On the Welfare Effects of Adverse Selection in Oligopolistic Markets*

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Abstract

We consider a principal-agent relationship with adverse selection. Principals pay informational rents due to asymmetric information and sell their output in a homogeneous Cournot-oligopoly. We find that asymmetric information may mitigate or more than compensate for the welfare reducing impact of market power. We further show that welfare in a setting with adverse selection may be higher than the maximized welfare level attainable in a world with perfect observability.

Keywords: Adverse Selection, Oligopoly, Welfare

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1 Introduction

The rent-efficiency trade-off implies a welfare loss in a principal-agent relationship with adverse selection. Usually, this result is derived for simple bilateral negotiations between, for instance, a firm or its owner (principal) and one manager (agent). Moreover, there generally is no interaction on the output market.[1] However, pure monopolies, perfect competition or a constant demand per firm are rare market characteristics. Instead, "(o)ligopoly is pervasive in our daily live" (Head and Spencer, 2017, p. 1415).

Therefore, in this paper we study an adverse selection problem in an oligopolistic market. We show that the characteristics of the output market can fundamentally affect the rent-efficiency trade-off. More precisely, welfare with asymmetric information can be higher than welfare, which results in a setting with perfect observability of a manager's type. This is because in an oligopoly the contracts offered by the owner ensure that the output of the low-cost manager increases in a setting with asymmetric information in comparison to a world with perfect observability. This response is in contrast to the usual result of no distortion at the top, which arises in a world with a single principal. It comes about because the informational rent declines with the number of firms. As it is standard, output of the high-cost manager falls. The net impact on expected aggregate output is negative. Moreover, the variance of aggregate output is higher in a setting with asymmetric information. These two effects - lower expected aggregate output and higher variance - imply a reduction in welfare, relative to a setting with perfect observability. However, the greater spread between the output produced by a high-cost and a low-cost manager results in a fall in expected production costs. This welfare-enhancing effect of a more efficient production in a world with asymmetric information can outweigh the welfare-reducing effects if the share of low-cost managers is not too high and the overall number of oligopolists exceeds a threshold level.

As the above reasoning clarifies, our analysis is primarily related to studies that investigate adverse selection in oligopolistic markets.[2] Piccolo (2011) focuses on entry decisions in a Salop (1979) type model with asymmetric information and derives conditions under

[1] Seminal papers are Stiglitz (1977), Guesnerie and Laffont (1984) and Demski and Sappington (1984).

[2] Perfectly competitive markets have been studied, for instance, by Hart (1983), Scharfstein (1988) and recently by Pouyet et al. (2008) and Guerrieri et al. (2010).

which greater uncertainty makes entry more costly. Etro and Cella (2013) investigate the effects of competitors on incentive schemes and R&D investment decisions. Bonazzi et al. (2020) analyze manufacturer-retailer hierarchies and compare the welfare consequences of different vertical price restraints under asymmetric information.[3] Furthermore, given that there is ex-ante uncertainty about productivity, our analysis features mechanisms, which are also present in investigations of the effects of price (or cost) uncertainty in the tradition of Waugh (1944), Oi (1961), and Massell (1969). Finally, one of the channels affecting the relationship between welfare and the number of firms is also present in analyses of excessive entry in Cournot-oligopolies, which build on the seminal contributions by von Weizsäcker (1980), Mankiw and Whinston (1986) and Suzumura and Kiyono (1987).[4] In contrast to our analysis, none of the contributions mentioned above compares the welfare effects of different informational structures in an oligopoly.

2 The Model

2.1 Set-up

There is an exogenously given number of n firms, which compete in a Cournot-oligopoly. Each firm is owned by a (male) principal. Lacking in managerial expertise, production is only feasible if the owner delegates governance of the firm to a manager (a female agent). We index owners and managers, who are both risk-neutral, by $i = 1, 2, ..., n$. Owners incur sunk costs $F, F > 0$, to settle productivity. Each unit of effort, which a manager provides, generates one unit of output, q.

To obtain closed-form solutions, the (inverse) demand schedule is linear, $P(Q) = a - bQ$, with P being the price, a the choke price, and b a positive parameter. Total output is denoted by Q and consumer surplus is given by $CS = bQ^2/2$. A manager's outside options is, for simplicity, normalized to zero. As the measure of welfare, we employ the sum of expected profits of all n firms, expected consumer surplus, and expected utility of

[3] There are also some studies that scrutinize the implications of adverse selection in duopoly and, consequently, cannot consider the effects of the number of firms, which play a decisive role in our analysis (see e.g., Martimort, 1996, Piccolo and Pagnozzi, 2013).

[4] Mukherjee and Tsai (2014) consider the excess entry proposition in the presence of managerial delegation. However, they neither incorporate adverse selection nor consider the welfare impact of delegation.

all n managers.

A manager's utility equals the wage w minus the costs of production θq. The probability that a manager has low marginal costs is $Pr[\theta = \underline{\theta}] = v$ and the converse probability is $Pr[\theta = \overline{\theta}] = 1 - v$, $\theta \in \{\underline{\theta}, \overline{\theta}\}$, $\overline{\theta} > \underline{\theta}$, $0 < v < 1$. We assume that types are uncorrelated and we denote the spread between marginal costs by $\Delta\theta \triangleq \overline{\theta} - \underline{\theta}$. For each owner and a θ-type manager, in realization, profits π_i and utility U_i are:

$$\pi_i(w_i(\theta), q_i(\theta), Q) = (a - bQ)q_i(\theta) - w_i(\theta) - F, \tag{1}$$

$$U_i(w_i(\theta), q_i(\theta), \theta) = w_i(\theta) - \theta q_i(\theta). \tag{2}$$

We consider two informational structures: The first presumes that the owner can perfectly observe his manager's type and, hence, conditions the contract he offers on her productivity. In the second setting with asymmetric information, marginal production costs are private knowledge of the manager. The timing is as follows:

1. Managers realize their type.
2. Each owner meets one manager and offers her a contract, i.e. determines the incentive scheme.
3. Each manager decides about the offer. If she accepts, the contract is executed. If the manager declines, she receives the outside option of zero and the owner incurs a loss of F.
4. All firms that have hired a manager compete in quantities in the product market.
5. Payments are made and profits are realized.

