Policymaking Precision and Electoral Accountability

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Abstract

A voter who dislikes uncertainty would be willing to sacrifice some ideological similarity with an elected official for greater precision in policymaking. This form of competence – policymaking precision – differs from traditional conceptions of politician competence in that it has implications for spatial policy choice. Accordingly, our paper explores the mechanisms and consequences of voters holding politicians accountable based on their competence at executing the policies they intend to. We find that the voter cannot induce an incumbent to choose a policy other than her ideal point in any equilibrium. The voter can, however, induce incumbents to put forward effort to improve their policymaking precision. Since policymaking precision provides a benefit to all risk-averse actors who care about policy, it is no surprise to see term-limited incumbents still make investments in their competence. Less expected is that the possibility of being retained despite being a high-variance lawmaker exerts downward pressure on incumbents' first-period investments in policymaking precision. Our model yields predictions about the variance of policies across periods as well as comparative statics about retention probabilities, all of which provide comparisons for and reinterpretations of extant findings in the electoral accountability literature.

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1 Introduction

Shepsle (1972) observed that the "act of voting, like gambling or purchasing insurance, is one involving 'risky' alternatives" (p. 560). When studying ways in which voters might overcome this risk, the electoral accountability literature has traditionally focused on two mechanisms. First, based on the spatial model of politics, scholars have examined whether voters can overcome adverse selection of politicians with respect to potentially dissimilar policy preferences. Second, much work has focused on voters' ability to select more competent politicians, where competence is a valence attribute orthogonal to policy preferences.

We take a hybrid approach in this paper, engaging with both competence and spatial preferences. While all policy-motivated voters evaluate their ideological similarity to candidates, risk-averse voters should also consider how precisely candidates are able to implement a chosen policy. This paper presents a model that incorporates the trade-off voters face between ideological congruence and a particular form of competence, namely, policymaking precision. We ask whether and how voters might select politicians who are able to implement their chosen policies more precisely.

In our model, voters develop a retention rule on the basis of policy and competence, defined as policymaking precision that reduces the variance of policy outcomes for a given implemented policy. A voter observes how close the realized policy hews to a particular policy, updates her beliefs about the incumbent's policy-making ability (i.e., precision), and retains the incumbent if and only if the realized policy is sufficiently close to the intended policy. Reminiscent of Fearon (1999) insight that selecting on competence interferes with voters' ability to enforce a given policy stance, we find that the only announced policy for which the voter can hold the politician accountable is the politician's ideal point.

While voters in our model cannot in induce policy convergence in equilibrium, they are able to select politicians on the basis of competence and, in so doing, incentivize politicians to invest in competence. In contrast to rent-seeking environments, policy-motivated politicians prefer competent office-holders, even when they are term-limited or when they lose. This motivates politicians to invest in competence in every period but also has a somewhat unexpected, additional effect. Investing in competence increases the likelihood an incumbent is retained despite being a low-precision type, which exerts downward pressure on their incentive to invest in their own policymaking precision.

In the next section, we situate this paper more fully within the literature on electoral accountability and discuss work on risk-aversion in political behavior. We then present the details of the model and proceed to derive and analyze the equilibrium, distilling predictions about policy precision across periods and comparative statics about the probability of incumbent retention. We briefly explore an extension involving durable investments in policymaking precision before concluding with a discussion of directions in which the remarkably tractable model below may be taken.

2 Literature Review

Studies on risk attitudes across a variety of disciplines depict individuals as being risk averse across a wide swath of behaviors. Holt and Laury (2002, 2005) conduct a series of experiments where participants are given a series of lottery choices for real cash payments and find that participants are largely risk averse even for small payments. As cash payments increase so does the level of risk aversion. Evidence of risk aversion is not confined to laboratory settings. Studies have shown individuals exhibit risk aversion in the context of insurance markets (Binswanger 1980, Cohen & Einav 2007, Bosch-Domènech & Silvestre 1999), financial decisions (Nelson 2015), and game shows (Gertner 1993, Metrick 1995).

Research in political science echoes these findings. Prior work has shown voters to be predominantly, but not exclusively, risk averse and, further, shown that voters' risk attitudes affect their vote choice. Risk-averse individuals are more likely to support incumbents over challengers (Morgenstern & Zechmeister 2001, Kam & Simas 2012), constituting a key determinant of the incumbency advantage (Eckles, Kam, Maestas & Schaffner 2014). Nadeau, Martin & Blais (1999) offers evidence that risk-averse voters give more weight to the perceived possibility of the worst case scenario while risk-loving voters engage in cost-benefit analysis when deciding among candidates. Tomz & Van Houweling (2009) finds that only risk-neutral or risk-loving voters do not punish candidates who take ambiguous policy positions. Individual's risk orientation also affects how they respond to policy frames, where those with higher levels of risk tolerance are more likely to select probabilistic policy outcomes over certain ones (Kam & Simas 2010). Bartels (1986) shows that voter's uncertainty of candidate positions has an effect similar to that of ideological distance on vote choice.

It stands to reason that members of Congress may also display risk-aversion. Lawmakers face uncertainty over the policies they choose to enact as well as over voters' responses to those already-uncertain policy outcomes. When taking actions that affect their electoral prospects members frequently engage in "worst case analysis" (Mayhew 1974, Fenno 1978, Arnold 1990). Such thinking often entails a greater desire to avoid blame for negative outcomes than to receive a reward for positive outcomes. In-so-far as members of Congress are both risk-averse and policy-motivated, high-variance policymaking will negatively impact them as much as it does voters.

Models of political accountability have proven to be a highly effective tool with which to study the consequences of uncertainty, and thus risk, in policymaking and elections. In this class of models, voters deciding whether to retain an incumbent politician or replace her with a challenger glean information about the incumbent's type that is relevant to future payoffs from actions or outcomes observed under the incumbent's previous governance. Work on electoral accountability may feature moral hazard with regards to politicians' actions and adverse selection when the incumbent possesses more knowledge about her type than the voter possesses. Politicians may also behave opportunistically and voters must sanction/reward behavior to limit shirking. Multi-task or pandering models, in the spirit of Holmstrom & Milgrom (1991), highlight the tension between expending effort towards activities that are objectively good for voters versus activities that are best for being re-elected (Ashworth 2005, Ashworth & Bueno de Mesquita 2006, Besley 2006, Canes-Wrone, Herron & Shotts 2001, Daley & Snowberg 2009).

In previous work, politicians may vary in either their competence or preferences, with voters seeking to select more capable or more ideologically-aligned politicians. Scholars have usually considered accountability with respect to competence in a rent-seeking environment, where politicians' type reflects their ability to produce valence outcomes or integrity (Berganza 2000, Fox & Shotts 2009) conditional on some investment of effort. Meanwhile, the study of voters' selection of ideologically-aligned candidates tended to invoke models of spatial politics. The traditional silos of rent-seeking contexts for studying competence-based selection and spatial-politics contexts for studying preferences-based selection have given way to some "mixing and matching" of assumptions. For example, well-motivated studies of accountability over politicians' preferences regarding the provision of valence goods in rent-seeking environments, such as the desire to partake in corruption, have produced new insights and profitably pushed the accountability literature forward.

By way of studying the consequences of risk-averse voters' desire for policymaking precision, this paper also adds to the accountability literature by modeling accountability with respect to politician competence in a spatial-politics context. Not all of the results below require a spatial context. Specifically, the final-period investment and especially the downward pressure on first-period investment only require the politician to benefit from the same competence from which the voter benefits. A spatial setting is particularly natural for grounding these assumptions, though, especially if motivated by voters and politicians sharing risk aversion with respect to policy outcomes. Furthermore, some of the results rely completely on our invoking a spatial context, specifically the implications for expected policy location or incumbent retention as a function of observed policy location.

