Delegation and Decision Process in Organizations*

Hideshi Itoh† Kimiyuki Morita‡

March 28, 2023

Abstract

We study a three-stage decision process that consists of information acquisition, project choice, and execution of the selected project. There are a principal and an agent whose "favorite" projects are different and each receives a higher private benefit from the success of the favorite one than that of the other. The principal is more "biased" toward her favorite project. The agent is responsible for the first and third stages, and at the beginning of the relationship the principal decides and commits herself to the allocation of the right to choose a project to herself or to the agent. We show that (i) under some conditions, it is optimal for the principal to delegate the authority over project choice to the agent and give him complete control over the decision process; (ii) delegation may be more likely to be optimal than when the principal and the agent have the same favorite project; and (iii) the principal always chooses to retain the authority under the reduced, two-stage decision process where either the information acquisition stage or the execution stage is exogenously fixed.

JEL Classification Number: D23, D82, D83, M11.
Keyword: Decision Process, Delegation, Information Acquisition, Execution, Incentives

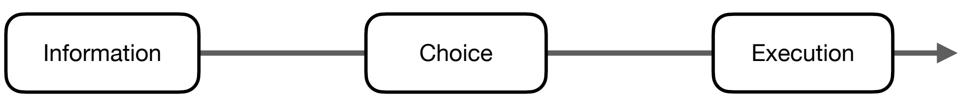
*Financial support from JSPS KAKENHI Grant Number 18H03640 (Itoh) is greatly appreciated.
†Waseda Business School, Waseda University, Tokyo 169-8050, Japan (e-mail: hideshi.itoh@waseda.jp)
‡School of Economics, Senshu University, Kanagawa 214-8580, Japan (e-mail: kimiyuki1229@gmail.com)

1 Introduction

Decisions in organizations take a number of steps. Mintzberg (1979) describes the decision process as the following four steps:[1] (i) collecting *information* about what can be done; (ii) presenting *advice* about what should be done; (iii) making the *choice*, i.e., deciding and authorizing what is intended to be done; (iv) *executing* what is in fact done. Based on his scheme, Gibbons et al. (2012) synthesize a number of models analyzing decisions in organizations. They first focus on the part of the process from advice to choice in a basic model, illustrate various models of strategic communication, and enrich the model to many directions, in particular, add information acquisition and reinterpret the model as the part of the process from choice to execution. They hence do not analyze the whole decision process, from information to execution at once.

In this paper we consider a three-stage decision process from information/advice to choice to execution (see Figure 1), in an organization that consists of a principal (she) and an agent (he). The agent exerts effort in information acquisition in the first stage as well as in execution decision in the third stages. It is often the case that the person who executes the decision is in a better position to acquire costly information valuable for decision making, and hence throughout the paper we fix these roles of the agent in the three-stage process.

Figure 1: Three-Stage Decision Process



Our main focus is on the allocation of the decision authority at the intermediate stage of choice, which we model as project choice. The right to choose a project at the second stage is contractible, and at the beginning of the relationship, the principal decides to retain the authority (P-authority) or delegate it to the agent (A-authority), and commits herself to the allocation.

Delegation in our model implies that the principal offers the agent complete control over the decision process, from information acquisition to project choice to implementation. We do not consider contingent monetary transfers in the main model, and hence the principal has no instrument to influence the agent's decisions once she commits herself to A-authority.

[1]There are actually five steps in Mintzberg (1979), who distinguishes between making the choice and authorizing the choice. Mintzberg himself attributes this framework to T. T. Paterson, *Management Theory*, Business Publications Ltd., 1969.

Does the principal still want to give complete control over the project selection and execution to the agent?[2]

Specifically, the principal and the agent have to select and implement one of two projects. A project succeeds if and only if it "fits" the true state of nature and the agent exerts high effort for execution of the project. The projects are ex ante symmetric and equally likely to fit the true state. The agent exerts information-gathering effort before project choice that raises the probability of generating an informative signal. While the principal and the agent both prefer success to failure, each of them favors one of two projects, *ceteris paribus*, and enjoys a higher private benefit from the success of the favorite project than that of the other, disfavored one. In the main model we focus on the *heterogeneous* case in which their favorite projects are different. Furthermore, the principal is more "biased" toward her favorite project than the agent. For example, project choice matters more at the principal, who is in charge of managing many projects and obtain larger benefits from the other ongoing projects if her preferred project is executed.

In this setting, we show three main results. First, under some conditions, it is optimal for the principal to delegate the authority over project choice to the agent. Second, under some conditions, delegation is more likely to be optimal in the heterogeneous organization than in the *homogeneous* organization where the principal and the agent have the same favorite project. Third, the three-stage decision process is necessary for delegation to be optimal: the principal always chooses to retain the authority under the reduced, two-stage decision process where either the information acquisition stage or the execution stage is exogenously fixed.

The first main result identifies two cases in which delegation can be optimal. The first case is that additional information obtained in the first stage is so noisy that the decision maker who selects project does not react to the information but sticks to her or his favorite project irrespective of the outcome of the information acquisition stage. The agent still has an incentive to collect costly information in order to improve his execution decision (called *the implementation motivation effect*), and his incentive is stronger when the project to be

[2] There are many anecdotes about big companies establishing independent units within the companies. For example, Sarah Lacy writes as follows:

> Google has a new policy..., according to several sources close to the company...of "autonomous units" within the company...who do have the freedom to run like independent startups with almost no approvals needed from HQ, according to our sources.

—Sarah Lacy, Google Takes Another Big Step to Retain Employees: Autonomous Business Units, *TechCrunch*, December 17, 2010.
https://techcrunch.com/2010/12/17/google-takes-another-big-step-to-retain-employees-autonomous-business-units/

executed is his favorite one than when it is his disfavored project. Delegation is optimal for the principal when its incentive effects on information acquisition and implementation are sufficiently strong, and the execution of her disfavored project is not very costly (her bias is not too high).

We also find another case where delegation can become optimal. Suppose that in contrast to the first case, the signal precision is intermediate and the principal is strongly biased toward her favorite project such that the agent under A-authority reacts to the signal indicating his disfavored project, but the principal under P-authority does not. This *reactivity effect* provides the agent under A-authority with stronger incentives to gather information in addition to the implementation motivation effect. Delegation is optimal if the advantage of delegation from higher information-gathering effort more than offset the disadvantage from the execution of the principal's disfavored project.

The second main result of the paper is that if the same project is favored by both principal and the agent, delegation may no longer be optimal. This is clear in the first case for the optimality of delegation explained above. In that case the principal chooses delegation since the agent is more motivated to acquire information and execute his favorite (and the principal's disfavored) project. This advantage no longer exists in the homogeneous organization since their favorite project is the same, and hence the principal has no reason for delegation.

The third main result is that the three-stage decision process is an essential feature behind the optimality of delegation. We consider two alternative decision processes. Suppose first that the decision process consists of information acquisition and project choice, and the selected project is implemented with some fixed probability. The agent then no longer exerts information-gathering effort if the decision maker does not react to additional information: There is no implementation motivation effect. There still exists an intermediate range of the signal precision where only the agent under A-authority is reactive and the information-gathering effort is higher under A-authority than under P-authority. However, the more biased principal does not want the agent to react, and hence retains the authority.

Suppose next that there is no information acquisition by the agent and instead informative signals realize with some fixed probability. The decision process is then two-stage, from project choice to execution. While the implementation motivation effect is at work and the agent is more highly motivated to execute the project under A-authority, the principal enjoys higher private benefit under P-authority, and the latter effect dominates the former one because she is more biased. Furthermore, the principal never delegates the authority even if the decision maker is reactive only under delegation. The reactivity advantage alone does not make delegation optimal for the principal.

We offer literature review in the rest of this section. In Section 2 we present the model of the three-stage decision process and the analysis and the first and second results are presented in Section 3. In Section 4 we study alternative two-stage decision processes. In Section 5 we summarize the results and discuss applications and future research.

Related literature While Gibbons et al. (2012) present quite a few papers that study either from information to choice or from choice to execution in the decision process, they do not introduce any work that cover all three stages of the decision process. One exception is our recent work Itoh and Morita (2023), which in turn extends the choice-execution model of Landier et al. (2009). Delegation is not an issue in these papers, where the party who does not engage in execution makes project choice.

As Gibbons et al. (2012) summarize, there are many reasons for delegation. The reason for delegation in this paper is to motivate the agent. In this respect, several existing papers are closely related. Aghion and Tirole (1997) and Che and Kartik (2009) study the information-choice part of the decision process and ask whether delegation motivates the agent's information acquisition. In the original model of Aghion and Tirole (1997) both the principal and the agent exert information-gathering efforts. Baker et al. (1999) and Gibbons et al. (2012) modify the model by fixing the probability that the principal is informed, and show that delegation raises the agent's marginal benefit of becoming informed and hence the agent is induced to choose higher information-gathering effort. In their model, however, the delegation decision does not matter when both the principal and the agent are uninformed. This is in contrast to our model where the agent is better off by choosing his favorite project conditional on the realization of the uninformed signal. Che and Kartik (2009) in fact show that with the "complacency" effect that under delegation the agent suffers less from the lack of additional information, delegation is never optimal. Although the model and driving forces are different from ours, it goes hand in hand with our result that the principal never chooses delegation under the information-choice decision process.

Bester and Krähmer (2008) and Zábojník (2002) instead analyze the role of delegation in motivating the agent to implement the selected project in the choice-execution decision process. Like us both papers argue that the principal's delegation decision takes into consideration its effects on the agent's implementation motivation. Zábojník (2002) shows that even if the principal is better informed than the agent, she may choose delegation because it can save the limited-liability rent that is necessary to induce the agent to choose high implementation effort. Bester and Krähmer (2008) show that providing the agent with ex post execution incentives works against delegation when incentive contracts are restricted with the limited liability constraints. Their result is also consistent with our result that the principal retains authority under the choice-execution decision process without incentive contracting accords with theirs.

Compared with these earlier contributions that study incentive effects of delegation under the information-choice or choice-execution decision process, our contribution is to point out the importance of *compound* effects of motivating information acquisition *and* execution as the reason for delegation, that can arise only from the three-stage decision process.

2 The Model

We consider an organization that consists of a principal (female) and an agent (male) to select and execute a project. There are two projects 1 and 2, and two states of nature $\theta \in \{1, 2\}$. Project $d \in \{1, 2\}$ is *efficient* if and only if a project fits the state ($\theta = d$). We assume that each state is equally likely to realize, implying that the environment is highly uncertain.

After a project is selected, the agent exerts effort to execute the project. For simplicity, we assume effort is binary ($e \in \{0, 1\}$) and call $e = 1$ ($e = 0$) implementation (no implementation, respectively). Implementation $e = 1$ costs $\tilde{c}$ to the agent, which is random and distributed according to a cumulative distribution function $F(\cdot)$ with the corresponding density function $f(\cdot)$. We assume $F(0) = 0$ and $F(\cdot)$ is strictly increasing. For expositional simplicity, we assume $F(\cdot)$ is uniformly distributed over $[0, 1]$. No implementation $e = 0$ costs zero. The agent chooses e after observing the realized cost $\tilde{c}$.

The selected project d succeeds if and only if it is efficient ($d = \theta$) and the agent chooses $e = 1$: project efficiency and the implementation decision are perfect complements. If the project succeeds, the principal and the agent obtain private benefits B and b, respectively. Otherwise, they obtain nothing. We can interpret their private benefits as intrinsic motivation, perks on the jobs, acquisition of human capital, benefits from other ongoing projects, and so on.

The principal and the agent enjoy higher private benefits if her or his favorite project succeeds. The principal obtains $B = B_H$ ($B = B_L$) if her favorite (respectively, disfavored) project succeeds, where $0 < B_L < B_H$. Similarly, the agent obtains $b = b_H$ ($b = b_L$)

if his favorite (respectively, disfavored) project succeeds, where $0 < b_L < b_H$. When the principal and the agent favor the same project, we call such an organization *homogeneous*. The organization where their favorite projects are different is called *heterogeneous*. Denote $\Gamma \equiv B_H/B_L > 1$ as the principal's "bias" and $\gamma \equiv b_H/b_L > 1$ as the agent's "bias" toward favorite project. Throughout the paper we assume the principal is at least as biased as the agent.

$$\Gamma \geq \gamma.$$

This assumption represents the situation where project choice matters more at the top, for example, the principal is in charge of managing many projects and obtains larger benefits from other ongoing projects if her favorite project succeeds. Hence larger Γ can be interpreted as the project managed by the agent is less important for the principal relative to the other projects.[3]

Before project choice, the agent engages in information acquisition and generates a signal σ about the state of nature. The agent chooses information-gathering effort $\pi \in [0, 1]$ by incurring cost $\psi(\pi)$. We assume $\psi'(\pi) > 0$ and $\psi''(\pi) > 0$ for all $\pi \in (0, 1)$, and $\psi'(\pi) \downarrow 0$ as $\pi \downarrow 0$ and $\psi'(\pi) \uparrow +\infty$ as $\pi \uparrow 1$. With probability π, an informative signal $\sigma \in \{1, 2\}$ realizes, and an uninformative signal $\sigma = \phi$ realizes with probability $1 - \pi$. If $\sigma = \phi$, the posterior probability is the same as the prior. If $\sigma \in \{1, 2\}$, each of which occurs with equal probability, the posterior probability becomes $\Pr[\theta \mid \sigma] = \alpha > 1/2$ for $\theta = \sigma$, and $\Pr[\theta \mid \sigma] = 1 - \alpha < 1/2$ for $\theta \neq \sigma$, where α represents the precision of the signal. We consider α as an exogenous parameter and focus on the agent's motivation to gather information. This setting is natural, for example, α represents the technological or environmental uncertainty relevant to the project.

The principal decides whether or not to delegate authority over project choice to the agent before information acquisition. If the principal retains authority, we refer to this case as *P-authority*. Instead, if the principal delegates authority to the agent, we refer to this case as *A-authority*. The one with the authority over project choice is called the *decision maker*. The principal can ex ante commit herself to her delegation decision, and retains authority if indifferent.

The timing of decisions and information structure are summarized as follows.

1. The principal selects either P-authority or A-authority, which is observable to the agent.

[3] In Appendix B, we show that delegation may be less likely to be optimal because the principal tends to choose the agent's favorite project to increase his implementation motivation under P-authority if $\Gamma < \gamma$.

2. The agent chooses information-gathering effort $\pi \in [0, 1]$.

3. Signal $\sigma \in \{\phi, 1, 2\}$ realizes. We assume σ is observable to both the principal and the agent.[4]

4. Given available information, the decision maker chooses a project $d \in \{1, 2\}$, which is observable to the other party. We assume that if indifferent, the favorite project of the decision maker is selected.

5. The cost of implementation $\tilde{c}$ realizes and the agent observes it.

6. The agent makes implementation decision $e \in \{0, 1\}$.

7. The outcome of the project realizes.

3 Main Analysis

We first focus on the heterogeneous organization where the principal's (agent's) favorite project is project 1 (project 2, respectively), and solve for the subgame perfect equilibrium of the model by moving backwards, in the order of project implementation, project choice, information acquisition, and delegation. In Subsection 3.5, we consider the homogeneous organization where their favorite project coincides, and show how our results change.

3.1 Project Implementation

Suppose a signal σ realizes and project d with the agent's private benefit b is selected. The agent then observes the realized implementation cost $\tilde{c}$ and chooses $e = 1$ if and only if

$$\Pr[\theta = d \mid \sigma]b \geq \tilde{c}.$$

Since $\tilde{c}$ is distributed according to $F(c) = c$, the probability of implementation is given by $\Pr[e = 1 \mid b, d, \sigma] = F(\Pr[\theta = d \mid \sigma]b) = \Pr[\theta = d \mid \sigma]b$. To ensure $F(\Pr[\theta = d \mid \sigma]b) < 1$, we assume $b_H < 1$.

[4]In Appendix B, we consider the case where the signal is the agent's private and soft information and show that delegation still can be optimal under certain conditions.

3.2 Project Choice

P-authority

First, we analyze project choice under P-authority. The principal's expected benefit from project d with private benefits (B, b) conditional on signal σ is given by

$$\Pr[\theta = d \mid \sigma] \times F\left(\Pr[\theta = d \mid \sigma]b\right) \times B = \left(\Pr[\theta = d \mid \sigma]\right)^2 bB,$$

where $\Pr[\theta = d \mid \sigma] \times F(\Pr[\theta = d \mid \sigma]b)$ represents the success probability of project d (probability of $\theta = d$ and $e = 1$) conditional on σ. The principal's project choice depends on the principal's private benefit and the success probability, which in turn depends on the project's efficiency and the agent's implementation motivation.

If $\sigma = \phi$, project's efficiency of each project is the same and the principal obtains higher private benefit from her favorite project. The principal then chooses project 1 if and only if $(1/2)F(b_L/2)B_H \geq (1/2)F(b_H/2)B_L$ or $(1/4)b_L B_H \geq (1/4)b_H B_L$, which is equivalent to $\Gamma \geq \gamma$. If $\sigma = 1$, the principal's favorite project is more likely to be efficient, and thus, she chooses project 1. If $\sigma = 2$, the principal's disfavored project is more likely to be efficient and the agent is more likely to implement it. Therefore, there exists $\alpha_{\mathrm{P}} \in (1/2, 1)$ such that the principal chooses her disfavored project if and only if $\alpha > \alpha_{\mathrm{P}}$, where α_{P} is the solution to $\alpha F(\alpha b_H)B_L = (1-\alpha)F((1-\alpha)b_L)B_H$, that is,

$$\alpha_{\mathrm{P}} = \frac{\sqrt{\Gamma}}{\sqrt{\gamma} + \sqrt{\Gamma}}.$$

A-authority

Next, we analyze project choice under A-authority. The agent's net expected benefit of project d with his private benefit b conditional on signal σ is calculated as follows:

$$\Pr[\theta = d \mid \sigma]F(\Pr[\theta = d \mid \sigma]b)b - \int_0^{\Pr[\theta = d|\sigma]b} cf(c)dc = \int_0^{\Pr[\theta = d|\sigma]b} F(c)dc,$$

which we denote by $K(\Pr[\theta = d \mid \sigma]b)$. Under the uniform distribution, it is rewritten as $K(\Pr[\theta = d \mid \sigma]b) = (1/2)(\Pr[\theta = d \mid \sigma]b)^2$, which is increasing and convex in $\Pr[\theta = d \mid \sigma]b$. The convexity of $K(\cdot)$ naturally arises from the implementation decision by the agent and is a key to our main results. In Subsection 4.1, we consider the case where the selected project is exogenously implemented, and show that the results substantially change.