For both informational structures, we study a Bayesian Cournot-Nash equilibrium, in which each firm i has a belief about the aggregate output produced by all other firms, Q_{-i}. Given that types are uncorrelated, knowledge of the own manager's type does not provide a firm with information about marginal costs of other firms. From firm i's point of view, in expectation $(n-1)v$ other firms are matched with a $\underline{\theta}$-type, while the remaining $(n-1)(1-v)$ firms employ a $\overline{\theta}$-type manager.

Focusing on a symmetric (separating) equilibrium, in which firms of the same type offer identical contracts, expected output of all other firms is:

$$\mathbb{E}_\theta[Q_{-i}] = \sum_{j \neq i} \mathbb{E}[q_j(\theta)] = (n-1)[v\underline{q} + (1-v)\overline{q}],$$

where $\underline{q} \triangleq q(\underline{\theta})$ and $\overline{q} \triangleq q(\overline{\theta})$ is used to simplify notation. Owner i's expected profits are:

$$\mathbb{E}_\theta[\pi_i] = v[(a - b\underline{q}_i - b\mathbb{E}_\theta[Q_{-i}])\underline{q}_i - \underline{w}_i] + (1-v)[(a - b\overline{q}_i - b\mathbb{E}_\theta[Q_{-i}])\overline{q}_i - \overline{w}_i] - F, \quad (3)$$

with $\underline{w}_i \triangleq w_i(\underline{\theta})$ and $\overline{w}_i \triangleq w_i(\overline{\theta})$. Since firms are (ex-ante) identical, we subsequently omit the index i. Moreover, we restrict our analysis to the case of strictly positive quantities and thereby implicitly assume that the choke price a is sufficiently high to ensure that no owner opts for a shut-down policy if he is matched with a high-cost manager.

Each firm stochastically produces $q(\theta)$. Thus, the aggregate quantity is the sum of n i.i.d. random variables, such that $\mathbb{E}_\theta[Q] = n\mathbb{E}[q(\theta)]$ and $\mathrm{Var}_\theta[Q] = n\mathrm{Var}[q(\theta)]$ hold. Expected welfare W is:

$$\mathbb{E}_\theta[W] = \mathbb{E}_\theta[(a - \frac{b}{2}Q)Q - n\theta q(\theta) - nF]. \quad (4)$$

To better highlight our result, we decompose expected welfare, making use of the variance identity $\mathrm{Var}_\theta[Q] = \mathbb{E}_\theta[Q^2] - \mathbb{E}_\theta[Q]^2$, and obtain:

$$\mathbb{E}_\theta[W] = \int_0^{\mathbb{E}_\theta[Q]} P(t)dt - \frac{b}{2}n\mathrm{Var}[q(\theta)] - n\mathbb{E}[\theta q(\theta)] - nF. \quad (5)$$

The first two terms of (5) would equal the society's expected surplus if there were no costs. In particular, the second term is the surplus reduction due to the variability in output (Shapiro, 1986). The expression clarifies that, other things being equal, the variance in aggregate output increases consumer surplus by less than it decreases the firms' expected aggregate revenues. The third term in (5) describes expected production costs, which also vary with the spread between quantities. The final term delineates total sunk costs.[5] Notably, an increase in the spread between the quantities could result in a

[5] An alternative expression for expected welfare can be obtained by adding and subtracting the term

more efficient use of resources and, thereby, enhance welfare. Other things being equal, a rise in the quantity provided by low-cost managers and a fall in the quantity of the high-cost managers reduce expected production costs.[6]

2.2 Perfect Observability

Under perfect observability, denoted by the subscript p, an owner observes the type of the manager that he is matched with, before a contract is offered. Each owner maximizes (3), subject to the participation constraints

$$U\left(w(\theta), q(\theta), \theta\right) = w(\theta) - \theta q(\theta) \geqslant 0 \qquad \forall\, \theta \in \left\{\underline{\theta}, \overline{\theta}\right\}. \qquad PC(\theta)$$

The wage compensates for the cost and, therefore, the manager is left with her reservation utility level of zero. The output choice equalizes marginal revenues with marginal costs. This yields:

Lemma 1.

Under perfect observability, quantities, expected aggregate output and expected price are:

$$\underline{q}_p = \frac{a-\underline{\theta}}{b(1+n)} - \frac{(1-n)(1-v)}{2b(1+n)}\Delta\theta, \tag{7}$$

$$\overline{q}_p = \frac{a-\overline{\theta}}{b(1+n)} + \frac{(1-n)v}{2b(1+n)}\Delta\theta, \tag{8}$$

$$\mathbb{E}_\theta[Q_p] = \frac{n(a-\mathbb{E}[\theta])}{b(1+n)}, \tag{9}$$

$$\mathbb{E}_\theta[P_p] = \frac{a+n\mathbb{E}[\theta]}{1+n}. \tag{10}$$

Proof Lemma 1.

