# The Inner Beauty of Firms: Internal Organization in Industry Equilibrium

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**Abstract**

This paper studies how the internal organization of firms interacts with labor and product markets using millions of task assignments within hundreds of hair salons. I develop a measure of organization complexity and provide evidence of firm-specific organization costs, which grant complex salons a comparative advantage in producing high-quality products. Based on these facts, I develop a model where oligopolistic firms with different organization costs choose their internal structure. Complexity is costly, but it allows firms to improve product quality by better matching workers with multidimensional skills to tasks. I characterize the profit-maximizing organization, and identify and estimate the model for Manhattan hair salons. Counterfactuals reveal that allowing internal organization to be heterogeneous and endogenous changes the equilibrium effects of policy. A sales tax cut increases specialization and therefore the productivity of all workers, while a minimum wage increase generates new types of wage spillovers.

"Of all the things I've done, the most vital is coordinating those who work with me and aiming their efforts at a certain goal." - **Walt Disney**

# 1   Introduction

Greater specialization allows markets to better use the unique talents of individuals. As early as Adam Smith's pin factory, economists have recognized that much of this division of labor occurs within the firm, a process often referred to as internal organization. In practice, firms differ in their ability to organize people and use a wide variety of organization structures. How do firms choose their internal organization, and how does this choice interact with product markets, labor markets and government policy?

To answer this question, I propose a framework to study firms' equilibrium choice of internal organization. Using a set of stylized facts from management software data, I model firms as deciding which workers to hire and how to assign them to tasks. More complex assignments are costly, but they improve product quality through a better match of skills to tasks. Because firms differ in their organizational capabilities, they choose different internal structures. Additionally, because firms share a product and a labor market, the internal organization structures of competing firms are intertwined in equilibrium. I estimate the model for Manhattan hair salons, and show that allowing internal organization to be heterogeneous and endogenous qualitatively changes the effect of counterfactual policies. For example, a minimum wage raises equilibrium specialization for minimum wage workers, reduces specialization for non-minimum wage workers, and causes wage spillovers which are not monotone in initial wage.

In the first part of this paper, I use novel data to establish empirical patterns in firm internal organization. The data, which come from a management software company, allow me to observe the assignment of millions of tasks to individual workers across hundreds of hair salons. I view firms as choosing organization structures, which are matrices where rows represent workers, columns represent tasks, and each element is the fraction of total time assigned to each worker-task pair. I create a measure called organization complexity, which quantifies the amount of information that must be communicated within a firm in order to implement a given organization structure.

I document three facts about salon internal organization. First, complexity varies sig-

nificantly across salons but very little across time, with few salons engaging in complete specialization. This is evidence of firm-specific and time-invariant organization costs which prevent full specialization. Second, complex firms have higher revenue and employment. This indicates firms with lower organizational costs have a competitive advantage in the product market. Third, complex firms have higher prices and more repeat customers. This is evidence the organizational competitive advantage operates through quality rather than quantity, meaning organizationally efficient salons have a comparative advantage at producing higher-quality products.

In the second part of the paper, I build a model consistent with these facts. In this model, firms with product market power choose product prices, the composition of their workforce, and worker task assignments. Workers differ in their skill at each task. Assigning tasks to the most skilled worker raises product quality but also increases organization complexity. Firms differ in the cost of complexity and their task-based production function, which causes them to choose different internal structures. Firms compete in a common product and labor market, so their choices of internal structures both shape and are shaped by wages, prices and qualities.

The main theoretical result is a characterization of the firm's optimal organization structure enabling analysis, identification and estimation. My model differs from past task-assignment models along three dimensions: firms face heterogeneous organization costs which prevent full specialization, firms have market power, and workers have horizontal skill heterogeneity. Because of these differences, I cannot use existing approaches to make the firm's problem tractable. Instead, I show that the profit-maximizing organization also solves an equivalent rational inattention problem with mutual information attention costs. This equivalence allows me to weave together existing results to prove the other propositions in the paper.

In the third part of the paper, I identify and estimate a structural version of the model for hair salons in New York City. I prove that even though firms are choosing the task assignment of each individual worker, at a high level, the firm is choosing a point along a convex frontier that divides two dimensions: organization complexity and wages adjusted for product quality. Thus the complexity of a firm's task assignments can be inverted to recover its organization cost parameter. This implies that organization costs

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and structures are known functions of the data and the other parameters, and do not need to be estimated. Variation in the interaction of task intensity and organization complexity across firms in the same market allows the identification of the other parameters. Intuitively, firms intense in task $k$ and organizationally complex hire a large share of task $k$ specialists and assign a large amount of task $k$ to these specialists. The quality of these firms identifies the skill of task $k$ specialists, while the cost of these firms identifies the wage of task $k$ specialists. I provide a computationally light, nested fixed-point estimation procedure which implements this identification strategy.

The estimated model reveals that even within a single industry (hair salons) and occupation (cosmetologists), variation in task specialization is large and depends on unobserved worker skills and unobserved firm organizational differences. Firms in the bottom quartile of organization costs (efficient salons) on average assign 90% of tasks to the associated specialist, while firms in the top quartile (inefficient salons), assign only 67%. Haircut specialists spend most of their time cutting, but blow-dry specialists spend less than half of their time blow-drying. I also show that internal organization is a large source of productivity differences across firms, accounting for 40% of variation in marginal costs.

In the fourth part of the paper, I study two counterfactual policy changes, one in the product market and one in the labor market. In both cases, the fact that internal organization is heterogeneous and endogenous introduces new economic forces and qualitatively changes the total economic impact of each policy. The structure of the model allows any policy to be cleanly decomposed into a reallocation effect, where labor shifts across firms but internal organization remains fixed, and a reorganization effect, where task assignments within the firm are allowed to adjust. The reallocation effect is driven by the heterogeneity of internal organization, while the reorganization effect is driven by the endogeneity of internal organization.

In the first counterfactual, I eliminate the 4.5% New York City sales tax on services. The reallocation effect improves the competitive position of complex salons who were initially providing high-quality services. The reorganization effect induces almost all salons to reorganize in order to increase quality. Both effects increase equilibrium task specialization across all workers and increase equilibrium labor productivity. Workers capture most of the productivity gains through higher wages.

In the second counterfactual, I increase the minimum wage from \$15 to \$20. The re-allocation effect reduces the competitive position of firms with internal structures that rely on minimum wage workers. Thus, non-minimum wage workers initially employed alongside minimum wage workers see a reduction in labor demand. The reorganization effect causes firms to lay off more minimum wage workers and shift their tasks onto other workers. This increases task specialization for minimum wage workers but reduces it for other workers. Although the labor market is competitive, organizational heterogeneity and endogeneity allow the model to generate aggregate labor-labor substitution patterns that are not possible with standard models. For Manhattan hair salons, reallocation and reorganization together produce wage spillovers that are non-monotonic in initial wage, with high- and low-wage workers seeing wage increases and workers in the middle seeing wage decreases.

In this paper I draw insights from organizational economics and the task-based literature in labor economics in order to understand how internal organization decisions shape economic outcomes. The primary contribution of the paper is to build and estimate a model where organizationally unique firms make task assignment decisions which have labor and product market consequences.

The literature in organizational economics provides many ways in which firms can allocate talent better than markets do. These include monitoring (Alchian and Demsetz 1972, Baker and Hubbard 2003), relational contracts (Baker, Gibbons, and Murphy 2002), knowledge (Garicano and Wu 2012), coordination (Dessein and Santos 2006), trust (Meier, Stephenson, and Perkowski 2019) and culture (Martinez et al. 2015). This papers views heterogeneity in these dimensions as a major source of productivity differences across firms, and studies the implications for market outcomes.

I model labor as being divisible into tasks which can be assigned to workers with different skills, a tradition that dates back to at least Sattinger (1975) but has seen growing use since Autor, Levy, and Murnane (2003). I incorporate features present in different parts of the literature, including multidimensional worker types (Lindenlaub 2017, Ocampo 2022), firms with multiple worker types (Haanwinckel 2020, Freund 2022), organization costs (Adenbaum 2021, Garicano 2000), and firm-specific task demands (Lazear 2009). I also incorporate product market power. This combination of features allows for

flexible labor-labor substitution patterns that are determined by the distribution of skills, organization costs and task demands in the economy. This flexibility is why I find that a minimum wage generates non-monotonic wage spillovers even in a competitive labor market. Additionally, my model generates jobs which are bundles of tasks and which vary from firm to firm even for the same type of worker. This makes my model more realistic than past models, which typically generate fully specialized jobs that are homogeneous within industry.

Finally, this paper makes a methodological contribution. In the majority of task-based models and hierarchy models (Garicano and Rossi-Hansberg 2006, Caliendo et al. 2012, Garicano and Hubbard 2016), workers are matched to tasks according to a single observable dimension, typically involving education. Since it is known prior to estimation that wages are increasing in this observed dimension, information on the wage distribution identifies features of the task-based production function. Direct information on tasks is then typically not used for estimation. I consider the opposite case, when information on wages is limited but information on tasks is rich. I show that the parameters of the task-based production function can be inferred based on differences in qualities and costs across firms intense in different tasks but operating in the same market. Further, task information allows the incorporation of workers that have unobserved horizontal differences in skill. This makes the model useful for incorporating skill differences which cannot be inferred from observed characteristics of workers.

## 2 Data

This section describes the salon management software data I use in this paper.

### 2.1 Context and Institutional Details

The data set was obtained from a data sharing agreement I negotiated with a salon management software company. The software facilitates running a beauty business, including scheduling, pricing, payments, inventory, staffing, business reporting, client profiling and marketing. As of July 19, 2022, a monthly subscription has a base price of $175. Although the company also markets its software to spas, tanning salons and massage parlors, hair

salons and barbers make up the majority of its clients. For this reason, I analyze only hair salons and barbershops.

The software is sold to beauty businesses throughout the United States, but the data indicate uptake is largest in Los Angeles (where the company was founded) and New York City. An important aspect of the data set is that it allows me to observe the internal organization of salons that are geographically close and therefore likely to be direct competitors in labor and product markets. For example, I observe 10 salons in the lower Manhattan zip code 10013, which is a 0.55 square mile area.

The data document which stylist is assigned to each task and client, and record the duration of the appointment, the price paid, and a custom text description of each task. If more than one employee is assigned to a single client, this is recorded as multiple entries describing what each employee contributed. Although the data are de-identified, unique IDs allow a researcher to track employees and clients across time within a salon.[1]

A sample from the data is provided in Table 1, with IDs replaced with pseudonyms. This sample shows the different ways two salons coordinate employees to meet customer demand. Blake requested a cut, highlights and a treatment at salon 1A. The salon had a single employee, Rosy, perform all three services. Grace requested a cut and a single process (color) at salon 2A. Unlike salon 1A, salon 2A chose to assign each of these tasks to two separate employees, Tyler and Ben. Both of these salons are in the same zip code.

Table 1: Salon Activity Data Sample

| Firm | Salon | App. | Cust. | Service | Staff | Time Stamp | Price | Duration |
|------|-------|------|-------|---------|-------|------------|-------|----------|
| 1 | 1A | 123 | Blake | Advanced Cut | Rosy | 3/26/2021 16:15 | 100 | 72 |
| 1 | 1A | 123 | Blake | Full Head - Highlights | Rosy | 3/26/2021 16:15 | 243 | 127 |
| 1 | 1A | 123 | Blake | Treatment Add On (Olaplex) | Rosy | 3/26/2021 16:15 | 39 | 72 |
| 2 | 2A | 9982 | Grace | Women's Cut | Tyler | 3/17/2021 11:00 | 225 | 43 |
| 2 | 2A | 9982 | Grace | Single Process | Ben | 3/17/2021 11:00 | 200 | 77 |

**Note:** This table is a snapshot displaying two actual appointments at salons in the same zip code from the data used for the estimation. Customer IDs are replaced by pseudonyms.

While the data are rich in terms of task content and worker assignments, information about employee compensation is sparse. The software can track some compensation information (tips, commissions and employment relationship, etc.), but these additional functions are not used consistently by client salons, as discussions with the company and analysis of internal data revealed. The data contain 20,560 unique text descriptions of ser-

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1. IDs are salon specific, so I cannot track employees or clients if they move across salons.

vices. A licensed cosmetologist was hired to group the tasks into a manageable number of categories. In the end, five mutually exclusive task categories were created. Appendix Section A.10 provides additional details about the process.

## 2.2 Descriptive Statistics

The data used in this section and the stylized facts include all observed firm-quarters where revenue per customer is positive. I exclude 2021 Q3, because I observe only part of the quarter. I also exclude an establishment in Kentucky with revenue that is implausibly high. The data contain information on 445 hair salon establishments, which represent 316 unique businesses, 9,179 hair stylists, 1,654,233 customers and 10.8 million services performed. Establishments first appear in the data when they adopt the management software. The last complete month with available data is August 2021. Although the software is used by salons across the country, users are concentrated in New York and California.

I aggregate the data to the firm-quarter level, for analysis. Descriptive statistics at this level are provided in Table 2. Throughout the paper, I refer to the price as the average revenue per customer per quarter. The salons have an average price of $200. Even though there is significant variation in the relative intensity of tasks at different salons, most salons offer at least four of the five task categories in a given quarter. Throughout the paper, I refer to the task mix of a salon as the fraction of total time spent on each of the five tasks. Firm-quarter heterogeneity in the task mix is illustrated in Figure 1. Firms vary in their intensity in each task.
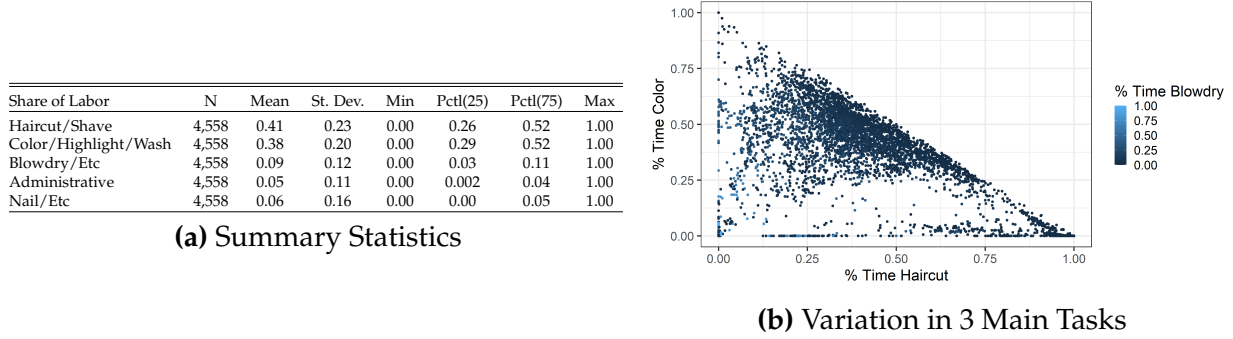
Table 2: Summary Statistics for All Salon-Quarters

| Statistic | N | Mean | St. Dev. | Min | Pctl(25) | Pctl(75) | Max |
|---|---|---|---|---|---|---|---|
| Revenue | 4,558 | 213,201.30 | 248,359.90 | 5 | 58,912.5 | 271,236.5 | 2,559,703 |
| Price | 4,558 | 199.73 | 135.16 | 0.20 | 111.71 | 261.88 | 3,180.44 |
| Employees | 4,558 | 13.38 | 10.79 | 1 | 6 | 17 | 92 |
| Customers | 4,558 | 1,159.23 | 1,098.45 | 1 | 397 | 1,619 | 16,768 |
| Task Categories | 4,558 | 4.45 | 0.86 | 1 | 4 | 5 | 5 |
| Labor per. Customer | 4,558 | 2.15 | 1.63 | 0.10 | 1.52 | 2.57 | 61.33 |

**Note:** The table displays summary statistics for the main variables of interest with data aggregated at the salon-quarter level. There is significant variation across salons in complexity, number of employees, revenue and many other dimensions.

The salons in the sample have an average quarterly revenue of $213,201 and an aver-

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**Figure 1:** Variation in Firm-Quarter Task Mix

| Share of Labor | N | Mean | St. Dev. | Min | Pctl(25) | Pctl(75) | Max |
|---|---|---|---|---|---|---|---|
| Haircut/Shave | 4,558 | 0.41 | 0.23 | 0.00 | 0.26 | 0.52 | 1.00 |
| Color/Highlight/Wash | 4,558 | 0.38 | 0.20 | 0.00 | 0.29 | 0.52 | 1.00 |
| Blowdry/Etc | 4,558 | 0.09 | 0.12 | 0.00 | 0.03 | 0.11 | 1.00 |
| Administrative | 4,558 | 0.05 | 0.11 | 0.00 | 0.002 | 0.04 | 1.00 |
| Nail/Etc | 4,558 | 0.06 | 0.16 | 0.00 | 0.00 | 0.05 | 1.00 |

**(a)** Summary Statistics



**(b)** Variation in 3 Main Tasks

**Note:** Panel A provides summary statistics about the share of time spent on each task across all firm-quarters. Panel B illustrates this variation for the three most common tasks. Each point is a firm-quarter.

age of 13 employees. Johnson and Lipsitz (2022) studies a sample of salon owners and reports an average annual (not quarterly) revenue of $233,000 and an average of seven stylists. It is important to be cautious when comparing self-reported survey estimates from other sources with management data (like this source), but given the subscription fee of the software, it is reasonable to conclude that the salons in my sample skew toward larger and higher-end salons. This suggests the heterogeneity found in this paper underestimates the heterogeneity in the universe of U.S. salons.

# 3   Stylized Facts

The model I use to study the effect of internal organization on product and labor markets is inspired by three stylized facts. These facts require the definition of two concepts which will be used throughout the paper. To begin, denote workers by the index $i$, firms by the index $j$, and tasks by the index $k$.

**Definition 1** *A firm's organization structure, denoted by $B_j$, is a matrix where element $B_j(i, k)$ is the fraction of labor assigned to worker $i$ and task $k$.*

An example of two different organizational structures is given in Figure 2. The left structure is staffed by specialists while the right structure is staffed by generalists.

The second concept, complexity, measures the minimum amount of information that must flow through the firm in order for it to implement a given structure, and it is based

**Figure 2:** Two Organization Structures

| | | Specialist Salon Tasks | | | | | | Generalist Salon Tasks | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | 1 | 2 | 3 | | | 1 | 2 | 3 | |
| Employee | A | 1/2 | 0 | 0 | 1/2 | A | 1/6 | 1/12 | 1/12 | 1/3 |
| | B | 0 | 1/4 | 0 | 1/4 | B | 1/6 | 1/12 | 1/12 | 1/3 |
| | C | 0 | 0 | 1/4 | 1/4 | C | 1/6 | 1/12 | 1/12 | 1/3 |
| | Tot. | 1/2 | 1/4 | 1/4 | 1 | Tot. | 1/2 | 1/4 | 1/4 | 1 |

**Note:** Two organizational structures a firm with a task mix of $1/2, 1/4, 1/4$ could choose. Column sums represent the task mix, and row sums represent the fraction of work performed by each employee.

on a literature in information theory starting with Shannon (1948).

