Interacting collective action problems *

Nicolas Quérou†

Abstract

We consider a setting where groups of agents interact, any group member's action inducing an externality in the same group, and aggregate action in one group induces an externality in other groups. The interplay between in-group and out-group interactions is shown to affect the comparison between the decentralized and the cooperative outcomes, and also the effect of the fundamentals on individual decisions and welfare, compared to the case where there is no in-group or out-group interaction. Moreover, group characteristics greatly influence the capacity of group-level cooperation to alleviate the inefficiency problems driven by decentralization. Finally, we identify cases where inter-group relocation policies result in efficiency gains, and highlight how this crucially depends on the existence and nature of in-group and out-group interactions. All results stress the importance to acknowledge interactions between potential collective action problems.

Key words: Group interactions; externalities; group-level cooperation; inter-group relocation policies.

JEL classification: C72, D62, D70.

1 Introduction

Many economic and socially-related situations involve groups of agents instead of individuals. Illustrative examples are problems faced by communities providing public goods available to members only and potentially detrimental to those outside, teams or departments interacting within an organization, groups of owners

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†CEE-M, Univ. Montpellier, CNRS, INRAE, Institut Agro, Montpellier, France. E-mail: nicolas.querou@umontpellier.fr

managing natural resources via neighbouring concessions, or groups of firms producing differentiated goods that may be substitutable. By contrast, most of the economic literature tends to consider that there is no interplay between in-group and out-group collective actions.

This contribution aims to reconcile these two dimensions. We consider a model where groups of agents face simultaneous in-group and out-group problems: any given group member's action induces an externality in the same group (in-group problem), and aggregate action in one group induces an externality on other groups (out-group problem). We analyze whether (and if so, how) the interplay between both types of problem affects the comparison between the decentralized and cooperative outcomes, and the effects of the fundamentals on individual actions and welfare, compared to classical models where such an interplay does not exist. Then we analyze the effect of two initiatives, group-level cooperation and inter-group relocation schemes, in order to understand how group characteristics influence their capacity to alleviate the inefficiency problems driven by decentralization.

The framework can account for (i) strategic substitutability both within and between groups (ii) in-group complementarity and out-group substitutability and (iii) asymmetric externalities between groups. The case where strategic substitutability exists both within and between groups might seem a little bit surprising, but it relates to important cases. For instance, it is consistent with oligopoly competitions where groups of firms produce differentiated goods (each group producing one good) that are substitutable at the aggregate level. Another consistent situation corresponds to the case where a common-pool resource is managed by different groups.[1] An agent might suffer both from an increase in her group members' extractions and from an increase in aggregate extraction in a neighbouring concession.[2] and a large literature suggests economic instruments as solutions.The case where there is in-group strategic complementarity and out-group strategic substitutability corresponds to situations where peer effects can have positive spillovers to members of the same group and negative spillovers to other groups. This is the case for social activities exhibiting forms of limited morality behavior (see Tabellini

[1]As is relevant for CPRs such as fisheries, forests, groundwater, when groups manage neighbouring CPR concessions, the resource is often used jointly within a given area, and each local area is managed by a different group. The case of oil concessions is also relevant, as a given concession may be managed by a joint venture involving several companies, and neighbouring concessions may exhibit different property rights structures.

[2]Seminal contributions about the management problems resulting from the existence of cost-externality include Gordon (5), Hardin (7), or Ostrom (16). The reader is referred to Stavins (19) for empirical evidence. These studies mostly abstract from group-decision making considerations (Gillet et al. (4) or Kotchen and Segerson (10) focus on the case of a unique group.).

(21)) applying to an agent's own group, or when an action benefiting an agent's own group negatively affects the performance (and the benefits) of other groups (see Markussen et al. (11) for a discussion of relevant examples).

This paper theoretically analyzes how the interaction of in-group and out-group collective action problems affects behaviors. The main research questions are: Does this interaction between collective action problems matter? If such interaction does matter, how does in-group collective action affect out-group collective action (and vice versa)? Finally, does this in turn impact the effect of cooperation and the emergence of inter-group relocation schemes?

Within this framework, the analysis shows that the conclusions drawn from classical models ignoring the interaction between in-group and out-group problems must be qualified. For instance, in case of in-group substitutability, non-cooperative individual action level may be inefficiently low compared to the cooperative outcome. In the context of CPR problems, this means that the classical *tragedy of the commons* conclusion can be reversed, even though the nature of spillover effects remains the same both within and between groups. Other qualifications are obtained under certain conditions: for instance, an agent's action or payoff level may actually increase with the size of his own group: this provides a different perspective on the existing discussion about the group size paradox (see Olson (15) or Esteban and Ray (2)). Second, the effect of potential solutions to the inefficiency problem may differ significantly.[3] Specifically, it is proved that the effect of group-level cooperation, where there is cooperation within one group and non cooperation both between groups and within the other group[4], is not always positive overall and strongly depends on the nature of strategic interactions.[5] Under in-group complementarity, group-level cooperation cannot result in a Pareto improvement, as non-cooperating group members are negatively impacted. By contrast, under in-group substitutability, a Pareto improvement can emerge provided the strength of the externality imposed by the cooperating group on others lies below a threshold value. Furthermore, accounting for the interplay between in-group and out-group collective action problems opens up avenues for the design

[3]Other ways to solve collective action problems have been analyzed in the literature, for instance the use of communication (see Ostrom et al. (17)), dynamic concerns (Heitzig et al. (9)), or explicit incentive mechanisms (for instance Gerber and Wichardt (3), Harstad (8) or Chen and Zeckhauser (1)). All these studies focus on the case of a unique group.

[4]This type of cooperation is considered as, among other reasons, experiments reporting high levels of cooperation are often related to group membership (McAdams (12)).

[5]The main point is to assess whether the emergence of cooperation may correspond to an overall improvement of the situation. This differs from situations where cooperation is equivalent to such an improvement, as in empirical works by Rustagi et al. (18) and Stoop et al. (20).

of innovative group-based solutions, such as inter-group relocation schemes. Under in-group complementarity, the relative size of the relocated sub-population and the magnitude of the in-group problem have first-order importance. By contrast, under in-group substitutability the comparison between the intensities of in-group and out-group problems is the main driving factor.

Providing the intuition about the effect of an increase in the size of one group on individual action levels may help to understand the persistent trade-off that will be one of the driving factors. The effect of an increase in the size of group A on individual action levels results from two effects. The first one is a direct effect that depends on the nature of in-group interactions. For instance, under in-group substitutability, an increase in the size of group A tends to increase in-group competition, which tends to decrease individual action levels within this group. The second effect is indirect and follows from the induced effect on individual action levels in the other groups. It results in this case in lower individual action levels in group B, which in turn tends to increase individual action levels in group A. There is thus a trade-off that depends on in-group and out-group characteristics, unlike the case where there is no interplay between in-group and out-group problems.

Before concluding this section, we briefly discuss the relationship with the literature on contests, which is not the focus of the present study for several reasons. First, most of the related contributions are not consistent with the assumptions of the present study.[6] Second, it mostly considers conflicts between individuals, and not between groups.[7] Third, this literature is not consistent with the research questions addressed here. Since rent-seeking efforts are by nature inefficient, it is impossible to obtain a qualified comparison between the decentralized equilibrium and the cooperative outcome, as in this contribution. Moreover, this literature abstracts from questions such as the overall effect of group-level cooperation, or the potential effectiveness of inter-group policies.

The remainder of the paper is organized as follows. The model is presented in Section 2, together with the characterizations of the decentralized and cooperative outcomes. The effects of fundamentals on agents' actions and payoffs are provided in section 3, together with the comparison between the decentralized and cooperative outcomes. Section 4 presents results on the potential solutions to the collective action problems. Section 5 concludes. The proofs of all results are provided in an

[6] We here consider situations where there is either in-group substitutability or in-group complementarity, and out-group substitutability.

[7] Few contributions focus on inter-group conflicts, most of them assume non-cooperation both between and within groups (Nitzan (13), Esteban and Ray (2)), even though agents may choose an intra-group sharing rule in a pre-contest stage (Nitzan and Ueda (14)).

Appendix at the end of the paper.

2 Model & benchmarks

2.1 The model

Because we want to highlight the important qualitative differences that emerge when considering that interactions may involve groups and not individuals, we focus on the simplest possible setting consistent with this assumption and allowing for heterogeneities between groups. So we consider two groups, A and B (of size N_A and N_B), which are subject to interacting collective action problems. Specifically, there is an in-group problem: for a given group i, each member's action imposes an externality on all members in the same group (modeled by parameter δ_{ii}). There is simultaneously an out-group problem: aggregate action in group i imposes an externality on each member in group j (modeled by parameter δ_{ij}). For a given group (say, A) the payoff of a given agent $i \in \{1, ..., N_A\}$ is thus specified as follows:

$$\Pi_i^A = ax_{iA} - \frac{b_{iA}}{2}(x_{iA})^2 - \delta_{AA}x_{iA}X_{-iA} - \delta_{BA}x_{iA}X_B, \tag{1}$$

where $x_{iA} \geq 0$ denotes this agent's action, $X_{-iA} = \sum_{j \in A, j \neq i} x_{jA}$ (respectively, X_B) the aggregate decision of other group A members (respectively, the aggregate decision in group B), and a and b_{iA} are positive parameters (measuring the private payoff function absent external effects). Parameter δ_{AA} captures the external effects of individual in-group actions on agent i's payoff, while δ_{BA} captures the degree of linkage between the two groups. The case of in-group substitutability corresponds to $\delta_{ii} > 0$ for any $i = A, B$ while the case of in-group complementarity is such that $\delta_{ii} < 0$ is satisfied for any $i = A, B$.

The example of common-pool resources helps to illustrate the case where there is in-group substitutability. This could model the resource extraction problems in local areas A and B, which are managed by two separate groups: neighbouring fisheries, groundwater or oil concessions are consistent examples. Here x_{iA} would denote the extraction level of agent i in group A. Any area owner's individual extraction induces a cost-type externality in the same area, and aggregate extraction in area i induces a cost-type externality on each agent in group j (inter-group externality modeled by parameter $\delta_{ij} > 0$). This specification of the model generalizes frameworks introduced in Walker et al. (23) and used in many other contributions, the main differences are that (i) "players" are groups and not individuals and (ii) externalities can be asymmetric.[8]

[8]See Walker and Gardner (22) for another seminal work. Some contributions introduce heterogeneity in CPR settings (Hackett (6)) but focus on a single collective action problem.

We now proceed with the analysis as follows. In order to keep the exposition of the results as simple as possible, we present some of them (namely Propositions 4, 9 and 10) assuming that $\delta_{AA} = \delta_{BB}$ and $\delta_{AB} = \delta_{BA}$ hold. Other findings will be provided in their most general form, as their exposition remains reasonably simple.

2.2 Decentralized versus cooperative outcome

We first derive the Nash equilibrium and the cooperative outcome. In order to focus on the most interesting cases, and to allow for meaningful comparative statics results and other comparisons, we provide the necessary and sufficient conditions ensuring existence and uniqueness of interior decentralized and cooperative outcomes. First, regarding the decentralized outcome, for a given agent i in group A, the corresponding optimality condition is:

$$a - b_A x_{iA} - \delta_{AA} X_{-iA} - \delta_{BA} X_B + \lambda_{iA} = 0, \tag{2}$$

where $\lambda_{iA} \geq 0$ denotes the corresponding lagrangian parameter, and a similar type of condition holds for any agent in group B. Solving for the equilibrium outcome, we obtain the following result:

Proposition 1. *Assume that one of the following set of conditions hold:*

- *In-group substitutability: for any $i, j = A, B$, $i \neq j$, we have*

$$b_i \geq \delta_{ii} > 0 \;\; and \;\; b_i + (N_i - 1)\,\delta_{ii} - N_i \delta ij > 0 \tag{3}$$

- *Weak in-group complementarity: for any $i, j = A, B$, $i \neq j$, we have*

$$b_i + (N_i - 1)\,\delta_{ii} - N_i \delta ij > 0 \tag{4}$$

- *Strong in-group complementarity and out-group strategic substitutability: for any $i, j = A, B$, $i \neq j$, we have*

$$b_i + (N_i - 1)\,\delta_{ii} < 0 \;\; and \;\; b_i + (N_i - 1)\,\delta_{ii} > -N_i \delta ij \tag{5}$$

Then the unique Nash equilibrium of the game corresponds to vectors of choices $\left(x^N_{1A}, ..., x^N_{N_A A}\right)$ and $\left(x^N_{1B}, ..., x^N_{N_B B}\right)$ characterized as follows:

$$\forall i \in A \;\; x^N_{iA} = x^N_A = \frac{b_B + (N_B - 1)\,\delta_{BB} - N_B \delta_{BA}}{\left[b_A + (N_A - 1)\,\delta_{AA}\right]\left[b_B + (N_B - 1)\,\delta_{BB}\right] - N_A N_B \delta_{AB} \delta_{BA}} a \tag{6}$$

and

$$\forall i \in B \;\; x_{iB}^{N} = x_{B}^{N} = \frac{b_A + (N_A - 1)\,\delta_{AA} - N_A \delta_{AB}}{[b_A + (N_A - 1)\,\delta_{AA}]\,[b_B + (N_B - 1)\,\delta_{BB}] - N_A N_B \delta_{AB} \delta_{BA}} a \tag{7}$$

The first proposition provides the conditions ensuring existence and uniqueness of the interior equilibrium. It is easily checked that there is no equilibrium where agents all choose a zero action level. Moreover, an equilibrium where agents in one group (say A) choose a positive action level, while the agents choose a zero action level in the other group, requires that the externality imposed on group B by group A is very strong, which is ruled out, for instance in the case of in-group and out-group strategic substitutability, when condition $b_A + (N_A - 1)\,\delta_{AA} - N_A \delta_{AB} > 0$ is satisfied. Finally, it is interesting to notice that the existence and uniqueness result holds in two separate cases when there is in-group complementarity, depending on whether this effect is weak or strong enough.[9] The characterization provided in Proposition 1 will be used extensively in the analysis. The next result provides the characterization of the cooperative outcome.