The agent's project choice depends on his private benefit and the project's efficiency. Similar to project choice under P-authority, if $\sigma \in \{\phi, 2\}$, the agent chooses his favorite and weakly more efficient project 2. On the other hand, if $\sigma = 1$, there exists $\alpha_{\mathrm{A}} \in (1/2, 1)$ such that the agent follows the signal $\sigma = 1$ and chooses project 1 if and only if $\alpha > \alpha_{\mathrm{A}}$, where α_{A} is the solution to $K(\alpha b_L) = K((1-\alpha)b_H)$, that is,

$$\alpha_{\mathrm{A}} = \frac{\gamma}{1+\gamma}.$$

Comparison

Table 1 summarizes the optimal project choice under each authority. We say the decision maker is *reactive* if he or she chooses his or her disfavored project following the signal indicating it is more likely to be efficient. We may also call P-authority (A-authority) reactive if the principal (the agent, respectively) is reactive. Which of them is more likely to be reactive depends on the biases as follows:

$$\alpha_{\mathrm{P}} > \alpha_{\mathrm{A}} \quad \text{if and only if} \quad \Gamma > \gamma^3.$$

The principal is less likely to be reactive if her bias is sufficiently larger than the agent's bias.

Table 1: Optimal Project Choice

	P-authority		A-authority	
	$\alpha \leq \alpha_{\mathrm{P}}$	$\alpha > \alpha_{\mathrm{P}}$	$\alpha \leq \alpha_{\mathrm{A}}$	$\alpha > \alpha_{\mathrm{A}}$
$\sigma = \phi$	project 1		project 2	
$\sigma = 1$	project 1		project 2	project 1
$\sigma = 2$	project 1	project 2	project 2	

3.3 Information Acquisition

This subsection analyzes how delegation affects the agent's incentive to gather information. His expected payoffs under P-authority and A-authority are, respectively, written by

$$\frac{\pi}{2}\left[K(\alpha b_L)+K((1-\alpha)b_L)+\mathbf{1}_{\alpha>\alpha_{\mathrm{P}}}\left(K(\alpha b_H)-K((1-\alpha)b_L)\right)\right]+(1-\pi)K\left(\frac{b_L}{2}\right)-\psi(\pi),$$
$$\frac{\pi}{2}\left[K((1-\alpha)b_H)+K(\alpha b_H)+\mathbf{1}_{\alpha>\alpha_{\mathrm{A}}}\left(K(\alpha b_L)-K((1-\alpha)b_H)\right)\right]+(1-\pi)K\left(\frac{b_H}{2}\right)-\psi(\pi),$$

where $\mathbf{1}_E$ is the indicator function, taking value one (zero) if E is true (false, respectively). The first-order condition for the optimal effort $\pi_i=\pi_i(\alpha)$, under i-authority for $i\in\{\mathrm{A},\mathrm{P}\}$ becomes, respectively, given as follows:

$$\frac{1}{2}\left[K(\alpha b_L)+K((1-\alpha)b_L)+\mathbf{1}_{\alpha>\alpha_{\mathrm{P}}}\left(K(\alpha b_H)-K((1-\alpha)b_L)\right)\right]-K\left(\frac{b_L}{2}\right)=\psi'(\pi), \tag{1}$$
$$\frac{1}{2}\left[K((1-\alpha)b_H)+K(\alpha b_H)+\mathbf{1}_{\alpha>\alpha_{\mathrm{A}}}\left(K(\alpha b_L)-K((1-\alpha)b_H)\right)\right]-K\left(\frac{b_H}{2}\right)=\psi'(\pi), \tag{2}$$

where the left-hand side (LHS) of each equation represents the marginal benefit from gathering additional information. Denote LHS of (1) by $y_{\mathrm{P}}(\alpha)$, and that of (2) by $y_{\mathrm{A}}(\alpha)$. Under P-authority, $y_{\mathrm{P}}(\alpha)$ discontinuously jumps at $\alpha=\alpha_{\mathrm{P}}$: Since the non-reactive principal always chooses the agent's disfavored project, the agent is strictly better off with the reactive principal given signal $\sigma=2$, even if the signal were very noisy (α close to $1/2$). Hence his incentive to gather information discontinuously rises at $\alpha=\alpha_{\mathrm{P}}$ beyond which the principal becomes reactive. Under A-authority, $y_{\mathrm{A}}(\alpha)$ does not jump at $\alpha=\alpha_{\mathrm{A}}$ since the agent himself makes project choice.

The agent's incentive to gather information depends on his implementation motivation and the reactivity of the decision maker. First, suppose the signal precision is so low that the decision maker is non-reactive under both authorities. The agent then exerts positive information-gathering effort even though additional information dose not change project choice. To see why, for example, consider A-authority. The marginal benefit under A-authority $y_{\mathrm{A}}(\alpha)$ is written by $(1/2)[K((1-\alpha)b_H)+K(\alpha b_H)]-K(b_H/2)$, which is equal to zero at $\alpha=1/2$ and strictly increasing in α. As α increases, the agent's motivation upon $\sigma=1$ decreases and that upon $\sigma=2$ increases. By the convexity of $K(\cdot)$, the latter positive effect dominates the former negative one. We call this effect the *implementation motivation effect*. The effect is similarly at work under P-authority.

Second, suppose the signal precision is so high that the decision maker is reactive under both authorities. Then, the reactivity increases the agent's motivation to gather information. For example, consider A-authority. If $\alpha>\alpha_{\mathrm{A}}$, the term $K(\alpha b_L)-K((1-\alpha)b_H)$, which is positive for $\alpha > \alpha_{\mathrm{A}}$, is added to the marginal benefit, increasing his motivation to gathering information. We call this effect the *reactivity effect.* The reactivity effect raises his marginal benefit even more under P-authority.

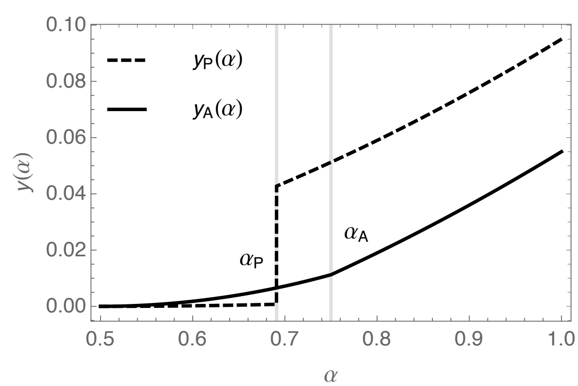
The following lemma shows which authority provides the agent with stronger incentive to acquire information.

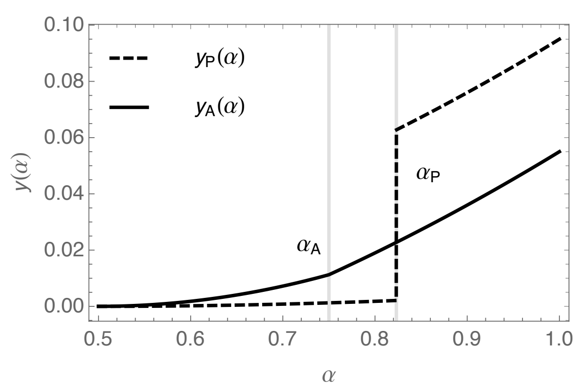
Lemma 1. *The agent's incentive to gather information differs between P-authority and A-authority as follows:*

- $\pi_{\mathrm{P}}(\alpha) < \pi_{\mathrm{A}}(\alpha)$ *if* $\alpha \in (1/2, \alpha_{\mathrm{P}}]$.
- $\pi_{\mathrm{P}}(\alpha) > \pi_{\mathrm{A}}(\alpha)$ *if* $\alpha \in (\alpha_{\mathrm{P}}, 1]$.

Lemma 1 shows that delegation increases the agent's initiative in terms of information gathering only if the precision of additional information is sufficiently low ($\alpha \le \alpha_{\mathrm{P}}$). Figure 2 illustrates the result by depicting the marginal benefit of information gathering under each authority. In both figures, we set $b_L = 0.2$, and the solid curve depicts $y_{\mathrm{A}}(\alpha)$ and the dashed curve $y_{\mathrm{P}}(\alpha)$. In the left-hand side figure, we set $\gamma = 3 < 15 = \Gamma$, and then, $\alpha_{\mathrm{A}} > \alpha_{\mathrm{P}}$. In the right-hand side figure, we set $\gamma = 3 < 65 = \Gamma$, and then, $\alpha_{\mathrm{A}} < \alpha_{\mathrm{P}}$.

Figure 2: Marginal Benefit of Information-Gathering Effort Under Two Authorities





To understand intuition behind the result, suppose first that the signal precision is so low that the decision maker does not react under either authority ($\alpha \le \min\{\alpha_{\mathrm{A}}, \alpha_{\mathrm{P}}\}$). Then, subtracting LHS of (1) from that of (2) yields the difference in the marginal benefits between the two authorities:

$$
\begin{aligned}
&y_{\mathrm{A}}(\alpha) - y_{\mathrm{P}}(\alpha)\\
&\quad = \frac{1}{2}\left[K((1-\alpha)b_H) + K(\alpha b_H)\right] - K\left(\frac{b_H}{2}\right) - \frac{1}{2}\left[K((1-\alpha)b_L) + K(\alpha b_L)\right] + K\left(\frac{b_L}{2}\right).
\end{aligned}
$$

When the decision maker is non-reactive, he or she chooses the favorite project upon the realization of all the signals, and the agent enjoys higher private benefit under A-authority: Since his private benefit b and the signal's precision α are complements, his favorite project provides him with higher marginal benefit from gathering information than the disfavored one.

Second, suppose that the signal precision is so high that the decision maker is reactive under both authorities ($\alpha > \max\{\alpha_{\mathrm{A}}, \alpha_{\mathrm{P}}\}$). Then, the difference in the marginal benefits between two authorities is written as follows:

$$y_{\mathrm{A}}(\alpha) - y_{\mathrm{P}}(\alpha) = K\left(\frac{b_L}{2}\right) - K\left(\frac{b_H}{2}\right) < 0.$$

Since the decision maker is reactive under both authorities, project choice upon the realization of informative signals $\sigma \in \{1, 2\}$ do not differ between two authorities, and project choice is different only upon the realization of the uninformative signal $\sigma = \phi$: While the agent chooses his favorite project under A-authority, the principal chooses the agent's disfavored project under P-authority. The agent hence has stronger incentive to gather information to avoid $\sigma = \phi$ under P-authority. We call this difference contingent on $\sigma = \phi$ the *ignorance-avoiding difference.*

Third, suppose that the principal has relatively strong bias ($\Gamma > \gamma^3$) and the decision maker is reactive only under A-authority ($\alpha \in (\alpha_{\mathrm{A}}, \alpha_{\mathrm{P}}]$). Then, the difference in the marginal benefits between two authorities is written as follows:

$$y_{\mathrm{A}}(\alpha) - y_{\mathrm{P}}(\alpha) = \frac{1}{2}\left[K(\alpha b_H) - K((1-\alpha)b_L)\right] + K\left(\frac{b_L}{2}\right) - K\left(\frac{b_H}{2}\right),$$

where $K(b_L/2) - K(b_H/2)$ is the ignorance-avoiding difference, and the first term, which is positive, reflects the difference in project choice contingent on $\sigma = 2$: While the non-reactive principal chooses the agent's disfavored project under P-authority, the agent chooses more efficient and his favorite project under delegation. The first term is increasing in α, and dominate the ignorance-avoiding difference for $\alpha > \alpha_{\mathrm{A}}$. The agent's information-gathering effort is thus higher under A-authority.

Finally, suppose that the principal has relatively weak bias ($\Gamma \leq \gamma^3$) and the decision maker is reactive only under P-authority ($\alpha \in (\alpha_{\mathrm{P}}, \alpha_{\mathrm{A}}]$). Then, the difference in the marginal benefit between two authorities is written as follows:

$$y_{\mathrm{A}}(\alpha) - y_{\mathrm{P}}(\alpha) = \frac{1}{2}\left[K((1-\alpha)b_H) - K(\alpha b_L)\right] + K\left(\frac{b_L}{2}\right) - K\left(\frac{b_H}{2}\right),$$

where $K(b_L/2) - K(b_H/2)$ is the ignorance-avoiding difference, and first term is positive for $\alpha \le \alpha_{\mathrm{A}}$, reflecting the difference in project choice upon $\sigma = 1$. In contrast to the previous case, the agent is non-reactive and chooses his favorite but less efficient project under delegation, and the first term is not high enough to offset the ignorance-avoiding difference. The agent's information-gathering effort is thus higher under P-authority.

3.4 Delegation

We now analyze under what conditions the principal delegates the authority over project choice to the agent. Let $U_i(\alpha)$ denote the principal's expected payoff under i-authority for $i \in \{\mathrm{P}, \mathrm{A}\}$, which is written as follows:

$$U_i(\alpha) = \pi_i(\alpha) u_i^{12}(\alpha) + (1 - \pi_i(\alpha)) u_i^{\phi},$$

where $u_i^{12}(\alpha)$ represents the principal's expected benefit conditional on $\sigma \in \{1, 2\}$ and u_i^{ϕ} the expected benefit conditional on $\sigma = \phi$. They are calculated as follows.

$$\begin{aligned} u_{\mathrm{P}}^{12}(\alpha) = \frac{1}{2} \Big[& \alpha F(\alpha b_L) B_H + (1-\alpha) F((1-\alpha) b_L) B_H \\ & + \mathbf{1}_{\alpha > \alpha_{\mathrm{P}}} \left(\alpha F(\alpha b_H) B_L - (1-\alpha) F((1-\alpha) b_L) B_H \right) \Big], \\ u_{\mathrm{A}}^{12}(\alpha) = \frac{1}{2} \Big[& (1-\alpha) F((1-\alpha) b_H) B_L + \alpha F(\alpha b_H) B_L \\ & + \mathbf{1}_{\alpha > \alpha_{\mathrm{A}}} \left(\alpha F(\alpha b_L) B_H - (1-\alpha) F((1-\alpha) b_H) B_L \right) \Big], \end{aligned}$$

and

$$u_{\mathrm{P}}^{\phi} = \frac{1}{2} F\left(\frac{b_L}{2}\right) B_H \ge u_{\mathrm{A}}^{\phi} = \frac{1}{2} F\left(\frac{b_H}{2}\right) B_L.$$

The principal's delegation decision depends on (i) projects' efficiency, (ii) the agent's implementation motivation and information-gathering efforts, and (iii) the principal's private benefit. Note that while the difference in project choice upon $\sigma = \phi$ between two authorities works as an advantage of delegation by raising the agent's motivation to acquire information, it works against delegation due to the principal's lower private benefit: $u_{\mathrm{P}}^{\phi} \ge u_{\mathrm{A}}^{\phi}$.

To keep the analysis tractable, we introduce a parameter $k > 0$ and adopt the following specific cost function:

$$\psi(\pi, k) = \frac{1}{k} \log_e \left(\frac{1}{1-\pi} \right), \tag{3}$$

where k is the parameter that reduces the marginal cost of information-gathering effort such as investment in information technology, the extent of the agent's discretion over his time allocation during work, the magnitude of organizational support for his activities, and so on. This cost function satisfies our assumption in Section 2, except for $\psi'(\pi) > 0$ for all $\pi > 0$. The optimal information-gathering effort is solved as

$$\pi_i(\alpha, k) = \max\left\{1 - \frac{1}{ky_i(\alpha)}, 0\right\},$$

where $y_i(\alpha)$ is the marginal benefit from gathering information under i-authority for $i \in \{\mathrm{P}, \mathrm{A}\}$. The optimal information-gathering effort is positive only for sufficiently high k,

$$\pi_i(\alpha, k) > 0 \quad \text{if and only if} \quad k > k_i(\alpha) \equiv 1/y_i(\alpha).$$

Note that $k_\mathrm{P}(\alpha) > k_\mathrm{A}(\alpha)$ for $\alpha \leq \alpha_\mathrm{P}$ and $k_\mathrm{A}(\alpha) > k_\mathrm{P}(\alpha)$ for $\alpha > \alpha_\mathrm{P}$. Under this specification, we rewrite the principal's expected payoff as $U_i(\alpha, k)$ under i-authority, $i \in \{\mathrm{P}, \mathrm{A}\}$. The principal delegates her authority if and only if $U_\mathrm{A}(\alpha, k) > U_\mathrm{P}(\alpha, k)$.

The following proposition, our first main result, shows under what conditions the principal delegates her authority to the agent.

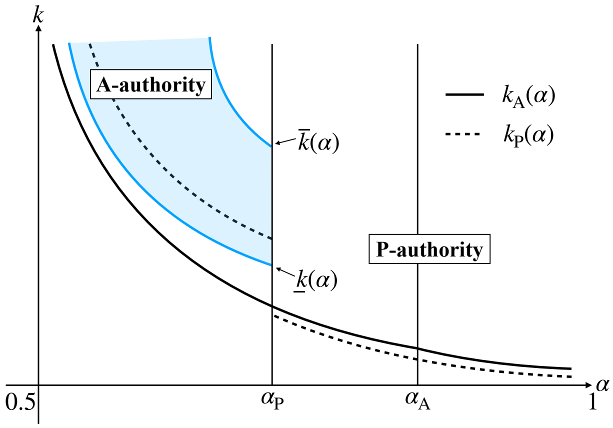
Proposition 1. *The principal chooses P-authority if $\alpha \in (\alpha_\mathrm{P}, 1]$ holds. She chooses A-authority only if $\alpha \in (1/2, \alpha_\mathrm{P}]$ and there are two sub-cases.*

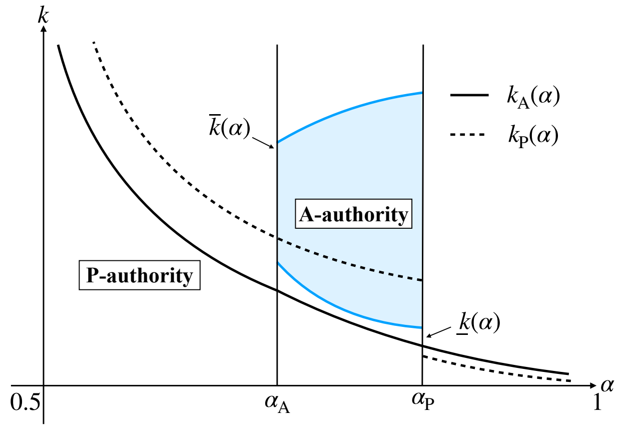
Case 1: *Suppose $\Gamma < \gamma^3$ and $\alpha \in (1/2, \alpha_\mathrm{P}]$. Then there exists $\overline{\Gamma}(\alpha)$ and an interval $(\underline{k}(\alpha), \overline{k}(\alpha))$, where $\overline{\Gamma}(\alpha) \in (\gamma, \gamma^3)$, $\underline{k}(\alpha) > k_\mathrm{A}(\alpha)$, and $\overline{k}(\alpha) > k_\mathrm{P}(\alpha)$, such that A-authority is optimal for the principal if and only if $\Gamma < \overline{\Gamma}(\alpha)$ and k belongs to the interval.*

Case 2: *Suppose $\Gamma > \gamma^3$ and $\alpha \in (\alpha_\mathrm{A}, \alpha_\mathrm{P}]$. Then there exists an interval $(\underline{k}(\alpha), \overline{k}(\alpha))$, where $\underline{k}(\alpha) > k_\mathrm{A}(\alpha)$ and $\overline{k}(\alpha) > k_\mathrm{P}(\alpha)$, such that if $\gamma > 3$ and k belongs to the interval, A-authority is optimal for the principal.*

Our first main result shows that delegation can be optimal if the signal precision is sufficiently low ($\alpha \leq \alpha_\mathrm{P}$). Figure 3 illustrates the result. The left figure depicts the optimal authority when $\Gamma < \overline{\Gamma}(\alpha)$ (Case 1) and shows A-authority is optimal for $\alpha \leq \alpha_\mathrm{P}$ and $k \in (\underline{k}(\alpha), \overline{k}(\alpha))$. The right figure illustrates optimal authority when $\Gamma > \gamma^3$ and $\gamma > 3$ (Case 2) and shows A-authority is optimal for $\alpha \in (\alpha_\mathrm{A}, \alpha_\mathrm{P}]$ and $k \in (\underline{k}(\alpha), \overline{k}(\alpha))$.