**Definition 2** *The complexity of an organization structure $B_j$ is*[2]

$$I(B_j) = \sum_{i,k} B_j(i,k) log \left( \frac{B_j(i,k)}{\underbrace{\sum_{k'} B_j(i,k')}_{\text{Worker i Labor Share}} \underbrace{\sum_{i'} B_j(i',k)}_{\text{Task k Labor Share}}} \right)$$

Consider the two structures in Figure 2. The firm can implement the chair-renter structure (right) by randomly assigning workers to tasks. This implementation does not require information about tasks or worker identities, so the complexity is 0 in this case. To implement the employee structure (left), the firm must tell each worker exactly which task to perform. The firm can write an employee manual, stating "assign the task to employee A if you observe '0', assign to B if you observe '01', and assign to C if you observe '10'." The expected number of bits (or amount of information) is $1 \times 1/2 + 2 \times 1/2 = 1.5$. This is the minimum information required to communicate this assignment, so the complexity in this case is 1.5.

I now present three stylized facts about internal organization. Throughout the rest of the paper, complexity is assumed to be measured without error. Appendix Section A.12 provides evidence that measurement error is small.

**Fact 1** *Complexity varies significantly across firms and little across time.*

To establish this fact, I first compute $I_j^{max}$, which is the maximum value of complexity given a firm's task mix in a given quarter. I construct normalized complexity $\bar{I}_j$ as raw

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2. When computing this measure, I assume that $0log(0) = 0$.

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**Figure 3:** Histogram of Normalized Complexity



**(a)** Raw                   **(b)** Normalized
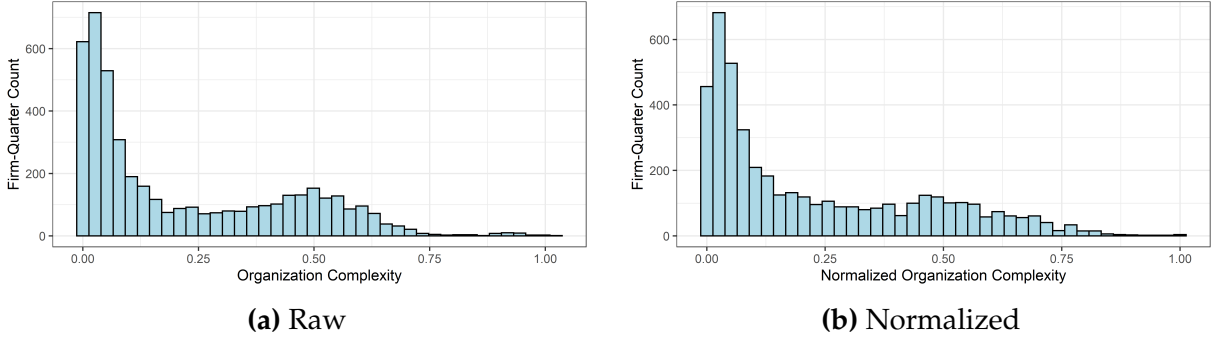
**Note:** Includes all firm-quarter observations. Both normalized and raw complexity vary significantly. Normalized complexity is observed achieving both its lower bound (0) and upper bound (1).

complexity divided by $I_j^{max}$. Normalized complexity $\bar{I}_j$ has a minimum of 0 (like raw complexity) and a maximum of 1 (unlike raw complexity). I plot a histogram of normalized complexity in Figure 3 and observe that complexity varies significantly across firm-quarters and has a long right tail. In particular, I observe that while some firms have very complex organizations (close to the upper bound), others have very simple organizations (complexity of 0). To understand whether the variation is across time or across salons, I decompose complexity into a salon-specific component, a time-specific component and a residual component:

$$\bar{I}_{j,t} = \bar{I}_j + \bar{I}_t + e_{j,t}$$

I estimate the firm and year components by regressing normalized complexity on time and salon fixed effects. This allows me to decompose the total variance of complexity into the three components:

$$\underset{.0516}{Var(I_{j,t})} = \underset{.0464}{Var(\bar{I}_j)} + \underset{.0002}{Var(\bar{I}_t)} + \underset{-.0009}{2Cov(\bar{I}_j, \bar{I}_t)} + \underset{0.0059}{Var(e_{j,t})}$$

These results demonstrate that 90 percent of the variance in normalized complexity is attributable to the firm component and only 0.4 percent to the time component. Therefore, complexity varies significantly across firms but little across time. This is evidence the choice of complexity is driven by a time-invariant, firm-specific organization cost.

**Fact 2** *Complex firms have higher revenue and employment.*

Complexity is positively correlated with revenue and employment, as well as several other measures of firm size. This correlation is depicted in Figure 4, which shows binned scatter plots of residualized complexity against residualized revenue employment, customers and visits. The plots control for the task mix, county, and quarter fixed effects. Appendix Table A1 demonstrates via a series of regressions that the correlation is positive for all firm size variables and statistically significant at the 5 percent level for revenue and employment. The positive relationship between revenue and complexity is robust; it remains when only Manhattan hair salons are analyzed and when employee count is interacted with complexity.

**Figure 4:** Organization Complexity and Firm Size



**(a)** Revenue ($)                                    **(b)** Employees
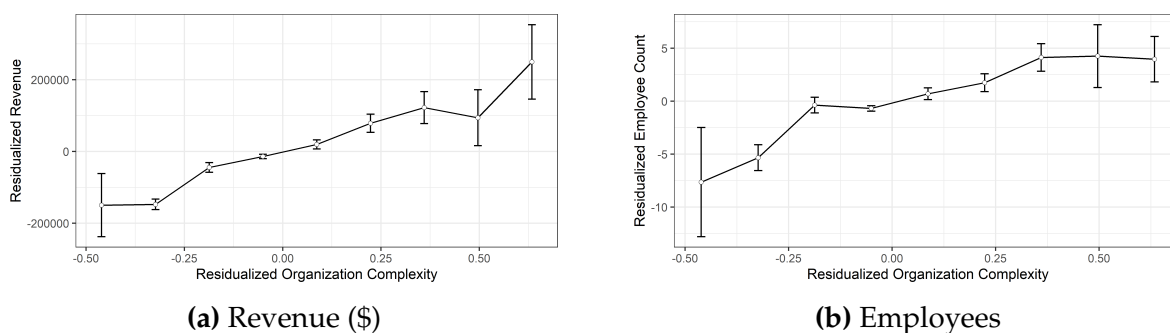
**Note:** All variables are residualized for quarter, county and task mix. Firm-quarters are grouped into equally spaced bins based on complexity.

The positive correlation between firm size and complexity suggests some salons have an organizational competitive advantage in that they find it easier than competitors to adopt productive organizational practices. This allows them to implement more complex task assignments at a lower cost.

**Fact 3** *Complex firms have higher prices and more repeat customers.*

Complexity is positively correlated with price, as shown in Panel A of Figure 5. Appendix Section A.9 proves that this pattern in the data is inconsistent with a model where organizational competitive advantages operate only through marginal cost reductions. In such a model, prices should be decreasing in complexity. The fact that the opposite is true suggests salons with higher internal complexity are producing services with higher

unobserved quality and thus higher costs.[3]

To test this quality channel, I use the share of repeat visits as a proxy for quality in Panel B of Figure 5. It is reasonable to assume that a customer who returns was satisfied with the quality of the original service. The fraction of visits by return customers rises with complexity, evidence of a link between quality and organization. This suggests that the organizational advantage described in Fact 2 operates through unobserved quality rather than quantity. In the next section, I build a model inspired by this and the other two facts. Appendix Section A.11 discusses the robustness of the stylized facts.

**Figure 5:** Organization Complexity, Prices and Repeat Customers



**(a)** Price

**(b)** Repeat Customers
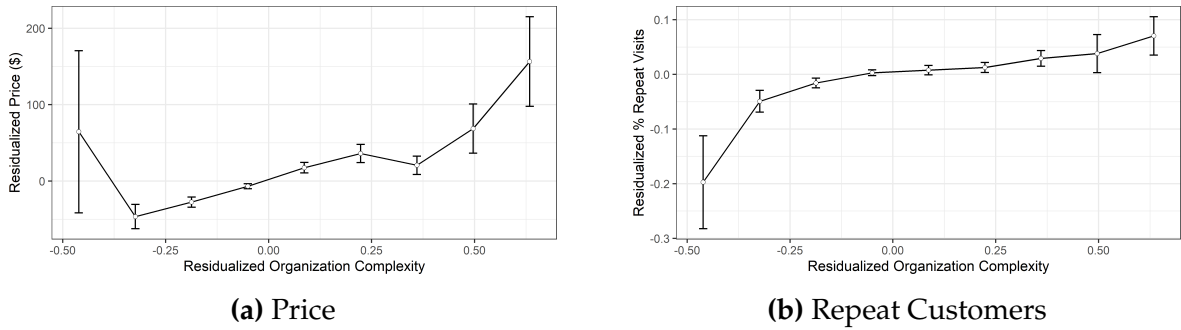
**Note:** The positive relationship between organization complexity and price (panel A), and the relationship between organization complexity and the fraction of customers that return (panel B). All variables are residualized for quarter, county and task mix. Firm-quarters are grouped into equally spaced bins based on complexity.

# 4   Model

This section specifies a model where firms choose prices and organizational structures simultaneously in order to compete for consumers. Consistent with the stylized facts, firms choose their organization structure subject to heterogeneous organization costs. The main benefit of a complex organization is the ability to produce a higher-quality product. There are three important groups of objects in the economy: firms, indexed by $j = 1, ..., J$; worker types, indexed by $i = 1, ..., N$; and tasks, indexed by $k = 1, ..., K$.

**Firms and Tasks.** The $J$ firms differ in their organization cost $\gamma_j \in \mathbb{R}_+$, discussed below. Each firm produces a single good using a Leontief task-based production function

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3. Kugler and Verhoogen (2012) use a similar argument to conclude that endogenous product quality is important.

described by $\alpha \in \mathbb{R}_+^K$, which I refer to throughout the paper as the task mix. The task mix is homogeneous in the theoretical section only for exposition: all results are obtained when it varies by firm. To produce one unit of the good, the firm must allocate $\alpha_k$ labor to task $k$, where I normalize $\sum_k \alpha_k = 1$. The firm can choose how these tasks are assigned to workers in a process that is described shortly.

**Workers and Labor Markets.** Each of the $N$ worker types is characterized by inelastic total labor supply $L_i$ and skill set vector $\theta_i$. Element $\theta_i(k)$ is the quality with which worker $i$ performs task $k$. The labor market is competitive with type-specific wages $w_i$, which I collect into a wage vector $w$.

**Firm Strategies.** Firms choose the price of their product $p_j \in \mathbb{R}_+$ and their organizational structure $B_j \in \Delta^{N \times K}$, where $\Delta^{N \times K}$ is a $N \times K$-dimensional unit simplex. Element $B_j(i, k)$ of an organization structure specifies the fraction of total labor allocated to worker type $i$ and task $k$. An organizational structure $B_j$ is feasible if it is consistent with the task-mix vector: $\sum_i B_j(i, k) = \alpha_k \, \forall k$. The workforce composition, $E_j(i) = \{E_j(1), ..., E_j(N)\}$, is the fraction of total labor demanded that is from each worker type. By definition, $E_j(i) = \sum_k B_j(i, k)$.

The cost of a firm's organization structure is the firm-specific parameter $\gamma_j$ multiplied by the complexity of the organization structure $I(B_j)$. Recall complexity is defined as[4]

$$I(B_j) = \sum_{i,k} B_j(i, k) log \left( \frac{B_j(i, k)}{\underbrace{\sum_{k'} B_j(i, k')}_{\text{Type } i \text{ Labor Share, } E_i} \underbrace{\sum_{i'} B_j(i', k)}_{\text{Task-mix, } \alpha_k}} \right).$$

A firm's organizational structure determines the match between worker skills and tasks. As a result, it determines product quality ($\xi(B_j)$). I specify that product quality is a weighted average of task quality: $\xi(B_j) = \sum_{i,k} B_j(i, k) \theta_i(k)$. Since quality is valued by consumers, increased quality is the main benefit of carefully assigning workers to tasks. A firm's organization structure also determines its per-unit wage bill: $W(B_j) = \sum_{i,k} w_i B_j(i, k)$.

**Demand.** Total market demand for good $j$ is given by a function $D_j$ which maps

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4. The mutual information is used because it is the only cost function in a certain class where complexity over types will be equal to complexity over worker identities under a general matching process (Bloedel and Zhong 2021).

the prices and qualities of all firms into a quantity demanded for firm $j$. I assume that demand for good $j$ depends on own-price and own-quality only through the quality-price index $\xi(B_j) - \rho p_j$, where $\rho$ is a consumer price sensitivity parameter. I also assume demand for good $j$ is strictly increasing in good $j$'s quality-price index. This implies the demand can be written as $D_j(\xi(B_j) - \rho p_j, p_{-j}, \xi_{-j})$.[5]

**The Firm's Problem.** Per-unit organization costs and competitive labor markets imply marginal costs are constant. I denote the feasible set of organization structures $\mathbb{B} = \{B \in \Delta^{N \times K} | \sum_i B(i,k) = \alpha_k \, \forall \, k\}$. The firm's problem can now be defined:

$$\max_{p_j \in \mathbb{R}_+, B_j \in \mathbb{B}} D_j(\underbrace{\xi(B_j)}_{\text{quality}} - \rho p_j, p_{-j}, \xi_{-j}) \left[ p_j - \underbrace{\left( \overbrace{\gamma_j I(B_j)}^{\text{org. cost}} + \overbrace{W(B_j)}^{\text{avg. wage}} \right)}_{\text{constant marginal cost, } MC_j} \right] \tag{1}$$

**Equilibrium.** An equilibrium consists of firm strategies $\{p_j, B_j\}_{j=1}^J$ and wages $w$ such that:

1. Firms choose prices $p_j$ and organizational structures $B_j$ to maximize (1).

2. Labor markets for each worker type clear:

$$\sum_j D_j(\xi(B_j) - \rho p_j, p_{-j}, \xi_{-j}) \sum_k B_j(i,k) = L_i \, \forall \, i = 1, ..., N.$$

**Model Summary.** Figure 6 illustrates the model from the perspective of a single firm. The firm chooses $B_j$ (i.e., determining who it hires and how hired workers are assigned to tasks) and prices taking into account internal factors (i.e., the task mix and organization costs), labor market factors (i.e., wages and skills), and product market factors (i.e., consumer price sensitivity, the prices and qualities of other products). The choice of $B_j$ feeds back into the product market by determining product quality and prices, and into the labor market by determining labor demand across worker types.

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5. Multinomial logit, nested logit and mixed logit with a non-random price coefficient all satisfy. Mixed logit with consumer price sensitivity heterogeneity would not.

**Figure 6:** Illustration of the Model
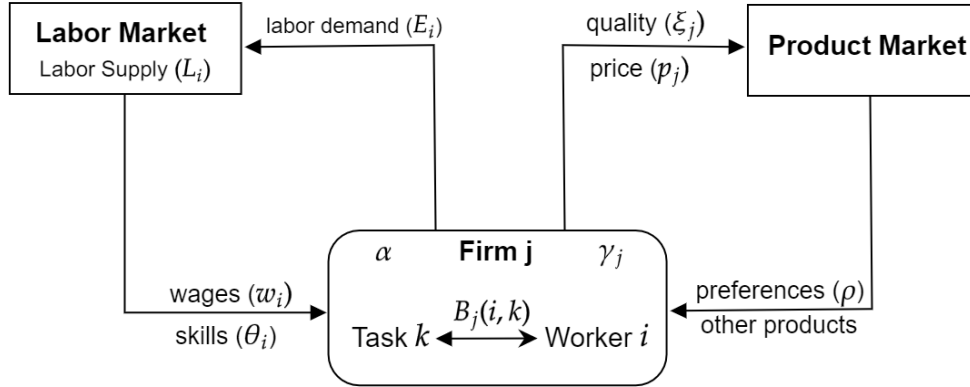


## 4.1 Discussion of Organization Costs

Because complexity is a measure of distance from the random assignment of workers to tasks,[6] $\gamma_j$ can be interpreted directly as a firm-specific specialization cost. However, $\gamma_j$ can also account for several other dimensions of organizational heterogeneity.

**Coordination Costs.** Under this interpretation, $\gamma$ represents the fact that firms are "second-best solutions to transactions plagued by various forms of contractual incompleteness" (Gibbons 2020) and that "firms can never succeed in conquering the nonrational dimensions of organizational behavior" (Williamson 1984). As $\gamma$ approaches 0, coordination costs disappear and a firm can design any organizational structure it chooses at 0 cost. When $\gamma$ becomes sufficiently large, firms will resort to assigning every worker the same job. In the latter case, workers are essentially firms, since they perform all of the tasks the firm performs and do not rely on coworkers.

**Rational Inattention.** The mutual information form of organization costs gives it a rational inattention micro-foundation. We can interpret $\gamma$ as the level of "managerial talent" (Lucas 1978) which determines the attention cost needed to allocate tasks to workers. Similarly, organization costs also capture contractual inattention, such as those described by Tirole (2009). Different firms may find it more or less costly to write down complex contracts in order to support complex organizational structures.

6. See Appendix Section A.7 for a proof.

# 5 Theoretical Results

This section analyzes the theoretical model. I first show the profit-maximizing organization structure is also the solution to a simpler problem that is well studied in information theory and behavioral economics. I use this equivalence to understand the economic forces which determine each firm's internal structure.

## 5.1 Main Characterization

The firm's problem as written in Equation (1) appears complicated at first glance; there are $1 + N \times K$ choice variables and the objective is highly non-linear. The following theorem reveals the firm's problem can be greatly simplified.

**Theorem 1** *An organizational structure ($B_j^*$) is profit-maximizing if and only if it solves*

$$\min_{B_j \in \mathbb{B}} \gamma_j I(B_j) + W(B_j) - \rho^{-1} \xi(B_j), \tag{2}$$

*which is a rate-distortion problem and a rational inattention problem.*

The proof of the result is provided in Appendix Section A.2. The main idea of the proof is that if an organization structure does not solve Equation (2), the firm can switch to a structure that does and adjust its prices to strictly improve its profit. In this way, even though price and organization structure appear entangled in the firm's problem, they can be separated during analysis. The result relies on the fact that the quality-price index $\xi(B_j) - \rho p_j$ is sufficient for price and organization structure in demand, and demand is strictly increasing in the quality-price index. The result does not rely on the functional form of organization costs.

Theorem 1 is useful for three reasons. First, it allows the model to be taken to the data. Because (2) is a rate distortion and rational inattention problem, and these problems are well studied in information theory and behavioral economics, I can weave together results across the two strands of literature to identify firm-specific organization costs, prove a form of equilibrium existence and uniqueness, construct an estimation algorithm, and solve for counterfactual equilibria.

Second, the pricing and organization decisions can be separated when solving for an equilibrium for fixed wages. Specifically, a firm's internal organization *directly* affects own and competitor prices, but these do not directly affect internal organization. Additionally, one firm's internal organization does not *directly* impact a competitor's internal organization; however, each firm's internal organization is *indirectly* impacted by all prices and all competitor internal organizations via wages.

The separation implied by Theorem 1 has practical implications: it means equilibria are robust to timing. Although I assume firms choose organizations and prices simultaneously, if firms chose organizations first and then competed in prices, the outcomes would be the same. The separation implied by Theorem 1 also greatly improves tractability because for fixed wages, the organization problem can be solved first and then used to derive equilibrium prices. This simplifies counterfactual analyses and allows for policies to be decomposed in useful ways.

