Positive Selection of Employees*

Michael Waldman† Zhenda Yin‡

January 2020

Abstract

This paper takes a step towards bridging the theory of internal labor market (ILM) and external labor market (ELM) by studying an integrative framework with job changes both inside and across firms. In particular, we consider a dynamic model, whereby firms use retention and promotion as the personnel policy to positively select the workforce (workers) concerning utility of working and abilities. The equilibrium captures three sets of empirical findings in a single setting. That is, (1) standard ILM findings (concerning the pay and promotion pattern inside firms); (2) standard ELM findings (concerning the pay and mobility pattern across firms); (3) standard, yet unexplained, findings on the "interaction" of ILM and ELM (concerning effects of promotion on exit rates and effects of entry routes on career paths and workforce compositions).

Keywords: Integrative Models of the Labor Market; Personnel Policy; Dynamic Learning

JEL Codes: J2, J3, J6, M5

---

*We thank Marco Battaglini, Yi Chen, David Easley, Justin Johnson, Rongzhu Ke, Armin Rick and, especially Jin Li, Michael Powell, and Vincenzo Quadrini, for their valuable comments and suggestions. All remaining errors are our own.

†Cornell University. Email: mw46@cornell.edu

‡Peking University. Email: zdyin@pku.edu.cn

1 Introduction

It is evident to even casual observers that job-to-job transitions are common in labor markets. According to the Current Population Survey (CPS), 2.6% of employed persons change employers each month. This accounts for almost 40% of the new jobs started between 1994 and 2003 (Fallick and Fleischman, 2004). The Longitudinal Employer Household Dynamics (LEHD) program suggests that the pace of employer-to-employer flow is high, representing approximately 4% of employment and 30% of separations each quarter (Bjelland et al., 2011). In fact, external recruits account for about 15% to 20% of all appointments to top management positions (Denis and Denis, 1995), while roughly 20% of CEOs of large U.S. firms are outsiders (Agrawal, Knoeber, and Tsoulouhas, 2006). These facts suggest that workers *often* change jobs both *inside* firms and *across* firms. Many research questions then arise. For example, (a) how should firms use the personnel policy—such as hiring, retention, and promotion/demotion—to staff opening positions and manage the existing workforce? (b) when workers plan careers in advance, how to evaluate the consequences of various job changes (e.g., maintaining an existing job or starting a new job at another firm)? (d) economists and management consulting firms want to understand both the personnel policy of firms and job-change decisions of workers.

Existing theoretical research has generally taken two separate approaches. One approach focuses on the internal labor market (ILM hereafter), which studies internal operations of organizations and highlights the importance of internal structure—job ladder or hierarchy—and various human resource management practices (Doeringer and Piore, 1971; Gibbons and Waldman, 1999a; Lazear and Oyer, 2013). However, this literature has mainly focused on job changes inside firms. The other approach centers on the external labor market (ELM hereafter) with emphases on the macroeconomic pattern of job changes across firms (Mortensen, 1986; Mortensen and Pissarides, 1999; Rogerson and Shimer, 2011).

Due to the dichotomy of existing literature, relatively little is known about the "interaction" of ILM and ELM. That is, as workers change jobs both inside and across firms, how will various pathways of entering jobs impact on future pay and career path? Noticeably, there are a number of empirical studies on the "interaction" of the two markets. Based on a single firm's personnel record, Baker, Gibbs, and Holmström (1994a,b) and Bidwell (2011) document a series of findings concerning the impact of entry routes on employment outcomes and career paths. For instance, employees recruited from outside are less likely to be later promoted than incumbent employees; employees become less to exit after being internally promoted. Thanks to the recently developed longitudinal linked employer-employee data, we see many related, yet more comprehensive, empirical studies reinforcing the previous findings (Van der Klaauw and Da Silva, 2011; Kauhanen and Napari, 2012; Frederiksen, Halliday, and Koch, 2016; DeVaro, Kauhanen, and Valmari, 2019).[1]

[1] We will refer to these findings as "interaction" findings of ILM and ELM. Due to the dichotomy of theoretical