Figure 3: Optimal Authority





To see the intuition behind the result, first suppose that the signal precision is so high that the decision maker is reactive at least under P-authority ($\alpha > \alpha_P$). Then, the principal retains authority for all k. This case is divided into two sub-cases. First, if the principal has relatively strong bias ($\Gamma \geq \gamma^3$) and $\alpha_P \geq \alpha_A$, the decision maker is reactive under both authorities for $\alpha > \alpha_P$, and then, project choice upon the realization of informative signals $\sigma \in \{1, 2\}$ do not differ between two authorities. Moreover, the agent exerts greater information-gathering effort under P-authority. Hence, delegation has no advantage.

Second, if the principal has relatively weak bias ($\Gamma < \gamma^3$) and $\alpha_P < \alpha_A$, the principal retains authority for $\alpha > \alpha_A$ as discussed above. For $\alpha \in (\alpha_P, \alpha_A]$, the agent exerts greater information-gathering effort under P-authority, and the principal obtains higher expected benefit upon the realization of informative signals since the decision maker is reactive only under P-authority, and hence, the principal does not delegate her authority.

Next, suppose that the signal precision is sufficiently low ($\alpha \leq \alpha_P$). Then, delegation can be optimal. This case is also divided into two sub-cases. First, if the principal has relatively weak bias ($\Gamma < \gamma^3$) and $\alpha_A > \alpha_P$ (Case 1), the decision maker is non-reactive under both authorities for $\alpha \leq \alpha_P$. The principal then obtains higher expected benefit under P-authority upon the realization of any signals $\sigma \in \{\phi, 1, 2\}$ since her favorite project is selected under P-authority. However, the agent exerts greater information-gathering effort under delegation. The agent exerts a positive information-gathering effort for $k > k_A(\alpha)$, and the difference $\pi_A(\alpha, k) - \pi_P(\alpha, k)$ is strictly decreasing in k, and the principal's expected benefit upon the realization of any signals are increasing in Γ. Hence, delegation is optimal if k is high but not too high and the principal's bias Γ is sufficiently low. Note that the interval $(\underline{k}(\alpha), \overline{k}(\alpha))$

where delegation can be optimal shrinks as α rises for sufficiently high γ.[5]

Second, if the principal has relatively strong bias ($\Gamma > \gamma^3$) and $\alpha_{\mathrm{P}} > \alpha_{\mathrm{A}}$ (Case 2), delegation can be optimal for $\alpha \leq \alpha_{\mathrm{A}}$ as discussed above. For $\alpha \in (\alpha_{\mathrm{A}}, \alpha_{\mathrm{P}}]$, the decision maker is reactive only under delegation, and the agent exerts greater information-gathering effort under delegation. For $\alpha \leq \alpha_{\mathrm{P}}$, the signal precision is not so high that the reactivity is beneficial for the principal. However, the reactivity advantage becomes sufficiently high for $\alpha > \alpha_{\mathrm{A}}$ if γ is sufficiently high since α_{A} increases as γ rises. The agent exerts a positive information-gathering effort for $k > k_{\mathrm{A}}(\alpha)$ and the difference $\pi_{\mathrm{A}}(\alpha, k) - \pi_{\mathrm{P}}(\alpha, k)$ is decreasing in k. Hence, the principal delegates if γ is sufficiently high and k is high but not too high. Moreover, $\underline{k}(\alpha)$ is decreasing and $\overline{k}(\alpha)$ is increasing in α, and thus the interval $(\underline{k}(\alpha), \overline{k}(\alpha))$ where delegation can be optimal expands as α rises.

3.5 Homogeneous Organization

This subsection considers the homogeneous organization where both the principal and the agent favor the same project. In particular, we assume that their favorite project is project 2. This is without loss of generality since two projects as well as additional information are ex ante symmetric. Then the project choice and the information-gathering effort under A-authority remain the same as in the main analysis. Under P-authority, project implementation is unaffected, and hence we first analyze project choice and information acquisition, and then, show when the principal delegates authority to the agent.

Project Choice

If $\sigma \in \{\phi, 2\}$, the principal chooses project 2 since her favorite project is weakly more likely to be efficient. If $\sigma = 1$, there exists $\alpha_{\mathrm{hom}} \in (1/2, 1)$ such that the principal reacts to the signal and chooses project 2 if and only if $\alpha > \alpha_{\mathrm{hom}}$, where α_{hom} is the solution to $\alpha F(\alpha b_L) B_L = (1-\alpha) F((1-\alpha) b_H) B_H$, that is,

$$\alpha_{\mathrm{hom}} \equiv \frac{\sqrt{\gamma \Gamma}}{1 + \sqrt{\gamma \Gamma}}.$$

By comparing α_{hom} with α_{A} and α_{P}, we can show that $\alpha_{\mathrm{hom}} > \alpha_{\mathrm{P}}$ and $\alpha_{\mathrm{hom}} \geq \alpha_{\mathrm{A}}$, where $\alpha_{\mathrm{hom}} = \alpha_{\mathrm{A}}$ holds only if $\Gamma = \gamma$. When both the principal and the agent favor project 2, the principal is more likely to ignore the signal $\sigma = 1$ and choose project 2. The principal is less

[5]The difference $\overline{k}(\alpha) - \underline{k}(\alpha)$ is strictly decreasing in α for $\Gamma < \overline{\Gamma}(\alpha)$ if $\gamma \geq (3 + \sqrt{5})/2 \approx 2.618$.

reactive than the agent in the homogeneous organization whenever she is more biased than the agent.

Information Acquisition

We next analyze the information-gathering effort in the homogeneous organization. Under P-authority, the agent's expected payoff is given by

$$\frac{\pi}{2}\left[K((1-\alpha)b_H + K(\alpha b_H)) + \mathbf{1}_{\alpha>\alpha_{\text{hom}}}\left(K(\alpha b_L) - K((1-\alpha)b_H)\right)\right] + (1-\pi)K\left(\frac{b_H}{2}\right) - \psi(\pi).$$

The first-order condition for the optimal effort, denoted by $\pi_{\text{hom}}(\alpha)$, is given as follows:

$$\frac{1}{2}\left[K((1-\alpha)b_H + K(\alpha b_H)) + \mathbf{1}_{\alpha>\alpha_{\text{hom}}}\left(K(\alpha b_L) - K((1-\alpha)b_H)\right)\right] - K\left(\frac{b_H}{2}\right) = \psi'(\pi).$$

Denote by $y_{\text{hom}}(\alpha)$ LHS of the condition. Since the agent also prefers project 1, the ignorance-avoiding difference disappears, resulting in $y_{\text{hom}}(\alpha) = y_{\text{A}}(\alpha)$ for $\alpha \leq \alpha_{\text{A}}$ or $\alpha > \alpha_{\text{P}}$.

The following lemma shows the comparison between $\pi_{\text{hom}}(\alpha)$ and $\pi_{\text{A}}(\alpha)$.

Lemma 2. *Suppose that the principal and the agent favor the same project. Then the agent's incentive to gather information differs with authority as follows:*

- $\pi_{\text{hom}}(\alpha) = \pi_{\text{A}}(\alpha)$ *if* $\alpha \in (1/2, \alpha_{\text{A}}]$ *or* $\alpha \in (\alpha_{\text{hom}}, 1]$.
- $\pi_{\text{hom}}(\alpha) < \pi_{\text{A}}(\alpha)$ *if* $\alpha \in (\alpha_{\text{A}}, \alpha_{\text{hom}}]$.

If the signal precision is so low that the decision maker is non-reactive under both authorities ($\alpha \leq \alpha_{\text{A}}$), their favorite project 2 is always selected under either authority, and hence the optimal information-gathering effort is identical. This result contrasts with the corresponding result in the main analysis where the agent exerts greater information-gathering effort under delegation since the agent can implement his favorite project only under A-authority.

If the signal precision is so high that the decision maker is reactive under both authorities ($\alpha > \alpha_{\text{hom}}$), there is no difference in project choice. Since there is no longer the ignorance-avoiding difference, the agent chooses the same information-gathering effort under two authorities.

In the final case where the decision maker is reactive only under A-authority ($\alpha \in (\alpha_{\text{A}}, \alpha_{\text{hom}}]$), the agent exerts greater information-gathering effort under delegation as in the main analysis, due to the reactivity effect.

Delegation

In the homogeneous organization with i-authority ($i \in \{\mathrm{P}, \mathrm{A}\}$), the principal's expected benefit $u^{12}_{i\mathrm{hom}}(\alpha)$ conditional on $\sigma \in \{1,2\}$ and $u^{\phi}_{i\mathrm{hom}}$ conditional on $\sigma = \phi$ are given as follows:

$$
\begin{aligned}
u^{12}_{\mathrm{Phom}}(\alpha) = \frac{1}{2}\Big[&\alpha F(\alpha b_H)B_H + (1-\alpha)F((1-\alpha)b_H)B_H \\
&+ \mathbf{1}_{\alpha > \alpha_{\mathrm{hom}}}\left(\alpha F(\alpha b_L)B_L - (1-\alpha)F((1-\alpha)b_H)B_H\right)\Big], \\
u^{12}_{\mathrm{Ahom}}(\alpha) = \frac{1}{2}\Big[&\alpha F(\alpha b_H)B_H + (1-\alpha)F((1-\alpha)b_H)B_H \\
&+ \mathbf{1}_{\alpha > \alpha_{\mathrm{A}}}\left(\alpha F(\alpha b_L)B_L - (1-\alpha)F((1-\alpha)b_H)B_H\right)\Big], \\
u^{\phi}_{\mathrm{Phom}} = u^{\phi}_{\mathrm{Ahom}} = \frac{1}{2}&F\left(\frac{b_H}{2}\right)B_H.
\end{aligned}
$$

Since the principal and the agent agree about the favorite project (project 2), the principal's expected benefit conditional on $\sigma = \phi$ is identical. Note further that $u^{12}_{\mathrm{Phom}}(\alpha) = u^{12}_{\mathrm{Ahom}}(\alpha)$ for $\alpha \leq \alpha_{\mathrm{A}}$ or $\alpha > \alpha_{\mathrm{hom}}$.

As in the main analysis, we introduce the parameter $k > 0$ and adopt the specific cost function (3). We rewrite the optimal information-gathering effort under P-authority as $\pi_{\mathrm{hom}}(\alpha, k)$, where $\pi_{\mathrm{hom}}(\alpha, k) > 0$ if and only if $k > k_{\mathrm{hom}}(\alpha)$.

The following proposition shows when the principal delegates in the homogeneous organization.

Proposition 2. *Suppose that the principal and the agent favor the same project. Then the principal retains authority for $\alpha \in (1/2, \alpha_{\mathrm{A}}]$ or $\alpha \in (\alpha_{\mathrm{hom}}, 1]$. If $\alpha \in (\alpha_{\mathrm{A}}, \alpha_{\mathrm{hom}}]$ and $\gamma > 1 + \sqrt{2}$, then there exists an interval $(k_{\mathrm{A}}(\alpha), \overline{k}_{\mathrm{hom}}(\alpha))$, where $\overline{k}_{\mathrm{hom}}(\alpha) > k_{\mathrm{P}}(\alpha)$, such that if $k \in (k_{\mathrm{A}}(\alpha), \overline{k}_{\mathrm{hom}}(\alpha))$, A-authority is optimal for the principal.*

If the signal precision is so low that the decision maker is non-reactive under both authorities ($\alpha \leq \alpha_{\mathrm{A}}$) or so high that the decision maker is reactive under both authorities ($\alpha > \alpha_{\mathrm{hom}}$), project choice and information-gathering effort do not differ with authority, and hence, the principal retains her authority.

If the signal precision is such that the decision maker is reactive only under A-authority ($\alpha \in (\alpha_{\mathrm{A}}, \alpha_{\mathrm{hom}}]$), delegation is optimal when the reactivity and information-gathering advantages are sufficiently beneficial as in the corresponding case in the heterogeneous organization. Note that $k_{\mathrm{A}}(\alpha)$ is decreasing and $\overline{k}_{\mathrm{hom}}(\alpha)$ is increasing in α, and thus, the interval $(k_{\mathrm{A}}(\alpha), \overline{k}_{\mathrm{hom}}(\alpha))$ where delegation can be optimal expands as α rises.

One might conjecture that the principal is more likely to delegate in the homogeneous organization. Indeed, the principal delegates her authority under potentially less stringent conditions when the decision maker is reactive only under delegation since there is no disagreement upon the realization of no information.[6] However, delegation can be optimal only in the heterogeneous organization if the signal precision is so low that the decision maker is not reactive under both authorities. In this case, the decision maker chooses the agent's favorite project under either authority in the homogeneous organization, and hence the two authorities are indifferent. In contrast, in the heterogeneous organization, the agent can implement his favorite project only under delegation, and hence delegation motivates him to gather information and implement a project, making delegation optimal.

This comparison implies that when the environment is so uncertain that the decision maker is non-reactive under both authorities, delegation is more likely to be optimal in the heterogenous organization.

4 Two-Stage Decision Process

In this section we analyze two alternative decision processes, the decision process without the implementation stage, and the decision process without the information acquisition stage. We show that under both two-stage processes, the principal's optimal authority is P-authority for all α.

4.1 Without the Implementation Stage

This subsection considers the two-stage process where the agent engages only in information acquisition and the selected project is implemented with an exogenously fixed probability, denoted by x, under both authorities. Without loss of generality, we assume $x = 1$ below. This alternative model illustrates the situation where, for example, the implementation cost is sufficiently low, and the agent's motivation to implement a project does not matter.

[6] Specifically, compared with the corresponding conditions for the heterogeneous organization in Proposition 1, delegation can become optimal for lower bias of the agent ($\gamma > 1 + 2\sqrt{2}$) and lower k ($k_{\mathrm{A}}(\alpha) < \underline{k}(\alpha)$) in the homogeneous organization, while whether $\overline{k}_{\mathrm{hom}}(\alpha)$ is greater than $\overline{k}(\alpha)$ or not is ambiguous.

Project Choice

We first analyze project choice under A-authority. The agent's expected benefit of project d with his private benefit b is given by $\Pr[\theta = d \mid \sigma]b$. If $\sigma \in \{\phi, 2\}$, the agent chooses project 2 as in the main analysis. Suppose $\sigma = 1$. Then the agent chooses project 1 if and only if $\alpha b_L > (1-\alpha)b_H$, which is equivalent to $\alpha > \alpha_{\mathrm{A}}$: The threshold level of precision does not change from the case of the three-stage decision process.

Next, we analyze project choice under P-authority. The principal's expected benefit of project d with her private benefit B conditional on signal σ is given by $\Pr[\theta = d \mid \sigma]B$, which does not depend on the agent's private benefit b. Hence whether the organization is homogeneous or heterogenous does not matter. The principal chooses her favorite project if the signal is ϕ or indicates that project. Otherwise, there exists $\hat{\alpha}_{\mathrm{P}} \in (1/2, 1)$ such that the principal chooses her disfavored project if and only if $\alpha > \hat{\alpha}_{\mathrm{P}}$, where $\hat{\alpha}_{\mathrm{P}}$ is the solution to $\alpha B_L > (1-\alpha)B_H$, that is,

$$\hat{\alpha}_{\mathrm{P}} = \frac{\Gamma}{1+\Gamma}.$$

Note that $\hat{\alpha}_{\mathrm{P}} > \alpha_{\mathrm{P}}$. The principal is less likely to react without the implementation stage because she does not take into account the agent's implementation motivation when she chooses a project.

Since $\Gamma \geq \gamma$, $\hat{\alpha}_{\mathrm{P}} \geq \alpha_{\mathrm{A}}$ holds, and thus the agent is more likely to be reactive without endogenous implementation since the agent has weaker bias toward the favorite project.