One question is whether the separation implied by the model is reasonable, or, equivalently, is it the case that wages are the main connection between different firms' internal organizations? The answer appears to come down to whether the labor market is well approximated by perfect competition and whether demand satisfies the index restriction. These assumptions seem reasonable in the case of hair salons, because they sell a horizontally differentiated product and are small in terms of employment, but they may not be in industries where differentiation is largely vertical (e.g., supermarkets) or where individual firms employ a large share of the labor market (e.g., manufacturing company towns).

Third, Theorem 1 reveals the forces that shape a firm's internal organization. Examining Equation (2) shows firms face a triple trade-off, as depicted in Figure 7. Each firm wishes to achieve the lowest complexity and wages while achieving the highest quality. How it navigates this trade-off depends on its internal organization cost $\gamma_j$, consumer price sensitivity $\rho$ and their interaction $\gamma_j \cdot \rho$.

If a firm wishes to increase quality, it has two options: (1) hire better workers and incur a wage cost or (2) rearrange its current workforce to better leverage existing worker skills and incur an organization cost. Intuitively, when consumers are price sensitive ($\rho$ is high), the firm cannot pass on costs to consumers via prices. Thus, firms prioritize minimizing

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**Figure 7:** The Complexity-Wage-Quality Trade-Off
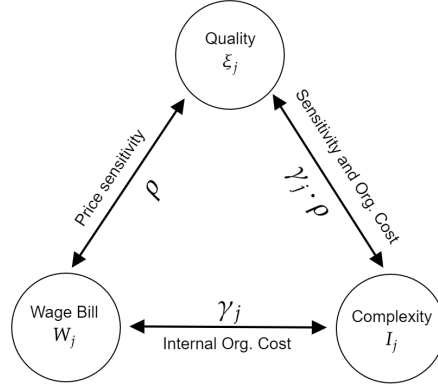


costs over maximizing quality by choosing less complex organizations.

To analyze how the firm navigates the complexity-wage-quality trade-off, I define the *organization frontier* as the set of all organization structures which minimize complexity for some quality-adjusted wages ($Q$). The frontier consists of the simplest organization that achieves some quality-adjusted wages. I wish to study the relationship between quality-adjusted wages and complexity along the frontier:

$$I^*(Q) = \min_{B_j \in \mathbb{B}} I(B_j) \text{ s.t. } W(B_j) - \rho^{-1}\xi(B_j) \leq Q.$$

The characterization provided in Theorem 1 allows me to apply existing results from information theory to understand the general shape of this relationship.

**Proposition 1** *Organization complexity along the organization frontier ($I^*(Q)$) is continuous, convex and decreasing in quality-adjusted wages.*

The proof is provided in Appendix Section A.3. The proposition implies the choice of a high-dimensional organization structure can be thought of as a two-dimensional choice, similar to a classic expenditure minimization problem from consumer theory, as illustrated in Figure 8. Although $B_j$ (i.e., how a firm chooses its workers and how they are assigned to tasks) is a high-dimensional object, the firm essentially solves a two-dimensional trade-off between complexity and quality-adjusted wages. The firm's optimal structure will be the point of tangency between the organization frontier and the best possible (leftmost) isoprofit curve. The firm's isoprofit curves have a slope equal to $-\gamma_j^{-1}$. As the organization cost parameter ($\gamma_j$) rises, the curves become flatter, causing

**Figure 8:** Choosing an Organizational Structure
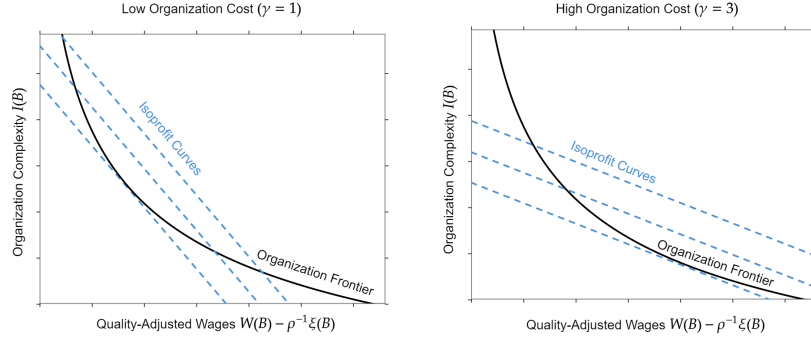


**Note:** Although $B_j$ (i.e., how a firm chooses its workers and how they are assigned to tasks) is a high-dimensional object, the firm essentially solves a two-dimensional trade-off between complexity and quality-adjusted wages. The firm's optimal structure will be the point of tangency between the organization frontier and the best possible isoprofit curve.

the tangent point to shift right and reducing organizational complexity while increasing quality-adjusted wages. A more complex organization allows a firm to produce a higher-quality good at a lower wage, but it requires a greater organization cost. An immediate consequence is that a lower organization cost parameter grants the firm an organizational competitive advantage in the product market.

**Proposition 2** *In equilibrium, firms with a lower organization cost ($\gamma_j$) have higher organization complexity, market share and profits.*

The proof is provided in Appendix Section A.3. Recall that $\gamma_j$ represents the management technology, relationships, knowledge and practices specific to the firm which make it easier or harder for the firm to implement a given organizational structure. Proposition 2 implies more organizationally efficient firms are larger and more profitable, and can produce better-quality goods at a lower cost. Importantly, this proposition confirms that the model is consistent with Fact 2: complexity should be positively correlated with measures of firm size. This is in line with the findings of Kuhn et al. (2022), who use surveys and administrative data to show that more coordinated or specialized firms are more profitable.

## 5.2 Workforce Heterogeneity

The model assumes that workers are perfect substitutes in production, both in terms of quantity and quality. To see this, set $\gamma_j = 0$ and examine Equation (2). Without organization costs, the firm minimizes a constrained linear objective with weights determined by wages and skill sets. All complementarities between workers arise endogenously via organization costs. Because these costs are firm specific, this allows for rich heterogeneity within a product and labor market, both in terms of labor-labor substitution patterns and workforce composition.

I illustrate this with a simple version of the model with three worker types. Suppose wages are fixed at $w = (21, 20, 15)$, the task mix is $\alpha = (1/3, 1/3, 1/3)$, price sensitivity is $\rho = 1$, and worker skill sets are given by $\Theta$ (defined shortly). Under this worker-type space, there are two worker types that are specialists in task 1 and 3 relative to each other, but that have higher absolute skill in all tasks compared to a third type. When I "adjust" skills for wages, it can be seen that in relative terms, there are two workers who are optimal to hire for task 1 and task 3, and one jack-of-all-trades who is a safe option for all tasks:[7]

$$\Theta = \begin{bmatrix} 15 & 19 & 26 \\ 23 & 19 & 15 \\ 15 & 15 & 15 \end{bmatrix} \implies \theta - \rho w = \begin{bmatrix} -6 & -2 & 5 \\ 3 & -1 & -5 \\ 0 & 0 & 0 \end{bmatrix}.$$

Firms facing the same market conditions and task mix can have heterogeneous workforce compositions, as illustrated in Figure 9 panel A. Organizationally efficient firms employ an equal share of each worker because they can fully utilize the specific skills of each worker type. Firms with intermediate organization costs hire only two worker types. Organizationally inefficient salons employ only type 3 workers (jacks-of-all-trades), because these firms cannot utilize the specific skills of the specialist types.

Additionally, firms facing the same market conditions and task mix can exhibit very different labor-labor substitution patterns, as demonstrated in Figure 9 panel B. When the wage of type 1 workers is increased by 1, firms with different organization costs react differently. Firms with very high or very low organization costs reduce the share of type 1 workers and increase the share of the other two types. Firms with intermediate costs

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7. These parameter values are based on an example in Csaba (2021).

reduce the share of both type 1 and type 2 workers. Thus, these two types are substitutes at extreme firms, but are complements at intermediate firms.

**Figure 9:** Organizational Heterogeneity



**(a)** Workforce Composition



**(b)** Labor-Labor Substitution
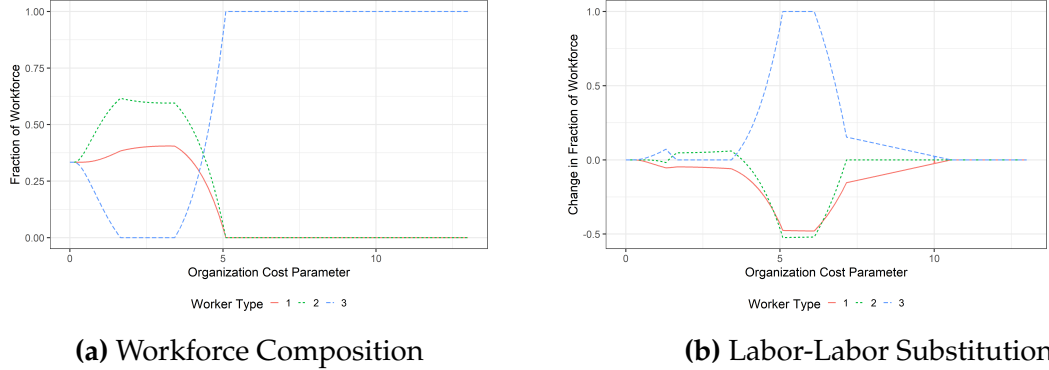
**Note:** Panel A illustrates that as organization costs change, the composition of a firm's workforce changes in a non-monotone fashion. Panel B illustrates the change in the share of each worker type due to a 1-unit increase in the wage of type 1 workers.

# 6 A Structural Model of Internal Organization

Understanding the quantitative relationship between internal organization and the labor and product market requires a structural model that can be taken to the data. This section describes such a model, which preserves the spirit of the theory developed in Section 5 while allowing for additional firm and worker heterogeneity. At a high-level, the econometric model takes in task assignments over worker identities and returns the equilibrium task assignments over worker skill sets. Appendix Figure A7 provides a visual example.

## 6.1 Econometric Model

I define labor markets and product markets as counties, and time periods as quarters. I estimate the model for New York County (Manhattan) 2021 Quarter 2, the last full quarter with available data in my sample. I add several types of heterogeneity to the theoretical model introduced in Section 4 to better fit the data. The theoretical results in Section 5 continue to apply to the econometric model.

**Consumers.** I assume a parametric form for demand. There is a mass $M$ of consumers interested in purchasing at most one of the $J$ final products, where $M$ is set to be the

population of Manhattan. Consumer $z$'s utility for good $j$ is represented by the logit utility function

$$u_{z,j} = \xi(B_j) - \rho p_j + \epsilon_{z,j},$$

where $\epsilon_{z,j}$ is distributed i.i.d. Type 1 extreme value across consumers and products. The outside option for consumers is assigned index $j = 0$, and its utility is normalized to $u_{z,0} = \epsilon_{z,0}$. As in McFadden (1973), market demand for good $j$ can be written as

$$D_j(\xi(B_j) - \rho p_j, p_{-j}, \xi_{-j}) = \frac{exp(\xi(B_j) - \rho p_j)}{\sum_{j'} exp(\xi(B_{j'}) - \rho p_{j'})}. \tag{3}$$

**Marginal Cost and Production Function Heterogeneity.** The task mix is firm-specific and therefore indexed by $j$ ($\alpha_j$). This allows firms in the same product and labor market to have different organizational frontiers. Since tasks are observed, the distribution of time across task categories can be computed. Marginal cost may depend on the firm-specific task mix ($\alpha_j$) to capture the costs of materials relating to specific tasks (e.g., dyes) as well as an idiosyncratic marginal cost shifter $\phi_j$. I measure $\bar{a}_j$ as the average number of hours salon $j$ spends on a customer in a quarter. I specify that organization costs and wages are per hour of labor. This allows each firm to have a different required labor per unit ($\bar{a}_j$) so that I can capture traditional productivity differences across firms. With these modifications, marginal cost can be expressed as $MC_j = \bar{a}_j\left[\gamma_j I(B_j) + W(B_j)\right] + \sum_k m_k \alpha_j(k) + \phi_j$.

**Quality Heterogeneity.** In addition to endogenously chosen quality $\xi(B_j)$, each firm also has exogenous unobserved quality $\nu_j$ whcih represents reputation and other attributes that impact quality but are fixed in a given period and unrelated to labor. Inclusion of $\nu_j$ ensures only quality differences correlated with observed organization complexity ($I_j$) will be attributed to internal organization. Quality is now $\xi(B_j) + \nu_j$.

**Worker Labor Supply and Matching.** Workers with the same skill set may differ in their labor supply. This clarifies the relationship between worker identities (observed in the data) and worker types in the model (unobserved). Specifically, in addition to being characterized by their skill set, workers are also characterized by an inelastic person-

specific labor supply.[8] I augment the game by specifying that firms first demand an amount of labor of each skill set, and then an unspecified process matches workers to firms. The only assumption I place on this process is that the firm's labor demand from the first stage is exactly met. Thus if a firm demands 10 hours of a skill set, this amount may be met by any combination and number of workers, but no more or less than 10 hours is supplied in total. Following the matching process, firms then select an organization structure $\tilde{B}_j$, which is an assignment of worker identities to tasks.[9]

**Worker Skills.** I assume there is one specialist worker type for each of the five tasks. Tasks are performed with a base skill level $\beta_k$ when assigned to a non-specialist, and are performed with an additive skill gap $\theta_k$ when assigned to a task $k$ specialist. The matrix of skill sets, where each row denotes a worker type and each column a task, can be written as

$$
\Theta = \begin{bmatrix}
\theta_1 + \beta_1 & \beta_2 & \beta_3 & \beta_4 & \beta_5 \\
\beta_1 & \theta_2 + \beta_2 & \beta_3 & \beta_4 & \beta_5 \\
\beta_1 & \beta_2 & \theta_3 + \beta_3 & \beta_4 & \beta_5 \\
\beta_1 & \beta_2 & \beta_3 & \theta_4 + \beta_4 & \beta_5 \\
\beta_1 & \beta_2 & \beta_3 & \beta_4 & \theta_5 + \beta_5
\end{bmatrix}.
$$

**Additional Details.** The state of New York does not tax hair services. However, New York City levies a 4.5 percent tax on beauty services. Therefore, I denote the sales tax $\tau$ and assume it is 4.5% initially. I define consumers' outside option as not buying services from a salon. I use Consumer Expenditure Survey micro-data to compute the share of individuals from a county in a quarter who spend \$0 at salons, and take this to be the share of people who choose the outside option. Based on this methodology, the share of New York County residents selecting the outside option in 2021 Q2 is 40%.

**Profit.** Under the econometric model, a firm's profit can be written as

$$
\frac{exp(\xi(B_j) - \rho(1+\tau)p_j + \beta\alpha_j + \nu_j)}{\sum_{j'} exp(\xi(B_{j'}) + -\rho(1+\tau)p_{j'} + \beta\alpha_{j'} + \nu_{j'})} \left[ p_j - \bar{a}_j\left(\gamma_j I(B_j) + W(B_j) + m\alpha\right) - \phi_j \right],
$$

where the features added to the theoretical model are written in blue. Fixing an equilibrium, the parameters of the model can be divided into two groups. The first group

8. If the set of labor supplies is $\Lambda$, the worker type space is now $\Theta \times \Lambda$.
9. A firm may employ several workers with the same skills and assign them different tasks.

is the firm-specific organization cost coefficients $\{\gamma_j\}_{j=1}^{J}$. The second group consists of worker skills (10 parameters), wages (5 parameters), material costs (5 parameters) and price sensitivity (1 parameter). I call these *market parameters* and denote them by $\Omega$.

## 6.2 Equilibrium Existence and Uniqueness with Fixed Wages

In the empirical application of the model, I treat wages as fixed parameters to be estimated. Prior to identification and estimation, I establish that for fixed wages, there almost always exists a unique equilibrium.

**Proposition 3** *Suppose wages are fixed parameters. A pure strategy equilibrium always exists, and it is unique except over a set of parameters with measure 0.*

The proof of this result is provided in Appendix Section A.5, and it relies on the equivalence to a rational inattention problem established in Theorem 1. This result means that multiplicity arises only in knife-edge cases. Proposition 3 does not establish equilibrium uniqueness or existence in the full model with wages determined endogenously by labor market clearing. Nevertheless, Proposition 3 is crucial for proving Proposition 4, the main identification result in this paper.

Several aspects of the model make Proposition 3 surprising. First, each firm has 26 choice variables, and quality and marginal cost are endogenous. Many models where product positioning is endogenous (including the canonical two-stage Hotelling model) suffer from equilibrium existence and uniqueness problems.

The result is also useful for counterfactual analysis, because it means the model almost always delivers one and only one internal organization structure for each firm. The model will almost never suffer from inverse identification problems, at least when wages are held fixed.

## 6.3 Identification of Firm-Specific Organization Costs

The organization costs $\{\gamma_j\}_{j=1}^{J}$ are important parameters, determining product quality, organization complexity and marginal costs for each firm. However, the fact that there

is one parameter per firm and that I place no restrictions on the economy-wide distribution raises concerns for identification and estimation. I alleviate this concern with the following result.

**Proposition 4** *Organization costs ($\gamma_j$) and organization structures ($B_j$) are a known function of firm task mixtures ($\alpha_j$), complexities ($I_j$) and market parameters ($\Omega$) for all firms with positive complexity, except for a set of market parameters with measure 0.*

The proof is fully described in Appendix Section A.6, and it makes use of the essential equilibrium uniqueness result given in Proposition 3. A key hurdle is that I do not observe worker types, but worker identities within firms. Because I allow for a flexible matching process, a given firm may in principle employ multiple workers of the same skill set and assign these workers different tasks. However, a property of the mutual-information based organization cost ensures that if firms do employ multiple workers with the same skill set, they assign these workers the same tasks. This implies that the observed organization complexity based on worker identities is equal to the true organization complexity based on worker skill sets.

The intuition for the identification of $\gamma_j$ is demonstrated in Figure 8. Suppose two firms with the same task mix ($\alpha_j$) are observed in the same product and labor market. This means they have the same organization frontier. If firm A has a higher complexity, it must be that the slope of firm A's isoprofit curve is steeper than B's, which can only be because A has a lower organization cost. Therefore, I can order the firms by organization cost without knowing the market parameters. Once these parameters are known, I can find the cardinal values of each firm's organization cost.

The proposition implies that organization costs do not need to be estimated in the statistical sense. For any market parameters, there are unique organization cost parameters which rationalize the observed organization complexities and task mixtures. This is similar to the way unobserved product quality is a known function of market shares in Berry, Levinsohn, and Pakes (1995).

Beyond estimation, Proposition 4 also implies that observing the task mix (a vector of length $K$) and organizational complexity (a scalar) is enough to estimate the model. It is not necessary to observe the individual assignments of workers to tasks; the researcher need only observe complexity. This means the model can be estimated in settings where

rich assignment data are not available.[10] It also means that a researcher who has assignment data can estimate the model using only complexity and the task mix and use the rest of the data to conduct validation exercises. This is precisely what I do in Section 7.2.

## 6.4 Estimation of Market Parameters

I have established that organization costs are a known function of the data and market parameters. This section derives a set of moments and assumptions under which the market parameters can be estimated via the generalized method of moments.