This paper takes a step towards bridging the theory of ILM and ELM. The main idea stems from the positive selection of workers in the long-term employment relationship. That is, with repeated interactions, firms gradually identify and make use of valuable workers. Existing labor studies have mostly focused on worker abilities (productivity), whereby those being found to be more capable (productive) are deemed as more valuable ones.[2] This paper, however, points out that utility of working—a measure of how much a worker likes a given job—also influences the long-term employment relationship. In particular, we model utility of working as a firm-specific matching quality, so a worker needs to work at a firm for certain periods to discover how much she likes a job. To fix ideas, consider a worker holding a job at a particular firm. Many non-financial aspects of employment can affect utility of working, e.g., how well she adapts to the organization environment, how much she likes the corporate culture, how well she gets along with the boss and coworkers, etc. In fact, we are not the first to consider utility of working in the workplace.[3] However, very few studies have paid attention to its role in the long-term employment. To the best of our knowledge, we are the first to study (dynamic) employer learning concerning utility of working.

We show that a broad set of empirical findings can be rationalized in a single framework. The main building blocks of our dynamic model include private information of utility and asymmetric employer learning of abilities. To study job changes both inside and across firms, we consider a competitive labor market, whereby all firms offer two jobs, labour and manager, and inherit the same production technology.

We model retention and promotion as the key personnel policy of firms, whereby the retention policy weeds out workers with low utility of working and the promotion policy sorts more (resp. less) able workers into managerial (resp. labour) positions, yet signals worker abilities to the market. The equilibrium of our model is shown to feature positive selection of workers over time concerning both utility of working and abilities. That is, firms positively select workers through retaining high-utility workers and promoting high-ability workers. Most importantly, the equilibrium of our model, in a single setting, captures three sets of empirical findings documented in, but not limited to, Baker, Gibbs, and Holmström (1994a,b) and Bidwell (2011). That is, (1)

literature, we are the first to use an integrative approach to offer an explanation to these "interaction" findings.

[2]There is an extensive literature on employer learning concerning worker abilities (productivity). For instance, employers could detect abilities (productivity) of existing employees from performances (Jovanovic, 1979; Harris and Holmström, 1982), job assignments (experimentation) (Gibbons and Waldman, 1999b; Pastorino, 2015, 2019), training (Chang and Wang, 1996; Acemoglu and Pischke, 1998), and rotation programs (Ortega, 2001; Deb and Stewart, 2018). The market could infer abilities (productivity) of new hires through educational attainments (Spence, 1973), job referrals (Montgomery, 1991), and employment records such as promotion (Waldman, 1984a) and job separations (Greenwald, 1986; Gibbons and Katz, 1991).

[3]For instance, Rosen (1974) in his hedonic wage theory considers a setting where firms, all else equal, offer different working conditions and workers hold different preferences for working conditions. It is established that compensating wage differentials arise in equilibrium to compensate new workers for non-financial characteristics of the job. In two-period frameworks, Acemoglu and Pischke (1998), Schönberg (2007), and Kahn (2013) show that utility of working can drive worker turnover in equilibrium, where workers change firms to find a better match.

standard ILM findings; (2) standard ELM findings; (3) standard findings on the "interaction" of ILM and ELM.

Specifically, our model nests standard ILM findings concerning the pay and promotion pattern inside firms. That is, the promotion policy is biased towards workers with high educational attainments; moreover, there is a pay gap across jobs and a pay dispersion within jobs. Our model also nests standard ELM findings concerning the pay and mobility pattern across firms. For instance, we find that the labor market features an "adverse selection" effect that exit rates of workers decline with abilities. Moreover, our model matches the macroeconomic pattern of job changes across firms. That is, exit rates decline with tenure; exit rates decline with seniority (labor-market experience); many new jobs end early, and long-term employment relationships are common; workers receive a pay increase for changing jobs across firms.

As to "interaction" findings concerning effects of promotion on exit rates, we find a negative effect of promotion on exit rates. That is, workers become less likely to exit after being promoted. We also establish a "fast-track exit" effect that those being promoted sooner generally have higher exit rates than those being promoted later. As to "interaction" findings concerning effects of entry routes on career paths, our model predicts that new hires have greater exit rates and fewer promotion prospects than incumbents. In terms of workforce compositions, we show that new hires typically receive more education than incumbents.

Overall, we find that asymmetric employer learning of abilities and private information of utility are two crucial factors in terms of driving the above three broad sets of findings. As existing models are, however, silent to the "interaction" of the internal and external labor markets, we are the first to use an integrative approach to offer an explanation to these "interaction" findings.