Proposition 2. *Assume that one of the following set of conditions hold:*

- *In-group substitutability: for any $i, j = A, B$, $i \neq j$, we have*

$$b_i \geq 2\delta_{ii} > 0 \;\; \text{and} \;\; b_i + 2\,(N_i - 1)\,\delta_{ii} - N_i\,(\delta_{AB} + \delta_{BA}) > 0 \tag{8}$$

- *In-group complementarity: for any $i, j = A, B$, $i \neq j$, we have*

$$b_i + 2\,(N_i - 1)\,\delta_{ii} - N_i\,(\delta_{AB} + \delta_{BA}) > 0 \tag{9}$$

Then the cooperative outcome corresponds to vectors of choices $\left(x_{1A}^{*}, ..., x_{N_A A}^{*}\right)$ and $\left(x_{1B}^{*}, ..., x_{N_B B}^{*}\right)$ characterized as follows:

$$\forall i \in A \;\; x_{iA}^{*} = x_{A}^{*} = \frac{b_B + 2\,(N_B - 1)\,\delta_{BB} - N_B\,(\delta_{AB} + \delta_{BA})}{[b_A + 2\,(N_A - 1)\,\delta_{AA}]\,[b_B + 2\,(N_B - 1)\,\delta_{BB}] - N_A N_B\,(\delta_{AB} + \delta_{BA})^2} a \tag{10}$$

and

$$\forall i \in B \;\; x_{iB}^{*} = x_{B}^{*} = \frac{b_A + 2\,(N_A - 1)\,\delta_{AA} - N_A\,(\delta_{AB} + \delta_{BA})}{[b_A + 2\,(N_A - 1)\,\delta_{AA}]\,[b_B + 2\,(N_B - 1)\,\delta_{BB}] - N_A N_B\,(\delta_{AB} + \delta_{BA})^2} a \tag{11}$$

[9] For expositional simplicity, the second part of the set of conditions (5) provides sufficient conditions: the necessary and sufficient condition would require that $[b_A + (N_A - 1)\,\delta_{AA}]\,[b_B + (N_B - 1)\,\delta_{BB}] - N_A N_B \delta_{AB} \delta_{BA} < 0$ be satisfied.

In the next Section we will rely on these characterizations to compare the full cooperation outcome with the case of non-cooperation. One can notice that no characterization is provided in the case of strong in-group complementarity: indeed, the aggregate payoff function is not concave (and not convex either) when the magnitude of complementarity effects lies above a threshold level. Thus, there is no easy way to characterize the efficient outcome without imposing some further assumptions that would not contribute to our understanding of the interplay between in-group and out-group interactions. Since the decentralized outcome is well-defined, we will still provide some properties related to this case.

Starting from Section 3, in order to highlight the differences driven by interacting collective action problems, we will sometimes refer to the *polar* or *classical* cases, which correspond to the models of collective action usually considered in the literature. The first case corresponds to $\delta_{AB} = \delta_{BA} = 0$ and refers to situations where there is no out-group collective action problem. The second case corresponds to either $N_A = N_1 = 1$ or $\delta_{AA} = \delta_{BB} = 0$ and refers to situations where there is no in-group collective action problem.

3 The effect of fundamentals

In this section we will analyze how group characteristics affect the agents' action levels and payoffs. First, we will provide results of comparative statics on the agents' equilibrium action levels. Second, the effect of parameters on the comparison between non cooperative and cooperative outcomes will be analyzed. Finally, the same analysis will be performed on the equilibrium payoffs.

3.1 Comparative statics

The first Proposition allows for detailed comparative statics results on the effects of the various fundamentals (externality parameters, size of the populations) on the non cooperative equilibrium outcome. Specifically, we have:

Proposition 3. *Under the assumptions of Proposition 1, we have the following comparative statics results: for $i, j = A, B$, $i \neq j$*

1. *The individual action level within group i decreases with an increase in the intensity of in-group externality in the same group:*

$$\frac{\partial x_i^N}{\partial \delta_{ii}} < 0;$$

Moreover, when there is strong in-group complementarity $\frac{\partial x_i^N}{\delta_{jj}} < 0$ *holds. By contrast, the effect of* δ_{jj} *on* x_i^N *is positive for the other cases.*

2. *The individual action level within group* i *decreases with an increase in the intensity of out-group externality from group* j *on group* i*:*

$$\frac{\partial x_i^N}{\partial \delta_{ji}} < 0;$$

Moreover, when there is strong in-group complementarity $\frac{\partial x_i^N}{\delta_{ij}} < 0$ *holds, while the effect of* δ_{ij} *on* x_i^N *is positive for the other cases.*

3. *The individual action level within group* i *increases as the number of agents in group* j *increases, that is,* $\frac{\partial x_i^N}{\partial N_j} > 0$ *is satisfied, when there is strong in-group complementarity. This effect is non-positive for the other cases.*

4. *When there is weak in-group complementarity* $\frac{\partial x_i^N}{\partial N_i} > 0$ *is always satisfied. By contrast, the effect is ambiguous in the other two cases. When there is in-group substitutability we have*

$$\frac{\partial x_i^N}{\partial N_i} > 0 \Longleftrightarrow \delta_{ii}\left[b_j + (N_j - 1)\,\delta_{jj}\right] < N_j \delta_{ij} \delta_{ji}$$

When there is strong in-group complementarity, we have

$$\frac{\partial x_i^N}{\partial N_i} < 0 \Longleftrightarrow \delta_{ii}\left[b_j + (N_j - 1)\,\delta_{jj}\right] < N_j \delta_{ij} \delta_{ji}$$

Points 1, 2 and 3 in Proposition 3 are fairly simple to understand. For instance, regarding the effects of in-group externality parameters, the main impact of an increase in δ_{ii} follows quite directly from the optimality condition, and tends to affect negatively the individual action level. By contrast, the main effect of a larger δ_{jj} is indirect, and follows from the resulting decrease in aggregate action level in area j: each agent in group i then decreases her own action level when there is strong in-group complementarity, while each agent increases her own action level in the other cases.

The most complex effect is related to an increase in the size of an agent's own group. Focusing on the case of group A, the optimality condition characterizing x_A^N highlights the existing trade-off that relates to the effect of an increase in N_A. Differentiating it with respect to N_A, we obtain:

$$-\left[b_A + (N_A - 1)\,\delta_{AA}\right]\frac{\partial x_A^N}{\partial N_A} - \delta_{AA} x_A^N - \delta_{BA} N_B \frac{\partial x_B^N}{\partial N_A} = 0$$

The first term corresponds to the direct in-group effect: it depends on the nature of in-group strategic interactions. The second term corresponds to the indirect in-group effect: it is positive when there is in-group complementarity, and negative otherwise. The last term corresponds to the indirect out-group effect, as an increase in the number of agents results in a change in the external effects imposed on the other group. Its relative effect depends on the nature of in-group strategic interactions, the magnitude of out-group externalities, and the size of the other group. Rearranging, we have:

$$[b_A + (N_A - 1)\,\delta_{AA}]\,\frac{\partial x_A^N}{\partial N_A} = -\delta_{AA} x_A^N - \delta_{BA} N_B \frac{\partial x_B^N}{\partial N_A}$$

This highlights that the effect is non-ambiguous when there is in-group complementarity and this effect is not too strong. Otherwise, the effect depends on the interplay between the size of the other group and the externality parameters.

We illustrate this interplay by relying on two polar cases. First, when $\delta_{AB} = \delta_{BA} = 0$ we obtain immediately that $\frac{\partial x_i^N}{\partial N_i} < 0$ when there is in-group substitutability and $\frac{\partial x_i^N}{\partial N_i} > 0$ when there is (strong) in-group complementarity. By contrast, when $\delta_{AB} = \delta_{BA} = \delta_{out}$ is high enough, then the conclusions are reversed.

These polar cases highlight that the interplay between in-group and out-group effects should be accounted for, as they deeply affect the conclusions. Indeed, the situation is degenerate when the in-group problem is not accounted for, and when $\delta_{AB} = \delta_{BA} = 0$ then the indirect effect disappears.

3.2 Comparison between cooperative and decentralized outcomes

We consider the cases where the cooperative and decentralized equilibrium outcomes are characterized by conditions (6)-(7) and (10)-(11). Thus, we now consider that the assumptions of Propositions 1 and 2 are satisfied simultaneously.

As the analysis will highlight it, the comparison between both outcomes is more complex than in classical models where either the in-group or the out-group dimension is not accounted for. Depending on how these dimensions interact, the comparison requires qualifications as the next result shows. In order to keep the analysis tractable and the exposition as simple as possible, we assume that $\delta_{AA} = \delta_{BB} = \delta_{in}$ and $\delta_{AB} = \delta_{BA} = \delta_{out}$ are satisfied. Moreover, in order to isolate the effect of each fundamental, we will further restrict value of each of them (nature and magnitude of in-group and out-group externalities, size of the groups).

Specifically, we obtain:

Proposition 4. *We have the following comparisons:*

1. *Let us first assume that* $N_A = N_B = N$ *is satisfied.*

   - *When there is in-group substitutability, assuming that conditions (3)-(8) hold, we have* $x_A^N > x_A^*$.
   - *When there is in-group complementarity, assuming that conditions (4)-(9) hold, we have:*

$$x_A^N \geq x_A^* \iff \delta_{out} \geq -\frac{N-1}{N}\delta_{in} \tag{12}$$

2. *Let us now assume that* $\delta_{in} = \delta_{out} = \delta$ *is satisfied under in-group substitutability, and that* $\delta_{out} = -\delta_{in} = \delta$ *under in-group complementarity, while both groups differ in size.*

   - *When there is in-group substitutability, assuming that conditions (3)-(8) hold, we have* $x_A^N > x_A^*$.
   - *When there is in-group complementarity, assuming that conditions (4)-(9) hold, then* $x_A^N \geq x_A^*$ *when* $N_A \leq N_B$ *and* $x_A^N < x_A^*$ *otherwise.*

3. *Let us now assume that* $N_A \neq N_B$ *and* $\delta_{in} \neq \delta_{out}$ *is satisfied under in-group substitutability. When* $N_A > N_B$ *there exist* $\bar{\delta} \in]0, \frac{b}{2}[$ *and* $\delta_{out}\left(\bar{\delta}\right) \in]0, \frac{b+2(N_B-1)\bar{\delta}}{2N_B}[$ *such that the following property holds:*

$$x_A^N < x_A^* \iff \delta_{in} < \bar{\delta} \text{ and } \delta_{out} > \delta_{out}\left(\bar{\delta}\right)$$

   In all other cases $x_A^N > x_A^*$ *is satisfied.*

This result highlights the qualitative differences driven by the fundamentals. When the size of groups is homogeneous, then the case of in-group substitutability is unambiguous. It is indeed similar to the classical model characterized by inefficiently high action levels under decentralized behaviors. When there is in-group complementarity, the existing trade-off is reasonably clear to characterize. As long as the magnitude of the in-group externality is not too large compared to that of the out-group externality, then decentralization results in inefficiently high action levels. By contrast, when the magnitude of the in-group effect becomes large enough, then decentralization results in inefficiently low action levels.

