Delegating War

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Abstract

Governments often delegate the fight for control over natural or political resources to local armed groups. This paper presents a model of proxy war in which governments delegate conflict by sending non-negotiable offers to militias. Contracts are composed of monetary transfers and of a sharing rule of political influence. Armed groups are positioned along a continuum representing the ideological misalignment between each militia and its government sponsor. Using a principal-agent model with two principals and two agents, I characterize the optimal contracts under complete and incomplete information about the militias' ideological positions. The analysis shows that with incomplete information armed groups receive lower transfers but are left with higher political independence. When governments strategically choose whether to fight by delegation or engage directly in conflict, the equilibria can be characterized in function of the local support to militias. If governments compete to recruit the same armed group, the militia generally carves out higher rents and pledges allegiance to the government ideologically closer.

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1 Introduction

Governments seeking to further their international strategic goals often delegate costly fighting to third parties. Sponsoring governments act as patrons and accord financial and military assistance to armed groups, which are in charge of fighting on their sponsors' behalf. Militias aligned with an external party receive resources to strengthen their political and military power, while giving up a portion of their political autonomy. Conflicts by proxy take a heavy toll on civilians and inflict heavy damages on human capital, hampering economic development and shaping the balance of internal and intrastate power for years to come (Esteban and Ray, 2017). Existing economic theories of conflict fail to explain observed varying patterns of militias' autonomy, the degrees of foreign assistance and fighting intensities. This paper fills precisely this gap and studies the contracting of militias by states.

History abounds with examples of delegated conflicts across time and geography. Since classical antiquity and the Middle Ages, through the Renaissance to modern history, proxies were exploited in internal and external conflicts. In early modern history, the Thirty Years' War represents a classic example of power struggle between two main powers – the royal houses of the Habsburg and Bourbon – for the hegemony over Europe. It is an emblematic representation of a proxy conflict and exemplifies its characteristics. Based on affiliations rooted in ideological and religious differences between Catholics and Protestants, this war involved proxies ranging from Sweden to the Italian city-states, and caused Germany to loose 40% of its population which dropped from roughly 20 million to 12 million between 1618 and 1648 (Wedgwood, 2016). The Treaty of Westphalia, which signed the end of the conflict, placed the following centuries of European politics in a new frame.

The end of World War II saw the US and the URSS confronting each other for more than four decades in a series of proxy wars. The delegated conflicts of the Cold War, which channeled the tensions between the two superpowers, have been the main causes of battle deaths in the 1946-1989 period. The end of the Cold War came with a sharp decrease in battle deaths, which decreased by two thirds when considering the years between 1990 and 2002 (Lacina and Gledistch, 2005)[1]. Consider, for example, how the Carter and then Reagan administrations responded to the 1979 Soviet invasion of Afghanistan by arming, funding and training the fledgling Afghan mujahedeen. More than 1 million civilians died and millions of Afghans fled

[1]This represent an underestimation of the number of deaths because of measurement difficulties. Moreover, it only considers battle deaths and not those inflicted on civilians, genocides and violence on the population.

the country as refugees to neighboring Pakistan and Iran. The instability caused by that war still resonates in contemporary Afghanistan. Similarly, the Soviet use of Cuban proxies during the civil war in Angola, where conflict first broke out in 1974, left 800,000 killed and 4 million displaced. Nearly 70,000 Angolans became amputees as a result of land mines. The wars of Korea between 1950 and 1953, the Vietnam war and the Nicaraguan civil war caused incalculable losses and can all be placed in this context.

In recent times, the Middle East and North Africa have become the main theaters of proxy wars. Since the civil war in Lebanon in the 70s and 80s, external powers acted as sponsors to a constellation of local militias by giving financial and military support in exchange for geopolitical influence. During the Lebanese civil war, the system of patron-client relations was firmly anchored in the religious dimension, whereby Hezbollah received strong support from Iran and many of the Christians militias were flanked by Israel and by Western countries. Approximately 120,000 died and more than one third of the entire population left the country. This conflict left its mark on the Lebanese political system where proxy relationships between armed parties and external sponsors are still at the heart of the balance of power. Nowadays, the struggle for regional supremacy between Iran, on one side, and Saudi Arabia on the other[2], permeates daily life and politics in Iraq, Yemen and Syria. Recent estimates show that more than 1 million human lives have been lost and several millions are displaced in these three conflicts combined[3]. In Iraq, after the invasion led by the United States, Iran sought a more friendly Shia-dominated government and supported sympathetic rebel factions as part of an effort to undermine the U.S. led coalition, which Iran feared would install a government hostile to its interests. Symmetrically, the United States and Saudi Arabia support the government of Kurdistan and its Sunni armed groups as a barrier to the expansion of Iranian influence in the region. In Yemen, a country historically in the sphere of influence of Saudi Arabia, the Shia dominated Houthi militias are used by Iran as a means to assert its influence in the southern Arabian Peninsula while the central Sunni government is armed and supported by Saudi Arabia. Another example of a proxy conflict in the region is represented by the war in Libya, where Russia and Turkey confront each other for geopolitical influence in the Southeastern Mediterranean.

The logic of proxy conflict is based on a mechanism of indirect engagement in a struggle for power whereby powers may use a local war to advance their global and regional strategic interests without the need to intervene with their own forces (Deutsch, 1964; Miller, 1967; Bar-Siman-Tov, 1984). States employ delegated fighters for two main reasons (Mumford, 2013;

[2]In turn supported by the United States.

[3]ACLED for Yemen, https://www.iraqbodycount.org/database, http://iraqdtm.iom.int/IDPsML.aspx., Global Conflict Tracker for Syria https://www.cfr.org

Ahram, 2011). First, hiring a proxy offers the possibility of achieving strategic goals more economically, with fewer political costs and less risk. Direct involvement in a war is a costly strategy, as the state burns resources and lives are lost. While it must spend resources to finance local armed groups, the sponsor bears neither direct ties nor the associated domestic war weariness and discontent. Second, when there is no international legitimacy for direct intervention, proxy wars might be the best way to advance one's own interests (Byman, 2007; Gleditsch et al., 2008; Regan, 2000 and 2002). The international community often looks the other way when states delegate conflict to local groups, and such sovereignty violations are not condemned as strongly as border violations by government troops. Symmetrically, armed groups are willing to accept an informal allegiance in return for two main rewards (Salehyan 2009 and 2010; Schultz, 2008; Bar-Siman-Tov 1984). First, they demand resources that can significantly augment the groups' military and political capabilities. Local armed groups face the challenge of mobilizing supporters, training recruits, finding sources of finances, and acquiring arms. Foreign patrons can help overcome large power asymmetries between local groups and, importantly, can help mobilizing resources quickly. Second, in case of victory, armed groups can take control of geopolitical and natural resources which will be shared with the sponsor government.

This mechanism has its own tradeoffs. Religious and ideological misalignment between a government and its sponsor is always costly – whether politically, financially or materially – and the long term strategic objectives of local armed groups and their state sponsors often diverge. Sponsor governments want to influence policymaking of their proxies, which in turn lose a portion of organizational and political independence (Salehyan, 2010). Moreover, as proxies begin to develop greater perceptions of autonomy, the political and strategic costs of delegating conflict to third parties are likely to become more important. Two main puzzling facts emerge from qualitative evidence. Contracted militias are left with considerable political autonomy by the sponsor[4] and governments often support groups that are initially weak and have low fighting capacity (Byman, 2018). The relevance of this phenomenon requires a formal discussion to better understand the strategic incentives underlying proxy conflicts.

I present here a simple model of conflict delegation. Two governments compete to exert influence in a third country, in the form of extracting resources at a low price, having preferential access to its markets for domestic firms or more generally want to expand their political and military influence in a wider geopolitical context. The territory which represents the battlefield

[4]See, for instance, the case of the National Defense Forces in Syria which were hired by the Assad regime (https://www.mei.edu/publications/all-presidents-militias-assads-militiafication-syria) or the Lashkal-i-Taiba group in Kashmir hired by Pakistan (Ahram, 2011)

is often a fragile or failed state, where institutions are weak and captured by interest groups and militias. In this political context, control over physical and political resources is held by these armed groups. I assume that the universe of groups is divided into two parties. Each governmental actor uses one of these two parties to fight on her behalf against the other[5]. In the model, sponsor governments act as principals that offer non-negotiable contracts to their militias which in turn act as their agents. In the baseline model, each militia receives offers of contracts from one single government, but this assumption is relaxed in the last section of the paper. The offered contracts are made up of two elements, a transfer of resources and a share of influence over militias' policymaking. The two governments send monetary transfers to their militias, which can be used to recruit fighters and buy weapons or can be invested in productive activities. In return, armed groups give away a share of their political independence to the government that supports them. Militias can either accept the contract and become the governments' delegated fighters or turn down the offer and remain neutral, keeping their whole political independence and receiving a fixed positive payoff. When they accept the contract, militias fight each other to appropriate the contested geopolitical and natural resources as well as the investments in productive activities. The total prize of the contest is then divided within the winning party[6], according to a fixed sharing rule which is the result of ex-post negotiations.

I formalize this situation with a two-stage game where in the first stage governments simultaneously set their proposed transfers and degrees of control over militias' policymaking. Contracts are offered to their respective militias, which can choose to either turn down the contract and remain neutral or accept the contract. If they accept, in the second stage militias strategically choose their optimal effort of combat and fight against each another for the control of resources. The key parameters are the ideological misalignments between each government and its militia, which influence the strategic considerations of all players in the game. The higher the misalignment, the higher is the sponsors' political cost of transferring resources to militias, due to deeper scrutiny in parliamentary committees and stronger critiques from the general public for the involvement in foreign conflicts through local groups. Secondly, it enters the utility of militias through the cost of recruiting fighters. The higher the misalignment, the more local groups have to reward local combatants to have them fight on behalf of a foreign power. This, in turn, influences the strategic decisions of fighting intensity and ultimately the equilibria of the game.

The aim of this paper is to bring the literature forward by offering a formalization of dele-

[5]Throughout the paper I use the terms "armed groups" and "militias" interchangeably.

[6]In what follows, a party is composed by the union of a militia and her sponsoring government

gated conflicts through a principal-agent model with two principals and two agents. I ask three main research questions. First, what are the optimal contracts offered by governments to local armed groups and what are the equilibrium fighting efforts of militias? In the presence of informational asymmetries whereby governments hold incomplete information on their militias' ideological position, how can governments design optimal contracts? Salehyan (2019) argues that governments often face adverse selection when contracting local armed groups, as these groups lack a fixed ideological structure and frequently change leadership and inspiration. Research in political science (e.g. Krieg, 2016; Pfaff, 2017) has shown that when states transfer the burden of warfare from their own military to local groups, they never really know – or at least sometimes misinterpret – the militia's ideology and interests. Governments may face domestic and international criticism as informational asymmetries may increase the reputation costs of delegating fighting to third parties. Militias can damage the sponsor's public image by taking unexpected violent actions motivated by their ideology (e.g. human rights abuses, Lauri and Suhrke 2020) or by adopting a rhetoric incompatible with that of the sponsor. While many scholars in international relations and conflict studies tackle this issue with the tools of qualitative principal-agent theory (Byman and Kreps, 2010; Innes, 2007; Berman and Lake, 2019), little has been done to formalize it.

Second, under which conditions is delegating conflict an equilibrium? States struggling for dominance are often faced with the dilemma of whether to wage war directly or to delegate it to third parties. Direct confrontation entails higher human and material costs, but avoids the negative consequences of being associated to a group that is ideologically misaligned. I describe this situation with a strategic delegation model where principals fully commit to the contracts they offer. As highlighted by Fershtman et al. (1991), the principals of a delegation game can strategically use agents to play on their behalf by offering contracts that are common knowledge. In their seminal paper, cooperative outcomes emerge as equilibria in the game with delegation, providing that each principal is fully committed to the contract he signed with his agent and the contracts are fully observed.

Third, what are the optimal contracts when two governments compete to hire the same militia? Weak polities are characterized by the fractiousness of its armed groups. When local groups first form (or after their partial disintegration), the choice of which external support to accept is a crucial decision. External sponsors, often allies, compete to hire local combatants and armed groups receive different offers. I model this situation with a common-agency model inspired by Bernheim and Whinston (1986), where two principals – the external governments – compete to hire a common agent – the militia.

The first main contribution of this paper is to formally highlight and quantify the role of asymmetric information in the contracting of proxies by their sponsors. I find that when sponsors have incomplete information on their militia's ideology, they can design a menu of second-best contracts that are incentive-compatible. Governments are able to elicit the militia's true ideology and each militia picks the contract that has been designed for her. In second-best contracts, offers are characterized by transfers that are lower compared to the case of complete information, while militias are left with more political independence. Since governments transfer more resources to groups ideologically closer, militias seek to exploit their informational advantage to receive higher transfers. By lowering the schedule of transfers for every ideological type, governments can offset the militias' incentive to falsely declare to be more ideologically aligned than they actually are. Lower transfers have to be balanced by leaving a higher degree of political independence to militias.

The second main contribution of this paper lies in the characterization of equilibria of the delegation game, which hinges on the relative ideological misalignments and on the level of support that armed groups receive from local populations. The delegation of conflict is the unique equilibrium when militias receive weak support from local populations. In this case, groups are at the fringe of society and lack the networks to recruit fighters and buy weapons. This finding is in accord with evidence, as highlighted by the qualitative literature on conflicts (e.g. Beaman, 2018). Militias that lack local support are in desperate need of external resources and are easily hired by foreign principals, which can spend little resources recruiting them. The equilibria are always unique, except when the ideological misalignments of the two parties are identical. When this happens, multiple equilibria could arise for intermediate levels of local support.

The third main contribution is the characterization of equilibria of the common agency game. Equilibria hinge on the relative ideological misalignments of each government vis-à-vis the militia, the value of resources at stake and the outside option of neutrality. In general, both governments try to get the allegiance of the militia by sending offers. The competition triggers a sequence of undercuttings of the control over the militia's independence in policymaking. If one of the two governments has a distinct ideological advantage over the other and the outside option is not much attractive, then the former is able to keep the latter out of the competition by leaving high political independence to the militia. When the option of remaining neutral becomes very attractive for the militia, the ideologically advantaged government must further increase the offered share of political power left to the militia until a point where it gives it all away to the delegated group. Moreover, when the two principals are in tight ideological competition – their ideological misalignments to the armed group are almost the same – or when the value of resources at stake is very high, neither government can keep a positive share of political influence. The recruited militia has complete independence in policymaking and the two governments compete on the offered transfers. For the most aligned principal it is sufficient to offer slightly more than its competitor to win the militia's allegiance.

1.1 Related Literature

This study naturally relates to the literature on the theory of incentives, to that on contests and third-party intervention, as well as to the literature on conflicts in international relations. Following the seminal papers by Baron and Myerson (1982) and Maskin and Riley (1984), the theory of incentives has found great success in modeling economic and political problems. While the traditional setting has been largely applied to the theory of the firm, most applications in political science have abstracted away from the game-theoretical formalization. Notable exceptions can be found, for instance, in Alesina and Tabellini (2005, 2006), which look at the strategic decision of politicians to delegate some tasks to bureaucrats. They find that politicians would not delegate tasks that would be socially optimal to delegate, but instead prefer to delegate risky policies to shift risk and blame on bureaucracies[7]. Despite a surge of recent studies in international relations and conflict studies that exploit the qualitative results of the theory of incentives (Carter and Pant, 2017; Gates, 2002; Meirowitz et al, 2019; Salehyan, 2010), none of these articles engages in the formal discussion of strategic decisions of actors involved in delegated conflicts. This paper departs from these studies by offering a full-fledged model of conflict delegation that takes inspiration by seminal papers in the theory of contracts (Fershtman et al., 1991; Fershtman and Kalai, 1997; Bernheim and Whinston, 1986). It looks at the literature on conflicts from a different perspective by offering a threefold contribution: it explores the role of asymmetric information in the government-militia relationship[8], it studies the problem of strategic delegation of war and introduces a common-agency framework to understand the effect of government competition on optimal contracting. This is also, for the best of my knowledge, the first paper to explicitly study the role of adverse selection in states' contracting of local armed groups. Baik and Kim (2014) studies a two-player contests in which, in order to win a prize, each player hires a delegate to expend effort on her behalf; neither party's delegation contract is revealed to the rival party when the delegates choose their effort levels. This study

[7]An excellent overview of delegation problems in political economy can be found in Persson and Tabellini (2002)

[8]Laffont and Martimort (2009) gives an excellent overview of methods and insights on general models in contract theory.

differs in many aspects. First, I introduce the differentiation of delegates along the ideological spectrum. Second, offers are composed by two contracting variables. Third, I introduce the possibility of competition between principals hiring a delegate. The literature on contests in economics[9] has investigated principal-agent problems only marginally (Hirshleifer, 1989, 1985, 2000; Tullock, 1980; Konrad, 2005; Skaperdas, 1996; Esteban and Ray, 1999; Wittman, 2000; Mehlum et al. 2004; Grossman and Kim, 1995), while research in political and development economics has looked at the policy implications of third-party interventions in the context of attempts to avoid civil conflicts spilling into neighboring countries (Silve and Verdier, 2018), as a means to quell a rebellion (Kathman, 2010 and 2011), to lower the overall level of conflict (Siqueira, 2003) or to maximize society's welfare (Amegashie and Kutsoati, 2007). A recent paper by Sambanis et al. (2020) shows how external intervention interacts with polarization of group identities to induce rebellion and civil war. While providing statistical evidence for the importance of this interaction, and proposing a model supporting their findings, their study differs significantly from this work in that it does not tackle the question of incentives. A recent literature in political science has looked at the strategic aspects of third party interventions and its impact on militarization and conflict outcomes. Meirowitz et al. (2020) use a game-theoretic model of intervention with strategic militarization and bargaining. While considering a wide variety of possible interventions that range from commitment to military assistance in case of war, to subsidizing a challenger's militarization, they do not consider the role of delegation of conflict per se and do not explore the ideological dimension in the issue of third party intervention.

The paper proceeds as follows: Section 2 first presents the baseline model and then it studies optimal contracting in complete and incomplete information on groups' ideology. Section 3 looks at the game of strategic delegation. Section 4 presents the analysis of a common agency game, where two principals want to recruit the same militia to fight on its behalf. The Conclusion summarizes the findings and outlines directions for future research.

2 The Model

The model formalizes the interaction between two governments and two militias that operate in a weakly institutionalized polity where the rule of law is weak or absent. Actions are taken sequentially. First, the two competing governments delegate fighting to militias which are positioned along a continuum of types $\theta_{k=i,j} \in (0,1]$ representing the ideological misalignment

[9] Garfinkel and Skaperdas (2007) gives a comprehensive overview of the main models.

between themselves and their governmental sponsors. Governments offer contracts made of two contracting variables: a transfer of resources and a demanded share of control over the militia's policymaking. Each armed group is affiliated to only one government[10]. Once offers are extended, in the second stage militias compete for control over resources by strategically choosing their fighting efforts $a_{k=i,j} \in \mathbb{R}_+$. Fighting between militias takes place and the winning party gets the whole prize. Formally, we have a set of players $(G_k, m_k)_{k=i,j}$ consisting in two governments $G_{k=i,j}$ and two militias $m_{k=i,j}$. Contracts are represented by a set of allocations $\mathcal{A} = \{(t_k, \gamma_k) : t_k \in \mathbb{R}_+, \gamma_k \in [0,1]\}_{k=i,j}$, consisting in a set of transfers and in the demanded shares of political power. Players play simultaneously within each stage of the game. We assume throughout the paper that governments have negligible budget constraints with regards to this type of intervention and that militias perfectly know each others' ideological types. This is a reasonable assumption, as armed groups are deeply rooted in the territory, often share a common language and have interacted before the contest takes place.