The remainder of this paper is structured as follows. After a review of the literature in Section 2, Section 3 presents the model. Section 4 and Section 5 provide the equilibrium analysis. Section 6 shows that the equilibrium of our model nests standard internal-labor-market findings and standard external-labor-market findings. Section 7 explores model implications concerning the interaction of internal and external labor markets, a.k.a., "interaction" findings. Section 8 concludes. Technical details are relegated to the appendix.

2 Related Literature

2.1 Theoretical literature

This paper mainly connects to two strands of theoretical literature. That is, (1) employer-learning models and integrative models of the labor market from the ILM literature; (2) macro-labor models from the ELM literature. In what follows, we provide an overview of how this paper fits into the ILM literature. More details concerning how our approach relates and differs from specific ILM and ELM models can be found in Section 6.

As mentioned above, existing models almost exclusively center on employer learning concerning worker abilities. Among them, one set of learning models study the information content of job separations. For instance, under the hypothesis of asymmetric employer learning, Greenwald (1986), Gibbons and Katz (1991), and Fan and DeVaro (2015) find that job separations are perceived as negative signals of abilities since incumbent firms will offer higher wages to retain more able workers. Another set of learning models explore the information content of job assignments. In particular, under the hypothesis of asymmetric learning, Waldman (1984a), Bernhardt (1995), Zábojník and Bernhardt (2001), Golan (2005), and DeVaro and Waldman (2012) consider promotion, in sorting more able workers to higher-level jobs, serves as positive signals of abilities to the market.

This paper incorporates the information content of both job separations and assignments. The main departure from the existing literature is that we also study employer learning concerning utility of working. In this regard, our framework is closest to Acemoglu and Pischke (1998), Schönberg (2007), and Kahn (2013), where utility of working drives worker turnover in equilibrium. We extend their analyses in two aspects. First, we consider a dynamic framework with multiple periods, where firms can infer utility through the mobility decision of workers. Second, our model features job ladders inside firms so that workers can change jobs both inside and across firms.

Overall, this paper belongs to the growing literature of integrative models (Harris and Holmström, 1982; MacLeod and Malcomson, 1988; Demougin and Siow, 1994; Gibbons and Waldman, 1999b, 2006; Ke, Li, and Powell, 2018), which attempts to provide an explanation for broad patterns of evidence rather than a few stylized facts. In particular, under the hypothesis of symmetric learning, Gibbons and Waldman (1999b, 2006) provide a dynamic framework to rationalize a series of documented facts in Baker, Gibbs, and Holmström (1994a,b) concerning wages and promotion dynamics inside firms. In a follow-up study, Ghosh (2007) shows that results therein are robust to job changes across firms. This paper, on the other hand, considers a dynamic framework with asymmetric learning. Furthermore, we focus on a different set of empirical findings in, but not limited to, Baker, Gibbs, and Holmström (1994a,b).

2.2 Empirical literature

Our model matches standard ILM findings concerning the pay and promotion pattern inside firms. That is, (i) promotion prospects increase with educational attainments of employees (Lluis, 2005; Belzil and Bognanno, 2010; DeVaro and Waldman, 2012; Bognanno and Melero, 2016); (ii) there is a pay gap across jobs (Baker, Gibbs, and Holmström, 1994a,b; McCue, 1996) and a pay dispersion within jobs (Baker, Gibbs, and Holmström, 1994a,b; Dato et al., 2016).

In terms of standard ELM findings, our model echoes the macroeconomic pattern concerning job changes across firms (Mincer and Jovanovic, 1981; Farber, 1994, 1998, 1999; Fallick and

Fleischman, 2004). That is, (i) exit rates decline with tenure; (ii) exit rates decline with seniority (labor-market experience); (iii) many new jobs end early, and long-term employment relationships are common. Moreover, our model features a "pay more for less" effect for recruiting from outside. That is, new hires initially under-perform incumbents (Baker, Gibbs, and Holmström, 1994a,b; Bartel, 1995; Bidwell, 2011; Van der Klaauw and Da Silva, 2011; Kauhanen and Napari, 2012) and receive a pay increase for changing jobs across firms (Bartel, 1995; Bidwell, 2011; Van der Klaauw and Da Silva, 2011; Frederiksen, Halliday, and Koch, 2016).