Things become more complex when the size of the groups are heterogeneous. When the magnitude of in-group and out-group problems is homogeneous, then the same conclusions hold for the case of in-group substitutability. Under in-group complementarity, the conclusion relies on the relative comparison of the size of both groups. When group A is smaller than group B, then the dominant effect (on group A) is related to the out-group effect, which tends to increase individual action levels in group A. By contrast, when group A is larger than group B, the dominant effect is the in-group effect, which tends to decrease individual action levels. The conclusions follow.

The proposition highlights that, under in-group substitutability, individual action levels in group A may be lower under decentralization, but this requires that both the group sizes and the in-group and out-group effects be heterogeneous. The group size has first-order importance: for this result to hold, group A must be larger than group B. Then it requires that the in-group effect, which tends to negatively impact action levels under decentralization, be small enough while the out-group effect, which tends to positively impact action levels under decentralization, be simultaneously large enough.

This last result highlights a notable difference that emerges when there is an interplay between in-group and out-group collective action problems. It is easily checked that the other conclusions also differ here compared to situations where either the in-group or the out-group problem is unaccounted for.

3.3 How does the size of groups affect welfare?

We will conclude this section by discussing the effect of the number of agents on the welfare of the different groups. This is done by simple comparative statics analysis. We have the following conclusion:

Proposition 5. *Under the assumptions of Proposition 1, let us denote Π_A^N and Π_B^N the equilibrium payoffs of any agent in, respectively, groups A and B. Then we have:*

1. *Regarding the effect on agents' payoffs in group A, we have $\frac{\partial \Pi_A^N}{\partial N_A} > 0$ under weak in-group complementarity. Under strong in-group complementarity, then $\frac{\partial \Pi_A^N}{\partial N_A} \geq 0$ if and only if $\frac{b_B+(N_B-1)\delta_{BB}}{N_B\delta_{BA}} \leq \frac{\delta_{AB}}{\delta_{AA}}$ holds.*

   Finally, under in-group substitutability, then $\frac{\partial \Pi_A^N}{\partial N_A} \geq 0$ if and only if $\frac{b_B+(N_B-1)\delta_{BB}}{N_B\delta_{BA}} \leq \frac{\delta_{AB}}{\delta_{AA}}$ holds.

2. *Regarding the effect on agents' payoffs in group B, we have $\frac{\partial \Pi_B^N}{\partial N_A} < 0$ under*

weak in-group complementarity. Finally, we have $\frac{\partial \Pi_B^N}{\partial N_A} > 0$ *under in-group substitutability or strong complementarity.*

The effect of changes in the size of the agents' population in one group is unambiguous when looking at the agents' payoffs in the other group. Under in-group substitutability, a lower number of agents results in higher payoffs (and the opposite conclusion holds under in-group complementarity). The marginal effect of a change in the size of the agents' population in group A can be derived as follows:

$$\frac{\partial \Pi_B^N}{\partial N_A} = -x_B^N \left[\delta_{BB} \left(N_B - 1 \right) \frac{\partial x_B^N}{\partial N_A} + \delta_{AB} \left[x_A^N + N_A \frac{\partial x_A^N}{\partial N_A} \right] \right]$$

The first part in the bracketed expression on the right-hand side of the equality reflects the effect due to changes in the aggregate action level in group B. Individual action level in group B decreases as N_A increases, which implies that the related effect on agents' payoffs in this group is positive. The second part in the bracketed expression characterizes the effect due to changes in the aggregate action level in group A. Aggregate action level increases as N_A increases, and the related effect on agents' payoffs in group B is thus negative. As long as x_A^N and x_B^N are positive, the first effect dominates the second one, and the overall effect on the agents' payoffs in group B is positive.

By contrast, the marginal effect on the agents' payoffs in group A depends on the interplay between the externality parameters and the size of agents' population in group B. Specifically, we have:

$$\frac{\partial \Pi_A^N}{\partial N_A} = x_A^N \left[\delta_{AA} \frac{\partial x_A^N}{\partial N_A} - \delta_{AA} \frac{\partial X_A^N}{\partial N_A} - \delta_{BA} N_B \frac{\partial x_B^N}{\partial N_A} \right]$$

where $X_A^N = N_A x_A^N$ which, after simplifications, yields:

$$\frac{\partial \Pi_A^N}{\partial N_A} = -x_A^N \left[\delta_{AA} x_A^N + \delta_{AA} \left(N_A - 1 \right) \frac{\partial x_A^N}{\partial N_A} + \delta_{BA} N_B \frac{\partial x_B^N}{\partial N_A} \right]$$

The first part in the bracketed expression on the right-hand side of the equality reflects the effect due to changes in the action levels in group A. The second part in the bracketed expression highlights the effect due to changes in the aggregate action level in group B. Aggregate action level decreases as N_A increases, and the related effect on agents' payoffs in group A is thus positive. As long as x_A^N and x_B^N are positive, the effect driven by group A dominates the second one, and the overall effect on the agents' payoffs in group A depends on the interplay between externality parameters and N_B.

Proposition 5 yields several interesting implications. First, there are cases for which there is a specific conflict of interest between agents in different groups. This is so when agents' payoffs in group A are higher, while agents' payoffs in group B are lower, as the number of agents increases in group A. Second, since classical models in the literature correspond to either $N_A = N_B = 1$ or $\delta_{AB} = \delta_{BA} = 0$, they are incompatible with a positive effect of a larger size of agents' population on profits (as depicted in the first case in Proposition 5). As in the fourth case in Proposition 4, this last conclusion requires a necessary condition on the comparison between the magnitude of in-group externalities and that of out-group externalities. Finally, Proposition 5 suggests a nuanced way to think about the relationship between group size and collective action (see Olson (15) or Esteban and Ray (2)). Indeed, following Esteban and Ray (2), if one uses the definition of group effectiveness that relates group size to per-capita payoffs, then Proposition 5 highlights that group effectiveness can increase as a group gets larger, and this result first depends on the nature of in-group interactions and also on other fundamentals of the situation. Specifically, looking at the case of group A, the payoff to an agent always increases with her group size under weak in-group complementarity. By contrast, under both in-group substitutability and strong complementarity, it increases with the agent's group size provided the following condition is satisfied:

$$\frac{b_B + (N_B - 1)\,\delta_{BB}}{N_B \delta_{BA}} \leq \frac{\delta_{AB}}{\delta_{AA}}$$

Under in-group substitutability, this is more likely to hold as the size of the other group increases, as the magnitude of external effects imposed by group A on group B gets larger, or as the in-group effect gets less severe in group A.

4 Potential solutions to the collective action problems

The main point of this contribution is to show the importance of existing interactions between collective action problems. As highlighted in the previous section, several results obtained in classical models can be reversed in such a setting.

The next section will highlight the importance of existing interactions by showing how it opens up new research avenues for the design of potential solutions to collective action problems. Indeed, compared to the classical models in the literature, certain solutions will be shown to have significantly different effects. Specifically, we now consider two potential solutions to the problem of collective action, namely (i) group-level cooperation and (ii) inter-group relocation schemes, and analyze whether they might be effective and if so, we will characterize the conditions under which they alleviate the problem. Case (i) assumes that agents in a given group are able to solve their internal collective action problem: there is cooperation within one group, and non cooperation both between the groups and within the other group. Case (ii) analyzes policies that aim at relocating some agents from one group (say, group B) to the other one.

4.1 Group-level cooperation

Again we consider the case where the fully non cooperative outcome is interior, that is, the situation where it is characterized by Proposition 1. In order to allow for the simplest comparison that is possible, we will characterize the conditions under which the group-level cooperation outcome is interior too. The main difference compared to the decentralized situation is that group A members maximize the aggregate payoffs within their group. Thus, for a given agent i in group A, the corresponding optimality condition is:

$$a - b_A x_{iA} - \delta_{AA} X_{-iA} - \delta_{AA} \sum_{l \neq i} x_{lA} - \delta_{BA} X_B + \lambda_{iA} = 0,$$

where $\lambda_{iA} \geq 0$ denotes the corresponding lagrangian parameter. For any agent in group B, the optimality condition is similar to that in the decentralized case. We now obtain the following results:

Proposition 6. *Assume that there is group-level cooperation within group A, while there is non cooperation both within group B and between the two groups. Assume that one of the following sets of conditions hold:*

- *In-group substitutability:*

$$b_A \geq 2\delta_{AA}, \;\; b_B \geq \delta_{BB}, \;\; b_A + 2\left(N_A - 1\right)\delta_{AA} > N_A \delta_{AB}, \;\; b_B + \left(N_B - 1\right)\delta_{BB} > N_B \delta_{BA} \tag{13}$$

- *In-group complementarity:*

$$b_A + 2\left(N_A - 1\right)\delta_{AA} > N_A \delta_{AB} \text{ and } b_B + \left(N_B - 1\right)\delta_{BB} > N_B \delta_{BA} \tag{14}$$

Then the unique group-level cooperation outcome corresponds to vectors of choices $\left(x_{1A}^{gc}, ..., x_{N_A A}^{gc}\right)$ and $\left(x_{1B}^{gc}, ..., x_{N_B B}^{gc}\right)$ characterized as follows:

$$\forall i \in A \;\; x_{iA}^{gc} = x_A^{gc} = \frac{b_B + \left(N_B - 1\right)\delta_{BB} - N_B \delta_{BA}}{\left[b_A + 2\left(N_A - 1\right)\delta_{AA}\right]\left[b_B + \left(N_B - 1\right)\delta_{BB}\right] - N_A N_B \delta_{AB} \delta_{BA}} a \tag{15}$$

and

$$\forall i \in B \;\; x_{iB}^{gc} = x_B^{gc} = \frac{b_A + 2\left(N_A - 1\right)\delta_{AA} - N_A \delta_{AB}}{\left[b_A + 2\left(N_A - 1\right)\delta_{AA}\right]\left[b_B + \left(N_B - 1\right)\delta_{BB}\right] - N_A N_B \delta_{AB}\delta_{BA}} a \tag{16}$$

The fact that group A members are assumed to be able to cooperate without extra costs is an important simplification. Indeed, the cooperation process is quite likely to be costly, and the related costs might depend on certain characteristics of the group, its size for instance. One way to think about this is to assume that there is a group leader whose influence is such that he or she is able to implicitly enforce cooperation. From a general point of view, we decide to ignore specific costs related to the cooperation process, because we want to focus solely on the potential effects that will be driven by the existence of interactions between the collective action problems. This will allow to characterize that the effects resulting from group-level cooperation depend notably on the nature of interacting collective action problems, even without cooperation-specific costs.

The main point is now to assess the effects of group-level cooperation on individual actions and payoffs. This is done by comparing the outcome of Proposition 1 and that of Proposition 6. Specifically, we obtain:

Proposition 7. *Assume that conditions (3)-(13) hold under in-group substitutability, and that conditions (4)-(14) hold under in-group complementarity.*

Then we obtain the following comparisons:

1. *With respect to individual action levels:*
   - $x_A^{gc} < x_A^N$ *and* $x_B^{gc} > x_B^N$ *under in-group substitutability*
   - $x_A^{gc} > x_A^N$ *and* $x_B^{gc} < x_B^N$ *under in-group complementarity*
2. *Regarding the effect on group-level welfare, we have:*
   - $\Pi_B^{gc} > \Pi_B^N$ *under strategic substitutability both within and between groups, while the effect on group-A payoffs is ambiguous:*

   $$\Pi_A^{gc} \geq \Pi_A^N \Longleftrightarrow \frac{b_A + 2\left(N_A - 1\right)\delta_{AA}}{b_A} \geq \left(\frac{x_A^N}{x_A^{gc}}\right)^2$$

   Specifically, when the magnitude of in-group substitutability is large enough ($\delta_{AA} \geq \frac{N_A}{N_A - 1}\delta_{AB}$) then group-level cooperation always results in a (strict) Pareto improvement. By contrast, when the magnitude *of substitutability lies below this threshold level, group-level cooperation results in a Pareto improvement if and only if the following inequality holds:*

$$\frac{b_B + (N_B - 1)\,\delta_{BB}}{N_B \delta_{BA}} \geq \frac{N_A \delta_{AB}}{(N_A - 1)\,\delta_{AA}}$$

- $\Pi_B^{gc} < \Pi_B^N$ *and* $\Pi_A^{gc} > \Pi_A^N$ *under in-group complementarity.*

It is again interesting to contrast these results with the classical cases in the literature. When $\delta_{AB} = \delta_{BA} = 0$ it is easily checked that group-level cooperation always improves the situation overall, as it results in a strict Pareto improvement in group A and leaves group B unaffected. Obviously, when $N_A = N_B = 1$ group-level cooperation does not have any meaning.