Once militias receive the transfers from their sponsors, they can choose how to allocate resources between two ways of generating income: peaceful production l_k or appropriative efforts to seize resources previously controlled by others – which costs $\mu\theta_k a_k$[11]. The marginal cost of fighting $\mu\theta_k$ represents the unit cost that militia leaders undergo to engage in combat-related activities. It represents the cost of recruiting fighters, mobilization and logistics. It is composed of two parameters, μ and θ_k. The former is constant across militias and it relates to the conditions of local labor markets and to the characteristics of the contested region's geography. The latter varies across armed groups and represents the ideological misalignment between the militia and the sponsor government. I assume the marginal cost of fighting $\mu\theta_k$ to be less than 1. It is a reasonable assumption since armed groups can engage in fighting activities at a discount with respect to regular armies, which I assume in this paper to have a marginal cost of fighting equal to one[12]. The higher is the ideological misalignment θ_k, the higher is the compensation militia leaders have to pay recruits to fight for an external entity that is not perfectly aligned to their ideology. The received transfer is strategically split between fighting and productive activities

$$t_k = l_k + \mu\theta_k a_k$$

[10] I relax this assumption in Section 4.

[11] This trade-off is widely documented by a wealth of studies in the economics of conflicts (Garfinkel and Skaperdas, 2006) and its appropriative nature is well captured by Vilfredo Pareto's (1927)

[12] Local armed groups engage in lighter forms of combat, and often use techniques of guerrilla due to their superior knowledge of the territory. Militias' fighters receive lower salaries compared to members of regular armies of major powers and their wage structure is quite flat. See, for an interesting study on Iraq, Bahney et al. (2013).

Rearranging we can see that what is invested in productive activities can be expressed in terms of the transfer received by the sponsor government net of what is spent in fighting effort

$$l_k = t_k - \mu\theta_k a_k$$

The stakes of the contest are represented by $V > 0$, which captures the value of disputed natural resources and geopolitical importance combined. The stakes V and the output of the two militias' joint production $\bar{L} = \sum_{k=i,j} l_k = \sum_k t_k - \mu\theta_k a_k$, are subject to dispute, which I assume to be resolved in a winner-take-all contest. The overall prize T of the contest writes

$$\begin{aligned} T &= V + \bar{L} \\ &= V + \sum_{k=i,j} l_k \\ &= V + \sum_{k=i,j} t_k - \mu\theta_k a_k \end{aligned}$$

Each militia's probability of winning is modeled by a standard Tullock contest function and depends on the militias' relative investment in fighting. The probability that militia k wins the contest is[13]

$$p(winner = k) = \frac{a_k}{a_k + a_{-k}}$$

In case of defeat, militias get a normalized payoff of 0 from the contest. Once the outcome of the contest is realized, the winning party splits the spoils of war between the sponsoring government and the delegated militia according to a sharing rule. A portion s^g of the total prize T goes to the government while s^m is received by the militia, such that $s^g + s^m = 1$. For the sake of simplicity, I assume the sharing rule of the spoils of war to be ex-ante homogenous across parties[14], as this can be considered the equilibrium outcome of negotiations occurring after the outcome of the contest is realized. Militias also enjoy from a degree of political independence P in the realm of policymaking, independently from the outcome of the conflict. This represents the autonomy in establishing internal laws and enforcing social norms, regulating the exploitation of resources under militias' control as well as the political influence over their electoral base. When a militia

[13] By construction, militia k's victory implies the victory of her sponsoring government and the probability of winning $p(winner = k)$ applies as well to governments. Another possible way to interpret p_k could be to consider it as the share of the total prize that is appropriated by k.

[14] As outlined above, a party is composed by a militia k and her sponsoring government k.

m_k becomes a proxy for an external entity, it gives up a share γ_k of its independence in policy-making P to its sponsor (Salehyan, 2010). Sponsoring governments thus capture a share of the internal decision-making process of militias over social and political matters by influencing its leaders, who in turn lose a portion of their political independence. Militias who fight on behalf of a foreign power maximize their utility

$$
\begin{aligned}
u_k^m &= (1-\gamma_k)P + \frac{a_k}{a_k + a_{-k}}\, s^m\, T \\
&= (1-\gamma_i)P + \frac{a_i}{a_i + a_j}\, s^m \left(V + \sum_{k=i,j}(t_k - \mu\theta_k a_k)\right)
\end{aligned}
$$

for $k = i, j$ and where I substituted the expression for the total prize $T = V + l_i + l_j$. Governments delegating fighting to local armed groups maximize the following utility function, where they extract some control of policy-making $\gamma_k P$ from their militias and expect to receive a share s^g of the total prize T

$$
u_k^G = \gamma_k P + \frac{a_k}{a_k + a_{-k}}\, s^g \left(V + \sum_k (t_k - \mu\theta_k a_k)\right) - \omega\theta_k t_k^2
$$

where $\omega\theta_k t_k^2$ represents the political and logistics cost of transferring resources to a local militia. It is reasonable to assume that the higher is the ideological misalignment between a government and its proxy, the higher is the marginal cost of financing it, either for domestic political resistance (parliamentary committees etc.), retaliation vis á vis allies or criticism from the public opinion. The assumption of increasing marginal costs of transferring funds is also realistic. The political damage of increasing the transferred funds by one unit is small when the level of assistance is also small – it can be dissimulated as humanitarian aid and can be more easily hidden from the public opinion. In other words, the possibility of plausible deniability of supporting foreign groups decreases as governments' involvement increases. When the level of support for the delegated militia is high, increasing the transfer by one unit entails a higher political damage potentially due to domestic and international criticism. From this functional specification, we see how a portion of sponsors' transfers that is not used in fighting can be recovered as part of the contest's prize. This is consistent with the fact that investments in productive activities by militias (e.g. financial institutions, social cooperatives, irrigation or health services) increase the political returns of such strategies of sponsor states by maximizing sentiments of loyalty of the local population towards the sponsor[15].

2.1 Baseline Model

Let us assume for now that the vector of ideological misalignments between both governments and their militias $\boldsymbol{\theta} = (\theta_i, \theta_j)$ is perfectly known by all players of this game. For now, I also assume γ_k to be an exogenous parameter, representing the equilibrium result of previous negotiations. I relax this assumption in the rest of the paper. Let us solve the model backward. Given the transfers, militias have to strategically decide how much to invest in fighting. They simultaneously maximize their utilities with respect to the fighting efforts (a_i, a_j) and the first order conditions for militias i and j respectively write

$$\begin{cases} \dfrac{a_j}{(a_i + a_j)^2}\Big(V + t_i + t_j - \mu\theta_i a_i - \mu\theta_j a_j\Big)\ s^m - \dfrac{a_i}{a_i + a_j}\mu\theta_i\ s^m \ = 0 \\ \dfrac{a_i}{(a_i + a_j)^2}\Big(V + t_i + t_j - \mu\theta_i a_i - \mu\theta_j a_j\Big)\ s^m - \dfrac{a_j}{a_i + a_j}\mu\theta_j\ s^m \ = 0 \end{cases}$$

where the first term represents the effect of a marginal increase in fighting effort on the increased probability of winning while the second term represents the negative impact on the final prize. Subject to the conflict technology and vector of ideological misalignments, each militia k chooses her effort in fighting a_k taking a_{-k} as given. Analyzing the best responses of the two militias, we see that there can be only one interior equilibrium where both militias actively fight[16]. Solving for the interior solution of the fighting stage, the optimal fighting efforts write

[15]Other, maybe simpler, functional forms of utilities fail to properly account for the non-military returns that sponsors receive from funding local groups. Sectarian identities are an important tool for geopolitical influence, and represent a widely exploited strategy. See, for instance, the report by the Brookings Doha Center Analysis report by G.Gause III (2014).

[16]It is possible to define the contest function $p(a_i = 0, a_j = 0) = 1/2$ where the outcome of the contest is random if none of the two militias fight actively. However, the conflict technology rules out the possibility that peace, i.e. $a_i + a_j = 0$, is a Nash equilibrium. As pointed out by Skaperdas and Syropoulos (1997), the existence of the equilibrium derives from the fact that $p_k \partial^2 p_k / \partial a_k^2 < \big(\partial p_k / \partial a_k\big)^2$ for $k = i, j$, and uniqueness of that equilibrium follows from the general characteristics of the contest function as specified above. Theorem 2 of Skaperdas and Syropoulos (1997) shows that if at least one pure-strategy equilibrium is in the interior of the strategy space, that equlibrium will be unique.

$$a_i^*(t_i,t_j) = \frac{t_i + t_j + V}{2\mu(\theta_i + \sqrt{\theta_i\theta_j})}$$
$$a_j^*(t_i,t_j) = \frac{t_i + t_j + V}{2\mu(\theta_j + \sqrt{\theta_i\theta_j})}$$

This equilibrium is characterized by fighting efforts that are always positive and that depend positively on the *sum* of resources transferred to militias i and j because a part of the resources transferred, i.e. the resources not invested in fighting but in productive activities, become subject to dispute and can be seized through fighting. Interestingly, the optimal fighting effort of militias is not directly dependent on the sharing rule of the spoils of war s^m and armed groups optimally respond to each other only taking into consideration their ideological positions and the total resources at stake in the contest. Solving backward the model, we maximize the governments' utilities given the militias' best responses. Substituting and simplifying we have,

$$\begin{cases} u_i^G(\mathbf{a}^*) = \gamma_i P + \dfrac{\theta_j}{2(\theta_j + \sqrt{\theta_i\theta_j})}\, s^g\ (t_i + t_j + V) - \omega\theta_i t_i^2 \\ u_j^G(\mathbf{a}^*) = \gamma_j P + \dfrac{\theta_i}{2(\theta_i + \sqrt{\theta_i\theta_j})}\, s^g\ (t_i + t_j + V) - \omega\theta_j t_j^2 \end{cases}$$

The problem of finding the optimal transfers is now completely decoupled between the two governments, since the strategic interaction between the two parties is fully taken into account at the fighting stage. The utilities of governments, when evaluated at the equilibrium of fighting efforts (a_i^*, a_j^*), depend additively on the transfers, and each government optimizes independently. Solving independently the two first order conditions of the the system in (t_i, t_j) we find the equilibrium values of the optimal transfers and of the optimal fighting efforts.

$$t_i^* = s^g \frac{\theta_j}{4\theta_i(\theta_j + \sqrt{\theta_i\theta_j})\omega} \qquad \textit{and} \qquad a_i^* = \frac{s^g\left(\theta_i + \theta_j - \sqrt{\theta_i\theta_j}\right) + 4\theta_i\theta_j V\omega}{8\theta_i\theta_j(\theta_i + \sqrt{\theta_i\theta_j})\mu\,\omega}$$
$$t_j^* = s^g \frac{\theta_i}{4\theta_j(\theta_i + \sqrt{\theta_i\theta_j})\omega} \qquad\qquad a_j^* = \frac{s^g\left(\theta_i + \theta_j - \sqrt{\theta_i\theta_j}\right) + 4\theta_i\theta_j V\omega}{8\theta_i\theta_j(\theta_j + \sqrt{\theta_i\theta_j})\mu\,\omega}$$

First, let us note that the equilibrium fighting efforts are always positive since $\theta_i + \theta_j > \sqrt{\theta_i\theta_j}$. Interestingly, the optimal transferred resources crucially depend on a combination of the ideological parameters and on the sharing rule of the spoils of war. The higher is the amount of the spoils that will be assigned to the governments, the more resources they transfer at equilibrium. This mechanism feeds into the optimal fighting between militias, which at optimum counter-intuitively increases with the share of the spoils going to governments because this in turn increases the contested amount of resources they try to appropriate. In other words, the higher is the share of the spoils of war that goes to governments, the higher are the optimal transfers and ultimately the fighting intensities of both groups.

Proposition 1.
In a sequential game of proxy conflict, the government most ideologically aligned to its militia transfers the highest amount of resources, its delegated militia exerts a higher fighting effort and it has a strictly higher probability of winning the contest, i.e. if $\theta_j > \theta_i$ then

$$p_i\Big(a_i^*(\theta_i,\theta_j),a_j^*(\theta_i,\theta_j)\Big) = \frac{\theta_j}{\theta_j+\sqrt{\theta_i\theta_j}} \qquad > \qquad p_j\Big(a_i^*(\theta_i,\theta_j),a_j^*(\theta_i,\theta_j)\Big) = \frac{\theta_i}{\theta_i+\sqrt{\theta_i\theta_j}}$$

Proposition 1 highlights that the main determinant of strategic decisions is the reciprocal position of militias and governments on the ideological spectrum. It crucially affects the equilibrium choices of transfers and fighting intensities and it ultimately represents the parameter influencing the probability of victory. The proof of Proposition 1 is in the Appendix[17].

2.1.1 Baseline: comparative statics

Let us conduct some comparative statics on the equilibrium transfers, on fighting intensities and on the probabilities of winning at equilibrium. The optimal transfers to the militias (t_i^*, t_j^*) are such that an increase in the misalignment between government k and its armed group decreases the amount of resources transferred at equilibrium, while an increase in the opposing party's misalignment impacts positively on government k's transfer through a net increase in its probability of winning. The following proposition studies the impact of changes in ideological

[17]The general results of the baseline model are robust to some changes in the functional form of utilities, provided that the cost of fighting of militias depends linearly on the ideological misalignment, i.e. the cost is $-\theta_k a_k$, and provided that the total prize for militia k depends on the transfer she receives t_k. Otherwise, it would not be possible to link the strategic decision of fighting by militias to the strategic transfer by sponsor states. One could think of another class of functional forms where the transfer increases the technology of fighting of militias. Even though such class of functions would yield the same qualitative dependencies between fighting efforts, ideological misalignments and transfers, it would fail to account properly for the non-military returns that sponsors receive by delegating conflict. Moreover, given the contest function, that would soon become too complex to be studied analytically.

misalignments on equilibrium transfers, fighting efforts and winning probabilities.

Proposition 2.
When the ideological misalignment θ_k between militia k and its sponsor increases, government k transfers less resources, the militia decreases its fighting and the probability of victory for party k decreases. When the ideological misalignment of the opposing party θ_{-k} increases, government k transfers more resources, the militia decreases its fighting effort less than its opponent and the probability of victory for party k increases.

We see that in a contest by proxy with complete information, the more severe is the misalignment of the militia to her government, the higher is the marginal cost of transferring resources due to the increased political cost of supporting a local armed group. A higher ideological misalignment in party k thus lowers the transfer militia k receives and disproportionately reduces the fighting effort of militia k compared to its opponent's, $\partial a_k^*/\partial\theta_k < \partial a_{-k}^*/\partial\theta_k < 0$. As a consequence, as θ_k increases party k faces a lower probability of winning the contest. In the same logic, the more misaligned is the opponent's militia to her sponsoring government, the higher is the incentive to transfer resources to his own militia and the higher is the probability of winning the war. A marginal increase in the misalignment between the opposing players θ_{-k}, brings a decrease in fighting effort a_{-k} that is in magnitude bigger than the decrease a_k, thus resulting in a higher probability of winning for government k. In this scenario, the total cost of transferring a higher amount of resources is thus more than counterbalanced by the larger expected prize from the contest, since the total transfers make up a part of the prize. When θ_{-k} increases, holding θ_k fixed, the unit cost of transferring resources is constant while the marginal expected benefit increases through the increased probability of winning.

Proposition 3.
When the marginal cost of recruiting for militias' fighters μ increases, the equilibrium amount of transferred resources does not change, while the optimal fighting efforts of both militias decrease equally. Moreover, an increase in the cost technology of transferring funds results in a decrease in total transfers and consequently in a decrease in equilibrium fighting for both militias.

An increase in the marginal cost of recruiting and mobilization due to changes in the labor market of fighters μ, e.g. a decrease of the unemployment rate[18], does not affect the optimal transfers while it impacts negatively the fighting effort of both militias. Also, an increase in the cost technology of transferring funds ω has obviously a negative impact on optimal transfers $\frac{\partial t_k}{\partial \omega} < 0$, and it also negatively impacts the fighting effort of militias through the combined effect on the transfers. For the sake of simplicity, in the rest of the paper I assume $\omega = 1$. This will not change any of the insights coming from the results, while it simplifies the exposition.

2.2 Contracts

2.2.1 Complete information

Let us now consider a situation where the control over militias' policy-making γ_k is no more an exogenous parameter, but it is determined strategically by governments. Militias know their own type and that of their opponent, and governments learn both their types perfectly. Governments now have the possibility to offer a menu of contracts conditional on the ideology position of both militias. They simultaneously offer contracts of the form

$$\Big(t_i(\theta_{i,}, \theta_j), \gamma_i(\theta_{i,}, \theta_j)\Big) \qquad \qquad and \qquad \qquad \Big(t_j(\theta_i, \theta_j), \gamma_j(\theta_{i,}, \theta_j)\Big)$$

A contract consists of a transfer $t_k(\cdot)$ and of a proposed control over policy-making $\gamma_k(\cdot)$ that are functions of the vector of ideological misalignments $\boldsymbol{\theta} = (\theta_i, \theta_j)$. Now, militias have the choice not to accept the contract. In this case, they do not receive any transfer from the government sponsor, do not enter the dispute for resources and adopt a strategy of neutrality which gives a fixed payoff of $N > P > 0$. This captures the fact that by choosing to remain neutral, militias retain full control over their political autonomy in policy-making and benefit from an additional fixed payoff deriving from not getting into fighting. I expand about the role of the outside option in the following sections of this paper. Since militias always have complete information about the ideological types, the second stage of the game is as before and the best responses of militias are

[18] See for instance Darden (2019), who highlights how worsening economic vulnerability can create an increase in terrorist groups' recruitment of youth.

$$a_i^*(t_i, t_j) = \frac{t_i(\theta_i, \theta_j) + t_j(\theta_i, \theta_j) + V}{2\mu(\theta_i + \sqrt{\theta_i\theta_j})} \qquad a_j^*(t_i, t_j) = \frac{t_i(\theta_i, \theta_j) + t_j(\theta_i, \theta_j) + V}{2\mu(\theta_j + \sqrt{\theta_i\theta_j})}$$

Substituting the best responses of militias in their utilities, the participation constraints of militias write [19]

$$\begin{cases} u_m^i(a_i^*, a_j^*) = (1 - \gamma_i(\theta_i, \theta_j))P + \dfrac{\theta_j}{2(\theta_j + \sqrt{\theta_i\theta_j})}\, s^m\, \left(t_i(\theta_i, \theta_j) + t_j(\theta_i, \theta_j) + V\right) \geq N \\ u_m^j(a_i^*, a_j^*) = (1 - \gamma_j(\theta_i, \theta_j))P + \dfrac{\theta_i}{2(\theta_i + \sqrt{\theta_i\theta_j})}\, s^m\, \left(t_i(\theta_i, \theta_j) + t_j(\theta_i, \theta_j) + V\right) \geq N \end{cases}$$

which implies that the participation constraints for the two militias can be written in terms of the proposed shared control over policy-making by the governments as

$$\begin{cases} \gamma_i(\theta_i, \theta_j) \leq 1 - \dfrac{1}{P}\left(N - \dfrac{\theta_j(t_i(\theta_i, \theta_j) + t_j(\theta_i, \theta_j) + V)}{2(\theta_j + \sqrt{\theta_i\theta_j})} s^m\right) & (PC_i) \\ \gamma_j(\theta_i, \theta_j) \leq 1 - \dfrac{1}{P}\left(N - \dfrac{\theta_i(t_i(\theta_i, \theta_j) + t_j(\theta_i, \theta_j) + V)}{2(\theta_i + \sqrt{\theta_i\theta_j})} s^m\right) & (PC_j) \end{cases}$$

The principals' programs in compact form are

$$\max_{\gamma_k, t_k} u_G^k \;=\; \max_{\gamma_k, t_k} \gamma_k(\theta_i, \theta_j)P + \frac{\theta_{-k}\Big(t_k(\theta_i, \theta_j) + t_{-k}(\theta_i, \theta_j) + V\Big)}{2(\theta_{-k} + \sqrt{\theta_k\theta_{-k}})}\, s^g - \omega\theta_k t_k(\theta_i, \theta_j)^2$$

subject to

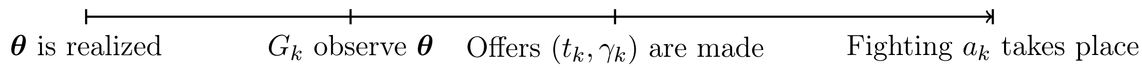
$$(1 - \gamma_k)P + \frac{\theta_{-k}\Big(t_k(\theta_i, \theta_j) + t_{-k}(\theta_i, \theta_j) + V\Big)}{2(\theta_{-k} + \sqrt{\theta_k\theta_{-k}})}\, s^m \geq N$$

for $k = i, j$.