As to "interaction" findings concerning effects of promotion on exit rates, our model matches the documented negative effect of promotion on exit rates (Carson et al., 1994; Trevor, Gerhart, and Boudreau, 1997; Saporta and Farjoun, 2003). That is, a worker becomes less likely to exit after being promoted. Our model also predicts a "fast-track exit" effect that those being promoted sooner generally have higher exit rates than those being promoted later (Baker, Gibbs, and Holmström, 1994a; Treble et al., 2001; Van der Klaauw and Da Silva, 2011). As to "interaction" findings concerning effects of entry routes on career paths, our model is consistent with stylized facts that new hires have greater exit rates (Baker, Gibbs, and Holmström, 1994a; Treble et al., 2001; Bidwell, 2011; Van der Klaauw and Da Silva, 2011) and fewer promotion prospects (Baker, Gibbs, and Holmström, 1994a; Treble et al., 2001; Chan, 2006; Oyer, 2007; Bidwell, 2011; Kauhanen and Napari, 2012) than incumbents. In terms of workforce compositions, our model captures the finding that external hires typically receive more education than incumbents (Baker, Gibbs, and Holmström, 1994a; Bidwell, 2011; Kauhanen and Napari, 2012; DeVaro, Kauhanen, and Valmari, 2019).

3 The Model

3.1 A dynamic framework integrating ILM and ELM

This section develops a three-period model integrating the internal labor market (ILM) and external labor market (ELM). Two main building blocks are asymmetric employer learning of worker abilities and private information of utility of working. To study job changes both inside and across firms, we consider a competitive labor market, whereby all firms offer two jobs, labour and manager, and inherit the same production technology.

The labor market

We consider a competitive labor market consisting of a finite number of firms and a continuum of workers with mass one. Due to free entry, firms will actively compete for worker services in each period. Each worker's career lasts for three periods. Upon entering the labor market, a worker has an innate ability $\theta \in \{l, h\}$ (i.e., low or high) and schooling $k \in [0, \bar{k}]$, both of which are fixed over time. It is natural that ability is ex ante unknown, yet positively related to schooling, which is publicly observable. In particular, we assume that the monotone likelihood ratio property (MLRP) holds, i.e., $\partial[p(h|k)/p(l|k)]/\partial k > 0$, and $p(h|k = 0) = 0$.

Production

Firms possess identical constant-return-to-scale (CRS) production technology. The supply of labor, as the only input of production, is inelastic and indivisible.[4] For a given firm, the period-t output of a (θ, k)-type hire of period $\tau \leq t$ (i.e., this worker has entered the firm since period τ and worked for the firm for $t - \tau$ periods) on job J is given by

$$(1 + n_{t-1} \cdot \mathbf{I}\{t - \tau \geq 1\}) \cdot x(\theta; J), \tag{1}$$

where $n_{t-1} > 0$ captures the return to tenure for a worker who has been hired for at least one period (so the indicator function becomes $\mathbf{I}\{t - \tau \geq 1\} = 1$). We assume that $n_2 > n_1$, so the return to tenure increases with seniority (labor-market experience). $x(\theta; J) \geq 0$ characterizes the output component on job J due to ability θ, which is independent of schooling.

Firms also maintain the same internal structure. That is, each firm offers two jobs, L (labour) and M (manager), and a worker is promoted (resp. demoted) whenever she is placed from job L to job M (resp. from job M to job L). Following Sattinger (1975), Rosen (1982), and Waldman (1984b), we assume that

$$x(\theta; L) = 1 \quad \text{and} \quad x(\theta; M) = \hat{\theta}^{-1} \cdot \theta,$$

where $\hat{\theta} \in (0, 1)$ is a constant. This means that jobs are ranked by the importance of abilities such that the labor output is not influenced by abilities, whereas the managerial output increases with abilities. We assume throughout that a worker is promoted when the firm is indifferent between promoting and not promoting the worker.

To ease expositions, we normalize abilities to $l = 0 < 1 = h$ as in Figure 1. This then suggests that an h type is more productive on job M and an l type is more productive on job L, i.e., $x(h; M) = \hat{\theta}^{-1} > 1 = x(h; L)$ and $x(l; L) = 1 > 0 = x(l, M)$. Moreover, $\hat{\theta}$ can be interpreted as the threshold ability level, with which a worker is equally productive on job L and job M, i.e., $x(\hat{\theta}; L) = x(\hat{\theta}; M)$.