Proposition 7 provides several interesting insights on the effect of group-level cooperation. First, the qualitative effect on individual action levels depends notably on the nature of in-group interactions. Indeed, while group-level cooperation results in lower action levels in the cooperating group under in-group substitutability, the opposite result follows under in-group complementarity. Intuitively, the effect of cooperation is to internalize part of the externality driven by the nature of in-group interactions, which is negative under in-group substitutability and positive under in-group complementarity.

Second, the overall effect on welfare is complex, and it markedly differs in all cases. Group-level cooperation may result in a Pareto improvement under in-group substitutability, while it cannot result in such an improvement under in-group complementarity. Under in-group substitutability, the effect of group-level cooperation induces members of the cooperating group to lower action levels, which tends to decrease payoffs in group A compared to the fully decentralized case. On the other side, cooperation allows to reduce in-group externalities, which tends to increase payoffs in group A compared to the fully decentralized outcome. Furthermore, due to the decrease in individual action levels in group A, an indirect effect of out-group interactions is that individual action levels in group B increase. This tends to lower payoffs in group A. The net effect first depends on the magnitude of in-group external effects in group A: if it is large enough, so that the effect of internalizing in-group externality be strong enough, it offsets the decrease in payoffs resulting from lower individual action levels in group A and higher action levels in group B. If the magnitude of in-group effects is not high enough, then there is a trade-off, and a Pareto improvement is more likely as the magnitude of out-group effects from group B on group A gets smaller.

When looking at the effect on group-level welfare, it is interesting to notice that the effect on the non-cooperating group differs markedly in both cases. Since there is out-group substitutability, this difference is also driven by the qualitative nature of individual behavioral adjustments in group A. Absent redistribution instruments, an implication of these results is that the emergence of self-voluntary group-level cooperation is unlikely under in-group complementarity, while it might actually be the case under in-group substitutability. Moreover, it is easily checked that group effectiveness increases as group A gets larger under in-group complementarity, and also under in-group substitutability provided the magnitude of (at least) one out-group effect is large enough. This is consistent with our discussion following Proposition 5 about the relationship between group size and collective action (see Olson (15) and Esteban and Ray (2)).

One of the conclusions in Proposition 7 has implications regarding the global efficiency effect of group-level cooperation under in-group substitutability. We conclude this section by showing that the nature of in-group interactions has first-order importance on the global effect of group-level cooperation.[10] Specifically, we have:

Proposition 8. *Assume that conditions (3)-(13) hold under in-group substitutability, and that conditions (4)-(14) hold under in-group complementarity. Moreover, assume that $N_A \neq N_B$ while $\delta_{AB} = \delta_{BA} = \delta = |\delta_{AA}| = |\delta_{BB}|$ hold. Then group-level cooperation always results in a global efficiency gain under in-group substitutability.*

By contrast, whether group-level cooperation results in a global efficiency gain under in-group complementarity depends both on group sizes and the magnitude of externalities. Specifically, when group A is sufficiently larger than group B, group-cooperation results in a global efficiency gain. By contrast, when group B is sufficiently larger than group A, then group-level cooperation results in a global efficiency loss.

This result highlights that the nature of in-group interactions has first-order importance on the global efficiency of group-level cooperation. Indeed, under in-group complementarity, the relative size of the groups has first-order importance on the global efficiency of group-level cooperation. By contrast, when groups are heterogeneous in size and homogeneous in terms of in-group and out-group externalities, group-level cooperation always results in a global efficiency gain. Under in-group complementarity, an improvement in global efficiency requires that the cooperating group be sufficiently larger than the non-cooperating one. In other words, the group characteristics do matter for the effectiveness of group-level cooperation when in-group interactions exhibit complementarities.

[10] As we want to focus on the effects of in-group and out-group characteristics, we here assume that $b_{iA} = b_{lB} = b$ for any group-A member i and any group-B member l.

4.2 Inter-group relocation scheme

The second policy that we consider consists in relocating agents from one group to the other: this might yield efficiency gains because of the existence of in-group and out-group effects. We thus consider a policy relocating k agents from group B to group A. The aim of this part of the analysis is to study whether such policy can achieve efficiency gains, whether efficiency gains occur at the global or group level, and how such conclusions depend on the features of the problem at hand.

First, Proposition 1 provides us with the conditions under which the post-relocation situation yields a unique equilibrium outcome, together with its characterization. Specifically, the decentralized outcome $(x_{1A}^R, ..., x_{(N_A+k)A}^R)$ and $(x_{1B}^R, ..., x_{(N_B-k)B}^R)$ is characterized as $x_{iA}^R = x_A^R$ for $i = 1, ..., N_A + k$ and $x_{jB}^R = x_B^R$ for $j = 1, ..., N_B - k$ where

$$x_A^R = \frac{b_B + (N_B - k - 1)\,\delta_{BB} - (N_B - k)\,\delta_{BA}}{[b_A + (N_A + k - 1)\,\delta_{AA}]\,[b_B + (N_B - k - 1)\,\delta_{BB}] - (N_A + k)\,(N_B - k)\,\delta_{AB}\delta_{BA}}$$

and

$$x_B^R = \frac{b_A + (N_A + k - 1)\,\delta_{AA} - (N_A + k)\,\delta_{AB}}{[b_A + (N_A + k - 1)\,\delta_{AA}]\,[b_B + (N_B - k - 1)\,\delta_{BB}] - (N_A + k)\,(N_B - k)\,\delta_{AB}\delta_{BA}}$$

if and only if the following conditions hold

$$b_A + (N_A - 1)\,\delta_{AA} - N_A\delta_{AB} > k\,(\delta_{AB} - \delta_{AA}) \tag{17}$$

and

$$b_B + (N_B - 1)\,\delta_{BB} - N_B\delta_{BA} > k\,(\delta_{BB} - \delta_{BA}) \tag{18}$$

together with $b_i \geq \delta_{ii}$ for $i = 1, 2$ under in-group substitutability.

We now analyze the efficiency effects of this relocation policy. In order to do so, we compare the pre and post-relocation decentralized outcomes. In order to isolate the effect of each fundamental on the results, we will consider several cases where groups are allowed to differ with respect to only one feature at a time. We first consider the case where groups have different sizes initially:

Proposition 9. *Assume that conditions (3)-(17)-(18) hold, that $\delta_{AA} = \delta_{BB} = \delta_{AB} = \delta_{BA} = \delta$ under in-group substitutability, and that $\delta_{AA} = \delta_{BB} = -\delta$ while $\delta_{AB} = \delta_{BA} = \delta$ under in-group complementarity. When $N_A \neq N_B$ we obtain the following conclusions:*

1. *The relocation policy has no efficiency effect under in-group substitutability.*

2. *Under in-group complementarity, the relocation policy results in an efficiency gain at the global level, but the group-level effects are heterogeneous. It has a positive effect on the payoffs of agents remaining in group A, and a negative effect on the payoffs of agents remaining in group B. The effect on relocated agents depends on the comparison between the pre-relocation group sizes:*
    - *When $N_B < N_A$ the policy has positive effects on the payoffs of agents relocated from group B to group A for any $1 \leq k \leq N_B$ when $N_A + N_B \leq \frac{b-\delta}{2\delta}$ and for any $1 \leq k \leq \frac{b-\delta}{2\delta} < N_B$ when $N_A + N_B > \frac{b-\delta}{2\delta}$ is satisfied.*
    - *When $N_B > N_A$ the policy has positive effects on the payoffs of agents relocated from group B to group A if and only if $k > N_B - N_A$ when $N_A + N_B \leq \frac{b-\delta}{2\delta}$ and for any $k \in]N_B - N_A, \frac{b-\delta}{2\delta}[$ when $N_A + N_B > \frac{b-\delta}{2\delta}$ is satisfied.*

When groups only differ with respect to their size, the effect of a relocation policy depends heavily on the nature of in-group interactions. Under in-group substitutability, the policy has basically no effect on individual decisions, and as such no efficiency effect. By contrast, under in-group complementarity, there is a globally positive efficiency effect. Looking at the effect on agents remaining in group A, adding k agents in group A has two effects. First, the size of this group increases, which tends to increase individual action level compared to the pre-relocation situation: this has a positive effect on these agents' payoffs. Second, the size of the other group decreases, and this again has a positive effect on payoffs (see Proposition 2). The case of the agents remaining in group B is exactly symmetric. The conclusion is more complex for agents relocated in group A. If group B is initially smaller than group A then the effect on a relocated agent's payoff is qualitatively to that affecting an agent remaining in group A. By contrast, if group B is initially larger than group A, then the size of the relocated sub-population must be large enough for this conclusion to hold: it should be so that the post-relocation size of group A be larger than that of group B.

We now move on to the case where groups are homogeneous in size, in-group externality parameters, and out-group externality parameters, while these last two parameters differ. We obtain:

Proposition 10. *Assume that conditions (3)-(17)-(18) hold, that $N_A = N_B = N$ and that $|\delta_{AA}| = |\delta_{BB}| = |\delta_{in}| \neq \delta_{out} = \delta_{AB} = \delta_{BA}$, where $|.|$ stands for the absolute value.*
Under in-group complementarity, the relocation policy has a positive effect on the payoffs of agents remaining in group A and on those of relocated agents, and a negative effect on the payoffs of agents remaining in group B.

The global efficiency effect depends on the fundamentals. When $k \geq \frac{N}{2}$ the policy results in an efficiency gain. When $k < \frac{N}{2}$ there is an efficiency gain when either $|\delta_{in}| \leq \max\{\frac{3N+2k}{N-2k}\delta_{out}, \frac{2b+N\delta_{out}}{3N-2}\}$ or the following inequality is satisfied:

$$N^3\left(\delta_{in}+\delta_{out}\right)\left[\left(\delta_{out}\right)^2-\left(\delta_{in}\right)^2\right]+2\left(b-\delta_{in}\right)^3$$

$$+N\left[\left(b-\delta_{in}\right)^2\left(3\delta_{in}+\delta_{out}\right)-k^2\left(\delta_{in}+\delta_{out}\right)\left[\left(\delta_{out}\right)^2-\left(\delta_{in}\right)^2\right]\right]>0$$

The effects of the relocation policy under in-group substitutability depend on how in-group effects compare to out-group effects. When $\delta_{out} > \delta_{in}$ the group effects are similar than under in-group complementarity. When $\delta_{out} < \delta_{in}$ the conclusions are reversed: the policy positively impacts the welfare of agents remaining in group B, and it negatively impacts the welfare of agents both remaining in group A and relocating in this group.

Regarding the global effects, there is an efficiency gain when $\delta_{out} > \delta_{in}$ is satisfied. By contrast, when $\delta_{out} < \delta_{in}$, there is an efficiency gain if and only if the following inequality is satisfied:

$$\left(b-\delta_{in}\right)^2\left[2\left(b-\delta_{in}\right)+N\left(3\delta_{in}+\delta_{out}\right)\right]+\left[\left(\delta_{out}\right)^2-\left(\delta_{in}\right)^2\right]N\left(N^2-k^2\right)\left(\delta_{in}+\delta_{out}\right)<0$$

which requires that k lies below a threshold value.

The polar cases allow to understand the effects of each fundamental. One could be induced to conclude that such a type of policy is unlikely to emerge voluntarily: in the different cases considered so far, the effectiveness of a relocation scheme would rely on the use of group transfers to ensure that all sub-groups are at least as well off ex post. We conclude the analysis by highlighting that there exist cases where relocation policies would actually result in a (strict) Pareto improvement:

Proposition 11. *Assume in-group substitutability and that conditions (3)-(17)-(18) hold, $N_A = N_B = N$ and $\delta_{AA} = \delta_{BB} = \delta_{in}$ while $\delta_{AB} \neq \delta_{BA}$ is satisfied. If $\delta_{in} > \max\{\delta_{AB}, \delta_{BA}\}$ then the relocation policy results in a (strict) Pareto improvement when k lies above a threshold value.*

Here one interesting implication is that more heterogeneous situations open up the possibility that relocation policies might be both grounded in efficiency and be acceptable as more heterogeneous situations might result in all sub-groups being made better off. There is here no need for group transfers to achieve such Pareto improvements. Acknowledging the existing interactions between the in-group and the inter-group problems thus opens up new avenues for policy design.