Fearon (1999) focuses on the trade off between sanctioning and selecting politicians in a spatial setting. As long as voters believe there is any chance a politician is a good type, voters focus on selecting the politicians with similar preferences rather than sanctioning the policy outcome to induce implementation of the voter's ideal point. As voters become less informed about politicians' types, politicians are more likely to respond to electoral incentives, improving voter welfare (Besley 2006, Ashworth & Bueno de Mesquita 2014). In Banks & Sundaram (1998), politicians vary in their cost at providing public goods and lower types (higher cost) provide more public goods than higher-types in order to be re-elected.

Duggan & Martinelli (2017) show that in a two-period adverse selection selection model all politician types have electoral incentives to select the median ideal policy. This result generally holds in the infinite horizon setting (Duggan et al. 2000, Banks & Duggan 2008). Convergence depends crucially on the fact voters cannot perfectly observe politician type allowing extreme candidates to behave like moderate ones. Politicians only have incentives to represent voters if it is possible to obscure their own preferences for voters. If their true preferences are known with certainty than voters will select the politician with the closest ideal policy. Somewhat paradoxically to the folk theory of democracy, "asymmetric information can facilitate responsive democracy, where as full transparency leads to shirking" (Duggan & Martinelli 2017, 935).

3 Model Preliminaries

The model has three players: an incumbent (I), a challenger (C), and a voter (V). The game consists of two periods, $t = 1, 2$, each of which entail policymaking that results in a realized policy. The realized policy is a function of the policy a politician intends to implement, the effort a politician may invest in her own policymaking precision, and a politician's type.

The politician $p \in \{I, C\}$ holding office in period t selects an intended policy, $x_{p,t} \in \mathbb{R}$, and invests effort in her own policymaking precision, $e_{p,t} \geq 0$. Each politician has a type, θ_p, which is constant across periods and which we often refer to as the politician's competence. All players know only that $\theta_p \sim Gamma(\alpha, \lambda)$, for each $p \in \{I, C\}$, where $\alpha > 1$ is the shape parameter and $\lambda > 0$ is the rate (inverse-scale parameter).[1]

[1] Ashworth & Bueno de Mesquita (2017, p. 1376) present a compelling argument in favor of assuming symmetric uncertainty around competence in accountability models, vis-à-vis policy preferences, for example, where an assumption of asymmetric information makes more sense.

Given an intended policy, $x_{p,t}$, and given effort, $e_{p,t}$, the realized policy is $y_{p,t} \sim \mathcal{N}(x_{p,t}, \tau_{p,t})$, where $\tau_{p,t} = e_{p,t}\theta_p$ and represents the implementing politician's policymaking precision (i.e., the inverse of the variance). All players incur payoffs from the realized $y_{I,1}$, and the voter decides at the end of the first period to retain I or replace her with C. In the second period, the winning politician selects $x_{p,2}$, the players incur payoffs from the realized $y_{p,2}$, and the game ends.

All players have an ideal point $\hat{x}_i \in \mathbb{R}$, $i = I, C, V$ and incur quadratic loss in the distance from their ideal points to realized policies. The cost of effort is linear in the amount put forward, $e_{p,t}$. None of the players discount second-period payoffs.

We consider two cases of ideal points, one of which embodies a "primary challenger" (1.a), the other of which embodies a "general challenger" (1.b), and both of which provide simplifications that streamline the presentation below.

Assumption 1. *Let the ideal points be arrayed in one of two cases:*

1.a: $\hat{x}_I = \hat{x}_C > \hat{x}_V = 0$,

1.b: $\hat{x}_I = -\hat{x}_C > \hat{x}_V = 0$.

In addition to the policy component of their preferences, politicians also receive a benefit $\beta > 0$ from holding office. We assume β is sufficiently large to guarantee the incumbent seeks to be reelected.[2]

Assumption 2. $\beta > \sqrt{\frac{\lambda}{\alpha-1}}$.

The ideal points and preferences are all common knowledge. The voter observes only $y_{I,1}$ when making her retention decision, but neither the policy choice, $x_{I,1}$, nor the effort put forth, $e_{I,1}$. Throughout, we employ the solution concept of weak perfect Bayesian equilibrium in pure strategies (henceforth, "equilibrium"). In the context of accountability models, the requirement of consistency of beliefs ensures that unobserved policies $x_{p,t}$ and precision investments $e_{p,t}$ are effectively known in equilibrium.

3.1 Discussion of the model's features

The voter's risk aversion is built in with the quadratic loss utility. While the politicians' utility functions are quasi-linear, they are still risk averse with regards to policy. The multiplicative form for the effect of effort on precision serves as an Inada condition of sorts, in that it will always be optimal for the politician to make a strictly positive investment in effort.

[2]We require some version of this assumption to ensure the existence of a pure-strategy equilibrium around the incumbent's policy decision (see the remark following the proof of Proposition 1 for additional detail).

We assume the politicians have, *ex ante*, the same distribution over possible policymaking precision. This is intended to capture the notion that, at the start of the game, the incumbent was drawn from the same distribution as the challenger.[3] Further, all players share the same prior beliefs over types.

3.2 Useful results from distribution theory

Before beginning the analysis in earnest, we state a few particularly relevant results. We will make repeated use of these calculations. They also demonstrate the usefulness of several of the modeling choices we invoked above. All proofs may be found in Appendix A.

The gamma distribution is closed under multiplication by scalars. If $\theta \sim Gamma(\alpha, \lambda)$, then $e \cdot \theta \sim Gamma(\alpha, \lambda/e)$. The gamma distribution also serves as a conjugate prior for the unknown precision underlying draws from a normal likelihood. As such, posterior beliefs about the precision will also be in the family of gamma distributions, with updated parameters.

Lemma 1. *Given an observed $y_{I,1}$ realized from a choice of first-period policy $x_{I,1}$ and first-period investment $e_{I,1}$, $\theta_I|(y_{I,1}; x_{I,1}, e_{I,1}) \sim Gamma(\alpha', \lambda' e_{I,1})$, where $\alpha' = \alpha + 1/2$ and $\lambda' = \lambda/e_{I,1} + (y_{I,1} - x_{I,1})^2/2$.*

Further, the posterior predictive (updated likelihood) distribution will be in the family of t-distributions. Throughout the paper, let $p_\nu(y|\mu, \sigma^2)$ denote the density and $P_\nu(y|\mu, \sigma^2)$ the distribution function of a random variable y that is distributed according to a t-distribution with ν degrees of freedom, location parameter μ, and scale parameter σ, i.e., $y \sim t_\nu(\mu, \sigma^2)$. Note that σ is not the standard deviation but rather the scale parameter for a non-standard t-distribution. The variance would be $\sigma^2 \frac{\nu}{\nu-2}$, leading to the result in Lemma 2.