Utility of working

During every period, each worker experiences some utility of working—sampled from a distribution G—which captures how much she likes a job. It is natural that utility is private information of workers. Moreover, utility is an "inspection" good at the incumbent firm and an "experience"

[4] This means that we abstract away from employment statuses such as being unemployed and being employed for multiple firms.

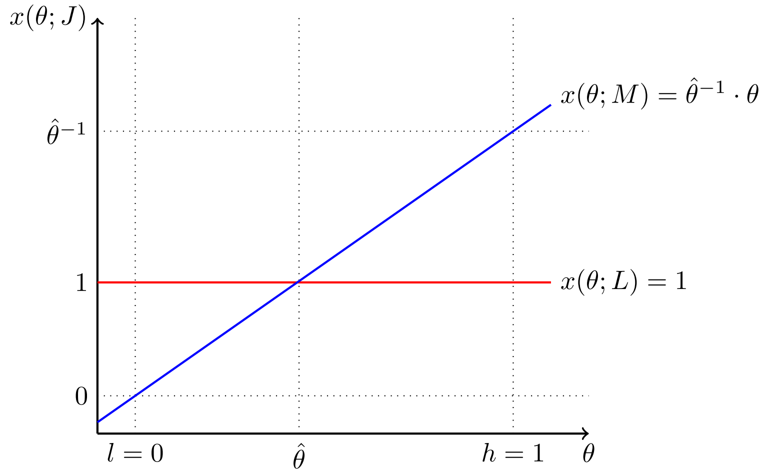


Figure 1: Plot of output component $x(\theta; J)$ for a θ-type worker on job J. There are two abilities, $\theta \in \{l, h\}$, and two jobs, $J \in \{L, M\}$, where abilities are normalized to $l = 0 < 1 = h$ and $\hat{\theta} \in (0, 1)$ represents the threshold ability level.

good at an outside firm. This means that, a worker, at the beginning of each period, can inspect utility at the incumbent firm, whereas she must work for one period to experience utility at an outside firm.

Specifically, consider a worker who has experienced utility η_t during period t. At the beginning of period $t+1$, she inspects utility η_{t+1} at the incumbent firm, which either remains unchanged (i.e., $\eta_{t+1} = \eta_t$) with probability q, or, is resampled (i.e., $\eta_{t+1} \sim G$) with probability $1-q$, where $q \in (0, 1)$.[5] During period $t+1$, she can either stay at the incumbent firm to experience the inspected utility (i.e., η_{t+1}) or move to an outside firm to experience some resampled utility (i.e., $\eta_{t+1} \sim G$).

The above specifications suggest that, whenever a worker dislikes a job, she may move to a new firm to resample utility. It is natural to assume that utility is independently distributed with abilities and schooling. Without loss of generality, we assume that G is log-concave and normalize its mean to zero.

3.2 The dynamic game

We assume that employer learning of abilities is asymmetric. That is, after one period of employment, abilities are revealed to incumbent firms, but not to outside firms. However, outside firms can infer abilities through employment (job-change) histories—such as turnover and promotion—

[5]One can enrich our model such that workers being promoted are more likely to resample utility (inside firms) than workers not being promoted. As long as the difference in probabilities of resampling utility (inside firms) is not prohibitively large, we conjecture that such extension will not change our results in a substantial way.

Time	Events
Beginning of period t	*incumbent firms assign jobs* workers inspect utility at incumbent firms *outside firms make poaching offers* incumbent firms make retention offers workers select a firm (i.e., "stay" or "move")
During period t	workers conduct production and experience utility *incumbent firms observe abilities*

Figure 2: Timing of the game.

which are public information.[6,7] To ensure that all outside firms have the same information, we assume that a worker, once exited from a firm, cannot come back in the future. This is equivalent to assume that, (a) employees cannot immediately recover the return to tenure with past employers; (b) firms forget abilities of past employees. This strong assumption is made to ease off-equilibrium analysis and to preclude the possibility that workers switch firms in order to convey information about their abilities to multiple firms.

Figure 2 depicts timing of the game.[8] To close the model, we assume that all players are rational, risk-neutral, and share a common discount factor $\delta \in (0, 1]$. Furthermore, there is no cost for workers to change firms or for firms to recruit new hires. In each period, a worker's payoff equals "wage plus utility", while a firm's payoff equals "output minus wage".