5 Conclusion

Until recent years, while many economic and socially-related situations involve groups of agents instead of individuals, most economic analyzes still do not consider the interplay between intra-group and inter-group interaction problems. This paper precisely analyzes how the interaction of collective action problems affect individual and group behavior.

We first show that the conclusions of classical models in the literature should be qualified. We highlight how the comparison between decentralized and cooperative outcomes and payoffs depends on the nature of in-group interactions and the relative magnitude of both types of collective action problems. This also provides a different perspective on group effectiveness and the group size paradox introduced by Olson (15). We then highlight how acknowledging the interplay between collective action problems yields new instruments to address the potential inefficiencies driven by decentralization. We first analyze the effect of group-level cooperation, and show how the effectiveness of this instrument relies heavily on the interplay between in-group and out-group problems. Group-level cooperation exhibits more potential under in-group substitutability: there are cases where it results in a Pareto improvement, and generally provides at least a global efficiency gain. By contrast, group-level cooperation is unlikely to emerge voluntarily as it never results in a Pareto improvement, and its global efficiency relies on the relative size of the cooperating group. Finally, we discuss the potential of group-relocation policies: the positive effect of such policies tends to rely mainly on the relative group size under in-group complementarity, while the comparison between the magnitude of in-group and out-group effects does impact the comparison under in-group substitutability. All together, these results highlight the importance of accounting for the existing interactions between collective action problems.

The goal of this work was to assess the main differences that emerge when such interactions are acknowledged. As such, the corresponding model has been kept relatively simple, and we abstracted from several issues. For instance, allowing for asymmetric information and introducing dynamic considerations would constitute interesting and important extensions that deserve future research.

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Appendix

Proof of Proposition 1

The optimality conditions are necessary and sufficient, and given by, for any $i \in A$ and $j \in B$:

$$a - b_A x_{iA} - \delta_{AA} X_{-iA} - \delta_{BA} X_B + \lambda_{iA} = 0$$

and

$$a - b_B x_{jB} - \delta_{BB} X_{-jB} - \delta_{AB} X_A + \lambda_{jB} = 0,$$

where $\lambda_{iA} \geq 0$ and $\lambda_{jB} \geq 0$ are the lagrangian parameters associated to each optimality condition. First, it is easily checked that there is no equilibrium where no agent in each group chooses a positive action level. Second, there cannot be an equilibrium outcome for which $\lambda_{iA} > 0$ and $x_{lA} > 0$ simultaneously (and the same holds for group B). Otherwise, we would have:

$$a - \delta_{AA} X_{-iA} - \delta_{BA} X_B = a - \delta_{AA} X_A - \delta_{BA} X_B < 0$$

while

$$a - \delta_{AA} X_{-lA} - \delta_{BA} X_B = b_A x_{lA} > 0$$

The second condition yields

$$a - \delta_{AA} X_A - \delta_{BA} X_B = (b_A - \delta_{AA})\, x_{lA}$$

which is non-negative when $b_A \geq \delta_{AA}$ holds. This is a contradiction.

Now, if there is one equilibrium such that $x_{iA}^N > 0$ for any $i \in A$ while $\lambda_{jB} > 0$ for any $j \in B$, then one has:

$$a - \delta_{AB} X_A^N < 0$$

while

$$a - \delta_{AA} X_A^N = (b_A - \delta_{AA})\, x_{iA}^N = (b_A - \delta_{AA})\, x_{lA}^N,$$

which in turn implies that $x_{iA}^N = x_{lA}^N = x_A^N$ for any i and $l \in A$. Rewriting, we obtain:

$$x_A^N = \frac{a}{b_A + (N_A - 1)\,\delta_{AA}} > 0$$

and

$$a \frac{b_A + (N_A - 1)\,\delta_{AA} - N_A \delta_{AB}}{b_A + (N_A - 1)\,\delta_{AA}} < 0$$

The expression of x_A^N implies that this case is already ruled out when there is in-group strong strategic complementarity. Otherwise, since $b_A + (N_A - 1)\,\delta_{AA} - N_A \delta_{AB} \geq 0$ by assumption, this case is also ruled out.

The symmetric case for group B is ruled out in a similar way. All together, this implies that one must have $\lambda_{iA} = 0 = \lambda_{jB}$ for all $i \in A$ and $j \in B$, and more specifically that $x_{iA}^N = x_{lA}^N = x_A^N > 0$ for any i and $l \in A$, and the same property holds within group B. Now, coming back to the optimality conditions and solving for x_A^N and x_B^N, we obtain the desired expressions, and the assumptions ensure that x_A^N and x_B^N are positive, which concludes the proof.

Proof of Proposition 2

The assumption $b_i \geq 2\,(N_i - 1)\,|\delta_{ii}|$ $(i = A, B)$ ensures that the aggregate payoff function is strictly concave. The rest of the proof is omitted, as it follows mainly from the same type of calculations than in the proof of Proposition 1, except that the problem here is to maximize the sum of all agents' payoffs over the two groups.

Proof of Proposition 3

We prove the proposition for the case of agents in group A, the conclusions will follow similarly for the case of agents in group B.
We use the expression of x_A^N provided in Proposition 1, and we denote

$$D := [b_A + (N_A - 1)\,\delta_{AA}]\,[b_B + (N_B - 1)\,\delta_{BB}] - N_A N_B \delta_{AB} \delta_{BA}$$

First, we differentiate x_A^N with respect to δ_{AA}, and we obtain:

$$\frac{\partial x_A^N}{\partial \delta_{AA}} = -\frac{a}{D^2}\,(N_A - 1)\,[b_B + (N_B - 1)\,\delta_{BB} - N_B \delta_{BA}]\,[b_B + (N_B - 1)\,\delta_{BB}]$$

Since $b_B + (N_B - 1)\,\delta_{BB}$ and $b_B + (N_B - 1)\,\delta_{BB} - N_B \delta_{BA}$ are positive when there is either in-group substitutatibility or in-group weak complementarity, we conclude that the effect of δ_{AA} is negative in these two cases. We obtain the same conclusion for in-group strong complementarity since $b_B + (N_B - 1)\,\delta_{BB}$ and $b_B + (N_B - 1)\,\delta_{BB} - N_B\delta_{BA}$ are negative then.

Now, differentiating with respect to δ_{BB} we obtain:

$$\frac{\partial x_A^N}{\partial \delta_{BB}} = \frac{a}{D^2}\,(N_B - 1)\,N_B\delta_{BA}\,[b_A + (N_A - 1)\,\delta_{AA} - N_A\delta_{AB}]$$

Since $b_A + (N_A - 1)\,\delta_{AA} - N_A\delta_{AB}$ is positive for in-group substitutability or in-group weak complementarity and negative otherwise, we conclude that the effect of δ_{BB} is positive for the first two cases and negative otherwise. This concludes the proof of the first point in the proposition. Differentiating with respect to δ_{AB} and δ_{BA} we obtain, respectively:

$$\frac{\partial x_A^N}{\partial \delta_{AB}} = \frac{N_A N_B \delta_{BA} a}{D^2}\,[b_B + (N_B - 1)\,\delta_{BB} - N_B\delta_{BA}]$$

and

$$\frac{\partial x_A^N}{\partial \delta_{BA}} = -\frac{aN_B}{D^2}\,[b_A + (N_A - 1)\,\delta_{AA} - N_A\delta_{AB}]\,[b_B + (N_B - 1)]$$

Arguments similar to those used in the first point imply that $\frac{\partial x_A^N}{\partial \delta_{BA}} < 0$ is satisfied in all cases, while $\frac{\partial x_A^N}{\partial \delta_{AB}} > 0$ for in-group substitutability or in-group weak complementarity and $\frac{\partial x_A^N}{\partial \delta_{AB}} < 0$ for in-group strong complementarity. This concludes the proof of the second point in the proposition. Differentiating with respect to N_B we obtain:

$$\frac{\partial x_A^N}{\partial N_B} = -\frac{a}{D^2}\,(b_B - \delta_{BB})\,[b_A + (N_A - 1)\,\delta_{AA} - N_A\delta_{AB}]$$

We conclude immediately that the effect of an increase in N_B is negative for in-group substitutability or in-group weak complementarity and positive for in-group strong complementarity. This concludes the proof of the third point in the proposition.
Finally, differentiating with respect to N_A we obtain:

$$\frac{\partial x_A^N}{\partial N_A} = -\frac{a}{D^2}\,[b_B + (N_B - 1)\,\delta_{BB} - N_B\delta_{BA}]\,[\delta_{AA}\{b_B + (N_B - 1)\,\delta_{BB}\} - N_B\delta_{AB}\delta_{BA}\delta_{AB}]$$

The first term between brackets on the right hand side of the equality is positive for in-group susbstitutability or in-group weak complementarity, and negative for in-group strong complementarity. The second term between brackets on the right hand side of the equality is negative for in-group weak complementarity, and this concludes the proof of this case. Now, the sign of $\frac{\partial x_A^N}{\partial N_A}$ depends on that of $\delta_{AA}\,[b_B + (N_B - 1)\,\delta_{BB}] - N_B\delta_{AB}\delta_{BA}\delta_{AB}$ for the other two cases, which yields the conclusion.

Proof of Proposition 4

We begin by considering the different cases related to the situation where there is strategic substitutability both within and between groups. When $N_A = N_B$ we quickly obtain: We have:

$$x_A^N - x_A^* = \frac{(N-1)\,\delta_{in} + N\delta_{out}}{[b + (N-1)\,\delta_{in} + N\delta_{out}]\,[b + 2\,(N-1)\,\delta_{in} + 2N\delta_{out}]} \tag{19}$$

Since both terms in the denominator are positive by assumption, and the numerator is also positive by definition of this case, we conclude immediately.

Secondly, if $N_A \neq N_B$ and $\delta_{in} = \delta_{out} = \delta$ then we can use the expressions of x_A^N and x_A^* to obtain the following condition:

$$x_A^N \geq x_A^* \Longleftrightarrow \phi(\delta) \geq 0$$

with

$$\phi(\delta) = \delta \left[N_A + N_B - 1\right] \{b^2 - 3\delta b + 2\delta^2\}$$

which is always non-negative as $b \geq 2\delta$ holds by assumption.

Finally, when $N_A \neq N_B$ while $\delta_{AA} = \delta_{BB} = \delta_{in}$ and $\delta_{AB} = \delta_{BA} = \delta_{out}$ are satisfied, we come back to the expressions of x_A^N and x_A^* and we obtain:

$$x_A^N \geq x_A^* \Longleftrightarrow f(\delta_{in}) \geq 0$$

with

$$f(\delta_{in}) = 2\left(N_A - 1\right)\left(N_B - 1\right)^2 \left(\delta_{in}\right)^3 + \left(N_A - 1\right)\left(N_B - 1\right)\left(\delta_{in}\right)^2 \left(3b - 2N_B\delta_{out}\right)$$
$$+\delta_{in}\left[\left(N_A - 1\right)b^2 - 2N_A N_B \left(N_B - 1\right)\left(\delta_{out}\right)^2\right] + N_B\delta_{out}\{b^2 - 3N_A\delta_{out}b + 2N_A N_B \left(\delta_{out}\right)^2\}$$

Differentiating $f(\delta_{in})$ twice we obtain:

$$f'(\delta_{in}) = 6\left(N_A - 1\right)\left(N_B - 1\right)^2 \left(\delta_{in}\right)^2 + 2\left(N_A - 1\right)\left(N_B - 1\right)\delta_{in}\left(3b - 2N_B\delta_{out}\right)$$
$$+ \left(N_A - 1\right)b^2 - 2N_A N_B \left(N_B - 1\right)\left(\delta_{out}\right)^2$$

and

$$f''(\delta_{in}) = 2\left(N_A - 1\right)\left(N_B - 1\right)\{6\left(N_B - 1\right)\delta_{in} + 3b - 2N_B\delta_{out}\}$$

which is positive as the term between brackets on the right hand side of the equality is positive by assumption. As such, we know that $f'(\delta_{in})$ increases as δ_{in} increases. Now, we obtain:

$$f'(\delta_{in} = 0) = \left(N_A - 1\right)b^2 - 2N_A N_B \left(N_B - 1\right)\left(\delta_{out}\right)^2$$

which is non-negative if and only if $b \geq \sqrt{\frac{2N_A N_B (N_B - 1)}{N_A - 1}}\delta_{out}$ holds, which is the case since $b > 2N_B\delta_{out}$ is satisfied by assumption and $2N_B \geq \sqrt{\frac{2N_A N_B (N_B - 1)}{N_A - 1}}$ always holds. Since $f'(.)$ is an increasing function we conclude that $f'(\delta_{in}) \geq 0$ always holds, and thus $f(\delta_{in})$ increases as δ_{in} increases.