To construct moment conditions, I follow a common approach in the industrial organization literature and use one demand-side and one supply-side equation. Starting with the supply side, the equilibrium pricing equation can be written as

$$p_j = \frac{1}{\rho(1+\tau)(1-s_j)} + \bar{a}_j \left[ \gamma(\Omega, I_j, \alpha_j) I_j + W(\Omega, I_j, \alpha_j) \right] + m\alpha_j + \phi_j. \tag{4}$$

Because the demand system takes a multinomial logit form, market shares can be expressed as

$$log(s_j) - log(s_0) = \xi(\Omega, I_j, \alpha_j) - \rho(1+\tau)p_j + \beta\alpha_j + \nu_j. \tag{5}$$

I interact firm-level covariates with Equations 4 and 5. I use covariates that are relevant to the determination of prices and market shares but also independent of $\nu_j, \phi_j$. The firm organizational complexity ($I_j$) and task-mix vector ($\alpha_j$) fit these requirements, because they change organization costs but do not depend on $\nu_j, \phi_j$. Additionally, I include their interaction, $\alpha_j \cdot I_j$. Section 6.6 discusses how this variation identifies specific parameters.

I add one additional wage moment. For each county and quarter, I compute the average wage bill of hair salons in Manhattan using the Quarterly Census of Employment and Wages. This corresponds to $W(\Omega, I_j, \alpha_j)$ multiplied by the number of customers. Allowing for classical measurement error ($e_j$) yields $W_j = Ms_j a_j W(\Omega, I_j, \alpha_j) + e_j$. Taken together, the moment conditions used for estimation are

$$\mathbb{E}\left[ \begin{pmatrix} \phi_j(\Omega, I_j, \alpha_j, p_j, s_j) \\ \nu_j(\Omega, I_j, \alpha_j, p_j, s_j) \end{pmatrix} \begin{pmatrix} \alpha_j & \alpha_j I_j \end{pmatrix} \right] = 0 \qquad \mathbb{E}[e_j(\Omega, I_j, \alpha_j)] = 0.$$

---

10. For example, privacy concerns may often prevent the disclosure of employee-client assignments.

For a single market and quarter, I obtain a total of 21 moments to estimate 21 market parameters. The model is globally identified if I assume that $\Omega$ is the unique vector of parameter values which satisfies the moment conditions. With this assumption, I estimate the model using the generalized method of moments (GMM). Denote the sample analogue of the moments as $G(\Omega)$. Then, to estimate $\Omega$, I solve

$$\arg\min_{\hat{\Omega}} G(\hat{\Omega})'WG(\hat{\Omega}), \tag{6}$$

where $W$ is a weighting matrix. Note that to evaluate this GMM objective, I must recover the vector of organization costs implied by the data and each guess of the market parameters. I take the weighting matrix $W$ to be a diagonal matrix, where each diagonal element is the sample variance of the independent variable involved in the moment. This standardizes the moments in the objective function, preventing variables with large nominal values (i.e., average hours per unit) from dominating during estimation. I constrain wages to be between \$15 (the minimum wage) and \$200 per hour.

## 6.5 A Computationally Light Estimation Procedure

Although a firm's organization costs is a known function of the data, there does not exist a closed-form expression for this function. This is a problem for estimation when there are many tasks and many firms, because it is necessary to numerically solve each firm's internal organization problem many times until the model-produced complexities match the complexities in the data.

I use the equivalence to a rate-distortion problem proved in Theorem 1 to provide a solution.

**Lemma 1** *Given market parameter values, the Blahut–Arimoto algorithm with Lagrangian multiplier $(\bar{a}_j\gamma_j)^{-1}$ delivers an organizational structure $B_j$ which maximizes firm profit.*

The lemma follows directly from Theorem 1 and well-known results in information theory.[11] The Lagrangian multiplier involves $\bar{a}_j$ because marginal costs are $\bar{a}_j(\sum_i w_i E_i +$

11. See Tishby, Pereira, and Bialek 2000 or Blahut 1972.

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$\gamma_j I(B_j)) - \rho^{-1}\xi(B_j)$. The Blahut–Arimoto algorithm (Blahut 1972) is a fixed-point algorithm which iterates on two optimality conditions and can be described as follows:

0. Guess some labor demand $E^0$. Create matrix $V$: $V_{i,k} = exp[(\bar{a}\gamma)^{-1}(\rho^{-1}\theta_{i,k} - w_i)]$.

1. Compute interim organization structure $B(i,k)^t = \alpha_k \frac{V_{i,k}E^t(k)}{\sum_i E^t(i)V_{i,k}}$.

2. Compute interim relative labor demands $E^{t+1}(i) = \sum_k B(i,k)^t$.

3. If converged, exit; else return to Step 1 and advance $t$.

The Blahut–Arimoto algorithm is proven to converge to a global optimum from any feasible starting point (Tishby, Pereira, and Bialek 2000). The algorithm also delivers the entire internal organization of the firm, $B_j$. Thus, the full estimation procedure is as follows:

1. Given a guess of the market parameters $\hat{\Omega}$, use the Blahut–Arimoto algorithm to find the organization costs $\gamma_j(\hat{\Omega})$ which rationalize each firm's observed organizational complexity $I_j$. Because complexity is monotone in $\gamma_j$ this is always feasible.

2. Using $B_j(\hat{\Omega}), \gamma_j(\hat{\Omega})$, compute firm-specific wage bills and endogenous quality.

3. Evaluate the GMM objective given by Equation 6. If the objective is minimized, stop; otherwise, return to step 1 with a new market parameter guess, $\hat{\Omega}$.

This estimation algorithm is similar in spirit to the demand estimation procedures that have become popular in industrial organization since Berry, Levinsohn, and Pakes (1995). Just as those procedures invert market shares using a contraction mapping to derive unobserved product qualities, my procedure inverts organization complexities using a contraction mapping to obtain unobserved organization costs. Implicit in this inversion procedure is that complexity is measured without error. Appendix Section A.12 provides evidence that measurement error is small.

## 6.6 Identifying Variation

Proposition 4 establishes that given fixed values for the market parameters, organization costs ($\gamma_j$) are identified by variation in complexity and the task mix across firms. The

purpose of this section is to discuss the sources of identifying variation for the market parameters.

Consumer price sensitivity is identified by the pass-through of average wages to consumers. If wage costs are passed through to consumers via higher prices, consumers are not price sensitive and $\rho$ is low. Once price sensitivity ($\rho$) is known, the marginal cost of each firm can be obtained by subtracting the markup from prices. Similarly, service quality can be obtained from observed prices and market shares. For this reason, I discuss identification of the other parameters as if quality and cost were observed.

The base skill parameters ($\beta$) and the material costs ($m$) are identified by variation in the task mixtures ($\alpha_j$) across firms. When I observe a firm that is intense in task $k$, I its cost is informative about $m_k$ and its quality is informative about $\beta_k$. This is why $\alpha$ is interacted with the demand supply side residuals to obtain a first set of moments.

Recall that a complex firm generally has a specialized workforce. When I observe a firm that is complex and is intense in a task $k$, this implies two things. First, the firm likely uses a large share of task $k$ specialists. Therefore, the observed price (and thus cost) of that firm largely reflects the wage of task $k$ specialists ($w_k$). Second, the firm assigns a large amount of task $k$ to those specialists. Therefore, the observed market share (and thus quality) largely reflects the skill gap of task $k$ specialists ($\theta_k$). This is why $\alpha_j \cdot I_j$ is interacted with the demand supply side residuals to obtain a second set of moments.

# 7   Empirical Results

This section summarizes parameter estimates and uses the model to analyze the sources of variation in task content for hair stylists in Manhattan.

## 7.1   Parameter Estimates

Estimated wages, skill parameters and material costs are organized by task in Table 3 while auxiliary parameters are available upon request. Standard errors are computed as the sample standard deviation of the parameter estimates from 500 bootstrap replications.

The coefficients associated with the color and haircut tasks are the most precisely estimated. This is not surprising, as these tasks are the most common and, as a consequence,

Table 3: Parameter Estimates, Tasks

| Task | Associated Specialist | | Skill Base | Material Cost |
|---|---|---|---|---|
| | Skill Gap | Wage | | |
| Administrative | 43.29* | 26.99 | -16.16 | -147.60* |
| | ( 21.66) | (63.75) | ( 14.58) | ( 13.47) |
| Blowdry/Etc. | 141.69* | 20.91 | -70.56* | 12.39 |
| | ( 36.67) | (40.22) | ( 13.57) | ( 16.65) |
| Color/Highlight/Wash | 60.03* | 37.75* | -9.69 | 56.49* |
| | ( 21.24) | ( 7.00) | ( 11.97) | ( 15.79) |
| Haircut/Shave | 32.45* | 16.96* | . | . |
| | ( 13.07) | ( 8.32) | . | . |
| Nail/Spa/Eye/Misc. | 66.48 | 81.16 | -252.58* | -1061.12* |
| | ( 37.72) | (53.52) | ( 11.47) | ( 10.73) |

**Note:** Standard errors from 500 bootstrap replications in parentheses; * indicates significance at the 0.05 level. For each task, the table lists the skill gap and wage of the associated specialist in 2021 dollars.

their associated parameters will have the most statistical power. Across all tasks, the skill-gap parameters are positive, indicating that assigning the task to the associated specialist increases quality. The skill-gap parameters can be interpreted as the dollar value to a consumer of increasing task specialization in that task by 4 percentage points. Wages are in 2021 dollars per hour. Material costs are in terms of 2021 dollars per service.

Wages for color specialists are more than double the wages for haircut specialists, and the skill gap for color specialists is nearly double the skill gap for haircut specialists. This is in line with folk wisdom in the industry that it is hard to master coloring. The material costs are largest for the color task, in line with the fact that coloring is intense in expensive non-labor supplies, such as hair dye.

The organization costs ($\gamma_j$) for each firm are shown in Figure 10a. To provide a magnitude for these estimates, I plot the cost of implementing the median-complexity organization structure across all firms in Figure 10b. There are large differences in organization costs. Firms in the bottom quartile of organization costs can implement the structure for less than \$50 an hour. It would cost firms in the top quartile over \$150 an hour. The estimates imply that variation in organization costs explains 40% of total variation in prices across firms.

For each firm, I recover the unobserved, equilibrium organization structure $B_j$. Four examples are visualized in Figure 11. These matrices represent the amount of time allocated to each task and each worker type.

**Figure 10:** Estimated Organization Costs



**(a)** Organization Costs ($\gamma_j$)

**(b)** Cost of Median Org. Structure
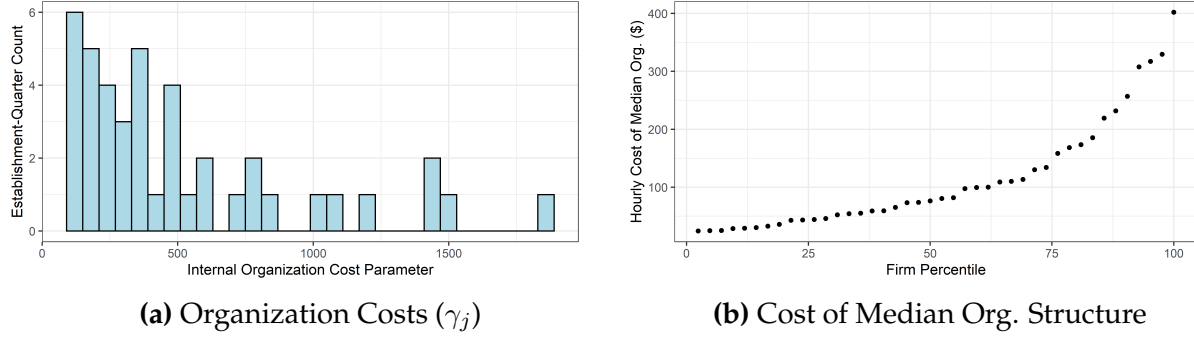
**Note:** Panel A displays the estimated organization cost ($\gamma_j$) parameters for Manhattan. These can be interpreted as a measure of organization frictions at each firm, with lower values indicating less friction. Panel B displays the magnitude of these differences, by plotting the cost (in dollars) to each firm of implementing the median-complexity organization structure.

## 7.2   Model Fit and Validation

I assess model fit by comparing the predicted and actual relationship between prices and various organizational variables in Appendix Figure A6. The model captures the shape of the relationships.

Although the model delivers an entire predicted organization structure ($B_j$) for each firm, the estimation procedure uses only some of this information. I use the additional predicted information to validate the model. In particular, I compare the model-generated distribution of task content to the observed distribution of task content. Recall that the jobs within firm $j$ are denoted by $b_j$, which is a matrix where element $i, k$ denotes the time worker type $i$ spends on task $k$. Using the model, I can compute $b_j$ for each firm among worker types that it hires. In the data, I can compute $\tilde{b}_j$, which are jobs within firm $j$, where element $i, k$ denotes the time worker $i$ spends on task $k$. The main difference between $\tilde{b}_j$ and $b_j$ is that the first is with respect to worker identity, and the second is with respect to worker types. To make them comparable, I can weight each job by the total amount of labor it represents. Combining all $J$ firms yields an unobserved and model-based distribution of job task content for each of the five tasks, where jobs are weighted by their effective labor.

Appendix Tables A4a, A4b and A4c compare the model and observed mean, median, variance, 25th percentile and 75th percentile of job task content. The estimated results exactly match the mean and between-firm variance of job task content because the model

**Figure 11:** Estimated Organization Structures
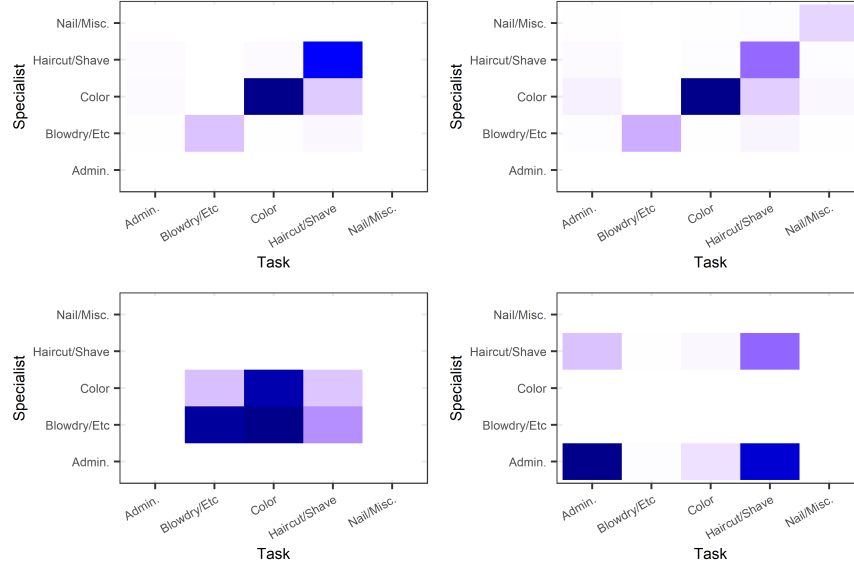


**Note:** Darker colors indicate a higher fraction of total labor was allocated to that worker-task pair. The first two structures are for organizationally efficient salons, while the last two are for organizationally inefficient salons.

imposes that organization structures must be consistent with the task mix $\alpha_j$, which is exactly the average amount of time spent on each task at each firm. The estimates are also reasonable approximations of the total variances of task content and the 75th percentile of the job task-content distribution. The model is not able to match the median and the 25th percentile.

## 7.3  The Determinants of Task Specialization

The estimated model allows the researcher to understand how worker skills and firm internal organization determine the task specialization of jobs. Measuring task specialization as the amount of time a worker spends on their specialty task, I find that 45% of the variation in task specialization is attributable to firms, while 55% is attributable to worker skills.

I calculate the variation in task specialization due to the firm component by computing the fraction of total firm labor spent on any worker's specialty. Firms with higher organization costs exhibit less task specialization. The magnitude of this effect is large: firms in the bottom quartile of organization costs (efficient firms) assign on average 90%

of tasks to the associated specialist, while firms in the top quartile (inefficient firms), only 67%.

There is also significant variation in specialization across worker types. Haircut/shave specialists work the most specialized jobs, spending 95% of their time on their specialty task. Blow dry/extension/style specialists work the most generalized jobs, spending only 48% of their time on their specialty task.

# 8   Counterfactual Policies

This section uses the estimated model to study two counterfactual policy changes, one impacting the product market and one impacting the labor market. Internal organization creates new responses to these well-studied policies.[12] The model allows me to decompose the effect of any policy into two parts: a reallocation and a reorganization effect.

To do this, I first define the reallocation equilibrium. It is the outcome when firms are allowed to adjust prices ($p_j$) but organization structures ($B_j$) are fixed at the initial equilibrium choices. Because prices control quantities, this equilibrium allows firms to adjust the total labor they hire, but not the division of labor within the firm. The *reallocation effect* of any policy change is the change in outcomes between the reallocation equilibrium and the initial equilibrium. It captures changes due to the reallocation of labor across firms. Because firms differ in their organization costs and task mixtures, reallocation will change the task content of jobs, relative wages, and other outcomes.

The *reorganization effect* of any policy is the change in outcomes between the full equilibrium and the reallocation equilibrium. It captures changes due to reorganization of labor within firms. I define the *total effect* of any policy change as the change in outcomes from the initial to the full equilibrium. In the reallocation equilibrium, firms are acting as if they employ a composite worker. The worker's skills and wage are determined exogenously by the initial internal organization of the firm, $B_j$. The firm has the option of adjusting the total amount of labor it demands from this composite worker, but cannot adjust the worker's skills and wages. In the full equilibrium, the firm is free to fully adjust its internal structure.

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12. The procedure used to solve for equilibria is omitted for brevity but available upon request.

## 8.1 Minimum Wage Increase

I study a counterfactual increase in the minimum wage in Manhattan from $15 (the minimum in 2021) to $20. An increase to $20 is similar in magnitude to the increases that would occur if the minimum wage were pegged to inflation, as proposed in several pending pieces of legislation.[13]

To implement the counterfactual, I require that all equilibrium wages be at least $20, and that markets clear for all worker types for which the wage is not binding. I allow there to be excess labor supply (unemployment) for those worker types facing a binding minimum wage. The model is well suited for studying large increases in the minimum wage because it allows salons to reorganize as well as raise prices.[14]

I find that the minimum wage binds for the haircut/shave specialist only. The new wages and employment levels across worker types are given in Appendix Table A5 (including values for the reallocation equilibrium). I first discuss the reallocation and reorganization effects of this policy change. I then analyze the overall impact of the new policy, using the reallocation and reorganization effects to understand the underlying forces.
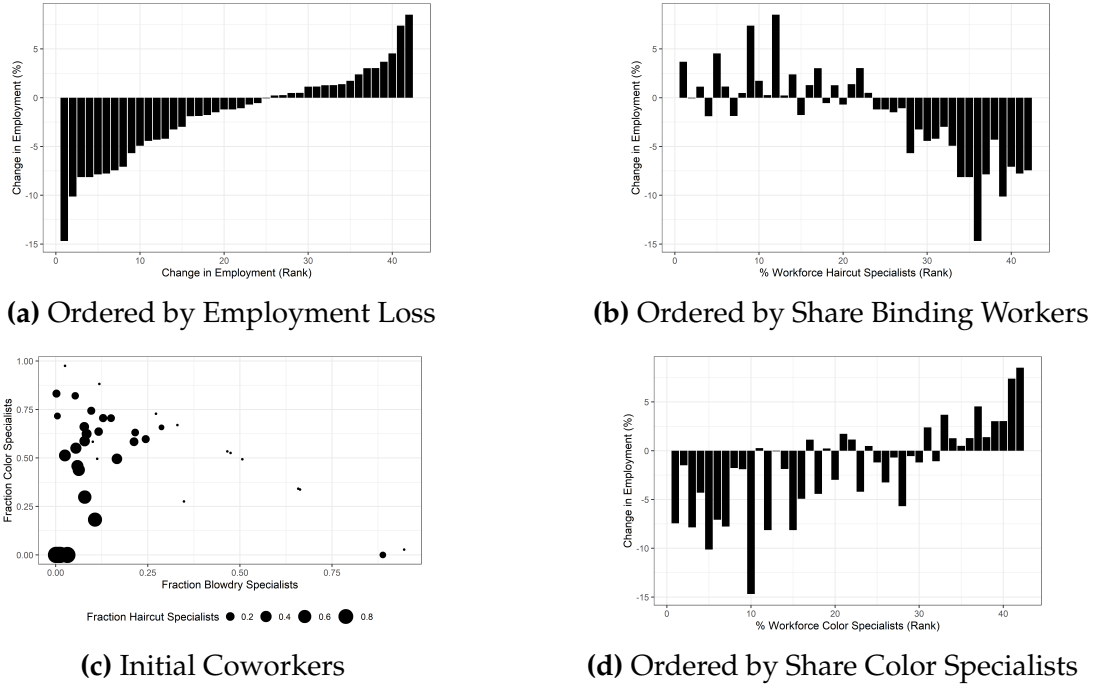
### 8.1.1 The Reallocation Effect

The impact of the minimum wage on individual salons depends partly on their initial internal structure. As a result, the minimum wage changes the competitive positions of salons and reallocates labor. By comparing the initial and reallocation equilibrium, I can hold each firm's internal structure fixed but allow firms to adjust prices. This captures the extensive margin adjustment of salons but prevents internal reorganization. Figure 12 presents the reallocation effect of the minimum wage in a series of three panels.