Lemma 2. *If $y|(x;\tau) \sim \mathcal{N}(x, \tau)$, and $\tau \sim Gamma(\alpha, \lambda)$, then $y|x \sim t_{2\alpha}(x, \lambda/\alpha)$, i.e., a t-distribution with 2α degrees of freedom, location parameter x, and scale parameter $\sqrt{\lambda/\alpha}$, with a mean of x and variance $\lambda/(\alpha - 1)$.*

These closed-form expressions for the moments of the posterior distributions for the unknown parameters are particularly helpful for calculating expected utility with respect to realized policies. Players will calculate the expected squared distance to their ideal point $\hat{x}_i$ of a realized policy centered

[3] This could be fruitfully relaxed in future iterations.

around x. If y is a random variable with mean x, then

$$\begin{aligned}\mathbb{E}[(y-\hat{x}_i)^2] =& \mathbb{E}[y^2] - 2\mathbb{E}[y]\cdot\hat{x}_i + \hat{x}_i^2\\ =& \mathbb{E}[y^2] - \mathbb{E}[y]^2 + \mathbb{E}[y]^2 - 2\mathbb{E}[y]\cdot\hat{x}_i + \hat{x}^2\\ =& Var(y) + (x-\hat{x}_i)^2.\end{aligned}$$

4 Equilibrium Analysis

We first characterize optimal politician behavior in the second period. Given second-period play, we may then determine the voter's retention rule, and given this retention rule, we are able to formulate the incumbent's first-period maximization problem. Finally, we address consistency of beliefs, ensuring that the voter's expectations of the incumbent's first-period behavior are correct in equilibrium.

Any politician setting policy in the second period will optimally implement her own ideal point, i.e., $x^*_{p,2} = \hat{x}_p$, $p \in \{I, C\}$. As effort towards policy precision benefits all players with risk-averse attitudes towards policy, it will not be optimal for a politician to exert zero effort in the second period. Applying Lemma 2, we note $y|(x,e) \sim t_{2\alpha}(x, \lambda/(\alpha e))$, and hence $\mathbb{E}(y|x,e) = x$, and $Var(y|x,e) = \frac{\lambda}{(\alpha-1)e}$. The second-period officeholder determines the optimal investment in policymaking precision by solving:

$$\begin{aligned}&\max_{e_{p,2}\in\mathbb{R}_+} -\left[Var(y|\hat{x}_p, e_{p,2}) + (\hat{x}_p - \hat{x}_p)^2\right] - e_{p,2} \Rightarrow\\ &\max_{e_{p,2}\in\mathbb{R}_+} -\frac{\lambda}{(\alpha-1)e_{p,2}} - e_{p,2} \Rightarrow\\ &e^*_{p,2} = \sqrt{\frac{\lambda}{\alpha-1}}.\end{aligned} \tag{1}$$

The second-period officeholder's equilibrium policymaking variance is given by the same quantity, and her value function is $-2\sqrt{\lambda/(\alpha-1)}$. While there are no electoral benefits of costly investment in the final stage of the game, it is still of benefit to the second-period officeholder.

The voter then calculates a payoff of $-\sqrt{\lambda/(\alpha-1)} - (\hat{x}_C - \hat{x}_V)^2$ from electing the challenger. If the incumbent is in office in period 2, however, the voter will have observed a policymaking outcome in period 1 and updated her beliefs about the incumbent's type. Since both the incumbent and the voter shared the same uncertainty about the incumbent's type, the incumbent will update in precisely the same manner as the voter, in a manner specified by in the next result.

Applying Lemmas 1 and 2, as well as the facts that the incumbent will optimally choose $x_{I,2} = \hat{x}_I$

and a level of effort denoted $e_{I,2}$, the incumbent's second period policy has a density given by:

$$p_{2\alpha+1}\left(y|\hat{x}_I, \frac{\lambda + e_{I,1}(y_{I,1} - x_{I,1})^2/2}{e_{I,2}(\alpha + 1/2)}\right).$$

The second-period policy realization thus has a mean of $\hat{x}_I$ and a variance of $\frac{\lambda + e_{I,1}(y_{I,1} - x_{I,1})^2/2}{e_{I,2}(\alpha - 1/2)}$, the scale parameter multiplied by $\frac{\nu'}{\nu'-2}$, where $\nu' = 2(\alpha + 1/2)$. By the formula for the optimal second-period investment in precision in (1), it follows that:

$$e^*_{I,2} = \sqrt{\frac{\lambda + e_{I,1}(y_{I,1} - x_{I,1})^2/2}{\alpha - 1/2}},$$

again resulting in an equilibrium variance of the same quantity.

The voter then retains the incumbent if:

$$-\sqrt{\frac{\lambda + e_{I,1}(y_{I,1} - x_{I,1})^2/2}{\alpha - 1/2}} - (\hat{x}_I - \hat{x}_V)^2 \geq -\sqrt{\frac{\lambda}{\alpha - 1}} - (\hat{x}_C - \hat{x}_V)^2.$$

By Assumption 1, $(\hat{x}_I - \hat{x}_V)^2 = \hat{(}x_C - \hat{x}_V)^2$. The voter thus retains the incumbent as long as the observed $y_{I,1}$ satisfies:

$$e_{I,1}(y_{I,1} - x_{I,1})^2 \leq \frac{\lambda}{\alpha - 1}. \tag{2}$$

Let this specify $y^*(x_{I,1}, e_{I,1})$ and $y_*(x_{I,1}, e_{I,1})$, the bounds of a retention region that is symmetric about $x_{I,1}$. Note that the greater first-period effort, the more closely the policy realization must hew to the implemented policy for the voter to retain the incumbent. Investment inflates the precision of the incumbent, so the voter must deflate her inference.

The incumbent takes the retention region as given, i.e., not as a function of her first-period choices. Indeed, her first-period choices must be optimal given a supposed retention region of $[\underline{y}, \overline{y}]$, a region which must in equilibrium equal $[y_*, y^*]$. The probability with which the incumbent believes she will be retained is thus $P_{2\alpha}(\overline{y}|x_{I,1}, \lambda/(\alpha e_{I,1})) - P_{2\alpha}(\underline{y}|x_{I,1}, \lambda/(\alpha e_{I,1}))$.

The incumbent then solves (omitting constant terms that do not affect the maximization problem):

$$
\begin{aligned}
&\max_{(x_{I,1},e_{I,1})\in\mathbb{R}\times\mathbb{R}_+} -(x_{I,1}-\hat{x}_I)^2 - \frac{\lambda}{(\alpha-1)e_{I,1}} - e_{I,1} + \\
&\left[P_{2\alpha}(\overline{y}|x_{I,1},\lambda/(\alpha e_{I,1})) - P_{2\alpha}(\underline{y}|x_{I,1},\lambda/(\alpha e_{I,1}))\right] * \\
&\left(\beta + (\hat{x}_I - \hat{x}_C)^2 + \sqrt{\frac{\lambda}{\alpha-1}} + \mathbb{E}\left[-2\sqrt{\frac{\lambda + e_{I,1}(y_{I,1}-x_{I,1})^2/2}{\alpha - 1/2}}\,\middle|\, y_{I,1}\in[\underline{y},\overline{y}]\right]\right).
\end{aligned}
$$

The expectation term is the incumbent's second-period value function, adjusted for the fact that $y_{I,1}$, which enters into her updated beliefs about her latent type, was sufficiently close to $x_{I,1}$ for her to have been retained.

The solution to this problem defines optimal $(x^*_{I,1}, e^*_{I,1})$ that are functions of the bounds of the retention region. The voter's problem implicitly defined optimal bounds for retention as a function of first-period policy location, $x_{I,1}$, and effort, $e_{I,1}$. Equilibrium then requires that these values are fixed points of the composed best-response functions.

Although we allow for arbitrary policies, the incumbent's optimal intended policy is her ideal point in the first period. This result may seem familiar to the insight from Fearon (1999) that the possibility of holding elected officials accountable precludes any residual enforcement of behavior that is irrelevant to drawing inference about politician type. In this setting, the incumbent's choice of policy certainly affects inferences about her type. The incumbent's policy motivation, however, leads any proposed equilibrium built around a policy $x_{I,1} \neq \hat{x}_I$ to unravel. We explore this further after the following proposition, which speaks to optimal intended policies across both periods.