Information Acquisition

The agent's expected payoff under homogeneous P-authority, heterogenous P-authority, and A-authority are, respectively, written by

$$\frac{\pi}{2}\left[(1-\alpha)b_H + \alpha b_H + \mathbf{1}_{\alpha > \hat{\alpha}_{\mathrm{P}}}\left(\alpha b_L - (1-\alpha)b_H\right)\right] + (1-\pi)\frac{b_H}{2} - \psi(\pi).$$
$$\frac{\pi}{2}\left[\alpha b_L + (1-\alpha)b_L + \mathbf{1}_{\alpha > \hat{\alpha}_{\mathrm{P}}}\left(\alpha b_H - (1-\alpha)b_L\right)\right] + (1-\pi)\frac{b_L}{2} - \psi(\pi),$$
$$\frac{\pi}{2}\left[(1-\alpha)b_H + \alpha b_H + \mathbf{1}_{\alpha > \alpha_{\mathrm{A}}}\left(\alpha b_L - (1-\alpha)b_H\right)\right] + (1-\pi)\frac{b_H}{2} - \psi(\pi).$$

The first-order condition for the optimal effort, denoted by $\hat{\pi}_{\mathrm{hom}}(\alpha)$ (under homogeneous P-authority), $\hat{\pi}_{\mathrm{P}}(\alpha)$ (under heterogenous P-authority), and $\hat{\pi}_{\mathrm{A}}(\alpha)$ (under A-authority) is, respectively, given as follows:

$$\frac{1}{2}\left[(1-\alpha)b_H + \alpha b_H + \mathbf{1}_{\alpha>\hat{\alpha}_{\mathrm{P}}}\left(\alpha b_L - (1-\alpha)b_H\right)\right] - \frac{b_H}{2} = \psi'(\pi), \tag{4}$$

$$\frac{1}{2}\left[\alpha b_L + (1-\alpha)b_L + \mathbf{1}_{\alpha>\hat{\alpha}_{\mathrm{P}}}\left(\alpha b_H - (1-\alpha)b_L\right)\right] - \frac{b_L}{2} = \psi'(\pi), \tag{5}$$

$$\frac{1}{2}\left[(1-\alpha)b_H + \alpha b_H + \mathbf{1}_{\alpha>\alpha_{\mathrm{A}}}\left(\alpha b_L - (1-\alpha)b_H\right)\right] - \frac{b_H}{2} = \psi'(\pi), \tag{6}$$

where LHS of each equation represents the marginal benefit. Denote LHS of (4) by $\hat{y}_{\mathrm{hom}}(\alpha)$, and so on.

Note that the agent's expected payoffs are linear in his private benefit. Hence if the decision maker is non-reactive, the agent does not exert positive information-gathering effort: The implementation motivation effect disappears. If the decision maker is reactive, the agent chooses a positive effort for information acquisition under either authority.

Comparing (4), (5), and (6) immediately yields the following result.

Lemma 3. *Suppose that the selected project is exogenously implemented with probability* 1. *Then, the agent's incentive to gather information differs with the two authorities as follows: In the heterogenous organization,*

- $\hat{\pi}_{\mathrm{P}}(\alpha) = \hat{\pi}_{\mathrm{A}}(\alpha) = 0$ *if* $\alpha \in (1/2, \alpha_{\mathrm{A}}]$,
- $\hat{\pi}_{\mathrm{P}}(\alpha) = 0 < \hat{\pi}_{\mathrm{A}}(\alpha)$ *if* $\alpha \in (\alpha_{\mathrm{A}}, \hat{\alpha}_{\mathrm{P}}]$,
- $\hat{\pi}_{\mathrm{P}}(\alpha) > \hat{\pi}_{\mathrm{A}}(\alpha)$ *if* $\alpha \in (\hat{\alpha}_{\mathrm{P}}, 1]$.

In the homogeneous organization,

- $\hat{\pi}_{\mathrm{hom}}(\alpha) = \hat{\pi}_{\mathrm{A}}(\alpha) = 0$ *if* $\alpha \in (1/2, \alpha_{\mathrm{A}}]$,
- $\hat{\pi}_{\mathrm{hom}}(\alpha) = 0 < \hat{\pi}_{\mathrm{A}}(\alpha)$ *if* $\alpha \in (\alpha_{\mathrm{A}}, \hat{\alpha}_{\mathrm{P}}]$,
- $\hat{\pi}_{\mathrm{hom}}(\alpha) = \hat{\pi}_{\mathrm{A}}(\alpha)$ *if* $\alpha \in (\hat{\alpha}_{\mathrm{P}}, 1]$.

As we have already pointed out, the agent does not exert a positive effort if the decision maker is non-reactive. If the decision maker is reactive only under A-authority, delegation raises the agent's initiative as in the main model. If α is so high that the decision maker is reactive under both P-authority and A-authority, whether the organization is homogeneous or heterogeneous matters. In the heterogenous organization, the agent exerts greater information-gathering effort under P-authority due to the ignorance-avoiding difference as in the main analysis. In the homogeneous organization, the ignorance-avoiding difference disappears, and hence the information-gathering effort is identical between P-authority and A-authority.

Delegation

In the following proposition we present the main result from the analysis of the two-stage decision process without endogenous implementation stage.

Proposition 3. *Suppose that the selected project is exogenously implemented with probability* 1*. Then, whether the organization is homogeneous or heterogeneous, the principal retains her authority (chooses P-authority) for all* $\alpha > 1/2$*.*

The principal does not delegate when the agent engages only in information acquisition. If α is so low that the decision maker is non-reactive under both authorities, the agent does not exert a positive information-gathering effort, and hence delegation has no advantage, and the principal is even worse off in the heterogenous organization due to their disagreement about favorite projects.

If α is so high that the decision maker is reactive under both authorities, the principal is once again indifferent between two authorities in the homogeneous organization. In the heterogenous organization the agent chooses greater information-gathering effort under P-authority, and project choice given informative signals $\sigma \in \{1, 2\}$ is identical between P-authority and A-authority, and hence the principal does not delegate as in the main analysis.

If α is in the intermediate interval so that the decision maker is reactive only under A-authority, delegation induces reactivity and increases the agent's information-gathering effort as in the main model. However, the agent's reactivity under A-authority is not beneficial for the more biased principal. Furthermore, in the heterogenous organization, the principal's expected benefit condition on no informative signal $\sigma = \phi$ is higher under P-authority. The principal hence retains authority. The result shows that the advantage in terms of higher information-gathering effort alone does not make delegation optimal for the principal without the endogenous implementation stage.

4.2 Without the Information Acquisition Stage

This subsection analyzes the two-stage decision process where the agent engages only in project implementation and there is no endogenous information acquisition stage. Both the principal and the agent obtain informative signals $\sigma \in \{1, 2\}$ with probability π and the uninformative signal $\sigma = \phi$ with probability $1-\pi$, where π is exogenously given and identical under each authority. This alternative model illustrates the situation, for example, in which it is not costly to acquire information, and the agent's motivation to gather information does not matter.

Given available signals, the results from project implementation and project choice remains the same as in the main analysis. We thus focus on the principal's delegation decision. In either homogeneous or heterogeneous organization, the principal can replicate A-authority under P-authority by choosing the same project as that under A-authority, and hence P-authority is weakly better off. This result shows that the reactivity advantage alone does not make delegation optimal for the principal. Proposition 4 summarizes the result.

Proposition 4. *Suppose that the probability of acquiring informative signal $\sigma \in \{1, 2\}$ is exogenously given under each authority. Then, whether the organization is homogeneous or heterogeneous, the principal retains authority (chooses P-authority) for all $\alpha > 1/2$.*

5 Conclusion

While we understand decisions in organizations typically take multiple steps from information acquisition, project choice, and execution of the selected project, almost all studies of internal organizations focus only on two-stage parts of the process, either from information to choice or choice to implementation. Our analysis suggests that it is important to explicitly model and study all three stages at once in order to understand the incentive reason for delegation. We identify compound incentive effects from information acquisition and execution, that play a critical role in determining whether or not to delegate the authority over project choice.

The results have several managerial implications. To determine the allocation of decision rights, organization designers should take all the decision process into consideration. Despite the disagreement about favorite projects between the manager and the subordinate, the manager should still offer complete control over project choice and execution to the subordinate if additional information is likely to be quite noisy and the project management by this subordinate is sufficiently important (the manager's relative "bias" is not too strong). Even if the project managed by the subordinate is not so important, delegation can still be desirable under the condition that additional information is modestly valuable and delegating authority only enables that information to be utilized for project choice: The manager with a strong bias can use delegation as the commitment device to react to valuable information.

Our results depend on several assumptions. We fix information acquisition and execution as the agent's task. We assume incentive contracts are infeasible and to focus on the motivation effects of delegation, there is no asymmetric information between the principal and the agent, for example, for the realization of the signal. It is an important next task to relax these assumptions and investigate how robust our results are.

References

Aghion, Philippe and Jean Tirole (1997) "Formal and Real Authority in Organizations," *Journal of Political Economy*, Vol. 105, pp. 1–29.

Baker, George, Robert Gibbons, and Kevin J. Murphy (1999) "Informal Authority in Organizations," *Journal of Law, Economics, and Organization*, Vol. 15, pp. 56–73.

Bester, Helmut and Daniel Krähmer (2008) "Delegation and Incentives," *Rand Journal of Economics*, Vol. 39, pp. 664–682.

Che, Yeon-Koo and Navin Kartik (2009) "Opinions as Incentives," *Journal of Political Economy*, Vol. 117, pp. 815–860.

Gibbons, Robert, Niko Matouschek, and John Roberts (2012) "Decisions in Organizations," in Robert Gibbons and John Roberts eds. *The Handbook of Organizational Economics*, Princeton NJ: Princeton University Press, Chap. 10, pp. 373–431.

Itoh, Hideshi and Kimiyuki Morita (2023) "Information Acquisition, Decision Making, and Implementation in Organizations," *Management Science*, Vol. 69, No. 1, pp. 446–463.

Landier, Augustin, David Sraer, and David Thesmar (2009) "Optimal Dissent in Organizations," *Review of Economic Studies*, Vol. 76, pp. 761–794.

Mintzberg, Henry (1979) *The Structuring of Organizations: A Synthesis of Research*, Englewood Cliffs, NJ: Prentice Hall.

Zábojník, Ján (2002) "Centralized and Decentralized Decision Making in Organizations," *Journal of Labor Economics*, Vol. 20, pp. 1–22.

Appendix A: Proofs

Proof of Lemma 1

Since $\pi_{\mathrm{A}}(\alpha) > \pi_{\mathrm{P}}(\alpha)$ if and only if $y_{\mathrm{A}}(\alpha) > y_{\mathrm{P}}(\alpha)$, we only need to compare the marginal benefits. First, suppose $\alpha \leq \min\{\alpha_{\mathrm{A}}, \alpha_{\mathrm{P}}\}$. Then, the difference in the marginal benefit between two authorities is written as follows:

$$
\begin{aligned}
&y_{\mathrm{A}}(\alpha) - y_{\mathrm{P}}(\alpha) \\
&\quad = \frac{1}{2}\left[K((1-\alpha)b_H) + K(\alpha b_H)\right] - K\left(\frac{b_H}{2}\right) - \frac{1}{2}\left[K((1-\alpha)b_L) + K(\alpha b_L)\right] + K\left(\frac{b_L}{2}\right), \\
&\quad = \gamma^2 \frac{b_L^2}{4}\left(\alpha^2 + (1-\alpha)^2 - \frac{1}{2}\right) - \frac{b_L^2}{4}\left(\alpha^2 + (1-\alpha)^2 - \frac{1}{2}\right), \\
&\quad = (\gamma^2 - 1)\frac{b_L^2}{4}\left(\alpha^2 + (1-\alpha)^2 - \frac{1}{2}\right), \\
&\quad > 0.
\end{aligned}
$$

Second, suppose $\alpha > \max\{\alpha_{\mathrm{A}}, \alpha_{\mathrm{P}}\}$. Then, the difference between $y_{\mathrm{A}}(\alpha)$ and $y_{\mathrm{P}}(\alpha)$ is strictly negative:

$$
y_{\mathrm{A}}(\alpha) - y_{\mathrm{P}}(\alpha) = K\left(\frac{b_L}{2}\right) - K\left(\frac{b_H}{2}\right) < 0.
$$

Third, suppose $\Gamma > \gamma^3$ and $\alpha \in (\alpha_{\mathrm{A}}, \alpha_{\mathrm{P}}]$. Then, the difference between $y_{\mathrm{A}}(\alpha)$ and $y_{\mathrm{P}}(\alpha)$ is written as follows:

$$
\begin{aligned}
y_{\mathrm{A}}(\alpha) - y_{\mathrm{P}}(\alpha) &= \frac{1}{2}\left[K(\alpha b_H) - K((1-\alpha)b_L)\right] + K\left(\frac{b_L}{2}\right) - K\left(\frac{b_H}{2}\right), \\
&= \frac{b_L^2}{4}\left[\alpha^2\gamma^2 - (1-\alpha)^2 - \frac{1}{2}(\gamma^2 - 1)\right],
\end{aligned}
$$

which is positive if the expression in the square bracket is positive. Because the expression in the square bracket is increasing in α, it is strictly positive for $\alpha > \alpha_{\mathrm{A}}$ if it is positive at

$\alpha = \alpha_{\mathrm{A}}$. The expression in the square bracket at $\alpha = \alpha_{\mathrm{A}}$ is written as follows:

$$
\begin{aligned}
\alpha_{\mathrm{A}}^2\gamma^2 - (1-\alpha_{\mathrm{A}})^2 - \frac{1}{2}(\gamma^2 - 1) &= \frac{\gamma^2}{(1+\gamma)^2}\gamma^2 - \frac{1}{(1+\gamma)^2} + \frac{1}{2}(1-\gamma^2), \\
&= \frac{1}{2(1+\gamma)^2}\left[2\gamma^4 - 2 + (1-\gamma^2)(1+\gamma)^2\right], \\
&= \frac{1}{2(1+\gamma)^2}\left[\gamma^4 - 1 + 2\gamma - 2\gamma^3\right], \\
&= \frac{1}{2(1+\gamma)^2}\left[\gamma^2(\gamma^2 - 2\gamma) + \gamma + \gamma - 1\right], \\
&= \frac{1}{2(1+\gamma)^2}(\gamma - 1)\left[\gamma^2(\gamma - 1) - \gamma + 1\right], \\
&= \frac{1}{2(1+\gamma)^2}(\gamma - 1)^2(\gamma^2 - 1) \\
&> 0.
\end{aligned}
$$

Finally, suppose $\Gamma \leq \gamma^3$ and $\alpha \in (\alpha_{\mathrm{P}}, \alpha_{\mathrm{A}}]$. Then, the difference between $y_{\mathrm{A}}(\alpha)$ and $y_{\mathrm{P}}(\alpha)$ is written as follows:

$$
\begin{aligned}
y_{\mathrm{A}}(\alpha) - y_{\mathrm{P}}(\alpha) &= \frac{1}{2}\left[K((1-\alpha)b_H) - K(\alpha b_L)\right] + K\left(\frac{b_L}{2}\right) - K\left(\frac{b_H}{2}\right), \\
&= \frac{b_L^2}{4}\left[(1-\alpha)^2\gamma^2 - \alpha^2 - \frac{1}{2}(\gamma^2 - 1)\right],
\end{aligned}
$$

which is decreasing in α and strictly negative at $\alpha = 1/2$, and hence, strictly negative for $\alpha > 1/2$. □

Proof of Proposition 1

As in the main text the principal's expected payoff under i-authority for $i \in \{\mathrm{A}, \mathrm{P}\}$ is written by

$$U_i(\alpha, k) = \pi_i(\alpha, k)\left(u_i^{12}(\alpha) - u_i^{\phi}\right) + u_i^{\phi}.$$

The difference between $U_{\mathrm{A}}(\alpha, k)$ and $U_{\mathrm{P}}(\alpha, k)$ is written as follows:

$$U_{\mathrm{A}}(\alpha, k) - U_{\mathrm{P}}(\alpha, k) = \pi_{\mathrm{A}}(\alpha, k)\left(u_{\mathrm{A}}^{12}(\alpha) - u_{\mathrm{A}}^{\phi}\right) - \pi_{\mathrm{P}}(\alpha, k)\left(u_{\mathrm{P}}^{12}(\alpha) - u_{\mathrm{P}}^{\phi}\right) + u_{\mathrm{A}}^{\phi} - u_{\mathrm{P}}^{\phi}.$$

For $k < \min\{k_{\mathrm{A}}(\alpha), k_{\mathrm{P}}(\alpha)\}$, $\pi_{\mathrm{A}}(\alpha, k) = \pi_{\mathrm{P}}(\alpha, k) = 0$, and thus, the difference between $U_{\mathrm{A}}(\alpha, k)$ and $U_{\mathrm{P}}(\alpha, k)$ is strictly negative:

$$U_{\mathrm{A}}(\alpha, k) - U_{\mathrm{P}}(\alpha, k) = u_{\mathrm{A}}^{\phi} - u_{\mathrm{P}}^{\phi} = -\frac{b_L B_L}{2}\frac{1}{2}(\Gamma - \gamma) < 0.$$

Hence, it is obvious that the principal retains the authority for all α. Below, we suppose $k > \min\{k_{\mathrm{A}}(\alpha), k_{\mathrm{P}}(\alpha)\}$. Then, at least one of $\pi_{\mathrm{A}}(\alpha, k)$ and $\pi_{\mathrm{P}}(\alpha, k)$ is strictly positive.

There are the following four cases to consider: (1) the decision maker is non-reactive under both authorities **(NN)**; (2) the decision maker is reactive under both authorities **(RR)**; (3) the decision maker is reactive only under A-authority **(RN)**; (4) the decision maker is reactive only under P-authority **(NR)**.

(NN)

First, suppose α is so low that the decision maker is non-reactive under both authorities ($\alpha < \min\{\alpha_{\mathrm{A}}, \alpha_{\mathrm{P}}\}$). Then, the principal's expected payoff under P-authority and A-authority are respectively written as follows:

$$\begin{aligned} U_{\mathrm{P}}(\alpha, k) &= \frac{b_L B_L}{2}\left[\pi_{\mathrm{P}}(\alpha, k)\left(\alpha^2 + (1-\alpha)^2 - \frac{1}{2}\right)\Gamma + \frac{\Gamma}{2}\right], \\ U_{\mathrm{A}}(\alpha, k) &= \frac{b_L B_L}{2}\left[\pi_{\mathrm{A}}(\alpha, k)\left(\alpha^2 + (1-\alpha)^2 - \frac{1}{2}\right)\gamma + \frac{\gamma}{2}\right]. \end{aligned}$$

The difference between $U_{\mathrm{P}}(\alpha, k)$ and $U_{\mathrm{A}}(\alpha, k)$ is written as follows:

$$U_{\mathrm{A}}(\alpha, k) - U_{\mathrm{P}}(\alpha, k) = \frac{b_L B_L}{2}\underbrace{\left[\left(\alpha^2 + (1-\alpha)^2 - \frac{1}{2}\right)(\pi_{\mathrm{A}}(\alpha, k)\gamma - \pi_{\mathrm{P}}(\alpha, k)\Gamma) - \frac{1}{2}(\Gamma - \gamma)\right]}_{\equiv \Delta U^{\mathrm{NN}}},$$

which is strictly positive if and only if $\Delta U^{\mathrm{NN}} > 0$. By Proposition 1, $\pi_{\mathrm{A}}(\alpha, k) \geq \pi_{\mathrm{P}}(\alpha, k)$ for $\alpha \leq \alpha_{\mathrm{P}}$, and thus, $k_{\mathrm{A}}(\alpha) < k_{\mathrm{P}}(\alpha)$. We consider the following two cases: (i) $k \in (k_{\mathrm{A}}(\alpha), k_{\mathrm{P}}(\alpha)]$ and (ii) $k > k_{\mathrm{P}}(\alpha)$. The analysis of these two cases provides the proof for **Case 1** in the proposition.