We can rewrite the programs of the principals only in terms of the transfers when the participation constraints are binding. In fact, with complete information the participation constraint must bind at equilibrium because the governments are able to extract all rents of militias by

[19] In the next section of the paper, I relax the assumption on the identical values of the outside option for both militias.

offering a combination $\big(t_k(\theta_i,\theta_j),\gamma_k(\theta_i,\theta_j)\big)$ to keep them slightly above the indifference point between accepting the contract and staying neutral. In this setting there is no uncertainty, so both governments maximize their utilities given $\boldsymbol{\theta}=(\theta_i,\theta_j)$ and the participation constraints. The game unfolds as below for both parties



When the constraint binds, and taking into account the fact that $s^g+s^m=1$ the utility of the governments is

$$u_G^k = P - N + \frac{\theta_{-k}}{2(\theta_{-k}+\sqrt{\theta_k\theta_{-k}})}\Big(t_k(\theta_i,\theta_j)+t_{-k}(\theta_i,\theta_j)+V\Big) - \omega\theta_k t_k(\theta_i,\theta_j)^2$$

and the first order condition in complete information writes

$$\frac{\partial u_G^k}{\partial t_k} = \frac{\theta_{-k}}{2(\theta_{-k}+\sqrt{\theta_k\theta_{-k}})} - 2\omega\theta_k t_k(\theta_i,\theta_j) = 0$$

We see that the strategic interaction between the two parties enters the game only in the second stage through the best responses of militias to one another. When governments optimize over their transfers, they internalize the reaction of militias to one another which depend on the vector of types $\boldsymbol{\theta}=(\theta_i,\theta_j)$ and on the transfers $\mathbf{t}=(t_i(\theta_i,\theta_j),t_j(\theta_i,\theta_j))$.

Proposition 4 (CI)**.**
The first best contracts when the both types of militias are perfectly observed are

$$\begin{cases} t_i^* &= \dfrac{\theta_j}{4\theta_i(\theta_j+\sqrt{\theta_i\theta_j})\omega} \\ \gamma_i^* &= 1-\dfrac{1}{P}\Bigg[N-\dfrac{s^m}{8\theta_i\omega}\Big(\dfrac{\theta_i+\theta_j+4\theta_i\theta_jV-\sqrt{\theta_i\theta_j}}{\theta_j+\sqrt{\theta_i\theta_j}}\Big)\Bigg] \end{cases}$$

and for party j

$$\begin{cases} t_j^* &= \dfrac{\theta_i}{4\theta_j(\theta_i + \sqrt{\theta_i\theta_j})\omega} \\ \gamma_j^* &= 1 - \dfrac{1}{P}\Bigg[N - \dfrac{s^m}{8\theta_j\omega}\Big(\dfrac{\theta_i + \theta_j + 4\theta_i\theta_j V - \sqrt{\theta_i\theta_j}}{\theta_i + \sqrt{\theta_i\theta_j}}\Big)\Bigg] \end{cases}$$

Two observations are in order. First, the amount transferred when militias have the outside option of remaining neutral and γ_k are contracting variables, is higher than the amount transferred when there is no participation constraint and γ_k are exogenous parameters. Second, the demanded share of political control by governments γ_k is highest for the most ideologically aligned militia. Overall, the government ideologically closest to its militia offers at equilibrium a higher monetary transfer than the opponent but also demands a higher share of political power. The value of the outside option N, enters the optimal contract only in the consideration of how much control of policymaking sponsor governments are willing to extract from militias. In the limit of $N \to 0$, it is easy to verify that below a certain threshold on N, militias always accept the contract even if governments take fully control over militias' policymaking.

2.2.2 Incomplete information on the opposing militia's ideology

Let us suppose now that the misalignment θ_k between each government k and its proxy is private information to government k. Consequently, governments ignore the ideological misalignment between the enemy government and its militia. The following analysis always assumes that militias perfectly know the ideological positions of all actors of the game, implying that the fighting stage of the game is unaffected and asymmetric information plays a role only in the strategic decisions of governments. This is a reasonable assumption, as armed groups are deeply rooted in the territory, often share a common history and have interacted before the delegated contest takes place. Also, I assume the ideological distances to be distributed uniformly $(0, 1]$. The programs of governments are then

$$\begin{aligned} \max_{t_k,\gamma_k} \mathbb{E}\, u_G^k(\mathbf{t}, \gamma_k|\theta_k) &= \max_{t_k,\gamma_k} \int_{\boldsymbol{\theta_{-k}}} u_G^k(\mathbf{t}, \gamma_k|\theta_k)\, d\theta_{-k} \\ &= \max_{t_k,\gamma_k} \int_{\boldsymbol{\theta_{-k}}} \gamma_k(\theta_k)P + \frac{\theta_{-k}}{2(\theta_{-k} + \sqrt{\theta_k\theta_{-k}})}\Big(t_k(\theta_k) + t_{-k}(\theta_{-k}) + V\Big)s^g - \omega\theta_k t_k^2 \;\, d\theta_{-k} \end{aligned}$$

subject to

$$(1-\gamma_k(\theta_k))P + \frac{\theta_{-k}}{2(\theta_{-k}+\sqrt{\theta_k\theta_{-k}})}\Big(t_k(\theta_k)+t_{-k}(\theta_{-k})+V\Big)s^m \geq N$$

for $k = i, j$.

The amount of information in possession of the two principals now is quite different. It is useful to draw a time line to visualize how the game unfolds. We have that

$\boldsymbol{\theta}$ is realized | G_k observes only θ_k | Offers (t_k, γ_k) are made | Fighting a_k takes place

Let us solve government i's program since j's will be perfectly symmetric. The optimal transfer for government k now can only depend explicitly on θ_k, since it does not receive any signal on the misalignment of the opponents. At the level of the militias' best responses

$$a_i^*(t_i, t_j) = \frac{t_i(\theta_i)+t_j(\theta_j)+V}{2\mu(\theta_i+\sqrt{\theta_i\theta_j})} \qquad a_j^*(t_i, t_j) = \frac{t_i(\theta_i)+t_j(\theta_j)+V}{2\mu(\theta_j+\sqrt{\theta_i\theta_j})}$$

We take into consideration the participation constraints of the two militias and we set them both to be binding. Governments are able to fully exploit the perfect knowledge of the type of their own militia to offer a combination of $\big(t_k(\theta_k), \gamma_k(\theta_k)\big)$ barely sufficient to convince her to accept the contract. Substituting into government's k utility function, the optimization program of governments is reduced to a problem in one variable, thus considerably reducing its complexity. The utility of government k writes,

$$u_G^k(\mathbf{t}, |\theta_k) = P - N + \frac{\theta_{-k}}{2(\theta_{-k}+\sqrt{\theta_k\theta_{-k}})}\Big(t_k(\theta_k)+t_{-k}(\theta_{-k})+V\Big) - \omega\theta_k t_k(\theta_k)^2$$

Taking the expectation over the opponent's type θ_{-k} and factoring out constant terms

$$\begin{aligned}\mathbb{E}u_G^k(\mathbf{t}, |\theta_k) = &\int_0^1 P - N + \frac{\theta_{-k}}{2(\theta_{-k}+\sqrt{\theta_k\theta_{-k}})}\Big(t_k(\theta_k)+t_{-k}(\theta_{-k})+V\Big) - \omega\theta_k t_k(\theta_k)^2 \, d\theta_{-k} \\ =&P - N + \int_0^1 \frac{\theta_{-k}}{2(\theta_{-k}+\sqrt{\theta_k\theta_{-k}})}\Big(t_k(\theta_k)+t_{-k}(\theta_{-k})+V\Big) - \omega\theta_k t_k(\theta_k)^2 \, d\theta_{-k}\end{aligned}$$

The first order condition writes

$$\int_0^1 \frac{\theta_{-k}}{2(\theta_{-k} + \sqrt{\theta_k \theta_{-k}})} - 2\omega\theta_k \; t_k(\theta_k) \; d\theta_{-k} = 0$$

where we have simply taken the derivative of the integrand with respect to the transfer t_k and set it equal to zero. Solving the integral we obtain the first best transfers offered by government k, when the opponent's ideology is unknown. This scenario is quite realistic and it implicitly assumes that the transparent communication between government k and its militia k contributes to the perfect knowledge of the misalignment parameter, while keeping some uncertainty on that of their opponents. While it is possible that militia k could communicate to its sponsor the ideological position of the opposing militia, that information could be discarded by government k as not credible. The proof of Proposition 3 is in the appendix.

Proposition 5 (IN_{-k})**.**
The first best transfers when each government k observes perfectly the type of his its own militia θ_k but does not observe the opponent's, θ_{-k}, is characterized by

$$\begin{cases} t_i^{*,IN} = \dfrac{1}{2\omega}\left[log\left(\dfrac{1+\sqrt{\theta_i}}{\sqrt{\theta_i}}\right) - \dfrac{2\sqrt{\theta_i}-1}{2\theta_i}\right] \\ \\ t_j^{*,IN} = \dfrac{1}{2\omega}\left[log\left(\dfrac{1+\sqrt{\theta_j}}{\sqrt{\theta_j}}\right) - \dfrac{2\sqrt{\theta_j}-1}{2\theta_j}\right] \end{cases}$$

Let us compare the optimal transfers in the incomplete information setting with the complete information. For the sake of exposition, I denote by CI the setting where the ideological positions of militias are publicly observed and all actors play a game of complete information. Moreover, I denote by IN_{-k} a situation where government k only observes the ideological type of its own delegate θ_k and holds incomplete information about that of the opponent θ_{-k}.

Proposition 6.
There exists a threshold $\widehat{\theta_j}$ such that if $\theta_j > \widehat{\theta_j}$, militia i receives a higher transfer in complete information than in incomplete information about the opposing militia's misalignment. Conversely if $\theta_j < \widehat{\theta_j}$, militia i receives a higher transfer in incomplete information than in complete information. More formally,

$$\begin{cases} t_i^{IN} > t_i^{CI} & \textit{if} \quad \theta_j < \widehat{\theta_j} \\ \\ t_i^{CI} > t_i^{IN} & \textit{if} \quad \theta_j > \widehat{\theta_j} \end{cases}$$

where $\widehat{\theta_j} = \dfrac{\left(1 + 2\theta_i log\left(\dfrac{\sqrt{\theta_i}+1}{\sqrt{\theta_i}}\right) - 2\sqrt{\theta_i}\right)^2}{4\left(\sqrt{\theta_i} log\left(\dfrac{\sqrt{\theta_i}+1}{\sqrt{\theta_i}}\right) - 1\right)^2}$. *The symmetric result applies for the opponent.*

Proposition 4 formalizes the fact that in incomplete information government i takes an average of the possible values of θ_j when computing the optimal transfer to its own militia. So, since t_i^{CI} is increasing and concave in θ_j and t_i^{IN-k} constant in θ_j, there must be a value of θ_j for which the two are equal. This is particularly interesting, because it means that if militia i is responsible of communicating to her sponsor the true type of militia j, it would indeed do so only if militia j's type is high, thus putting the government in a setting of complete information. In this way, militia i would be able to receive a higher transfer. On the contrary, if the true type of militia j were lower than the $\widehat{\theta_j}$, militia i would have no interest of communicating it to her sponsor and would prefer to leave her government in a setting of incomplete information[20].

2.2.3 Incomplete information on the ideology of both militias: second best contracts

I now study a situation where governments do not perfectly observe the ideological type of their proxies and militias have the possibility to misreport their ideology to their governmental sponsors. In this setting it is not clear a priori whether militias have an incentive to report a lower misalignment in order to get a higher transfer and possibly get a lower share of political power or report a higher misalignment and retain a higher political independence. Thus, militias are not only strategic in their mutual fighting effort a_k but also in the revelation of the ideological type vis à vis their respective governments. I always assume that militias perfectly know each others' ideology position. This is a reasonable assumption considering these armed

[20]This reasoning would apply only if we assume that militias cannot misreport the opposing militia's ideology and only if government i cannot strategically draw information on the opposing militia's ideology from the decision of militia i whether to reveal or not j's type.

groups share the same territory, language, history and share a good deal of informal contacts and networks. The revelation principle is valid in this context, hence we can focus our attention to direct revelation mechanisms of the form $\{\ t(\widetilde{\theta_k}), \gamma(\widetilde{\theta_k})\ \}$, whereby governments are able to incentivize their militias to reveal their true ideology. These mechanisms are then truthful, i.e., such that

$$u_m^k(\theta_k, \theta_{-k}) \geq u_m^k(\widetilde{\theta_k}, \theta_{-k})$$

which implies that

$$\Big(1-\gamma_k(\theta_k)\Big)P+C(\theta_k,\theta_{-k})\Big(t_k(\theta_k)+t_{-k}(\theta_{-k})+V\Big) \geq \Big(1-\gamma_k(\widetilde{\theta_k})\Big)P+C(\theta_k,\theta_{-k})\Big(t_k(\widetilde{\theta_k})+t_{-k}(\theta_{-k})+V\Big)$$

for any $(\theta_k, \widetilde{\theta_k}) \in (0,1]^2$. In particular, for milia k this implies that

$$\Big(1-\gamma_k(\theta_k)\Big)P+C(\theta_k,\theta_{-k})\Big(t_k(\theta_k)+t_{-k}(\theta_{-k})+V\Big) \geq \Big(1-\gamma_k(\theta_k')\Big)P+C(\theta_k,\theta_{-k})\Big(t_k(\theta_k')+t_{-k}(\theta_{-k})+V\Big)$$

and similarly

$$\Big(1-\gamma_k(\theta_k')\Big)P+C(\theta_k',\theta_{-k})\Big(t_k(\theta_k')+t_{-k}(\theta_{-k})+V\Big) \geq \Big(1-\gamma_k(\theta_k)\Big)P+C(\theta_k',\theta_{-k})\Big(t_k(\theta_k)+t_{-k}(\theta_{-k})+V\Big)$$

for all pairs $(\theta_k, \theta_k') \in (0,1]^2$ where $C(\theta_k, \theta_{-k}) = \dfrac{\theta_{-k}}{2(\theta_{-k}+\sqrt{\theta_k\theta_{-k}})}s^m$ and $C(\theta_k', \theta_{-k}) = \dfrac{\theta_{-k}}{2(\theta_{-k}+\sqrt{\theta_k'\theta_{-k}})}s^m$. Adding the last two inequalities we obtain the monotonicity constraint

$$\Big(C(\theta_k, \theta_{-k}) - C(\theta_k', \theta_{-k})\Big)(t_k(\theta_k) - t_k(\theta_k')) \geq 0$$

Incentive compatibility alone requires that the schedule of transfers should be non-increasing in θ_k. The fact that we can restrict our attention to direct revelation mechanisms that are truthful implies that the following first-order condition must hold for every $\theta_k \in (0,1]$

$$\Big(1-\dot{\gamma_k}(\theta_k)\Big)P + C(\theta_k, \theta_{-k})\Big(\dot{t_k}(\theta_k) + t_{-k}(\theta_{-k}) + V\Big) = 0$$

since announcing its true ideology θ_k is an optimal response for militia $k = i, j$ (revelation principle). Now, thanks to the envelope theorem and the condition above, the local incentive constraint can be written

$$\dot{u}^k_m(\theta_k, \theta_{-k}) = \dot{C}(\theta_k, \theta_{-k})\Big(t_k(\theta_k) + t_{-k}(\theta_{-k}) + V\Big)$$

Integrating both sides of the last equation yields

$$u^k_m(\overline{\theta_k}, \theta_{-k}) - u^k_m(\theta_k, \theta_{-k}) = \int_{\theta_k}^{\overline{\theta_k}} \dot{C}(\theta_k, \theta_{-k})\Big(t_k(\theta_k) + t_{-k}(\theta_{-k}) + V\Big)\ d\theta_k$$

or

$$\begin{aligned} u^k_m(\theta_k, \theta_{-k}) &= u^k_m(\overline{\theta_k}, \theta_{-k}) - \int_{\theta_k}^{\overline{\theta_k}} \dot{C}(\theta_k, \theta_{-k})\Big(t_k(\theta_k) + t_{-k}(\theta_{-k}) + V\Big)\ d\theta_k \\ &= N - \int_{\theta_k}^{\overline{\theta_k}} \dot{C}(\theta_k, \theta_{-k})\Big(t_k(\theta_k) + t_{-k}(\theta_{-k}) + V\Big)\ d\theta_k \\ &= N + \int_{\theta_k}^{\overline{\theta_k}} \frac{\theta_{-k}^2}{2\sqrt{\theta_k\theta_{-k}}(\theta_{-k} + \sqrt{\theta_k\theta_{-k}})^2}\Big(t_k(\theta_k) + t_{-k}(\theta_{-k}) + V\Big)s^m\ d\theta_k \end{aligned}$$

since incentive compatibility implies that only the participation constraint of the most inefficient type can be binding. Because the principals wants to minimize the militias' rents, at least one participation constraint must be binding (otherwise, the principal could decrease all rents uniformly without affecting neither the incentive constraints nor the monotonicity requirement). The participation constraint must be binding for $\theta_k = 1$ and every type of militia is willing to participate whenever the worst type is willing to do so. We also know that the shared control over policymaking can be written as

$$\gamma_k(\theta_k) = 1 - \frac{1}{P}\left(u^k_m(\theta_k) - \int_{\boldsymbol{\theta_{-k}}} C(\theta_k, \theta_{-k})\big(t_k(\theta_k) + t_{-k}(\theta_{-k}) + V\big)\ d\theta_{-k}\right)$$

and I use this expression to write the optimal $\gamma_k(\theta_j)^{SB}$ that takes into consideration the local incentive constraint and the optimal transfers at the second best optimum t_k^{SB} and t_{-k}^{SB}. Writing the program of governments in terms of the rents of militias u^k_m and taking the expectation over θ_{-k}, the program of governments writes

$$\max_{\{(t_k, u^k_m)\}} \int_{\boldsymbol{\theta_{-k}}} \int_{\boldsymbol{\theta_k}} P + \frac{\theta_{-k}}{2(\theta_{-k} + \sqrt{\theta_k\theta_{-k}})}\Big(t_k(\theta_k) + t_{-k}(\theta_{-k}) + V\Big) - \omega\theta_k t_k^2(\theta_k) - u^k_m(\theta_k)\ \ f(\theta_{-k})f(\theta_k)d\theta_{-k}\ d\theta_k$$

subject to

$$\begin{cases} \dot{u}(\theta_k, \theta_{-k}) = & \dot{C}(\theta_k, \theta_{-k})\Big(t_k(\theta_k) + t_{-k}(\theta_{-k}) + V\Big) \\ \\ \dot{t}(\theta_k) \leq & 0 \\ \\ u_m^k(\theta_k) \geq & N \end{cases}$$

The first constraint is the local incentive constraint, the second is the monotonicity constraint and the third is the participation constraint of militias. Since it is easy to see that the Spence-Mirlees property holds with a negative sign, local incentive constraints imply the global incentive constraints and the optimization program of governments is well defined as above. The monotone hazard rate is also respected since

$$\frac{\partial}{\partial \theta_k} \frac{1 - F(\theta_k)}{f(\theta_k)} = \frac{\partial}{\partial \theta_k}(1 - \theta_k) \leq 0$$

The Proposition below characterizes the second best contracts.