Proposition 1. *In both periods, the politician in office optimally seeks to implement her ideal point as the policy.*

Players' inference about the incumbents latent competence take into account the intended first-period policy. In equilibrium, then, the accountability mechanism cannot induce the incumbent away from attempting to implement her ideal point, and this holds for any $e_{I,1} > 0$. The first proposition states that no politician in this model will intend – in equilibrium – to implement anything other than her most-preferred policy.

The first-order condition corresponding to the maximization problem above compares straightforwardly, albeit equivocally, to single-period considerations. The first part of the objective function is simply the maximization problem that a politician limited to a single term would face. Our main result establishes that, while first-period investment will be greater if the incumbent were limited to just one term, the possibility of retention exerts both upward and downward pressure on the extent of this increased investment.

Proposition 2. *First-period incumbent investment with the possibility of retention will be higher than without the possibility of retention. The extent to which first-period investment is greater increases in the size of the office-holding benefit, the ideological distance between the incumbent and the challenger, and the expected variance of the challenger's policy realizations; it decreases in the expected variance of the incumbent's own policy realizations.*

The possibility of retention exerts upward pressure on the incumbent's first-period effort because it increases the probability of the policy realization lying in the retention region, and she seeks to be retained in order to reap the office-holding benefit, the ability to implement her own ideal policy rather than the challenger's, and to avoid the variance in policy realizations associated with the challenger. Yet the incumbent understands that her effort increases the chance that she will be retained despite being of low type, i.e., low policymaking precision. The incentive effect suffers from the benefit the incumbent derives from being of high type. The downward pressure on the incentive effect from the incumbent's desire to be of high type may be counter-intuitive for at least two reasons. First, effort by the incumbent that enhances reelection prospects in other accountability settings does not confer any cost other than the cost of the effort itself. In our setting, however, because the incumbent herself benefits from the same selection effects that the voter does, she prefers not to inflate the estimate of her quality simply to improve her chance of being reelected. Second, it may seem odd that a politician would reduce their own reelection prospects for the sake of being retained only if high enough quality. If we truly believe this lacks verisimilitude, then the implication is that β is quite large relative to policy motivations, in which case the incentive effect of reelection on effort would be strong.

[Additional analysis section(s) to come.]

5 Discussion

This paper presents a spatial model where voters hold politicians accountable for their competence at executing the policies they intend to. Voters, while knowledgeable about politicians' ideal points, are uncertain about their competence and use realized policies as the basis for evaluating policymaking precision relative to politicians' implemented policies. We find that voters use a retention rule that serves as a selection mechanism on the basis of politician competence, though the incumbent's own desire to be retained only when she is a high type tempers the effectiveness of this mechanism.

While politician errors might sometimes work in the voter's favor, the realized policy is just as likely to work against them. Voters therefore are willing to trade this policy loss for less erratic politicians. This result stands in stark contrast to the majority of electoral accountability that show politicians implementing the voter's ideal policy or some policy between the voter's and politician's ideal points. Electoral incentives may encourage politicians to put forward effort to improve their policymaking precision, and indeed politician's may put forth this sort of effort even without the possibility of retention. Other models feature the unrealistic prediction that term-limited politicians expend zero effort. That is not the case in this model, where investments in precision make all players in our model – the voters, elected politicians, and losing politicians – better off.

While at first glance the inability of voters to induce congruence suggests poor government accountability, policymaking precision is as important for government performance as the policies that incumbents promise. If voters are risk averse and care about policy outcomes, then they are best served implementing an accountability mechanism on the basis of policymaking precision. This result perhaps reconciles high incumbent reelection rates with increasingly polarized politicians.

Finally, it is worth noting that our model is fully compatible with citizen-candidate models of electoral competition. As with all accountability models, we assume politicians are citizen candidates in the sense that binding platforms are not possible. Further, we assume ideal points are known, as they would be in any citizen-candidate model – an assumption not common in accountability models with spatial preferences. Moreover, and seemingly unique to our model of accountability with spatial preferences, all politicians implement their ideal policy in all periods. Such behavior is the premise of citizen-candidate models of electoral competition. The citizen-candidate class of models has yielded a number of insights, especially around entry into elections. Our accountability model could be merged with such models to speak to both self-selection by politicians into campaigns and voters' selection of incumbents in subsequent elections.

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A Proofs of In-Text Results

Proof of Lemma 1. The density of $y|(x_{I,1}, e_{I,1})$ is

$$\sqrt{\frac{\tau_{I,1}}{2\pi}} e^{-\tau_{I,1}(y-x_{I,1})^2/2} \frac{(\lambda/e_{I,1})^\alpha}{\Gamma(\alpha)} \tau_{I,1}^{\alpha-1} e^{-(\lambda/e_{I,1})\tau_{I,1}}.$$

The distribution of $\tau_{I,1}|y_{I,1}$ (where $y_{I,1}$ is a realization of $y|x_{I,1}$) is then

$$\propto \tau_{I,1}^{\alpha-1/2} e^{-\left(\lambda/e_{I,1}+(y_{I,1}-x_{I,1})^2/2\right)\tau_{I,1}},$$

from which we observe $\tau_{I,1}|y_{I,1}$ must be distributed $Gamma(\alpha+1/2, \lambda/e_{I,1}+(y_{I,1}-x_{I,1})^2/2)$. Since $\tau_{I,1} = \theta_I e_{I,1}$, $\theta_I|y_{I,1} \sim Gamma(\alpha+1/2, \lambda+e_{I,1}(y_{I,1}-x_{I,1})^2/2)$. ■

Proof of Lemma 2. Recall the density of $y|(\tilde{x};\tau)$ is $\sqrt{\frac{\tau}{2\pi}} e^{-\tau(y-x)^2/2}$, and the density of τ is $\frac{\lambda^\alpha}{\Gamma(\alpha)}\tau^{\alpha-1}e^{-\lambda\tau}$. Integrating τ out, the density of $y|\tilde{x}$ is

$$\begin{aligned}
&\int_0^\infty \sqrt{\frac{\tau}{2\pi}} e^{-\tau(y-\tilde{x})^2/2} \frac{\lambda^\alpha}{\Gamma(\alpha)} \tau^{\alpha-1} e^{-\lambda\tau} d\tau \\
&\frac{1}{\sqrt{2\pi}} \frac{\lambda^\alpha}{\Gamma(\alpha)} \int_0^\infty \tau^{\alpha-1/2} e^{-\tau(\lambda+(y-\tilde{x})^2/2)} d\tau \\
&\frac{1}{\sqrt{2\pi}} \frac{\lambda^\alpha}{\Gamma(\alpha)} \frac{\Gamma(\alpha+1/2)}{(\lambda+(y-\tilde{x})^2/2)^{\alpha+1/2}} \\
&\frac{\Gamma(\alpha+1/2)}{\sqrt{2\pi\lambda}\,\Gamma(\alpha)} (1+(y-\tilde{x})^2/(2\lambda))^{-(\alpha+1/2)} \\
&(\text{letting } \nu = 2\alpha, \mu = \tilde{x}, \sigma^2 = \lambda/\alpha) \\
&\frac{\Gamma((\nu+1)/2)}{\sqrt{2\alpha\sigma^2\pi}\,\Gamma(\nu/2)} (1+(y-\tilde{x})^2/(2\alpha\sigma^2))^{-(\nu+1)/2} \\
&\frac{1}{\sqrt{\nu}\sigma} \frac{\Gamma((\nu+1)/2)}{\Gamma(1/2)\Gamma(\nu/2)} \left(1+\left(\frac{y-\tilde{x}}{\sqrt{\nu}\sigma}\right)^2\right)^{-(\nu+1)/2}
\end{aligned}$$

This is the density of a random variable y distributed according to a t-distribution with 2α degrees of freedom, location parameter $\tilde{x}$, and scale parameter λ/α. It follows that $\mathbb{E}(y) = \tilde{x}$ and $\mathbb{E}(y-\tilde{x})^2 = \frac{\nu}{\nu-2}\sigma^2 = \frac{2\alpha}{2\alpha-2}\frac{\lambda}{\alpha} = \frac{\lambda}{\alpha-1}$.[4] ■

[4] ***Casella & Berger cite needed, or Severini, or Ferguson even, maybe.