The minimum wage has a disproportionately negative impact on salons whose internal organization relies heavily on minimum wage workers. These salons see the largest increases in marginal costs and thus the largest decreases in output and employment. Because the minimum wage increases some salons' costs more than others', it changes the competitive position of firms in the product market. As can be seen in the figure, this effect is heterogeneous enough that some salons see employment increases.

13. Senate Bill S3062C and Assembly Bill A7503B.
14. Technical details are available upon request.

**Figure 12:** The Minimum Wage Reallocation Effect



**(a)** Ordered by Employment Loss



**(b)** Ordered by Share Binding Workers



**(c)** Initial Coworkers



**(d)** Ordered by Share Color Specialists

**Note:** In panels A, B and D each bar is a firm, and employment changes are comparing the reallocation equilibrium to the initial equilibrium, holding fixed internal organization. Panel A orders firms by employment losses. Panel B reorders firms by the fraction of the workers that are haircut specialists in the initial equilibrium. Panel C plots firms by their initial workforce composition. Panel D orders firms by their share of color specialists.

Workers that are often employed alongside minimum wage workers initially see negative wage spillovers because the minimum wage erodes the competitive position of these firms. In the opposite way, workers employed at salons with few minimum wage workers initially see positive wage spillovers, because the minimum wage improves the competitive position of these firms. The effects of the minimum wage is contagious, and are spread across workers based on firm internal organization. In equilibrium, the minimum wage reallocates labor towards high-complexity, task-specialized salons and away from low-complexity, task-generalized salons, raising industry task specialization and average worker productivity.

### 8.1.2 The Reorganization Effect

By comparing the full equilibrium and the equilibrium where firms can adjust only prices, I can study the effect of internal reorganization. Almost all salons reduce relative em-

ployment and increase task specialization of minimum wage workers. Salons reduce task specialization and increase relative employment for workers above the minimum wage. I call this a "pick-up-the-slack" effect. Intuitively, the minimum wage reduces the comparative advantage of workers for which the minimum wage binds in all tasks relative to other (non-binding) workers. Firms respond by laying off minimum wage workers and shifting tasks performed by them onto the relatively less expensive non-binding workers. Only minimum wage workers that are sufficiently productive survive, which are those who are task specialized. This implies that the minimum wage increases the absolute productivity of binding workers, but decreases the absolute productivity of non-binding workers.

### 8.1.3 Total Effect

Although the minimum wage is binding for only one worker type, all workers see wage changes. The largest positive spillover is for administrative specialists, who see a wage increase of 4.2% (+$1.13). Color/highlight/wash specialists see a small wage decrease, of 0.7% (-$0.23). What is notable about these spillovers is that they are non-monotonic in initial wage. To see this, I plot the wage change experienced by different workers ordered by initial wage in Figure 13. Non-monotone spillovers occur because labor-labor substitution patterns in the model are determined endogenously based on the distribution of firm organization costs and task mixtures.

**Figure 13:** Minimum Wage Spillovers Across the Initial Wage Distribution
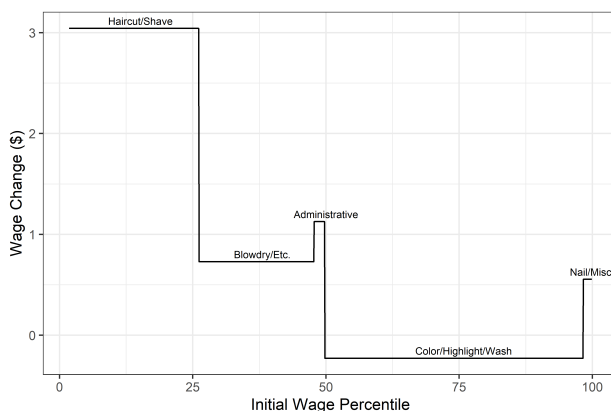


**Note:** This figure plots the wage change experienced by different workers ordered by the initial wage of the worker.

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These non-monotonic wage spillovers illustrate that internal organization can link workers that are very far apart in the initial wage distribution. Workers that differ horizontally in their specialty may be quite likely to work alongside each other, and they may have quite different wages depending on other factors. In this way the reallocation effect can cause large wage increases or decreases even for high-wage workers: indeed, this is exactly what I observe with haircut and nail specialists. Similarly, because the initial wage distribution is not determined by vertical skill differences, the reorganization effect will induce firms to shift tasks from minimum wage workers to workers across the wage distribution.

In Table 4 I decompose wage spillovers into those arising from the reallocation and the reorganization effect. As discussed in the prior two subsections, spillovers for each worker type are a combination of forces, with the reorganization and reallocation effects sometimes moving in opposite directions. For example, color specialists see negative wage spillovers because they are employed alongside minimum wage workers and the minimum wage increase disadvantages the salon where they work. But they also see positive wage spillovers because firms shift tasks from minimum workers to them during reorganization. The total wage spillover for color specialists is negative, as the reallocation effect is about double the reorganization effect. For binding workers (haircut/shave specialists) the two effects work in the same direction, increasing unemployment. In this sense, internal reorganization amplifies unemployment losses.

Table 4: Spillovers from an Increase in the Minimum Wage

(a) Reallocation Effect

| Type | Reallocation Change | | |
| --- | --- | --- | --- |
| | Employment | Task-Spec. | Wage |
| Haircut/Shave | -5.85% | -0.04% | 17.95% |
| Color/Highlight/Wash | 0% | -0.17% | -1.13% |
| Blowdry/Style/Treatment/Extension | 0% | -0.40% | 4.63% |
| Administrative | 0% | 0.09% | 5.22% |
| Nail/Spa/Eye/Misc. | 0% | -0.03% | 0.58% |

(b) Reorganization Effect

| Type | Reorganization Change | | |
| --- | --- | --- | --- |
| | Employment | Task-Spec. | Wage |
| Haircut/Shave | -0.73% | 0.12% | 0% |
| Color/Highlight/Wash | 0% | -0.33% | 0.52% |
| Blowdry/Style/Treatment/Extension | 0% | 0.03% | -1.15% |
| Administrative | 0% | 0.03% | -1.05% |
| Nail/Spa/Eye/Misc. | 0% | -0.00% | 0.10% |

**Note:** The minimum wage increase has positive spillovers for some workers and negative spillovers for others.

Table 4b shows that reorganization wage spillovers follow a pattern. Workers that see an increase in task specialization see a wage decrease (or unemployment increase), while workers that see a decrease in task specialization see a wage increase. Because task specialization determines worker productivity, this implies that internal reorganization

causes absolute productivity and wages to move in opposite directions.

## 8.2 Sales Taxes

New York City is unique in that it levies a 4.5% sales tax on certain services, including those performed at hair salons. This section studies the effect of eliminating this sales tax. Formally, I estimate a new equilibrium with $\tau^{NEW} = 0$. The wages in this new equilibrium are provided in Table A7.
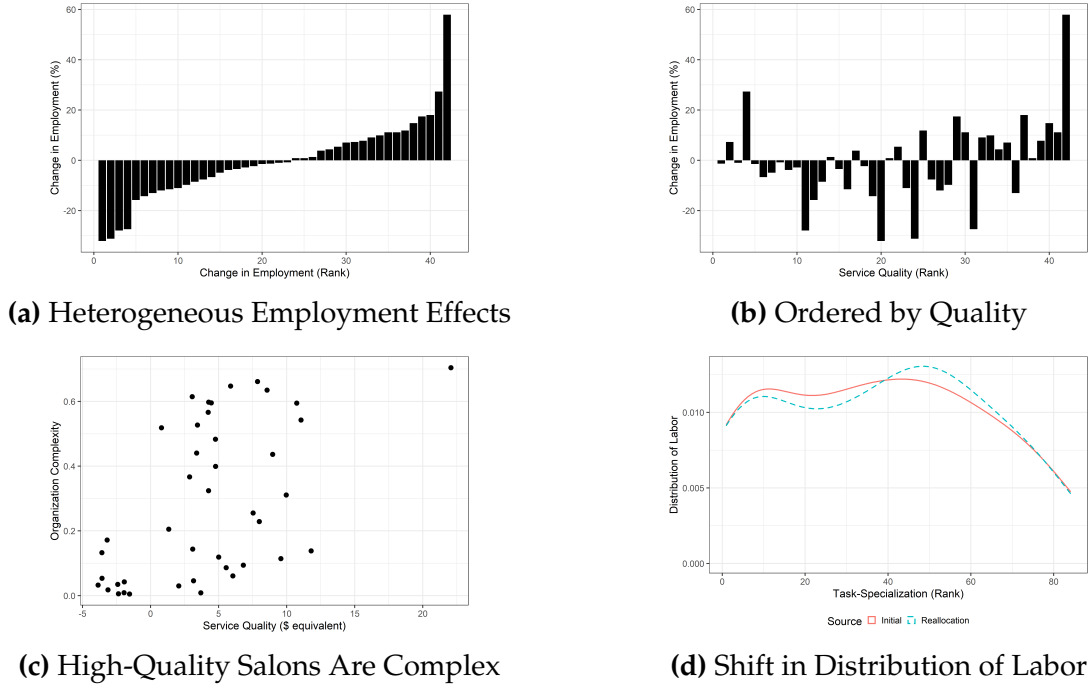
### 8.2.1 Reallocation Effect

Eliminating the sales tax confers a competitive advantage on firms producing high-quality services in the initial equilibrium. Since salons with low organization costs tend to produce high-quality services, eliminating the sales tax reallocates labor towards organizationally efficient firms, as seen in Figure 14. These firms produce high-quality services using a more task-specialized internal structure. Thus the reallocation effect increases market-wide task specialization because more workers are working at task-specialized firms.

### 8.2.2 Reorganization Effect

Eliminating the sales tax makes producing higher-quality products more attractive. In order to produce higher-quality products, firms choose internal organizations which are on average 5.5% more complex, increasing average labor market task specialization by 0.9%. In terms of the three-way trade-off introduced in Figure 7, eliminating the sales tax has the same effect as reducing consumer price sensitivity ($\rho$). Average firm service quality rises by 10%. This is consistent with the quality-complexity-wage three-way trade-off discussion in the theoretical section.

Figure 15 illustrates that these market-wide patterns also happen at the firm level. However, the extent to which salons increase quality and increase task specialization depends on the firm's internal organization costs and its particular task mix. Thus, the slopes and lengths of the arrows in Figure 15 differ. Changing sales tax, a product market policy, influences what workers do and what workers are paid in the labor market.

**Figure 14:** Sales Tax Reallocation Effect



**(a)** Heterogeneous Employment Effects



**(b)** Ordered by Quality



**(c)** High-Quality Salons Are Complex



**(d)** Shift in Distribution of Labor

**Note:** Each bar is a salon. The sales tax elimination decreases employment at some salons and increases it at others (Panel A). Salons with high-quality service see an improvement in their competitive position (Panel B). These salons have a complex, task-specialized internal structure (Panel C). As a result, labor is reallocated to task-specialized firms, and workers become more specialized in equilibrium (Panel D).

### 8.2.3 Total Impact

Table 5a summarizes the effect of the policy on wages and task specialization. All worker types see wage increases and task-specialization increases. Wage increases are not proportional to task-specialization increases: even though blow-dry specialists see the largest increase in specialization, they see the lowest increase in wages. This is because the size of wage increases is partly driven by how the policy impacts the competitive position of firms.

Overall, eliminating the sales tax leads to a small welfare increase, of 0.19%. However, the effects are quite different for different actors in the model. Firms respond to the sales tax elimination by increasing quality by 10%. Firms capture the surplus from improved quality and reduced taxes from consumers by raising prices by 8.7%. Firm profit increases by a modest 0.58% because workers capture most of the surplus from firms through higher wages, which rise by a dollar amount that is comparable to the total lost tax revenue. This is consistent with workers capturing almost all of the productivity

**Figure 15:** Reorganization Effect Under a Sales Tax
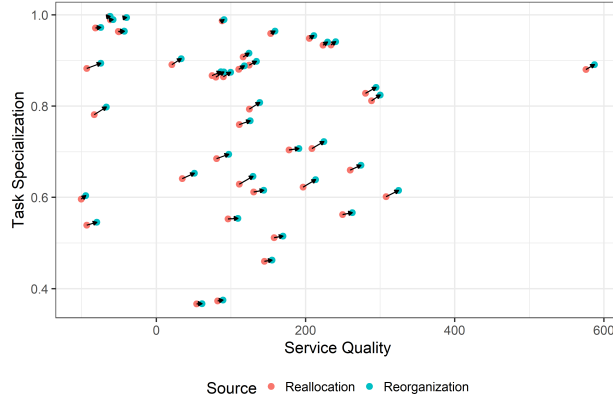


**Note:** Each pair of dots connected by an arrow represents a firm, with red representing the firm before the sales tax and blue representing the firm after the sales tax. The direction of the arrows indicates that most salons increase quality by raising task specialization internally.

Table 5: Total Effects of a Sales-Tax Elimination

(a) Wage Changes by Worker Type

| Type | Wage Change | Task-Spec. Change |
|---|---|---|
| Haircut/Shave | 31.99% | 0.29% |
| Color/Highlight/Wash | 20.09% | 2.57% |
| Blowdry/Style/Treatment/Extension | 6.06% | 3.01% |
| Administrative | 17.99% | 1.03% |
| Nail/Spa/Eye/Misc. | 12.74% | 2.39% |

(b) Welfare Breakdown

| Source | Change | Percent Change |
|---|---|---|
| Salon Profit | $942,740 | 0.58% |
| Consumer Welfare | -$494,199 | -0.30% |
| Wages | $11,603,777 | 7.12% |
| Tax Revenue | -$11,739,300 | -7.20% |
| Total Welfare | $313,017 | 0.19% |

**Note:** Eliminating the sales tax raises wages most in percentage terms for haircut specialists. Workers gain the most from the elimination of the sales tax: wage increases are almost equal to the lost revenue to the government.

improvements from increased task specialization.

# 9  Conclusion

This paper studies how internal organization decisions within firms interact with markets outside firms. I develop a structural model, grounded in a set of stylized facts, which allows firms to differ in their internal organization and to change it in response to market conditions. The counterfactual exercises illustrate that allowing internal organization to be endogenous and heterogeneous qualitatively changes the impact of policy. Minimum wage increases generate new types of wage spillovers that cannot occur in many other models of the labor market. Sales tax cuts induce firms to reorganize their workforce, changing the task composition of jobs. Although these effects are specific to the salon in-

dustry, they indicate that internal organization is an important force that deserves careful study in a variety of contexts.

The framework in this paper provides a starting point for researchers to do exactly this. The approach in this paper can be extended to accommodate quantity-based (rather than quality-based) productivity, continuous task spaces, labor market power and more sophisticated demand systems. These extensions, combined with traditional employer-employee matched data will be important to answer future questions, including the effect of internal organization on human capital accumulation and the welfare implications of endogenous task assignment for workers.

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# A   Online Appendix

## A.1   Rate Distortion and Rational Inattention Equivalence

Equation (2) from Theorem 1 can be rewritten as

$$\gamma_j \min_{B_j \in \mathbb{B}} \left\{ I(B_j) + \gamma_j^{-1} \left[ W(B_j) - \rho^{-1}\xi(B_j) \right] \right\}. \tag{7}$$

I can rewrite (7) as a maximization problem:

$$\max_{B_j \in \mathbb{B}} \left\{ \sum_{i,k} B_j(i,k)(\rho^{-1}\theta_{i,k} - W_i) \right] - \gamma_j I(B_j) \right\}. \tag{8}$$

Comparing (8) to formulations in papers such as Jung et al. (2019) illustrates that this is a rational inattention problem with mutual information attention costs. I rewrite Equation 7 one last time:

$$\gamma_j \min_{B_j \in \mathbb{B}} \left\{ I(B_j) + \gamma_j^{-1} \sum_{i,k} B_j(i,k)(W_i - \rho^{-1}\theta_{i,k}) \right] \right\}. \tag{9}$$

Comparing Equation (9) to formulations such as Equation 6 in Tishby, Pereira, and Bialek (2000) demonstrates this is a well-understood minimization problem from information theory called a rate-distortion problem.

## A.2   Proof of Theorem 1

For any given organization structure, the firm will choose prices only weakly above marginal cost; otherwise, it receives negative profit. Without loss, I therefore restrict the set of price-structure pairs considered to be those where price exceeds marginal cost.

First, I prove that if an organization structure $B_j^*$ solves the simpler problem (Equation 2), then it is profit-maximizing ("only if" direction). I need to show that for any price-organization structure pair $(p_j', B_j')$ there exists $p$ such that profit under $(p_j, B_j^*)$ is weakly higher than profit under $(p_j', B_j')$. I do this by construction. Denote $B_j^*$ as a structure which solves Equation (2). Such a structure always exists because Equation (2) is a rate-distortion/rational inattention problem, as shown in Appendix Section A.1.