Proposition 7.
When government k holds incomplete information on its own militia's ideology it offers a menu of contracts $\{\big(t_k(\theta_k), \gamma_k(\theta_k)\big)\}$ *such that*

- *There is no distortion for the most misaligned militia.*
- *The second best transfers are* $t_k^{SB} = t_k^{IN_{-k}} - \frac{s^m}{4\theta_k \omega}\left(1 + \frac{(1-2\theta_k)}{\sqrt{\theta_k}} - (1-\theta_k) log\Big(\frac{(1+\sqrt{\theta_k})^2}{\sqrt{\theta_k}}\Big)\right)$

Each militia θ_k picks the contract that is designed for her by revealing her true type, and there is no bunching of types, since the monotonicity constraint is always satisfied. The governments transfer a sub efficient quantity of resources to militias of all ideologies, except of the most extreme ones, thus reducing the fighting efforts of both proxy militias. Governments have an incentive to offer this menu of contracts because they want to decrease the incentive that militias have to mimic less misaligned armed groups in order to receive a higher transfer. Moreover, t_k^{SB}

is clearly decreasing, all types choose therefore different allocations and there is no bunching in the optimal contract. It is also interesting to note that, while the share of the spoils of war did not enter the optimal contract IN_{-k}, the second best solution depends negatively on it. The higher is the share of the spoils of war supposed to go the armed groups, the lower would the transfer be at the second best optimum. Because of the incentive constraint, s^m enters into the optimal contract for all ideological types except for the most misaligned militiia.

Proposition 8.
When government k holds incomplete information on its own militia's ideology, it transfers less resources compared to a situation where it perfectly knows its militia's ideology, but it leaves her a higher share of political power, i.e. the second best contract is such that

$$\begin{cases} t_k^{SB}(\theta_k) & \leq t_k^{IN_{-k}}(\theta_k) \\ \\ 1-\gamma_k^{SB}(\theta_k) & \geq 1-\gamma_k^{IN_{-k}}(\theta_k) \end{cases}$$

Compared to the setting of full information on their own delegated armed groups, asymmetric information alters the governments' optimization problem by the subtraction of the expected rent that has to be given to militias. We note that there is no distortion for the most misaligned militia – since the hazard rate $\frac{1-F(\theta_k)}{f(\theta_k)} = (1-\theta_k)$ equals 0 when $\theta_k = 1$ – and a downward distortion for all the other types. At the optimal contract, governments have no incentive to increase the transferred resources to $t_k^{SB}(\theta_k)+\epsilon$ because, even though it would bring an increase in total surplus through type θ_k, they would have to give higher information rents to all types $\theta_k^{'} < \theta_k$. Thus, all types have a positive information rent except the most aligned type of militia. Governments *could* implement the first best and offer instead $\{\big(t_k(\theta_k)^{IN_{-k}}, \gamma_k(\theta_k)(\theta_k)^{IN_{-k}}\big)\}$, since it also respects the monotonicity requirement. However, it is not optimal for principals to do so, because the expected rent she would have to pay is greater: while the rent to a type $\theta_k = 0$ is zero, the rent to any higher type buyer is determined by the quantity assigned to its strictly lower type, and increases with that quantity. Hence, it is optimal for governments to depart from the first best IN_{-k} and reduce the quantity assigned to a every type $\theta_k < 1$, so as to save on the rent left to all higher types.

3 Strategic Delegation of War

The involvement of governments in conflicts can be of two general types: either direct intervention or indirect engagement through third parties. We now turn to studying the strategic decision of governments about whether or not to delegate conflict when the ideological types of both available militias are public information. This situation can be represented as a normal form game, where the action space of government k is $I_k = (D, ND)$, i.e. governments choose whether to delegate (D) or not to delegate conflict (ND). We also assume, as before, that governments fully commit to the offered contracts. When governments enter conflict directly, they avoid the political and monetary cost of transferring resources to local proxies $-\omega\theta_k t_k^2$ while renouncing to capturing a share of political influence over the armed group's policymaking $\gamma_k P$. In this case, government k directly exerts a fighting effort that influences both the probability of winning $\frac{a_k}{a_k+a_{-k}}$ and the total prize. The outside options of militias to remain neutral can be decomposed, for the sake of exposition, as $N = P + R$, representing the fact that militias that do not fight enjoy support by local populations R [21] and enjoy full independence in the formulation and implementation of policymaking P. In this framework, the governments' strategic considerations of whether to delegate conflict to their militias are twofold. First, they have to decide whether delegation is payoff maximizing given the opposing government's type of involvement into the conflict – delegated or direct. Secondly, they have to either optimize over the contracting variables (t_k, γ_k) or to optimally choose the fighting efforts a_{G_k}, for every given combination of the action space $I = I_i \times I_j = (D, ND)^2$. This situation can be represented in a normal form as

		Government i	
		D	ND
Government j	D	$u_i^G(D,D), u_j^G(D,D)$	$u_i^G(ND,D), u_j^G(ND,D)$
	ND	$u_i^G(D,ND), u_j^G(D,ND)$	$u_i^G(ND,ND), u_j^G(ND,ND)$

where $u_k^G(\cdot,\cdot)$ is the payoff government k gets given the optimal contracts offered to the militias and their mutual best responses in fighting efforts. Governments engaging directly in conflict do not invest in productive activities locally and face a unit cost of fighting -1 that is higher than that of militias $-\mu\theta_k$. This is a reasonable assumption given that the mobilization of a regular army is an extremely expensive operation – including transportation, provisioning and

[21] Support can either be material, e.g. financial resources and food, or non material, e.g. shelter and medical assistance to militia members

arming – while local militias are already located in the territory, can recruit fighters at a low cost and generally employ a lighter type of warfare. In what follows, I assume $\mu = 1$ and $\omega = 1$ for the sake of expositional simplicity. Considering more general values does not change the fundamental characterization of equilibria of the game.

We have now to specify the payoffs of governments for all four possible combinations of actions. Let us focus on government i since the payoffs of j are symmetric. When both governments delegate, i.e. the action profile is (D, D) we resume the results we found in the complete information setting. In this case, militias optimally respond to each other choosing

$$\begin{cases} a_i^*(D,D) = \dfrac{t_i + t_j + V}{2(\theta_i + \sqrt{\theta_i \theta_j})} \\ a_j^*(D,D) = \dfrac{t_i + t_j + V}{2(\theta_j + \sqrt{\theta_i \theta_j})} \end{cases}$$

which implies that the optimal transfers are

$$\begin{cases} t_i^*(D,D) = \dfrac{\theta_j}{4\theta_i(\theta_j + \sqrt{\theta_i \theta_j})} \\ t_j^*(D,D) = \dfrac{\theta_i}{4\theta_j(\theta_i + \sqrt{\theta_i \theta_j})} \end{cases}$$

Substituting in government's i payoff we obtain at equilibrium

$$u_i^G(D,D) = \frac{\frac{\theta_j^2}{\theta_i} + 8\theta_j V \left(\sqrt{\theta_i\theta_j} + \theta_j\right) + 2\sqrt{\theta_i\theta_j}}{16\left(\sqrt{\theta_i\theta_j} + \theta_j\right)^2} - R$$

where $R = N - P$. When computing $u_i^G(ND, D)$, it is necessary to be a little more careful. In this case, government i enters conflict with its proper means and human resources, does not transfer resources to third party combatants and fights against the militia sponsored by G_j. Militia i is not involved anymore and we denote by $a_{G_i}^*$ the optimal fighting effort by government i. At the fighting stage, governments i maximizes

$$u_i^G(ND,D) = \frac{a_i}{a_i + a_j}(V + t_j - \theta_j a_j - a_i)$$

where government i does not have to share the spoils of war with its delegated armed group, since it enters conflict directly. Militia j optimizes [22]

$$u_m^j(ND,D) = (1-\gamma_j)P + \frac{a_j}{a_i+a_j}(V + t_j - \theta_j a_j - a_i)s^m$$

The equilibrium of the fighting stage implies that

$$\begin{cases} a^*_{g_i}(ND,D) = \dfrac{t_j+V}{2(1+\sqrt{\theta_j})} \\ a^*_j(ND,D) = \dfrac{t_j+V}{2(\theta_j+\sqrt{\theta_j})} \end{cases}$$

and the utility of government i writes

$$u_i^G(ND,D) = \frac{\sqrt{\theta_j}}{2(1+\sqrt{\theta_j})}\Big(V+t_j\Big)$$

The same reasoning applied in the previous section implies that government j offers γ_j in a way that the participation constraint always binds and

$$\gamma_j^*(ND,D) = 1 - \frac{1}{P}\Big(N - \frac{V+t_j}{2(1+\sqrt{\theta_j})}\, s^m\Big)$$

When the participation constraint binds, and taking into account the fact that $s^m + s^g = 1$ the utility of government j writes

$$u_j^G(ND,D) = P - N + \frac{V+t_j}{2(1+\sqrt{\theta_j})} - \theta_j t_j^2 = -R + \frac{V+t_j}{2(1+\sqrt{\theta_j})} - \theta_j t_j^2$$

from which we can compute the optimal transfer of government j,

$$t_j^*(ND,D) = \frac{1}{4\theta_j(1+\sqrt{\theta_j})}$$

Substituting the equilibrium values in the utility of government i we find

[22]It is possible to envisage other ways of representing the cost of fighting incurred by government i, for instance writing $u_i^G(ND,D) = \frac{a_i}{a_i+a_j}(V+t_j-\theta_j a_j) - a_i$. This would slightly change the optimal fighting efforts and the optimal contract, but it would not change the characterization of the equilibria presented in this section of the paper.

$$u_i^G(ND,D) = \frac{1+4(1+\sqrt{\theta_j})\theta_j V}{8(1+\sqrt{\theta_j})^2\sqrt{\theta_j}}$$

Following the same reasoning for the reverse situation, where government i delegates and government j enters directly in the conflict we obtain

$$\begin{cases} a_i^*(D,ND) = \dfrac{t_i+V}{2(\theta_i+\sqrt{\theta_i})} \\ a_{g_j}^*(D,ND) = \dfrac{t_i+V}{2(1+\sqrt{\theta_i})} \end{cases}$$

and the participation constraint for militia i is

$$\gamma_i^* = 1 - \frac{1}{P}\Big(N - \frac{V+t_i}{2(1+\sqrt{\theta_i})}s^m\Big)$$

When the participation constraint binds, government i is able to put its delegated armed group's utility down to its reservation value N_i, the utility of government i is then

$$u_i^G(D,ND) = -R + \frac{V+t_i}{2(1+\sqrt{\theta_i})} - \theta_i t_i^2$$

From this expression, we can write the first order conditions and compute the optimal transfer

$$t_i^*(D,ND) = \frac{1}{4\theta_i(1+\sqrt{\theta_i})}$$

Substituting the equilibrium values in the utility of government i we find

$$u_i^G(D,ND) = \frac{1+8(1+\sqrt{\theta_i})\theta_i V}{16(1+\sqrt{\theta_i})^2\theta_i} - R$$

Finally, when both countries do not delegate and fight each other directly we go back to the classical model of conflict between two states without delegation. In this simple situation, governments only choose their fighting effort. The functional form of the utilities of governments is now identical. For government $k=i,j$ it writes $u_k^G(ND,ND) = \frac{a_k}{a_k+a_{-k}}(V-a_k-a_{-k})$ [23].The two competing governments exert identical fighting efforts

[23] As before, we could write the utilities of governments using different specifications, e.g. $u_k^G(N,N) = \frac{a_k}{a_k+a_{-k}}V - a_k$. The optimal fighting efforts would remain unchanged as well as the results of this section.

$$a^*_{G_i} = a^*_{G_j} = \frac{V}{4}$$

Substituting in the utility of government i we obtain

$$u^G_i(ND, ND) = u^G_j(ND, ND) = \frac{V}{4}$$

Having computed the payoffs of government $k = i, j$ for all strategy profiles, we can study what are the conditions under which government $k = i, j$ has profitable deviations, given the strategy of the opponent. In particular, we are seeking the conditions under which government i has an incentive to delegate given that government j delegates as well, and the conditions under which it prefers to enter conflict directly given that the opponent does the same. The symmetric reasoning applies to government j.

In order to illustrate the mechanism, let us first consider the strategic decision of government i to delegate conflict or not, given that government j delegates: we focus on the payoffs $u^G_i(D, D)$ and $u^G_i(ND, D)$. We observe that $u^G_i(D, D)$ is linearly decreasing in R while $u^G_i(ND, D)$ does not depend on R. Moreover, when $R = 0$ we see that $u^G_i(D, D) > u^G_i(ND, D)$, implying that there exists a threshold of R, such that for values of R smaller than that threshold, it is profitable for government i to deviate from ND and choose D instead, given that the opponent also delegates. Similarly, we see that $u^G_i(D, ND)$ is linearly decreasing in R while $u^G_i(ND, ND)$ is constant. Also, in $R = 0$ it is easy to see that $u^G_i(D, ND) > u^G_i(ND, ND)$. This implies that there exists another threshold of R such that for values larger than the threshold, government i has an incentive to deviate from D and choose ND, given that the opponent also does not delegate and enters conflict directly. We can thus define for government i two thresholds on R, λ^N_i and λ^D_i, where λ^N_i represents the minimum value of R for which i's dominant strategy is to enter conflict directly – choosing ND – given that j also does not delegate. Similarly, λ^D_i is the maximum value of R such that i's dominant strategy is to choose D when also the opponent delegates. A symmetrically analysis applies for government j. Once defined the thresholds, I can compute them by setting $u^G_i(D, D) = u^G_i(ND, D)$ to compute λ^D_i and by setting $u^G_i(D, ND) = u^G_i(ND, ND)$ to compute λ^N_i. The following Lemma establishes the order of the thresholds λ^N_k and λ^D_k for $k \in \{i, j\}$. In what follows, we assume that the value of the contest prize V is large, $V >> 0$. This is a quite natural assumption since we are considering a situation where countries engage in war to capture the resources, be it natural or geopolitical, of an external territory. It just means that for both parties the stakes are high, which is consistent with the subject and the multidisciplinary literature on conflict.

Lemma 1 (Order of Thresholds).

Let the value of the contested resources V be large. Then

$$\begin{cases} \lambda_j^D > \lambda_j^N > \lambda_i^N > \lambda_i^D & \textit{if} \quad \theta_i > \theta_j \\ \\ \lambda_i^D > \lambda_i^N > \lambda_j^N > \lambda_j^D & \textit{if} \quad \theta_j > \theta_i \\ \\ \lambda_i^N = \lambda_j^N > \lambda_i^D = \lambda_j^D & \textit{if} \quad \theta_i = \theta_j \end{cases}$$

This Lemma just says that, given the respective order of ideological misalignments, the order of the thresholds on the value of R defines disjoint intervals for each party. It defines the intervals on R such that government k has a dominant strategy to delegate if the opponent delegates and not to delegate if the opponent doe not delegate as well. The respective order of λ_k^N and λ_k^D is different for parties i and j if $\theta_i \neq \theta_j$ where the most misaligned party enjoys the most intuitive $\lambda^N > \lambda^D$. Moreover, if parties are characterized by the same ideological misalignment, the fours threshold values collapse to only two. It interesting to notice that in this case, the most natural order of $\lambda^N > \lambda^D$ is restored for both parties. It is interesting to note that in the special case when $\theta_i = \theta_j = 1$, the two thresholds further collapse into only one, i.e. $\lambda^D = \lambda^N = \lambda$ and we obtain only two possible equilibria, depending on the value of R. For $R > \lambda$ the unique equilibrium is characterized by both governments entering conflict directly and for $R < \lambda$ both delegate fighting. The proof is in the Appendix. I use this Lemma to prove Proposition 9, which gives a characterization of pure strategy Nash equilibria in relation to the thresholds λ_k^D and λ_k^N for $k = i, j$. In case $\theta_i = \theta_j$, the four thresholds defined above collapse to only two, which we denote by λ^D and λ^N.

Proposition 9.

There exist multiple equilibria of the delegation game, depending on the relative ideological misalignments θ_k and the value of local support to militias R.

- *When $\theta_i > \theta_j$*
  - *If $R < \lambda_i^D$ the unique equilibrium is (D,D)*
  - *If $\lambda_i^D < R < \lambda_j^N$ the unique equilibrium is (ND,D)*

- *If $R > \lambda_j^N$ the unique equilibrium is (N,N)*

- *When $\theta_j > \theta_i$*
  - *If $R < \lambda_j^D$ the unique equilibrium is (D,D)*
  - *If $\lambda_j^D < R < \lambda_i^N$ the unique equilibrium is (D,ND)*
  - *If $R > \lambda_i^N$ the unique equilibrium is (N,N)*

- *When $\theta_i = \theta_j$*
  - *If $R < \lambda^D$ the unique equilibrium is (D,D)*
  - *If $\lambda^D < R < \lambda^N$ there exist two equilibria (ND,D) and (D,ND)*
  - *If $R > \lambda^N$ the unique equilibrium is (N,N)*

The proposition shows that the ex-ante strength of local support to militias R_k is an important parameter characterizing the strategic delegation game. In general, a weak support from the local population implies armed groups are at the fringe of society, and for government sponsors it is cost effective to recruit militias and engage in indirect conflict. On the contrary, militias that are endowed with strong local support demand wider political autonomy and higher transfers of resources from the sponsor, hence making direct confrontation optimal for the external government. More in particular, the respective ideological misalignment is crucial in the characterization of equilibria where one government delegates fighting and the opponent enters conflict directly. For intermediate values of R, only the most aligned government delegates fighting at equilibrium while the opponent does not. Only the relative position between θ_i and θ_j matters. The unicity of equilibria is lost when $\theta_i = \theta_j$ and for intermediate values of R.