Lemma 3. *For $q \in [0,1]$:*

$$\mathbb{E}\left[\left(\frac{\lambda + e_{I,1}(y_{I,1}-x_{I,1})^2/2}{\alpha - 1/2}\right)^q \middle| y_{I,1} \in [\underline{y}, \overline{y}]\right] \cdot$$
$$\left[P_{2\alpha}(\overline{y}|x_{I,1}, \lambda/(\alpha e_{I,1})) - P_{2\alpha}(\underline{y}|x_{I,1}, \lambda/(\alpha e_{I,1}))\right]$$
$$= \left(\frac{\lambda}{\alpha - 1/2}\right)^q \frac{\Gamma(\alpha+1/2)}{\Gamma(\alpha)} \frac{\Gamma(\tilde{\alpha})}{\Gamma(\tilde{\alpha}+1/2)} \left[P_{2\tilde{\alpha}}(\overline{y}|x_{I,1}, \lambda/(\tilde{\alpha} e_{I,1})) - P_{2\tilde{\alpha}}(\underline{y}|x_{I,1}, \lambda/(\tilde{}_{I,1}))\right],$$

where $\tilde{\alpha} = \alpha - q$.

For $q = \frac{1}{2}$, $\sqrt{\frac{\lambda}{\alpha-1/2}} \frac{\Gamma(\alpha+1/2)}{\Gamma(\alpha)} \frac{\Gamma(\alpha-1/2)}{\Gamma(\alpha)} = \sqrt{\frac{\lambda}{\alpha-1/2}} (\alpha - 1/2) \frac{\Gamma(\alpha-1/2)}{\Gamma(\alpha)} \frac{\Gamma(\alpha-1/2)}{\Gamma(\alpha)}$
$= \sqrt{\lambda(\alpha-1/2)} \left(\frac{\Gamma(\alpha-1/2)}{\Gamma(\alpha)}\right)^2$.

Proof of Lemma 3.

$$\int_{\underline{y}}^{\overline{y}} \left(\frac{\lambda + \tilde{e}(\tilde{y}-\tilde{x})/2}{\alpha - 1/2}\right)^q \int_0^\infty \sqrt{\frac{\theta\tilde{e}}{2\pi}} e^{-\theta\tilde{e}(\tilde{y}-\tilde{x})^2/2} \frac{\lambda^\alpha}{\Gamma(\alpha)} \theta^{\alpha-1} e^{-\lambda\theta} d\theta d\tilde{y}$$
$$= \int_{\underline{y}}^{\overline{y}} \left(\frac{\lambda + \tilde{e}(\tilde{y}-\tilde{x})/2}{\alpha - 1/2}\right)^q \frac{\lambda^\alpha \sqrt{\tilde{e}}}{\sqrt{2\pi}\Gamma(\alpha)} \Gamma(\alpha+1/2)(\lambda + \tilde{e}(\tilde{y}-\tilde{x})^2/2)^{-(\alpha+1/2)} d\tilde{y}$$
$$= \int_{\underline{y}}^{\overline{y}} \left(\frac{1}{\alpha - 1/2}\right)^q \frac{\lambda^\alpha \sqrt{\tilde{e}}}{\sqrt{2\pi}\Gamma(\alpha)} \Gamma(\alpha+1/2)(\lambda + \tilde{e}(\tilde{y}-\tilde{x})^2/2)^{-(\alpha+1/2-q)} d\tilde{y}$$
$$= \int_{\underline{y}}^{\overline{y}} \left(\frac{1}{\alpha - 1/2}\right)^q \frac{\lambda^\alpha \sqrt{\tilde{e}} \lambda^{-(\alpha+1/2-q)}}{\sqrt{2\pi}\Gamma(\alpha)} \Gamma(\alpha+1/2)(1 + \tilde{e}(\tilde{y}-\tilde{x})^2/(2\lambda))^{-(\alpha+1/2-q)} d\tilde{y}$$
$$= \left(\frac{\lambda}{\alpha - 1/2}\right)^q \frac{\Gamma(\alpha+1/2)}{\Gamma(\alpha)} \int_{\underline{y}}^{\overline{y}} \frac{\sqrt{\tilde{e}}}{\sqrt{2\pi\lambda}} (1 + \tilde{e}(\tilde{y}-\tilde{x})^2/(2\lambda))^{-(\alpha+1/2-q)} d\tilde{y}$$
$$= \left(\frac{\lambda}{\alpha - 1/2}\right)^q \frac{\Gamma(\alpha+1/2)}{\Gamma(\alpha)} \frac{\Gamma(\alpha-q)}{\Gamma(\alpha-q+1/2)} \cdot$$
$$\left[P_{2(\alpha-q)}(\overline{y}|x_{I,1}, \lambda/((\alpha-q)e_{I,1})) - P_{2(\alpha-q)}(\underline{y}|x_{I,1}, \lambda/((\alpha-q)e_{I,1}))\right]$$

Setting $\tilde{\alpha} = \alpha - q$ yields the desired result. The derivations for $q = 1/2$ follow from simplifications using the definition of the Gamma function, viz., $\Gamma(z) = (z-1)\Gamma(z-1)$. ■

Remark Lemma 3 establishes a rather remarkable simplification of the expectation term in the incumbent's maximization problem, leading to the following, equivalent maximization problem:

$$
\begin{aligned}
&\max_{(x_{I,1},e_{I,1})\in\mathbb{R}\times\mathbb{R}_+} -(x_{I,1}-\hat{x}_I)^2 - \frac{\lambda}{(\alpha-1)e_{I,1}} - e_{I,1} + \\
&\left[P_{2\alpha}(\overline{y}|x_{I,1},\lambda/(\alpha e_{I,1})) - P_{2\alpha}(\underline{y}|x_{I,1},\lambda/(\alpha e_{I,1}))\right] \cdot \left(\beta + (\hat{x}_I - \hat{x}_C)^2 + \sqrt{\frac{\lambda}{\alpha-1}}\right) - \\
&\left[P_{2\tilde{\alpha}}(\overline{y}|x_{I,1},\lambda/(\tilde{\alpha} e_{I,1})) - P_{2\tilde{\alpha}}(\underline{y}|x_{I,1},\lambda/(\tilde{\alpha} e_{I,1}))\right] \cdot \left(2\sqrt{\lambda(\alpha-1/2)}\left(\frac{\Gamma(\alpha-1/2)}{\Gamma(\alpha)}\right)^2\right)
\end{aligned}
$$

where $\tilde{\alpha} = \alpha - 1/2$.