The solution concept we use is Perfect Bayesian Equilibrium (PBE). That is, for any employ-

[6]Earlier papers that build upon the hypothesis of asymmetric employer learning include Waldman (1984a), Greenwald (1986), Lazear (1986), Milgrom and Oster (1987), Waldman (1990), Bernhardt and Scoones (1993), Laing (1994), Bernhardt (1995), Chang and Wang (1996), Acemoglu and Pischke (1998), Zábojník and Bernhardt (2001), etc. See Carrillo-Tudela and Kaas (2015), Ferreira and Nikolowa (2017), and Friedrich (2019) for more recent models of such.

[7]Although early studies such as Farber and Gibbons (1996) and Altonji and Pierret (2001) show that the hypothesis of symmetric learning is consistent with documented facts concerning wage dynamics inside firms, a number of papers, including Gibbons and Katz (1991) and Acemoglu and Pischke (1998), develop other predictions and find supporting evidence for the hypothesis of asymmetric learning. Recently, there is a growing literature that specially disentangles whether employer learning is symmetric or asymmetric in labor markets. For instance, Schönberg (2007), Pinkston (2009), and Kahn (2013) consider tests that distinguish between symmetric and asymmetric learning. Overall, empirical evidence supports an important role for asymmetric learning. See Waldman (2013) for a discussion on that.

[8]There are three comments concerning job offers in our model. (i) Worker receive job offers prior to production. Thus, we consider offers in the form of spot contracts, as opposed to piece-rate contracts and dynamic contracts. In addition, workers are price takers. See Ricart-I-Costa (1988) and Golan (2009) for related models where employees can offer contracts to firms or bargain wages with firms. (ii) There is no distinction between layoffs and quits since incumbent firms can use retention offers to counter-offer against poaching offers. The counter-offer assumption can be found in early papers such as Greenwald (1986) and Milgrom and Oster (1987) and, under the hypothesis of asymmetric employer learning, leads to a winner's curse type result, i.e., a worker's wage equals the productivity at an outside firm of the worst worker with the same labor market signal. See Barron, Berger, and Black (2006) for empirical evidence on the common use of counter-offers in labor markets. (iii) Following promotional signaling models such as Golan (2005) and DeVaro and Waldman (2012), poaching offers consist of both jobs and wages, whereas counter-offers only include wages. This is because, if counter-offers also specify jobs, incumbent firms will choose to promote workers only in the counter-offer stage, so promotional signaling (which elicits information to the market) will be delayed by one period.

ment history, given correct beliefs about strategies of outside firms and workers, every incumbent firm assigns jobs and makes retention offers to maximize its profits; each worker chooses a firm to work for (she chooses to stay at the incumbent firm whenever the payoff to stay is no less than the payoff to move); all players update beliefs according to the Bayes' rule.

Parametric restrictions

Throughout this paper, we will assume that,

Assumption 1. *The sampling distribution of utility, G, has a sufficiently large support.*

First, this assumption guarantees that the cutoff utility (defined below) are characterized by first-order conditions in equilibrium. Second, this assumption suggests that incumbent firms will earn some *rents due to under-bidding*. Intuitively, when utility (i.e., non-financial aspects of working) plays a big role in determining worker payoffs, retention wages (i.e., financial aspects of working) will not largely affect the separation probability of workers, so incumbent firms can under-bid the market by offering existing workers some retention wages below poaching wages. Moreover, as utility becomes more disperse valued, firms will earn more rents due to under-bidding.[9]

4 Preliminaries

We have at hand a large and complex game, which features dynamic employer learning of both worker abilities and utility of working. That is, incumbent firms infer utility of existing workers through their mobility decisions, while job assignments—reflecting incumbent firms' advantageous information—signal abilities to the market.