For any price $p_j'$ and any structure $B_j'$, I can construct $p_j = p_j' + \gamma_j I(B_j^*) + W(B_j^*) -$

$\gamma_j I(B'_j) - W(B'_j)$. The price $p_j$ is positive and therefore feasible. Recall that profit evaluated at $(p_j, B^*_j)$ is

$$D_j(\xi(B^*_j) - \rho p_j, p_{-j}, \xi_{-j})\left[p_j - \gamma_j I(B^*_j) - W(B^*_j)\right].$$

The second multiplicative term of profit is equal under $(p_j, B^*_j)$ and $(p'_j, B'_j)$. The first term (demand) is strictly increasing in the quality-price index $\xi(B_j) - \rho p_j$; therefore, it is sufficient to show that this index is weakly higher for $(p_j, B^*_j)$. I show this by rewriting $\xi(B^*_j) - \rho p_j$:

$$= \xi(B^*_j) - \rho[p'_j + \gamma_j I(B^*_j) + W(B^*_j) - \gamma_j I(B'_j) - W(B'_j)] \tag{10}$$

$$= \xi(B^*_j) - \rho[p'_j + \gamma_j I(B^*_j) + W(B^*_j) - \gamma_j I(B'_j) - W(B'_j)] + \xi(B'_j) - \xi(B'_j) \tag{11}$$

$$= \xi(B'_j) - \rho[p'_j + \gamma_j I(B^*_j) + W(B^*_j) - \gamma_j I(B'_j) - W(B'_j) - \rho^{-1}\xi(B^*_j) + \rho^{-1}\xi(B'_j)] \tag{12}$$

$$= \xi(B'_j) - \rho p'_j - \rho[\underbrace{\gamma_j I(B^*_j) + W(B^*_j) - \rho^{-1}\xi(B^*_j) - \{\gamma_j I(B'_j) + W(B'_j) - \rho^{-1}\xi(B'_j)\}}_{\leq 0 \text{ because } B^*_j \text{ minimizes}}] \tag{13}$$

$$\geq \xi(B'_j) - \rho p'_j. \tag{14}$$

This proves the "only if" direction. I now prove that if a structure $B^*_j$ is profit maximizing, it solves Equation (2) (the "if" direction). Suppose for sake of contradiction there exists $B'_j$ which is profit maximizing but does not solve Equation (2). Then, as in the first part of the proof, there exists $B^*_j$ which does solve Equation (2). Then I can construct $p_j$ as before for any $p'_j$ that is weakly higher than marginal cost under $B'_j$. However, because $B'_j$ does not minimize Equation (2), $\xi(B^*_j) - \rho p_j > \xi(B'_j) - \rho p'_j$, and thus profit is strictly higher under $B^*_j, p_j$. This contradicts optimality of $B'_j$ and concludes the proof.

## A.3   Proof of Proposition 1 and 2

I have already shown in Theorem 1 that optimal $B$ solves a rate-distortion problem.

- Denote by $Q$ the quality-adjusted wages. Denote by $I^*(Q)$ the optimal complexity as a function of quality-adjusted wages.

- RD equivalence $\implies I^*(Q)$ is continuous, convex and decreasing. It is also strictly decreasing above some threshold $\bar{Q}$ (Chen, n.d.).

- The firm's choice of quality-adjusted wages solves

$$V := \min_Q \gamma I^*(Q) + Q.$$

- The envelope theorem implies the index and thus profit are increasing in $\gamma$:

$$\frac{\partial V}{\partial \gamma} = I^*(Q) \geq 0.$$

- Examine the FOC:

$$\frac{dI^*(Q) + \gamma^{-1}Q}{dQ} = \frac{dI^*(Q)}{dQ} + \gamma^{-1} = 0 \implies \frac{dI^*(Q)}{dQ} = -\gamma^{-1}.$$

- Because $I^*$ is decreasing and convex, its derivative is negative and increasing.

- Therefore, $Q$ which solves is increasing in $\gamma$.

- Thus profit and complexity will be positively correlated via $\gamma$.

## A.4  Optimal Jobs Within the Firm

The last result shows the originally high-dimensional problem of the firm can be reduced to a tractable two-dimensional trade-off. However, one of the goals of the model is to understand how firms assign workers to tasks. This section describes the properties of task assignments within the firm and shows that the firm customizes the bundles of tasks it assigns individual worker types. For this, I define the job of worker type $i$ at firm $j$ as a vector $(b_j(i, \cdot))$, where element $k$ denotes the amount of $i$'s time spent on task $k$. The jobs at a firm are the rows of the organization structure divided by the total labor of worker type $i$:

$$b_j(i, k) = \frac{B_j(i, k)}{\sum_{k'} B_j(i, k')}.$$

**Proposition 5** *The profit-maximizing organizational structure satisfies the following properties.*

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1. **Law of Demand:** *The share of workers of type $i$ ($E_j(i)$) decreases as their wage increases.*

2. **Incomplete Specialization:** *All hired worker types spend a positive amount of time on each task whenever $\gamma_j > 0$.*

3. **Optimal Jobs:** *Jobs take the following logit-like form:*

$$b_j(i,k) = \alpha_k \frac{exp(-\gamma^{-1}w_i + (\rho\gamma)^{-1}\theta_{i,k}))}{\sum_{i'} E_j(i')exp(-\gamma^{-1}w_{i'} + (\rho\gamma)^{-1}\theta_{i',k}))}.$$

I prove this result by appealing to the rational inattention literature. I derive the expression for optimal jobs by manipulating the first-order conditions and the constraints. The proof is provided in Appendix Section A.4.1. Even though at a high level the firms are trading off complexity and quality-adjusted wages, under the surface, they customize jobs for individual workers and tasks. The proposition illustrates that task assignments depend on skills through $\theta_{i,k}$, wages through $w_i$, consumer price sensitivity through $\rho$, the task mix through $\alpha_k$, and organization costs through $\gamma_j$. This proposition highlights two important features of the model. First, whenever there are some organizational frictions within a firm, complete specialization will not occur. Every "job" will be a bundle of multiple tasks. Second, because jobs depend on organization costs, where someone works matters for what they do. That is, two identical workers will not perform the same tasks even in the same product and labor market. The tasks included in any job will depend on the firm where a worker is employed.

### A.4.1   Proof of Proposition 5

For the purposes of this proof only, I define $h_{i,k}$ as the fraction of task $k$ performed by worker $i$. Then the optimal job of worker $i$ is given by

$$h_{i,k} = \frac{E_i}{Z(k,\lambda)}exp\left(-\lambda(\rho w_i - \theta_{i,k})\right).$$

Summing over $i$ yields

$$\sum_i h_{i,k} = \frac{1}{Z(k,\lambda)}\sum_i E_i exp\left(-\lambda(\rho w_i - \theta_{i,k})\right) = 1.$$

Therefore,

$$Z(k,\lambda) = \sum_i E_i exp(-\lambda(\rho w_i - \theta_i \delta^{\mathbb{I}\{\kappa_i \neq k\}}))$$

and

$$h_{i,k} = \frac{e_i exp(\lambda(-\rho w_i + \theta_{i,k}))}{\sum_{i'} e_{i'} exp(\lambda(-\rho w_{i'} + \theta_{i,k}))}.$$

Substituting for $\lambda$ yields

$$h_{i,k} = \frac{E_i exp(-\gamma^{-1} w_i + (\rho\gamma)^{-1}\theta_{i,k}))}{\sum_{i'} E_{i'} exp(-\gamma^{-1} w_{i'} + (\rho\gamma)^{-1}\theta_{i',k}))}.$$

By the definition of $h_{i,k}$,

$$B_{i,k} = \alpha_k h_{i,k}.$$

To get to jobs, I divide by $E_i$:

$$b_{i,k} = B_{i,k}/E_i = \alpha_k/E_i h_{i,k} = \frac{\alpha_k exp(-\gamma^{-1} w_i + (\rho\gamma)^{-1}\theta_{i,k}))}{\sum_{i'} E_{i'} exp(-\gamma^{-1} w_{i'} + (\rho\gamma)^{-1}\theta_{i',k}))}.$$

This illustrates that optimal jobs take a multinomial logit form. I can also derive this result by applying Theorem 1 from Matêjka and McKay (2015).

The fact that all hired worker types spend a positive amount of time on each task is a direct application of Lemma 1 from Jung et al. (2019). An increase in wage corresponds to a decrease in the "payoff" to the firm of using workers of type $i$ in all tasks (i.e., states of the world in the rational inattention literature). This means I can apply Proposition 3 from Matêjka and McKay (2015) to say that an increase in $w_i$ leads to a decrease in $E_i$ all else constant. I can even say that $E_i$ is strictly decreasing in $w_i$ whenever the initial share of worker $i$ is strictly interior, i.e., $0 < E_i < 1$.

## A.5   Proof of Proposition 3

To recover the best responses of the firm's problem, I use the fact that the joint maximization of any function is equivalent to the sequential maximization. Thus I can write the firm's problem as

$$\max_{B_j \in \mathbb{B}} \max_{p_j \in \mathbb{R}_+} \underbrace{\frac{exp(\overbrace{\xi(B_j)}^{\text{quality}} - \rho p_j)}{\sum_{j'} exp(\xi(B_{j'}) - \rho p_{j'})}}_{\text{market share, } s_j} \left[ p_j - \underbrace{\left( \overbrace{\gamma_j I(B_j)}^{\text{org.}} + \overbrace{\sum_{i,k} w_i B_j(i,k)}^{\text{avg. wage}} \right)}_{\text{constant marginal cost, } MC_j} \right].$$

I first study the inner pricing problem. Fixing an organization structure, the model reduces to a logit Bertrand game with heterogeneous costs and qualities. Proposition 7 of Caplin and Nalebuff (1991) proves that such a game has a unique pure-strategy Nash equilibrium in prices. Therefore, for any chosen organizational structure, there is a single best-response price. In the proof of Theorem 1, I substituted the equation characterizing the optimal price into profit, and showed that the best response $B_j$ also solves

$$\min_{B_j \in \mathbb{B}} I(B_j) + \gamma_j^{-1} \sum_{i,k} B_j(i,k)(W_i - \rho^{-1}\theta_{i,k}).$$

The best-response structure will therefore depend on other actions of the firm only through wages. The theorem also establishes that this is equivalent to a rational inattention problem with a mutual information cost function. With the equivalence to a rational inattention problem, I can establish existence. I can then appeal to Matêjka and McKay (2015) to say that there exists an organization structure which maximizes profit for each firm. This establishes equilibrium existence.

To obtain uniqueness, note that a rational inattention problem with mutual information costs is a special case of the problems considered by Lipnowski and Ravid (2022). A stochastic choice rule in their language is an organization structure in mine. Proposition 1 of their paper implies that if $\gamma_j$ is known, the set of quality-adjusted wages which generate multiple organization structures is "meager and shy." Since I consider the case of finite tasks (finite $\Omega$ in their language), "meager and shy" implies a null set. This is only for one firm with a specific $\gamma_j$. The set of quality-adjusted wages which generate multiple profit-maximizing organization structures for at least one firm will be the union of all sets which generate multiplicity for each individual firm. The union of countable null sets is also null; therefore, the set of quality-adjusted wages that generate multiplicity is null.

Denote the set of quality-adjusted wages which generate multiplicity as $\mathbb{M}$. The mapping from market parameters $\Omega$ to quality-adjusted wages is defined by a multivariate,

vector-valued function $F : \mathbb{R}_+^{N \times K + N + 1} \to \mathbb{R}_+^{N \times K}$. It can be shown that if $F$ is smooth and the rank of the Jacobian of $F$ is at least $N \times K$, then the measure of the pre-image of any measure 0 set is 0.

I now prove that $F$ satisfies the rank condition. Recall that the quality-adjusted wage of worker $i$ and task $k$ has the form $w_i - \rho^{-1}\theta_{i,k}$. Collapse $i, k$ into a single index, $y = 1, ..., N \times K$, where $\mathcal{I}(\cdot)$ and $\mathcal{K}(\cdot)$ return the task and worker type associated with the index $y$. Then that element $y$ of $F$ is $F(\Omega) = w_{\mathcal{I}(y)} - \rho^{-1}\theta_{\mathcal{I}(y),\mathcal{K}(y)}$. The Jacobian of this function has a rank of at least $N \times K$ because each skill parameter $\theta_{i,k}$ impacts only one quality-adjusted wage. Formally, there exist at least $N \times K$ columns of the Jacobian which are linearly independent of each other. Thus the pre-image of the null set $\mathbb{M}$ on $F$ will be measure 0. Since the pre-image is the set of parameters which generate multiplicity, the set of parameters which cause at lease one firm to have multiple profit-maximizing organization structures is measure 0.

Whenever all firms have a unique organization structure, they also have a unique cost and quality. It remains to be shown that equilibrium prices are also unique. To do this, I appeal to Caplin and Nalebuff (1991) and note that demand is multinomial logit, so whenever organization structures are unique, so are Nash equilibrium prices.

## A.6 Proof of Proposition 4

For simplicity, firm index $j$ is suppressed throughout this section. I denote by $I(\tilde{B})$ the organization complexity based on worker identities. This is observed in the task assignment data. I denote by $I(B)$ the organization complexity based on worker skill sets. This is unobserved. I denote by $I^*(\gamma)$ the firm's complexity predicted by the model, where market parameters $\Omega$ and the task mix $\alpha$ are assumed to be known and thus are incorporated into the function and not left as arguments.

First, I prove that observed organization complexity based on worker identities ($I(B)$) is equal to unobserved true complexity based on worker skill sets ($I(\tilde{B})$). Consider the augmented model proposed in Section 6.1. In particular, recall that workers with different labor supplies match to firms by some unspecified matching process. I then prove the following:

**Lemma 2** *All workers with the same skill set are assigned the same distribution of tasks regardless*

*of their labor supply.*

**Proof of Lemma.** A well-known property of mutual information attention costs is that they satisfy compression monotonicity or are "distraction-free " (Tian 2019). I will use this in the proof.

Suppose for the sake of contradiction the firm assigned two workers of the same skill set different distributions of tasks. Consider a different assignment of work such that the same amount of each task is accomplished, and both workers still are assigned the same total amount of work. Such an assignment always exists: I can just take the total time spent on each task by both workers and split it based on effective units of labor. By the strict distraction-free property of mutual information, this new assignment reduces organization costs. This does not impact the wage bill, since both workers have the same wage. Also, it does not impact quality, because the total amount of each task accomplished remains the same, and both workers have the same skill set. Thus quality-adjusted cost strictly decreases, so profit strictly decreases, contradicting the optimality of the original assignment. Therefore, all workers with the same skill set are assigned the same distribution of tasks regardless of their effective units of labor. ∎

This lemma means that the firm treats workers with different labor supplies but the same skill sets as if they were a single, aggregate worker. Denote worker identities as indexed by $n$, and worker skill sets by $i$. Denote the organizational structure over worker identities as $\tilde{B}$. Then

$$\frac{\tilde{B}_{n,k}}{\sum_{k'} \tilde{B}_{n,k'}} = \frac{B_{i,k}}{\sum_{k'} B_{i,k'}} \forall i, k \text{ s.t. } \theta_n = \theta_i.$$

Because the total amount of each task is fixed at $\alpha_k$,

$$\sum_{n'} \tilde{B}_{n',k} = \alpha_k = \sum_{i'} B_{i',k}.$$

Plugging these results into organization complexity yields

$$I(\tilde{B}) = \sum_{n,k} \tilde{B}_{n,k} log\left(\frac{\tilde{B}_{n,k}}{\sum_{k'} \tilde{B}_{n,k'} \sum_{n'} \tilde{B}_{n',k}}\right) = \sum_{n,k} \sum_{i} \frac{B_{i,k} \sum_{k'} \tilde{B}_{n,k'}}{\sum_{k'} B_{i,k'}} \mathbb{I}\{\theta_n = \theta_i\} log\left(\frac{B_{i,k}}{\sum_{k'} B_{i,k'} \sum_{i'} B_{i',k}}\right).$$

And rearranging terms yields

$$= \sum_{i,k} \frac{B_{i,k}}{\sum_{k'} B_{i,k'}} log \left( \frac{B_{i,k}}{\sum_{k'} B_{i,k'} \sum_{i'} B_{i',k}} \right) \sum_{n,k'} \tilde{B}_{n,k'} \mathbb{I}\{\theta_n = \theta_i\}.$$

The sum of all $\tilde{B}_{n,k}$ of workers with the same skill set but different labor supply is $E_i$, which is exactly equal to $\sum_{k'} B_{i,k'}$. Therefore, I can write

$$I(\tilde{B}) = \sum_{i,k} \frac{B_{i,k}}{\sum_{k'} B_{i,k'}} log \left( \frac{B_{i,k}}{\sum_{k'} B_{i,k'} \sum_{i'} B_{i',k}} \right) \sum_{k'} B_{i,k'} = \sum_{i,k} B_{i,k} log \left( \frac{B_{i,k}}{\sum_{k'} B_{i,k'} \sum_{i'} B_{i',k}} \right) = I(B).$$

Therefore, organization complexity based on worker identities is equal to organization complexity based on worker skill sets. Since I observe identities, this implies that I can compute organization complexity as the mutual information between worker identities and tasks.

I next show that $\gamma$ is identified. This requires that there be a unique $\gamma$ such that $I^*(\gamma) = I(\tilde{B})$. Define $Q_j := W(B_j) - \rho^{-1}\xi(B_j)$. Applying Theorem 1, I can write the firm's problem in the following way:

$$V := \min_{B \in \mathbb{B}} \gamma I(B) + W(B) - \rho^{-1}\xi(B) = \min_{Q \in \mathbb{Q}} \gamma \tilde{I}(Q) + Q,$$

where $\tilde{I}$ is a continuous, decreasing and convex function. Further, it is strictly decreasing whenever $\tilde{I}(Q) > 0$ (Chen, n.d.). Consider only the case when $\tilde{I}(Q) > 0$. Then the FOC $\frac{dV}{dQ} = \gamma \frac{d\tilde{I}(Q)}{dQ} + 1 = 0$ and convexity imply the optimally chosen $Q$ is strictly increasing in $\gamma$. This implies $\tilde{I}(B)$ is strictly decreasing in $\gamma$. Since $\tilde{I}(B) = I^*(\gamma)$ for optimal $B$, $I^*(\gamma)$ is strictly decreasing and identification is achieved whenever $I(\tilde{B}) > 0$.[15]

Theorem 1 established that the firm's problem is a rate-distortion problem. As a result, Blahut (1972) provides an algorithm that can be used to arbitrarily approximate $I^*(\gamma)$. Thus, because $I^*$ is strictly decreasing, I can use this algorithm to invert complexity to retrieve $\gamma$ as a known function of complexity and all other parameters.

To identify organization structures ($B_j$), I appeal to Proposition 3. Since wages are parameters during estimation, the proposition can be applied exactly, and I have that all organization structures are identified except over a set of market parameters with measure 0. Further, the algorithm given in Blahut (1972) constructs optimal $B_j$ for each firm

15. Whenever complexity is 0 (it cannot be negative), any sufficiently large $\gamma$ is consistent with the data.

in the process of computing $I^*$. In the knife-edge cases where more than one structure is optimal for a firm, the algorithm will return one of them. Thus, organization structures are also a known function of the data and market parameters, except for a set of market parameters with measure 0.

## A.7   Organization Complexity as Task Specialization

This section illustrates that complexity is a measure of average task specialization. To see this, first define a job as a vector, where component $k$ is the fraction of a worker's total labor spent performing task $k$:

$$b_i(k) = \frac{B(i, k)}{E_i}.$$

I can measure the specialization of any job by comparing it to a benchmark "generalist job." I define the generalist job as the job where all workers are assigned exactly the task mix:

$$b_j^G(k) = \alpha_k.$$

Notice that when the firm gives all workers the generalist job, each worker is working as a miniature version of the firm itself. There is no sense in which a worker needs a coworker in order to produce output. With these two concepts in hand, I obtain the following result.