See Appendix A.1.

$n\mathbb{E}[\theta]\mathbb{E}[q(\theta)]$ in (5). This leads to a decomposition reminiscent of Shapiro (1986):

$$\mathbb{E}_\theta[W] = (a - \frac{b}{2}n\mathbb{E}[q(\theta)] - \mathbb{E}[\theta])n\mathbb{E}[q(\theta)] - \frac{b}{2}n\text{Var}[q(\theta)] + n(\mathbb{E}[\theta]\mathbb{E}[q(\theta)] - \mathbb{E}[\theta q(\theta)]) - nF. \tag{6}$$

In (6), the first term equals the surplus for society if marginal costs and the quantity are non-stochastic and equal to their respective means. The third term is the negative covariance between the output of the firm and the marginal costs of the manager a firm is matched with.

[6] The fact that a higher variance of output raises expected profits is well-established. See, for instance, Bergstrom and Varian (1985) in Cournot settings and Waugh (1944), Oi (1961), or Massell (1969) for the basic idea focusing on price variability.

Since managers obtain zero utility, expected welfare is given by:

$$\mathbb{E}_\theta[W_p] = n\mathbb{E}_\theta[\pi_p] + \mathbb{E}_\theta[CS_p]. \tag{11}$$

Substituting in accordance with Lemma 1, and decomposing expected welfare as in (5), we can express $E_\theta[W_p]$ as a function of exogenous parameters and the number of firms, n:

$$\begin{aligned}\mathbb{E}_\theta[W_p(n)] =& \frac{n(a-\overline{\theta}+v\Delta\theta)(a(2+n)+n\mathbb{E}[\theta])}{2b(1+n)^2} - \underbrace{\frac{nv(1-v)\Delta\theta^2}{8b}}_{\frac{b}{2}\mathrm{Var}_\theta[Q_p(n)]} \\ & - n\underbrace{\left(\frac{(a-\overline{\theta}+v\Delta\theta)\mathbb{E}[\theta]}{b(1+n)} - \frac{v(1-v)\Delta\theta^2}{2b}\right)}_{\text{expected cost } \mathbb{E}[\theta q_p(\theta,n)]} - nF.\end{aligned} \tag{12}$$

Expected welfare is strictly concave in n as there are two countervailing effects. On the one hand, more firms intensify competition, such that expected aggregate output and, therefore, the first term in (12) rise. Moreover, expected variable production costs decline. Inspection of the first and third terms in (12) clarifies that the resulting increase in expected welfare is not linear in the number of firms, but becomes smaller the more oligopolists there are. On the other hand, the variance in output increases in the number of firms, n. The net impact of a higher variability in output on welfare is negative. Furthermore, each competitor incurs the fixed costs, F, of settling productivity. The effects captured by the second and fourth terms in (12) imply a welfare reduction, which increases linearly in the number of firms. As the number of competitors grows, the benefit for the society resulting from higher expected output and lower expected production costs can outweigh the detrimental impact due to a greater variance and higher fixed cost.[7]

2.3 Asymmetric Information

In the presence of asymmetric information, denoted by the subscript a, a manager's type is private information when the owner offers contracts. Each owner defines a mechanism $\langle q(\theta), w(\theta)\rangle$, which entails a transfer w for the observable and verifiable output. By the

[7] A similar trade-off is underlying the determination of optimal entry in the settings considered by, for example, Mankiw and Whinston (1986) and Suzumura and Kiyono (1987).

revelation principle, we analyze a direct revelation mechanism where managers truthfully reveal their types.

Formally, owners maximize expected profits, as defined in (3), by choosing wages and quantities, subject to the participation constraints of managers ($PC(\theta)$) and the incentive compatibility (IC) constraints

$$U\left(w\left(\underline{\theta}\right),q\left(\underline{\theta}\right),\underline{\theta}\right)\geqslant U\left(w\left(\overline{\theta}\right),q\left(\overline{\theta}\right),\underline{\theta}\right), \qquad IC\left(\underline{\theta}\right)$$

$$U\left(w\left(\overline{\theta}\right),q\left(\overline{\theta}\right),\overline{\theta}\right)\geqslant U\left(q\left(\underline{\theta}\right),w\left(\underline{\theta}\right),\overline{\theta}\right). \qquad IC\left(\overline{\theta}\right)$$

As it is standard (see e.g., Laffont and Martimort, 2002), the participation constraint of the low-cost type, $\underline{\theta}$, is satisfied and the incentive constraint of the high-cost type, $\overline{\theta}$, is slack at the optimum. Moreover, the other two constraints bind and, therefore, an owner's problem reduces to:

$$\underset{\{\underline{q},\overline{q}\}}{Max}\,\mathbb{E}_\theta\left[\pi_a\right]=\begin{pmatrix} v\left[(a-b\underline{q}-b\mathbb{E}_\theta[Q_{-i}]-\underline{\theta})\underline{q}\right]+ \\ (1-v)\left[(a-b\overline{q}-b\mathbb{E}_\theta[Q_{-i}]-\overline{\theta})\overline{q}\right]-v\Delta\theta\overline{q}\end{pmatrix}-F. \tag{13}$$

Making use of the first-order conditions and the binding constraints, contracts are implicitly defined as:

$$\begin{array}{ll} a-b\mathbb{E}_\theta[Q_{-i}]-2b\underline{q}_a=\underline{\theta} & a-b\mathbb{E}_\theta[Q_{-i}]-2b\overline{q}_a=\overline{\theta}+\frac{v}{1-v}\Delta\theta \\ \underline{w}_a=\underline{\theta}\underline{q}_a+\Delta\theta\overline{q}_a & \overline{w}_a=\overline{\theta}\overline{q}_a \end{array}. \tag{14}$$

Therefore, each low-cost type obtains a rent, $\Delta\theta\overline{q}_a$, and each firm pays an expected rent, which is given by $\mathbb{E}_\theta[rent_a]\triangleq v\Delta\theta\overline{q}_a$. This yields:

Lemma 2.

