# Adaptive Diarchy: Authority between Two Heads

Zanhui Liu [†] Yucheng Qiu [‡]

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**Abstract**

An organization is referred to as diarchy if it appoints two executives with the same responsibility. This paper explores rationale behind diarchial organizations, starting from their most intriguing feature: A diarchy is often composed of two agents with ambiguous authority allocation. We study a model where a principal must rely on a biased agent for information and execution. To seek extra advice, she can appoint a supervisor with intrinsic preferences over dissenting voice. However, the supervisor may collude with the agent and advocate the latter's advice in communication. For the principal, the only way to prevent potential collusion is to make the supervisor roving. Nevertheless, the signal disclosed by the roving supervisor may become completely uninformative. We show that when vertical conflict of interest is moderate, adaptive diarchy emerges in equilibrium, and is characterized by three features. First, there exists a stationary supervisor. Second, although ex ante contingent delegation is impossible with the principal's lack of necessary expertise, the allocation of real authority is adaptive ex post. Third, the supervisor and the agent are of slight difference of opinion and similar relative status. We then embed our baseline setting in an infinite-horizon model to explore the historical regime shift from roving to stationary supervisors. We show that adaptive diarchy emerges in the proceed of the principal's learning about the extent to which enhancement of control erodes agents' participation.

[†] Tsinghua University. Email: zhliu2018@nsd.pku.edu.cn.
[‡] Peking University. Email: ycqiu2018@nsd.pku.edu.cn.

"Two heads were better than one."

– John Whitehead, former co-chairman of Goldman Sachs

"A body with two heads is in the social as in the animal sphere a monster, and has difficulty in surviving."[1]

– Henri Fayol (1841-1925)

# 1 Introduction

An organization is referred to as *diarchy* if it appoints two executives with the same responsibility. Despite the criticism of inefficiency, as quoted in our second epigraph, diarchy can be widely found among not only business but also political organizations. A number of well-known firms, including Goldman Sachs and Google, have appointed two people to lead a department or even the whole corporation. John Whitehead, who shares chairmanship of Goldman Sachs with John Weinberg during 1970s and 1980s, once commented on the arrangement of dual chiefs, "Two heads were better than one." When we turn to public sectors, a government under two heads is reminiscent of Roman Republic when two people jointly performed consulship. The quintessence of political diarchy, however, must be the Chinese state. In imperial era, emperors appointed two high-rank officials to jointly govern a province, such as *Zongdu* and *Xunfu* in the Ming and Qing dynasties. In modern China, the regime of diarchy prevails with the establishment of party-state. Each province (city) is governed by a party secretary and a governor (mayor); Universities are under the administration of both presidents and chiefs of party committee; Troops are led by commanders and commissars......

This paper explores rationale behind diarchy in lens of organizational economics, starting from the most intriguing feature of the regime: *A diarchy is often composed of two agents with ambiguous authority allocation and similar administrative ranks*. A straightforward rationale for an organization to appoint two agents is to introduce check and balance between them. The purpose seems to be very limited in both practice and theory. In many ex-

[1] Fayol (2016)

amples in China, the party secretary in a city concentrates de facto power, leaving no real authority to the mayor. Moreover, potential collusion between the two agents may erode mutual checks, the target that the regime of diarchy should have been expected to achieve. Another common justification focuses on division of work associated with diarchial arrangements. Some conventions indicate that in China, while party secretaries focus on political dimension including personnel and party affairs, mayors are mainly in charge of economic development. However, in numerous cases, party secretaries initiate and dominate the process of policymaking concerning business and industry in fact. The complexity stems from two features underlying most diarchial arrangements. One the one hand, the allocation of authority between the two agents is often specified in an ambiguous way; on the other hand, the two agents have similar ranks or status, especially due to the fact that neither agent has the right to fire or remove the other one. Consequently, de facto authority allocation within a diarchy is subject to flexible adjustments in reality.

Hence, we investigate how real authority is allocated ex post within a diarchial organization, given that delegation of formal decision-making authority is rigid ex ante. We attempt to describe four features observed from Chinese diarchial regimes. *First, one of the agent is the de jure stationary supervisors appointed by the principal.* Main provincial officials, *Xunfu* in the Qing dynasty and party secretaries in modern China, are in fact centrally appointed supervisor, who stays in their jurisdictions and work together with their local colleagues (*Buzhengshi* or governors). *Second, the allocation of authority between the two agents is ex ante ambiguous.* As already seen, authority allocation and task assignment between party secretaries and mayors are always conditional on specific contexts. Third, the two agents have similar status, ranks, and even other relevant types. In a province, *Zongdu* and *Xunfu*, or party secretaries and governors, share the same or closely similar administrative ranks. In each case, neither a party has the right to remove or demote his counterpart. More importantly, because the Chinese bureaucracy is highly fluid with frequent rotation and promotion, those provincial officials are greatly homogeneous. *Finally, it is often the case that roving supervisors become stationary, and de facto diarchy emerges.* Literally, *Xunfu* means "roving supervisors on behalf of the Emperor", and it was not a permanent position in provinces at the outset in the Ming dynasty. Only after decades since its appearance had the position become standing in provinces. We think that the four features, especially the first three ones, apply in broader situations where an organization adopts some form of diarchial arrangement, such as co-chairmanship in Goldman Sachs.

We study in a setting in which an uninformed (female) principal (the P, for short) consults an informed (male) manager (the M, for short) before adopting a project. Although both individuals want to avoid the disastrous choice, their ideal projects are different, so that their interests are imperfectly aligned. Lack of necessary expertise in identities of possible projects, the P has to seek advice from the M. She also finds it is prohibitively costly to implement a project, so that she always cedes execution of her decision to the M. The chosen project delivers private benefits to the agent, but he must exert costly effort for its success. Implementation costs are random, drawn from a distribution depending on some exogenous binary state of the world which only the M privately knows. In this introduction, we refer to the two possible states as simply "crisis times" and "normal times". Compared to normal times, the chosen project is more likely to fail in crisis times. Hence, relative stakes of the agent's initiative in execution is higher in crisis times. In a complete-information benchmark where the P knows both project identities and the state, she is faced with the fundamental trade-off: adopting her ideal project maximizes her interests ex post, but decreases the M's incentives for execution of her decision. The optimal adoption of a project depends on conflict of interest. When conflict of interest is small, the P prefers the agent's ideal project which empowers the M to execute her decision. In this case, her *initiative concerns* dominate. By contrast, costly loss of control lead to the adoption of the P's ideal project when their interests are extremely misaligned so that her *control concerns* dominate. More interestingly, when conflict of interest is moderate, the optimal adoption is contingent on the state. Specifically, in crisis times, the P chooses the M's ideal project to increase his initiative in execution. In contrast, in normal times, she adopt her ideal project to avoid costly loss of control. Hence, in this case, the optimal decision turns out to be *contingent adoption*, and her *adaptation concerns* emerges.

Under private information, the P may appoint a (male) supervisor (the S, for short) to seek extra advice. The S is of exogenously given two-dimensional type. The first one is *difference of opinion*. That is, he obtains an intrinsic gain if he dissents from the M in communication, advising the P on the adoption of a different project. The second one is his *relative status*, which determines his bargaining power with the M. The S has the same access to information as that of the M. With full commitment, th P can initially choose among three possible formal regimes. The first formal regime is the *Absent Supervisor* ($AS$) where the P does not appoint a supervisor ex ante. The organization is thus composed of only the P and the M. Under the second possible formal regime, the P appoints a supervisor who exits the game instantly after his advise is received. In this scenario, the S has no access to implementation of the chosen project, and thus cannot be granted any effective claim on private benefits delivered by it. We refer to this regime as the *Roving Supervisor* ($RS$). Finally, in contrast to $RS$, the third formal regime is the *Stationary Supervisor* ($SS$) under which the S stays in the continuation game when the P's decision is made and executed. Therefore, the key difference between $RS$ and $SS$ is the timing when the S exits the game, which in turn determines whether or not the S can potentially obtain control over final execution. After the S is appointed, he makes a take-it-or-leave-it proposal, suggesting that the M share control over execution with him and he advocate the M's advice in communication. If the M accepts it, the S obtains partial control over execution, which ensures him a credible part of claim on private benefits yielded by the chosen project. Then, the two agents (the S and the M) simultaneously give their advice on project adoption which is soft or nonverifiable, and the P makes her decision after communication. In the final stage, if the S stays and has reached agreement with the M, the two agents jointly execute the P's decision and divide the private benefits by their relative status. The P receives a larger share the higher his relative status is. Never can the P use monetary transfer that is contingent on collusion, their disclosure behavior, or execution effort.

This framework captures fundamental themes encountered by many organizations which adopt diarchial arrangements. In particular, we take into account collusion between the two agents and elaborate the essential role it plays in diarchy. Analogous to Aghion and Tirole (1997), in our setting, superior knowledge of agents entails the difference between formal and real authority. However, there are three noteworthy differences. First, agents are exogenously informed in the current setting. Instead of considering costly information acquisition, we focus on agents' incentives for execution of the P's decision. Second, in our setting, the fundamental initiative and loss of control trade-off depends on the state of the world. Hence, besides initiative and control concerns, we introduce adaptation concerns when analyzing the allocation of authority. Third, and the most subtle, whereas Aghion and Tirole studies the choice between delegation and centralization, we investigate the allocation of real authority. In our setting, the P retains formal authority, and ceded real authority to agents with superior expertise. Nevertheless, faced with two agents with difference of opinion under diarchy, the P is able to allocate real authority ex post, choosing whose recommendation to rubber stamp. This captures the substantive notion that authority allocation is ex ante ambiguous and subject to ex post adjustment within a diarchial organization.

Our main result is that when conflict of interest between the P and agents is moderate, *adaptive diarchy* emerges in equilibrium, characterized by three features: *existence of a stationary supervisor*, *adaptive allocation of real authority*, and *the combination of slight difference of opinion and similar relative status*. Let us discuss each one. First, appointing a supervisor with intrinsic preferences over dissenting voice, the P can make information about her ideal project revealed. However, given the cheap-talk nature of communication, she can only observe whether she receives dissenting or unanimous advice. If the supervisor is roving without any access to private benefits delivered by the chosen project, the S will never advocate the M's advice. Regardless of the state, communication always exhibits dissension. Consequently, the P retains her prior belief after receiving agents' advice, and thus her decision can never be tailored to the state of the world. Hence, $RS$ is diarchial, but not adaptive. By contrast, if the S is stationary, collusion between agents become possible. From the perspective of the M, whereas accepting the S's proposal on collusion limits his ability to extract rents in the execution stage, advocacy from the S ensures the adoption of his ideal project, increasing the total expected gains. In equilibrium, collusion succeeds in crisis times, but fails in normal times. Therefore, depending on the state of the world, their strategic disclosure exhibits dissension or unanimity, from which the P acquires relevant information. Having updated her beliefs, she is then able to make her decision that is contingent on the state. Hence, under some conditions, existence of a stationary supervisor leads to adaptive diarchy. Second, in equilibrium, the allocation of real authority is adapted to the state of the world under $SS$. Specifically, in crisis times, she cedes real authority to the M who has reached agreement with the S to ensure unanimous voice in communication. By contrast, in normal times, real authority within the organization is held by the S who, having failed to have his proposal accepted, never advocates the advice of the M. Hence, although ex ante contingent delegation is impossible with the P's lack of necessary expertise, the allocation of real authority is adaptive ex post when adaptive diarchy emerges. Finally, were the intrinsic gain that the S obtains from dissenting voice too large, he could never commit to advocacy in communication, so that collusion between the two agents would always fail. On the other hand, were the S's relative status too low or too high, the M would always accept or reject his proposal regardless of the state of the world. Hence, adaptive diarchy emerges with two agents of slight difference of opinion and similar relative status.

Equipped with these results, we finally embed our baseline setting in an infinite-horizon model to explore the historical regime shift from roving to stationary supervisors. Our main ideal is that *adaptive diarchy emerges in the process of the P's learning about the extent to which enhancement of control erodes agents' participation*. To introduce the learning concerns, we assume that the P is now uncertain about the feasibility of her ideal project. In this scenario, both $RS$ and $SS$ can end up with learning about the feasibility, albeit through distinct mechanisms. Specifically, if the P chooses $RS$, she can choose her ideal project and learn its feasibility directly from the outcome of execution. We refer to this mechanism as delegated experimentation. On the other hand, under $SS$, dissension or unanimity in communication reveal information about it, which we refer to as adaptive communication. We show that given the prior belief of the P, if conflict of interest is sufficiently large, she will choose $RS$ at the outset. However, if execution of her initial decision turns out to fail, she downgrades her belief. Should she witness so many failures associated with her enhancing control, she will choose SS to make her decision adaptive, tolerating collusion between agents to avoid costly violation of their participation constraint. Consequently, we can observe the regime shift from roving to stationary supervisors.

The paper is organized as follows. The next section reviews the literature. Section 3 presents the baseline model and discuss the P's different concerns in a complete-information benchmark. Section 4 analyzes the outcome of strategic communication in subgames of each formal regimes, describing the ex post allocation of real authority. In Section 5, we establish the condition under which adaptive diarchy emerges in equilibrium, identifying the key features of the regime. Finally, we embed our baseline setting in an infinite-horizon model to investigate the dynamic regime shift in Section 6, and conclude in Section 7.