To make progress, this section modifies the game in Figure 2 to a simplified version. That is, we now consider the game in Figure 3, where we assume that a worker, during each period, either stays at the incumbent firm or moves to a new firm to obtain some payoff to exit. For instance, denote the mobility decision of "stay" and "move" respectively as "s" and "m", a period-1 hire faces e_2 as the payoff to exit in period 2 and produces y_2^s if she stays in period 2. Likewise, a period-2 hire (i.e., a mover in period 2) faces e_3^m as the payoff to exit in period 3 and produces $y_3^{m,s}$ if she stays in period 3, while a period-1 hire (i.e., a stayer in period 2) faces e_3^s as the payoff to exit in period 3 and produces $y_3^{s,s}$ if she stays in period 3. By taking both outputs and payoffs to exit as given, we can study the mobility decision of workers and the retention strategy of firms.[10]

[9] In the equilibrium analysis, we will use Assumption 1 to rule out (uninteresting) cases where the promotion policy in period 2 is completely efficient. That is, firms place all h types as managers and all l types as labours in period 2 and consequently there will be no promotion in period 3. See the appendix for more details.

[10] Note, however, that in equilibrium both outputs and payoffs to exit will be determined by job assignments

Time	Events
Beginning of period t	workers inspect utility at incumbent firms incumbent firms make retention offers workers select a firm (i.e., "stay" or "move")
During period t	workers conduct production and experience utility

Figure 3: Timing of the simplified game.

The simplified game abstracts away from events in *italic* in Figure 2, i.e., job assignments and employer learning, by assuming that workers, upon moving to new firms, can derive some payoffs to "exit".

The mobility decision and retention strategy

Let us first consider the mobility decision of workers. During each period, a worker will move to a new firm whenever the payoff to exit exceeds the payoff to stay, where the latter incorporates both the retention wage and utility of working. This suggests a basic principle that the turnover rate—probability of "move"—increases with the payoff to exit and decreases with the retention wage. Since utility is private information of workers, the retention strategy of firms will take a simple *cutoff* form whereby a worker with utility above a cutoff level will be retained.

We will study the retention strategy—cutoff(s)—for each cohort, given outputs and payoffs to exit. That is, (i) for a period-2 hire, we derive the cutoff for period 3, $\hat{\eta}_3^m$, given the output and payoff to exit, $(y_3^{m,s}, e_3^m)$; (ii) for a period-1 hire, we derive the cutoffs for period 2 and period 3, $\hat{\eta}_2$ and $\hat{\eta}_3^s$, respectively, given the (lifetime) outputs and payoffs to exit, $\{(y_2^s, e_2), (y_3^{s,s}, e_3^s)\}$. Formally, we solve two screening problems. That is,

$$\underbrace{\mathcal{R}^1 : \{(y_2^s, e_2), (y_3^{s,s}, e_3^s)\} \overset{(w_2, w_3^s)}{\longmapsto} (\hat{\eta}_2, \hat{\eta}_3^s)}_{\text{(dynamic) screening for a period-1 hire}} \quad \text{and} \quad \underbrace{\mathcal{R}^2 : (y_3^{m,s}, e_3^m) \overset{w_3^m}{\longmapsto} \hat{\eta}_3^m}_{\text{(static) screening for a period-2 hire}} ,$$

The (dynamic) screening problem for a period-1 hire will capture the dynamics of employer learning concerning utility.[11,12]

and employer learning. Specifically, the output specification in (1) suggests that outputs will depend on job placement and, due to asymmetric employer learning, payoffs to exit (i.e., expected payoffs to moving to new firms) will incorporate poaching offers, which depend on job assignments as signals of abilities.

[11] One can interpret $\mathcal{R}^1$ and $\mathcal{R}^2$ as direct mechanisms, where the "price" is the retention wage and the "allocation rule" is the cutoff. Note, however, that we solve for the PBE rather than the dynamic mechanism. This means that, as mobility decisions in the past reveal information concerning utility, a job stayer may receive a different wage than a job mover, i.e., $w_3^s \neq w_3^m$. See the online appendix for how our analysis relates to behavior-based price discrimination (BBPD) (Fudenberg and Villas-Boas, 2006) and dynamic mechanism design (Tirole, 2016).

[12] In the full model, wages will be contingent on the history of both job separations and assignments. See Bernhardt (1995) for a related model (without worker turnover), where wages only depend on the history of job assignments.

The retention strategy in equilibrium

In what follows, we provide an overview of the preliminary results—formally verified in the appendix—concerning the retention strategy of firms in equilibrium.

For a period-2 hire, the equilibrium cutoff, $\hat{\eta}_3^m$, trades-off the output and payoff to exit in period 3. We show that firms will increase the cutoff for period 3, $\hat{\eta}_3^m = \Phi^{-1}(y_3^{m,s} - e_3^m)$, whenever "output less payoff to exit" in period 3, $y_3^{m,s} - e_3^m$, decreases.[13] This is an intuitive result since firms will retain a larger proportion of workers—implement a lower cutoff via raising the retention wage—given a higher output or a lower payoff to exit.