Under asymmetric information, quantities, expected aggregate output and expected price *are:*

$$\underline{q}_a = \frac{a-\underline{\theta}}{b(1+n)} - \frac{(1-n)}{2b(1+n)}\Delta\theta, \tag{15}$$

$$\overline{q}_a = \frac{a-\overline{\theta}}{b(1+n)} - \frac{1}{2b}\frac{v}{1-v}\Delta\theta, \tag{16}$$

$$\mathbb{E}_\theta[Q_a] = \frac{n(a-\overline{\theta})}{b(1+n)}, \tag{17}$$

$$\mathbb{E}_\theta[P_a] = \frac{a+n\overline{\theta}}{1+n}. \tag{18}$$

Proof Lemma 2.

See Appendix A.2.

A comparison of (7), (8), (15), and (16) clarifies that $\underline{q}_a(n) > \underline{q}_p(n) > \overline{q}_p(n) > \overline{q}_a(n)$ for $n > 1$. Furthermore, (16) indicates that the informational rent, $\Delta\theta\overline{q}_a$, declines with the number of firms.

Expected welfare in case of asymmetric information is:

$$\mathbb{E}_\theta[W_a] = n\left(\mathbb{E}_\theta[\pi_a] + \mathbb{E}_\theta[rent_a]\right) + \mathbb{E}_\theta[CS_a]. \tag{19}$$

Substituting in accordance with Lemma 2, and decomposing expected welfare as in (5), we can express $\mathbb{E}_\theta[W_a]$ as a function of the number of firms, n:

$$\begin{aligned}\mathbb{E}_\theta[W_a(n)] =& \frac{n(a-\overline{\theta})(a(2+n)+n\overline{\theta})}{2b(1+n)^2} - \underbrace{\frac{nv\Delta\theta^2}{8b(1-v)}}_{\frac{b}{2}\mathrm{Var}_\theta[Q_a(n)]} \\ &- n\underbrace{\left(\frac{(a-\overline{\theta})\mathbb{E}[\theta]}{b(1+n)} - \frac{v\Delta\theta^2}{2b}\right)}_{\text{expected cost } \mathbb{E}[\theta q_a(\theta,n)]} - nF.\end{aligned} \tag{20}$$

Again, expected welfare is strictly concave in n, reflecting the same trade-off as in the case of perfect observability.

2.4 Welfare Comparison

Given an oligopolistic market structure, we focus our analysis on $n > 1$. Provided it exists, we denote with $\bar{n}$ the unique critical number of firms for which $\mathbb{E}_\theta[W_a(\bar{n})] = \mathbb{E}_\theta[W_p(\bar{n})]$ holds, i.e. expected welfare, which results in the two informational structures, is the same. It is given by [see (12) and (20)]:

$$\overline{n} = \frac{v^2\Delta\theta + 2\sqrt{v(1-v)\Delta\theta(2(a-\overline{\theta})(2-3v) + v\Delta\theta(3-4v))}}{v\Delta\theta(2-3v)}. \tag{21}$$

Moreover, we denote $n_k^* \triangleq \text{argmax}_n \, \mathbb{E}_\theta[W_k(n_k)]$, $k = \{a, p\}$, and consider an interior solution.[8] Then, we have:

Proposition 1.

Assume that $v < 2/3$ holds, i.e. $\bar{n} > 0$ exists.

(i) If $n > \overline{n}$, $\mathbb{E}_\theta[W_a(n)] > \mathbb{E}_\theta[W_p(n)]$.

(ii) If the variance of marginal costs, $\text{Var}[\theta] = v(1-v)\Delta\theta^2$, is sufficiently large, $\mathbb{E}_\theta[W_a(n_p^*)] > \mathbb{E}_\theta[W_p(n_p^*)]$.

Proof Proposition 1.

See Appendix A.4.

Figure 1 illustrates Proposition 1, assuming n to be a continuous variable. Consider initially a monopoly ($n = 1$). Welfare (in expectation) is unambiguously lower in the asymmetric information setting than in the case of perfect observability. This is because an owner saves on informational rent payments by lowering the quantity produced by the high-cost manager, while the quantity produced by the low-cost manager remains the same, i.e. there is no distortion at the top. This implies $\mathbb{E}_\theta[W_a(1)] < \mathbb{E}_\theta[W_p(1)]$.[9]

[8] In Appendix A.3, we compute the f.o.c. for a welfare maximum under both informational structures. If $8bF - 3\text{Var}[\theta] \leqslant 0$, the solution to the maximization of expected welfare under perfect observability admits a corner solution. The result as in Proposition 1 would remain valid if we allowed for such a case.

[9] For this standard result see, for example, Laffont and Martimort (2002, in particular chapter 2.6.) and the references therein.