## 2 Related Literature

Our paper builds on the literature of authority allocation in organizations and focuses on the importance of adaptation(see Gibbons and Roberts (2013) and Bolton and Dewatripont (2013) for a review). The seminal contribution of Aghion and Tirole (1997) suggests that the delegation of authority faces a basic trade-off between initiative and loss of control. Such trade-off is state-dependent (Aghion and Reenen (2021)) and state-contingent delegation is therefore preferred (Aghion and Bolton (1992),Aghion and Dewatripont (2004) and Krähmer (2006)). The state, however, is sometimes non-contractible and state-contingent delegation is impossible. Organizations, both in private sector and in public sector, therefore benefit from adaptation and flexibility (see Bernheim and Whinston (1999), Dessein and Santos (2006), Alonso and Matouschek (2008), Rantakari (2008), Callander and Krehbiel (2014) and Garicano and Rayo (2016)). Our paper contributes to the literature by showing that diarchy allows for adaptive allocation of real authority and achieves ex post state-contingent delegation even when the state is non-contractible.

Our paper contributes to the literature concerning vertical strategic communication and emphasizes that dissent among agents is necessary but not sufficient condition for effective communication. It is accepted in the literature that dissent among agents fosters costly information acquisition (Che and Kartik (2009), Dessein and Gertner (2010) and Itoh and Morita (2023)) and the use of objective information (Dewatripont and Tirole (1999) and Landier and Thesmar (2009)). Such opinion is also supported by empirical evidence (Landier and Thesmar (2013)). In cheap talk models (Crawford and Sobel (1982) and Farrell and Rabin (1996)), endogenous conflict among multi-senders is also important and the receiver benefits from resulting increased information (Battaglini (2002), Ambrus and

Takahashi (2008), Chakraborty and Harbough (2010) and Sobel (2013)). In principle, we model diarchy in a multi-dimensional cheap talk setting with multi-senders, and we show that state-contingent collusion between agents, along with dissent, plays an important role in effective communication.

In political economy, diarchy (or power duality) is popular and is always considered as means for ”divide and rule” or ”check and balance” (Acemoglu and Verdier (2004), Dragu and Kuklinski (2014) and Li (2019)). Our paper provides an alternative rationale for understanding diarchy, focusing on adaptation it allows. Moreover, a large strand of literature documents the performance of party-state in China (e.g., Bai and Song (2020)), but less is known about the organizational structure behind it. Our paper provides a preliminary research on the topic of party-state through lens of organizational economics.

# 3 Model

## 3.1 The Basic Setting

An organization composed of a (female) principal (P) and a (male) manager (M) must implement one project whose cost of execution depends on a state of the world, $\theta \in \{0, 1\}$. The P retains formal authority to make the decision, but lacks the necessary expertise in both the identities of possible projects and the state. She also finds it prohibitively costly to execute her choice. Hence, she has to rely on the M for information and implementation. The P may additionally appoint a (male) supervisor (S) to gain extra advice, whereas potential collusion between the M and the S cannot be prevented. We shall later consider further details. Throughout, subscripts of $P$, $M$, and $S$ refer to the principal, the manager, and the supervisor, respectively. We refer to the two subordinates (*i.e.* the M and the S) commonly as agents whenever we do not distinguish between them. All players are risk-neutral.

**Projects.** There are four possible projects, indexed by $n = 0, 1, 2, 3$. The payoff of project $n$ consists of three parts: $(\Pi_n, B_n, F_n)$. If successfully implemented, the project deliver benefits $\Pi_n$ to the $P$ and $B_n$ to agents. We shall later consider how $B_n$ is divided between the M and the S. If agents put no effort into execution, the project will fail and all players are paid the same amount $F_n$. The set of projects is denoted by $\mathcal{N} := \{0, 1, 2, 3\}$. Project $0$ is the default project with $\Pi_0 = B_0 = F_0 = 0$. Formally treated as a project, it corresponds to status quo or no project's being implemented. Among the remaining projects are a disastrous project, the P's ideal project and the agents' ideal project. The disastrous project is terrible for all parties: $\Pi_n = B_n = F_n = -\infty$, regardless of implementation. If succeeding, the P's ideal project yields $\Pi_n = R_1$ and $B_n = r_1$, while the agents' ideal project yields $\Pi_n = R_2$ and $B_n = r_2$, where $R_1 > R_2 > 0$ and $r_2 > r_1 > 0$. The imperfect alignment of the P's and the agents' interests can be thus captured by the parameter $\kappa := R_1/R_2 > 1$.

A key difference between our setting and that of Aghion and Tirole (1997) is that we assume that agents can privately observe identities of each project without costly investigation. We refer to identities of each project as the link between projects and associated payoff vectors. Formally, the set of possible payoff vectors is denoted by $\mathcal{R} := \{(0,0,0), (R_1, r_1, 0), (R_2, r_2, 0), (-\infty, -\infty, -\infty)\}$. Agents initially have the private expertise in the bijective map $\chi : \mathcal{N} \to \mathcal{R}$. The identity of the default project (*i.e.* project $0$) is public information. Specifically, the P believes that each bijective map from $\mathcal{N}$ to $\mathcal{R}$ with $\chi(0) = (0,0,0)$ is equally likely. The prior beliefs of the P is common knowledge. Although ex post information acquisition in Aghion and Tirole (1997) is replaced by ex ante superior knowledge of agents, we can also expect the difference between formal and real authority in our setting, as discussed in details later. In particular, there is still an underlying congruence between the P and agents in our setting. That is, the P prefers agents' preferred decision to her either doing nothing (*i.e.* choosing project $0$) or making a decision at random (because of the existence of the disastrous project). This partial congruence is a key condition to delegation of real authority.

**Implementation Costs.** Implementation of project $n \in \mathcal{N}$ commonly incurs a cost $c \in \mathbb{R}_+$ which is realized only after the decision is made by the P. Once realized, $c$ is privately observed by agents. Initially, all parties know that implementation costs $c$ are drawn from a state-dependent distribution with CDF $F(\cdot|\theta)$. The state of the world $\theta \in \{0, 1\}$ is initially privately observed by agents. The prior belief of the P is

$$\Pr(\theta = 0) = \pi \in (0, 1),$$

which is also common knowledge. For project $n$ with payoff $(\Pi_n, B_n, F_n)$, the probability of successful execution, $F(B_n|\theta)$, measures agents' incentives to implement the project. Hence, agents' initiative depends on the state. The vague discussion about state-dependent initiative may be clarified by the following assumption that we shall maintain hereafter:

**Assumption 1** (Monotone Likelihood Ratio Property). *For all* $\theta \in \{0, 1\}$, $r_1, r_2 \in \text{Support}(F(\cdot|\theta))$ *and* $0 \notin \text{Support}(F(\cdot|\theta))$. *For all* $x \in \text{Support}(F(\cdot|\theta))$, *the likelihood ratio*

$$\frac{f(x|\theta = 0)}{f(x|\theta = 1)}$$

*is strictly increasing in* $x$.

The monotone likelihood ratio property implies that higher implementation costs are more likely to realized at state $\theta = 0$ than at state $\theta = 1$. Intuitively, this means that stakes of agents' initiative in execution are relatively higher for the organization at state $\theta = 0$. Heuristically, the realization of the state $\theta = 0$ could be understood as some "crisis times", whereas the state $\theta = 1$ corresponds to "normal times". Decisions of an organization are presumably more likely to fail in crisis times, so that agents' initiative in execution become more crucial for its success than in normal times. One prominent view of Aghion and Tirole (1997) is that delegation of authority engenders initiative versus loss of control trade-off. Although there is no costly information acquisition in our setting, we can still analyze implications of this trade-off on authority allocation, focusing on agents' initiative in execution and implementation.

**Communication.** After privately observing the state $\theta$ and identities of each project $\chi$, agents (both the M and the S) strategically discloses information to the P. Communication is soft or nonverifiable. Given the simple setting of our model, any soft claims about either the state or project identities could be expected to be inflated. Hence, we restrict attention to cheap-talk recommendations: each agent $i = M, S$ advises the P on the adoption of one project $m_i \in \mathcal{M} := \{1, 2, 3\}$. Note that since the identity of the default project is common knowledge and information acquisition is not considered, we assume that neither agent recommends non-adoption (*i.e.* $m_i = 0$). Signals $m_M$ and $m_S$ are, at least partially, informative, for two reasons. First, as discussed before, the underlying congruence between parties implies that the disastrous project will be never recommended, so that $\chi(m_M), \chi(m_S) \in \{(R_1, r_1, 0), (R_2, r_2, 0)\}$. Hence, the signals are partially informative of project identities. Second, if the S is appointed, communication will exhibits either dissent (*i.e.* $m_M \neq m_S$) or unanimity (*i.e.* $m_M = m_S$), revealing information about the true state. We postpone the formal analysis of this point until the next section. In a nutshell, dissent/unanimity of agents' advice may become perfectly informative of the state in equilibrium under some conditions.

**Supervisor.** The P may appoint a supervisor to seek extra advice. Specifically, she designs ex ante two contractible aspects of the appointment with full commitment. *First, the P chooses whether or not to appoint a supervisor.* The S will be rewarded with $v > 0$ should his recommendation be different from that of the M (*i.e.* $m_S \neq m_M$). As a minimum way to model difference of preferences between the two agents, we assume that $v$ is exogenously given and sufficiently small (*i.e.* $v = \epsilon > 0$). The small reward $v$ can be interpreted as monetary transfer contingent on dissent/unanimity of agents' advice, or simply as intrinsic gains that the S obtains from exhibiting different opinion in communication.[2] *Second, the P chooses whether the S is roving or stationary.* Specifically, the S could either exit the game instantly after his advice has been received (in which case we refer to the S as a "roving" one), or stay in the continuation game when the decision is made by the P and implemented by agents (in which case we refer to the S as a "stationary" one). There is no asymmetric information between the M and the S, both of whom privately observes the state $\theta$ and project identities $\chi$.

[2]We can allow the P to choose the magnitude of $v$ ex ante. It can be shown that in equilibrium, $v$ is set to be positive and sufficiently small so that diarchy remains adaptive. We postpone this discussion until after the equilibrium choice of the formal regime is established. If we interpret $v$ as the S's intrinsic gains from exhibiting different opinion, this result means that the P prefers to appoint a supervisor of some slightly different type from that of the M.

A key feature of our model is that collusion between the S and the M is allowed. Although formal authority remains in the hand of the P, the M controls implementation of her decision. For expository simplicity, we refer to his control over execution as power. Power translates to a credible claim on the net private benefit, $B_k - c$, of project $k$ once it is chosen and successfully implemented. If the S is appointed, he can propose that the M share the power with him in return for his manipulating his advice. We model the potential collusion between them as veto bargaining (Romer and Rosenthal (1978)) over power and associated claims, where the S acts as a proposer, and the M as a vetoer. Specifically, the S may choose to make a take-it-or-leave-it proposal ($d_S \in \{0, 1\}$), suggesting that the M share the power with him and he recommend the agent's ideal project (*i.e.* $\chi(m_S) = (R_2, r_2, 0)$) in return. The M then decides whether to accept the proposal or reject it and preserve the full claim on $B_k - c$ ($d_M \in \{0, 1\}$). So long as agreement is reached, the S obtains credibly partial control over execution of the chosen project, which ensures his claim on $\gamma \in (0, 1]$ of $B_k - c$. Interpreted intuitively as the S's *bargaining power*, or *status* relative to the M, $\gamma$ is exogenously given and common knowledge. Given the state $\theta$, if agreement is reached and project $k$ is chosen by the P, the S and the M therefore have the following expected payoffs from implementation of the project respectively: $b_S(B_k, \theta) \coloneqq \gamma V(B_k|\theta)$ and $b_M(B_k, \theta) \coloneqq (1 - \gamma) V(B_k|\theta)$, where

$$V(x|\theta) \coloneqq \mathbb{E}\left[(x - c) \mathbb{1}\{x - c \geq 0\}\right]. \tag{1}$$

Note that since it is the net private benefit that is divided, collusion between the two agents is irrelevant to their aggregate initiative in execution, measured by $F(B_k|\theta)$, regardless of the S's status $\gamma$. Hence, agents' initiative depends only on the P's decision $k \in \mathcal{N}$ and the state $\theta \in \{0, 1\}$. It is also worth noting that we only assume that agreement on power and claims is credible. The S commits to his proposal on advising the agents' ideal project for more subtle reasons, which we shall elaborate in the next section.

**Contracts and Formal Regimes.** We adopt the common approach of incomplete contracting (Grossman and Hart (1986)) by positing that the state, projects, and agents' effort cannot be described and contracted on ex ante. In particular, both contingent transfer and contingent delegation are infeasible. Specifically, the P can neither use monetary transfer that is contingent on agents' effort exerted towards execution, nor choose the allocation of authority and the arrangement of a supervisor that are contingent on the state. Only two issues are contractible. First, the P can commit to the initially specified arrangement with respect to the supervisor (*i.e.* whether the S is appointed and whether the S is roving or stationary). Second, agreement on power and claims is credible once collusion between the S and the M succeeds. Arrangements with respect to the supervisor give rise to three possible formal regimes, among which the P selects ex ante with full commitment. Let us discuss each one:

- *Absent Supervisor* ($AS$): The P appoints no supervisor, so that the organization consists of the P and the M. The P retains decision-making authority and relies on the M for recommendation. The M controls implementation of the project chosen by the P.

- *Roving Supervisor* ($RS$): The P appoints a supervisor who exits the game instantly after his advice has been received. With no access to execution, the S cannot be granted any effective claim on private benefits of the chosen project. The P retains formal authority and solicits advice from both the M and the S. The M alone has control over execution of the P's decision.