We finally obtain:

$$f(\delta_{in} = 0) = N_B\delta_{out}[b^2 - 3N_A\delta_{out}b + 2N_A N_B \left(\delta_{out}\right)^2]$$

The term between brackets is a polynomial expression of δ_{out} and thus we deduce that it is always positive when $9N_A - 8N_B \leq 0$ or $N_A \leq \frac{8}{9}N_B$ is satisfied. Moreover, when $N_A > \frac{8}{9}N_B$ holds, defining:

$$\underline{\delta}_{out} = \frac{3N_A - \sqrt{N(9N_A - 8N_B)}}{4N_A N_B}$$

we quickly conclude that $f(\delta_{in} = 0) \geq 0$ when $N_B \geq N_A > \frac{8}{9}N_B$ is satisfied, which implies $f(\delta_{in}) \geq 0$ by monotonicity of f. By constrast, when $N_A > N_B$ holds, we deduce that

$f(\delta_{in} = 0) \geq 0$ when $\delta_{out} \leq \underline{\delta}_{out}$ is satisfied, and that $f(\delta_{in} = 0) < 0$ when $\delta_{out} \in]\underline{\delta}_{out}, \frac{b}{2N_B}[$ holds.

Due to the monotonicity of f the only case that remains is when $N_A > N_B$ holds. We compute:

$$f(\delta_{in} = \frac{b}{2}) = b^3 \frac{N_A - 1}{2}[1 + (N_B - 1)\frac{N_B + 2}{2}] + N_B \delta_{out} b^2 \frac{2 - (N_A - 1)(N_B - 1)}{2}$$
$$-N_A N_B (\delta_{out})^2 b (N_B + 2) + 2N_A (N_B)^2 (\delta_{out})^3$$

Differentiating with respect to δ_{out} we obtain:

$$6N_A (N_B)^2 (\delta_{out})^2 - 2N_A N_B (N_B + 2) \delta_{out} b + N_B b^2 \frac{2 - (N_A - 1)(N_B - 1)}{2}$$

We quickly deduce that this polynomial expression is negative when $\delta_{out} \in]0, \frac{b}{2}[$ holds. As $f(\delta_{in} = \frac{b}{2}) > 0$ when δ_{out} gets arbitrarily small and $f(\delta_{in} = \frac{b}{2})$ gets close to zero when δ_{out} gets arbitrarily close to $\frac{b}{2}$ we conclude that $f(\delta_{in} = \frac{b}{2})$ remains non-negative for all feasible values of δ_{out}. The monotonicity of $f(.)$ and a continuity arguments allows to conclude.

Moving on to the cases of in-group complementarities, we first conclude quickly that the proof of the case where $N_A = N_B = N$ follows directly from expression (19) for both weak and strong complementarity.

Now, if $\delta_{out} = \delta = -\delta_{in}$ then computing $x_A^N - x_A^*$ yields the conclusion that:

$$x_A^N \geq x_A^* \Longleftrightarrow f(\delta) \geq 0$$

with

$$f(\delta) = -2(N_A - 1)(N_B - 1)^2 \delta^3 + (N_A - 1)(N_B - 1)\delta^2 (3b - 2N_B\delta)$$
$$-\delta\left[(N_A - 1) b^2 - 2N_A N_B (N_B - 1)\delta^2\right] + N_B \delta\{b^2 - 3N_A \delta b + 2N_A N_B \delta^2\}$$

Rewriting, we obtain:

$$x_A^N \geq x_A^* \Longleftrightarrow \delta u(\delta) \geq 0$$

where

$$u(\delta) = 2(2N_B - 1)(N_A + N_B - 1)\delta^2 - 3b(N_A + N_B - 1)\delta + b^2[N_B - (N_A - 1)]$$

Function u is a polynomial expression of degree two, so we obtain quickly that $u(\delta) > 0$ when $9(N_A + N_B - 1)^2 - 8(N_A + N_B - 1)(2N_B - 1)(N_B + 1 - N_A) \leq 0$ or

$$N_A \leq \frac{16(N_B)^2 - N_B + 1}{16N_B + 1}$$

In this case we conclude that $x_A^N > x_A^*$ under weak or strong complementarity. Now, if $N_A > \frac{16(N_B)^2 - N_B + 1}{16N_B + 1}$ holds, we know that $u(\delta) \leq 0$ if and only δ lies in between δ_1 and δ_2 with

$$\delta_1 = \frac{3(N_A + N_B - 1)b - b\sqrt{9(N_A + N_B - 1)^2 - 8(N_A + N_B - 1)(2N_B - 1)(N_B + 1 - N_A)}}{4(2N_B - 1)(N_A + N_B - 1)}$$

and

$$\delta_2 = \frac{3\left(N_A+N_B-1\right)b + b\sqrt{9\left(N_A+N_B-1\right)^2 - 8\left(N_A+N_B-1\right)\left(2N_B-1\right)\left(N_B+1-N_A\right)}}{4\left(2N_B-1\right)\left(N_A+N_B-1\right)}$$

When $N_B+1 \leq N_A$ then $\delta_1 < 0$ and we deduce that $u(\delta) \leq 0$ if and only if $\delta \in [0, \delta_2]$ is satisfied. Under weak complementarity, we know that $\delta < \frac{b}{2(2N_B-1)}$ by assumption. It is easily checked that $\frac{b}{2(2N_B-1)} < \delta_2$ and we conclude that $u(\delta) < 0$ is always satisfied: thus $x_A^N < x_A^*$ holds. Under strong complementarity, we know that $\delta > \frac{b}{N_B-1}$ by assumption. It is easily checked that $\frac{b}{N_B-1} > \delta_2$ and we conclude that $u(\delta) > 0$ is always satisfied: thus $x_A^N > x_A^*$ holds.

We finally consider the case where $\frac{16(N_B)^2-N_B+1}{16N_B+1} < N_A \leq N_B$ holds. Under strong complementarity the inequality $\frac{b}{N_B-1} > \delta_2$ still holds and we again conclude that $u(\delta) > 0$ is always satisfied: thus $x_A^N > x_A^*$ holds. Now, under weak complementarity, it is easily checked that $\frac{b}{2(2N_B-1)} \leq \delta_1$ and we conclude that $u(\delta) \geq 0$ is always satisfied: thus $x_A^N \geq x_A^*$ holds.

Proof of Proposition 5

We first differentiate the agents' payoffs in group A with respect to N_A, accounting for the optimality conditions characterizing x_A^N:

$$\frac{\partial \Pi_A^N}{\partial N_A} = x_A^N \left[-\delta_{AA} x_A^N - \delta_{AA}\left(N_A-1\right)\frac{\partial x_A^N}{\partial N_A} - \delta_{BA} N_B \frac{\partial x_B^N}{\partial N_A}\right]$$

Differentiating the expressions of x_A^N and x_B^N with respect to N_A and simplifying, we obtain:

$$\frac{\partial \Pi_A^N}{\partial N_A} = -x_A^N \frac{b_A}{D^2}\left[\delta_{AA}\{b_B + (N_B-1)\delta_{BB}\} - N_B\delta_{AB}\delta_{BA}\right]\left[b_B + (N_B-1)\delta_{BB} - N_B\delta_{BA}\right]$$

with $D := \left[b_A + (N_A-1)\delta_{AA}\right]\left[b_B + (N_B-1)\delta_{BB}\right] - N_A N_B \delta_{AB}\delta_{BA}$ and we can now conclude as follows. By assumption, the second term between brackets on the right hand side of the equality is positive under in-group substitutability or weak complementarity, and negative under in-group strong complementarity. The sign of the first term between brackets is negative under weak in-group complementarity, and we conclude that $\frac{\partial \Pi_A^N}{\partial N_A} > 0$ in this case. Now, if the first term between brackets is non-negative, then $\frac{\partial \Pi_A^N}{\partial N_A} \geq 0$ under strong complementarity and $\frac{\partial \Pi_A^N}{\partial N_A} \leq 0$ under weak complementarity. The first term between brackets is non-negative if and only if $\frac{b_B+(N_B-1)\delta_{BB}}{N_B\delta_{BA}} \leq \frac{\delta_{AB}}{\delta_{AA}}$ under strong complementarity and $\frac{b_B+(N_B-1)\delta_{BB}}{N_B\delta_{BA}} \geq \frac{\delta_{AB}}{\delta_{AA}}$ under substitutability.

Similar calculations yield:

$$\frac{\partial \Pi_B^N}{\partial N_A} = x_B^N \frac{\delta_{AB} b_B}{D^2}\left[\delta_{AA} - b_A\right]\left[b_B + \left(N_B-1\right)\delta_{BB} - N_B\delta_{BA}\right]$$

Under intra-group substitutability, the first term and the second term between brackets are positive, and $\frac{\partial \Pi_B^N}{\partial N_A}$ is positive. Under weak intra-group complementarity, the first term between brackets is negative, the second term between brackets is positive, and $\frac{\partial \Pi_B^N}{\partial N_A}$ is negative. Finally, under strong intra-group complementarity, the first term between brackets is negative, the second term between brackets is negative, and $\frac{\partial \Pi_B^N}{\partial N_A}$ is positive.

Proof of Proposition 6

It is immediately checked that the first order conditions are necessary and sufficient, and given by:

$$a - b_A x_{lA}^{gc} - \delta_{AA} X_{-lA}^{gc} - \delta_{AA} \sum_{k \neq l} x_{kA}^{gc} - \delta_{BA} X_B^{gc} + \lambda_{lA} = 0$$

for any agent $l \in A$ and

$$a - b_B x_{iB}^{gc} - \delta_{AB} X_A^{gc} - \delta_{BB} X_{-iB}^{gc} + \lambda_{iB} = 0$$

for any agent $i \in B$. Assume first that $\lambda_{lA} > 0$ for agent $l \in A$, then

$$a - \delta_{AA} X_A^{gc} - \delta_{AA} X_A^{gc} - \delta_{BA} X_B^{gc} < 0$$

Let us first assume that $x_{jA}^{gc} > 0$ for $j \neq l$ then

$$a - b_A x_{jA}^{gc} - \delta_{AA} X_{-jA}^{gc} - \delta_{AA} X_{-jA}^{gc} - \delta_{BA} X_B^{gc} = 0$$

And thus necessarily

$$a - \delta_{AA} X_A^{gc} - \delta_{AA} X_A^{gc} - \delta_{BA} X_B^{gc} = (b_A - 2\delta_{AA}) \, x_{jA}^{gc} < 0$$

which is a contradiction by assumption. Thus, if $\lambda_{lA} > 0$ for agent l then necessarily this must hold for all agents in group A. This in turn implies that the only case that could be consistent is that $x_{jB} > 0$ for at least one agent in group B. Then condition $b_B \geq \delta_{BB}$ rules out the possibility that some agents in B choose a positive action level while some others choose a zero action level. In other words, if agents in group A choose a zero action level then any agent $j \in B$ must necessarily choose $x_{jB}^{gc} > 0$ and we have

$$a - \delta_{BA} X_B^{gc} < 0$$

and

$$a - b_B x_{jB}^{gc} - \delta_{BB} X_{-jB}^{gc} = 0$$

Combining these conditions imply that necessarily $\frac{b_B + (N_B - 1)\delta_{BB} - N_B \delta_{BA}}{b_B + (N_B - A)\delta_{BB}} < 0$ which contradicts the set of assumptions corresponding to each case. As such we conclude that $x_{lA}^{gc} > 0$ for any agent l in group A. A symmetric reasoning allows to conclude that, under the respective set of conditions, the group-level cooperation outcomes satisfy necessarily that $x_{iB}^{gc} > 0$ for any agent i in group B. Finally, solving the optimality conditions we obtain the unique group-level cooperation outcome characterized by expressions (15) and (16). This concludes the proof.

Proof of Proposition 7

Concerning individual action levels, using the expressions provided in Proposition1 and 6 we deduce that:

$$x_A^{gc} \geq x_A^N \iff -(N_A - 1)\, \delta_{AA} \left[b_B + (N_B - 1)\, \delta_{BB} \right] \left[b_B + (N_B - 1)\, \delta_{BB} - N_B \delta_{BA} \right] \geq 0$$

The conclusions then follow from the assumptions. Regarding action levels in group B, we obtain: and

$$x_B^{gc} \geq x_B^N \iff (N_A - 1)\, \delta_{AA} N_A \delta_{AB} \left[b_B + (N_B - 1)\, \delta_{BB} - N_B \delta_{BA} \right] \geq 0$$

and again the conclusions follow from the assumptions.