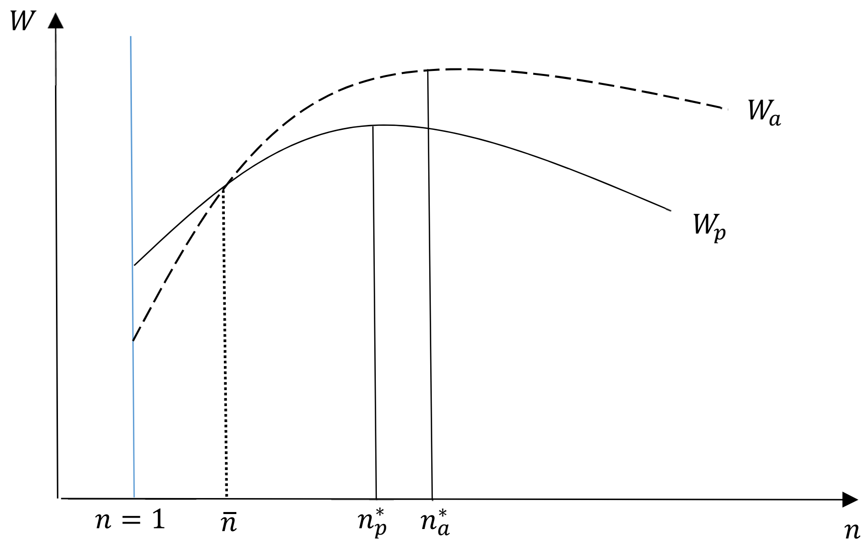

Figure 1: Comparing Welfare Levels

Moving from a monopoly to an oligopoly, i.e. (marginally) increasing the number of firms n, alters the structure of contracts. As also indicated by Etro and Cella (2013), owners set contracts such that the output of the high-cost manager falls by more in a setting with perfect observability than if there is asymmetric information [see (8) and (16)]. Moreover, output of the low-cost manager in a world with asymmetric information rises with the number of firms, relative to the output if there is perfect observability [see (7) and (15)]. Consequently, there are distortions at the bottom and the top. This latter effect becomes more pronounced with the number of firms because the informational rent declines in n.

The contractual adjustments have conflicting consequences for welfare. First, expected aggregate output is lower in an oligopoly with asymmetric information than in a world with perfect observability [see the first terms in (12) and (20)], which has a negative impact on the ratio $W_a(n)/W_p(n)$. Second, the variance of aggregate output is higher. This raises expected consumer surplus and reduces the firms' expected revenues. Since the revenue impact dominates, the variance effect reduces the ratio $W_a(n)/W_p(n)$ [as evidenced by the second terms in (12) and (20)]. Third, asymmetric information results in a more efficient production, since it reduces expected production costs [compare the third terms in (12) and (20)], which, c.p., raises $W_a(n)/W_p(n)$. The welfare-enhancing effect of a more efficient production can outweigh the two welfare-reducing effects, such that $W_a(n)/W_p(n) > 1$ holds, if the probability v of being matched with a low-cost type is not too high ($v < 2/3$), as long as the number of firms exceeds $\bar{n}$.

To gain further intuition, note that a higher variance of aggregate output reduces welfare, as elucidated above, while a greater variance of marginal costs lowers total expected production costs and, thereby, has the opposite impact.[10] Under perfect observability, the variance of the aggregate quantity is proportional to the variance of marginal costs and this ratio is independent of the probability v of having a low-cost manager, since $\mathrm{Var}_\theta[Q_p] = (n/(2b)^2)\mathrm{Var}[\theta]$. In case of asymmetric information, the ratio of variances rises with v. Accordingly, the welfare loss due to greater output variability increases, relative to the welfare gain resulting from the more efficient mode of production. Therefore, when we juxtapose the two effects we observe that if the odds of meeting a low-cost type are high, welfare with perfect observability exceeds welfare under asymmetric information, irrespective of the number of firms n. Conversely, if v is relatively small, for some critical $\overline{n}$, the welfare-increasing effect of a more efficient production is sufficiently high to compensate for the loss of surplus due the output reduction and the variance of the aggregate quantity. For any $n > \overline{n}$, welfare in a setting with asymmetric information exceeds welfare resulting in a situation with perfect observability.

Finally, the higher the variance of marginal costs is, the more similar in magnitude the variances of aggregate output in both informational settings are. If, therefore, the variance of marginal costs is sufficiently high, the variances of aggregate output are comparable in size. In consequence, expected welfare in a setting with asymmetric information exceeds the maximal level attainable in case of perfect observability of managers' type, i.e. $W_a(n_p^*) > W_p(n_p^*)$.

3 Discussion

Our paper assumes the co-existence of a principal-agent relationship in the production sphere with a Cournot-oligopoly on the output market. We distinguish two informational

[10]This result is implicit in Bergstrom and Varian (1985) and made explicit in Salant and Shaffer (1999).

settings. In a situation with perfect observability, an owner observes the type of manager he is matched with. In case of asymmetric information, only the manager is aware of her type at the contracting stage. As our main result, we establish the existence of a separating equilibrium, which can result in a higher level of welfare in case of asymmetric information than resulting in a world with perfect observability. It is even possible that welfare with asymmetric information exceeds the maximum welfare level obtainable with perfect observability. In short: Having less knowledge can raise welfare.

From the perspective of society, given that informational frictions are a widespread phenomenon, our result supports the idea of pro-competitive policy measures in order to overcome the welfare-reducing combination of sufficiently high market power and asymmetric information. In addition, our paper provides another example for a second-best world.