- *Stationary Supervisor* ($SS$): The P appoints a supervisor with relative status $\gamma \in (0, 1]$ who stays in the continuation game when the decision is made by the P and implemented by agents. If the S reaches agreement with the M, he obtains an effective claim on $\gamma$ of net private benefits of the chosen project. The P retains formal authority and solicits advice from both the M and the S. If collusion between the S and the M succeeds, the two agents jointly control execution of the P's decision and divide the net private benefits by their relative status. Otherwise, the M preserves his power and associated full claim.

Let $f \in \{AS, RS, SS\}$ denote the P's ex ante choice of the formal regime. Note that as discussed in the introduction, we focus on ex post allocation of real authority under diarchy.

Hence, in contrast to Aghion and Tirole (1997) who consider implications of delegation of formal authority, we maintain the presumption throughout that the P retains decision-making authority and never delegates it to agents.

**Timing.** The sequence of events is as follows.

1. The P chooses the formal regime, $f \in \{AS, RS, SS\}$. It is publicly observed.

2. The state of the world $\theta \in \{0, 1\}$ and identities of each project $\chi : \mathcal{N} \to \mathcal{R}$ are realized. Both $\theta$ and $\chi$ are privately observed by agents. The P has prior belief that $\Pr(\theta = 0) = \pi$.

3. If an supervisor is appointed, the S chooses whether to make a proposal ($d_S \in \{0, 1\}$), suggesting that the M share control over execution with him. It is privately observed by agents.

4. The M decides whether to accept the S's proposal ($d_M = 1$) or reject it and preserve his complete power ($d_M = 0$). It is privately observed by agents.

5. The M and the S (if appointed) simultaneously advise the P on the adoption of a project, $m_M, m_S \in \mathcal{M}$. Their recommendations are publicly observed.

6. If $f = RS$, the S exits the game. If $f = SS$, the S stays in the continuation game.

7. The P makes the decision, $k \in \mathcal{N}$.

8. The implementation cost $c$ is drawn from the state-dependent distribution with CDF $F(\cdot|\theta)$. It is privately observed by agents.

9. If $f = SS$ and collusion between the S and the M succeeds (*i.e.* $d_S d_M = 1$), the two agents jointly decide whether or not to implement project $k$. Otherwise, the M alone decides whether or not to implement it. Their execution effort is noncontractible.

10. If successfully implemented, project $k$ delivers $\Pi_k$ to the P and $B_k$ to agents. The two agents divide $B_k - c$ by their relative status, and the S obtains $\gamma(B_k - c)$ (only if $f = SS$ and $d_S d_M = 1$). If it fails, all players are paid $F_k$.

The timing of the game can be recapitulated as four sequential parts: *organization design*, *power sharing*, *decision making*, and *implementation stage*. During the organization design, the P selects the formal regime. During the power sharing, the S and the M chooses whether or not to reach agreement on power and claims. During the decision making, the P communicates with agents and then makes the decision. Finally, during the implementation stage, agents choose whether or not to implement the chosen project. Our solution concept is Perfect Bayesian Equilibrium, which we refer to as, for short, equilibrium hereafter. We restrict attention to pure strategy equilibria.

## 3.2 Initiative, Control, and Adaptation

As a prelude to our analysis, it is useful to identify the trade-off faced by the P and discuss her different concerns in a complete-information benchmark. We maintain the presumption that the P finds it prohibitively costly to execute her decision, so that she cedes control over implementation to agents. If project $k \in \mathcal{N}$ is chosen, she has the expected gain $\Pi_k F(B_k|\theta)$. Recall that the P and agents have preference conflicts over projects because $R_1 > R_2$ and $r_2 > r_1$. Hence, the expression sheds light on the *initiative* versus *loss of control* trade-off: *selection of agents' ideal project creates incentives for them to execute the P's decision, but fails to maximize her interests ex post*. Note that in contrast to Aghion and Tirole (1997), the trade-off is engendered by the agents' control over decision execution rather than information acquisition. Moreover, while the choice between delegation and centralization is the main theme of them, we focus primarily on implications of the trade-off on the allocation of real authority. We return to this point later.

Recall that agent's initiative in execution, measured by $F(B_k|\theta)$, depends on the true state $\theta$. As discussed before, the state of the world determines stakes of effort exerted into implementing the chosen project, with failure being more likely to happen at state $\theta = 0$. Thus, the fundamental trade-off between initiative and loss of control is state-dependent. Specifically, the following Lemma turns out to play an essential role in elaborating the impact of the state; its proof, and all subsequent proofs not in the text, are in the Appendix.

**Lemma 1** *Define* $V(r|\theta) := \mathbb{E}\left[(r-c)\mathbb{1}\{r-c \geq 0\}|\theta\right]$ *for all* $\theta \in \{0,1\}$. *Then,*

$$1 < \frac{F(r_2|\theta=1)}{F(r_1|\theta=1)} < \frac{F(r_2|\theta=0)}{F(r_1|\theta=0)}, \tag{2}$$

*and*

$$0 < \frac{V(r_1|\theta=0)}{V(r_2|\theta=0)} < \frac{V(r_1|\theta=1)}{V(r_2|\theta=1)} < 1. \tag{3}$$

We postpone the second inequality until when we discuss the relative status of the S, $\gamma$. To understand the lemma, note that the ratio $F(r_2|\theta)/F(r_1|\theta)$ captures the magnitude of increased initiative associated with the P's selecting agents' ideal project. Hence, the inequality (2) establishes a comparison of increased initiative at state $\theta = 0$ with that at state $\theta = 1$. The result confirms the intuition that the increase in incentives for execution should be larger at the state when it is more likely that a project fails to deliver payoffs. Define the two critical values:

$$\kappa_l := \frac{F(r_2|\theta=1)}{F(r_1|\theta=1)} \text{ and } \kappa_u := \frac{F(r_2|\theta=0)}{F(r_1|\theta=0)}. \tag{4}$$

Lemma 1 therefore establishes that $1 < \kappa_l < \kappa_u$.

We are now in a position to elaborate the P's possible concerns entailed by agents' control over execution and their state-dependent initiative to exert effort. It is useful to first ignore strategic communication, assuming that both the state $\theta$ and project identities $\chi$ are publicly observed. In this benchmark, formal regimes are irrelevant and contingent adoption of projects becomes possible. Nevertheless, the P is still faced with the trade-off between initiative and loss of control, relying on agents for executing her decision with conflict of interest. Recall that the parameter $\kappa := R_1/R_2 > 1$ captures misalignment of interest between the two parties. If conflict of interest is sufficiently small that $\kappa < \kappa_l < \kappa_u$, we have $R_2 F(r_2|\theta) > R_1 F(r_1|\theta)$ for all $\theta \in \{0,1\}$. That is, when conflict of interest between the two parties is small, the P always chooses the agents' ideal project regardless of the state of the world. In this case, the benefit from improved incentives for execution outweighs the the costly loss of control. We refer to $(1, \kappa_l)$ as *the initiative region*, where, faced with small conflicts, the P would prefer to empower agents by adopting agents' ideal project were information complete. Hence, her *initiative concerns* dominates in this region. Let us now consider the opposite case where interests between two parties are extremely misaligned so that $\kappa > \kappa_u > \kappa_l$. Then, for all $\theta \in \{0, 1\}$, $R_1 F(r_1|\theta) > R_2 F(r_2|\theta)$. Thus, the P chooses her ideal project independent of the state. In this case, conflict of interest entails highly costly loss of control. We refer to $(\kappa_u, +\infty)$ as *the control region*, where the P's *control concerns* dominate. Finally, we turn to the case where conflict of interest is moderate: $\kappa_l < \kappa < \kappa_u$. In this region, contingent adoption emerges. Specifically, we have $R_2 F(r_2|\theta = 0) > R_1 F(r_1|\theta = 0)$ and $R_1 F(r_1|\theta = 1) > R_2 F(r_2|\theta = 1)$. Therefore, while the P prefers to empower agents to execute her decision at state $\theta = 0$, she would like to choose her ideal project at state $\theta = 1$ even though it decreases the initiative gains. In other words, *were the state of the world and project identities publicly known, the optimal adoption would be adapted to $\theta$ when conflict of interest is moderate*. Contingent adoption stems from implications the state of the world has on stakes of agents' initiative. Assumption 1 implies that failure is more likely to happen at state $\theta = 0$, so that stakes of effort exerted into execution are higher than those at state $\theta = 1$. Thus, it is reasonable that when interests between two parties are moderately misaligned, initiative concerns outweigh control concerns at state $\theta = 0$ whereas loss of control becomes too costly at state $\theta = 1$. We refer to $(\kappa_l, \kappa_u)$ *the adaptation region* where moderate conflict of interest gives rise to *adaptation concerns*. We summarize as follows:

**Claim 1** (Benchmark). *If the state of the world $\theta$ and project identities $\chi$ are publicly observed, then:*

(a) *If $1 < \kappa < \kappa_l$, the P prefers to choose project $k(\theta)$ such that $\chi(k(\theta)) = (R_2, r_2, 0)$ for all $\theta \in \{0, 1\}$. Hence, initiative concerns dominate in the initiative region $(1, \kappa_l)$;*

(b) *If $\kappa_l < \kappa < \kappa_u$, the P prefers to choose project $k(\theta)$ such that $\chi(k(\theta = 0)) = (R_2, r_2, 0)$ and $\chi(k(\theta = 1)) = (R_1, r_1, 0)$. Hence, adaptation concerns emerge in the adaptation region $(\kappa_l, \kappa_u)$;*

(c) *If $\kappa > \kappa_u > \kappa_l$, the P prefers to choose project $k(\theta)$ such that $\chi(k(\theta)) = (R_1, r_1, 0)$ for all $\theta \in \{0, 1\}$. Hence, control concerns dominate in the control region $(\kappa_u, +\infty)$.*

Under private information, agents are endowed with superior expertise, and the P has to rely on them for recommendation. Given noncontractibility of information provided by agents, formal regimes specified ex ante matter now, determining how informative strategic communication is of the state of the world and project identities. Could the uninformed P design formal regimes ex ante to satisfy her concerns (*initiative*, *control*, or *adaptation*) ex post? In particular, when conflict of interest is moderate, how will a formal regime allow her decision to be adapted to the true state?

# 4 Absent, Roving, and Stationary Supervisors

This section analyzes the outcome of strategic communication in subgames of each formal regime. As discussed in Section 3, there are three possible formal regimes. Under *Absent Supervisor* $(AS)$, the P relies solely on the M for advice and execution. Under *Roving Supervisor* $(RS)$, a supervisor of type $(v, \gamma)$ is appointed, where $v = \epsilon$ denote his intrinsic gains from exhibiting dissent and $\gamma \in (0, 1]$ his relative status. The S exits the game instantly after his advice is received. Finally, under *Stationary Supervisor* $(SS)$, the S stays during the decision making and the implementation stage. Our objective is to characterize the ex post allocation of real authority, and the adoption of projects under different formal regimes.

**Absent Supervisor.** We start from the *Absent Supervisor* $(AS)$ where the organization consists of the P and the M. The timing under $AS$ involves the decision making and the implementation stage. In the absence of necessary expertise in project identities, a blind decision is likely to result in the adoption of the disastrous project. Hence, the P either solicits advice from the M who has better access to information, or preserves the status quo (*i.e.* choosing the project 0). Recall that because of the partial congruence between the two parties, the P prefers the M's ideal project than her doing nothing. Consequently, regardless of the state, she always rubber stamps the recommendation of the M who in turn discloses a signal $m_M$ such that $\chi(m_M) = (R_2, r_2, 0)$. We have proved:

**Proposition 1** (Absent Supervisor Subgame). *Under Absent Supervisor* $(AS)$*, there exists a unique Perfect Bayesian Equilibrium so that* $\forall \theta \in \{0, 1\}$,

(a) *The $M$ recommends $m_M(\theta) \in \mathcal{M}$ such that $\chi(m_M(\theta)) = (R_2, r_2, 0)$;*

(b) *The $P$ rubber stamps the recommendation of $M$, i.e. $k(m_M) = m_M(\theta)$.*

This result is reminiscent of the equilibrium rubber-stamping in Aghion and Tirole (1997). As in Aghion and Tirole (1997) and Dessein (2002), the difference between formal and real authority stems from superior knowledge of the M. While retaining formal authority, the P cedes complete real authority to the M who privately observes $\theta$ and $\chi$. Consequently, while the P maximizes initiative from the M, her interests are undesirable ex post. Under $AS$, strategic communication is completely uninformative of the state, and thus the P's decision $k$ can never be tailored to $\theta$.

**Roving Supervisor.** Now consider the *Roving Supervisor* ($RS$). The P appoints a supervisor of type $(v, \gamma)$ and have the S exit instantly after she receives his advice. The S privately observes the state $\theta$ and project identities $\chi$. With no access to execution in the final stage, the S cannot be granted any effective claim on private benefits yielded by the chosen project. Therefore, under $RS$, he never makes the proposal on power sharing. The regime thus proceeds analogously with $AS$, except that during the decision making, the M and the S simultaneously disclose their signals, $m_M$ and $m_S$, respectively. The S obtains the intrinsic gain $v = \epsilon$ from exhibiting difference of opinion (*i.e.* $m_S \neq m_M$), but never recommends the disastrous project which delivers $-\infty$ to all players. As in $AS$, the uninformed P has to rubber stamp the advice of agents. However, faced now with two informed advisers, she can choose whose recommendation to rubber stamp. Hence, even though she still has to cede real authority, the P is now able to allocate it ex post between the two agents with slightly different types.