NN-(i) Fix $k \in (k_{\mathrm{A}}(\alpha), k_{\mathrm{P}}(\alpha)]$. Then, $\pi_{\mathrm{A}}(\alpha, k) > \pi_{\mathrm{P}}(\alpha, k) = 0$, and

$$\Delta U^{\mathrm{NN}} = \left(\alpha^2 + (1-\alpha)^2 - \frac{1}{2}\right)\pi_{\mathrm{A}}(\alpha, k)\gamma - \frac{1}{2}(\Gamma - \gamma),$$

which is negative at $k = k_{\mathrm{A}}(\alpha)$, and increasing in k. For $\Delta U^{\mathrm{NN}} > 0$ to ever hold, it must be positive at $\pi_{\mathrm{A}}(\alpha, k) = 1$, that is,

$$\Gamma < \overline{\Gamma}_1^{\mathrm{NN}}(\alpha) \equiv 2\gamma(\alpha^2 + (1-\alpha)^2)$$

By substituting $\pi_{\mathrm{A}}(\alpha, k) = 1 - (k y_{\mathrm{A}}(\alpha))^{-1}$ into ΔU^{NN} we obtain $\Delta U^{\mathrm{NN}} > 0$ if and only if $k > \underline{k}(\alpha)$, where $\underline{k}(\alpha) \in (k_{\mathrm{A}}(\alpha), +\infty)$ is defined by

$$\begin{aligned}
\underline{k}(\alpha) &= \frac{1}{y_{\mathrm{A}}(\alpha)} \frac{(\alpha^2 + (1-\alpha)^2 - 1/2)\gamma}{(\alpha^2 + (1-\alpha)^2 - 1/2)\gamma - (1/2)(\Gamma - \gamma)}, \\
&= \frac{1}{\gamma(b_L^2/4)} \frac{1}{(\gamma(\alpha^2 + (1-\alpha)^2) - \Gamma/2)}.
\end{aligned}$$

Note that $\underline{k}(\alpha)$ is decreasing in α. Since we fix $k < k_{\mathrm{P}}(\alpha)$, $\underline{k}(\alpha) < k_{\mathrm{P}}(\alpha)$ must hold. This inequality holds if

$$(\gamma^2 - 1)\left(\alpha^2 + (1-\alpha)^2 - \frac{1}{2}\right) - \frac{\gamma}{2}(\Gamma - \gamma) > 0,$$

which is rewritten as follows.

$$\Gamma < \overline{\Gamma}_2^{\mathrm{NN}}(\alpha) \equiv \frac{2(\gamma^2 - 1)}{\gamma}\left(\alpha^2 + (1-\alpha)^2 - \frac{1}{2}\right) + \gamma.$$

Note that $\overline{\Gamma}_2^{\mathrm{NN}}(\alpha)$ is increasing in α and $\overline{\Gamma}_2^{\mathrm{NN}}(\alpha) > \gamma$ for $\alpha > 1/2$. Moreover, $\overline{\Gamma}_1^{\mathrm{NN}}(\alpha) > \overline{\Gamma}_2^{\mathrm{NN}}(\alpha)$ for all $\alpha > 1/2$. Hence define $\overline{\Gamma}(\alpha) = \overline{\Gamma}_2^{\mathrm{NN}}(\alpha)$. Furthermore, we can show that $\gamma^3 > \overline{\Gamma}(\alpha)$.[7]

Summarizing the case $k \in (k_{\mathrm{A}}(\alpha), k_{\mathrm{P}}(\alpha)]$, we show that ΔU^{NN} is increasing in k and negative at $k = k_{\mathrm{A}}(\alpha)$. It is positive at $k = k_{\mathrm{P}}(\alpha)$ if and only if $\Gamma < \overline{\Gamma}(\alpha)$ which

[7]To see this,

$$\begin{aligned}
\gamma^3 - \overline{\Gamma}(\alpha) &= \gamma^3 - \frac{2(\gamma^2-1)}{\gamma}\left(\alpha^2 + (1-\alpha)^2 - \frac{1}{2}\right) - \gamma \\
&= \gamma(\gamma^2 - 1) - \frac{2(\gamma^2-1)}{\gamma}\left(\alpha^2 + (1-\alpha)^2 - \frac{1}{2}\right), \\
&= \frac{(\gamma^2-1)}{\gamma}\left[\gamma^2 - 2\left(\alpha^2 + (1-\alpha)^2 - \frac{1}{2}\right)\right] \\
&> 0,
\end{aligned}$$

where the last inequality holds for all $\alpha > 1/2$ because $2(\alpha^2 + (1-\alpha)^2 - 1/2) \leq 1$ for all $\alpha > 1/2$.

is smaller than γ^3. Hence the principal delegates the authority ($\Delta U^{\mathrm{NN}} > 0$) if and only if $\Gamma < \overline{\Gamma}(\alpha)$ and k is sufficiently close to $k_{\mathrm{P}}(\alpha)$, that is, $k \in (\underline{k}(\alpha), k_{\mathrm{P}}(\alpha)]$ where $\underline{k}(\alpha) > k_{\mathrm{A}}(\alpha)$.

NN-(ii) Fix $k > k_{\mathrm{P}}(\alpha)$. Then, $\pi_{\mathrm{A}}(\alpha, k) > 0$ and $\pi_{\mathrm{P}}(\alpha, k) > 0$, and

$$\Delta U^{\mathrm{NN}} = \left(\alpha^2 + (1-\alpha)^2 - \frac{1}{2}\right)(\pi_{\mathrm{A}}(\alpha, k)\gamma - \pi_{\mathrm{P}}(\alpha, k)\Gamma) - \frac{1}{2}(\Gamma - \gamma),$$

which is positive at $k = k_{\mathrm{P}}(\alpha)$ if $\Gamma < \overline{\Gamma}(\alpha)$. At the limit $k \to +\infty$,

$$\lim_{k \to +\infty} \Delta U^{\mathrm{NN}} = -(\Gamma - \gamma)\left(\alpha^2 + (1-\alpha)^2 - \frac{1}{2}\right) - \frac{1}{2}(\Gamma - \gamma) < 0.$$

Differentiation yields

$$\frac{d\Delta U^{\mathrm{NN}}}{dk} = \left(\alpha^2 + (1-\alpha)^2 - \frac{1}{2}\right)\frac{1}{k^2}\frac{(\gamma y_{\mathrm{P}}(\alpha) - \Gamma y_{\mathrm{A}}(\alpha))}{y_{\mathrm{A}}(\alpha) y_{\mathrm{P}}(\alpha)} < 0.$$

Hence there exists $\overline{k}(\alpha) \in (k_{\mathrm{P}}(\alpha), +\infty)$ such that $\Delta U^{\mathrm{NN}} > 0$ if and only if $k < \overline{k}(\alpha)$, where $\overline{k}(\alpha)$ is given by

$$\begin{aligned}\overline{k}(\alpha) &= \frac{(\alpha^2 + (1-\alpha)^2 - 1/2)}{(\Gamma - \gamma)(\alpha^2 + (1-\alpha)^2)}\frac{(\Gamma y_{\mathrm{A}}(\alpha) - \gamma y_{\mathrm{P}}(\alpha))}{y_{\mathrm{A}}(\alpha) y_{\mathrm{P}}(\alpha)}, \\ &= \frac{1}{(\Gamma - \gamma)}\frac{1}{(\alpha^2 + (1-\alpha)^2)}\frac{(\Gamma\gamma - 1)}{\gamma(b_L^2/4)}.\end{aligned}$$

Note that $\overline{k}(\alpha)$ is decreasing in α.

Let us summarize the result for $\alpha < \min\{\alpha_{\mathrm{A}}, \alpha_{\mathrm{P}}\}$ (neither authority is reactive). The principal chooses delegation if and only if $\Gamma < \overline{\Gamma}(\alpha)$ and $k \in (\underline{k}(\alpha), \overline{k}(\alpha))$ where $\underline{k}(\alpha) \in (k_{\mathrm{A}}(\alpha), k_{\mathrm{P}}(\alpha))$ and $\overline{k}(\alpha) > k_{\mathrm{P}}(\alpha)$. Note that $\overline{\Gamma}(\alpha)$ is increasing, $\underline{k}(\alpha)$ is decreasing, and $\overline{k}(\alpha)$ is decreasing in α. Moreover, we can show that the difference $\overline{k}(\alpha) - \underline{k}(\alpha)$ is strictly decreasing in α for sufficiently high γ. Differentiation yields

$$\begin{aligned}&\frac{d(\overline{k}(\alpha) - \underline{k}(\alpha))}{d\alpha} \\ &\quad = \frac{(4\alpha - 2)}{(b_L^2/4)}\left[\frac{(\Gamma - \gamma)\gamma(\alpha^2 + (1-\alpha)^2)^2 - (\Gamma\gamma - 1)(\gamma(\alpha^2 + (1-\alpha)^2) - \Gamma/2)^2}{(\gamma(\alpha^2 + (1-\alpha)^2) - \Gamma/2)^2(\Gamma - \gamma)\gamma(\alpha^2 + (1-\alpha)^2)^2}\right],\end{aligned}$$

which is negative if

$$(\Gamma-\gamma)\gamma(\alpha^2+(1-\alpha)^2)^2-(\Gamma\gamma-1)(\gamma(\alpha^2+(1-\alpha)^2)-\Gamma/2)^2<0.$$

Because $(\Gamma\gamma-1)>(\Gamma-\gamma)\gamma$, the condition holds if $(\gamma(\alpha^2+(1-\alpha)^2)-\Gamma/2)^2>(\alpha^2+(1-\alpha)^2)^2$, which can be reduce to $2(\gamma-1)(\alpha^2+(1-\alpha)^2)>\Gamma$. This inequality holds for $\Gamma<\overline{\Gamma}(\alpha)$ if

$$2(\gamma-1)(\alpha^2+(1-\alpha)^2)>\overline{\Gamma}(\alpha) \quad\Longleftrightarrow\quad 2(\gamma-1)\left[\frac{\gamma}{2}+\frac{1}{2}-(\alpha^2+(1-\alpha)^2)\right]>\gamma,$$

which holds for all α if it holds at $\alpha=1$. By plugging $\alpha=1$ and summarizing, we can rewrite the inequality as follows:

$$\gamma>\frac{\sqrt{5}+3}{2}\approx 2.618.$$

(RR)

Second, suppose α is so high that the decision maker is reactive under both authorities ($\alpha>\max\{\alpha_{\mathrm{A}},\alpha_{\mathrm{P}}\}$). Then, the principal's expected payoff under P-authority and A-authority are respectively written as follows:

$$\begin{aligned}
U_{\mathrm{P}}(\alpha,k)&=\frac{b_LB_L}{2}\left[\pi_{\mathrm{P}}(\alpha,k)\left(\alpha^2(\Gamma+\gamma)-\frac{\Gamma}{2}\right)+\frac{\Gamma}{2}\right],\\
U_{\mathrm{A}}(\alpha,k)&=\frac{b_LB_L}{2}\left[\pi_{\mathrm{A}}(\alpha,k)\left(\alpha^2(\gamma+\Gamma)-\frac{\gamma}{2}\right)+\frac{\gamma}{2}\right].
\end{aligned}$$

The difference between $U_{\mathrm{A}}(\alpha,k)$ and $U_{\mathrm{P}}(\alpha,k)$ is written by

$$\begin{aligned}
&U_{\mathrm{A}}(\alpha,k)-U_{\mathrm{P}}(\alpha,k)\\
&\quad=\frac{b_LB_L}{2}\underbrace{\left[\pi_{\mathrm{A}}(\alpha,k)\left(\alpha^2(\gamma+\Gamma)-\frac{\gamma}{2}\right)-\pi_{\mathrm{P}}(\alpha,k)\left(\alpha^2(\Gamma+\gamma)-\frac{\Gamma}{2}\right)-\frac{1}{2}(\Gamma-\gamma)\right]}_{\equiv\Delta U^{\mathrm{RR}}},
\end{aligned}$$

which is strictly positive if and only if $\Delta U^{\mathrm{RR}}>0$. Since $\pi_{\mathrm{P}}(\alpha,k)\geq\pi_{\mathrm{A}}(\alpha,k)$ for $\alpha>\alpha_{\mathrm{P}}$, $k_{\mathrm{P}}(\alpha)<k_{\mathrm{A}}(\alpha)$. There are the following two cases to consider: (i) $k\in(k_{\mathrm{P}}(\alpha),k_{\mathrm{A}}(\alpha)]$ and (ii) $k>k_{\mathrm{A}}(\alpha)$.

RR-(i) Fix $k \in (k_{\mathrm{P}}(\alpha), k_{\mathrm{A}}(\alpha)]$. Then, $\pi_{\mathrm{P}}(\alpha, k) > \pi_{\mathrm{A}}(\alpha, k) = 0$, and

$$\Delta U^{\mathrm{RR}} = -\pi_{\mathrm{P}}(\alpha, k)\left(\alpha^2(\Gamma+\gamma) - \frac{\Gamma}{2}\right) - \frac{1}{2}(\Gamma - \gamma) < 0,$$

where the inequality holds for all $k \in (k_{\mathrm{P}}(\alpha), k_{\mathrm{A}}(\alpha)]$.

RR-(ii) Fix $k > k_{\mathrm{A}}(\alpha)$. Then, $\pi_{\mathrm{A}}(\alpha, k) > 0$ and $\pi_{\mathrm{P}}(\alpha, k) > 0$, and

$$\Delta U^{\mathrm{RR}} = \pi_{\mathrm{A}}(\alpha, k)\left(\alpha^2(\gamma+\Gamma) - \frac{\gamma}{2}\right) - \pi_{\mathrm{P}}(\alpha, k)\left(\alpha^2(\Gamma+\gamma) - \frac{\Gamma}{2}\right) - \frac{1}{2}(\Gamma - \gamma).$$

$\Delta U^{\mathrm{RR}} < 0$ at $k = k_{\mathrm{A}}(\alpha)$, and $\Delta U^{\mathrm{RR}} = 0$ as $k \to +\infty$. Differentiation yields

$$\frac{d\Delta U^{\mathrm{RR}}}{dk} = \frac{\partial \pi_{\mathrm{A}}(\alpha, k)}{\partial k}\left(\alpha^2(\gamma+\Gamma) - \frac{\gamma}{2}\right) - \frac{\partial \pi_{\mathrm{P}}(\alpha, k)}{\partial k}\left(\alpha^2(\Gamma+\gamma) - \frac{\Gamma}{2}\right),$$

where $\partial \pi_{\mathrm{A}}(\alpha, k)/\partial k > \partial \pi_{\mathrm{P}}(\alpha, k)/\partial k$ since $\partial \pi_i/\partial k$ is decreasing in $y_i(\alpha)$ for $i \in \{\mathrm{A}, \mathrm{P}\}$, and $(\alpha^2(\gamma+\Gamma)-\gamma/2) > (\alpha^2(\Gamma+\gamma)-\Gamma/2)$, and thus, $d\Delta U^{\mathrm{RR}}/dk > 0$. Hence, $\Delta U^{\mathrm{RR}} \leq 0$ for all $k > k_{\mathrm{A}}(\alpha)$.

In summary, the principal never delegates for $\alpha > \max\{\alpha_{\mathrm{A}}, \alpha_{\mathrm{P}}\}$ in which both A-authority and P-authority are reactive.

(RN)

Third, suppose $\Gamma \geq \gamma^3$ and α is such that the decision maker is reactive only under A-authority ($\alpha \in (\alpha_{\mathrm{A}}, \alpha_{\mathrm{P}}]$). Then, the principal's expected payoff under P-authority and A-authority are respectively written as follows:

$$\begin{aligned}
U_{\mathrm{P}}(\alpha, k) &= \frac{b_L B_L}{2}\left[\pi_{\mathrm{P}}(\alpha, k)\left(\alpha^2 + (1-\alpha)^2 - \frac{1}{2}\right)\Gamma + \frac{\Gamma}{2}\right], \\
U_{\mathrm{A}}(\alpha, k) &= \frac{b_L B_L}{2}\left[\pi_{\mathrm{A}}(\alpha, k)\left(\alpha^2(\gamma+\Gamma) - \frac{\gamma}{2}\right) + \frac{\gamma}{2}\right].
\end{aligned}$$

The difference between $U_{\mathrm{A}}(\alpha, k)$ and $U_{\mathrm{P}}(\alpha, k)$ is written by

$$\begin{aligned}
&U_{\mathrm{A}}(\alpha, k) - U_{\mathrm{P}}(\alpha, k) \\
&\quad = \frac{b_L B_L}{2}\underbrace{\left[\pi_{\mathrm{A}}(\alpha, k)\left(\alpha^2(\gamma+\Gamma) - \frac{\gamma}{2}\right) - \pi_{\mathrm{P}}(\alpha, k)\left(\alpha^2 + (1-\alpha)^2 - \frac{1}{2}\right)\Gamma - \frac{1}{2}(\Gamma - \gamma)\right]}_{\equiv \Delta U^{\mathrm{RN}}},
\end{aligned}$$

which is strictly positive if and only if $\Delta U^{\mathrm{RN}} > 0$. Since $\pi_{\mathrm{A}}(\alpha, k) \geq \pi_{\mathrm{P}}(\alpha, k)$ for $\alpha \in (\alpha_{\mathrm{A}}, \alpha_{\mathrm{P}}]$, $k_{\mathrm{A}}(\alpha) < k_{\mathrm{P}}(\alpha)$. There are the following two cases to consider: (i) $k \in (k_{\mathrm{A}}(\alpha), k_{\mathrm{P}}(\alpha)]$ and (ii) $k > k_{\mathrm{P}}(\alpha)$. The analysis provides the proof for **Case 2** of the proposition.