We compute the difference between agents' payoffs in group B under the two outcomes. Regarding the fully decentralized case, accounting for the first order condition satisfied by x_B^N and simplifying, we obtain:

$$\Pi_B^N = \frac{b_B}{2}\left(x_B^N\right)^2.$$

Regarding the case of group-level cooperation, accounting for the first order conditions satisfied by x_B^{gc} and simplifying, we obtain:

$$\Pi_B^{gc} = \frac{b_B}{2}\left(x_B^{gc}\right)^2.$$

As such $\Pi_B^{gc} - \Pi_B^N = \frac{b_B}{2}\left[\left(x_B^{gc}\right)^2 - \left(x_B^N\right)^2\right]$ and the conclusions follow from the conclusions of the first part of the proposition.

Finally, we compute the difference between agents' payoffs in group A under the two outcomes. Regarding the fully decentralized case, accounting for the first order conditions satisfied by x_A^N and simplifying, we obtain:

$$\Pi_A^N = \frac{b_A}{2}\left(x_A^N\right)^2.$$

Regarding the case of group-level cooperation, accounting for the first order conditions satisfied by x_A^{gc} and simplifying, we obtain:

$$\Pi_A^{gc} = \left[\frac{b_A}{2} + \delta_{AA}\left(N_A - 1\right)\right]\left(x_A^{gc}\right)^2.$$

We obtain:

$$\Pi_A^{gc} - \Pi_A^N = \frac{b_A}{2}\left[\left(x_A^{gc}\right)^2 - \left(x_A^N\right)^2\right] + \delta_{AA}\left(N_A - 1\right)\left(x_A^{gc}\right)^2$$

Under in-group substitutability, the first term on the right-hand side of the equality is negative, while the second term is positive as $\delta_{AA} > 0$ is satisfied. Under both weak and strong in-group complementarity, the first term on the righ-hand side of the equality is positive, but the second term is negative as $\delta_{AA} < 0$ is satisfied. Rewriting the expression of $\Pi_A^{gc} - \Pi_A^N$ yields the appropriate condition:

$$\Pi_A^{gc} \geq \Pi_A^N \iff \frac{b_A + 2\left(N_A - 1\right)\delta_{AA}}{b_A} \geq \left(\frac{x_A^N}{x_A^{gc}}\right)^2$$

We now use this condition to obtain the final conclusions in both cases. Using the expressions of x_A^N and x_A^{gc} we obtain:

$$\frac{x_A^N}{x_A^{gc}} = \frac{\left[b_A + 2\left(N_A - 1\right)\delta_{AA}\right]\left[b_B + \left(N_B - 1\right)\delta_{BB}\right] - N_A N_B \delta_{AB}\delta_{BA}}{\left[b_A + \left(N_A - 1\right)\delta_{AA}\right]\left[b_B + \left(N_B - 1\right)\delta_{BB}\right] - N_A N_B \delta_{AB}\delta_{BA}}$$

and thus

$$\frac{x_A^N}{x_A^{gc}} = 1 + \frac{\left(N_A - 1\right)\delta_{AA}\left[b_B + \left(N_B - 1\right)\delta_{BB}\right]}{\left[b_A + \left(N_A - 1\right)\delta_{AA}\right]\left[b_B + \left(N_B - 1\right)\delta_{BB}\right] - N_A N_B \delta_{AB}\delta_{BA}}$$

Using this expression, we obtain:

$$\frac{b_A + 2\left(N_A - 1\right)\delta_{AA}}{b_A} \geq \left(\frac{x_A^N}{x_A^{gc}}\right)^2 \iff$$

$$2\delta_{AA} \geq b_A \frac{(N_A-1)(\delta_{AA})^2[b_B+(N_B-1)\delta_{BB}]^2 + 2\delta_{AA}[b_B+(N_B-1)\delta_{BB}]\left([b_A+(N_A-1)\delta_{AA}][b_B+(N_B-1)\delta_{BB}]\right.}{\left([b_A+(N_A-1)\,\delta_{AA}]\,[b_B+(N_B-1)\,\delta_{BB}]-N_AN_B\delta_{AB}\delta_{BA}\right)^2}$$

Simplifying by δ_{AA} and rewriting, this is equivalent under in-group complementarity to:

$$2\left([b_A+(N_A-1)\delta_{AA}][b_B+(N_B-1)\delta_{BB}]-N_AN_B\delta_{AB}\delta_{BA}\right)^2$$

$$\leq b_A(N_A-1)\delta_{AA}[b_B+(N_B-1)\delta_{BB}]^2+2b_A[b_B+(N_B-1)\delta_{BB}]([b_A+(N_A-1)\delta_{AA}][b_B+(N_B-1)\delta_{BB}]-N_AN_B\delta_{AB}\delta_{BA})$$

After simplifications, this is equivalent to:

$$2\left([b_A+(N_A-1)\delta_{AA}][b_B+(N_B-1)\delta_{BB}]-N_AN_B\delta_{AB}\delta_{BA}\right)\left((N_A-1)\delta_{AA}[b_B+(N_B-1)\delta_{BB}]-N_AN_B\delta_{AB}\delta_{BA}\right)$$

$$\leq b_A\,(N_A-1)\,\delta_{AA}[b_B+(N_B-1)\delta_{BB}]^2$$

Simplifying this condition once again, we finally obtain:

$$\left([b_A+2(N_A-1)\delta_{AA}][b_B+(N_B-1)\delta_{BB}]-N_AN_B\delta_{AB}\delta_{BA}\right)\left[(N_A-1)\delta_{AA}\,[b_B+(N_B-1)\delta_{BB}]-N_AN_B\delta_{AB}\delta_{BA}\right] \leq 0$$

The first factor on the left-hand side of the inequality is positive by assumption, and the second factor is negative as $\delta_{AA} < 0$ by assumption. Thus, the inequality always holds and this concludes the proof of the corresponding part of the result.

Finally, under in-group substitutability, the right hand-side term in the equivalence can be rewritten as:

$$2\left([b_A+(N_A-1)\delta_{AA}][b_B+(N_B-1)\delta_{BB}]-N_AN_B\delta_{AB}\delta_{BA}\right)^2$$

$$\geq b_A(N_A-1)\delta_{AA}[b_B+(N_B-1)\delta_{BB}]^2+2b_A[b_B+(N_B-1)\delta_{BB}]([b_A+(N_A-1)\delta_{AA}][b_B+(N_B-1)\delta_{BB}]-N_AN_B\delta_{AB}\delta_{BA})$$

After simplifications we obtain:

$$[(N_A-1)\delta_{AA}[b_B+(N_B-1)\delta_{BB}]-N_AN_B\delta_{AB}\delta_{BA}]\,T \geq 0$$

where

$$T = [([b_A+(N_A-1)\delta_{AA}][b_B+(N_B-1)\delta_{BB}]-N_AN_B\delta_{AB}\delta_{BA})+(N_A-1)\delta_{AA}[b_B+(N_B-1)\delta_{BB}]]$$

The factor T is positive, and the sign of $\Pi_A^{gc}-\Pi_A^N$ is thus given by that of the first factor in the inequality. Since $b_B+(N_B-1)\delta_{BB} > N_B\delta_{BA}$ by assumption, then $\delta_{AA} \geq \frac{N_A}{N_A-1}\delta_{AB}$ is sufficient to obtain the conclusion. If $\delta_{AA} < \frac{N_A}{N_A-1}\delta_{AB}$ then group-level cooperation results in higher payoffs in group A if and only if $\frac{b_B+(N_B-1)\delta_{BB}}{N_B\delta_{BA}} \geq \frac{N_A\delta_{AB}}{(N_A-1)\delta_{AA}}$ and this concludes the proof.

Proof of Proposition 8

Under in-group substitutability, the difference in aggregate payoffs under group-level cooperation and under decentralization is computed as follows:

$$E = N_A\frac{b}{2}\left(x_A^{gc}-x_A^N\right)\left(x_A^{gc}+x_A^N\right)+N_A\left(N_A-1\right)\delta\left(x_A^{gc}\right)^2+N_B\frac{b}{2}\left(x_B^{gc}-x_B^N\right)\left(x_B^{gc}+x_B^N\right)$$

Using the expressions of the equilibrium decisions, we obtain that the sign of E is given by that of $N_A\left(N_A-1\right)\delta\left(b-\delta\right)H$ with:

$$H = -\left(b-\delta\right)^2\frac{b}{2}\{2\left[b+(N_A-1)\delta\right]\left[b+(N_B-1)\delta\right]-2N_AN_B\delta^2+(N_A-1)\delta\left[b+(N_B-1)\delta\right]\}$$

$$+ (b-\delta)\left\{[b+(N_A-1)\delta]\,[b+(N_B-1)\delta] - N_A N_B \delta^2\right\} + N_B\frac{b}{2}(N_A-1)\delta^2\left\{[b+(N_A-1)\delta]\,[b+(N_B-1)\delta] - N_A N_B \delta^2\right\}$$

Rewriting and simplifying the expression of H, we finally obtain:

$$H = \delta(b-\delta)\left\{(b-\delta)^2\left[b\frac{N_A-1+2N_B}{2} + \delta(N_A+N_B-1)^2\right] + \delta^2(N_A+N_B)(N_A-1)N_B\frac{b}{2}\right\}$$

The term between brackets is positive, and this implies that H is positive, which in turn implies that E is positive. This concludes the proof of this case.

Now, under in-group complementarity, the difference in aggregate payoffs under group-level cooperation and under decentralization is computed as follows:

$$E = N_A\frac{b}{2}\left(x_A^{gc} - x_A^N\right)\left(x_A^{gc} + x_A^N\right) - N_A(N_A-1)\delta\left(x_A^{gc}\right)^2 + N_B\frac{b}{2}\left(x_B^{gc} - x_B^N\right)\left(x_B^{gc} + x_B^N\right)$$

Using the expressions of the equilibrium decisions, we obtain that the sign of E is given by that of $N_A(N_A-1)\delta(b-\delta)[b+\delta-2N_B\delta]\,H$ with:

$$H = [b+\delta-2N_B\delta](b+\delta)[b+\delta-(N_A+N_B)\delta][N_A b - [b+\delta-(N_A+N_B)\delta]]$$

$$-(N_A-1)\frac{b}{2}[b+\delta-N_B\delta]^2[b+\delta-2N_B\delta]$$

$$-N_B\frac{b}{2}[2(b+\delta)[b+\delta-2N_A\delta][b+\delta-(N_A+N_B)\delta] - (N_A-1)\delta[b+\delta-2N_A\delta][b+\delta-N_B\delta]]$$

When group A is sufficiently larger than group B, the sign of H is driven by that of the following expression:

$$(N_A-1)(b+\delta)[b+\delta-2N_B\delta](b+\delta)[b+\delta-(N_A+N_B)\delta] - (N_A-1)\frac{b}{2}[b+\delta-N_B\delta]^2[b+\delta-2N_B\delta]$$

which is equal to

$$(N_A-1)[b+\delta-2N_B\delta]\,f(\delta)$$

with

$$f(\delta) = \frac{(b+\delta)^2}{2}[b+2\delta-2N_A\delta] - N_B\frac{\delta^2}{2}[2(b+\delta)+N_B b]$$

It is easily checked that $f(.)$ decreases as δ increases. Moreover, the highest feasible value of δ is $\min\{\frac{b}{2N_B-1}, \frac{b}{3N_A-2}\}$ and it is also easily checked that $f\left(\min\{\frac{b}{2N_B-1}, \frac{b}{3N_A-2}\}\right) \geq 0$ when N_A is sufficiently larger than N_B. Thus, we conclude that H is positive in this case. Now, when group B is sufficiently larger than group A, then the sign of H is driven by that of the following expression:

$$-N_B\frac{b}{2}[2(b+\delta)[b+\delta-2N_A\delta][b+\delta-(N_A+N_B)\delta] - (N_A-1)\delta[b+\delta-2N_A\delta][b+\delta-N_B\delta]]$$

It is negative, and so H is negative in this case. This concludes the proof.

Proof of Proposition 9

From the expressions of the equilibrium outcomes, we obtain quickly that $x_A^N = x_B^N = x_A^R = x_B^R$ under in-group substitutability, and the conclusion follows.