Let us define the following critical value:

$$\kappa_m := \frac{\pi F(r_2|\theta = 0) + (1-\pi) F(r_2|\theta = 1)}{\pi F(r_1|\theta = 0) + (1-\pi) F(r_1|\theta = 1)} \in (\kappa_l, \kappa_u), \tag{5}$$

and consider the case where $\kappa > \kappa_m$. In the absence of information about the state, with her belief that $\Pr(\theta = 0) = \pi$, the P prefers her ideal project. That is, $R_1 \mathbb{E}[F(r_1|\theta)] > R_2 \mathbb{E}[F(r_2|\theta)]$ if $\kappa > \kappa_m$. Since the M retains his complete control over execution and thus the full claim associated with his power, rubber-stamping his recommendation will lead to the adoption of agents' ideal project regardless of the state: $\chi(k(\theta)) = \chi(k(m_M)) = (R_2, r_2, 0)$. By contrast, any project delivers nothing to the roving supervisor. So long as the M recommends her ideal project, the S benefits from disclosure of a different signal $m_S$ such that $\chi(m_S) = (R_1, r_1, 0)$. Hence, it is likely that the P rubber stamps the recommendation of the S who advises her on the adoption of her ideal project. The following result establishes that the rubber-stamping and disclosure behavior characterizes the unique equilibrium:

**Proposition 2** (Roving Supervisor Subgame). *Suppose that $\kappa > \kappa_m$. Under Roving Supervisor $(RS)$, there exists a unique Perfect Bayesian Equilibrium so that $\forall \theta \in \{0, 1\}$,*

(a) *Collusion between the $S$ and the $M$ fails, i.e. $d_S(\theta) d_M(\theta) = 0$;*

(b) *The $M$ recommends $m_M(\theta) \in \mathcal{M}$ such that $\chi(m_M(\theta)) = (R_2, r_2, 0)$;*

(c) *The $S$ recommends $m_S(\theta) \in \mathcal{M}$ such that $\chi(m_S(\theta)) = (R_1, r_1, 0)$;*

(d) *The $P$ rubber stamps the recommendation of $S$, i.e. $k(m_M, m_S) = m_S(\theta)$.*

In this case, there are conflicts between the P and the M in preferred projects. As discussed before, without a supervisor, the P is compelled to rubber stamps the M's advice in spite of costly loss of control. Nevertheless, in contrast to $AS$, the P is now faced with two informed agents with different opinions and able to allocate real authority between them ex post. In equilibrium, she cedes real authority to the roving supervisor who has a intrinsic tendency to exhibit dissent in communication and can never benefit explicitly from any project. Consequently, the P chooses her ideal project, losing the initiative gain from empowering the M. However, under $RS$, although signals become more informative of project identities, they remain completely uninformative of the state. Communication always exhibits dissension regardless of the state. The P retains her prior belief about $\theta$ when making her decision, and thus the ex post allocation of real authority cannot be adapted to the state. Hence, under $RS$, contingent adoption is still infeasible with private information.

In general, we cannot rule out there being multiple equilibria under $RS$ if $\kappa < \kappa_m$. In this case, both the P and the M prefers agents' ideal project ex ante. As discussed before, the P retains her prior belief about $\theta$ after communication which always takes the form of dissension independent of the state. Hence, conflict of interest between the P and the M vanishes. It is then straightforward that the P can rubber stamps the recommendation of the M, leading to the regime of de facto $AS$. Alternatively, one can verify that the following rubber-stamping and disclosure behavior constitutes another equilibrium: regardless of $\theta$, the M and the S disclose signals $m_M$, and $m_S$, respectively, such that $\chi(m_M) = (R_1, r_1, 0)$ and $\chi(m_S) = (R_2, r_2, 0)$; the P always rubber stamps the recommendation of the S, $k = m_S$. Note that both equilibria lead to the adoption of the agents' ideal project. In this case, $AS$ and $RS$ are equivalent from the perspective of the P. Hence, for the remainder of the paper, we focus on $AS$ whenever $\kappa < \kappa_m$.

**Stationary Supervisor.** We are now in a position to characterize the equilibrium under the *Stationary Supervisor* ($SS$). Under this regime, the supervisor of type $(v, \gamma)$ stays in the continuation game when the P's decision is made and executed. With access to execution, the S can choose whether or not to initiate a veto bargaining, making a proposal that the M share the power with him and he recommend the agents' ideal project. If he reaches agreement with the M, he obtains credibly partial control over implementation, which ensures his effective claim on $\gamma \in (0, 1]$ of $B_k - c$ delivered by the chosen project. Thus, the sequence of events consists of the power sharing, the decision making, and the implementation stage. If collusion between the two agents succeeds during the power sharing (*i.e.* $d_S d_M = 1$), they jointly execute the P's decision and then divide the net private benefits by their relative status. As already noted, we assume that $v = \epsilon > 0$. That is, despite losing the intrinsic gain $v$ from dissension, the S of type $(v, \gamma)$ can benefit from colluding with the M.[3] Analogous to $RS$, faced with two informed agents, the P is able to allocate real authority ex post. On the other hand, in contrast to $RS$, with possibility of collusion, not only dissension but also unanimity may arise in communication under $SS$, which provides extra information to the P.

It suffices to focus on the case where $\kappa > \kappa_l$. When $\kappa < \kappa_l$ (*i.e.* in the initiative region), there is no conflict of interest between the P and the M, both of whom prefer to adopt the

[3]Specifically, it suffices to presume that $0 < v < \overline{v}(\gamma) := \gamma (V(r_2|\theta = 0) - V(r_1|\theta = 1))$. See Remark 1.

agents' ideal project regardless of the state. Consequently, the P can always rubber stamp the recommendation of the M, and the latter in turn disclose $m_M$ such that $\chi(m_M) = (R_2, r_2, 0)$. Hence, in this case, whatever the formal regime is, the de facto regime reduces to $AS$ where the P cedes complete real authority to the M. Define the two critical values:

$$\underline{\gamma} := 1 - \frac{V(r_1|\theta = 1)}{V(r_2|\theta = 1)} \text{ and } \overline{\gamma} := 1 - \frac{V(r_1|\theta = 0)}{V(r_2|\theta = 0)}. \tag{6}$$

Inequality (3) in Lemma 1 establishes that $0 < \underline{\gamma} < \overline{\gamma} < 1$. Our first main result is:

**Proposition 3** (Stationary Supervisor Subgame in the Adaptation and Control Region). *Suppose that $\kappa > \kappa_l$. Under Stationary Supervisor ($SS$), if $\gamma < \underline{\gamma} < \overline{\gamma}$, $\forall \theta \in \{0, 1\}$, collusion between $S$ and $M$ always succeeds, i.e. $d_S(\theta) d_M(\theta) = 1$, so that the equilibrium is equivalent to that of $AS$ subgame. If $\gamma > \overline{\gamma} > \underline{\gamma}$, $\forall \theta \in \{0, 1\}$, collusion between $S$ and $M$ always fails, so that the equilibrium is equivalent to that of $RS$ subgame. If $\underline{\gamma} < \gamma < \overline{\gamma}$, there exists a unique Perfect Bayesian Equilibrium so that:*

(i) *If $\theta = 0$,*

  (a) *Collusion between the $S$ and the $M$ succeeds, i.e. $d_S(\theta = 0) d_M(\theta = 0) = 1$;*

  (b) *The $M$ recommends $m_M(\theta = 0) \in \mathcal{M}$ such that $\chi(m_M(\theta = 0)) = (R_2, r_2, 0)$;*

  (c) *The $S$ recommends $m_S(\theta = 0) \in \mathcal{M}$ such that $\chi(m_S(\theta = 0)) = (R_2, r_2, 0)$;*

  (d) *The $P$ rubber stamps the recommendations of $M$, i.e. $k(m_M, m_S) = m_M(\theta = 0)$.*

(ii) *If $\theta = 1$,*

  (a) *Collusion between the $S$ and the $M$ fails, i.e. $d_S(\theta = 1) d_M(\theta = 1) = 0$;*

  (b) *The $M$ recommends $m_M(\theta = 1) \in \mathcal{M}$ such that $\chi(m_M(\theta = 1)) = (R_2, r_2, 0)$;*

  (c) *The $S$ recommends $m_S(\theta = 1) \in \mathcal{M}$ such that $\chi(m_S(\theta = 1)) = (R_1, r_1, 0)$;*

  (d) *The $P$ rubber stamps the recommendation of $S$, i.e. $k(m_M, m_S) = m_S(\theta = 1)$.*

To see the intuition, first assume that the state of the world is publicly observed. Consider the case where $\theta = 1$. Claim 1 shows that in this case, the P prefers to select her ideal project. Nevertheless, suffering from lack of information about project identities, she can only choose among advice she receives from the informed agents. Consequently, if agreement is reached during the power sharing and both agents disclose $m = m_M = m_S$ such that $\chi(m) = (R_2, r_2, 0)$, the P has to rubber stamp their joint recommendation. Given that his intrinsic gain from dissent is small, the S has no incentive to deviate his proposal on disclosure. Hence, collusion between the S and the M leads to the adoption of the agents' ideal project. The M's expected payoff from sharing his power is thus $(1-\gamma) V(r_2|\theta = 1)$. On the other hand, if the M rejects the S's proposal, the chosen project delivers nothing to the S. The continuation game proceeds similarly with that under $RS$. Specifically, after observing dissension in the communication, the P rubber stamps the recommendation of the S who discloses $m_S$ such that $\chi(m_S) = (R_1, r_1, 0)$. Therefore, the M's expected payoff from rejecting is $V(r_1|\theta = 1)$. Hence, in this case, the M's benefit of sharing power with the S of type $(v, \gamma)$, denoted $\Delta_M(\theta = 1)$, is given by

$$\Delta_M(\theta = 1) = (1-\gamma) V(r_2|\theta = 1) - V(r_1|\theta = 1) = \left(\underline{\gamma} - \gamma\right) V(r_2|\theta = 1). \tag{7}$$

That is, the M accepts the S's proposal at state $\theta = 1$ if and only if the S's status $\gamma$, which determines the division of net private benefits, is sufficiently small that $0 < \gamma < \underline{\gamma}$. Analogously, consider the case where $\theta = 0$. When $\kappa_l < \kappa < \kappa_u$, by Claim 1, the P prefers the agents' ideal project. As discussed before, in this case, she always rubber stamps the recommendation of the M, ignoring the S's advice. Consequently, retaining complete control over execution, the M never shares his power with the S. In contrast, when $\kappa > \kappa_u$, misaligned interests entail costly loss of control, so that the P prefers her ideal project. Then, similar to earlier analysis, the M's benefit of sharing power with the S of type $(v, \gamma)$ is

$$\Delta_M(\theta = 0) = (1-\gamma) V(r_2|\theta = 0) - V(r_1|\theta = 0) = (\overline{\gamma} - \gamma) V(r_2|\theta = 0). \tag{8}$$

Therefore, at state $\theta = 0$, the M colludes with the S if and only if $0 < \gamma < \overline{\gamma}$. We summarize as follows:

**Observation 1** (Collusion when the State is Publicly Observed). *Suppose that $\kappa > \kappa_l$. If the* *state of the world $\theta$ is publicly observed, then:*

(i) *If* $0 < \gamma < \underline{\gamma} < \overline{\gamma}$*,*

(a) *When* $\theta = 0$*, collusion between the S and the M fails* (*i.e.* $d_M(\theta = 0) = 0$) *if* $\kappa_l < \kappa < \kappa_u$*; and succeeds* (*i.e.* $d_M(\theta = 0) = 1$) *if* $\kappa > \kappa_u$*;*

(b) *When* $\theta = 1$*, collusion between the S and the M succeeds* (*i.e.* $d_M(\theta = 1) = 1$).

(ii) *If* $\underline{\gamma} < \gamma < \overline{\gamma}$*,*

(a) *When* $\theta = 0$*, collusion between the S and the M fails* (*i.e.* $d_M(\theta = 0) = 0$) *if* $\kappa_l < \kappa < \kappa_u$*; and succeeds* (*i.e.* $d_M(\theta = 0) = 1$) *if* $\kappa > \kappa_u$*;*

(b) *When* $\theta = 1$*, collusion between the S and the M fails* (*i.e.* $d_M(\theta = 1) = 0$).

(iii) *If* $\gamma > \overline{\gamma} > \underline{\gamma}$*,*

(a) *When* $\theta = 0$*, collusion between the S and the M fails* (*i.e.* $d_M(\theta = 0) = 0$)*;*

(b) *When* $\theta = 1$*, collusion between the S and the M fails* (*i.e.* $d_M(\theta = 1) = 0$).

Collusion depends on the state of the world. From the perspective of the M, sharing his power with the S ensures the adoption of the agents' ideal project, but limits his ability to extract surplus in the final stage. This trade-off depends on the supervisor's type and the state. When the M is faced with a supervisor of status $\gamma$, collusion entails a costly loss of $\gamma$ of his claim on the net private benefits. On the other hand, inequality (3) in Lemma 1 establishes how increases in surplus induced by collusion change between states. Specifically, the effect of collusion on increasing the total net private benefits can be captured by the ratio $V(r_2|\theta)/V(r_1|\theta)$. Lemma 1 shows that the magnitude of this effect is larger at state $\theta = 0$ than at state $\theta = 1$. The intuition is transparent: when $\theta = 0$, failure of the chosen project is more likely to happen, which increases relative stakes of aggregate initiative, and thus the magnitude of the incentive effect of collusion. Hence, consider the case where $\underline{\gamma} < \gamma < \overline{\gamma}$ and $\kappa > \kappa_u$. In this case, the status of the S is moderate, and failure of collusion leads to the adoption of the P's ideal project. When $\theta = 0$, compared to disagreement, increased total surplus delivers more expected gain to the M under collusion, even though he has to relinquish $\gamma$ of his claim. Consequently, he accepts the proposal of the S. In contrast, insofar as the incentive effect of collusion is of smaller magnitude at state $\theta = 1$, limited ability to extract surplus makes the M worse off under collusion despite the adoption of his ideal project. Therefore, when $\theta = 1$, the M never reaches agreement with the S.