Lemma 4. *Let* $\tilde{\alpha} = \alpha - 1/2$.

$$
\frac{p_{2\tilde{\alpha}}\left(y; x_{I,1}, \lambda/(\tilde{\alpha} e_{I,1})\right)}{p_{2\alpha}\left(y; x_{I,1}, \lambda/(\alpha e_{I,1})\right)} = \frac{\left(\frac{\Gamma(\alpha)}{\Gamma(\alpha-1/2)}\right)^2}{\alpha - 1/2}\sqrt{1 + \frac{e_{I,1}\cdot(y-x_{I,1})^2}{2\lambda}}.
$$

Proof of Lemma 4. The identify follows directly from the closed-form expression for the density of the t-distribution and the fact that $\Gamma(\alpha+1/2) = (\alpha-1/2)\Gamma(\alpha-1/2)$.

$$
\begin{aligned}
&\frac{p_{2\tilde{\alpha}}\left(y; x_{I,1}, \lambda/(\tilde{\alpha} e_{I,1})\right)}{p_{2\alpha}\left(y; x_{I,1}, \lambda/(\alpha e_{I,1})\right)} \\
&= \frac{\frac{\sqrt{e_{I,1}}}{\sqrt{2\pi\lambda}}\frac{\Gamma(\alpha)}{\Gamma(\alpha-1/2)}\left(1 + \frac{e_{I,1}\cdot(y-x_{I,1})^2}{2\lambda}\right)^{-\alpha}}{\frac{\sqrt{e_{I,1}}}{\sqrt{2\pi\lambda}}\frac{\Gamma(\alpha+1/2)}{\Gamma(\alpha)}\left(1 + \frac{e_{I,1}\cdot(y-x_{I,1})^2}{2\lambda}\right)^{-(\alpha+1/2)}}
\end{aligned}
$$

■

Remark Consider the expression:

$$
B \cdot p_{2\alpha}\left(y; x_{I,1}, \lambda/(\alpha e_{I,1})\right) - 2K \cdot p_{2\tilde{\alpha}}\left(y; x_{I,1}, \lambda/(\tilde{\alpha} e_{I,1})\right),
$$

where $\tilde{\alpha} = \alpha - 1/2$, $B = \beta + (\hat{x}_I - \hat{x}_C)^2 + \sqrt{\frac{\lambda}{\alpha-1}}$, and $K = \sqrt{\lambda(\alpha-1/2)}\left(\frac{\Gamma(\alpha-1/2)}{\Gamma(\alpha)}\right)^2$.

With Lemma 4, we may rewrite the above as:

$$
\begin{aligned}
&p_{2\alpha}(y; x_{I,1}, \lambda/(\alpha e_{I,1}))\left[B - 2K \cdot \frac{\left(\frac{\Gamma(\alpha)}{\Gamma(\alpha-1/2)}\right)^2}{\alpha-1/2}\sqrt{1 + \frac{e_{I,1}\cdot(y-x_{I,1})^2}{2\lambda}}\right] \\
&= p_{2\alpha}(y; x_{I,1}, \lambda/(\alpha e_{I,1}))\left[\beta + (\hat{x}_I - \hat{x}_C)^2 + \sqrt{\frac{\lambda}{\alpha-1}} - 2\sqrt{\frac{\lambda + e_{I,1}\cdot(y-x_{I,1})^2/2}{\alpha-1/2}}\right].
\end{aligned}
$$

Proof of Proposition 1. We first seek to characterize the incumbent's optimal choice of policy in the first stage.

$$\begin{aligned}
&\max_{x_{I,1}\in\mathbb{R}} -(\hat{x}_I - x_{I,1})^2 - \frac{\lambda}{(\alpha-1)e_{I,1}} - e_{I,1} + \\
&\left[P_{2\alpha}(\overline{y}|x_{I,1}, \lambda/(\alpha e_{I,1})) - P_{2\alpha}(\underline{y}|x_{I,1}, \lambda/(\alpha e_{I,1}))\right] \cdot B - \\
&\left[P_{2\tilde{\alpha}}(\overline{y}|x_{I,1}, \lambda/(\tilde{\alpha} e_{I,1})) - P_{2\tilde{\alpha}}(\underline{y}|x_{I,1}, \lambda/(\tilde{\alpha} e_{I,1}))\right] \cdot K
\end{aligned}$$

where $\tilde{\alpha} = \alpha - 1/2$, $B = \beta + (\hat{x}_I - \hat{x}_C)^2 + \sqrt{\frac{\lambda_C}{\alpha_C - 1}}$, and $K = \sqrt{\lambda(\alpha - 1/2)}\left(\frac{\Gamma(\alpha-1/2)}{\Gamma(\alpha)}\right)^2$.

If $t \sim F(\cdot)$ is a member of a location-scale family, then $s = \mu + \sigma \cdot t$ has cdf $F((s-\mu)/\sigma)$ and pdf $f((s-\mu)/\sigma)\frac{1}{\sigma}$. Deriving $F(\cdot)$ with respect μ, we have $-f((s-\mu)/\sigma)\frac{1}{\sigma}$.

Differentiating with respect to $x_{I,1}$ and setting equal to zero implicitly defines the incumbent's optimal (set of) intended policies, $x^*_{I,1}$:

$$\begin{aligned}
0 =& 2(\hat{x}_I - x^*_{I,1}) \\
&+ B \cdot \left[-p_{2\alpha}\left(\overline{y}; x^*_{I,1}, \lambda/(\alpha e_{I,1})\right) + p_{2\alpha}\left(\underline{y}; x_{I,1}, \lambda/(\alpha e_{I,1})\right)\right] \\
&- 2K \cdot \left[-p_{2\tilde{\alpha}}\left(\overline{y}; x^*_{I,1}, \lambda/(\tilde{\alpha} e_{I,1})\right) + p_{2\tilde{\alpha}}\left(\underline{y}; x_{I,1}, \lambda/(\tilde{\alpha} e_{I,1})\right)\right]
\end{aligned}$$

From here, we may replace $\underline{y}$ and $\overline{y}$ with y_* and y^*, respectively. More specifically, we employ the knowledge that, in any equilibrium, $y^* - x_{I,1} = x_{I,1} - y_*$, a quantity we denote by z^*. Plugging in z^*, the density terms cancel out, leaving us with $x^*_{I,1} = \hat{x}_I$.

Finally, we check that the problem is indeed concave at the maximum we have identified. Employing rearrangement made possible by Lemma 4 and calculating the second derivative of the maximand with respect to $x_{I,1}$, we have:

$$
\begin{aligned}
&\frac{\partial}{\partial x_{I,1}} 2(\hat{x}_I - x^*_{I,1}) \\
&\qquad + B \cdot \left[-p_{2\alpha}\left(\overline{y}; x^*_{I,1}, \lambda/(\alpha e_{I,1})\right) + p_{2\alpha}\left(\underline{y}; x_{I,1}, \lambda/(\alpha e_{I,1})\right)\right] \\
&\qquad - 2K \cdot \left[-p_{2\tilde{\alpha}}\left(\overline{y}; x^*_{I,1}, \lambda/(\tilde{\alpha} e_{I,1})\right) + p_{2\tilde{\alpha}}\left(\underline{y}; x_{I,1}, \lambda/(\tilde{\alpha} e_{I,1})\right)\right] \\
= &\frac{\partial}{\partial x_{I,1}} 2(\hat{x}_I - x^*_{I,1}) \\
&\qquad - p_{2\alpha}(\overline{y}; x_{I,1}, \lambda/(\alpha e_{I,1})) \left[B - 2K \cdot \frac{\left(\frac{\Gamma(\alpha)}{\Gamma(\alpha-1/2)}\right)^2}{\alpha - 1/2} \sqrt{1 + \frac{e_{I,1} \cdot (\overline{y} - x_{I,1})^2}{2\lambda}}\right] \\
&\qquad + p_{2\alpha}(\underline{y}; x_{I,1}, \lambda/(\alpha e_{I,1})) \left[B - 2K \cdot \frac{\left(\frac{\Gamma(\alpha)}{\Gamma(\alpha-1/2)}\right)^2}{\alpha - 1/2} \sqrt{1 + \frac{e_{I,1} \cdot (\underline{y} - x_{I,1})^2}{2\lambda}}\right] \\
= &\frac{\partial}{\partial x_{I,1}} 2(\hat{x}_I - x^*_{I,1}) \\
&\qquad - p_{2\alpha}(\overline{y}; x_{I,1}, \lambda/(\alpha e_{I,1})) \left[\beta + (\hat{x}_I - \hat{x}_C)^2 + \sqrt{\frac{\lambda}{\alpha - 1}} - 2\sqrt{\frac{\lambda + e_{I,1} \cdot (\overline{y} - x_{I,1})^2/2}{\alpha - 1/2}}\right] \\
&\qquad + p_{2\alpha}(\underline{y}; x_{I,1}, \lambda/(\alpha e_{I,1})) \left[\beta + (\hat{x}_I - \hat{x}_C)^2 + \sqrt{\frac{\lambda}{\alpha - 1}} - 2\sqrt{\frac{\lambda + e_{I,1} \cdot (\underline{y} - x_{I,1})^2/2}{\alpha - 1/2}}\right] \\
&\quad = -2 \\
&\qquad - \frac{(\alpha + 1/2) e_{I,1} (\overline{y} - x_{I,1})}{\lambda + e_{I,1}(\overline{y} - x_{I,1})^2/2} p_{2\alpha}(\overline{y}; x_{I,1}, \lambda/(\alpha e_{I,1})) \left[\beta + (\hat{x}_I - \hat{x}_C)^2 + \sqrt{\frac{\lambda}{\alpha - 1}} - 2\sqrt{\frac{\lambda + e_{I,1} \cdot (\overline{y} - x_{I,1})^2/2}{\alpha - 1/2}}\right] \\
&\qquad + \frac{(\alpha + 1/2) e_{I,1} (\underline{y} - x_{I,1})}{\lambda + e_{I,1}(\underline{y} - x_{I,1})^2/2} p_{2\alpha}(\underline{y}; x_{I,1}, \lambda/(\alpha e_{I,1})) \left[\beta + (\hat{x}_I - \hat{x}_C)^2 + \sqrt{\frac{\lambda}{\alpha - 1}} - 2\sqrt{\frac{\lambda + e_{I,1} \cdot (\underline{y} - x_{I,1})^2/2}{\alpha - 1/2}}\right] \\
&\qquad + p_{2\alpha}(\overline{y}; x_{I,1}, \lambda/(\alpha e_{I,1})) \left(\frac{\lambda + e_{I,1} \cdot (\overline{y} - x_{I,1})^2/2}{\alpha - 1/2}\right)^{-1/2} \frac{-e_{I,1} \cdot (\overline{y} - x_{I,1})}{\alpha - 1/2} \\
&\qquad - p_{2\alpha}(\underline{y}; x_{I,1}, \lambda/(\alpha e_{I,1})) \left(\frac{\lambda + e_{I,1} \cdot (\underline{y} - x_{I,1})^2/2}{\alpha - 1/2}\right)^{-1/2} \frac{-e_{I,1} \cdot (\underline{y} - x_{I,1})}{\alpha - 1/2}.
\end{aligned}
$$

In equilibrium, we know $\overline{y} = y^*$, $\underline{y} = y_*$, and $(y^* - x_{I,1}) = -(y_* - x_{I,1})$, a quantity we may replace with $z^* > 0$. Further, we know from (2) that $e_{I,1}(z^*)^2 = \lambda/(\alpha - 1)$. In what follows, we consider the last five lines of the derivation above and seek to show the expression is negative, and thus the original problem was concave around the solution played in equilibrium.

$$
\begin{aligned}
&-2\frac{(\alpha+1/2)}{\alpha-1/2}\frac{\alpha-1/2}{\lambda+e_{I,1}(\overline{y}-x_{I,1})^2/2}ezp\left[\beta+(\hat{x}_I-\hat{x}_C)^2+\sqrt{\frac{\lambda}{\alpha-1}}-2\sqrt{\frac{\lambda+e_{I,1}\cdot(\overline{y}-x_{I,1})^2/2}{\alpha-1/2}}\right]\\
&-2ezp\frac{1}{\alpha-1/2}\left(\frac{\lambda+e_{I,1}\cdot(\overline{y}-x_{I,1})^2/2}{\alpha-1/2}\right)^{-1/2}\leq 0\\
&(\alpha+1/2)\left(\frac{\lambda+e_{I,1}\cdot(\overline{y}-x_{I,1})^2/2}{\alpha-1/2}\right)^{-1/2}\left[\beta+(\hat{x}_I-\hat{x}_C)^2+\sqrt{\frac{\lambda}{\alpha-1}}-2\sqrt{\frac{\lambda+e_{I,1}\cdot(\overline{y}-x_{I,1})^2/2}{\alpha-1/2}}\right]+1\geq 0\\
&\left[\beta+(\hat{x}_I-\hat{x}_C)^2+\sqrt{\frac{\lambda}{\alpha-1}}-2\sqrt{\frac{\lambda+e_{I,1}\cdot(\overline{y}-x_{I,1})^2/2}{\alpha-1/2}}\right]\geq-\frac{1}{\alpha+1/2}\left(\frac{\lambda+e_{I,1}\cdot(\overline{y}-x_{I,1})^2/2}{\alpha-1/2}\right)^{1/2}\\
&\beta+(\hat{x}_I-\hat{x}_C)^2\geq\left(\frac{\alpha-1/2}{\alpha+1/2}\right)\sqrt{\frac{\lambda}{\alpha-1}}
\end{aligned}
$$

■

Proof of Proposition 2. With the result that $x^*_{I,1}=\hat{x}_I$ for any choice of $e_{I,1}$, the incumbent solves the following to determine her optimal effort:

$$
\begin{aligned}
&\max_{e_{I,1}\in\mathbb{R}_*}\ -(\hat{x}_I-\hat{x}_I)^2-\frac{\lambda}{(\alpha-1)e_{I,1}}-e_{I,1}+\\
&\left[P_{2\alpha}(\overline{y}|\hat{x}_I,\lambda/(\alpha e_{I,1}))-P_{2\alpha}(\underline{y}|\hat{x}_I,\lambda/(\alpha e_{I,1}))\right]\cdot B-\\
&\left[P_{2\tilde{\alpha}}(\overline{y}|\hat{x}_I,\lambda/(\tilde{\alpha} e_{I,1}))-P_{2\tilde{\alpha}}(\underline{y}|\hat{x}_I,\lambda/(\tilde{\alpha} e_{I,1}))\right]\cdot 2K,
\end{aligned}
$$

where $\tilde{\alpha}=\alpha-1/2$, $B=\beta+(\hat{x}_I-\hat{x}_C)^2+\sqrt{\frac{\lambda_C}{\alpha_C-1}}$, and $K=\sqrt{\lambda(\alpha-1/2)}\left(\frac{\Gamma(\alpha-1/2)}{\Gamma(\alpha)}\right)^2$.