RN-(i) Fix $k \in (k_{\mathrm{A}}(\alpha), k_{\mathrm{P}}(\alpha)]$. Then, $\pi_{\mathrm{A}}(\alpha, k) > \pi_{\mathrm{P}}(\alpha, k) = 0$, and

$$\Delta U^{\mathrm{RN}} = \pi_{\mathrm{A}}(\alpha, k) \left(\alpha^2(\gamma + \Gamma) - \frac{\gamma}{2} \right) - \frac{1}{2}(\Gamma - \gamma),$$

which is increasing in k and is negative at $k = k_{\mathrm{A}}(\alpha)$. Substituting $\pi_{\mathrm{A}}(\alpha, k) = 1 - (k y_{\mathrm{A}}(\alpha))^{-1}$ into ΔU^{RN} and summarizing yield $\Delta U^{\mathrm{RN}} > 0$ if and only if $k > \underline{k}(\alpha)$, where $\underline{k}(\alpha)$ is given by

$$\begin{aligned}\underline{k}(\alpha) &= \frac{1}{y_{\mathrm{A}}(\alpha)} \frac{(\alpha^2(\gamma + \Gamma) - \gamma/2)}{(\alpha^2(\gamma + \Gamma) - \gamma/2) - (1/2)(\Gamma - \gamma)}, \\ &= \frac{1}{(b_L^2/4)(\alpha^2(\gamma^2 + 1) - \gamma^2/2)} \frac{(\alpha^2(\gamma + \Gamma) - \gamma/2)}{(\alpha^2(\gamma + \Gamma) - \Gamma/2)}.\end{aligned}$$

Note the denominator must be positive, implying

$$\alpha > \frac{\sqrt{\Gamma}}{\sqrt{2(\gamma + \Gamma)}}.$$

The condition is satisfied at $\alpha = \alpha_{\mathrm{P}}$. Moreover, this condition holds at $\alpha = \alpha_{\mathrm{A}}$ if $\gamma > 1 + \sqrt{2}$, which we assume hereafter. Then, the denominator of $\underline{k}(\alpha)$ is strictly positive for all $\alpha \in (\alpha_{\mathrm{A}}, \alpha_{\mathrm{P}}]$. Furthermore, we can show that $\underline{k}(\alpha)$ is strictly decreasing in α.[8]

Since we fix $k \leq k_{\mathrm{P}}(\alpha)$, $\underline{k}(\alpha) < k_{\mathrm{P}}(\alpha)$ must hold. This inequality holds if

$$\left(\alpha^2(\gamma^2 + 1) - \frac{\gamma^2}{2} \right) \left(\alpha^2(\gamma + \Gamma) - \frac{\Gamma}{2} \right) - \left(\alpha^2(\gamma + \Gamma) - \frac{\gamma}{2} \right) \left(\alpha^2 + (1 - \alpha)^2 - \frac{1}{2} \right) > 0, \tag{A1}$$

[8]By the implicit function theorem, we obtain

$$\frac{\partial \underline{k}(\alpha)}{\partial \alpha} = -\frac{\partial \Delta U^{\mathrm{RN}}/\partial \alpha}{\partial \Delta U^{\mathrm{RN}}/\partial k},$$

where $\partial \Delta U^{\mathrm{RN}}/\partial k > 0$, and $\partial \Delta U^{\mathrm{RN}}/\partial \alpha$ is written by

$$\frac{\partial \Delta U^{\mathrm{RN}}}{\partial \alpha} = \frac{\partial \pi_{\mathrm{A}}(\alpha, k)}{\partial \alpha} \left(\alpha^2(\gamma + \Gamma) - \frac{\gamma}{2} \right) + \pi_{\mathrm{A}}(\alpha, k) 2\alpha(\gamma + \Gamma) > 0.$$

where the left-hand side (LHS) is positive at $\Gamma = \gamma^3$. To observe that, plugging $\Gamma = \gamma^3$ into LHS of (A1) yields

$$\gamma \underbrace{\left[\left(\alpha^2(\gamma^2+1) - \frac{\gamma^2}{2} \right)^2 - \left(\alpha^2(\gamma^2+1) - \frac{1}{2} \right) \left(\alpha^2 + (1-\alpha)^2 - \frac{1}{2} \right) \right]}_{\equiv D},$$

which is strictly positive if $D > 0$. Differentiation D with respect to α yields

$$\frac{\partial D}{\partial \alpha} = \alpha(\gamma^2+1) \left[2\gamma^2(2\alpha^2-1) + 4\alpha(1-\alpha) + 2\alpha - 1 \right] + (2\alpha - 1),$$

which is strictly positive for $\alpha > \alpha_{\mathrm{A}}$ because $\alpha_{\mathrm{A}} > 1/\sqrt{2}$ for $\gamma > 1+\sqrt{2}$. Thus, $D > 0$ for all $\alpha > \alpha_{\mathrm{A}}$ if $D \geq 0$ at $\alpha = \alpha_{\mathrm{A}}$.

$$\begin{aligned}
D|_{\alpha=\alpha_{\mathrm{A}}} &= \left(\frac{\gamma^2}{(1+\gamma)^2}(\gamma^2+1) - \frac{\gamma^2}{2} \right)^2 - \left(\frac{\gamma^2}{(1+\gamma)^2}(\gamma^2+1) - \frac{1}{2} \right) \left(\frac{\gamma^2}{(1+\gamma)^2} + \frac{1}{(1+\gamma)^2} - \frac{1}{2} \right), \\
&= \frac{\gamma^4(\gamma-1)^4}{(2(1+\gamma)^2)^2} - \frac{(2\gamma^2(\gamma^2+1) - (1+\gamma)^2)(\gamma-1)^2}{(2(1+\gamma^2))^2}, \\
&= \frac{(\gamma-1)^2}{(2(1+\gamma)^2)^2} \left[\gamma^4(\gamma-1)^2 - 2\gamma^2(\gamma^2+1) + (1+\gamma)^2 \right], \\
&= \frac{(\gamma-1)^2}{(2(1+\gamma)^2)^2} \left[\gamma^4(\gamma^2 - 2\gamma - 1) - (\gamma^2 - 2\gamma - 1) \right], \\
&= \frac{(\gamma-1)^2}{(2(1+\gamma)^2)^2} ((\gamma-1)^2 - 2)(\gamma^4 - 1), \\
&> 0,
\end{aligned}$$

where the last inequality holds since $(\gamma-1)^2 - 2 > 0$ for $\gamma > 1+\sqrt{2}$. Hence $D > 0$ for all $\alpha > \alpha_{\mathrm{A}}$.

The remaining proof is to show (A1) holds for $\Gamma > \gamma^3$. Differentiating LHS of (A1) with respect to Γ yields

$$\left(\alpha^2(\gamma^2+1) - \frac{\gamma^2}{2} \right) \left(\alpha^2 - \frac{1}{2} \right) - \alpha^2 \left(\alpha^2 + (1-\alpha)^2 - \frac{1}{2} \right) = \gamma^2 \left(\alpha^2 - \frac{1}{2} \right)^2 - \alpha^2(1-\alpha)^2,$$

which is strictly increasing in γ and negative at $\gamma = 1$. Hence, there exists $\gamma_1(\alpha) > 1$ such that $\gamma^2 \left(\alpha^2 - 1/2 \right)^2 - \alpha^2(1-\alpha)^2 > 0$ if and only if $\gamma > \gamma_1(\alpha)$. Note that $\gamma_1(\alpha)$ is strictly decreasing in α for $\gamma > 1+\sqrt{2}$, and $\gamma_1(\alpha_{\mathrm{A}}) = 3 > 1+\sqrt{2}$. Define $\underline{\gamma} = 3$.

To summarize the case $k \in (k_{\rm A}(\alpha), k_{\rm P}(\alpha)]$, we show that $\Delta U^{\rm RN}$ is increasing in k and strictly negative at $k = k_{\rm A}(\alpha)$. For $\gamma \geq \underline{\gamma}$, the principal delegates her authority, that is, $\Delta U^{\rm RN}$ is strictly positive, if and only if $k > \underline{k}(\alpha)$ where $\underline{k}^{\rm RN}(\alpha) < k_{\rm P}(\alpha)$ for $\gamma \geq \underline{\gamma}$. Note that $\underline{k}(\alpha)$ is strictly decreasing in α.

RN-(ii) Fix $k > k_{\rm P}(\alpha)$. Then, $\pi_{\rm A}(\alpha, k) > 0$ and $\pi_{\rm P}(\alpha, k) > 0$, and

$$\Delta U^{\rm RN} = \pi_{\rm A}(\alpha, k)\left(\alpha^2(\gamma + \Gamma) - \frac{\gamma}{2}\right) - \pi_{\rm P}(\alpha, k)\left(\alpha^2 + (1-\alpha)^2 - \frac{1}{2}\right)\Gamma - \frac{1}{2}(\Gamma - \gamma).$$

For $\gamma \geq \underline{\gamma}$, $\Delta U^{\rm RN} > 0$ at $k = k_{\rm P}(\alpha)$. At the limit $k \to +\infty$,

$$\lim_{k \to +\infty} \Delta U^{\rm RN} = \alpha^2\gamma - (1-\alpha)^2\Gamma \leq 0,$$

for $\alpha \leq \alpha_{\rm P}$. Differentiation yields

$$\begin{aligned}
\frac{d\Delta U^{\rm RN}}{dk} &= \frac{\partial \pi_{\rm A}(\alpha, k)}{\partial k}\left(\alpha^2(\gamma + \Gamma) - \frac{\gamma}{2}\right) - \frac{\partial \pi_{\rm P}(\alpha, k)}{\partial k}\left(\alpha^2 + (1-\alpha)^2 - \frac{1}{2}\right)\Gamma, \\
&= \frac{1}{k^2 y_{\rm A}(\alpha) y_{\rm P}(\alpha)}\left[\left(\alpha^2(\gamma + \Gamma) - \frac{\gamma}{2}\right) y_{\rm P}(\alpha) - \left(\alpha^2 + (1-\alpha)^2 - \frac{1}{2}\right)\Gamma y_{\rm A}(\alpha)\right], \\
&= \frac{1}{k^2 y_{\rm A}(\alpha) y_{\rm P}(\alpha)} \frac{b_L^2}{4}\left(\alpha^2 + (1-\alpha)^2 - \frac{1}{2}\right)\left[\gamma(\gamma\Gamma - 1)\left(\frac{1}{2} - \alpha^2\right)\right],
\end{aligned}$$

which is negative if and only if $\alpha \geq 1/\sqrt{2}$. Since $\alpha_{\rm A} \geq 1/\sqrt{2}$ if $\gamma \geq 1 + \sqrt{2}$, $\alpha_{\rm A} > 1/\sqrt{2}$ for $\gamma > \underline{\gamma}$. If $\gamma > \underline{\gamma}$, $d\Delta U^{\rm RN}/dk < 0$ for $\alpha > \alpha_{\rm A}$, and hence, there exists $\overline{k}(\alpha) \in (k_{\rm P}(\alpha), +\infty)$ such that $\Delta U^{\rm RN} > 0$ if $k < \overline{k}(\alpha)$, where $\overline{k}(\alpha)$ is given by

$$\begin{aligned}
\overline{k}(\alpha) &= \frac{(\alpha^2 + (1-\alpha)^2 - 1/2)\Gamma y_{\rm A}(\alpha) - (\alpha^2(\gamma + \Gamma) - \gamma/2) y_{\rm P}(\alpha)}{y_{\rm P}(\alpha) y_{\rm A}(\alpha)} \\
&\quad \times \frac{1}{((1/2)(\Gamma - \gamma) - (\alpha^2(\gamma + \Gamma) - \gamma/2) + (\alpha^2 + (1-\alpha)^2 - 1/2)\Gamma)}, \\
&= \frac{1}{((1-\alpha)^2\Gamma - \alpha^2\gamma)} \frac{\gamma(\gamma\Gamma - 1)(\alpha^2 - 1/2)}{(\alpha^2(\gamma^2 + 1) - \gamma^2/2)(b_L^2/4)}.
\end{aligned}$$

Note that $\overline{k}(\alpha)$ is strictly increasing in α.[9]

[9] By the implicit function theorem, we have

$$\frac{\partial \overline{k}^{\rm RN}(\alpha)}{\partial \alpha} = -\frac{\partial \Delta U^{\rm RN}/\partial \alpha}{\partial \Delta U^{\rm RN}/\partial k},$$

We summarize the result for $\Gamma \geq \gamma^3$ and $\alpha \in (\alpha_{\mathrm{A}}, \alpha_{\mathrm{P}}]$ where A-authority is reactive but P-authority is not. The principal delegates if $\gamma > \underline{\gamma}$ and $k \in (\underline{k}(\alpha), \overline{k}(\alpha))$ where $\underline{k}(\alpha) \in (k_{\mathrm{A}}(\alpha), k_{\mathrm{P}}(\alpha))$ and $\overline{k}(\alpha) > k_{\mathrm{P}}(\alpha)$. Note that $\underline{k}(\alpha)$ is decreasing in α and $\overline{k}(\alpha)$ is increasing in α, and hence, the interval $(\underline{k}(\alpha), \overline{k}(\alpha))$ expands as α increases.

(NR)

Finally, suppose $\Gamma < \gamma^3$ and $\alpha \in (\alpha_{\mathrm{P}}, \alpha_{\mathrm{A}}]$ and the decision maker is reactive only under P-authority. The principal's expected payoff under P-authority and A-authority are respectively written as follows:

$$\begin{aligned}
U_{\mathrm{P}}(\alpha, k) &= \frac{b_L B_L}{2}\left[\pi_{\mathrm{P}}(\alpha, k)\left(\alpha^2(\Gamma+\gamma) - \frac{\Gamma}{2}\right) + \frac{\Gamma}{2}\right] \\
U_{\mathrm{A}}(\alpha, k) &= \frac{b_L B_L}{2}\left[\pi_{\mathrm{A}}(\alpha, k)\left(\alpha^2 + (1-\alpha)^2 - \frac{1}{2}\right)\gamma + \frac{\gamma}{2}\right].
\end{aligned}$$

The difference between $U_{\mathrm{A}}(\alpha, k)$ and $U_{\mathrm{P}}(\alpha, k)$ is written by

$$\begin{aligned}
&U_{\mathrm{A}}(\alpha, k) - U_{\mathrm{P}}(\alpha, k) \\
&= \frac{b_L B_L}{2} \underbrace{\left[\pi_{\mathrm{A}}(\alpha, k)\left(\alpha^2 + (1-\alpha)^2 - \frac{1}{2}\right)\gamma - \pi_{\mathrm{P}}(\alpha, k)\left(\alpha^2(\Gamma+\gamma) - \frac{\Gamma}{2}\right) - \frac{1}{2}(\Gamma - \gamma)\right]}_{\equiv \Delta U^{\mathrm{NR}}}.
\end{aligned}$$

Since $\pi_{\mathrm{P}}(\alpha, k) \geq \pi_{\mathrm{A}}(\alpha, k)$ for $\alpha > \alpha_{\mathrm{P}}$, $k_{\mathrm{P}}(\alpha) < k_{\mathrm{A}}(\alpha)$. There are the following two cases to consider: (i) $k \in (k_{\mathrm{P}}(\alpha), k_{\mathrm{A}}(\alpha)]$ and (ii) $k > k_{\mathrm{A}}(\alpha)$.

NR-(i) Fix $k \in (k_{\mathrm{P}}(\alpha), k_{\mathrm{A}}(\alpha)]$. Then, $\pi_{\mathrm{P}}(\alpha, k) > \pi_{\mathrm{A}}(\alpha, k) = 0$, and

$$\Delta U^{\mathrm{NR}} = -\pi_{\mathrm{P}}(\alpha, k)\left(\alpha^2(\Gamma+\gamma) - \frac{\Gamma}{2}\right) - \frac{1}{2}(\Gamma - \gamma) < 0,$$

where the inequality holds for all $k \in (k_{\mathrm{P}}(\alpha), k_{\mathrm{A}}(\alpha)]$.

where $\partial \Delta U^{\mathrm{RN}}/\partial k < 0$ for $\alpha > \alpha_{\mathrm{A}}$ and $\gamma \geq \underline{\gamma}$, and $\partial \Delta U^{\mathrm{RN}}/\partial \alpha$ is written by

$$\frac{\partial \Delta U^{\mathrm{RN}}}{\partial \alpha} = 2\alpha\gamma + 2(1-\alpha) + \frac{1}{k(b_L^2/4)} \frac{\alpha\gamma(\gamma\Gamma - 1)}{(\alpha^2(\gamma^2+1) - \gamma^2/2)^2} > 0.$$

NR-(ii) Fix $k > k_{\mathrm{A}}(\alpha)$. Then, $\pi_{\mathrm{P}}(\alpha, k) > 0$ and $\pi_{\mathrm{A}}(\alpha, k) > 0$, and

$$\Delta U^{\mathrm{NR}} = \pi_{\mathrm{A}}(\alpha, k)\left(\alpha^2 + (1-\alpha)^2 - \frac{1}{2}\right)\gamma - \pi_{\mathrm{P}}(\alpha, k)\left(\alpha^2(\Gamma+\gamma) - \frac{\Gamma}{2}\right) - \frac{1}{2}(\Gamma-\gamma),$$

which is negative at $k = k_{\mathrm{A}}(\alpha)$. Since $\pi_{\mathrm{P}}(\alpha, k) > \pi_{\mathrm{A}}(\alpha, k)$, $\Delta U^{\mathrm{NR}} < 0$ if

$$\left(\alpha^2(\Gamma+\gamma) - \frac{\Gamma}{2}\right) > \left(\alpha^2 + (1-\alpha)^2 - \frac{1}{2}\right)\gamma,$$

which can be reduce to

$$\alpha^2\Gamma - (1-\alpha)^2\gamma + \frac{\gamma}{2} - \frac{\Gamma}{2} > 0, \tag{A2}$$

where LHS is increasing in α. At $\alpha = \alpha_{\mathrm{P}}$, LHS of (A2) is written by

$$\begin{aligned}(\alpha_{\mathrm{P}})^2\Gamma - (1-\alpha_{\mathrm{P}})^2\gamma + \frac{\gamma}{2} - \frac{\Gamma}{2} &= \frac{2\Gamma^2 - 2\gamma^2 - (\Gamma-\gamma)(\sqrt{\gamma}+\sqrt{\Gamma})^2}{2(\sqrt{\gamma}+\sqrt{\Gamma})^2}, \\ &= \frac{(\Gamma-\gamma)\left[2(\Gamma+\gamma) - (\sqrt{\Gamma}+\sqrt{\gamma})^2\right]}{2(\sqrt{\gamma}+\sqrt{\Gamma})^2}, \\ &= \frac{(\Gamma-\gamma)\left[\Gamma - 2\sqrt{\gamma}\sqrt{\Gamma} + \gamma\right]}{2(\sqrt{\gamma}+\sqrt{\Gamma})^2}, \\ &= \frac{(\Gamma-\gamma)(\sqrt{\Gamma}-\sqrt{\gamma})^2}{2(\sqrt{\gamma}+\sqrt{\Gamma})^2}, \\ &> 0.\end{aligned}$$

Hence, $\Delta U^{\mathrm{NR}} < 0$ for $k > k_{\mathrm{A}}(\alpha)$.

In summary, the principal always retains the authority if $\alpha \in (\alpha_{\mathrm{P}}, \alpha_{\mathrm{A}}]$ where P-authority is reactive but A-authority is not.