Return to our setting where information about the state is private, and focus on the nontrivial case where $\underline{\gamma} < \gamma < \overline{\gamma}$. Analogous to the situation under public information, collusion leads to an increase in the total expected private gain so long as the two agents disclose $m = m_M = m_S$ such that $\chi(m) = (R_2, r_2, 0)$. In addition, when the state $\theta$ is privately observed by the agents, a second motive is added to the M's incentive for collusion: *to avoid an adverse inference that the P attaches to dissension in communication*. As already noted, for the M, when $\theta = 1$, the loss from limited ability to extract surplus outweighs the benefit from the adoption of his ideal project under collusion. Hence, dissension in communication (*i.e.* $m_M \neq m_S$) would indicate that collusion between the agents fails, and thus the state is very likely to be $\theta = 1$. Proposition 3 confirms the intuition, showing that in equilibrium, the P's posterior beliefs about the state of the world are given by

$$\Pr(\theta = 1 | m_M \neq m_S) = \frac{(1-\pi)\Pr(m_M \neq m_S | \theta = 1)}{\pi \Pr(m_M \neq m_S | \theta = 0) + (1-\pi)\Pr(m_M \neq m_S | \theta = 1)} = 1, \quad (9)$$

$$\Pr(\theta = 1 | m_M = m_S) = \frac{(1-\pi)\Pr(m_M = m_S | \theta = 1)}{\pi \Pr(m_M = m_S | \theta = 0) + (1-\pi)\Pr(m_M = m_S | \theta = 1)} = 0. \quad (10)$$

If collusion between the S and the M fails, the P will observe dissension in communication and make her decision given her updated belief that $\theta = 1$. Because $\kappa > \kappa_l$, the P prefers her ideal project ex post. Therefore, given that the M rejects the proposal of the S, the continuation game proceeds analogously to $RS$. Faced with two different signals, the P rubber stamps the recommendation of the S, and the S discloses $m_S(\theta = 1)$ such that $\chi(m_S(\theta = 1)) = (R_2, r_2, 0)$. When $\theta = 1$, the M never accept the S's proposal even though his rejection leads to the adoption of the P's ideal project, and thus the collusion, disclosure, and rubber-stamping behavior constitutes an equilibrium. More subtly, consider the case where $\theta = 0$ and $\kappa_l < \kappa < \kappa_u$. Recall that in this case, were the state publicly observed, both the M and the P would prefer the agents' ideal project. Consequently, the P would rubber stamps the recommendation of the M, and the M would reject the S's proposal. Under private information, however, dissension associated with disagreement makes the P believe falsely that $\theta = 1$, which leads to the adoption of the P's ideal project. As discussed before, when $\theta = 0$, even if he has to lose $\gamma$ of his claim, the M still prefers her ideal project. Hence, to avoid such an adverse inference that the P attaches to dissension, the M shares his power with the S, ensuring that communication exhibits unanimity. By the same token, once agreement is reached, the S has no incentive to disclose a different signal given that his intrinsic gain from dissent is small. Therefore, when $\theta = 0$, collusion between the S and the M succeeds and both agents disclose $m = m_M = m_S$ such that $\chi(m) = (R_2, r_2, 0)$ in equilibrium. Observing unanimity in communication, the P rubber stamps their joint recommendation.

In contrast to $RS$, signals disclosed by the two agents are perfectly informative of the state under $SS$ if the status of the S is moderate (*i.e.* $\underline{\gamma} < \gamma < \overline{\gamma}$). In equilibrium, the P observes dissension in communication when $\theta = 1$ and unanimity when $\theta = 0$. Hence, she is able to update her beliefs about $\theta$ when making her decision, and thus the ex post allocation of real authority can be adapted to the true state. Specifically, when $\theta = 0$, the P cedes to the M real authority which he shares with the S under collusion. On the other hand, when $\theta = 1$, she cedes it to the stationary supervisor who gains intrinsically from dissent and fails to obtain control over execution. Consequently, at state $\theta = 0$, she maximizes aggregate initiative from the agents despite costly loss of control; at state $\theta = 1$, she chooses her ideal project, while losing the initiative gain. Hence, under $SS$ where the uninformed P is faced with two informed agents with slightly different opinion and similar relative status, strategic communication allows for equilibrium adoption that is contingent on the state of the world.

# 5 Equilibrium Choice of the Formal Regime

This section studies the P's equilibrium choice of the formal regime during the organization design, and the emergence of *adaptive diarchy*. As before, the foregoing analysis presumes that the type of the supervisor, $(v, \gamma)$, is exogenously given. We postpone the justification for this assumption until Remark 1 at the end of this section. The three propositions in the previous section establishes the equilibrium adoption of projects under different formal regimes. Compared to the complete-information benchmark summarized in Claim 1, the optimal adoption in the initiative region and the control region can be achieved under $AS$ and $RS$, respectively. That is, in equilibrium, $f(\kappa) = AS$ if $1 < \kappa < \kappa_l$, and $f(\kappa) = RS$ if $\kappa > \kappa_u$. Hence, we focus on the adaptation region $(\kappa_l, \kappa_u)$.

By Proposition 3, if $\gamma < \underline{\gamma} < \overline{\gamma}$ or $\gamma > \overline{\gamma} < \underline{\gamma}$, from the perspective of the P, $SS$ reduces to $AS$ or $RS$. Therefore, in the two cases, it suffices to compare $AS$ with $RS$. As already noted, when $\kappa_l < \kappa < \kappa_m$, $RS$ is equivalent to $AS$ in terms of the adoption of a project. For expository convenience, we suppose that $f(\kappa) = AS$ in this case. When $\kappa_m < \kappa < \kappa_u$, the P's expected payoffs from $AS$ and $RS$ are respectively:

$$\mathcal{U}_P^{AS}(\kappa) = \pi R_2 F(r_2|\theta = 0) + (1 - \pi) R_2 F(r_2|\theta = 1), \tag{11}$$

$$\mathcal{U}_P^{RS}(\kappa) = \pi R_1 F(r_1|\theta = 0) + (1 - \pi) R_1 F(r_1|\theta = 1). \tag{12}$$

The difference between $\mathcal{U}_P^{RS}(\kappa)$ and $\mathcal{U}_P^{AS}(\kappa)$ is thus given by

$$\mathcal{U}_P^{RS} - \mathcal{U}_P^{AS} = R_2 \mathbb{E}[F(r_1|\theta)](\kappa - \kappa_m) > 0. \tag{13}$$

Hence, when $\kappa_m < \kappa < \kappa_u$, the P chooses $RS$. In this scenario where the status of the S is too large or too small, the P fails to solicits information about the state under $SS$, and she has to compare costly loss of control with the initiative gain ex ante given her prior belief. Impossibility of ex post adaptation entails a welfare loss for the P. Specifically, she addresses neither her control concerns at state $\theta = 1$ when $\kappa_l < \kappa < \kappa_m$ nor her initiative concerns at state $\theta = 0$ when $\kappa_m < \kappa < \kappa_u$.

Now consider the case where the status of the S is moderate. That is, $\underline{\gamma} < \gamma < \overline{\gamma}$. By Proposition 3, under $SS$, signals disclosed by the two agents are now perfectly informative of the state, and thus the P can make decisions tailored to $\theta$. The expected payoff of the P from $SS$ is

$$\mathcal{U}_P^{SS}(\kappa) = \pi R_2 F(r_2|\theta = 0) + (1-\pi) R_1 F(r_1|\theta = 1). \tag{14}$$

It is straightforward that from the perspective of the P, $SS$ dominates either $AS$ or $RS$ in the initiative region $(\kappa_l, \kappa_u)$. Specifically, we have

$$\mathcal{U}_P^{SS}(\kappa) - \mathcal{U}_P^{AS}(\kappa) = \underbrace{(1-\pi) R_2 F(r_1|\theta = 1)(\kappa - \kappa_l)}_{\text{the control gain at } \theta = 1} > 0, \tag{15}$$

and, when $\kappa_m < \kappa < \kappa_u$,

$$\mathcal{U}_P^{SS}(\kappa) - \mathcal{U}_P^{RS}(\kappa) = \underbrace{\pi R_2 F(r_1|\theta = 0)(\kappa_u - \kappa)}_{\text{the initiative gain at } \theta = 0} > 0. \tag{16}$$

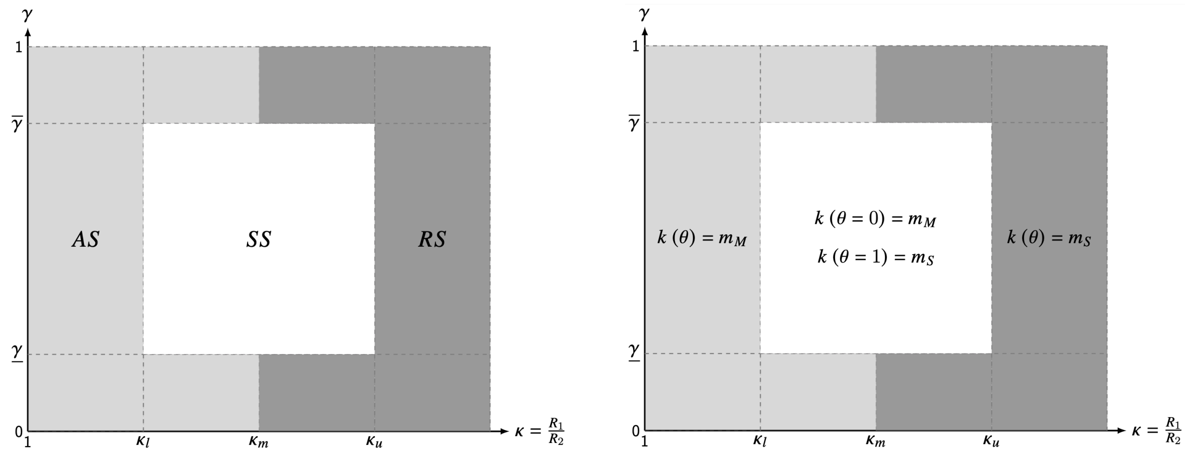
Allowing for contingent adoption ex post, $SS$ addresses both the control concerns and the initiative concerns when $\kappa_l < \kappa < \kappa_u$. On the one hand, compared to $AS$, the P knows that $\theta = 1$ when observing dissension in advice, and is able to adopt her ideal project at this state. On the other hand, compared to $RS$, collusion becomes possible, and the agents' aggregate incentives for execution are maximized at state $\theta = 0$ even when $\kappa_m < \kappa < \kappa_u$. Hence, our main result concerning the equilibrium choice of the formal regime is:

**Proposition 4** (Equilibrium Choice of the Formal Regime). *In equilibrium,*

(a) *If $\gamma < \underline{\gamma} < \overline{\gamma}$ or $\gamma > \overline{\gamma} > \underline{\gamma}$, the P chooses the Absent Supervisor, i.e. $f(\kappa) = AS$, when $\kappa < \kappa_m$; and chooses the Roving Supervisor, i.e. $f(\kappa) = RS$, when $\kappa > \kappa_m$;*

(b) *If $\underline{\gamma} < \gamma < \overline{\gamma}$, the P chooses the Absent Supervisor, i.e. $f(\kappa) = AS$, when $\kappa < \kappa_l$; and chooses the Stationary Supervisor, i.e. $f(\kappa) = SS$, when $\kappa_l < \kappa < \kappa_u$; and chooses the Roving Supervisor, i.e. $f(\kappa) = RS$, when $\kappa > \kappa_u$.*

Figure 1 illustrates Proposition 4. We say that *adaptive diarchy* emerges if the P chooses $SS$ during the organization design in equilibrium. Emergence of adaptive dairchy depends on the type of the supervisor, $(v, \gamma)$, and misalignment of interests, $\kappa$. That is, the P chooses $SS$ when the S is of moderate status and conflict of interest is of moderate magnitude. By Proposition 3, adaptive diarchy is characterized by three features: *existence of a stationary*




(a) Equilibrium ex ante choice of the formal regime

(b) Equilibrium ex post allocation of real authority

Figure 1: Equilibrium choice of the formal regime $f(\kappa)$ and ex post allocation of real authority $k(\theta)$

*supervisor*, *adaptive allocation of real authority*, and *the combination of slight difference of opinion and similar relative status*. Let us discuss each one:

- *Existence of a stationary supervisor:* Appointment of a supervisor with a tendency to exhibit dissension makes information about her ideal project revealed in communication. As discussed earlier, under both $RS$ and $SS$, the P ends up choosing her ideal project under some conditions. However, if the S is roving without any access to execution in the final stage, the P retains her prior belief about the state, and thus her decision $k(m_M, m_S)$ can never be tailored to $\theta$. That is, $RS$ is diarchial but not adaptive. In contrast, when the S is stationary, collusion between the two agents becomes possible. Consequently, depending on $\theta$, their strategic disclosure exhibits dissension or unanimity, from which the P in turn acquires information about $\theta$. Under $SS$, the two signals $m_M$ and $m_S$ are perfectly informative of the state. The P is then able to make her decision $k$ that is contingent on the true state. Hence, under appropriate conditions specified in Proposition 3 and Proposition 4, *existence of a stationary supervisor leads to adaptive diarchy*.