If $t\sim F(\cdot)$ is a member of a location-scale family, then $s=\mu+\sigma\cdot t$ has cdf $F((s-\mu)/\sigma)$ and pdf $f((s-\mu)/\sigma)\frac{1}{\sigma}$. Deriving $F(\cdot)$ with respect e, with $\sigma=\sqrt{\lambda/(\alpha e)}$, we have $-f((s-\mu)/\sigma)\frac{1}{\sigma^2}\sigma'(e)=f((s-\mu)/\sigma)\frac{1}{\sigma^2}\sqrt{\lambda/\alpha}e^{-3/2}/2)=f((s-\mu)/\sigma)\frac{1}{\sigma}\frac{s-\mu}{2e}$.

Differentiating with respect to $e_{I,1}$:

$$
\begin{aligned}
0=&\frac{\lambda}{(\alpha-1)(e_{I,1})^2}-1\\
&+B\cdot\left[p_{2\alpha}\left(\overline{y};x_{I,1},\lambda/(\alpha e_{I,1})\right)\frac{\overline{y}-x_{I,1}}{2e_{I,1}}-p_{2\alpha}\left(\underline{y};x_{I,1},\lambda/(\alpha e_{I,1})\right)\frac{\underline{y}-x_{I,1}}{2e_{I,1}}\right]\\
&-2K\cdot\left[p_{2\tilde{\alpha}}\left(\overline{y};x_{I,1},\lambda/(\tilde{\alpha} e_{I,1})\right)\frac{\overline{y}-x_{I,1}}{2e_{I,1}}-p_{2\tilde{\alpha}}\left(\underline{y};x_{I,1},\lambda/(\tilde{\alpha} e_{I,1})\right)\frac{\underline{y}-x_{I,1}}{2e_{I,1}}\right]
\end{aligned}
$$

With the optimal $e_{I,1}$ implicitly defined, we may employ knowledge of $\overline{y}-x_{I,1}=x_{I,1}-\underline{y}=y^*-\hat{x}_I=$

$\hat{x}_I - y^* = z^*$ in characterizing $e^*_{I,1}$. The first-order conditions may then be rewritten as:

$$
\begin{aligned}
0 =& \frac{\lambda}{(\alpha-1)(e_{I,1})^2} - 1 \\
&+ B \cdot p_{2\alpha}\left(z^*; 0, \lambda/(\alpha e_{I,1})\right) \frac{z^*}{e_{I,1}} - 2K \cdot p_{2\tilde{\alpha}}\left(z^*; 0, \lambda/(\tilde{\alpha} e_{I,1})\right) \frac{z^*}{e_{I,1}}
\end{aligned}
$$

By Lemma 4:

$$
\begin{aligned}
0 =& \frac{\lambda}{(\alpha-1)(e_{I,1})^2} - 1 \\
&+ \frac{\Gamma(\alpha+1/2)}{\sqrt{2\pi\lambda}\,\Gamma(\alpha)} \frac{z^*}{\sqrt{e_{I,1}}} \left(1 + \frac{e_{I,1}\cdot(z^*)^2}{2\lambda}\right)^{-(\alpha+1/2)} \left[B - 2K \cdot \frac{\left(\frac{\Gamma(\alpha)}{\Gamma(\alpha-1/2)}\right)^2}{\alpha - 1/2} \sqrt{1 + \frac{e_{I,1}\cdot(z^*)^2}{2\lambda}}\right].
\end{aligned}
$$

Inputting the values of B, K:

$$
\begin{aligned}
0 =& \frac{\lambda}{(\alpha-1)(e_{I,1})^2} - 1 \\
&+ \frac{\Gamma(\alpha+1/2)}{\sqrt{2\pi\lambda}\,\Gamma(\alpha)} \frac{z^*}{\sqrt{e_{I,1}}} \left(1 + \frac{e_{I,1}\cdot(z^*)^2}{2\lambda}\right)^{-(\alpha+1/2)} \left[\beta + (\hat{x}_I - \hat{x}_C)^2 + \sqrt{\frac{\lambda}{\alpha-1}} - 2\sqrt{\frac{\lambda + e_{I,1}\cdot(z^*)^2/2}{\alpha-1/2}}\right].
\end{aligned}
$$

In equilibrium, $-\sqrt{\frac{\lambda + e_{I,1}(z^*)^2/2}{\alpha-1/2}} = -\sqrt{\frac{\lambda}{\alpha-1}}$, such that $e\cdot z^2 = \lambda/(\alpha-1)$, yielding:

$$
0 = \frac{\lambda}{(\alpha-1)(e_{I,1})^2} - 1 + \frac{\Gamma(\alpha+1/2)}{\sqrt{2\pi}\,\Gamma(\alpha)} \frac{1}{e_{I,1}\sqrt{\alpha-1}} \left(\frac{\alpha-1/2}{\alpha-1}\right)^{-(\alpha+1/2)} * \left[\beta + (\hat{x}_I - \hat{x}_C)^2 - \sqrt{\frac{\lambda}{\alpha-1}}\right]
$$

Multiplying by $e_{I,1}^2$:

$$
0 = \frac{\lambda}{\alpha-1} + \frac{\Gamma(\alpha+1/2)}{\sqrt{2\pi}\,\Gamma(\alpha)} \frac{e_{I,1}}{\sqrt{\alpha-1}} \left(\frac{\alpha-1/2}{\alpha-1}\right)^{-(\alpha+1/2)} * \left[\beta + (\hat{x}_I - \hat{x}_C)^2 - 2\sqrt{\frac{\lambda}{\alpha-1}}\right] - e_{I,1}^2
$$

Letting $M := \frac{\Gamma(\alpha+1/2)}{\sqrt{2\pi}\,\Gamma(\alpha)} \frac{1}{\sqrt{\alpha-1}} \left(\frac{\alpha-1}{\alpha-1/2}\right)^{\alpha+1/2}$:

$$
0 = \frac{\lambda}{\alpha-1} + M\left[\beta + (\hat{x}_I - \hat{x}_C)^2 - \sqrt{\frac{\lambda}{\alpha-1}}\right] e_{I,1} - e_{I,1}^2
$$

and hence

$$e^*_{I,1} = \frac{M}{2}\left[\beta + (\hat{x}_I - \hat{x}_C)^2 - \sqrt{\frac{\lambda}{\alpha-1}}\right] + \frac{\sqrt{M^2\left[\beta + (\hat{x}_I - \hat{x}_C)^2 - \sqrt{\frac{\lambda}{\alpha-1}}\right]^2 + 4\lambda/(\alpha-1)}}{2}.$$

Examining the first-order condition again,

$$\begin{aligned}0 = &\frac{\lambda}{(\alpha-1)(e_{I,1})^2} - 1\\ &+ \frac{\Gamma(\alpha+1/2)}{\sqrt{2\pi\lambda}\,\Gamma(\alpha)}\frac{z^*}{\sqrt{e_{I,1}}}\left(1 + \frac{e_{I,1}\cdot(z^*)^2}{2\lambda}\right)^{-(\alpha+1/2)}\left[\beta + (\hat{x}_I - \hat{x}_C)^2 + \sqrt{\frac{\lambda_C}{\alpha_C - 1}} - 2\sqrt{\frac{\lambda + e_{I,1}\cdot(z^*)^2/2}{\alpha - 1/2}}\right].\end{aligned}$$

each component is positive and each component's derivative is negative. The second derivative of the objective function is thus negative, so the first-order condition defines a maximum level of positive effort. ■