□

Proof of Lemma 2

Since $\pi_{\mathrm{hom}}(\alpha) > \pi_{\mathrm{A}}(\alpha)$ if and only if $y_{\mathrm{hom}}(\alpha) > y_{\mathrm{A}}(\alpha)$, we compare the marginal benefit. For $\alpha \in (1/2, \alpha_{\mathrm{A}}]$ or $\alpha \in (\alpha_{\mathrm{hom}}, 1]$, $y_{\mathrm{hom}}(\alpha) = y_{\mathrm{A}}(\alpha)$ by the definitions, and hence, $\pi_{\mathrm{hom}}(\alpha) = \pi_{\mathrm{A}}(\alpha)$. For $\alpha \in (\alpha_{\mathrm{A}}, \alpha_{\mathrm{hom}}]$, $y_{\mathrm{A}}(\alpha) > y_{\mathrm{hom}}(\alpha)$ if and only if $K(\alpha b_L) > K((1-\alpha)b_H)$, which holds for $\alpha > \alpha_{\mathrm{A}}$. $\square$

Proof of Proposition 2

The principal's expected payoffs under P-authority and A-authority are rewritten as follows:

$$\begin{aligned} U_{\mathrm{Phom}}(\alpha, k) &= \pi_{\mathrm{hom}}(\alpha, k)\left(u^{12}_{\mathrm{Phom}}(\alpha) - u^{\phi}_{\mathrm{Phom}}\right) + u^{\phi}_{\mathrm{Phom}}, \\ U_{\mathrm{Ahom}}(\alpha, k) &= \pi_{\mathrm{A}}(\alpha, k)\left(u^{12}_{\mathrm{Ahom}}(\alpha) - u^{\phi}_{\mathrm{Ahom}}\right) + u^{\phi}_{\mathrm{Ahom}}. \end{aligned}$$

where $u^{12}_{i\mathrm{hom}}(\alpha)$ and $u^{\phi}_{i\mathrm{hom}}$ are defined in Subsection 3.5.

For $\alpha \in (1/2, \alpha_{\mathrm{A}}]$ or $\alpha \in (\alpha_{\mathrm{hom}}, 1]$, $\pi_{\mathrm{A}}(\alpha, k) = \pi_{\mathrm{hom}}(\alpha, k)$ by Lemma 2, and both $u^{12}_{\mathrm{Phom}}(\alpha) = u^{12}_{\mathrm{Ahom}}$ and $u^{\phi}_{\mathrm{Phom}}(\alpha) = u^{\phi}_{\mathrm{Ahom}}$ hold by the definitions, and hence $U_{\mathrm{Phom}}(\alpha, k) = U_{\mathrm{Ahom}}(\alpha, k)$: The principal is indifferent between P-authority and A-authority, and hence retains authority.

We thus consider the remaining case $\alpha \in (\alpha_{\mathrm{A}}, \alpha_{\mathrm{hom}}]$. The difference between $U_{\mathrm{Ahom}}(\alpha, k)$ and $U_{\mathrm{Phom}}(\alpha, k)$ is given by

$$\begin{aligned} U_{\mathrm{Ahom}}(\alpha, k) - U_{\mathrm{Phom}}(\alpha, k) &= \pi_{\mathrm{A}}(\alpha, k)\left(u^{12}_{\mathrm{Ahom}}(\alpha) - u^{\phi}_{\mathrm{Ahom}}\right) + u^{\phi}_{\mathrm{Ahom}} \\ &\quad - \pi_{\mathrm{hom}}(\alpha, k)\left(u^{12}_{\mathrm{Phom}}(\alpha) - u^{\phi}_{\mathrm{Phom}}\right) - u^{\phi}_{\mathrm{Phom}}, \\ &= \pi_{\mathrm{A}}(\alpha, k)\left(u^{12}_{\mathrm{Ahom}}(\alpha) - u^{\phi}_{\mathrm{Ahom}}\right) - \pi_{\mathrm{hom}}(\alpha, k)\left(u^{12}_{\mathrm{Phom}}(\alpha) - u^{\phi}_{\mathrm{Phom}}\right), \end{aligned}$$

where the last equality follows from $u^{\phi}_{\mathrm{Ahom}} = u^{\phi}_{\mathrm{Phom}}$.

Since $\pi_{\mathrm{A}}(\alpha, k) \geq \pi_{\mathrm{hom}}(\alpha, k)$ for $\alpha \in (\alpha_{\mathrm{A}}, \alpha_{\mathrm{hom}}]$, $k_{\mathrm{A}}(\alpha) < k_{\mathrm{hom}}(\alpha)$ holds. Consider the following three cases separately: (i) $k \leq k_{\mathrm{A}}(\alpha)$, (ii) $k \in (k_{\mathrm{A}}(\alpha), k_{\mathrm{hom}}(\alpha)]$, and (iii) $k > k_{\mathrm{hom}}(\alpha)$.

(i) Fix $k \leq k_{\mathrm{A}}(\alpha)$. Then, $\pi_{\mathrm{A}}(\alpha, k) = \pi_{\mathrm{hom}}(\alpha, k) = 0$, and hence $U_{\mathrm{Ahom}}(\alpha, k) - U_{\mathrm{Phom}}(\alpha, k) = 0$.

(ii) Fix $k \in (k_{\mathrm{A}}(\alpha), k_{\mathrm{hom}}(\alpha)]$. Then, $\pi_{\mathrm{A}}(\alpha, k) > \pi_{\mathrm{hom}}(\alpha, k) = 0$, and the difference between $U_{\mathrm{Ahom}}(\alpha, k)$ and $U_{\mathrm{hom}}(\alpha, k)$ is given as follows:

$$U_{\mathrm{Ahom}}(\alpha, k) - U_{\mathrm{hom}}(\alpha, k) = \pi_{\mathrm{A}}(\alpha, k)\left(u^{12}_{\mathrm{Ahom}}(\alpha) - u^{\phi}_{\mathrm{Ahom}}\right),$$

which is positive if

$$u^{12}_{\text{Ahom}}(\alpha) - u^{\phi}_{\text{Ahom}} = \frac{b_L B_L}{2}\left[\alpha^2(\gamma\Gamma+1) - \frac{\gamma\Gamma}{2}\right] > 0.$$

This inequality holds if and only if

$$\alpha > \frac{\sqrt{\gamma\Gamma}}{\sqrt{2(\gamma\Gamma+1)}} \equiv \underline{\alpha}_{\text{hom}}(\Gamma).$$

We show that if $\gamma \geq 1+\sqrt{2}$, then this condition holds for all $\alpha \in (\alpha_{\text{A}}, \alpha_{\text{hom}}]$ and $\Gamma \geq \gamma$. Note that $\underline{\alpha}_{\text{hom}}(\Gamma)$ is increasing in Γ, $\underline{\alpha}_{\text{hom}}(\Gamma) < \alpha_{\text{A}}$ at $\Gamma = \gamma$, and $\underline{\alpha}_{\text{hom}}(\Gamma) \to 1/\sqrt{2}$ as $\Gamma \to \infty$. However, the condition for $1/\sqrt{2} < \alpha_{\text{A}}$ turns out to be equivalent to $\gamma > 1+\sqrt{2}$.

(iii) Fix $k > k_{\text{hom}}(\alpha)$. Then, $\pi_{\text{A}}(\alpha,k) > \pi_{\text{hom}}(\alpha,k) > 0$, and the difference between $U_{\text{Ahom}}(\alpha,k)$ and $U_{\text{Phom}}(\alpha,k)$ is given as follows:

$$U_{\text{Ahom}}(\alpha,k) - U_{\text{Phom}}(\alpha,k) = \pi_{\text{A}}(\alpha,k)\left(u^{12}_{\text{Ahom}}(\alpha) - u^{\phi}_{\text{Ahom}}\right) - \pi_{\text{hom}}(\alpha,k)\left(u^{12}_{\text{Phom}}(\alpha) - u^{\phi}_{\text{Phom}}\right).$$

Differentiating with respect to k yields

$$\begin{aligned}\frac{dU_{\text{Ahom}}(\alpha,k) - U_{\text{Phom}}(\alpha,k)}{dk} = & \frac{\partial \pi_{\text{A}}(\alpha,k)}{\partial k}\left(u^{12}_{\text{Ahom}}(\alpha) - u^{\phi}_{\text{Ahom}}\right) \\ & - \frac{\partial \pi_{\text{hom}}(\alpha,k)}{\partial k}\left(u^{12}_{\text{Phom}}(\alpha) - u^{\phi}_{\text{Phom}}\right).\end{aligned}$$

Since $\partial\pi_{\text{A}}(\alpha,k)/\partial k < \partial\pi_{\text{hom}}(\alpha,k)/\partial k$, $dU_{\text{Ahom}}(\alpha,k) - U_{\text{Phom}}(\alpha,k)/dk < 0$ holds if

$$u^{12}_{\text{Phom}}(\alpha) - u^{\phi}_{\text{Phom}} > u^{12}_{\text{Ahom}}(\alpha) - u^{\phi}_{\text{Ahom}},$$

which can be reduced to

$$u^{12}_{\text{Phom}}(\alpha) > u^{12}_{\text{Ahom}}(\alpha),$$

This last condition holds for all $\alpha < \alpha_{\text{hom}}$. Hence for $\alpha \in (\alpha_{\text{A}}, \alpha_{\text{hom}}]$, $U_{\text{Ahom}}(\alpha,k) - U_{\text{Phom}}(\alpha,k)$ is decreasing in k. From the proof of case (ii), if $\gamma > 1+\sqrt{2}$, $U_{\text{Ahom}}(\alpha,k) - U_{\text{Phom}}(\alpha,k) > 0$ holds at $k = k_{\text{hom}}(\alpha)$. Then there exists $\overline{k}_{\text{hom}}(\alpha) \in (k_{\text{hom}}(\alpha), +\infty)$ such that $U_{\text{Ahom}}(\alpha,k) - U_{\text{Phom}}(\alpha,k) > 0$ if and only if $k < \overline{k}_{\text{hom}}(\alpha)$. This $\overline{k}_{\text{hom}}(\alpha)$ is given by

$$\overline{k}_{\text{hom}}(\alpha) = \frac{\left[(u^{12}_{\text{Phom}}(\alpha) - u^{\phi}_{\text{Phom}})y_{\text{A}}(\alpha) - (u^{12}_{\text{Ahom}}(\alpha) - u^{\phi}_{\text{Ahom}})y_{\text{hom}}(\alpha)\right]}{(u^{12}_{\text{Phom}}(\alpha) - u^{12}_{\text{Ahom}}(\alpha))y_{\text{hom}}(\alpha)y_{\text{A}}(\alpha)},$$
$$= \frac{(\Gamma-\gamma)\alpha^2}{((1-\alpha)^2\gamma\Gamma - \alpha^2)\gamma(b_L^2/4)(\alpha^2(\gamma^2+1) - \gamma^2/2)}.$$

Note that $\overline{k}_{\text{hom}}(\alpha)$ is strictly increasing in α.[10]

$\square$

Proof of Proposition 3

Heterogenous organization

We first analyze the heterogenous organization where the principal favors project 1 and the agent favors project 2. The principal's expected benefit conditional on $\sigma \in \{1, 2\}$, denoted by $\hat{u}^{12}_i(\alpha)$, and that conditional on $\sigma = \phi$, denoted by $\hat{u}^{\phi}_i$, under i-authority for $i \in \{\text{A}, \text{P}\}$ are given as follows.

$$\hat{u}^{12}_{\text{P}}(\alpha) = \frac{1}{2}\left[\alpha B_H + (1-\alpha)B_H + \mathbf{1}_{\alpha > \hat{\alpha}_{\text{P}}}(\alpha B_L - (1-\alpha)B_H)\right],$$
$$\hat{u}^{12}_{\text{A}}(\alpha) = \frac{1}{2}\left[(1-\alpha)B_L + \alpha B_L + \mathbf{1}_{\alpha > \alpha_{\text{A}}}(\alpha B_H - (1-\alpha)B_L)\right]$$
$$\hat{u}^{\phi}_{\text{P}} = \frac{1}{2}B_H > \frac{1}{2}B_L = \hat{u}^{\phi}_{\text{A}}.$$

Note that $\hat{u}^{12}_i(\alpha)$ and $\hat{u}^{\phi}_i$ do not depend on the agent's private benefit (and his bias γ).

Under the assumption on the cost function, we rewrite the principal's expected benefit as $\hat{U}_i(\alpha, k)$ for $i \in \{\text{A}, \text{P}\}$. The difference between $\hat{U}_{\text{A}}(\alpha, k)$ and $\hat{U}_{\text{P}}(\alpha, k)$ is given by

$$\hat{U}_{\text{A}}(\alpha, k) - \hat{U}_{\text{P}}(\alpha, k) = \hat{\pi}_{\text{A}}(\alpha, k)\left(\hat{u}^{12}_{\text{A}}(\alpha) - \hat{u}^{\phi}_{\text{A}}\right) - \hat{\pi}_{\text{P}}(\alpha, k)\left(\hat{u}^{12}_{\text{P}}(\alpha) - \hat{u}^{\phi}_{\text{P}}\right) + \hat{u}^{\phi}_{\text{A}} - \hat{u}^{\phi}_{\text{P}}.$$

[10]By the implicit function theorem, we have

$$\frac{\partial \overline{k}_{\text{hom}}(\alpha)}{\partial \alpha} = -\frac{\partial(U_{\text{Ahom}}(\alpha, k) - U_{\text{Phom}}(\alpha, k))/\partial\alpha}{\partial(U_{\text{Ahom}}(\alpha, k) - U_{\text{Phom}}(\alpha, k))/\partial k},$$

where $\partial(U_{\text{Ahom}}(\alpha, k) - U_{\text{Phom}}(\alpha, k))/\partial k > 0$, and

$$\frac{\partial(U_{\text{Ahom}}(\alpha, k) - U_{\text{Phom}}(\alpha, k))}{\partial\alpha} = \frac{b_L B_L}{2}\left[2\alpha + 2(1-\alpha)\gamma\Gamma + \frac{2\alpha(\gamma^3/2)(\Gamma-\gamma)}{k(b_L^2/4)(\alpha^2(\gamma^2+1) - \gamma^2/2)^2}\right] > 0.$$

We show that $\hat{U}_{\mathrm{A}}(\alpha,k)-\hat{U}_{\mathrm{P}}(\alpha,k)\leq 0$ holds for all α and k, by considering the following three cases depending on the value of α. First, suppose $\alpha\in(1/2,\alpha_{\mathrm{A}}]$. Then, $\hat{\pi}_{\mathrm{A}}(\alpha,k)=\hat{\pi}_{\mathrm{P}}(\alpha,k)=0$ for all k, and thus $\hat{U}_{\mathrm{A}}(\alpha,k)-\hat{U}_{\mathrm{P}}(\alpha,k)=\hat{u}^{\phi}_{\mathrm{A}}-\hat{u}^{\phi}_{\mathrm{P}}<0$.

Second, suppose $\alpha\in(\alpha_{\mathrm{A}},\hat{\alpha}_{\mathrm{P}}]$. Then, $\hat{\pi}_{\mathrm{A}}(\alpha,k)\geq\hat{\pi}_{\mathrm{P}}(\alpha,k)=0$, and hence

$$
\begin{aligned}
\hat{U}_{\mathrm{A}}(\alpha,k)-\hat{U}_{\mathrm{P}}(\alpha,k)&=\hat{\pi}_{\mathrm{A}}(\alpha,k)\left(\hat{u}^{12}_{\mathrm{A}}(\alpha)-\hat{u}^{\phi}_{\mathrm{A}}\right)+\hat{u}^{\phi}_{\mathrm{A}}-\hat{u}^{\phi}_{\mathrm{P}}\\
&=\frac{B_L}{2}\left[\hat{\pi}_{\mathrm{A}}(\alpha,k)\left(\alpha(1+\Gamma)-1\right)-(\Gamma-1)\right],
\end{aligned}
$$

which never becomes positive because

$$\hat{\pi}_{\mathrm{A}}(\alpha,k)\left(\alpha(1+\Gamma)-1\right)-(\Gamma-1)\leq\alpha(1+\Gamma)-1-(\Gamma-1)=\alpha(1+\Gamma)-\Gamma\leq 0$$

for $\alpha\leq\hat{\alpha}_{\mathrm{P}}$.

Finally, suppose $\alpha\in(\hat{\alpha}_{\mathrm{P}},1]$. Then $\hat{u}^{12}_{\mathrm{P}}(\alpha)=\hat{u}^{12}_{\mathrm{A}}(\alpha)$, which we denote by $\hat{u}^{12}(\alpha)$. Then

$$
\begin{aligned}
\hat{U}_{\mathrm{A}}(\alpha,k)-\hat{U}_{\mathrm{P}}(\alpha,k)&=\hat{\pi}_{\mathrm{A}}(\alpha,k)\left(\hat{u}^{12}(\alpha)-\hat{u}^{\phi}_{\mathrm{A}}\right)-\hat{\pi}_{\mathrm{P}}(\alpha,k)\left(\hat{u}^{12}(\alpha)-\hat{u}^{\phi}_{\mathrm{P}}\right)+\hat{u}^{\phi}_{\mathrm{A}}-\hat{u}^{\phi}_{\mathrm{P}}\\
&=-(\hat{\pi}_{\mathrm{P}}(\alpha,k)-\hat{\pi}_{\mathrm{A}}(\alpha,k))\left(\hat{u}^{12}(\alpha)-\hat{u}^{\phi}_{\mathrm{P}}\right)+(1-\hat{\pi}_{\mathrm{A}}(\alpha,k))\left(\hat{u}^{\phi}_{\mathrm{A}}-\hat{u}^{\phi}_{\mathrm{P}}\right)\\
&\leq 0.
\end{aligned}
$$

The last inequality holds since $\hat{\pi}_{\mathrm{P}}(\alpha,k)\geq\hat{\pi}_{\mathrm{A}}(\alpha,k)$, $\hat{u}^{12}(\alpha)>\hat{u}^{\phi}_{\mathrm{P}}$ for $\alpha>\hat{\alpha}_{\mathrm{P}}$, and $\hat{u}^{\phi}_{\mathrm{A}}<\hat{u}^{\phi}_{\mathrm{P}}$. Hence $\hat{U}_{\mathrm{A}}(\alpha,k)-\hat{U}_{\mathrm{P}}(\alpha,k)\leq 0$ for all k. □

Homogeneous organization

The principal's expected benefit conditional on information signal $\sigma\{1,2\}$, denoted by $\hat{u}^{12}_{i\mathrm{hom}}(\alpha)$, and that conditional on uninformative signal $\sigma=\phi$, denoted by $u^{\phi}_{i\mathrm{hom}}$, are calculated as follows:

$$
\begin{aligned}
\hat{u}^{12}_{\mathrm{Phom}}(\alpha)&=\frac{1}{2}\left[\alpha B_H+(1-\alpha)B_H+\mathbf{1}_{\alpha>\hat{\alpha}_{\mathrm{P}}}\left(\alpha B_L-(1-\alpha)B_H\right)\right]\\
\hat{u}^{12}_{\mathrm{Ahom}}(\alpha)&=\frac{1}{2}\left[(1-\alpha)B_H+\alpha B_H+\mathbf{1}_{\alpha>\alpha_{\mathrm{A}}}\left(\alpha B_L-(1-\alpha)B_H\right)\right]\\
\hat{u}^{\phi}_{\mathrm{Phom}}&=\hat{u}^{\phi}_{\mathrm{Ahom}}=\frac{1}{2}B_H.
\end{aligned}
$$

Since $\hat{u}^{\phi}_{\mathrm{Phom}}=\hat{u}^{\phi}_{\mathrm{Ahom}}$ holds, and $\hat{u}^{12}_{\mathrm{Phom}}(\alpha)=\hat{u}^{12}_{\mathrm{Ahom}}(\alpha)$ for $\alpha>\hat{\alpha}_{\mathrm{P}}$ or $\alpha\leq\alpha_{\mathrm{A}}$, the principal is indifferent between two authorities for $\alpha>\hat{\alpha}_{\mathrm{P}}$ or $\alpha\leq\alpha_{\mathrm{A}}$ as in the three-stage model in the homogeneous organization. We thus focus on her delegation decision for $\alpha \in (\alpha_{\mathrm{A}}, \hat{\alpha}_{\mathrm{P}}]$.