- *Adaptive allocation of real authority:* Abusing notation, we write the equilibrium ex post allocation of real authority as $k(\theta) \in \{m_M(\theta), m_S(\theta)\}$. During the decision making, the P cedes real authority to the informed agent whose recommendation she rubber stamps. Figure 1 depicts $k(\theta)$ associated with different equilibrium choices of formal regimes. Notice that when the P chooses $SS$ in equilibrium and adaptive diarchy emerges, $k(\theta)$ is contingent on $\theta$. Specifically, when $\theta = 0$, the P cedes real authority to the M who has shared his power with the S to ensure unanimous advice in communication. On the other hand, when $\theta = 1$, real authority within the organization is held by the S who never advocates the M's advice with his proposal on collusion being rejected by the latter. Under $SS$, the P makes her decision after having known the true state and success/failure of collusion, which causes the ex post allocation of real authority to be tailored to $\theta$. Hence, *although ex ante contingent delegation is impossible with the P's lack of necessary expertise, the allocation of real authority is adaptive ex post when adaptive diarchy emerges*.

- *The combination of slight difference of opinion and similar relative status:* As already seen, the type of the S, $(v, \gamma)$, matters for disclosure behavior under $SS$. We say that $SS$ is *effective* if signals $m_M$ and $m_S$ are perfectly informative of the state in equilibrium; otherwise, it becomes ineffective and reduces to $AS$ or $RS$. Figure 2 illustrates how the equilibrium choice of the formal regime depends on the type of the S in the initiative region, with details discussed in Remark 1. As shown in the figure, $SS$ is effective under two conditions concerning the supervisor's type. First, the intrinsic gain that the S obtains from dissension is positive but sufficiently small that $0 < v < \bar{v}(\gamma)$. Were difference of opinion too large, the S could never commit to unanimous advice in communication so that collusion between the two agents would always fail. $SS$ would reduce to $RS$. Second, the S is of similar relative status in the sense that $\underline{\gamma} < \gamma < \overline{\gamma}$. Were $\gamma < \underline{\gamma}$ or $\gamma > \underline{\gamma}$, the M would accept or reject the S's proposal regardless of the state. Hence, *adaptive diarchy emerges with two agents of slight difference of opinion and similar relative status*.

**Remark 1.** We have thus far assumed that the supervisor's type $(v, \gamma)$ is exogenously given and, in particular, $v = \epsilon$. In contrast, we can suppose that the P selects a supervisor from an available pool when choosing $SS$ during the organization design. Because adaptive diarchy

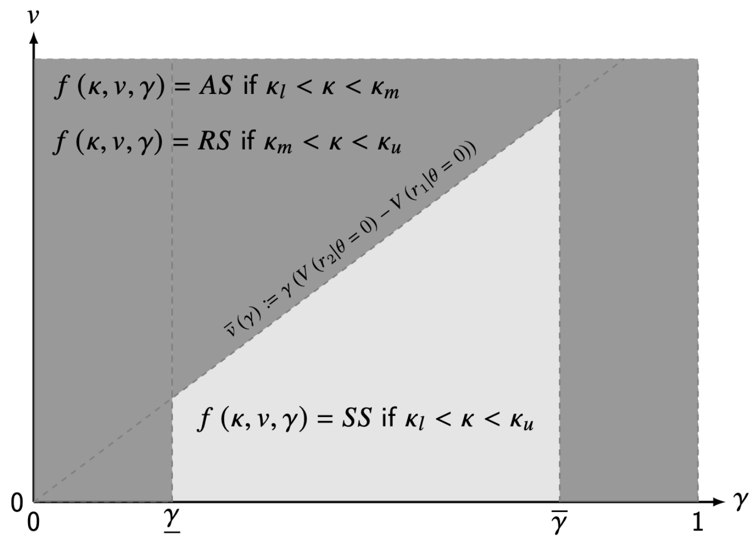



Figure 2: Type of the supervisor $(v, \gamma)$ and equilibrium choice of the formal regime $f(\kappa, v, \gamma)$ in the adaptation region $(\kappa_l, \kappa_u)$

leads to optimal adoption of projects in the initiative region, when there is a rich pool of possible supervisors in terms of their types $(v, \gamma)$, the P prefers to appoint a supervisor to make $SS$ effective. Specifically, by Proposition 3, the necessary condition for it is that the two agents have similar relative status. That is, $\underline{\gamma} < \gamma < \overline{\gamma}$. Given the supervisor's status $\gamma$, should his proposal be accepted by the M at state $\theta = 0$, he commits to recommending the agents' ideal project and ensuring unanimous advice if

$$v < \bar{v}(\gamma) := \gamma\left(V(r_2|\theta = 0) - V(r_2|\theta = 0)\right). \tag{17}$$

Hence, $SS$ is effective in the adaptation region if and only if the supervisor is of type $(v, \gamma)$ such that

$$0 < v < \bar{v}(\gamma) \quad \text{and} \quad \underline{\gamma} < \gamma < \overline{\gamma}. \tag{18}$$

If the available pool is sufficiently rich, in the adaptation region, the P will appoint a supervisor whose type satisfies (18). However, if the P is faced with a very limited pool of potential supervisors, an appropriate one may never exist, so that she can only choose between $AS$ and $RS$ during the organization design. It is often the case that an organization fails to find a supervisor who has only slight difference of opinion. Hence, to keep matters simple, we either assume that the P can select a supervisor from a rich pool or that the type of the S is exogenously given and, especially, $\upsilon = \epsilon > 0$.

# 6 Dynamic Extension and Regime Shift

We have studied the conditions that adaptive diarchy emerges in equilibrium, highlighting how strategic communication allows for ex post allocation of real authority under effective $SS$. However, as already seen in the introduction, it is often the case that adaptive diarchy appears as a result of regime shift, with roving supervisors being initially chosen and gradually becoming stationary. In this section, we embed our baseline setting in an infinite-horizon model to rationalize this historical regime shift. Specifically, we introduce the need of experimentation for the P (Bergemann and Hege (2005)), and investigate implications this has on the regimes shift. The main idea is that given that the P cedes execution of her decisions to agents, she is now uncertain about the feasibility of her ideal project at the outset. Consequently, she has to adjust her choices of formal regimes when learning about the actual extent to which enhancement of control erodes agents' participation. Adaptive diarchy emerges following some histories of outcomes.

**Learning Concerns.** To introduce the *learning concerns* of the P, we assume that she is initially uncertain about the feasibility of her ideal project. Specifically, the payoff of the project is now $(R_1, \widehat{r_1}, 0)$ with $\widehat{r_1} \in \{0, r_1\}$. By Assumption 1, if $\widehat{r_1} = 0$, the project will never be implemented by agents regardless of the state. The prior belief of the P is

$$\Pr\left(\widehat{r_1} = r_1\right) = \alpha_0 \in (0,1)\,. \tag{19}$$

To keep things simple, we assume that the P lives infinitely, whereas all agents (the M and the S) lives only one period. Therefore, in each period, the P chooses the formal regimes and appoints new agents. We suppose that the available pool of possible agents is so rich that we can treat their types as exogenously given (see Remark 1). Hence, our analysis in this section presumes that all supervisors have difference of opinion, $\upsilon = \epsilon > 0$, and relative status, $\underline{\gamma} < \gamma < \overline{\gamma}$. All agents privately observes $\widehat{r_1}$. Because all signals are soft or nonverifiable, agents always claim that $\widehat{r_1} = 0$, so that communication about $\widehat{r_1}$ is completely uninformative. Hence, we can still restrict attention to cheap-talk recommendation. Moreover, we assume that in each period, the state of the world $\theta$ and project identities $\chi$ are independently redrawn. Hence, *the P can only sequentially learn the feasibility of her ideal project, $\widehat{r_1}$.*

**Stage Game.** We focus on the case where $\kappa > \kappa_u$, so that the P prefers to adopt her ideal project should she know that $\widehat{r_1} = r_1$. Consequently, she always appoints a supervisor, choosing between $RS$ and $SS$. In each period $t = 0, 1, 2, ...$, the P's belief about $\widehat{r_1}$ is $\alpha_t$. Analogous to the baseline model, the timing of the stage game is as follows.

1. *Organization design:* The P chooses the formal regime, $f_t \in \{RS, SS\}$, and appoints new agents. The S is of type $(v, \gamma)$ satisfying $v = \epsilon > 0$ and $\underline{\gamma} < \gamma < \overline{\gamma}$. Agents privately observes $\widehat{r_1}$.

2. The state of the world $\theta_t \in \{0, 1\}$ and project identities $\chi_t : \mathcal{N} \to \mathcal{R}$ are redrawn. Both $\theta_t$ and $\chi_t$ are privately observed by agents. The P has prior belief that Pr $(\theta_t = 0) = \pi$.

3. *Power sharing:* The S and the M choose whether or not to reach agreement on power sharing in a veto bargaining.

4. *Decision making:* The M and the S simultaneously disclose $m_{M,t}, m_{S,t} \in \mathcal{M}$. The P then makes her decision $k_t \in \mathcal{N}$.

5. *Implementation stage:* Observing the implementation cost $c_t$ drawn from distribution $F(\cdot|\theta_t)$, agents choose whether or not to implement the chosen project.

Denote by $x_t \in \{0, 1\}$ the failure $(x_t = 0)$ or success $(x_t = 1)$ of the P's decision $k_t$. Notice that in each period $t$, the stage game depends only on the P's belief $\alpha_t$. Hence, our solution concept is Markov Perfect Equilibrium, which we refer to as simply equilibrium in this section.

**Two mechanisms of Learning.** Consider the stage game in period $t$ with the P's belief $\alpha_t$ given. Under both $RS$ and $SS$, the P can end up with learning the feasibility of her ideal project, albeit through distinct mechanisms. Let us outline each one:

- *Delegated experimentation under $RS$:* If $f_t = RS$, the agents disclose two different signals $m_{M,t}$ and $m_{S,t}$ such that $\chi(m_{M,t}) = (R_2, r_2, 0)$ and $\chi(m_{S,t}) = (R_1, \widehat{r_1}, 0)$. Hence, rubber-stamping the recommendation of the S, the P can learn $\widehat{r_1}$ directly from the outcome of execution. While failure of her ideal project $(x_t = 0)$ provide evidence indicating that $\widehat{r_1} = 0$, success $(x_t = 1)$ is a vindication of its feasibility. Denote $\overline{\pi} := 1 - \mathbb{E}_\theta [F(r_1|\theta)]$. Under $RS$ in period $t$, the P's posterior belief about $\widehat{r_1}$, conditional on the outcome of execution $x_t$, is computed via Bayes rule:

$$\alpha_{t+1} = \alpha^{RS}(\alpha_t) = \Pr(\widehat{r_1} = r_1 | x_t = 0) = \frac{\alpha_t \overline{\pi}}{1 - \alpha_t + \alpha_t \overline{\pi}} < \alpha_t, \tag{20}$$

$$\alpha_{t+1} = \Pr(\widehat{r_1} = r_1 | x_t = 1) = 1 \geq \alpha_t. \tag{21}$$

The speed of learning from delegated experimentation under $RS$ depends on the extent to which the P can attribute failure to the event $\widehat{r_1} = 0$ rather than the event $c_t > r_1$. Notice that $\overline{\pi}$ captures the probability that the P's project fails when $\widehat{r_1} = r_1$. Hence, it is straightforward that learning under $RS$ is more rapid the smaller $\overline{\pi}$ is.

- *Adaptive communication under $SS$:* Because the two agents are of slight difference of opinion and similar relative status, communication under $SS$ can be expected to take the form of either dissension or unanimity, depending on $\theta$ and $\widehat{r_1}$. The logic proceeds similarly with that underlying Proposition 3. Should $\widehat{r_1} = 0$, the M will accept the S's proposal, regardless of the state, to ensure unanimous advice, lest the P chooses to adopt her ideal project with a false inference about its feasibility that she attaches to dissension. On the other hand, if $\widehat{r_1} = r_1$, as already seen, collusion between the S and the M succeeds when $\theta = 0$, and fails when $\theta = 1$. Hence, signals disclosed by the two agents are informative of not only $\theta$ but also $\widehat{r_1}$ under $SS$. Whereas dissension in communication vindicates the viability of her ideal project, unanimous advice by agents implicitly warns the P against enhancement of control. Specifically, her posterior belief following communication is

$$\alpha_{t+1} = \alpha^{SS}(\alpha_t) = \Pr(\widehat{r_1} = r_1 | m_{M,t} = m_{S,t}) = \frac{\alpha_t \pi}{1 - \alpha_t + \alpha_t \pi} < \alpha_t, \tag{22}$$

$$\alpha_{t+1} = \Pr\left(\widehat{r_1} = r_1 | m_{M,t} \neq m_{S,t}\right) = 1 \geq \alpha_t. \tag{23}$$

The speed of learning from adaptive communication under $SS$ depends on the extent to which the P can attribute unanimous advice to the event $\widehat{r_1} = 0$ rather than the event $\theta = 0$. Hence, learning under $SS$ is more rapid the smaller $\pi$ is. It is worth noting that the P's ideal project is adopted only after she has observed dissension in communication and known that $\widehat{r_1} = r_1$. Consequently, in contrast to $RS$, the outcome of execution reveals no extra information to the P under $SS$.