4 Competing for a Common Militia

Governments often compete with one another to "hire" a local armed group whose task is to fight a third party to seize resources or exert geopolitical influence. Consider for instance a situation where the governments of two countries are willing to destabilize a given (external) territory in order to obtain preferential access to its resources. One strategy would be to delegate costly fighting to a local armed group which in turn would fight against the central power of the contested region. I model this situation by allowing two governments G_i and G_j to send simultaneous, non-negotiable offers to one local militia m. The offers are, as before, made of a transfer t_k and of a share of political power γ_k, for $k = i, j$. Once the militia receives the two offers, she evaluates which one makes her better off, selects one and declares publicly her allegiance. The government whose offer is turned down gets a normalized payoff of 0, which is also the value of the outside option for both competing governments. Afterwards, fighting between the militia and the central power C takes place and payoffs are realized.

When evaluating the optimal contracts, both governments and the militia take into consideration the costs and benefits of getting into this type of contracts.. Similarly to our previous discussion, θ_i and θ_j represent the religious and ideological distances respectively of governments G_i and G_j to the militia m and this is perfectly known to all players. Let us assume for the sake of exposition that $\theta_i > \theta_j$, i.e. government G_j is ideologically closer to the militia than government G_i. The central authority C of the contested region is also a strategic player: it can decide how much of some given stock of resources S to invest in productive activities and how much to invest in costly fighting. Let us assume that the outside option of C is $-\infty$, so that it is always willing to fight against the militia. Thus, at the fighting stage, the militia and the central government C simultaneously maximize

$$\begin{cases} u_m^k = (1-\gamma_k)P + \dfrac{a_k}{a_k + a_C}(V + t_k - \theta_k a_k + S - a_C)\ s^m \\ u_C = \dfrac{a_C}{a_k + a_C}(V + t_k - \theta_k a_k + S - a_C) \end{cases}$$

where $k = i, j$, i.e. depends on whether the militia pledges allegiance to government i or j, a_C is the central power's fighting effort which enters both the probability of victory and the total stakes of the contest. The central power also has to split its resources between investing an amount $S - a_C$ in productive activities or fighting, which implies disbursing financial resources to recruit, mobilize and supply its army. Solving for the best responses of the fighting stage, I obtain that

$$a_k^* = \frac{t_k + V + S}{2(\theta_k + \sqrt{\theta_k})}$$

$$a_C^* = \frac{t_k + V + S}{2(1 + \sqrt{\theta_k})}$$

We focus on the first equilibrium, where both parties actively fight, because the second equilibrium implies that the optimal transfer by any of the two external governments will be $t_k = 0$. In this case, for any proposed share of political independence $\gamma > 0$, the militia will be better off not getting into any contract and prefers getting the value of the outside option N, since $N > P$. In this equilibrium, fighting does not occur and the central power C keeps the status quo. The utility of the militia computed at equilibrium writes

$$u_m^k(\mathbf{a}^*) = (1-\gamma_k)P + \frac{t_k + \widetilde{V}}{2(1+\sqrt{\theta_k})}\, s^m$$

where $\widetilde{V} = V + S$ and depending on whether she declares allegiance to government $k = i$ or $k = j$. The program of government k takes into account two constraints. First, that the militia eventually prefers to declare allegiance to government k and not to government $-k$. I denote this constraint coming from the presence of a common agent – the militia – as CA. Second, that her utility at the optimal contract is higher than the outside option N, which represents the participation constraint and I denote this constraint as PC. More formally, each government maximizes the following program

$$\max_{\gamma_k, t_k} \quad \gamma_k P + \frac{t_k + \widetilde{V}}{2(1+\sqrt{\theta_k})}\, s^g - \theta_k t_k^2$$

$$s.t.$$

$$\begin{cases} (1-\gamma_k)P + \dfrac{t_k + \widetilde{V}}{2(1+\sqrt{\theta_k})}\, s^m \geq (1-\gamma_{-k})P + \dfrac{t_{-k} + \widetilde{V}}{2(1+\sqrt{\theta_{-k}})}\, s^m & (CA) \\ \\ (1-\gamma_k)P + \dfrac{t_k + \widetilde{V}}{2(1+\sqrt{\theta_k})}\, s^m \geq N & (PC) \end{cases}$$

The two governments maximize over the transfers when the first constraint is binding and I check ex-post whether the participation constraint is respected. Analyzing for instance the program of G_i, from CA I am able to write the second element of the contract γ_i in function of all the other contracting variables

$$\gamma_i = \gamma_j + \frac{s^m}{2P}\left(\frac{t_i + \widetilde{V}}{1+\sqrt{\theta_i}} - \frac{t_j + \widetilde{V}}{1+\sqrt{\theta_j}}\right)$$

and symmetrically for G_j. The utility of government k computed at the equilibrium of the fighting stage and when the first constraints (CA) binds, depends directly on $-k$'s offer of control over policy making γ_{-k}. The more control over policymaking γ_{-k} government $-k$ retains for itself, the less would be left to the militia $(1-\gamma_{-k})$, thus lowering the likelihood that the armed group pledges allegiance to government $-k$ and increasing the likelihood that it will choose k instead. Writing $s^m = 1 - s^g$, the utilities of governments are

$$\begin{cases} u_G^i = P\left[\gamma_j + \dfrac{1-s^g}{2P}\left(\dfrac{t_i+\widetilde{V}}{1+\sqrt{\theta_i}} - \dfrac{t_j+\widetilde{V}}{1+\sqrt{\theta_j}}\right)\right] + \dfrac{t_i+\widetilde{V}}{2(1+\sqrt{\theta_i})}\, s^g - \theta_i t_i^2 \\ \\ u_G^j = P\left[\gamma_i + \dfrac{1-s^g}{2P}\left(\dfrac{t_j+\widetilde{V}}{1+\sqrt{\theta_j}} - \dfrac{t_i+\widetilde{V}}{1+\sqrt{\theta_i}}\right)\right] + \dfrac{t_j+\widetilde{V}}{2(1+\sqrt{\theta_j})}\, s^g - \theta_j t_j^2 \\ \end{cases}$$

At this stage, the maximization over the optimal transfers is completely decoupled between the two sponsoring governments. Moreover, we can see that the additional term that derives from the competition between the two governments brings a markup in the optimal transfers, whose cost is borne by governments. The first order conditions write

$$\begin{cases} \dfrac{1-s^g}{2(1+\sqrt{\theta_i})} + \dfrac{s^g}{2(1+\sqrt{\theta_i})} - 2\theta_i t_i = 0 \\ \\ \dfrac{1-s^g}{2(1+\sqrt{\theta_j})} + \dfrac{s^g}{2(1+\sqrt{\theta_j})} - 2\theta_j t_j = 0 \end{cases}$$

which imply that the optimal transfers are

$$\begin{cases} t_i^{*CA} &= \dfrac{1}{4\theta_i(1+\sqrt{\theta_i})} \\ \\ t_j^{*CA} &= \dfrac{1}{4\theta_i(1+\sqrt{\theta_i})} \end{cases}$$

The optimal transfers when there is a common agent – the militia – and the governments compete to "hire" it, turns out to be exactly what governments would have transferred had they had the monopoly over the contracted militia. This is because the new binding constraint CA has the same impact on transfers than the binding participation constraint in the case of monopoly contracting. At the same time, governments compete on the offered share of control over policymaking γ_k and militias receive higher rents from this contracting variable. Assuming without loss of generality that $\theta_i > \theta_j$, implies that $t_i^{*CA} < t_j^{*CA}$, i.e. the government that is most aligned with the local militia offers a higher monetary transfer in equilibrium and, given the binding constraint (CA),

$$\gamma_i = \gamma_j + \frac{s^m}{2P} \underbrace{\left(\frac{t_i^* + \widetilde{V}}{1 + \sqrt{\theta_i}} - \frac{t_j^* + \widetilde{V}}{1 + \sqrt{\theta_j}} \right)}_{<0} \Longrightarrow \gamma_j > \gamma_i$$

Thus, the government ideologically closest to the militia, G_j, offers a strictly higher transfer but demands a higher share of political power compared to government G_i. This is driven by the fact that government j faces a lower marginal cost of transferring funds and is able to incentivize a higher fighting effort through the anticipated militia's best responses in the fighting stage. The binding constraint CA, which imposes a condition of indifference for the militia's choice of allegiance, implies that j is able to demand a higher share of political power γ_j to the detriment of the militia's interests. Thus, the two governments play a game where they offer a contract where the optimal transfers are computed maximizing independently over the monetary transfers, taking into account the best responses at the fighting stage and the ideological parameters θ_k. However, if the two governments offered respectively γ_i and γ_j as above, the militia would be indifferent between choosing either of the two governments, by construction of the constraint CA. This is clearly not an equilibrium because both governments can profitably deviate by a series of undercutting of the proposed share of political power γ_k, resulting in a Bertrand-like competition.

The government that is most aligned to the militia, G_j in this instance, has a clear advantage, because it has a stronger position to exert a lower level of political influence γ_j whose loss would be compensated by a higher expected reward from the prize of the contest itself – resources $\widetilde{V}$ and the output of productive activities – compared to a situation of monopoly contracting. This implies it can cut its proposed γ_j until such a level where the opposing government will be indifferent between entering the contest and offering a contract to the militia or stay out and get a normalized payoff of zero. I define γ_j^{CA} as the minimum value of control over policymaking offered by G_j such that government G_i is indifferent between entering the contest or staying out. For every proposed share of power γ_j smaller than γ_j^{CA}, government i stays out of the competition. Assuming as before $\theta_i > \theta_j$, which implies $\gamma_i < \gamma_j$, government j is able to put i's utility down to zero by choosing γ_j^{CA} such that

$$u_G^i(\gamma_j^{CA}) = P\left[\gamma_j^{CA} + \frac{1-s^g}{2P}\left(\frac{t_i^*+\widetilde{V}}{1+\sqrt{\theta_i}} - \frac{t_j^*+\widetilde{V}}{1+\sqrt{\theta_j}}\right)\right] + \frac{t_i^*+\widetilde{V}}{2(1+\sqrt{\theta_i})}\,s^g - \theta_i t_i^{*2} = 0$$

which, computing it at the optimal values t_i^* and t_j^*, implies

$$\begin{aligned}
\gamma_j^{CA} &= \frac{1}{P}\left(\theta_i t_i^{*2} - \frac{t_i^*+\widetilde{V}}{2(1+\sqrt{\theta_i})}\,s^g + \frac{1-s^g}{2}\left(\frac{t_j^*+\widetilde{V}}{1+\sqrt{\theta_j}} - \frac{t_i^*+\widetilde{V}}{1+\sqrt{\theta_i}}\right)\right)\\
&= \frac{1}{16P}\left(8V\left(\frac{1-s^g}{1+\sqrt{\theta_j}} - \frac{1}{1+\sqrt{\theta_i}}\right) + \frac{2(1-s^g)}{\theta_j(1+\sqrt{\theta_j})^2} - \frac{1}{\theta_i(1+\sqrt{\theta_i})^2}\right)
\end{aligned}$$

Assuming for now that $0 \leq \gamma_j^{CA} \leq 1$, the series of undercutting of γ_k by both governments $k = i, j$ stops when $\gamma_j = \gamma_j^{CA}$. For this value of the proposed share of power, government G_i is indifferent between trying to contract the militia or stay out of the contest. At the same time, government i offers γ_i which by definition is smaller than γ_j^{CA}, but not small enough to put government j out of competition. Now, it sufficient for government G_j to offer $\gamma_j^{CA} - \varepsilon$, get the allegiance of the militia and the prize of the contest with probability $p(winner = j) = \frac{1}{1+\sqrt{\theta_j}}$, while government G_i is out of competition.

I have put aside the participation constraint and we have now to check whether the participation constraint is satisfied when offering the share of power of the common agency game γ_j^{CA}. The participation constraint alone would imply that the optimal share of power offered in the contract should be found when the constraint binds. In that situation, the contracting government would have to give up as much political power as necessary to incentivize the militia to take up the contract, given the optimal transfers. However, when there is competition between governments, the militia is able to demand more political power and gain more rents. From the participation constraint evaluated at $t_{j,CA}^*$ when it binds

$$\begin{aligned}
\gamma_j^{PC} &= 1 - \frac{1}{P}\left(N - \frac{t_j^*+\widetilde{V}}{2(1+\sqrt{\theta_j})}\,s^m\right)\\
&= 1 - \frac{1}{P}\left(N - \frac{1+4(1+\sqrt{\theta_j})\theta_j\widetilde{V}}{8(1+\sqrt{\theta_j})^2\theta_j}\,s^m\right)
\end{aligned}$$

The proposed share of power derived from the participation constraint is bigger than that coming from the common agency game only if the total value of the outside option of remaining neutral N is small enough. For values of N bigger than a threshold N_j^*, γ_j^{PC} is smaller than γ_j^{CA}. In this situation the ability of government j to concede more power to the militia in order to keep the opposing government out is somehow limited by the attractiveness of the outside option. The attractiveness of remaining neutral and getting a fixed payoff independent of the outcome of the context has to be counterbalanced by offering to keep a share of control over policymaking small enough. The following Lemma formalized this idea and finds the value of the outside option N^* such that it equalized γ_j^{PC} and γ_j^{CA}.

Lemma 2.
Let us assume that $\theta_i > \theta_j$. Then, there exists a value of the outside option $N_j^* > 0$ such that

- *if $N < N_j^*$ then $\gamma_j^{PC} > \gamma_j^{CA}$*
- *if $N > N_j^*$ then $\gamma_j^{PC} < \gamma_j^{CA}$*

where $N^* = P + \dfrac{1 + 8(1+\sqrt{\theta_i})\theta_i V}{16(1+\sqrt{\theta_i})^2\theta_i}$. This applies symmetrically to party i.

Interestingly, from the point of view of government j the threshold value of the outside option N^* that equalizes the binding participation constraint γ_j^{PC} and γ_j^{CA} depends only on the opponent's misalignment to the militia θ_i and not on its own θ_j. The following Lemma establishes that when government j's misalignment is much smaller than that of government i's, the former has a distinct advantage over the latter and is able, in some cases, to demand a positive share of political power, depending on the value of $\widetilde{V}$. In this case, it characterizes for which values of the total stakes $\widetilde{V}$, the offered γ_j^{CA} is indeed in $[0, 1]$. On the contrary, when competition is too tight and $\theta_i \sim \theta_j$, government j has to give up demanding any positive share of power and can only offer $\gamma_j^{CA} = 0$.

Lemma 3.
Let us assume $\theta_i > \theta_j$ and let us define $\Delta = \dfrac{\theta_j}{\theta_i}\left(\dfrac{1+\sqrt{\theta_j}}{1+\sqrt{\theta_i}}\right)^2$.

- *When $\Delta < 2\ s^m$, government j is ideologically much closer to the militia than government i. There generally exist two thresholds $\widetilde{V}'$ and $\widetilde{V}''$ such that $1 \geq \gamma_j^{CA} \geq 0$ iff $\widetilde{V}'' \geq \widetilde{V} \geq$*

$\widetilde{V}'$. Moreover, if $\widetilde{V} > \widetilde{V}''$ government j can only offer $\gamma_j^{CA} = 0$ and leaves all the political power to the militia. Similarly, if $\widetilde{V} < \widetilde{V}'$ government j can offer $\gamma_j^{CA} = 1$.

- *When $1 > \Delta > 2\ s^m$, the competition between the governments is too tight because $\theta_j \sim \theta_i$, and government j can only offer to leave all political power to the militia by offering $\gamma_j^{CA} = 0$ for any value of $\widetilde{V}$.*

Focusing on the case where $\Delta < 2\ s^m$, Lemma 3 just says that if the total value of the prize is smaller than a certain amount $\widetilde{V}'$, government i gets a small benefit from getting into the conflict. When this happens, government j is able to extract all the control of policymaking from the militia, $\gamma_j^{CA} = 1$, since it is easy to prevent the opposing government from sending offers. In the opposite scenario, when the value of the prize is very large, government i enjoys a high utility from getting into the conflict trying to get the militia to its side. In this situation, government j does its best to get the militia's allegiance and thus offers a contract where it leaves all the political power to its proxy, $\gamma_j^{CA} = 0$, knowing that if $\widetilde{V}$ is too large it will not be enough to restrain the opponent to send offers. Now we have all the elements to find the equilibrium of the common agency game.

Proposition 10.
When two governments compete to hire a common armed group, the militia carves out higher rents by receiving more transfers and by keeping more political power, compared to a situation of monopolistic contracting. Let us assume that $\theta_i > \theta_j$. We have two regimes.

(i) When government j has a decisive ideological advantage and the stakes are not too high, i.e. $\Delta < 2\ s^m$ and $\widetilde{V} < \widetilde{V}''$, government j is able to put the opposing government out of competition by keeping a positive share of control over policymaking $\gamma_j \in (0, 1]$.

(ii) When the stakes are very high or the ideological competition is too tight, i.e. $\widetilde{V} > \widetilde{V}''$ or $1 > \Delta > 2\ s^m$, government j its opponent just by offering a low γ_j. The whole political power is left to the militia, $\gamma_j = 0$, and j wins the competition by offering a transfer higher than that offered by government i.

Proposition 7 shows that when two governments compete to hire a common militia, the rents offered to the armed group are higher than if governments monopolistically contract the militia. This effect is even stronger when the two governments are in tight competition or the stakes are very high, i.e. $1 > \Delta > 2\ s^m$ or $\widetilde{V} > \widetilde{V}''$: the militia is able to extract the maximum amount of rent from the competition and enjoys the full political power P. In this case, the government with the smallest ideological misalignment is not able to throw the opponent out of the race for hiring militia just by proposing a small γ_j. Both contenders offer a contract where they leave all political power to the militia $\gamma_i = \gamma_j = 0$, but government j always offers a higher transfer since he has a lower cost of transferring funds. The Proposition below exactly characterizes the optimal contracts in function of the relevant parameters of the problem: the value of the outside option of staying neutral N and the value of the prize $\widetilde{V}$.

Proposition 11.
Assume that $\theta_i > \theta_j$. The equilibria can be then characterized in function of the prize $\widetilde{V}$ and of the value of neutrality N.

Let us consider the case where government j has a decisive ideological advantage over government i, i.e. $\Delta < 2\ s^m$, and $\widetilde{V} < \widetilde{V}''$.

(i) Let us consider $\widetilde{V}' < \widetilde{V} < \widetilde{V}''$. If $N < N^*$, government j optimally offers $(t_j^{*CA}, \gamma_j^{CA})$. If $N^* < N < N_{out}$, government j optimally offers $(t_j^{*CA}, \gamma_j^{PC})$. If $N^{max} > N > N_{out}$, government j offers $\gamma_j = 0$ and increases the transfer to match the outside option until t_j^{max}. The militia pledges allegiance to j. For $N > N^{max}(t_j^{max})$ neither government is willing to contract the militia.

(ii) Let us consider $\widetilde{V} < \widetilde{V}'$. If $N < N^{out}$ Government j offers $(t_j^{*CA}, \gamma_j = 1)$ if $\gamma_j^{PC} \geq 1$ and $(t_j^{*CA}, \gamma_j^{PC})$ otherwise. If $N^{max} > N > N_{out}$, government j offers $\gamma_j = 0$ and increases the transfer until t_j^{max}. The militia pledges allegiance to j. For $N > N^{max}(t_j^{max})$ neither government is willing to contract the militia.

Let us consider the case where the governments are in tight ideological competition $1 > \Delta > 2\ s^m$ or $\widetilde{V} > \widetilde{V}''$. Then,

(i) Both governments send offers characterized by $\gamma_j = \gamma_i = 0$ for any value of N. Despite government i offering its maximum possible transfer, for j it is sufficient to offer slightly more to win the militia's allegiance.

Proposition 12.
Let us assume that $\theta_i > \theta_j$. The equilibria can be then characterized in function of the prize $\widetilde{V}$ and of the value of neutrality N.