Let $\hat{U}_{i\mathrm{hom}}(\alpha, k)$ denote the principal's expected payoff under i-authority for $i \in \{\mathrm{A}, \mathrm{P}\}$. For $\alpha \in (\alpha_{\mathrm{A}}, \hat{\alpha}_{\mathrm{P}}]$, $\hat{\pi}_{\mathrm{A}}(\alpha, k) \geq \hat{\pi}_{\mathrm{hom}}(\alpha, k) = 0$ by Lemma 3. If $k \leq \hat{k}_{\mathrm{A}}(\alpha)$, $\hat{\pi}_{\mathrm{A}}(\alpha, k) = \hat{\pi}_{\mathrm{hom}}(\alpha, k) = 0$, and hence $\hat{U}_{\mathrm{Ahom}}(\alpha, k) = \hat{U}_{\mathrm{Phom}}(\alpha, k)$. If $k > \hat{k}_{\mathrm{A}}(\alpha)$, then $\hat{\pi}_{\mathrm{A}}(\alpha, k) > \hat{\pi}_{\mathrm{hom}}(\alpha, k) = 0$, and the difference between $\hat{U}_{\mathrm{Ahom}}(\alpha, k)$ and $\hat{U}_{\mathrm{Phom}}(\alpha, k)$ is written by

$$\begin{aligned}
\hat{U}_{\mathrm{Ahom}}(\alpha, k) - \hat{U}_{\mathrm{Phom}}(\alpha, k) &= \hat{\pi}_{\mathrm{A}}(\alpha, k) \left(\hat{u}^{12}_{\mathrm{Ahom}}(\alpha) - \hat{u}^{\phi}_{\mathrm{A}} \right) + \hat{u}^{\phi}_{\mathrm{A}} - \hat{u}^{\phi}_{\mathrm{P}} \\
&= \hat{\pi}_{\mathrm{A}}(\alpha, k) \left(\hat{u}^{12}_{\mathrm{Ahom}}(\alpha) - \hat{u}^{\phi}_{\mathrm{A}} \right) \\
&= \hat{\pi}_{\mathrm{A}}(\alpha, k) \frac{B_L}{2} \left(\alpha(\Gamma + 1) - \Gamma \right),
\end{aligned}$$

which is negative for $\alpha \leq \hat{\alpha}_{\mathrm{P}}$. $\square$

Appendix B: Online Appendix

In Subsection B0.1, we consider the case where the principal has strictly weaker bias than the agent ($\Gamma < \gamma$) and show that delegation is less likely to be optimal.

B0.1 Less Biased Principal

In the main analysis, we assume that the principal has stronger bias than the agent ($\Gamma \geq \gamma$). In this subsection, we instead assume $\Gamma < \gamma$ and show when delegation is optimal if the principal has weaker bias than the agent. Under A-authority, all of project implementation, project choice, and information acquisition are unchanged. Under P-authority, project implementation is unaffected, and thus we first analyze project choice and information acquisition, and then, show when the principal delegates her authority to the agent.

Project Choice In contrast to the main analysis, the principal chooses the agent's favorite project 2 upon the realization of $\sigma \in \{\phi, 2\}$ for all α since she prefers to increase the agent's implementation motivation more than her own private benefit. If $\sigma = 1$, there exists $\check{\alpha}_{\mathrm{P}} \in (1/2, 1)$ such that the principal chooses project 1 if and only if $\alpha > \check{\alpha}_{\mathrm{P}}$, where $\check{\alpha}_{\mathrm{P}}$ is the solution to $\alpha F(\alpha b_L) B_H > (1-\alpha) F((1-\alpha) b_H) B_L$, that is,

$$\check{\alpha}_{\mathrm{P}} = \frac{\sqrt{\gamma}}{\sqrt{\Gamma} + \sqrt{\gamma}}.$$

We can show that $\alpha_{\mathrm{A}} > \check{\alpha}_{\mathrm{P}}$, and thus, the principal is more likely to be reactive.

Information Acquisition The agent's expected payoff under P-authority is given by

$$\frac{\pi}{2}\left[K((1-\alpha)b_H) + K(\alpha b_H) + \mathbf{1}_{\alpha > \check{\alpha}_{\mathrm{P}}}(K(\alpha b_L) - K((1-\alpha)b_H))\right] + (1-\pi)K\left(\frac{b_H}{2}\right) - \psi(\pi).$$

The first-order condition for the optimal effort, denoted by $\check{\pi}_{\mathrm{P}}(\alpha)$, is given as follows:

$$\frac{1}{2}\left[K((1-\alpha)b_H) + K(\alpha b_H) + \mathbf{1}_{\alpha > \check{\alpha}_{\mathrm{P}}}(K(\alpha b_L) - K((1-\alpha)b_H))\right] - K\left(\frac{b_H}{2}\right) = \psi'(\pi),$$

where LHS, denoted by $\check{y}_{\mathrm{P}}(\alpha)$, represents the marginal benefit from gathering information. By comparing the marginal benefit under each authority, we can show that the agent exerts weakly greater information-gathering effort under delegation. From the definitions, $\check{y}_{\mathrm{P}}(\alpha) =$

$y_{\mathrm{A}}(\alpha)$ for $\alpha \leq \check{\alpha}_{\mathrm{P}}$ and for $\alpha > \alpha_{\mathrm{A}}$. For $\alpha \in (\check{\alpha}_{\mathrm{P}}, \alpha_{\mathrm{A}}]$, $y_{\mathrm{A}}(\alpha) > \check{y}_{\mathrm{P}}(\alpha)$ if and only if $K((1-\alpha)b_H) > K(\alpha b_L)$, which holds for $\alpha < \alpha_{\mathrm{A}}$. Hence, $\pi_{\mathrm{A}}(\alpha) > \check{\pi}_{\mathrm{P}}(\alpha)$ for $\alpha \in (\check{\alpha}_{\mathrm{P}}, \alpha_{\mathrm{A}}]$.

Intuitively, when the signal precision is so low that the decision maker is not reactive under either authority or so high that the decision maker is reactive under both authorities, project choice does not differ with authority, and hence the agent chooses the same information-gathering effort under both authorities. If the signal precision is such that the decision maker is reactive only under P-authority, the principal reacts to the signal even though the signal precision is not high enough for the agent to react, and hence, the reactivity by the principal hurts the agent's motivation to gather information.

Delegation Under P-authority, the principal's expected benefit $\check{u}_{\mathrm{P}}^{12}(\alpha)$ conditional on $\sigma \in \{1,2\}$ and $\check{u}_{\mathrm{P}}^{\phi}$ conditional on $\sigma = \phi$ are given as follows:

$$\begin{aligned}
\check{u}_{\mathrm{P}}^{12}(\alpha) = \frac{1}{2}\Big[&(1-\alpha)F((1-\alpha)b_H)B_L + \alpha F(\alpha b_H)B_L \\
&+ \mathbf{1}_{\alpha > \check{\alpha}_{\mathrm{P}}}(\alpha F(\alpha b_L)B_H - (1-\alpha)F((1-\alpha)b_H)B_L)\Big], \\
\check{u}_{\mathrm{P}}^{\phi} = \frac{1}{2}&F\left(\frac{b_H}{2}\right)B_L = u_{\mathrm{A}}^{\phi}.
\end{aligned}$$

The principal's expected benefit conditional on $\sigma \in \{1,2\}$ is the same as that under A-authority for $\alpha \leq \check{\alpha}_{\mathrm{P}}$ or $\alpha > \alpha_{\mathrm{A}}$ since only the threshold of the signal precision is different. Note further that $\check{u}_{\mathrm{P}}^{\phi} = u_{\mathrm{A}}^{\phi}$ since the principal chooses project 2 without information.

As in the main analysis, we introduce a parameter k and adopt the specific cost function (3). Then, the optimal information-gathering effort under P-authority is rewritten as $\check{\pi}_{\mathrm{P}}(\alpha, k)$, where $\check{\pi}_{\mathrm{P}}(\alpha, k) > 0$ if and only if $k > \check{k}_{\mathrm{P}}(\alpha)$.

The following proposition shows when the less biased principal delegates her authority.

Proposition B1. *Suppose the principal has less bias than the agent ($\Gamma < \gamma$). Then the principal retains her authority for $\alpha \in (1/2, \check{\alpha}_{\mathrm{P}}]$ or $\alpha \in (\alpha_{\mathrm{A}}, 1]$. If $\alpha \in (\check{\alpha}_{\mathrm{P}}, \alpha_{\mathrm{A}}]$, there exists an interval $(k_{\mathrm{A}}(\alpha), \overline{k}_{\mathrm{less}}(\alpha))$, where $\overline{k}_{\mathrm{less}}(\alpha) > \check{k}_{\mathrm{P}}(\alpha)$, such that A-authority is optimal if and only if k belongs to the interval.*

Proof. The principal's expected payoff under P-authority is written as follows:

$$\check{U}_{\mathrm{P}}(\alpha, k) = \check{\pi}_{\mathrm{P}}(\alpha, k)\left(\check{u}_{\mathrm{P}}^{12}(\alpha) - \check{u}_{\mathrm{P}}^{\phi}\right) + \check{u}_{\mathrm{P}}^{\phi}.$$

For $\alpha \in (1/2, \check{\alpha}_{\mathrm{P}}]$ or $\alpha \in (\alpha_{\mathrm{A}}, 1]$, $\pi_{\mathrm{A}}(\alpha, k) = \check{\pi}_{\mathrm{P}}(\alpha, k)$ and both $\check{u}_{\mathrm{P}}^{12}(\alpha) = u_{\mathrm{A}}^{12}(\alpha)$ and $\check{u}_{\mathrm{P}}^{\phi} = u_{\mathrm{A}}^{\phi}$ by the definitions, and thus $\check{U}_{\mathrm{P}}(\alpha, k) = U_{\mathrm{A}}(\alpha, k)$. Hence, the principal is indifferent between P-authority and A-authority, and retains her authority.

We thus consider the remaining case $\alpha \in (\check{\alpha}_{\mathrm{P}}, \alpha_{\mathrm{A}}]$. The difference between $U_{\mathrm{A}}(\alpha, k)$ and $\check{U}_{\mathrm{P}}(\alpha, k)$ is given by

$$\begin{aligned} U_{\mathrm{A}}(\alpha, k) - \check{U}_{\mathrm{P}}(\alpha, k) &= \pi_{\mathrm{A}}(\alpha, k)\left(u_{\mathrm{A}}^{12}(\alpha) - u_{\mathrm{A}}^{\phi}\right) + u_{\mathrm{A}}^{\phi} - \check{\pi}_{\mathrm{P}}(\alpha, k)\left(\check{u}_{\mathrm{P}}^{12}(\alpha) - \check{u}_{\mathrm{P}}^{\phi}\right) - \check{u}_{\mathrm{P}}^{\phi}, \\ &= \pi_{\mathrm{A}}(\alpha, k)\left(u_{\mathrm{A}}^{12}(\alpha) - u_{\mathrm{A}}^{\phi}\right) - \check{\pi}_{\mathrm{P}}(\alpha, k)\left(\check{u}_{\mathrm{P}}^{12}(\alpha) - \check{u}_{\mathrm{P}}^{\phi}\right), \end{aligned}$$

where the last equality holds by $u_{\mathrm{A}}^{\phi} = \check{u}_{\mathrm{P}}^{\phi}$.

Since $\pi_{\mathrm{A}}(\alpha, k) \geq \check{\pi}_{\mathrm{P}}(\alpha, k)$ for $\alpha \in (\check{\alpha}_{\mathrm{P}}, \alpha_{\mathrm{A}}]$, $k_{\mathrm{A}}(\alpha) < \check{k}_{\mathrm{P}}(\alpha)$. Then, there are the following three cases: (i) $k \leq k_{\mathrm{A}}(\alpha)$, (ii) $k \in (k_{\mathrm{A}}(\alpha), \check{k}_{\mathrm{P}}(\alpha)]$, and (iii) $k > \check{k}_{\mathrm{P}}(\alpha)$.

(i) Fix $k \leq k_{\mathrm{A}}(\alpha)$. Then, $\pi_{\mathrm{A}}(\alpha, k) = \check{\pi}_{\mathrm{P}}(\alpha, k) = 0$, and hence $U_{\mathrm{A}}(\alpha, k) - \check{U}_{\mathrm{P}}(\alpha, k) = 0$.

(ii) Fix $k \in (k_{\mathrm{A}}(\alpha), \check{k}_{\mathrm{P}}(\alpha)]$. Then, $\pi_{\mathrm{A}}(\alpha, k) > \check{\pi}_{\mathrm{P}}(\alpha, k) = 0$, and the difference between $U_{\mathrm{A}}(\alpha, k)$ and $\check{U}_{\mathrm{P}}(\alpha, k)$ is given as follows:

$$U_{\mathrm{A}}(\alpha, k) - \check{U}_{\mathrm{P}}(\alpha, k) = \pi_{\mathrm{A}}(\alpha, k)\left(u_{\mathrm{A}}^{12}(\alpha) - u_{\mathrm{A}}^{\phi}\right) = \pi_{\mathrm{A}}(\alpha, k)\frac{b_H B_L}{2}\left(\alpha^2 + (1-\alpha)^2 - \frac{1}{2}\right),$$

which is strictly positive for $\alpha > 1/2$. Hence, $U_{\mathrm{A}}(\alpha, k) - \check{U}_{\mathrm{P}}(\alpha, k) > 0$.

(iii) Fix $k > \check{k}_{\mathrm{P}}(\alpha)$. Then, $\pi_{\mathrm{A}}(\alpha, k) > 0$ and $\check{\pi}_{\mathrm{P}}(\alpha, k) > 0$, and the difference between $U_{\mathrm{A}}(\alpha, k)$ and $\check{U}_{\mathrm{P}}(\alpha, k)$ is given by

$$U_{\mathrm{A}}(\alpha, k) - \check{U}_{\mathrm{P}}(\alpha, k) = \pi_{\mathrm{A}}(\alpha, k)\left(u_{\mathrm{A}}^{12}(\alpha) - u_{\mathrm{A}}^{\phi}\right) - \check{\pi}_{\mathrm{P}}(\alpha, k)\left(\check{u}_{\mathrm{P}}^{12}(\alpha) - \check{u}_{\mathrm{P}}^{\phi}\right),$$

which is strictly positive at $k = \check{k}_{\mathrm{P}}(\alpha)$. Differentiation yields

$$\frac{dU_{\mathrm{A}}(\alpha, k) - \check{U}_{\mathrm{P}}(\alpha, k)}{dk} = \frac{\partial \pi_{\mathrm{A}}(\alpha, k)}{\partial k}\left(u_{\mathrm{A}}^{12}(\alpha) - u_{\mathrm{A}}^{\phi}\right) - \frac{\partial \check{\pi}_{\mathrm{P}}(\alpha, k)}{\partial k}\left(\check{u}_{\mathrm{P}}^{12}(\alpha) - \check{u}_{\mathrm{P}}^{\phi}\right),$$

which is strictly negative since $\partial \pi_{\mathrm{A}}(\alpha, k)/\partial k < \partial \check{\pi}_{\mathrm{P}}(\alpha, k)/\partial k$, and $\check{u}_{\mathrm{P}}^{12}(\alpha) - \check{u}_{\mathrm{P}}^{\phi} > u_{\mathrm{A}}^{12}(\alpha) - u_{\mathrm{A}}^{\phi}$ for $\alpha \in (\check{\alpha}_{\mathrm{P}}, \alpha_{\mathrm{A}}]$. Hence, there exists $\overline{k}_{\mathrm{less}}(\alpha) \in (\check{k}_{\mathrm{P}}(\alpha), +\infty)$ such that $U_{\mathrm{A}}(\alpha, k) - \check{U}_{\mathrm{P}}(\alpha, k) > 0$ if and only if $k \in (k_{\mathrm{A}}(\alpha), \overline{k}_{\mathrm{less}}(\alpha))$, where $\overline{k}_{\mathrm{less}}(\alpha)$ is given by

$$\overline{k}_{\mathrm{less}}(\alpha) = \frac{\alpha^2(\gamma\Gamma - 1)}{(\alpha^2\Gamma - (1-\alpha)^2\gamma)(b_L^2/4)\gamma(\alpha^2(\gamma^2+1) - \gamma^2/2)}.$$

$\square$

If the signal precision is so low that the decision maker is not reactive under either authority or it is so high that the decision maker is reactive under both authorities, project choice and the information-gathering effort do not differ with authority. Hence, the principal is indifferent between P-authority and A-authority, and retains her authority.

If the signal precision is such that the decision maker is reactive only under P-authority, the principal delegates if k is high but not too high. Under P-authority, the principal reacts even though the signal precision is so low that the reactivity is not beneficial to the agent, demotivating the agent to gather information. Hence, if the information-gathering advantage of delegation is sufficiently high, that is, k is high but not too high, delegation is optimal for the principal. We can say that the less biased principal uses delegation as the commitment devise not to react to information.