**Dynamic Choices of Formal Regimes.** Let $W : [0,1] \to \mathbb{R}$ denote the P's value function. Suppose that $\kappa > \kappa_u$. In each period $t$, her belief about $\widehat{r_1}$ is given by $\alpha_t$. If choosing $RS$, she rubber stamps the recommendation the S who discloses $m_{S,t}$ such that $\chi_t\left(m_{S,t}\right) = \left(R_1, \widehat{r_1}, 0\right)$.[4] Hence, her continuation expected payoff from $f_t = RS$ is given by

$$\begin{aligned} W^{RS}\left(\alpha_t\right) &= \alpha_t \pi R_1 F\left(r_1\right) + \alpha_t\left(1-\pi\right) R_1 F\left(r_1\right) \\ &\quad + \delta\alpha_t\left(1-\overline{\pi}\right) W\left(1\right) + \delta\left(1-\alpha+\overline{\pi}\alpha_t\right) W\left(\alpha^{RS}\left(\alpha_t\right)\right). \end{aligned} \tag{24}$$

If choosing $SS$, the P rubber stamps the recommendation of the M if she receives unanimous advice ($m_{M,t} = m_{S,t}$), and that of the S if she observes dissension in communication ($m_{M,t} \neq m_{S,t}$). Consequently, her continuation expected payoff from $f_t = SS$ is

$$\begin{aligned} W^{SS}\left(\alpha_t\right) &= \pi R_2 F\left(r_2 | \theta = 0\right) + \alpha_t\left(1-\pi\right) R_1 F\left(r_1 | \theta = 0\right) \\ &\quad + \left(1-\alpha_t\right)\left(1-\pi\right) R_2 F\left(r_2 | \theta = 0\right) \\ &\quad + \delta\alpha\left(1-\pi\right) W\left(1\right) + \delta\left(1-\alpha_t+\alpha_t\pi\right) W\left(\alpha^{SS}\left(\alpha_t\right)\right). \end{aligned} \tag{25}$$

The equilibrium choices of formal regimes can be characterized by the following condition:

$$W\left(\alpha_t\right) = \max\left\{W^{RS}\left(\alpha_t\right), W^{SS}\left(\alpha_t\right)\right\} \quad \text{for all} \quad \alpha_t \in [0,1]. \tag{26}$$

[4] Rigorously, the P may choose to rubber stamps the recommendation of the M should her belief $\alpha_t$ is sufficiently low. However, in this case, $SS$ strictly dominates $RS$ in terms of both adaptation concerns and learning concerns of the P. Hence, to keep matters simple, we ignore this case in the text, presuming either that $\alpha_t$ is not too low or that $\kappa$ is very large.

We can now state:

**Proposition 5** (Dynamic Choices of Formal Regimes). *There exists $\alpha^*(\kappa)$ such that $\alpha^*(\kappa)$ is decreasing in $\kappa$ and $\lim_{\kappa \to \kappa_u} \alpha^*(\kappa) = 1$. Suppose $\kappa > \kappa_u$. There exist a Markov Perfect Equilibrium so that in each period $t$, the P chooses Roving Supervisor, i.e. $f_t = RS$, if $\alpha_t > \alpha^*(\kappa)$; and chooses Stationary Supervisor, i.e. $f_t = SS$, if $\alpha_t < \alpha^*(\kappa)$.*

To understand this result, notice that when $\kappa > \kappa_u$, the P has two concerns: the initiative concerns and the control concerns. In contrast to the static model, the former concerns stem from not only the state $\theta$, but her ideal project's feasibility $\widehat{r_1}$ as well. That is, on the one hand, faced with the state-dependent trade-off between initiative and loss of control, the P may prefer to make her decision that is contingent on $\theta$; on the other hand, she has to ensure the participation constraint should $\widehat{r_1} = 0$. In Claim 1 and Proposition 4, the relative magnitude of the two concerns depends solely on conflict of interest $\kappa$. Were $\widehat{r_1} = r_1$, the control concerns should have dominated when $\kappa > \kappa_u$. However, because of the uncertain feasibility, it is now determined by the P's belief $\alpha_t$ in addition. If her belief is sufficiently low, it matters more to ensure the participation constraint of agents. Consequently, the adaptation concerns emerges, and the P prefers to choose $SS$ under which she is able to make her decision tailored to $\widehat{r_1}$. To the contrast, if she believes that it is very likely that her ideal project is feasible, the P will prioritise her control concerns, appointing a roving supervisor to solicit dissenting advice.

To see the implication that Proposition 5 has on the regime shift and the emergence of adaptive diarchy, the following corollary may be useful.

**Corollary 1** (Regime Shift). *There exists $\kappa^*(\alpha_t)$ such that $\kappa^*(\alpha_t)$ is decreasing in $\alpha_t$ and $\lim_{\alpha_t \to 0} \kappa^*(\alpha_t) = +\infty$. Suppose $\kappa > \kappa_u$. There exist a Markov Perfect Equilibrium so that in each period $t$, The P chooses Roving Supervisor, i.e. $f_t = RS$, if $\kappa > \kappa^*(\alpha_t)$; and chooses Stationary Supervisor, i.e. $f_t = SS$, if $\kappa < \kappa^*(\alpha_t)$.*

Figure 3 depicts a possible equilibrium path through which formal regimes evolves from $RS$ to $SS$. From the perspective of the P who is uncertain about the feasibility of her ideal project, the adaptation region covers $(\kappa_u, \kappa^*(\alpha_t))$. If $\kappa > \kappa^*(\alpha_0)$, she chooses $RS$ at

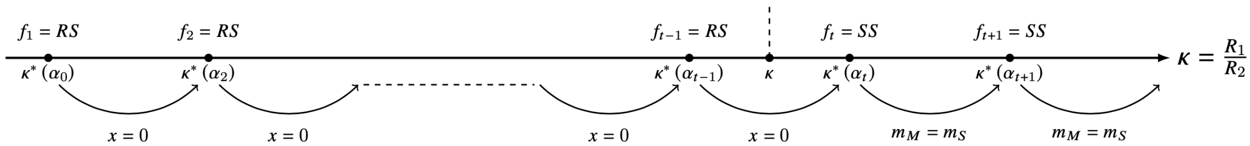



Figure 3: Equilibrium Regime Shift from $RS$ to $SS$

the outset. However, if execution of her initial decision turns out to fail, she downgrades her belief and extends the perceived adaptation region rightward. Should she witness so many failures associated with her enhancing control that $\kappa^*(\alpha_t) > \kappa$, she will choose $SS$ to make her decision adaptive, tolerating collusion between agents to avoid costly violation of their participation constraint. Hence, *adaptive diarchy emerges in the process of the P's learning about the extent to which enhancement of control erodes agents' participation*. Notice that once the P observes a success under $RS$ or receives dissenting advice under $SS$, the feasibility of her ideal project will be vindicated. Henceforth, she will return to $RS$ and never adjust formal regimes. In this sense, $SS$ evolving from $RS$ in the region $(\kappa_u, +\infty)$ is not stable. However, one can think that the P may be faced with adjustment costs increasing with time. Then, at some point of time, even if she observes dissension in communication, changing the formal regime from $SS$ to $RS$ might impose high adjustment costs to her that outweigh the benefits from eliminating collusion between agents. Consequently, in this scenario, the P chooses to preserve $SS$.

# 7 Conclusion

This paper explores the rationale behind diarchy in lens of organizational economics. We start from four intriguing features observed in diarchial regimes prevailing in China. First, one of the agent is the de jure stationary supervisor appointed by the principal. Second, and most importantly, the allocation of authority is ex ante ambiguous. Third, the two agents have similar status, ranks, and even other types. Finally, it is often the case that roving supervisors become stationary, and de facto diarchy emerges. Our model focuses on the ex post allocation of real authority within a diarchial organization, allowing for collusion between the agents. In equilibrium, we establish the conditions under which adaptive diarchy emerges in equilibrium. We show that by appointing a stationary supervisor with slight difference of opinion and similar relative status to oversee an informed agent, the principal can makes her decision tailored to the state of the world. Although ex ante contingent delegation is impossible, the ex post allocation of real authority is adaptive when diarchy is effective. Our results exemplify how bureaucracy can accommodate adaptation and control in the face of rigid formal institutions. By introducing some seemingly inefficient regimes with inherent ambiguity, the allocation of de facto authority within an organization can be surprisingly adapted to a changing world.

# References


Acemoglu, Daron, J. A. R. and Verdier, T. (2004). Kleptocracy and divide-and-rule: A model of personal rule. *Journal of the European Economic Association*, 2(2/3):162–192.

Aghion, Philippe, N. B. B. L. R. S. and Reenen, J. V. (2021). Turbulence, firm decentralization, and growth in bad times. *American Economic Journal: Applied Economics*, 13(1):133–169.

Aghion, P. and Bolton, P. (1992). An incomplete contracts approach to financial contracting. *The Review of Economic Studies*, 59(3):473–494.

Aghion, P. and Tirole, J. (1997). Formal and real authority in organizations. *Journal of Political Economy*, 105(1):1–29.

Aghion, Philippe, P. R. and Dewatripont, M. (2004). Transferable control. *Journal of the European Economic Association*, 2(1):115–138.

Alonso, Ricardo, W. D. and Matouschek, N. (2008). When does coordination require centralization? *The American Economic Review*, 98(1):145–179.

Ambrus, A. and Takahashi, S. (2008). Multi-sender cheap talk with restricted state spaces. *Theoretical Economics*, 3(1):1–27.

Bai, Chong-en, C.-T. H. and Song, Z. (2020). Special deals with chinese characteristics. *NBER Macroeconomics Annual*, 34(1):341–379.

Battaglini, M. (2002). Multiple referrals and multidimensional cheap talk. *Econometrica*, pages 1379–1401.

Bergemann, D. and Hege, U. (2005). The financing of innovation: Learning and stopping. *The RAND Journal of Economics*, 36(4):719–752.

Bernheim, B. D. and Whinston, M. D. (1999). Incomplete contracts and strategic ambiguity. *American Economic Review*, 88(4):902–932.

Bolton, P. and Dewatripont, M. (2013). Authority in organizations. In *Authority in Organizations*, pages 342–372. Princeton University Press.

Callander, S. and Krehbiel, K. (2014). Gridlock and delegation in a changing world. *American Journal of Political Science*, 58(4):819–834.

Chakraborty, A. and Harbough, R. (2010). Persuasion by cheap talk. *The American Economic Review*, 100(5):2361–2382.

Che, Y.-K. and Kartik, N. (2009). Opinions as incentives. *Journal of Political Economy*, 117(5):815–860.

Crawford, V. P. and Sobel, J. (1982). Strategic information transmission. *Econometrica*, 50(6):1431–1451.

Dessein, Wouter, L. G. and Gertner, R. (2010). Organizing for synergies. *American Economic Journal: Microeconomics*, 2(4):77–114.

Dessein, W. (2002). Authority and communication in organizations. *The Review of Economic Studies*, 69(4):811–838.

Dessein, W. and Santos, T. (2006). Adaptive organizations. *Journal of Political Economy*, 114(5):956–995.

Dewatripont, M. and Tirole, J. (1999). Advocates. *Journal of Political Economy*, 107(1):1–39.

Dragu, Tiberiu, X. F. and Kuklinski, J. (2014). Designing checks and balances. *Quarterly Journal of Political Science*, 9(1):45–86.

Farrell, J. and Rabin, M. (1996). Cheap talk. *Journal of Economic Perspectives*, 10(3):103–118.

Fayol, H. (2016). *General and Industrial Management*. Ravenio Books.

Garicano, L. and Rayo, L. (2016). Why organizations fail: Models and cases. *Journal of Economic Literature*, 54(1):137–192.

Gibbons, Robert, N. M. and Roberts, J. (2013). Decisions in organizations. In *The Handbook of Organizational Economics*, pages 373–431. Princeton University Press.

Grossman, S. J. and Hart, O. D. (1986). The costs and benefits of ownership: A theory of vertical and lateral integration. *Journal of Political Economy*, 94(4):691–719.

Itoh, H. and Morita, K. (2023). Information acquisition, decision making, and implementation in organizations. *Management Science*, 69(1):446–463.

Krähmer, D. (2006). Message-contingent delegation. *Journal of Economic Behavior Organization*, 60:490–506.

Landier, Augustin, D. S. and Thesmar, D. (2009). Optimal dissent in organizations. *The Review of Economic Studies*, 76(2):761–794.

Landier, Augustin, J. S.-D. S. and Thesmar, D. (2013). Bottom-up corporate governance. *Review of Finance*, 17(1):161–201.

Li, W. (2019). Meritocracy and dual leadership: Historical evidence and and interpretation. *Working Paper*.

Rantakari, H. (2008). Governing adaptation. *The Review of Economic Studies*, 75(4):1257–1285.

Romer, T. and Rosenthal, H. (1978). Political resource allocation, controlled agendas, and the status quo. *Public Choice*, 33(4):27–43.

Sobel, J. (2013). Giving and receiving advice. In *Advances in Economics and Econometrics*, pages 305–341. Cambridge University Press.

# Appendix

## Proof for Lemma 1

Suppose $Support(F(\cdot|\theta)) = [\underline{x}, \overline{x}] \subseteq (0, +\infty)$.