- *When government j has a decisive ideological advantage over government i, i.e. $\Delta < 2\ s^m$, and $\widetilde{V} < \widetilde{V}''$.*

- *If* $\widetilde{V}' < \widetilde{V} < \widetilde{V}''$ *and* $N < N^*$, *government* j *optimally offers* $(t_j^{*CA}, \gamma_j^{CA})$. *If* $N^* < N < N_{out}$, *government* j *optimally offers* $(t_j^{*CA}, \gamma_j^{PC})$. *If for* $N^{max} > N > N_{out}$, *government* j *offers* $\gamma_j = 0$ *and increases the transfer to match the outside option until* t_j^{max}. *The militia pledges allegiance to* j. *For* $N > N^{max}(t_j^{max})$ *neither government is willing to contract the militia.*
- *If* $\widetilde{V} < \widetilde{V}'$. *If* $N < N^{out}$ *Government* j *offers* $(t_j^{*CA}, \gamma_j = 1)$ *if* $\gamma_j^{PC} \geq 1$ *and* $(t_j^{*CA}, \gamma_j^{PC})$ *otherwise. If* $N^{max} > N > N_{out}$, *government* j *offers* $\gamma_j = 0$ *and increases the transfer until* t_j^{max}. *The militia pledges allegiance to* j. *For* $N > N^{max}(t_j^{max})$ *neither government is willing to contract the militia.*

- *Wehn the governments are in tight ideological competition* $1 > \Delta > 2\ s^m$ *or* $\widetilde{V} > \widetilde{V}''$ *. Then, both governments send offers characterized by* $\gamma_j = \gamma_i = 0$ *for any value of* N. *Despite government* i *offering its maximum possible transfer, for* j *it is sufficient to offer slightly more to win the militia's allegiance.*

This Proposition analyzes the effects of competition between two governments willing to "hire" a common armed group on the optimal contracting. The presence of competing principals modifies the optimal contracts on two fronts thus bestowing higher rents to the militia. First, it adds a constraint in the maximization program of each principal due to the imposed condition on the militia's choice of allegiance. This term puts an extra pressure on both governments to transfer more financial resources compared to a situation with monopolistic contracting. The militia gets higher financial rents simply because she has the possibility of aligning itself with the opposing government. Second, it also modifies the optimal contracting regarding the proposed share of political power γ, by triggering a sequence of reciprocal undercutting à la Bertrand ending when the most misaligned government becomes indifferent between entering the competition or staying out and receive a fixed payoff of zero. The government that is most closely aligned has a distinct advantage with respect to the opponent. Its marginal cost of transferring funds is strictly lower than that of the competing government, and this allows larger room for undercutting γ in order to put the opposing government out of competition and ensuring the militia will accept its contract.

In such a situation, the militia is able to extract considerably higher rents along both dimensions of the contract. The offered transfer is at least double what it would have received with no competition. Moreover, with competition, governments are forced to limit the offered political control over the militia. The lower the proposed γ, the lower it is the competing government's utility and the margin over its own offered share of power. This triggers a set of non-cooperative under-cuttings until the equilibrium is reached.

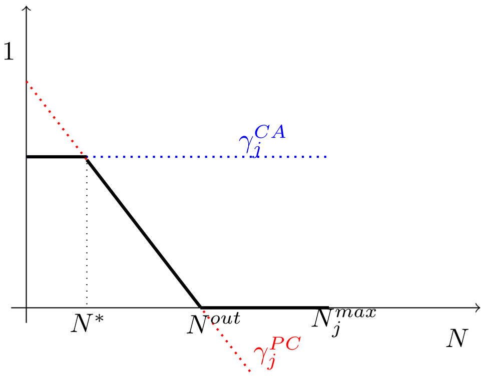


Figure 1: *Optimal* γ_j *when* $\Delta < 2\ s^m$ *and* $\widetilde{V} < \widetilde{V}''$

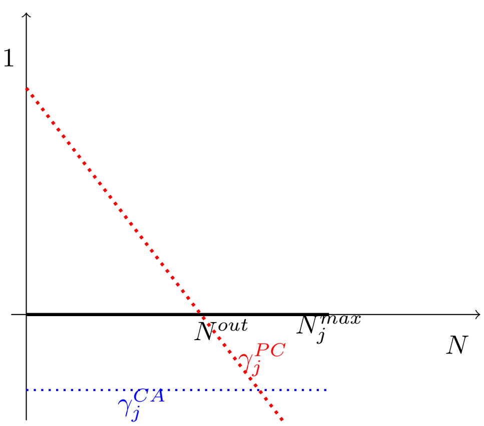


Figure 2: *Optimal* γ_j *when* $1 > \Delta > 2\ s^m$ *or* $\widetilde{V} > \widetilde{V}''$

5 Conclusion

This paper builds a model of conflict delegation, where two states competing for dominance over a given territory contract local armed groups to wage war on their behalf. Sponsoring governments transfer resources but demand a share of control over militias' policymaking. Armed groups are positioned on a continuum of ideological types. The analysis shows that sponsors' incomplete information on their militias' true ideology has the effect of lowering the transfers offered in second-best contracts, resulting in a net improvement in the welfare of local populations due to a lower fighting intensity. This comes at cost of a higher political independence left to delegates, and in governments' weaker control over militias' policymaking. This sheds some light on a possible mechanism underlying the governments' lack of control of armed groups that is observed in real conflicts. The analysis also explores the conditions for which delegating conflict is indeed an equilibrium of the strategic delegation game. When militias have little resources and weak support, sponsoring governments do not have an incentive to deviate from contracting them, and delegation emerges as the unique equilibrium. Finally, this paper analyzes a setting where two governments compete to recruit the same armed group. The sponsor that is ideologically closer has a distinct advantage due to the lower cost of transferring funds and to the better capacity of the armed group to recruit and motivate fighters. However, competition creates space for more demands from the militia, which generally carves out higher rents. This model can easily applied to a more general setting where two principals struggle for influence through their delegated agents who receive a transfer of resources and give up a portion of their organizational independence, resulting in agency loss.

This model could serve as a starting point for further investigation in a number of directions. First, it could be integrated in the study of the role of international institutions acting as brokers of peace as inspired by Myerson (1979, 1982) and Meirowitz et al. (2020). Waging war through proxies gives states the benefit of plausible deniability and it presents the international community with new challenges in the mediation efforts for peace. Secondly, it would be interesting to allow the model to integrate a new set of actors who have emerged to be the proxy war-wagers of the future, including private military companies and internet hackers. These new warriors are able to be co-opted by states with low appetite for direct military action and are predicted to have an increasingly important role in international political confrontations (Mumford, 2013, 2017; Maurer, 2016). Furthermore, it could guide future research on the micro determinants of militias' existence, and link it to the role of labor market conditions and of local institutions.

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A Appendix: Proofs

Proof of Proposition 1

Proof. We want to show that

$$t_i^* = s^g \frac{\theta_j}{4\theta_i(\theta_j + \sqrt{\theta_i\theta_j})\omega} > t_j^* = s^g \frac{\theta_i}{4\theta_j(\theta_i + \sqrt{\theta_i\theta_j})\omega}$$

Rearranging the terms we can write

$$\frac{\theta_i + \sqrt{\theta_i\theta_j}}{\theta_j + \sqrt{\theta_i\theta_j}} = \sqrt{\frac{\theta_i}{\theta_j}} > \left(\frac{\theta_i}{\theta_j}\right)^2$$

which is always true for $\theta_j > \theta_i$. For the optimal fighting effort are

$$a_i^* = \frac{s^g \left(\theta_i + \theta_j - \sqrt{\theta_i\theta_j}\right) + 4\theta_i\theta_j V\omega}{8\theta_i\theta_j(\theta_i + \sqrt{\theta_i\theta_j})\mu\,\omega} \qquad and \qquad a_j^* = \frac{s^g \left(\theta_i + \theta_j - \sqrt{\theta_i\theta_j}\right) + 4\theta_i\theta_j V\omega}{8\theta_i\theta_j(\theta_j + \sqrt{\theta_i\theta_j})\mu\,\omega}$$

we can see that they are identical except for one term at the denominator. Simple algebra gives us the result when $\theta_j > \theta_i$ □

Proof of Proposition 2

Proof. Computing the derivatives of equilibrium transfers we have that

$$\frac{\partial t_k^*}{\partial \theta_k} = -\frac{\theta_{-k}\left(2\theta_{-k} + 3\sqrt{\theta_k\theta_{-k}}\right)}{8\omega\theta_k^2\left(\sqrt{\theta_k\theta_{-k}} + \theta_{-k}\right)^2}\, s^g < 0$$

$$\frac{\partial t_k^*}{\partial \theta_{-k}} = \frac{\theta_{-k}}{8\omega\sqrt{\theta_k\theta_{-k}}\left(\sqrt{\theta_k\theta_{-k}} + \theta_{-k}\right)^2} s^g > 0$$

while for the winning probabilities we have

$$\frac{\partial p_k^*}{\partial \theta_k} = -\frac{\theta_{-k}^2}{2\sqrt{\theta_k\theta_{-k}}(\theta_{-k} + \sqrt{\theta_k\theta_{-k}})} < 0$$

$$\frac{\partial p_k^*}{\partial \theta_{-k}} = \frac{\sqrt{\theta_{-k}\theta_j}}{2(\theta_{-k} + \sqrt{\theta_k\theta_{-k}})} > 0$$

Computing the derivatives of the fighting efforts at equilibrium

$$\frac{\partial a_k}{\partial \theta_k} = \frac{-2\theta_k\left(4\omega V\theta_{-k}\sqrt{\theta_k\theta_{-k}} + s^g\sqrt{\theta_k\theta_{-k}} + 2\omega V\theta_{-k}^2 - s^g\theta_{-k}\right) - s^g\theta_{-k}\left(2\sqrt{\theta_k\theta_{-k}} + 3\theta_{-k}\right)}{16\mu\omega(\theta_k\theta_{-k})^{3/2}\left(\sqrt{\theta_k\theta_{-k}} + \theta_k\right)^2} < 0$$

$$\frac{\partial a_k}{\partial \theta_{-k}} = \frac{\theta_{-k}s^g\left(2\sqrt{\theta_k\theta_{-k}} - \theta_{-k}\right) - 2\theta_k\left(s^g\sqrt{\theta_k\theta_{-k}} + 2\theta_{-k}^2\omega V + \theta_{-k}s^g\right)}{16\theta_{-k}^2\mu\omega\sqrt{\theta_k\theta_{-k}}\left(\sqrt{\theta_k\theta_{-k}} + \theta_k\right)^2} < 0$$

□

Proof of Proposition 3

Proof. Computing the derivatives of the optimal transfers and fighting efforts with respect with μ and ω it is easy to see that

$$\frac{\partial t_k}{\partial \mu} = 0$$

$$\frac{\partial a_k}{\partial \mu} = -\frac{\frac{\theta_k}{4\theta_{-k}\omega\left(\sqrt{\theta_k\theta_{-k}}+\theta_k\right)}s^g + \frac{\theta_{-k}}{4\theta_k\omega\left(\sqrt{\theta_k\theta_{-k}}+\theta_{-k}\right)}s^g + V}{2\mu^2\left(\sqrt{\theta_k\theta_{-k}} + \theta_k\right)} < 0$$

$$\frac{\partial a_k}{\partial \omega} = -\frac{\frac{\theta_k}{4\theta_{-k}\omega^2\left(\sqrt{\theta_k\theta_{-k}}+\theta_k\right)}s^g + \frac{\theta_{-k}}{4\theta_k\omega^2\left(\sqrt{\theta_k\theta_{-k}}+\theta_{-k}\right)}s^g}{2\mu\left(\sqrt{\theta_k\theta_{-k}} + \theta_k\right)} < 0$$

□

Proof of Proposition 3

Proof. First, from the optimization problem in one variable, the first order condition is

$$\begin{aligned}
\frac{\partial u_G^k(\mathbf{t},|\theta_k)}{\partial t_k} =& \frac{\partial}{\partial t_k}\int_0^1 \frac{\theta_{-k}}{2(\theta_{-k}+\sqrt{\theta_k\theta_{-k}})}\Big(t_k(\theta_k)+t_{-k}(\theta_{-k})+V\Big) - \omega\theta_k t_k(\theta_k)^2 \; d\theta_{-k} \\
=& \int_0^1 \frac{\partial}{\partial t_k}\frac{\theta_{-k}}{2(\theta_{-k}+\sqrt{\theta_k\theta_{-k}})}\Big(t_k(\theta_k)+t_{-k}(\theta_{-k})+V\Big) - \omega\theta_k t_k(\theta_k)^2 \; d\theta_{-k} \\
=& \int_0^1 \frac{\theta_{-k}}{2(\theta_{-k}+\sqrt{\theta_k\theta_{-k}})} - 2\omega\theta_k \; t_k(\theta_k) \; d\theta_{-k} = 0
\end{aligned}$$

Let us take this integral for the utility maximization of government i and solve it in isolation. The case for government j is perfectly symmetric. Let us start by substituting $u = \sqrt{\theta_i\theta_j}$ and $du = \frac{\theta_i}{2\sqrt{\theta_i\theta_j}}$ and we can rewrite the integral as

$$\begin{aligned}
\frac{2}{\theta_i^2}\int_u \frac{u^3}{u^2/\theta_i+u}du =& \frac{2}{\theta_i}\int_u \frac{u^2}{\theta_i+u}du \\
=& \frac{2}{\theta_i}\left(\int_u \frac{\theta_i^2}{u+\theta_i} + u - \theta_i du\right)
\end{aligned}$$

I can now solve the three integrals separately. By substituting $s = u+\theta_i$ and $ds = du$ I can write the three integral above as

$$2\theta_i \int_s \frac{1}{s}ds + \frac{2}{\theta_i}\int_u udu - 2\int_u du = 2\theta_i log(s) + \frac{u^2}{\theta_i} - 2u$$

and by substituting back $s = u+\theta_i$ and $u = \sqrt{\theta_i\theta_j}$ we get to

$$\begin{aligned}
\int_0^1 \frac{\theta_j}{(\theta_j+\sqrt{\theta_i\theta_j})}d\theta_j =& 2\theta_i log(\sqrt{\theta_i\theta_j}+\theta_i) + \theta_j - 2\sqrt{\theta_i\theta_j}\Big|_0^1 \\
=& 1 + 2\theta_i log\left(\frac{\sqrt{\theta_i}+1}{\sqrt{\theta_i}}\right) - 2\sqrt{\theta_i}
\end{aligned}$$

Inserting the integral in the first order condition and simplifying we find that

$$t_i^{*,IN} = \frac{1}{2\omega}\left[log\left(\frac{1+\sqrt{\theta_i}}{\sqrt{\theta_i}}\right) - \frac{2\sqrt{\theta_i}-1}{2\theta_i}\right]$$

□

Proof of Proposition 4

Proof. I prove Proposition 3 for party i, but the proof is perfectly symmetrical for j. First, we know that

$$\frac{\partial t_i^{CI}}{\partial \theta_j} = \frac{\theta_j}{4\theta_i(\theta_j + \sqrt{\theta_i \theta_j})\omega} > 0$$

and that

$$\lim_{\theta_j \to 0} t_i^{CI} = 0$$

Second, we also know that with incomplete information about θ_j

$$t_i^{IN} = \frac{1}{2\omega}\left[log\left(\frac{1+\sqrt{\theta_i}}{\sqrt{\theta_i}}\right) - \frac{2\sqrt{\theta_i}-1}{2\theta_i}\right] > 0$$

for every $\theta_i \in (0,1]$ and we know that t_i^{IN} is constant in θ_j. This implies that there always exists a value $\widehat{\theta_j}$ of θ_j, such that $t_i^{CI}(\widehat{\theta_j}) = t_i^{IN}$. Moreover, for the monotonicity of t_i^{CI}, we know that for $\theta_j > \widehat{\theta_j}$ we have that $t_i^{CI} > t_i^{IN}$ and for $\theta_j < \widehat{\theta_j}$ we obtain that $t_i^{CI} < t_i^{IN}$. In order to find the value $\widehat{\theta_j}$ we set the condition $t_i^{CI}(\widehat{\theta_j}) = t_i^{IN}$ and we solve in $\widehat{\theta_j}$. Apart from the constant factor $\frac{1}{2\omega}$

$$\begin{aligned}
& log\left(\frac{1+\sqrt{\theta_i}}{\sqrt{\theta_i}}\right) - \frac{2\sqrt{\theta_i}-1}{2\theta_i} = \frac{\theta_j}{2\theta_i(\theta_j + \sqrt{\theta_i\theta_j})} \\
& \Rightarrow \theta_j - 2\theta_i(\theta_j + \sqrt{\theta_i\theta_j})\left(log\left(\frac{1+\sqrt{\theta_i}}{\sqrt{\theta_i}}\right) - \frac{2\sqrt{\theta_i}-1}{2\theta_i}\right) = 0
\end{aligned}$$

which can be rearranged as

$$\begin{aligned}
& 2\theta_j\sqrt{\theta_i} - 2\theta_i\theta_j log\left(\frac{1+\sqrt{\theta_i}}{\sqrt{\theta_i}}\right) + \sqrt{\theta_i\theta_j}\left(2\sqrt{\theta_i} - 2\theta_i log(\frac{1+\sqrt{\theta_i}}{\sqrt{\theta_i}}) - 1\right) \\
& = \theta_i\theta_j\left(\frac{2}{\sqrt{\theta_i}} - 2log(\frac{1+\sqrt{\theta_i}}{\sqrt{\theta_i}})\right) + \sqrt{\theta_i\theta_j}\left(2\sqrt{\theta_i} - 2\theta_i log(\frac{1+\sqrt{\theta_i}}{\sqrt{\theta_i}}) - 1\right) \\
& = \sqrt{\theta_j}\left(\sqrt{\theta_i} - 2\theta_i - 2\sqrt{\theta_i\theta_j} + 2\theta_i^{3/2} log(\frac{1+\sqrt{\theta_i}}{\sqrt{\theta_i}}) + 2\theta_i\sqrt{\theta_j} log(\frac{1+\sqrt{\theta_i}}{\sqrt{\theta_i}})\right) \\
& = 0
\end{aligned}$$

which is true if $\theta_j = 0$, which is not acceptable, or if the term inside the parenthesis equals zero, which happens when

$$\theta_j = \widehat{\theta_j} = \frac{\left(1 + 2\theta_i log\left(\frac{\sqrt{\theta_i}+1}{\sqrt{\theta_i}}\right) - 2\sqrt{\theta_i}\right)^2}{4\left(\sqrt{\theta_i} log\left(\frac{\sqrt{\theta_i}+1}{\sqrt{\theta_i}}\right) - 1\right)^2}$$