A Appendix

A.1 Proof of Lemma 1

Under perfect observability of a manager's marginal costs, each firm owner solves the following problem:

$$\max_{\{\underline{q},\underline{w},\overline{q},\overline{w}\}} \mathbb{E}_\theta[\pi_p]$$

$$s.t.$$

$$U\left(\underline{w},\underline{q},\underline{\theta}\right) = \underline{w} - \underline{\theta}\underline{q} \geqslant 0, \qquad PC(\underline{\theta})$$

$$U\left(\overline{w},\overline{q},\overline{\theta}\right) = \overline{w} - \overline{\theta}\overline{q} \geqslant 0. \qquad PC(\overline{\theta})$$

Since types are observable, both participation constraints bind, i.e., each owner sets the wage in such a manner that it just compensates the manager for her costs and makes her indifferent between working and obtaining the outside option of zero. Using wages for the binding PCs in the objective function, the maximization problem consists of finding quantities:

$$\underset{\{\underline{q},\overline{q}\}}{Max}\,\mathbb{E}_\theta[\pi_p] = \begin{pmatrix} v\left[(a - b\underline{q} - b\mathbb{E}_\theta[Q_{-i}] - \underline{\theta})\underline{q}\right] + \\ (1-v)\left[(a - b\overline{q} - b\mathbb{E}_\theta[Q_{-i}] - \overline{\theta})\overline{q}\right] \end{pmatrix} - F. \tag{A.1}$$

Computing the f.o.c.s, which are also sufficient, yields:

$$\frac{d\mathbb{E}_\theta[\pi_p]}{d\underline{q}} = -b\underline{q} + (a - b\underline{q} - b\mathbb{E}_\theta[Q_{-i}] - \underline{\theta}) = 0, \tag{A.2}$$

$$\frac{d\mathbb{E}_\theta[\pi_p]}{d\overline{q}} = -b\overline{q} + (a - b\overline{q} - b\mathbb{E}_\theta[Q_{-i}] - \overline{\theta}) = 0. \tag{A.3}$$

Together with the expression for the aggregate quantity, we have a linear system with three unknowns:

$$\begin{cases} -b\underline{q} + (a - b\underline{q} - b\mathbb{E}_\theta[Q_{-i}] - \underline{\theta}) = 0 \\ -b\overline{q} + (a - b\overline{q} - b\mathbb{E}_\theta[Q_{-i}] - \overline{\theta}) = 0 \\ \mathbb{E}_\theta[Q_{-i}] = (n-1)[v\underline{q} + (1-v)\overline{q}]. \end{cases} \tag{A.4}$$

It is straightforward to obtain the solutions:

$$\underline{q}_p = \frac{a - \underline{\theta}}{b(1+n)} - \frac{(1-n)(1-v)}{2b(1+n)}\Delta\theta, \tag{A.5}$$

$$\overline{q}_p = \frac{a - \overline{\theta}}{b(1+n)} + \frac{(1-n)v}{2b(1+n)}\Delta\theta. \tag{A.6}$$

Then, the expected aggregate quantity is:

$$\mathbb{E}_\theta[Q_p] = n[v\underline{q}_p + (1-v)\overline{q}_p] = \frac{n(a - \mathbb{E}[\theta])}{b(1+n)}. \tag{A.7}$$

Given the demand schedule, we find:

$$\mathbb{E}_\theta[P_p] = a - b\mathbb{E}_\theta[Q_p] = \frac{a + n\mathbb{E}[\theta]}{1+n}. \tag{A.8}$$

Equations (A.5)-(A.8) are the equations as in Lemma 1.

A.2 Proof of Lemma 2

To derive the menu of contracts that each owner posts, we follow standard techniques, change variables as in Laffont and Martimort (2002), and express everything in terms of rents:

$$\underline{U} = \underline{w} - \underline{\theta}\underline{q}, \tag{A.9}$$

$$\overline{U} = \overline{w} - \overline{\theta}\overline{q}. \tag{A.10}$$

Therefore, the (PC) constraints are $\underline{U} \geqslant 0$, $\overline{U} \geqslant 0$. The (IC) constraints are:

$$\underline{w} - \underline{\theta}\underline{q} \geqslant \overline{w} - \underline{\theta}\overline{q}, \qquad IC\left(\underline{\theta}\right)$$

$$\overline{w} - \overline{\theta}\overline{q} \geqslant \underline{w} - \overline{\theta}\underline{q}, \qquad IC\left(\overline{\theta}\right)$$

which in terms of rents can be written as:

$$\underline{U} \geqslant \overline{U} + \Delta\theta\overline{q}, \qquad IC\left(\underline{\theta}\right)$$

$$\overline{U} \geqslant \underline{U} - \Delta\theta\underline{q}. \qquad IC\left(\overline{\theta}\right)$$

After substitutions of wages from (A.9) and (A.10), the maximization problem reads:

$$\underset{\{\underline{q},\underline{U},\overline{q},\overline{U}\}}{Max} \ \mathbb{E}_\theta[\pi_a] = \begin{pmatrix} v\left[(a - b\underline{q} - b\mathbb{E}_\theta[Q_{-i}] - \underline{\theta})\underline{q}\right] + \\ (1-v)\left[(a - b\overline{q} - b\mathbb{E}_\theta[Q_{-i}] - \overline{\theta})\overline{q}\right] \end{pmatrix} - (v\underline{U} + (1-v)\overline{U}) - F \tag{A.11}$$

$$s.t. \quad PC\left(\underline{\theta}\right), PC\left(\overline{\theta}\right), IC\left(\underline{\theta}\right), IC\left(\overline{\theta}\right).$$