**Proposition 6** *Complexity ($I(B_j)$) is the weighted-average Kullback-Leibler divergence between the jobs at a firm and the firm's generalist job $b_j^G(k)$, where the weights are the share of each worker type.*

**Proof.** Using the definition of mutual information, I can write complexity as

$$
\begin{aligned}
I(B_j) &= \sum_{i,k} B(i,k) log \left( \frac{B(i,k)}{\sum_{k'} B(i,k') \sum_{i'} B(i',k)} \right) \\
&= \sum_{i,k} E_i \frac{B(i,k)}{E_i} log \left( \frac{B(i,k)}{E_i \alpha_k} \right) \\
&= \sum_i E_i \sum_k b_i(k) log \left( \frac{b_i(k)}{\alpha_k} \right) \\
&= \sum_i E_i \sum_k b_i(k) log \left( \frac{b_i(k)}{b_j^G(k)} \right) \\
&= \sum_i E_i D_{KL}(b_i || b_j^G).
\end{aligned}
$$

## A.8   Closed-Form Logit Price Expression

Demand for a product $j$ is given by

$$
s_j(p_j) = \frac{exp(-\rho p_j + \xi_j)}{\sum_{j'=0}^J exp(-\rho p_{j'} + \xi_{j'})}.
$$

Optimal pricing in a Bertrand Nash equilibrium with single-product firms is then given by

$$
p_j = MC_j + \frac{1}{\rho(1 - s_j(p_j))}.
$$

I now follow the arguments laid out in Aravindakshan and Ratchford (2011). I rewrite this expression as

$$
p_j = c_j + \frac{1}{\rho(1 - \frac{exp(-\rho p_j + \xi_j)}{exp(-\rho p_j + \xi_j) + \sum_{j' \neq j} exp(-\rho p_{j'} + \xi_{j'})})}.
$$

I rewrite it again as

$$
p_j = c_j + \frac{1}{\rho} + \frac{exp(-\rho p_j + \xi_j)}{\rho \sum_{j' \neq j} exp(-\rho p_{j'} + \xi_{j'})}.
$$

Multiplying by $\rho$ and subtracting $\xi_j$ yields

$$
\rho p_j - \xi_j = \rho c_j + 1 + \frac{exp(-\rho p_j + \xi_j)}{\sum_{j' \neq j} exp(-\rho p_{j'} + \xi_{j'})} - \xi_j.
$$

Now denote

$$E_j = \sum_{j' \neq j} exp(-\rho p_{j'} + \xi_{j'})$$

$$\frac{exp(-\rho p_j + \xi_j)}{E_j} + \xi_j - \rho p_j = -1 - \rho c_j + \xi_j$$

$$exp\left(\frac{exp(\xi_j - \rho p_j)}{E_j}\right) exp\left(\xi_j - \rho p_j\right) E_j^{-1} = exp\left(-1 + \xi_j - \rho c_j\right) E_j^{-1}$$

and

$$\tilde{W} = exp\left(\xi_j - \rho p_j\right) E_j^{-1}.$$

Then the expression becomes

$$\tilde{W} e^{\tilde{W}} = exp\left(-1 + \xi_j - \rho c_j\right) E_j^{-1}.$$

The left-hand side expression is the form required by Lambert's W, so the $\tilde{W}$ which solves is given by Lambert's W function of the right-hand side by definition. Thus optimal price solves

$$W\left(exp\left(-1 + \xi_j - \rho c_j\right) E_j^{-1}\right) = exp\left(\xi_j - \rho p_j\right) E_j^{-1}.$$

A property of this function is that $log(W(x)) = log(x) - W(x)$. Using this fact yields

$$-1 + \xi_j - \rho c_j - log(E_j) - W\left(exp\left(-1 + \xi_j - \rho c_j\right) E_j^{-1}\right) = \xi_j - \rho p_j - log(E_j),$$

which can be solved for the optimal price:

$$\frac{1}{\rho} + c_j + \rho^{-1} W\left(exp\left(-1 + \xi_j - \rho c_j\right) E_j^{-1}\right) = p_j^*. \tag{15}$$

## A.9   A Quantity-Based Model

In some contexts, such as manufacturing, one may wish to model organizational effi-
ciency as allowing firms to produce greater quantity rather than greater quality. Indeed,
this is the default definition of productivity in economics. The model can also be extended
to accommodate this: one can simply interpret the skill sets as denoting the amount of
time required by the worker to complete task $k$ (therefore smaller $\theta_{i,k}$ are better). Then the

production function becomes a function of organization structure:

$$F_{\alpha,B}(a_j) = \min \left\{ \frac{a_1}{\alpha_1 \sum_i \theta_{i,1} B_j(i,1)}, \dots \frac{a_k}{\alpha_k \sum_i \theta_{i,k} B_j(i,k)}, \dots, \frac{a_K}{\alpha_K \sum_i \theta_{i,K} B_j(i,K)} \right\}.$$

Given any fixed organizational structure, the efficient way to produce a single unit of output is to set $a_k = \alpha_k \sum_i \theta_{i,k} B_j(i,k)$. Thus the per-unit wage bill is given by

$$\sum_i W_i \sum_k \alpha_k \sum_i \theta_{i,k} B_j(i,k).$$

Marginal costs are constant and consist of the per-unit wage bill and organization costs:

$$MC_j = \sum_i w_i \sum_k \alpha_k \sum_i \theta_{i,k} B_j(i,k) + \gamma_j I(B_j).$$

All of the benefits of a more complex organization come through a reduction in the per-unit wage bill. In this way, the intuition from the original model extends directly to the quantity case: firms with greater organizational efficiency (lower $\gamma_j$) can produce more of the good with the same workforce. I did not use this as the main model because the following property is not compatible with the empirical application to hair salons:

**Proposition 7** *Under a quantity model with multinomial logit demand, prices are decreasing with organizational complexity.*

The proof of this proposition is given in the next paragraph. Intuitively, under the quantity model with logit demand, all the benefits of a complex organization come from greater output rather than from greater revenue per unit. The reduction in marginal cost outpaces the increase in the markup, resulting in lower prices. This implies a negative correlation between prices and complexity, which is shown not to be true for hair salons. However, for manufacturing firms, it appears to be true. Caliendo et al. (2020) finds that prices (revenue-based productivity) decline when manufacturing firms reorganize.

**Proof.** Equation 15 from Appendix Section A.8 provides a closed-form expression for price in any Nash Equilibrium under logit demand:

$$\frac{1}{\rho} + c_j + \rho^{-1} W \left( exp \left( -1 + \xi_j - \rho c_j \right) E_j^{-1} \right) = p_j^*.$$

Taking the derivative w.r.t. $c_j$ yields

$$\frac{\partial p_j^*}{\partial c_j} = 1 - exp\left(-1 + \xi_j - \rho c_j\right) E_j^{-1} W'\left(exp\left(-1 + \xi_j - \rho c_j\right) E_j^{-1}\right).$$

A property of the Lambert W function is that

$$W'(x) = \frac{W(x)}{(1 + W(x))x}.$$

Thus, I can simplify the expression to

$$\frac{\partial p_j^*}{\partial c_j} = 1 - \frac{W(exp[-1 + \xi_j - \rho c_j] E_j^{-1})}{1 + W(exp[-1 + \xi_j - \rho c_j] E_j^{-1})}.$$

The Lambert W function is weakly positive for values which are weakly positive; there-fore, the derivative is positive, and price is decreasing in cost. The firm minimizes cost:

$$\min_{B \in \mathbb{B}} \gamma I(B_j) + W(B_j).$$

This is again a rate-distortion problem. Denoting the optimal wage-bill as $D = W(B_j^*)$, I can reformulate the problem as before, with the firm choosing $D$ given some optimal organization cost and wage bill:

$$\min_D \gamma I(D) + W(D),$$

where $I$ and $W$ are expressed as functions of $D$ instead of $B_j$. Then, as before, there is a negative cross-partial derivative:

$$\frac{\partial \gamma I(D) + W(D)}{\partial D \partial \gamma} = I'(D) < 0$$

with strict inequality whenever $I(D)$ is strictly positive. This establishes strict decreasing differences of $D$ in $\gamma$; thus $D$ is strictly decreasing in $\gamma$, and since $I(D)$ is a strictly decreas-ing function, it is also strictly decreasing in $\gamma$. Therefore, prices should be decreasing as $\gamma$ decreases, while complexity should be increasing.

## A.10 Task Classification Process: Further Details

A licensed cosmetologist was paid to categorize 20,560 salon services performed according to their descriptions. As part of the agreement, the person provided a picture of their cosmetology license. The cosmetologist was provided with a blank spreadsheet with pre-defined subcategories and was instructed to mark all subcategories where the description matched with a 1. They were instructed that some subcategories may not be mutually exclusive, so they should mark all that applied. The initial job description was as follows:

> I have a list of approx. 20,560 short descriptions of salon services (mainly hair salons, but also some nail/spas). I would like someone with knowledge of the industry to mark whether each descriptions fits into one of several categories (male/female service, coloring, cutting, highlighting, washing, etc). This amounts to putting a 1 in each column that fits the description.

In a follow-up message I further clarified the instructions:

> Here are the descriptions. I did the first few to give you a sense of the task. Basically read the description and then put a 1 in all categories that fit. Sometimes a description may match many, sometimes 1, rarely none. If you start reading them and see that it may be worth adding a separate category let me know. The idea though is to capture the core "tasks" or services performed at hair salons, like cut, color, highlight, style, etc and also to get some info on gender and typos.

After the first draft was submitted, I checked the coding, looking for any mistakes or missed descriptions, and sent the document back to the cosmetologist several times for revision. A sample from the final spreadsheet is displayed in Figure A1.



**Figure A1:** Final Task Subcategorization Spreadsheet from Cosmetologist

Since the subcategories were very detailed, I hired the same cosmetologist, at a rate of $100, to classify the subcategories into six task categories. The specific instructions given to the cosmetologist were as follows:

Please categorize the 13 tasks from before into "groups." For the 6 group column, put the 13 tasks into 6 groups that are most similar in terms of who would do them/tasks they would require. for example, if color and highlight are similar, mark both as number 1. Number the groups 1 through 6. For the four group column, make 4 groups, etc. Underneath, please write a small note describing why you put the tasks together the way you did.

I use the six-category grouping provided by the cosmetologist with one modification: I combine the extension task with the blow-dry task to create five final task categories, because the extension task is very sparse–for Manhattan in 2021 Q2, fewer than 10 hours were dedicated to this task. This sparsity leads to estimation problems, as parameters tied to this task have a negligible effect on observable outcomes.

## A.11  Robustness of Stylized Facts

The first concern is one of reverse causality. Perhaps firm size allows firms to be organizationally complex and thus have a product market advantage.[16] Appendix Section A.13 shows that while this cannot be ruled out, it is not generating all of the observed relationships. Even among firm-quarters with the same number of employees, there is significant variation in complexity, and there is a positive association between complexity and the main market outcomes (i.e., revenue, prices and repeat customers).

A second concern is that the correlations are driven by demand-side factors, such as consumer preferences for particular stylists rather than firm choices. The software records when customers request a particular staff member. It would be concerning if there was a strong positive correlation between the request rate at a salon and complexity. Appendix Section A.14 shows that while many customers request specific staff, the rate of requests across salons is not correlated with organization complexity. Further, the correlation between the request rate and firm size is either zero or negative.

A third concern is that the correlations are driven by the specific functional form chosen for complexity. Appendix Section A.15 shows that the main patterns persist when complexity is replaced by within-visit specialization. Within-visit specialization is measured as the fraction of multi-service visits which are performed by a team (i.e., more than one employee).

16. Li and Tian (2013) provide a theoretical mechanism for such an effect.

## A.12    Measurement Error in Organization Complexity

Complexity is estimated based on the observed task assignments within firm, yet the empirical part of this paper treats complexity as if it were observed or measured without error. One justification is that many assignments are observed per firm per quarter, so estimation error should be small. If estimation error at the quarter level is small, the correlation between complexity measures at the month level within quarter should be large. This section illustrates that this is indeed the case.

To do this, I recompute complexity for each month within a quarter so that I have three measurements of complexity per firm-quarter observation. In the full sample, the pairwise correlation between the first and second month is 0.945, the first and third is 0.98, and the second and third is 0.939. When 2020 (the onset of the coronavirus pandemic) is excluded, the pairwise correlations are 0.978, 0.962 and 0.976, respectively. The high correlation between complexity measurements within quarters suggests that complexity at the quarter level is measured precisely.

## A.13    Complexity Relationships Among Similar-Size Firms

The main text of the paper established that complexity is correlated with the number of employees as well as other outcomes. This raises concerns about the direction of causality: are firms larger because they are more internally complex, or are larger firms naturally able to design more internally complex structures? The model in this paper specifies a common organization cost, which generates jointly both larger and more complex firms. In this sense, complexity does not cause a firm to be larger; rather a common, unobserved productivity heterogeneity generates both.

This answer still leaves several questions open. In particular, perhaps organization costs are more like fixed costs, so larger firms are better able to afford more complex organizations. Additionally, maybe larger firms have more organizational possibilities, and thus the relationships discussed are mechanical. I alleviate this concern by analyzing many of the outcomes among firms with the same number of employees.

The positive correlation between complexity and revenue, prices and repeat customers persists among firm-quarters with the same number of employees. There is a positive

correlation within almost all firm sizes and for almost all variables. The exception is repeat customers among firms with 2–5 employees. In general, the positive correlation is larger in magnitude for firm-quarters with 13 or more employees.

Essentially, while complexity is correlated with both employee count and other market outcomes, and employee count is correlated with the other market outcomes, there seems to be a large, direct effect of complexity on market outcomes. Another way to see this is that when firm-size fixed effects are added to a regression of revenue on complexity, the point estimate for complexity decreases by around 60 percent, but remains economically and statistically significant. So while much of the effect of complexity on other outcomes seems to come through size, a sizable amount does not.

## A.14 Consumers Requesting Particular Staff

The stylized facts and the model treat the assignment of workers to tasks as a choice of the firm. In practice, some customers directly request particular stylists. The software allows salons to record when a staff member is requested for a task, and this information is captured in a variable titled "Was Staff Requested." This section establishes that although there is heterogeneity in how often staff are requested at different salons, this heterogeneity is not correlated with organization complexity.

I start by examining the variation in requests across salon-quarters in Figure A2. A large number of salon-quarters have no requests observed in a quarter (Panel A). Among those salon-quarters with at least one request, the request rate varies significantly, spanning close to 0 all the way to 1 with a mode around 0.8 (Panel B). Much of this heterogeneity comes form an aggregate increase in the request rate over time (Panel C). Therefore, I also run analyses excluding quarters before the first observed request for a salon. I call this sample "after adoption."

The primary question is whether consumer requests are driving observed organization complexity. I test this using binned scatter plots in Figure A3. Both unconditionally (Panel A) and among salon-quarters with one request (Panel B), complexity does not appear to have a systematic relationship with the request rate.

Regressions with standard errors clustered at the salon level also reveal mixed results. In the full sample, the coefficient on the request rate is statistically insignificant
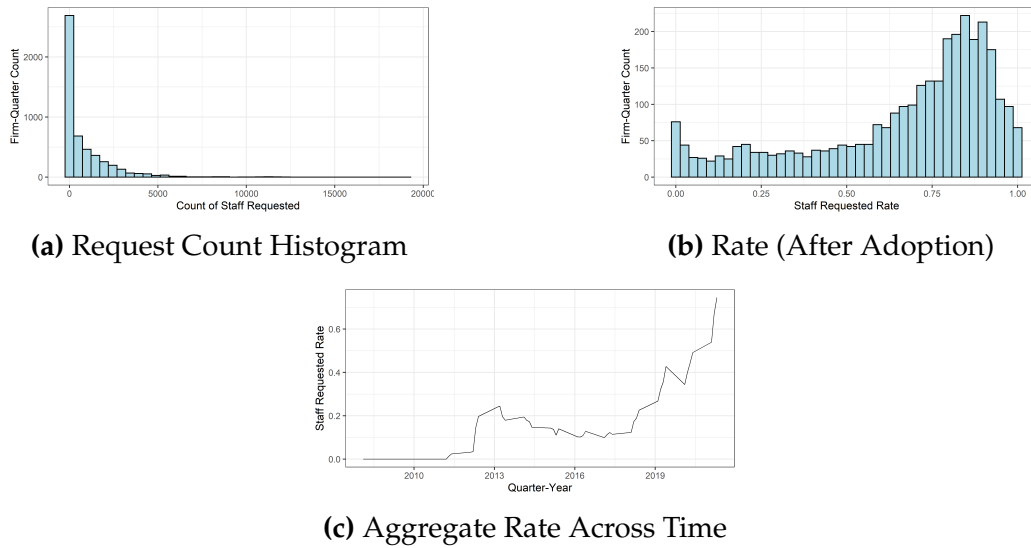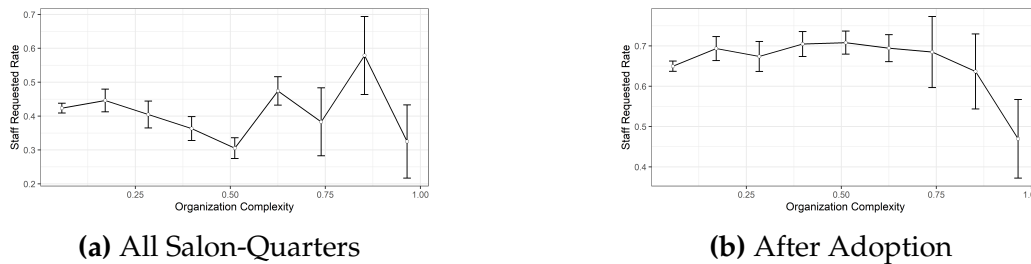
**Figure A2:** Was Staff Requested?



**(a)** Request Count Histogram



**(b)** Rate (After Adoption)



**(c)** Aggregate Rate Across Time

**Figure A3:** Request Rate and Organization Complexity


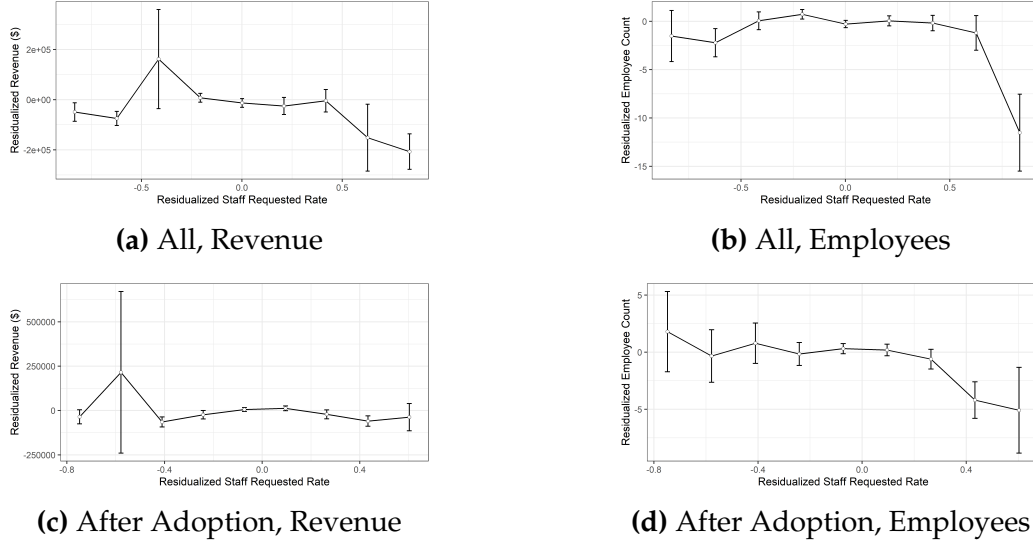
**(a)** All Salon-Quarters



**(b)** After Adoption

and negative. In the after-adoption sample, the coefficient is statistically insignificant and positive. In both cases, the coefficients are economically insignificant: they imply that a 1-standard-deviation increase in the request rate is associated with less than a 0.08-standard-deviation change in complexity.