Now, under in-group complementarity, we obtain:

$$x_A^R - x_A^N = \frac{2k\delta}{\left[b-(N_A-1)\delta\right]\left[b-(N_B-1)\delta\right]-N_AN_B\delta^2}a > 0$$

$$x_B^R - x_B^N = -\frac{2k\delta}{\left[b-(N_A-1)\delta\right]\left[b-(N_B-1)\delta\right]-N_AN_B\delta^2}a < 0$$

and finally

$$x_A^R - x_B^N = \frac{2\left[N_A-N_B+k\right]\delta}{\left[b-(N_A-1)\delta\right]\left[b-(N_B-1)\delta\right]-N_AN_B\delta^2}a$$

and we conclude that $x_A^R - x_B^N > 0$ if and only if $k > N_B - N_A$ is satisfied. Finally, computing the difference in global welfare Δ between the post-relocation and pre-relocation cases we obtain

$$\Delta = \frac{b}{2}\left[N_A\left(x_A^R - x_A^N\right)\left(x_A^R + x_A^N\right) + k\left(x_A^R - x_B^N\right)\left(x_A^R + x_B^N\right) + (N_B-k)\left(x_B^R - x_B^N\right)\left(x_B^R + x_B^N\right)\right]$$

We obtain that $\Delta > 0$ if and only if k satisfies $Ak^2 + Bk + C > 0$ with $A = -2\delta$, $B = 2\left[b-(N_B-1)\delta + b - (N_A-1)\delta\right]$ and $C = 2\left(N_A - N_B\right)\left[b-(N_B-1)\delta + b-(N_A-1)\delta\right]$. Solving for k we deduce that any feasible value $k < \frac{b-(2N_A-1)\delta}{2\delta}$ satisfies this inequality. Finally $N_A + N_B \leq \frac{b-\delta}{2\delta}$ is equivalent to $\frac{b-(2N_A-1)\delta}{2\delta} \geq N_B$ and $k \leq N_B$ is then the binding constraint, while $N_A + N_B > \frac{b-\delta}{2\delta}$ is equivalent to $\frac{b-(2N_A-1)\delta}{2\delta} < N_B$ and $k < \frac{b-(2N_A-1)\delta}{2\delta}$ is then the binding constraint. This concludes the proof.

Proof of Proposition 10

From the expressions of the equilibrium outcomes, we obtain quickly that $x_A^N = x_B^N$ under both in-group substitutability and in-group complementarity.

Now we obtain:

$$x_A^R - x_A^N = \frac{k\left(\delta_{out}-\delta_{in}\right)\left[b+(N-1)\delta_{in}-N\delta_{out}\right]\left[b+(N-1)\delta_{in}+N\delta_{out}-k\left(\delta_{out}+\delta_{in}\right)\right]}{\{\left[b+(N-1)\delta_{in}\right]^2 - N^2\left(\delta_{out}\right)^2\}\{\left[b+(N-1)\delta_{in}\right]^2 - N^2\left(\delta_{out}\right)^2 \ + k^2\left[\left(\delta_{out}\right)^2 - \left(\delta_{in}\right)^2\right]\}}a$$

$$x_B^R - x_B^N = -\frac{k\left(\delta_{out}-\delta_{in}\right)\left[b+(N-1)\delta_{in}-N\delta_{out}\right]\left[b+(N-1)\delta_{in}+N\delta_{out}+k\left(\delta_{out}+\delta_{in}\right)\right]}{\{\left[b+(N-1)\delta_{in}\right]^2 - N^2\left(\delta_{out}\right)^2\}\{\left[b+(N-1)\delta_{in}\right]^2 - N^2\left(\delta_{out}\right)^2 \ + k^2\left[\left(\delta_{out}\right)^2 - \left(\delta_{in}\right)^2\right]\}}a$$

and finally

$$x_A^R - x_B^N = \frac{k\left(\delta_{out}-\delta_{in}\right)\left[b+(N-1)\delta_{in}-N\delta_{out}\right]\left[b+(N-1)\delta_{in}+N\delta_{out}-k\left(\delta_{out}+\delta_{in}\right)\right]}{\{\left[b+(N-1)\delta_{in}\right]^2 - N^2\left(\delta_{out}\right)^2\}\{\left[b+(N-1)\delta_{in}\right]^2 - N^2\left(\delta_{out}\right)^2 \ + k^2\left[\left(\delta_{out}\right)^2 - \left(\delta_{in}\right)^2\right]\}}a$$

The denominator is positive in all cases, and so are the second and third terms between brackets in the numerators. As such the sign of each expression is driven by that of $(\delta_{out} - \delta_{in})$ and this concludes the first part of the proof.

Finally, computing the difference in global welfare Δ between the post-relocation and pre-relocation cases we obtain

$$\Delta = \frac{b}{2}\left[N_A\left(x_A^R - x^N\right)\left(x_A^R + x^N\right) + k\left(x_A^R - x^N\right)\left(x_A^R + x^N\right) + (N_B - k)\left(x_B^R - x^N\right)\left(x_B^R + x^N\right)\right]$$

$$= \frac{b}{2}\left[(N_A + k)\left(x_A^R - x^N\right)\left(x_A^R + x^N\right) + (N_B - k)\left(x_B^R - x^N\right)\left(x_B^R + x^N\right)\right]$$

where $x^N = x_A^N = x_B^N$ as mentioned before. We obtain that $\Delta > 0$ if and only if

$$2k^2\left(\delta_{out} - \delta_{in}\right)\left[b + (N-1)\delta_{in} - N\delta_{out}\right]^2 T > 0 \tag{20}$$

where

$$T := \left(b - \delta_{in}\right)^2\left[2\left(b - \delta_{in}\right) + N\left(3\delta_{in} + \delta_{out}\right)\right] + \left[\left(\delta_{out}\right)^2 - \left(\delta_{in}\right)^2\right] N\left(N^2 - k^2\right)\left(\delta_{in} + \delta_{out}\right) \tag{21}$$

Under in-group complementarity the sign of Δ is that of expression T. Moreover, when $\delta_{out} > |\delta_{in}|$ is satisfied it is easily checked that T is positive. Now, when $\delta_{out} < |\delta_{in}|$ is satisfied then the second term in expression (21) is positive. We know by assumption that $2\left[\left(b - \delta_{in}\right) + (N+k)\left(\delta_{in} - \delta_{out}\right)\right] > 0$ and it is easily checked that

$$2\left(b - \delta_{in}\right) + N\left(3\delta_{in} + \delta_{out}\right) \geq 2\left[\left(b - \delta_{in}\right) + (N+k)\left(\delta_{in} - \delta_{out}\right)\right] > 0$$

if and only if either $N \leq 2k$ or $N > 2k$ and $\frac{3N+2k}{N-2k}\delta_{out} \geq |\delta_{in}|$ is satisfied. This also implies that T is positive. Finally, when $N > 2k$ and $\frac{3N+2k}{N-2k}\delta_{out} < |\delta_{in}|$ the first-term in expression (21) is still non-negative if and only if $|\delta_{in}| \leq \frac{2b+N\delta_{out}}{3N-2}$ is satisfied. Finally, when $|\delta_{in}| > \max\{\frac{3N+2k}{N-2k}\delta_{out}, \frac{2b+N\delta_{out}}{3N-2}\}$ we conclude as $\Delta > 0$ is satisfied if and only if $T > 0$ is satisfied. This concludes the proof for the case of in-group complementarity.

Now, when there is in-group substitutability and $\delta_{out} > \delta_{in}$ then the sign of Δ is that of T which is positive. When $\delta_{out} < \delta_{in}$ we deduce from expression (20) that $\Delta > 0$ if and only if $T < 0$ is satisfied: noticing that the first-term in the expression of T is always positive, and that the second term is a decreasing function of k (which equals zero if $k = N$) we conclude the proof.

Proof of Proposition 11

The expressions of x_A^R and x_B^R are

$$x_A^R = \frac{b + (N-1)\delta_{in} - N\delta_{BA} + k\left(\delta_{BA} - \delta_{in}\right)}{\left[b + (N-1)\delta_{in}\right]^2 - N^2\delta_{AB}\delta_{BA} + k^2\left[\delta_{AB}\delta_{BA} - \left(\delta_{in}\right)^2\right]} a$$

and

$$x_B^R = \frac{b + (N-1)\delta_{in} - N\delta_{AB} - k\left(\delta_{AB} - \delta_{in}\right)}{\left[b + (N-1)\delta_{in}\right]^2 - N^2\delta_{AB}\delta_{BA} + k^2\left[\delta_{AB}\delta_{BA} - \left(\delta_{in}\right)^2\right]} a$$

Using these expressions together with those of x_A^N and x_B^N we deduce that $x_A^R - x_A^N \geq 0$ if and only if

$$k\left[\left(\delta_{in}\right)^2 - \delta_{AB}\delta_{BA}\right] > \left(\delta_{in} - \delta_{BA}\right)\frac{\left[b + (N-1)\delta_{in}\right]^2 - N^2\delta_{AB}\delta_{BA}}{b + (N-1)\delta_{in} - N\delta_{BA}} \tag{22}$$

We also obtain that $x_B^R - x_B^N \geq 0$ if and only if

$$k\left[(\delta_{in})^2 - \delta_{AB}\delta_{BA}\right] > (\delta_{AB} - \delta_{in})\frac{[b + (N-1)\,\delta_{in}]^2 - N^2\delta_{AB}\delta_{BA}}{b + (N-1)\,\delta_{in} - N\delta_{AB}} \tag{23}$$

Finally, we obtain that $x_A^R - x_B^N \geq 0$ if and only if

$$k^2\left[(\delta_{in})^2 - \delta_{AB}\delta_{BA}\right][b + (N-1)\,\delta_{in} - N\delta_{AB}] + k\,(\delta_{BA} - \delta_{in})\left\{[b + (N-1)\,\delta_{in}]^2 - N^2\delta_{AB}\delta_{BA}\right\}$$

$$+N\,(\delta_{AB} - \delta_{BA})\left\{[b + (N-1)\,\delta_{in}]^2 - N^2\delta_{AB}\delta_{BA}\right\} \geq 0 \tag{24}$$

The last condition ensures that the relocation policy results in higher payoffs for agents relocating from group B to group A. The numerator and denominator of the term on the right hand side of conditions (22) and (23) are positive by assumptions.

One can quickly notice that cases where $\delta_{AB}\delta_{BA} \geq (\delta_{in})^2$ are incompatible with in-group complementarity (as it would require $\delta_{BA} > \delta_{in} > \delta_{AB}$) and also with in-group substitutability (as it would also require $k > N\frac{\delta_{BA}-\delta_{AB}}{\delta_{BA}-\delta_{in}} \geq N$). Now, assuming $\delta_{AB}\delta_{BA} < (\delta_{in})^2$ and more specifically $\delta_{in} > \max\{\delta_{AB}, \delta_{BA}\}$ with $\delta_{AB} > \delta_{BA}$ we quickly conclude that condition (23) is satisfied. Moreover, condition (22) holds if and only if

$$k > \frac{\delta_{in} - \delta_{BA}}{(\delta_{in})^2 - \delta_{AB}\delta_{BA}}\frac{[b + (N-1)\,\delta_{in}]^2 - N^2\delta_{AB}\delta_{BA}}{b + (N-1)\,\delta_{in} - N\delta_{BA}}$$

The threshold value on the right hand side of this inequality is feasible as it is smaller than $\frac{b+(N-1)\delta_{in}-N\delta_{BA}}{\delta_{in}-\delta_{in}}$ (which is the upper limit of k by assumption). Now it remains to check that large enough values of k satisfy condition (24) and we check that $k = \frac{b+(N-1)\delta_{in}-N\delta_{BA}}{\delta_{in}-\delta_{in}}$ satisfy the inequality. Plugging the expression of k into inequality (24) and simplifying, we obtain that condition (24) is satisfied for this value of k if and only if:

$$\delta_{BA}N\,(b - \delta_{in})\left[2\,(\delta_{in})^2 - 2\delta_{in}\,(\delta_{AB} + \delta_{BA}) + 2\delta_{AB}\delta_{BA}\right] + \delta_{BA}\,(b - \delta_{in})^2\,[2\delta_{in} - (\delta_{AB} + \delta_{BA})] > 0$$

The two terms between brackets on the left hand side of this inequality are positive as $\delta_{in} \geq \delta_{AB} > \delta_{BA}$ by assumption. Since $b \geq \delta_{in}$ also by assumption, the left hand side is positive as the sum of two positive terms. As such this inequality is satisfied, and we conclude that it is also satisfied for values of k lying above a threshold value. Since this is also the case for condition (22) and that condition (23) is always satisfied, this concludes the proof.