As it is standard, $IC(\underline{\theta})$ and $PC(\overline{\theta})$ imply $PC(\underline{\theta})$. Disregarding $IC(\overline{\theta})$ and checking it at the optimum, the remaining two constraints are $PC(\overline{\theta})$ and $IC(\underline{\theta})$, which have to bind at the optimum. As a consequence, the relaxed problem is:

$$\underset{\{\underline{q},\overline{q}\}}{Max}\, \mathbb{E}_\theta[\pi_a] = \begin{pmatrix} v\left[(a - b\underline{q} - b\mathbb{E}_\theta[Q_{-i}] - \underline{\theta})\underline{q}\right] + \\ (1-v)\left[(a - b\overline{q} - b\mathbb{E}_\theta[Q_{-i}] - \overline{\theta})\overline{q}\right] - v\Delta\theta\overline{q} \end{pmatrix} - F. \tag{A.12}$$

Computing the f.o.c.s, which are also sufficient, yields:

$$\frac{d\mathbb{E}_\theta[\pi_a]}{d\underline{q}} = -b\underline{q} + (a - b\underline{q} - b\mathbb{E}_\theta[Q_{-i}] - \underline{\theta}) = 0, \tag{A.13}$$

$$\frac{d\mathbb{E}_\theta[\pi_a]}{d\overline{q}} = (1-v)[-b\overline{q} + (a - b\overline{q} - b\mathbb{E}_\theta[Q_{-i}] - \overline{\theta})] - v\Delta\theta = 0. \tag{A.14}$$

Together with the expression for the aggregate quantity, we have a linear system with three unknowns:

$$\begin{cases} -b\underline{q} + (a - b\underline{q} - b\mathbb{E}_\theta[Q_{-i}] - \underline{\theta}) = 0 \\ (1-v)[-b\overline{q} + (a - b\overline{q} - b\mathbb{E}_\theta[Q_{-i}] - \overline{\theta})] - v\Delta\theta = 0 \\ \mathbb{E}_\theta[Q_{-i}] = (n-1)[v\underline{q} + (1-v)\overline{q}]. \end{cases} \tag{A.15}$$

Hence, we obtain:

$$\underline{q}_a = \frac{a-\underline{\theta}}{b(1+n)} - \frac{(1-n)}{2b(1+n)}\Delta\theta, \tag{A.16}$$

$$\overline{q}_a = \frac{a-\overline{\theta}}{b(1+n)} - \frac{1}{2b}\frac{v}{1-v}\Delta\theta, \tag{A.17}$$

$$\mathbb{E}_\theta[Q_a] = \frac{n(a-\overline{\theta})}{b(1+n)}, \tag{A.18}$$

$$\mathbb{E}_\theta[P_a] = \frac{a+n\overline{\theta}}{1+n}. \tag{A.19}$$

Note that $IC(\overline{\theta})$ is satisfied because $\underline{q}_a(n) > \overline{q}_a(n)$ for every $n \geqslant 1$. Equations (A.16)-(A.19) are the equations as in Lemma 2.

A.3 Welfare Maximization

The number of firms, which maximize welfare, are denoted by n_a^* and n_p^*, respectively. They are implicitly given by:

$$\frac{d\mathbb{E}_\theta[W_p]}{dn} = 0 \Longleftrightarrow \frac{(a-\overline{\theta}+v\Delta\theta)^2}{b(1+n_p^*)^3} + \frac{3v(1-v)\Delta\theta^2}{8b} - F = 0, \tag{A.20}$$

$$\frac{d\mathbb{E}_\theta[W_a]}{dn} = 0 \Longleftrightarrow \frac{(a-\overline{\theta})(a-\overline{\theta}+v\Delta\theta(1+n_a^*))}{b(1+n_a^*)^3} + \frac{v\Delta(3-4v)\theta^2}{8b(1-v)} - F = 0. \tag{A.21}$$

W.l.o.g. we focus on parametrizations which ensure interior solutions.

A.4 Proof of Proposition 1

Proposition 1 can be (re-) stated as follows:

(a) If $v \geqslant 2/3$, $\mathbb{E}_\theta[W_p(n)] > \mathbb{E}_\theta[W_a(n)]$ holds for every n.

(b) If $v < 2/3$, $\mathbb{E}_\theta[W_a(n)] > \mathbb{E}_\theta[W_p(n)]$ holds for every $n > \overline{n}$. For $1 \leqslant n < \overline{n}$, we have $\mathbb{E}_\theta[W_a(n)] < \mathbb{E}_\theta[W_p(n)]$.

(c) If $\mathrm{Var}[\theta]$ is relatively large, i.e. $8bF - 3\mathrm{Var}[\theta]$ is positive, but relatively small, $\mathbb{E}_\theta[W_a(n_p^*)] > \mathbb{E}_\theta[W_p(n_p^*)]$ holds.

As asserted in the main text, expected welfare is strictly concave in n, irrespective of the informational structure:

$$\frac{d^2\mathbb{E}_\theta[W_p]}{dn^2} = -3\frac{(a - \overline{\theta} + v\Delta\theta)}{b(1+n)^4} < 0, \tag{A.22}$$

$$\frac{d^2\mathbb{E}_\theta[W_a]}{dn^2} = -(a - \overline{\theta})\frac{[3(a - \overline{\theta}) + 2(1+n)v\Delta\theta]}{b(1+n)^4} < 0. \tag{A.23}$$

Given $\underline{q}_a(n) > \underline{q}_p(n) > \overline{q}_p(n) > \overline{q}_a(n)$, a separating equilibrium in which firms contract with both types requires $\overline{q}_a(n) > 0$. For $n = 1$, we have:

$$\overline{q}_a(n=1) = \frac{(a - \overline{\theta})(1-v) - v\Delta\theta}{2b(1-v)}. \tag{A.24}$$

Evaluating $\mathbb{E}_\theta[W_a(\bar{n})] - \mathbb{E}_\theta[W_p(\bar{n})]$ at $n = 1$ and using (A.24) shows that $\mathbb{E}_\theta[W_a(1)] - \mathbb{E}_\theta[W_p(1)] < 0$ holds.

