1}^{\theta_2} \phi(g(\theta)) - \bar{v}'_i(\theta_i, g^\circ) d\theta_i &\geq \int_{\theta_1}^{\theta_2} \int_g \phi(g(\theta)) \mu(\theta_1, dg) d\theta_i - \int_{\theta_1}^{\theta_2} \bar{v}'_i(\theta_i, g^\circ) d\theta_i \\ &= \int_{\theta_1}^{\theta_2} \phi(g(\theta_1, \theta_{-i})) d\theta_i - \bar{v}'_i(\theta_i, g^\circ) d\theta_i, \end{aligned}$$

where the first inequality follows from Jensen's inequality and the last equality by the linearity in types of the utility function. Therefore, $g(\theta)$ is incentive compatible.

Moreover, by concavity of the objective function, we have that

$$\sigma(\theta, \gamma(\theta), g) \geq \sigma(\theta, \gamma(\theta), \mathbb{E}_\mu[g]).$$

$\square$

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