This expression is always positive and

$$\frac{\partial\widehat{\theta_j}}{\partial\theta_i} > 0$$

but it is easy to show that is bounded below one $\widehat{\theta_j} \leq 1$, as $\widehat{\theta_j}(0) = 0$ and

$$\widehat{\theta_j}(1) = \frac{(log(4)-1)^2}{4(log(2)-1)^2} < 1$$

. $\square$

Proof of Proposition 5

Let us derive the results above. By taking into account the incentive constraint in the governments' programs, I can write

$$\max_{t_k} \int_{\boldsymbol{\theta_{-k}}} \int_{\boldsymbol{\theta_k}} P - N + \frac{\theta_{-k}}{2(\theta_{-k} + \sqrt{\theta_k \theta_{-k}})}\Big(t_k(\theta_k) + t_{-k}(\theta_{-k}) + V\Big) - \omega\theta_k t_k^2(\theta_k) + \\ \int_{\theta_k}^1 \left(\dot{C}(\tau, \theta_{-k})\Big(t_k(\tau) + t_{-k}(\theta_{-k}) + V\Big)\, d\tau\right)\; d\theta_{-k} d\theta_k =$$

$$\max_{t_k} \int_{\boldsymbol{\theta_{-k}}} \int_{\boldsymbol{\theta_k}} P - N + \frac{\theta_{-k}}{2(\theta_{-k} + \sqrt{\theta_k \theta_{-k}})}\Big(t_k(\theta_k) + t_{-k}(\theta_{-k}) + V\Big) - \omega\theta_k t_k^2(\theta_k) + \\ \frac{1-F(\theta_k)}{f(\theta_k)} \dot{C}(\theta_k, \theta_{-k})\Big(t_k(\theta_k) + t_{-k}(\theta_{-k}) + V\Big)\; d\theta_{-k}\; d\theta_k$$

where we have applied the Fubini Theorem and integrated by parts. Substituting $\frac{1-F(\theta_k)}{f(\theta_k)} =$

$(1-\theta_k)$ since the ideological types are distributed uniformly in $(0,1]$, the governments' program writes

$$\max_{t_k} \int_{\boldsymbol{\theta_{-k}}} \int_{\boldsymbol{\theta_k}} P - N + \frac{\theta_{-k}}{2(\theta_{-k} + \sqrt{\theta_k \theta_{-k}})} \Big(t_k(\theta_k) + t_{-k}(\theta_{-k}) + V \Big) - \omega \theta_k t_k^2(\theta_k) +$$
$$(1-\theta_k)\dot{C}(\theta_k, \theta_{-k}) \Big(t_k(\theta_k) + t_{-k}(\theta_{-k}) + V \Big) \; d\theta_{-k} d\theta_k =$$

$$\max_{t_k} \int_{\boldsymbol{\theta_{-k}}} \int_{\boldsymbol{\theta_k}} P - N + \frac{\theta_{-k}}{2(\theta_{-k} + \sqrt{\theta_k \theta_{-k}})} \Big(t_k(\theta_k) + t_{-k}(\theta_{-k}) + V \Big) - \omega \theta_k t_k^2(\theta_k) +$$
$$-(1-\theta_k) \frac{\theta_{-k}^2}{2\sqrt{\theta_k \theta_{-k}}(\theta_{-k} + \sqrt{\theta_k \theta_{-k}})^2} \Big(t_k(\theta_k) + t_{-k}(\theta_{-k}) + V \Big) s^m \; d\theta_{-k} d\theta_k$$

where we substituted

$$\dot{C}(\theta_k, \theta_{-k}) = -\frac{\theta_{-k}^2}{4\sqrt{\theta_k \theta_{-k}}(\theta_{-k} + \sqrt{\theta_k \theta_{-k}})^2} \; s^m$$

We can maximize pointwise the integrand since the principal's payoff function is globally concave in t_k and we can thus focus on the relaxed problem. The monotone hazard rate is also respected since

$$\frac{\partial}{\partial \theta_k} \frac{1 - F(\theta_k)}{f(\theta_k)} = \frac{\partial}{\partial \theta_k}(1-\theta_k) \leq 0$$

We can see that there is no distortion in the optimal amount transferred to the most misaligned militia, i.e. when $\theta_k = 1$. For all other types of ideologies there is a downward distortion of the optimal proposed transfer. The first order condition is then

$$
\int_{\boldsymbol{\theta_{-k}}}\int_{\boldsymbol{\theta_k}} \frac{\partial}{\partial t_k} P - N_k + \frac{\theta_{-k}}{2(\theta_{-k}+\sqrt{\theta_k\theta_{-k}})}\Big(t_k(\theta_k)+t_{-k}(\theta_{-k})+V\Big) - \omega\theta_k t_k^2(\theta_k) +
(1-\theta_k)\dot{C}(\theta_k)\Big(t_k(\theta_k)+t_{-k}(\theta_{-k})+V\Big)\, d\theta_{-k}d\theta_k
$$

$$
= \int_{\boldsymbol{\theta_{-k}}}\int_{\boldsymbol{\theta_k}} \frac{\theta_{-k}}{2(\theta_{-k}+\sqrt{\theta_k\theta_{-k}})}\, t_k(\theta_k) - 2\omega\theta_k t_k(\theta_k) - (1-\theta_k)\frac{\theta_{-k}^2}{4\sqrt{\theta_k\theta_{-k}}(\theta_{-k}+\sqrt{\theta_k\theta_{-k}})^2} s^m\, t_k(\theta_k)\, d\theta_{-k}d\theta_k
$$

I now solve the integral of the first order condition with respect to θ_{-k}. The integration of the first term with respect to θ_{-k} is identical to the previous case of Proposition 3. On the other hand, the second term is new and comes from the incentive constraint. That is, up to a factor s^m,

$$
\int_{\boldsymbol{\theta_{-k}}} -(1-\theta_k)\frac{\theta_{-k}^2}{2\sqrt{\theta_k\theta_{-k}}(\theta_{-k}+\sqrt{\theta_k\theta_{-k}})^2}\, t_k(\theta_k) d\theta_{-k} =
$$
$$
= -\frac{t_k(\theta_k)}{2}(1-\theta_k)\int_{\boldsymbol{\theta_{-k}}} \frac{\sqrt{\theta_{-k}}}{\sqrt{\theta_k}(\sqrt{\theta_k}+\sqrt{\theta_{-k}})^2} d\theta_{-k} =
$$
$$
= -\frac{t_k(\theta_k)}{2}\Big(\frac{1}{\sqrt{\theta_k}}-\sqrt{\theta_k}\Big)\int_{\boldsymbol{\theta_{-k}}} \frac{\sqrt{\theta_{-k}}}{(\sqrt{\theta_k}+\sqrt{\theta_{-k}})^2} d\theta_{-k}
$$

Substituting $u=\sqrt{\theta_{-k}}$ and $du = \frac{1}{2\sqrt{\theta_{-k}}}$ we can rewrite the integral as

$$
-\frac{t_k(\theta_k)}{2}\Big(\frac{1}{\sqrt{\theta_k}}-\sqrt{\theta_k}\Big)\int_{\boldsymbol{\theta_{-k}}} \frac{u^2}{(u+\sqrt{\theta_k})^2}\, du
$$
$$
= -t_k(\theta_k)\Big(\frac{1}{\sqrt{\theta_k}}-\sqrt{\theta_k}\Big)\int_{\boldsymbol{\theta_{-k}}} \frac{\theta_k}{(u+\sqrt{\theta_k})^2} - \frac{2\sqrt{\theta_k}}{u+\sqrt{\theta_k}} + 1\, du
$$
$$
= -t_k(\theta_k)\left((2\theta_k-2)\int_{\boldsymbol{\theta_{-k}}} \frac{1}{u+\sqrt{\theta_k}}\, du + (\sqrt{\theta_k}-\theta_k^{3/2})\int_{\boldsymbol{\theta_{-k}}} \frac{1}{(u+\sqrt{\theta_k})^2}\, du + \Big(\frac{1}{\sqrt{\theta_k}}-\sqrt{\theta_k}\Big)\int_{\boldsymbol{\theta_{-k}}} 1\, du\right)
$$

Substituting again $s = u+\sqrt{\theta_k}$, $p = u+\sqrt{\theta_k}$ we can rewrite

$$- t_k(\theta_k)\Bigg((2\theta_k - 2)\int_{\boldsymbol{\theta_{-k}}} \frac{1}{s}\, ds + (\sqrt{\theta_k} - \theta_k^{3/2})\int_{\boldsymbol{\theta_{-k}}} \frac{1}{p^2}\, dp + (\frac{1}{\sqrt{\theta_k}} - \sqrt{\theta_k})\int_{\boldsymbol{\theta_{-k}}} 1\, du \Bigg) =$$

$$- t_k(\theta_k)\Bigg((2\theta_k - 2) log(s) - \frac{(\sqrt{\theta_k} - \theta_k^{3/2})}{p} + (\frac{1}{\sqrt{\theta_k}} - \sqrt{\theta_k})u \Bigg) + const.$$

Now, substituting back and computing the definite integral for $\theta_k \in (0, 1]$

$$-t_k(\theta_k)\frac{(1-\theta_k)\Big(- \sqrt{\theta_k\theta_{-k}} + 2(\theta_k + \sqrt{\theta_k\theta_{-k}}) log(\sqrt{\theta_k} + \sqrt{\theta_{-k}}) + \theta_k - \theta_{-k}\Big)}{\theta_k + \sqrt{\theta_k\theta_{-k}}}\Bigg|_0^1$$

$$= (1-\theta_k) log\Big(\frac{(1+\theta_k)^2)}{\theta_k}\Big) - \frac{1-2\theta_k}{\sqrt{\theta_k}} - 1$$

Substituting back in the first order condition

$$\int_{\boldsymbol{\theta_k}} \frac{1}{2}\Big[1 + 2\theta_i log\Bigg(\frac{\sqrt{\theta_i}+1}{\sqrt{\theta_i}}\Bigg) - 2\sqrt{\theta_i}\Big] t_k^{SB}(\theta_k) - 2\omega\theta_k t_k^{SB}(\theta_k) + \frac{s^m}{2}\Big[(1-\theta_k) log\Big(\frac{(1+\sqrt{\theta_k})^2}{\sqrt{\theta_k}}\Big) - \frac{(1-2\theta_k)}{\sqrt{\theta_k}} - 1\Big]\, t_k^{SB}(\theta_k) d\theta_k = 0$$

where $t_k^{SB}(\theta_k)$ is the second best optimal transfer. Maximizing pointwise I obtain

$$\begin{aligned} t_k^{SB} &= t_k^{IN_{-k}} + A(\theta_k) \\ &= t_k^{IN_{-k}} + \frac{s^m}{4\omega\theta_k}\Bigg((1-\theta_k) log\Big(\frac{(1+\sqrt{\theta_k})^2}{\sqrt{\theta_k}}\Big) - \frac{(1-2\theta_k)}{\sqrt{\theta_k}} - 1 \Bigg) \end{aligned}$$

In particular, we can easily check that the additional term $A(\theta_k)$ is always negative for $\theta_k \in (0, 1)$ and equals 0 when $\theta_k = 1$, i.e. there is no distortion for the most misaligned militia. Moreover, t_k^{SB} is clearly decreasing, all types choose therefore different allocations and there is no bunching in the optimal contract.

Proof of Proposition 6

We can now see that the contracting variable γ_k^{SB} of the share of power can be written as

$$\begin{aligned}\gamma_k^{SB}(\theta_k) &= 1 - \frac{1}{P}\Bigg(u_m^{k,SB}(\theta_k) - \int_{\theta_{-k}} C(\theta_k,\theta_{-k})\Big(t_k^{SB}(\theta_k) + t_{-k}^{SB}(\theta_{-k}) + V\Big)\Bigg)\ d\theta_{-k} \\ &= 1 - \frac{1}{P}\Bigg(N - \int_{\boldsymbol{\theta_{-k}}}\int_{\theta_k}^{1} \dot{C}(\tau,\theta_{-k})\Big(t_k(\tau) + t_{-k}(\theta_{-k}) + V\Big)d\tau - C(\theta_k,\theta_{-k})\Big(t_k^{SB}(\theta_k) + t_{-k}^{SB}(\theta_{-k}) + V\Big)d\theta_{-}\end{aligned}$$

where I substituted

$$u_m^{k,SB}(\theta_k) = \int_{\boldsymbol{\theta_{-k}}} N - \int_{\theta_k}^{1} \dot{C}(\tau,\theta_{-k})\Big(t_k(\tau) + t_{-k}(\theta_{-k}) + V\Big)d\tau$$

I want to show that at equilibrium $\gamma_k^{SB} < \gamma_k^{IN_{-k}}$ and, by neglecting the integration over θ_{-k} we have

$$\begin{aligned}\gamma_k^{SB} =& 1 - \frac{1}{P}\Bigg(N - \int_{\theta_k}^{1} \dot{C}(\tau)(t_k(\tau) + t_{-k} + V)d\tau - C(\theta_k)(t_k^{SB}(\theta_k) + t_{-k}^{SB}(\theta_{-k}) + V)\Bigg) < \\ & 1 - \frac{1}{P}\Bigg(N - C(\theta_k)(t_k^{IN_{-k}}(\theta_k) + t_{-k}^{IN_{-k}}(\theta_{-k}) + V)\Bigg) \\ &= \gamma_k^{IN_{-k}}\end{aligned}$$

Rearranging and simplifying we obtain

$$\int_{\theta_k}^{1} \dot{C}(\tau)(t_k(\tau)+t_{-k}+V)d\tau + \frac{C(\theta_k)(t_k^{SB}(\theta_k) + t_{-k}^{SB}(\theta_{-k}) + V)}{P} < \frac{C(\theta_k)(t_k^{IN_{-k}}(\theta_k) + t_{-k}^{IN_{-k}}(\theta_{-k}) + V)}{P}$$

which is always true because

$$\int_{\theta_k}^{1} \dot{C}(\tau)(t_k(\tau) + t_{-k} + V)d\tau < 0$$

where $\dot{C}(\tau) < 0$. The fact that $t_k^{IN_{-k}} > t_k^{SB}$ and $t_{-k}^{IN_{-k}} > t_{-k}^{SB}$ completes the proof.

Proof of Lemma 1

Proof. Let us assume without loss of generality that $\theta_i > \theta_j$ and let us focus on party i. The proof for $\theta_j > \theta_i$ is symmetric. As a preparatory step, I find the thresholds on λ_i^D and λ_i^N by setting respectively $u_i^G(D, D) = u_i^G(ND, D)$ and $u_i^G(D, ND) = u_i^G(ND, ND)$. First, it easy to see that λ_k^N and λ_k^D are positive in $V = 0$, for $\theta_j \in (0, 1]$. Second, I show that both λ_k^i and λ_i^D are strictly increasing in V. Third, showing that the slope of λ_i^N is greater than the slope of λ_i^D when $\theta_i > \theta_j$, implies that for V large enough $\lambda_i^N > \lambda_i^D$. This will imply that for party j the reverse condition applies, i.e. $\lambda_j^D > \lambda_i^N$. Fourth, showing that $\lambda_j^N > \lambda_i^N$ means that the intervals defined by λ_k^D and λ_k^N are disjoint for the two parties. Finally, I show that when $\theta_i = \theta_j = \theta$ only two thresholds are obtained and $\lambda^N > \lambda^D$.

0. The preliminary step to the proof is to find the thresholds λ_k^D and λ_k^N as defined above. This yields

$$\lambda_i^N = \frac{1 + 4(1-\theta_i)\theta_i V}{16\left(1+\sqrt{\theta_i}\right)^2 \theta_i}$$

$$\lambda_i^D = \frac{1}{16}\left(\frac{\frac{\theta_j^2}{\theta_i} + 8\theta_j V\left(\theta_j + \sqrt{\theta_i\theta_j}\right) + 2\sqrt{\theta_i\theta_j}}{\left(\theta_j + \sqrt{\theta_i\theta_j}\right)^2} - \frac{2\Big(1 + 4\theta_j\left(1+\sqrt{\theta_j}\right)\theta_j V\Big)}{\left(1+\sqrt{\theta_j}\right)^2\sqrt{\theta_j}}\right)$$

and symmetrically for government j

$$\lambda_j^N = \frac{1 + 4(1-\theta_j)\theta_j V}{16\left(1+\sqrt{\theta_j}\right)^2 \theta_j}$$

$$\lambda_j^D = \frac{1}{16}\left(\frac{\frac{\theta_i^2}{\theta_j} + 8\theta_i V\left(\theta_i + \sqrt{\theta_i\theta_j}\right) + 2\sqrt{\theta_i\theta_j}}{\left(\theta_i + \sqrt{\theta_i\theta_j}\right)^2} - \frac{2\Big(1 + 4\theta_i\left(1+\sqrt{\theta_i}\right)\theta_i V\Big)}{\left(1+\sqrt{\theta_i}\right)^2\sqrt{\theta_i}}\right)$$

1. Let us start by showing that both quantities are increasing in V. We have that

$$\frac{\partial\lambda_i^N}{\partial V} = \frac{1-\theta_i}{4(1+\sqrt{\theta_i})^2} > 0$$

when $1 > \theta_i > 0$.

and, up to a constant factor,

$$
\begin{aligned}
\frac{\partial \lambda_i^D}{\partial V} &= \frac{\theta_j}{\theta_j + \sqrt{\theta_i\theta_j}} - \frac{\sqrt{\theta_j}}{1+\sqrt{\theta_j}} \\
&= \frac{\theta_j(1-\sqrt{\theta_i})}{(\theta_j+\sqrt{\theta_i\theta_j})(1+\sqrt{\theta_j})} > 0
\end{aligned}
$$

for $1 > \theta_i > 0$.

2. Now, I show that the slope of λ_i^N is greater than the slope of λ_i^D when $\theta_i > \theta_j$. Up to a constant factor, I want to show that

$$
\frac{\partial \lambda_i^N}{\partial V} = -\frac{1}{2} + \frac{1}{1+\sqrt{\theta_i}} > -1 + \frac{1}{1+\sqrt{\theta_j}} + \frac{\theta_j}{\theta_j+\sqrt{\theta_i\theta_j}} = \frac{\partial \lambda_i^D}{\partial V}
$$

Rearranging and writing $\frac{\theta_j}{\theta_j+\sqrt{\theta_i\theta_j}} = \frac{\theta_j^2 - \theta_j\sqrt{\theta_i\theta_j}}{\theta_j(\theta_j-\theta_i)} = \frac{\theta_j - \sqrt{\theta_i\theta_j}}{\theta_j - \theta_i}$ we have

$$
\begin{aligned}
\frac{\partial \lambda_i^N}{\partial V} - \frac{\partial \lambda_i^D}{\partial V} &= \frac{1}{2} + \frac{\sqrt{\theta_j}-\sqrt{\theta_i}}{(1+\sqrt{\theta_i})(1+\sqrt{\theta_j})} - \frac{\sqrt{\theta_j}}{\sqrt{\theta_i}+\sqrt{\theta_j}} \\
&= \frac{(1-\sqrt{\theta_i})(1-\sqrt{\theta_j})(\sqrt{\theta_i}-\sqrt{\theta_j})}{2(1+\sqrt{\theta_i})(1+\sqrt{\theta_j})(\sqrt{\theta_i}+\sqrt{\theta_j})} > 0
\end{aligned}
$$

if $\theta_i > \theta_j$.

3. Now, I show that $\lambda_j^N > \lambda_i^N$, i.e. the intervals defined by λ_k^D and λ_k^N are disjoint for the two parties. More formally this implies that

$$
\frac{1+4(1-\theta_j)\theta_j V}{16\left(1+\sqrt{\theta_j}\right)^2\theta_j} - \frac{1+4(1-\theta_i)\theta_i V}{16\left(1+\sqrt{\theta_i}\right)^2\theta_i} > 0
$$

Rearranging and simplifying, we can write that this condition is true if and only if

$$
\frac{\sqrt{\theta_i}-\sqrt{\theta_j}}{\theta_i\theta_j(1+\sqrt{\theta_i})^2(1+\sqrt{\theta_j})^2}\left(8V\theta_i\theta_j(1+\sqrt{\theta_i})(1+\theta_j)+\theta_i^{3/2}+\theta_j^{3/2}+\theta_i\sqrt{\theta_j}+\theta_j\sqrt{\theta_i}+2\sqrt{\theta_i\theta_j}+2(\theta_i+\theta_j)+\theta_i+\theta_j\right) > 0
$$

which is verified if $\theta_i > \theta_j$.