Further, Figure A4 shows there is no evidence of a positive relationship between firm size and the request rate (if anything, there may be a negative relationship), which suggests the positive relationship between complexity and firm size documented in the stylized facts is not driven by customer request.

## A.15 Within-Visit Specialization

This section shows that many of the correlations between complexity and market outcomes persist when complexity is replaced with a simpler measure of within-visit spe-

**Figure A4:** Request Rate and Firm Size



**(a)** All, Revenue



**(b)** All, Employees



**(c)** After Adoption, Revenue



**(d)** After Adoption, Employees

cialization. I compute within-visit specialization as the number of customer visits[17] with two or more employees assigned divided by the number of customer visits with two or more services performed.

A histogram of this measure shows that it follows a similar power-law distribution as organization complexity, with observed values spanning the support and a long right tail. Like organization complexity, within-visit specialization is positively correlated with revenue, price and the share of repeat visits. However, unlike organization complexity, it has a non-monotone relationship with the number of employees.

These findings are further support that more internally specialized firms command a competitive advantage. To finish this section, I study the connection between complexity and within-visit specialization. A simple regression of complexity on within-visit specialization yields an R-squared of 0.50, suggesting that nearly half of the variation in complexity can be accounted for by specialization within-visit.

## A.16   Supplementary Tables and Figures

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17. Visits are the number of unique customer-date pairs in a quarter.

**Figure A5:** Within-Visit Specialization



**(a)** Revenue



**(b)** Employees



**(c)** Price



**(d)** Repeat Customers



**(e)** Histogram
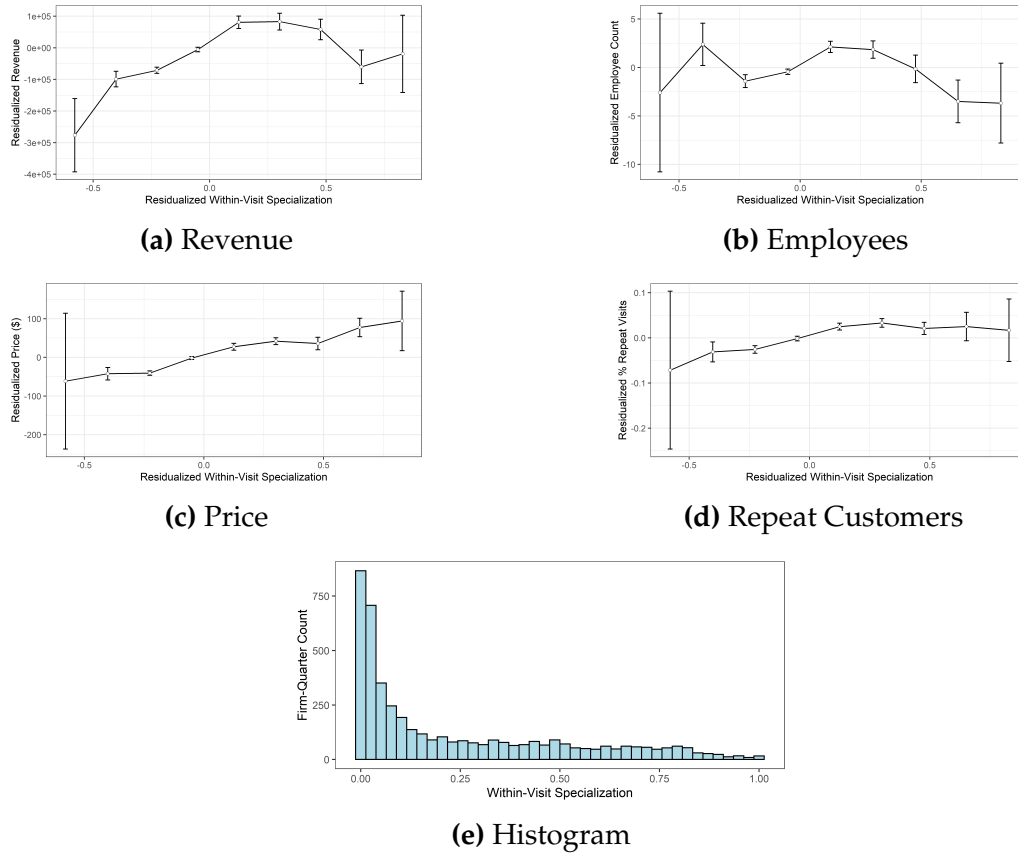
**Note:** Within-visit specialization is the share of visits with multiple services that are assigned to multiple employees.

Table A1: Regressions of Salon Size on Organization Complexity

| Dependent Variables:<br>Model: | Revenue<br>(1) | Employees<br>(2) | Utilized Labor<br>(3) | Customers<br>(4) | Visits<br>(5) |
|---|---|---|---|---|---|
| *Variables* | | | | | |
| Org. Complexity | 347549.2*** | 9.75** | 26481 | 334.6 | 731.7 |
| | (79546.2) | (3.016) | (35653.2) | (259.6) | (450.1) |
| *Fixed-effects* | | | | | |
| Quarter-Year | Yes | Yes | Yes | Yes | Yes |
| County | Yes | Yes | Yes | Yes | Yes |
| *Fit statistics* | | | | | |
| Observations | 4,558 | 4,558 | 4,558 | 4,558 | 4,558 |
| $R^2$ | 0.32465 | 0.34319 | 0.28918 | 0.34901 | 0.35004 |

*Standard-errors clustered at the salon level.*
*Signif. Codes: \*\*\*: 0.001, \*\*: 0.01, \*: 0.05*

**Note:** Observations are salon-quarters. Regressions illustrate a positive correlation between complexity and several measures of salon size after controlling for county and quarter fixed effects and the composition of tasks performed at the salon in the quarter.

## Table A2: Regressions of Revenue on Complexity

| Model: | (1) | (2) | (3) | (4) | (5) | (6) |
|---|---|---|---|---|---|---|
| Org. Complexity | 456571.3*** | 440904.1*** | 485026.4*** | 486995.5*** | 271694.6** | 261697** |
| | (100394.8) | (108427.1) | (116918.9) | (125004.8) | (87031.1) | (80920.6) |
| Task Mix 2 | | | | -19070.7 | -7609.7 | 14482.9 |
| | | | | (93817.4) | (78597) | (67354.5) |
| Task Mix 3 | | | | -8011.8 | 116011.4 | 98022 |
| | | | | (81014.1) | (106735) | (98077.1) |
| Task Mix 4 | | | | -24893.1 | 76296.2 | 67131.1 |
| | | | | (113959) | (96547) | (95768.9) |
| Task Mix 5 | | | | 43954.8 | 14593.5 | 33562.4 |
| | | | | (50238.8) | (47813) | (56691.1) |
| Staff Request Rate | | | | | | -94370.7 |
| | | | | | | (89112.9) |
| *Fixed-effects* | | | | | | |
| Quarter-Year | | Yes | Yes | Yes | Yes | Yes |
| County | | | Yes | Yes | Yes | Yes |
| Firm Size | | | | | Yes | Yes |
| *Fit statistics* | | | | | | |
| Observations | 5,116 | 5,116 | 5,116 | 5,116 | 5,116 | 5,116 |
| $R^2$ | 0.01475 | 0.01915 | 0.3104 | 0.31047 | 0.34273 | 0.34365 |

*Clustered standard-errors in parentheses*
*Signif. Codes: \*\*\*: 0.001, \*\*: 0.01, \*: 0.05*

**Note:** This table reports the regressions of revenue on complexity under various specifications, including controlling for the rate of staff requested.

## Table A3: Variance Decomposition: Without a Model

### Across Firms

| Task | Share of Labor | Share of Variance Firm | Within-Firm |
|---|---|---|---|
| Haircut/Shave | 0.4049 | 0.3744 | 0.6256 |
| Color/Highlight/Wash | 0.3902 | 0.2899 | 0.7101 |
| Blowdry/Style/Treatment/Extension | 0.0850 | 0.5056 | 0.4944 |
| Administrative | 0.0590 | 0.4900 | 0.5100 |
| Nail/Spa/Eye/Misc. | 0.0610 | 0.4124 | 0.5876 |

### Across Quarters

| Task | Share of Labor | Share of Variance Quarter | Within-Quarter |
|---|---|---|---|
| Haircut/Shave | 0.4049 | 0.0057 | 0.9943 |
| Color/Highlight/Wash | 0.3902 | 0.0062 | 0.9938 |
| Blowdry/Style/Treatment/Extension | 0.0850 | 0.0111 | 0.9889 |
| Administrative | 0.0590 | 0.0193 | 0.9807 |
| Nail/Spa/Eye/Misc. | 0.0610 | 0.0118 | 0.9882 |

## Table A4: Model Validation: Estimated vs. Observed Job Task Content

### (a) Mean and Median

| Task | Mean Model | Mean Observed | Median Model | Median Observed |
|---|---|---|---|---|
| Haircut/Shave | 0.4094 | 0.4094 | 0.2816 | 0.3357 |
| Color/Highlight/Wash | 0.4058 | 0.4058 | 0.3067 | 0.4042 |
| Blowdry/Style/Treatment/Extension | 0.1179 | 0.1179 | 0.0162 | 0.0704 |
| Administrative | 0.0278 | 0.0278 | 0.0050 | 0.0040 |
| Nail/Spa/Eye/Misc. | 0.0391 | 0.0391 | 0.0049 | 0.0000 |

### (b) Variance

| Task | Total Variance Model | Total Variance Observed | Between Firm Variance Model | Between Firm Variance Observed |
|---|---|---|---|---|
| Haircut/Shave | 0.1110 | 0.1268 | 0.0597 | 0.0597 |
| Color/Highlight/Wash | 0.1127 | 0.1105 | 0.0365 | 0.0365 |
| Blowdry/Style/Treatment/Extension | 0.0472 | 0.0194 | 0.0111 | 0.0111 |
| Administrative | 0.0098 | 0.0080 | 0.0063 | 0.0063 |
| Nail/Spa/Eye/Misc. | 0.0120 | 0.0171 | 0.0050 | 0.0050 |

### (c) Interquartile Range

| Task | p25 Model | p25 Observed | p75 Model | p75 Observed |
|---|---|---|---|---|
| Haircut/Shave | 0.1583 | 0.0469 | 0.8013 | 0.7577 |
| Color/Highlight/Wash | 0.0417 | 0.0388 | 0.7020 | 0.6383 |
| Blowdry/Style/Treatment/Extension | 0.0004 | 0.0110 | 0.0726 | 0.1892 |
| Administrative | 0.0027 | 0.0000 | 0.0166 | 0.0108 |
| Nail/Spa/Eye/Misc. | 0.0000 | 0.0000 | 0.0329 | 0.0106 |

**Note:** The table compares model-generated and observed job task content along several dimensions. The model is designed to exactly match the average market-wide amount of time spent on each task and the between-firm variance. The other moments were not targeted, and assessing their match serves as a validation exercise.

## Table A5: Minimum Wage Counterfactual Type-Specific Wages, Employment and Specialization

| Worker Type | Initial | | | Reallocation | | | Reorganization | | |
|---|---|---|---|---|---|---|---|---|---|
| | Hours | Wage | Task-Spec. | Hours | Wage | Task-Spec. | Hours | Wage | Task-Spec. |
| Haircut/Shave | 537550 | $16.96 | 0.9463 | 506090 | $20.00 | 0.9459 | 502152 | $20.00 | 0.947 |
| Color/Highlight/Wash | 997053 | $37.75 | 0.7245 | 997053 | $37.33 | 0.7233 | 997053 | $37.52 | 0.7209 |
| Blowdry/Style/Treatment/Extension | 444040 | $20.91 | 0.4837 | 444040 | $21.88 | 0.4817 | 444040 | $21.64 | 0.4819 |
| Administrative | 41860 | $26.99 | 0.6801 | 41860 | $28.40 | 0.6807 | 41860 | $28.12 | 0.6809 |
| Nail/Spa/Eye/Misc. | 34844 | $81.16 | 0.8262 | 34844 | $81.63 | 0.826 | 34844 | $81.71 | 0.826 |

**Note:** This table displays employment and wage levels across the initial, reallocation and full equilibrium under a $20 minimum wage. It provides context for the main counterfactual results, which are reported in percentages.


## Table A6: Total Effects of Increasing the Minimum Wage

### (a) Wage Changes by Worker Type

| Type | Wage Change | Total Wages Gained/Lost |
|---|---|---|
| Haircut/Shave - UNEMPLOYED | -100.00% | -$600,240 |
| Haircut/Shave - EMPLOYED | 17.95% | $1,528,205 |
| Color/Highlight/Wash | -0.61% | -$228,453 |
| Blowdry/Style/Treatment/Extension | 3.48% | $323,374 |
| Administrative | 4.17% | $47,154 |
| Nail/Spa/Eye/Misc. | 0.68% | $19,319 |

### (b) Welfare Breakdown

| Source | Change | Percent Change |
|---|---|---|
| Salon Profit | -$714,413 | -0.472% |
| Consumer Welfare | -$2,528,784 | -1.671% |
| Employed Wages | $1,689,600 | 1.116% |
| Unemployed Wages | -$600,240 | -0.397% |
| Total Welfare | -$2,153,838 | -1.423% |


## Table A7: Sales Tax Counterfactual Type-Specific Wages, Employment and Specialization

| Worker Type | Initial | | | Reallocation | | | Reorganization | | |
|---|---|---|---|---|---|---|---|---|---|
| | Hours | Wage | Task-Spec. | Hours | Wage | Task-Spec. | Hours | Wage | Task-Spec. |
| Haircut/Shave | 537550 | $16.96 | 0.9463 | 537550 | $21.18 | 0.9471 | 537550 | $22.38 | 0.9491 |
| Color/Highlight/Wash | 997053 | $37.75 | 0.7245 | 997053 | $45.99 | 0.7326 | 997053 | $45.34 | 0.7432 |
| Blowdry/Style/Treatment/Extension | 444040 | $20.91 | 0.4837 | 444040 | $21.01 | 0.4946 | 444040 | $22.18 | 0.4982 |
| Administrative | 41860 | $26.99 | 0.6801 | 41860 | $30.15 | 0.6786 | 41860 | $31.85 | 0.6872 |
| Nail/Spa/Eye/Misc. | 34844 | $81.16 | 0.8262 | 34844 | $90.75 | 0.8351 | 34844 | $91.49 | 0.846 |

**Note:** This table displays employment and wage levels across the initial, reallocation and full equilibrium under the elimination of the service sales tax. It provides context for the main counterfactual results, which are reported in percentages.


## Table A8: Model-Based Decomposition of Job Task-Content Variance

| Task | Share of Task-Content Variance | |
|---|---|---|
| | Firm | Worker |
| Haircut/Shave | 0.0761 | 0.9239 |
| Color/Highlight/Wash | 0.1194 | 0.8806 |
| Blowdry/Style/Treatment/Extension | 0.2180 | 0.7820 |
| Administrative | 0.0965 | 0.9035 |
| Nail/Spa/Eye/Misc. | 0.0865 | 0.9135 |

**Note:** The table displays a variance decomposition which uses the model to separate the variance of job task content into a worker and firm component.
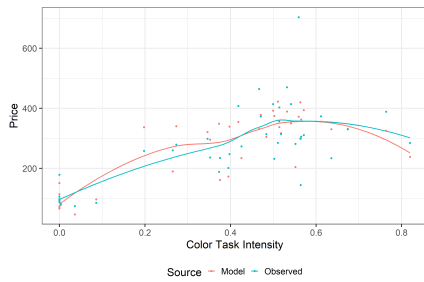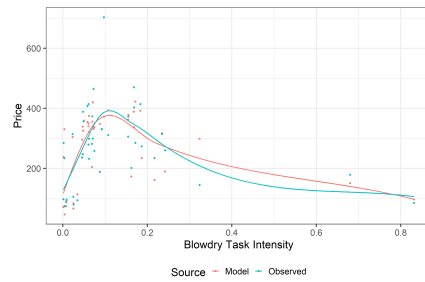
**Figure A6:** Model Fit
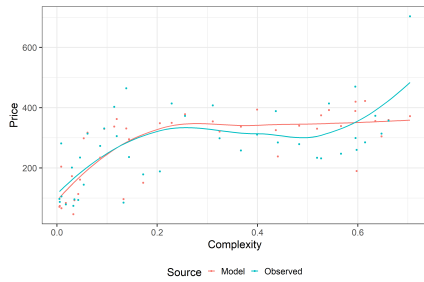


**(a)** Complexity
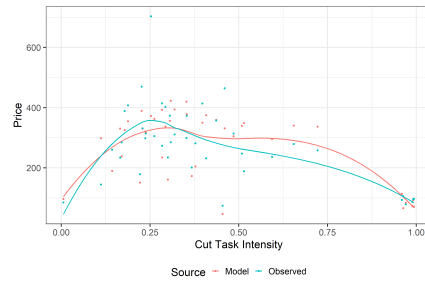
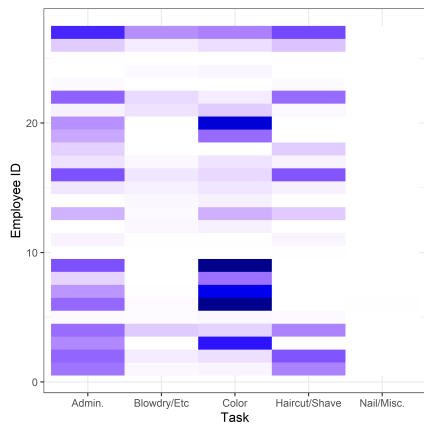**(b)** Task Mix 1 (Haircut/Shave Task)
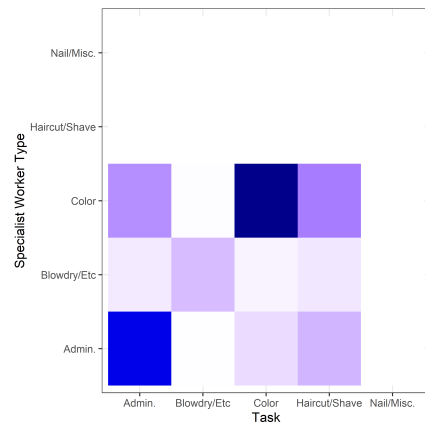
**(c)** Task Mix 2 (Haircut/Shave Task)

**(d)** Task Mix 3 (Haircut/Shave Task)

**Note:** Each panel plots the model and observed relationship between price and different firm variables. Dots represent individual firms, while lines are Loess smoothed fitted curves.

**Figure A7:** Utilization of Task Assignment Data



**(a)** Observed

**(b)** Estimated

**Note:** Darker colors indicate a higher fraction of total work at the salon. The model in this paper takes in establishment-level data about the task assignments of employees with unknown skills (Panel A) and returns the task assignments of worker types with known skills (Panel B).