To prove (a) and (b), observe from (21) that $v > 2/3$ ($v < 2/3$) implies that $\overline{n} < 0$ ($\overline{n} > 0$). To prove (c), given $\mathbb{E}_\theta[W_p(1)] > \mathbb{E}_\theta[W_a(1)]$ and the feature that $\mathbb{E}_\theta[W_p]$ and $\mathbb{E}_\theta[W_a]$ intersect only once at $n = \overline{n}$, it is therefore sufficient to show that $n_p^* > \overline{n}$. It is straightforward to see that $\mathrm{Var}[\theta] = v(1-v)\Delta\theta^2$. Inspection of (21) and (A.20) shows that the inequality $n_p^* > \overline{n}$ can be obtained if the difference $8bF - v(1-v)\Delta\theta^2$ is small enough. Since $\overline{n}$ is independent of b and F, variations in these parameters can always ensure such outcome. It remains to be proven that market fundamentals, that is, profits and quantities, are positive. For quantities, it is enough to observe that $\overline{q}_a(n) > 0$ holds if the choke price a is high enough. For profits, application of the envelope theorem clarifies that they can be enhanced by increasing a and/or decreasing either or b or F or both.

References

Bergstrom, T. C. and Varian, H. R. (1985). Two remarks on cournot equilibria, *Economics Letters* **19**(1): 5 – 8.

Bonazzi, L. M., Fiocco, R. and Piccolo, S. (2020). Vertical Price Restraints and Free Entry Under Asymmetric Information, *CSEF Working Papers 564*, Centre for Studies in Economics and Finance (CSEF), University of Naples, Italy.

Demski, J. S. and Sappington, D. (1984). Optimal incentive contracts with multiple agents, *Journal of Economic Theory* **33**(1): 152 – 171.

Etro, F. and Cella, M. (2013). Equilibrium principal-agent contracts: Competition and R&D incentives, *Journal of Economics & Management Strategy* **22**(3): 488–512.

Guerrieri, V., Shimer, R. and Wright, R. (2010). Adverse selection in competitive search equilibrium, *Econometrica* **78**(6): 1823–1862.

Guesnerie, R. and Laffont, J.-J. (1984). A complete solution to a class of principal-agent problems with an application to the control of a self-managed firm, *Journal of Public Economics* **25**(3): 329 – 369.

Hart, O. D. (1983). The market mechanism as an incentive scheme, *The Bell Journal of Economics* **14**(2): 366–382.

Head, K. and Spencer, B. J. (2017). Oligopoly in international trade: Rise, fall and resurgence, *Canadian Journal of Economics/Revue canadienne d'économique* **50**(5): 1414–1444.

Laffont, J.-J. and Martimort, D. (2002). *The Theory of Incentives: The Principal-Agent Model*, Princeton University Press, Princeton.

Mankiw, N. G. and Whinston, M. D. (1986). Free entry and social inefficiency, *The RAND Journal of Economics* **17**(1): 48–58.

Martimort, D. (1996). Exclusive dealing, common agency, and multiprincipals incentive theory, *The RAND Journal of Economics* **27**(1): 1–31.

Massell, B. F. (1969). Price stabilization and welfare, *The Quarterly Journal of Economics* **83**(2): 284–298.

Mukherjee, A. and Tsai, Y. (2014). Managerial delegation, cost asymmetry and social efficiency of entry, *Economic Record* **90**(288): 90–97.

Oi, W. Y. (1961). The desirability of price instability under perfect competition, *Econometrica* **29**(1): 58–64.

Piccolo, S. (2011). A note on free entry under uncertainty: The role of asymmetric information, *Economics Letters* **111**(3): 256 – 259.

Piccolo, S. and Pagnozzi, M. (2013). Information sharing between vertical hierarchies, *Games and Economic Behavior* **79**: 201 – 222.

Pouyet, J., Salanié, B. and Salanié, F. (2008). On competitive equilibria with asymmetric information, *The B.E. Journal of Theoretical Economics* **8**(1): article 13.

Salant, S. W. and Shaffer, G. (1999). Unequal treatment of identical agents in cournot equilibrium, *American Economic Review* **89**(3): 585–604.

Salop, S. C. (1979). Monopolistic competition with outside goods, *The Bell Journal of Economics* **10**(1): 141–156.

Scharfstein, D. (1988). Product-market competition and managerial slack, *The RAND Journal of Economics* **19**(1): 147–155.

Shapiro, C. (1986). Exchange of cost information in oligopoly, *The Review of Economic Studies* **53**(3): 433–446.

Stiglitz, J. E. (1977). Monopoly, non-linear pricing and imperfect information: The insurance market, *The Review of Economic Studies* **44**(3): 407–430.

Suzumura, K. and Kiyono, K. (1987). Entry barriers and economic welfare, *The Review of Economic Studies* **54**(1): 157–167.

von Weizsäcker, C. C. (1980). A welfare analysis of barriers to entry, *The Bell Journal of Economics* **11**(2): 399–420.

Waugh, F. V. (1944). Does the consumer benefit from price instability?, *The Quarterly Journal of Economics* **58**(4): 602–614.