4. Finally, it is easy to see that when $\theta_i = \theta_j$, we have that $\lambda_i^N = \lambda_j^N = \lambda^N$ and $\lambda_i^D = \lambda_j^D = \lambda^D$. We have still to show that $\lambda^N > \lambda^D$. When $\theta_i = \theta_j = \theta$ I can write that the difference between the two thresholds is

$$\lambda^N - \lambda^D = \frac{\theta\left(1 + 2\sqrt{\theta} - 3\theta\right)}{64\theta^2\left(1+\sqrt{\theta}\right)^2} > 0$$

for any $\theta \in (0, 1)$ since $\left(1 + 2\sqrt{\theta} - 3\theta\right)$ is monotonically decreasing in θ, and while in $\theta = 0$ it equals 1, for $\theta = 1$ it equals 0. Moreover, this also shows that when $\theta = 1$, then $\lambda^N = \lambda^D$ and there exists only one threshold. This concludes the proof. □

Proof of Proposition 7

Proof. Let us assume without loss of generality that $\theta_i > \theta_j$. The proof for $\theta_j > \theta_i$ is perfectly symmetric. Thanks to Lemma 1 we know that the order of the thresholds is such that $\lambda_j^D > \lambda_j^N > \lambda_i^N > \lambda_i^D$.

When $R < \lambda_i^D$, we know that R is also smaller than λ_j^N and consequently also smaller than λ_j^D. By definition of the threshold $R < \lambda_i^D$, government i's dominant strategy is to delegate, no matter what is the opponent's action. At the same time, government j delegates as well if i delegates, since $R < \lambda_j^D$ and it also delegates if i does not delegate since $R < \lambda_j^N$. Neither government has a profitable deviation from this action profile and the only equilibrium is (D, D).

When $\lambda_i^N > R > \lambda_i^D$ government i does not have a dominant strategy anymore. When the opponent j delegates government i has a profitable deviation to change his strategy from playing D to playing ND. On the contrary, if the opponent j does not delegate, i does not have a profitable deviation from playing D, since $R < \lambda_i^N$. At the same time, government j does not deviate from his dominant strategy and it plays D because $R < \lambda_j^N < \lambda_j^D$. Hence, the unique equilibrium is (ND, D)

When $\lambda_j^N > R > \lambda_i^N$ government i has, by definition of the thresholds, a dominant strategy not to delegate fighting. It has no profitable deviation from this strategy since $R > \lambda_i^N$. Similarly, also government j has a dominant strategy to delegate fighting no matter what its opponent does. This implies that the only possible equilibrium in this case is (ND, D).

When $\lambda_j^D > R > \lambda_j^N$ government i has, again, a dominant strategy not to delegate fighting since $R > \lambda_j^N$ implies also that $R > \lambda_i^N$. Thus, government i's optimal strategy is to directly enter conflict no matter what its opponent does. On the other hand, government j does not have a dominant strategy. If government i delegates, government j would also delegate since $\lambda_j^D > R$ and it has no profitable deviation by the definition of λ_j^D. If government i does not delegate, j has no profitable deviation from choosing N given the definition of $\lambda_j^N < R$. Hence, the unique equilibrium is (N, N).

Finally, when $R > \lambda_j^D$ both governments have always a dominant strategy not to delegate since $R > \lambda_j^D$ implies, by Lemma 1, both $R > \lambda_j^N$ and $R > \lambda_i^N$.

When $\theta_i = \theta_j$ Lemma 1 shows that there exist only two thresholds λ^N and λ^D such that $\lambda^N > \lambda^D$. □

Proof of Lemma 2

Proof. Since γ_j^{CA} is constant with respect to N and γ_j^{PC} is decreasing linearly with respect with N, it is sufficient to show that in $N = 0$, $\gamma_j^{PC} > \gamma_j^{CA}$ to prove the Lemma. If this is the case, γ_j^{PC} and γ_j^{CA} cross for a unique value of $N = N^*$.
The participation constraint can be written then

$$\gamma_j^{PC} = 1 - \frac{1}{P}\left(N - s^m\left(\frac{\widetilde{V}}{2(1+\sqrt{\theta_j})} + \frac{1}{8\theta_j(1+\sqrt{\theta_j})^2}\right)\right)$$

I want to show that the difference $\gamma_j^{PC} - \gamma_j^{CA} > 0$ in $N = 0$. I can write the difference as,

$$\begin{aligned}\gamma_j^{PC} - \gamma_j^{CA} &= \frac{1 - 16(\theta_i + \sqrt{\theta_i})^2(N-P) + 8\theta_i V(1+\sqrt{\theta_i})}{16\theta_i(1+\sqrt{\theta_i})^2 P} \\ &> 0\end{aligned}$$

if and only the numerator is positive. This condition is always satisfied because when $N = 0$ all terms in the numerator are positive. Moreover, we can find the exact expression for N^* by setting $\gamma_j^{CA} = \gamma_j^{PC}$

$$1 - 16(\theta_i + \sqrt{\theta_i})^2(N^* - P) + 8\theta_i V(1+\sqrt{\theta_i}) = 0$$

which implies

$$N^* = P + \frac{1 + 8(1+\sqrt{\theta_i})\theta_i V}{16(1+\sqrt{\theta_i})^2\theta_i}$$

□

Proof of Lemma 3

Proof. First, I have to show that

$$\frac{\partial \gamma_j^{CA}(\boldsymbol{t}^*)}{\partial \widetilde{V}} < 0$$

Second, I find the condition for which $\gamma_j^{CA}(\widetilde{V} = 0) > 0$ and the two thresholds $\widetilde{V}'$ and $\widetilde{V}''$ can exist. Finally, I find the conditions on $\widetilde{V}$ such that $1 \geq \gamma_j^{CA} \geq 0$.

1. I want to show that the offered γ_j^{CA} is downward sloping in $\widetilde{V}$. I have that

$$\frac{\partial \gamma_j^{CA}(\boldsymbol{t}^*)}{\partial \widetilde{V}} = \frac{1}{2P}\left(\frac{1-s^g}{1+\sqrt{\theta_j}} - \frac{1}{1+\sqrt{\theta_i}}\right)$$

which is always negative since

$$\frac{1-s^g}{1+\sqrt{\theta_j}} - \frac{1}{1+\sqrt{\theta_i}} < 0 \Longleftrightarrow 1 - s^g = s^m < \frac{1+\sqrt{\theta_j}}{1+\sqrt{\theta_i}}$$

which is true for any $\theta_i > \theta_j$ and any $s^m < 1/2$. In fact, the right hand side of the last inequality ranges between 1/2, when $\theta_j = 0$ and $\theta_i = 1$, and 1 when $\theta_i \sim \theta_j$.

2. Now, to find the conditions for which the two thresholds $\widetilde{V}'$ and $\widetilde{V}''$ exist and γ_j^{CA} is positive for some range of $\widetilde{V}$ it is sufficient to compute $\gamma_j^{CA}(\widetilde{V} = 0)$.

$$\gamma_j^{CA}(\widetilde{V} = 0) = -\frac{1}{16P}\left(-\frac{2(1-s^g)}{\theta_j(1+\sqrt{\theta_j})^2} + \frac{1}{\theta_i(1+\sqrt{\theta_i})^2}\right) > 0$$

which is true if and only if

$$\left(-\frac{2(1-s^g)}{\theta_j(1+\sqrt{\theta_j})^2} + \frac{1}{\theta_i(1+\sqrt{\theta_i})^2}\right) < 0$$

which can be written, substituting $s^m = 1 - s^g$, as

$$\frac{\theta_j}{\theta_i}\left(\frac{1+\sqrt{\theta_j}}{1+\sqrt{\theta_i}}\right)^2 < 2\ s^m$$

This condition tells us that when θ_j is considerably smaller than θ_i, government j has a distinct advantage over its opponent and can demand a positive share of power from the militia. If the condition is not satisfied, the competition is too tight and government j can only offer $\gamma_j^{CA} = 0$.

3. Now, I focus on γ_j^{CA} when $\Delta < 2\ s^m$. In this case, there exist a range of values of the stakes of war $\widetilde{V}$ such that the offered γ_j^{CA} is indeed in the interval $[0, 1]$. The conditions that make γ_j^{CA} be in the desired interval can be characterized in function of $\widetilde{V}$ as follows

$$\gamma_j^{CA} = \frac{1}{16P}\left(8V\left(\frac{1-s^g}{1+\sqrt{\theta_j}} - \frac{1}{1+\sqrt{\theta_i}}\right) + \frac{2(1-s^g)}{\theta_j(1+\sqrt{\theta_j})^2} - \frac{1}{\theta_i(1+\sqrt{\theta_i})^2}\right) > 0$$

when

$$\widetilde{V} < \frac{2(1-s^g)(1+\sqrt{\theta_i})\theta_i - (1+\sqrt{\theta_j})^2\theta_j}{8\theta_i\theta_j(1+\sqrt{\theta_i})(1+\sqrt{\theta_j})\left(s^g(1+\theta_i) - \sqrt{\theta_j} - \sqrt{\theta_i}\right)} = \widetilde{V}''$$

Similarly

$$\gamma_j^{CA} = \frac{1}{16P}\left(8V\left(\frac{1-s^g}{1+\sqrt{\theta_j}} - \frac{1}{1+\sqrt{\theta_i}}\right) + \frac{2(1-s^g)}{\theta_j(1+\sqrt{\theta_j})^2} - \frac{1}{\theta_i(1+\sqrt{\theta_i})^2}\right) < 1$$

when

$$\widetilde{V} > \frac{(1+\sqrt{\theta_i})(1+\sqrt{\theta_j})}{(1-s^g)(1+\sqrt{\theta_i}) - (1+\sqrt{\theta_j})}\left[2P\left(1 + \frac{\frac{1-s^g}{(1+\sqrt{\theta_j})^2\theta_j} - \frac{1}{(1+\sqrt{\theta_i})\theta_i}}{16P}\right)\right] = \widetilde{V}'$$

□

The following Corollary just shows the relative position of N^{out} and N^*, where N^{out} is the value of the outside option of neutrality that makes the militia indifferent between accepting the contract and leaving the contest, i.e. when $\gamma_j^{PC}(t_j^{CA}) = 0$. Now we have all the elements to find the equilibrium of the common agency game.

Corollary.

Let us assume that government j has a clear ideological advantage, i.e. $\Delta > 2$. The value of the outside option $N^* \leq N^{out}$ if $\widetilde{V} \leq \widetilde{V}'$. Otherwise, if $\widetilde{V} \geq \widetilde{V}'$ then $N^* \geq N^{out}$

Proof. We know from Lemma 2 that if $N < N^*$ then $\gamma_j^{PC} > \gamma_j^{CA}$ and $N^* \leq N^{out}$ when γ_j^{CA} is positive. This occurs when $\widetilde{V} \leq \widetilde{V}'$ by Lemma 3. When $\widetilde{V}$ increases γ_j^{CA} decreases, until a point where it becomes equal to zero for any value of N. At this point $N^* = N_{out}$. Infact

$$\frac{\partial \gamma_j^{CA}}{\partial \widetilde{V}} = -\frac{3}{1+\sqrt{\theta_i}} + \frac{1}{1+\sqrt{\theta_j}} < 0$$

since $\sqrt{\theta_i} < 2 + 3\sqrt{\theta_j}$. When $\widetilde{V}$ further increases then $N^* \geq N^{out}$ because

$$\frac{\partial N^*}{\partial \widetilde{V}} = \frac{3P}{\sqrt{\theta_i}+1} + \frac{1-2P}{2\sqrt{\theta_j}+2} >$$
$$\frac{\partial N_{out}}{\partial \widetilde{V}} = \frac{1}{2(1+\sqrt{\theta_j})}$$

□

Proof of Proposition 8

Proof. As Lemma 3 shows, if $\Delta < 2\ s^m$ and $\widetilde{V} < \widetilde{V}''$ then γ_j^{CA} is indeed positive. In this case, government j is able to put i's utility down to zero by offering a share of power smaller or equal to γ_j^{CA}, since u_G^i increases with γ_j. Government i then abstains from sending offers since he is put out of competition by the contract offered by j. The militia receives higher rents because she receives $\gamma_j^{CA} < \gamma_j^{PC}$. On the other hand, if $\widetilde{V} > \widetilde{V}''$ or $1 > \Delta > 2\ s^m$ then $\gamma_j^{CA} < 0$, meaning that government j is not able to put i out of competition by offering a low γ_j, which is bounded by definition to be at least 0. In this situation, both governments send offers characterized by $\gamma_i = \gamma_j = 0$ and they compete on the transfers. Government i can increase the transfer until a point where $u_G^i(t_i^{max}) = 0$, which defines a maximum value of N that can be counterbalanced by i's offer. Since government j has a lower cost of transferring funds (and the militia has a lower cost of paying its fighters when affiliated to government j), it can easily offer $t_j = t_i^{max} + \varepsilon$ to get the militia's allegiance. □

Proof of Proposition 9

Proof. Let us start with noticing that u_G^i and u_G^j are concave with respect to t_i and t_j. They maximize with respect to the transfers when the constraint CA is binding. It is easy to prove that the constraint CA must be binding. Suppose it is not. Then, for government i there exists a contract (γ_i, t_i) such that $u_m^i > u_m^j$ strictly. Following the same reasoning, government j also offers an optimal contract (γ_j, t_j) and $u_m^j > u_m^i$ strictly. Since the offers are simultaneous it implies that both $u_m^i > u_m^j$ and $u_m^j > u_m^i$, a contradiction. Then the constraint CA must bind and the two principals maximize over the optimal transfers when the constraint binds with equality. I find the optimal $t_{k,CA}^*$ when CA is binding. It is easy that $u_G^j(t_{j,CA}^*) > 0$ for any value of γ_j and of $\widetilde{V}$.

Let us assume without loss of generality that $\theta_i > \theta_j$. I divide the proof in two parts: first when $\Delta < 2\ s^m$ and $\widetilde{V} < \widetilde{V}''$, second when $1 > \Delta > 2\ s^m$ or $\widetilde{V} > \widetilde{V}''$.

(i) $\Delta < 2\ s^m$ and $\widetilde{V} < \widetilde{V}''$

As Lemma 3 shows, when $\Delta < 2\ s^m$ there exist two thresholds $\widetilde{V}'$ and $\widetilde{V}''$ for which γ_j^{CA} is between 0 and 1. Let us first look at the case where $\widetilde{V}'' > \widetilde{V} > \widetilde{V}'$ which implies $1 > \gamma_j^{CA} > 0$.

I first consider the case where $N < N^*$. Using Lemma 2 and Lemma 3 we know that in $N = 0$, $\gamma_j^{PC} > \gamma_j^{CA}$. Since γ_{CA} is constant in N and γ_{PC} linearly decreasing, we showed that there is a threshold level of the outside option $N^* > 0$ such that for values smaller than N^*, $\gamma_{PC} > \gamma_{CA}$ and for values bigger than N^*, $\gamma_{PC} < \gamma_{CA}$. Let us first analyze the case where $N < N^*$. In this case, government j offers γ_j^{CA} and, by definition of γ_j^{CA}, government i is indifferent between entering or not. Suppose this is not an equilibrium, and government j deviates by offering $\gamma_j^{dev} > \gamma_j^{CA}$. In this case, government i still participates to the contest and tries to hire the militia offering t_i^{CA} and possibly undercutting γ_i to such a low level that makes the militia indifferent between pledging allegiance to i or to j. This is clearly not an equilibrium, because j has now an incentive to undercut γ_j by offering $\gamma_j < \gamma_j^{dev}$ and by triggering a sequence of mutual undercuttings until a point where j offers $\gamma_j^{CA} - \varepsilon$, it is able to throw the opponent i out of competition and gets the militia. Similarly, j has no incentive to deviate by offering less than γ_j^{CA}. If it does, it just reduces the benefit from political control over the militia, in a situation where it does not face any competition and the militia accepts the offer for sure. Finally, let us consider a deviation on both contracting variables whereby j changes by ε the transfer. This entails a loss of order ε^2 that has to be compensated by an increase in the demanded γ_j. By doing so, government i re-enters the competition. This is not an equilibrium because it would start a new series of undercuttings of γ_i and γ_j until a point where j offers γ_j^{CA} and i is put at the indifference level. Now, j wins the competition for the militia and changes the transfer to its optimal value.

When $N^* < N < N^{out}$, γ_j^{CA} is no more optimal. For those values of N, $\gamma_j^{CA} > \gamma_j^{PC}$ and the participation constraint has to bind. Government j optimally offers the participation constraint itself and i always stays out of the competition since $\gamma_j^{PC} < \gamma_j^{CA}$. Suppose this is not an equilibrium and j demands a higher share of γ_j. Then the militia does not accept the offer because the participation constraint is violated and the government j receives a payoff of 0; this is not an equilibrium because by lowering its demanded share of power until γ_j^{PC} it gets a positive payoff. By offering less than γ_j^{PC}, it uselessy gives away some political power to the militia. This is clearly not an equilibrium because it can increase γ_j until γ_j^{PC} and still get the militia's allegiance.

When $N > N^{out}$ the share of power derived from the participation constraint touches its floor, $\gamma_j^{PC} = 0$, and the militia is indifferent between accepting and refusing the contract. Now, I can define a N_j^{max} as the value of outside option N that can be hired by government j with the maximum transfer t_j^{max} such that $u_G^j(t_j^{max}) > 0$. For every $N_j^{max} > N > N^{out}$ government j is able increase the transfer to $t_j(N)$ in order to keep the militia at the indifference level. Suppose this is not an equilibrium. For any deviation away from $(t_j(N), \gamma_j = 0)$, the militia will prefer to remain neutral and government j gets a payoff of 0 which is strictly less of its payoff when offering $(t_j(N), \gamma_j = 0)$. Moreover, since government j's utility is concave in the transfers, it can increase the transfer only until a point defined by a maximum value of the outside option N^{max}. If $N = N_{max} + \varepsilon$, the required transfer $t_j^{max} + \varepsilon$ to hire the militia is too high, $u_G^j(t_j^{max} + \varepsilon) < 0$ and it prefers to stay out of the contest.

(ii) $1 > \Delta > 2\, s^m$ or $\widetilde{V} > \widetilde{V}''$.

These conditions imply that $\gamma_j^{CA} < 0$ and, by Lemma 3, that government j is not able to put i's utility down to zero by offering the minimum possible value of $\gamma_j = 0$. Government i also enters the competition and offers $\gamma_i = 0$. Now, both governments can compete only by increasing the transfers. Government i has a higher cost of transferring funds and stops when N is too large, i.e. when $N = N_i^{max} < N_j^{max}$, which is the maximum value of N that j can afford. For government j it is sufficient to offer t_j such that $u_m^j > u_m^i$ which implies

$$\frac{t_j + \widetilde{V}}{1 + \sqrt{\theta_j}} > \frac{t_i^{max} + \widetilde{V}}{1 + \sqrt{\theta_i}}$$

$$\implies t_j > (t_i^{max} + \widetilde{V})\frac{1 + \sqrt{\theta_j}}{1 + \sqrt{\theta_i}} - \widetilde{V}$$

and the militia pledges allegiance to j.

$\square$