For $\forall x \in (\underline{x}, \overline{x}]$, if $x' < x$, then MLRP implies that $f(x|\theta = 0)f(x'|\theta = 1) > f(x|\theta = 1)f(x'|\theta = 0)$. Therefore, $f(x|\theta = 0)\int_{\underline{x}}^{x} f(x'|\theta = 1)dx' > f(x|\theta = 1)\int_{\underline{x}}^{x} f(x'|\theta = 0)dx'$. Hence, $f(x|\theta = 0)F(x|\theta = 1) > f(x|\theta = 1)F(x|\theta = 0)$.

$\frac{F(x|\theta=0)}{F(x|\theta=1)}$ is thus increasing: $\frac{d}{dx}(\frac{F(x|\theta=0)}{F(x|\theta=1)}) = \frac{f(x|\theta=0)F(x|\theta=1) - f(x|\theta=1)F(x|\theta=0)}{F(x|\theta=1)^2} > 0$. Hence, $\frac{F(r_1|\theta=0)}{F(r_1|\theta=1)} < \frac{F(r_2|\theta=0)}{F(r_2|\theta=1)} \implies 1 < \frac{F(r_2|\theta=1)}{F(r_1|\theta=1)} < \frac{F(r_2|\theta=0)}{F(r_1|\theta=0)}$.

$\frac{d}{dx}(V(x|\theta)) = F(x|\theta) \implies V(x|\theta) = \int_{\underline{x}}^{x} F(s|\theta)ds$. Therefore, $\frac{F(x|\theta=0)}{F(x|\theta=1)} > \frac{\int_{\underline{x}}^{x} F(s|\theta=0)ds}{\int_{\underline{x}}^{x} F(s|\theta=1)ds} = \frac{V(x|\theta=0)}{V(x|\theta=1)}$ (here we use the fact that $\frac{F(x|\theta=0)}{F(x|\theta=1)}$ is increasing). Hence, $\frac{d}{dx}(\frac{V(x|\theta=0)}{V(x|\theta=1)}) > 0 \implies 0 < \frac{V(r_1|\theta=0)}{V(r_2|\theta=0)} < \frac{V(r_1|\theta=1)}{V(r_2|\theta=1)} < 1$.

## Proof for Proposition 2

For $\kappa > \kappa_m$, $P$ prefers the project with payoff $(R_1, r_1, 0)$. $S$ will leave the game before the implementation stage. Hence, collusion will surely fail ($d_M d_S = 0$). In equilibrium, $m_S \neq m_M$ (otherwise $S$ cannot receive rewards from communication and has an incentive to diverge).

If $\chi(m_M) = (R_1, r_1, 0)$ and $\chi(m_S) = (R_2, r_2, 0)$, then $k = m_M$ is $P$'s optimal strategy. $M$, however, prefers to diverge given $k = m_M$. If $\chi(m_M) = (R_2, r_2, 0)$ and $\chi(m_S) = (R_1, r_1, 0)$, then $k = m_S$ is $P$'s optimal strategy. Together, this is a perfect Bayesian equilibrium and is unique.

For $\kappa < \kappa_m$, $P$ prefers the project with payoff $(R_2, r_2, 0)$. Similarly, there exist two perfect Bayesian equilibria: (1) $\chi(m_M) = (R_2, r_2, 0)$, $\chi(m_S) = (R_1, r_1, 0)$ and $k = m_M$; (2) $\chi(m_M) = (R_1, r_1, 0)$, $\chi(m_S) = (R_2, r_2, 0)$ and $k = m_S$.

## Proof for Proposition 3

As long as $\gamma > 0$, $S$ always prefers $d_S = 1$ to $d_S = 0$. Therefore, whether or not collusion succeeds depends on the value of $d_M$. For $P$, her strategies are characterized by her belief of $\theta$ when $m_M \neq m_S$. Her belief is shown as follows:

$\theta = 0$ **if** $m_M \neq m_S$

For $\kappa \in (\kappa_l, \kappa_u)$, $P$ prefers the project with payoff $(R_2, r_2, 0)$ if she believes $\theta = 0$, which is the preferred project of $M$. Therefore, $M$ has no incentive to collude with $S$ and $m_M \neq m_S$ regardless of $\theta$. $P$ thus forms a wrong belief.

For $\kappa > \kappa_u$, $P$ always prefers the project with payoff $(R_1, r_1, 0)$. Therefore, $M$ has an incentive to collude with $S$. If $(1-\gamma)V(r_2|\theta) > V(r_1|\theta)$, then $d_M(\theta) = 1$ and $m_M(\theta) = m_S(\theta)$. The belief of $P$ suggests that $m_M(\theta = 1) = m_S(\theta = 1)$, which implies that $(1-\gamma)V(r_2|\theta = 1) > V(r_1|\theta = 1)$. Therefore, $(1-\gamma)V(r_2|\theta = 0) > V(r_1|\theta = 0)$ (according to Lemma 1) and $m_M(\theta = 0) = m_S(\theta = 0)$. Hence, $m_M = m_S$ regardless of $\theta$. $P$ thus forms a wrong belief.

$\theta = 1$ **if** $m_M \neq m_S$

When $\theta = 1$, $P$ prefers the project with payoff $(R_1, r_1, 0)$. Therefore, $M$ has an incentive to collude with $S$. As $\gamma \in (\underline{r}, \overline{r})$, then $d_M(\theta = 0) = 1$ and $d_M(\theta = 1) = 0$. Hence, there exists an only perfect Bayesian equilibrium as described in the proposition.

$P$ **cannot infer the value of** $\theta$

For $\kappa \in (\kappa_l, \kappa_u)$, with this belief, the best that $P$ can achieve is $max\{R_1E[F(r_1|\theta)], R_2E[F(r_2|\theta)]\}$, which is smaller than $\pi R_2F(r_2|\theta = 0) + (1-\pi)R_1F(r_1|\theta = 1)$ (the expected payoff achieved under the equilibrium described in the proposition).

For $\kappa > \kappa_u$, $P$ always prefers the project with payoff $(R_1, r_1, 0)$. Therefore, $M$ has an incentive to collude with $S$ and they will collude if and only if $\theta = 0$ (since $\gamma \in (\underline{r}, \overline{r})$). This suggests that $P$ can infer the value of $\theta$. $P$ thus forms a wrong belief.

In sum, the equilibrium described in the proposition is the unique perfect Bayesian equilibrium.

## Proof for Proposition 5

Denotes $W(\alpha_t)$ as the value function (i.e., the expected utility of $P$ under equilibrium path at period $t$ with belief $\alpha_t$).

If $P$ chooses $RS$ at period $t$, then the expected utility $W_1(\alpha_t)$ equals:

$$\begin{aligned} W_1(\alpha_t) &= \alpha_t[(1-\overline{\pi})R_1 + \delta(1-\overline{\pi})W(1) + \delta\overline{\pi}W(\frac{\overline{\pi}\alpha_t}{1-\alpha_t+\overline{\pi}\alpha_t})] + (1-\alpha_t)\delta W(\frac{\overline{\pi}\alpha_t}{1-\alpha_t+\overline{\pi}\alpha_t}) \\ &= \alpha_t(1-\overline{\pi})R_1 + \alpha_t\delta(1-\overline{\pi})W(1) + \delta(1-\alpha_t+\overline{\pi}\alpha_t)W(\frac{\overline{\pi}\alpha_t}{1-\alpha_t+\overline{\pi}\alpha_t}). \end{aligned}$$

If $P$ chooses $SS$ at period $t$, then the expected utility $W_2(\alpha_t)$ equals:

$$\begin{aligned} W_2(\alpha_t) =& \alpha_t\{\pi[R_2F(r_2|\theta=0) + \delta W(\frac{\pi\alpha_t}{1-\alpha_t+\pi\alpha_t})] + (1-\pi)[R_1F(r_1|\theta=1) + \delta W(1)]\} \\ &+ (1-\alpha_t)[R_2E[F(r_2|\theta)] + \delta W(\frac{\pi\alpha_t}{1-\alpha_t+\pi\alpha_t})] \\ =& \alpha_t(1-\overline{\pi})R_1 + R_2E[F(r_2|\theta)] - \alpha_t[\pi R_1F(r_1|\theta=0) + (1-\pi)R_2F(r_2|\theta=1)] \\ &+ \alpha_t\delta(1-\pi)W(1) + \delta(1-\alpha_t+\pi\alpha_t)W(\frac{\pi\alpha_t}{1-\alpha_t+\pi\alpha_t}). \end{aligned}$$

Hence, we have:

$$W(\alpha_t) = max\{W_1(\alpha_t), W_2(\alpha_t)\}. \tag{27}$$

By setting $\alpha_t = 0$, we get: $W(0) = \frac{1}{1-\delta}R_2E[F(r_2|\theta)]$. By setting $\alpha_t = 1$, we get: $W(1) = \frac{1}{1-\delta}(1-\overline{\pi})R_1$. Substitute $W(1)$ and we get:

$$W_1(\alpha_t) = \frac{1-\delta\overline{\pi}}{1-\delta}(1-\overline{\pi})R_1\alpha_t + \delta(1-\alpha_t+\overline{\pi}\alpha_t)W(\frac{\overline{\pi}\alpha_t}{1-\alpha_t+\overline{\pi}\alpha_t}).$$

$$\begin{aligned} W_2(\alpha_t) =& \{\frac{1-\delta\pi}{1-\delta}(1-\overline{\pi})R_1 - [\pi R_1F(r_1|\theta=0) + (1-\pi)R_2F(r_2|\theta=1)]\}\alpha_t \\ &+ \delta(1-\alpha_t+\pi\alpha_t)W(\frac{\pi\alpha_t}{1-\alpha_t+\pi\alpha_t}) + R_2E[F(r_2|\theta)]. \end{aligned}$$

Define $C[0,1] = \{G|G : [0,1] \to \mathcal{R} \text{ and } G \text{ is continuous}\}$. We now prove that there exists a unique $W \in C[0,1]$ that satisfies equation (27). The existence of $W$ will be shown later. We first prove that the solution to equation (27) is unique.

Suppose $W$ is a solution to equation (27) and $W_1$ and $W_2$ are constructed as above.

$W_2(0) > W_1(0)$, $W_2(1) < W_1(1)$ and $W(\cdot)$ is continuous. Therefore, $x_0 = inf\{x|W_1(x) \geq W_2(x)\} \in (0,1)$. For $x \in [0, x_0]$, $W_2(x) \geq W_1(x)$, so $W(x) = W_2(x)$. Denotes $g(x) = \frac{\pi x}{1 - x + \pi x}$. $g(x) < x$ and $\lim_{n\to\infty} g^{(n)}(x) = 0$ $(x \in (0,1))$. Therefore, by iteration, we have: $W(x) = W_2(x) = A_0 x + B_0 (x \in [0, x_0])$, where

$$A_0 = \frac{1}{1-\delta}(1-\overline{\pi})R_1 - \frac{1}{1-\delta\pi}[\pi R_1 F(r_1|\theta=0) + (1-\pi)R_2 F(r_2|\theta=1)] - \frac{\delta(1-\pi)}{(1-\delta)(1-\delta\pi)} R_2 E[F(r_2|\theta)].$$

$$B_0 = \frac{1}{1-\delta} R_2 E[F(r_2|\theta)].$$

For any $x > x_0$, by iteration, we can also settle down the value of $W(x)$. $W$ is therefore unique and is a piece-wise linear function.

We can solve $x_0$ by noticing that $W_1(x_0) = W_2(x_0)$:

$$x_0 = \frac{R_2 E[F(r_2|\theta)]}{\frac{1-\delta\overline{\pi}}{1-\delta\pi}[\pi R_1 F(r_1|\theta=0) + (1-\pi)R_2 F(r_2|\theta=1)] + \frac{\delta(\overline{\pi}-\pi)}{1-\delta\pi} R_2 E[F(r_2|\theta)]}.$$

It is easy to verify that $x_0$ is a decreasing function of $\kappa = \frac{R_1}{R_2}$. Denote the "implied misaligned interest" $\kappa^*$ as the inverse function of $x_0$ and we have $\kappa^*(x_0)$ which is a decreasing function of $x_0$.

Denote $\phi(x) = \frac{x}{\overline{\pi} + (1-\overline{\pi})x}$. Then $\phi(x)$ is increasing and $\lim_{n\to\infty} \phi^{(n)}(x) = 1$ $(x \in (0,1))$. Define $x_n = \phi(x_{n-1})(n = 1, 2, ...)$. Define $A_n = \frac{1-\delta\overline{\pi}}{1-\delta}(1-\overline{\pi})R_1 + \delta\overline{\pi}A_{n-1} - \delta(1-\overline{\pi})B_{n-1}$ and $B_n = \delta B_{n-1}$ $(n = 1, 2, ...)$. It is easy to see that $A_n$ is increasing and $B_n$ is decreasing.

Set $W(x) = A_n x + B_n$ for $x \in [x_{n-1}, x_n]$ $(x_{-1} = 0)$. We can easily verify that $W(\cdot)$ is a solution to equation (27).

By construction, for $x \in [0, x_0)$, $W_2(x) > W_1(x)$; for $x \in (x_0, 1]$, $W_1(x) > W_2(x)$. Therefore, $x_0$ is the threshold which satisfies: when $\alpha_t < x_0$, $P$ chooses $SS$; when $\alpha_t > x_0$, $P$ chooses $RS$. On the other hand, $\kappa^*(\alpha_t)$ is the threshold which satisfies: given $\alpha_t$, when $\kappa > \kappa^*(\alpha_t)$, $P$ chooses $RS$; when $\kappa < \kappa^*(\alpha_t)$, $P$ chooses $SS$.