For a period-1 hire, there are two possible regimes, one with *intensive* screening and the other with *non-intensive* screening. Relative to non-intensive screening, intensive screening features a lower retention wage (i.e., wage savings from retained workers), yet entails a higher cutoff (i.e., output losses from separated workers who could otherwise be retained) in both period 2 and period 3. In other words, the regime with intensive screening is more intensive in weeding out low-utility workers than the regime with non-intensive screening. We show that the equilibrium cutoffs, $\hat{\eta}_2$ and $\hat{\eta}_3^s$, trade-off wage savings and output losses. That is, intensive screening is the equilibrium regime whenever the effective "output less payoff to exit" of period 3 outweighs that of period 2.

The detailed arguments are as follows. Imagine that utility is always resampled at the beginning of each period. Similar to a period-2 hire (i.e., a mover), the cutoff in period 3 for a period-1 hire (i.e., a stayer) is simply given by $\hat{\eta}_3^s = \Phi^{-1}\left(y_3^{s,s} - e_3^s\right)$, which is referred to as the *benchmark cutoff*. Note, however, in our model that utility remains unchanged with probability $q > 0$, so a stayer's utility distribution will be truncated from below if his or her utility is not resampled in period 3, i.e., $\eta_3 = \eta_2 \geq \hat{\eta}_2$. This then suggests that, when stayers are positively selected in the sense of having already sampled high utility in period 2, firms can exploit some *informational rents* in period 3. In particular, firms can first lower the retention wage in period 2 to weed out the workers with low utility and later lower the retention wage in period 3 to the stayed workers with high utility.[14]

Specifically, there are two possible cases (regimes). That is, either $\hat{\eta}_2 \leq \Phi^{-1}(y_3^{s,s} - e_3^s)$ (regime with non-intensive screening) or $\hat{\eta}_2 > \Phi^{-1}(y_3^{s,s} - e_3^s)$ (regime with intensive screening). If stayers are sufficient positively selected (in terms of utility), the cutoff for period 3 will be above the benchmark cutoff, i.e., $\hat{\eta}_2 > \Phi^{-1}(y_3^{s,s} - e_3^s) \Rightarrow \hat{\eta}_3^s > \Phi^{-1}(y_3^{s,s} - e_3^s)$; otherwise, the cutoff for period 3 will coincide with the benchmark cutoff, i.e., $\hat{\eta}_2 \leq \Phi^{-1}(y_3^{s,s} - e_3^s) \Rightarrow \hat{\eta}_3^s = \Phi^{-1}(y_3^{s,s} - e_3^s)$.

[13] In the appendix, we formally derive the cutoff from the first-order condition, i.e., $\Phi(\hat{\eta}) = r$ where $\hat{\eta}$ is the cutoff utility and r is the value of "output less payoff to exit". Since Φ is monotone, we can define $\hat{\eta} = \Phi^{-1}(r)$ as the cutoff, which decreases with r.

[14] Although both arise in equilibrium due to utility of working, informational rents are fundamentally different from rents due to under-bidding (as described in the discussion following Assumption 1) because dynamic screening (learning) is necessary for incumbent firms to exploit informational rents from job stayers.

t	job assignment (incumbent firm)	poaching offer: wage and job (outsider firm)	retention offer: wage (incumbent firm)
1	all firms offer wage $v_1(k)$ and job $\tilde{J}_1(k)$		
2	$J_2(\theta;k)$	$v_2(k,J_2)$ and $\tilde{J}_2(k,J_2)$	$w_2(\theta;k,J_2)$
3	$J_3^s(\theta;k,J_2)$	$v_3^s(k,J_2,J_3^s)$ and $\tilde{J}_3^s(k,J_2,J_3^s)$	$w_3^s(\theta;k,J_2,J_3^s)$
	$J_3^m(\theta;k,J_2)$	$v_3^m(k,J_2,J_3^m)$ and $\tilde{J}_3^m(k,J_2,J_3^m)$	$w_3^m(\theta;k,J_2,J_3^m)$

Table 1: A complete description of strategies for a (θ,k) type.

For period 3, the top row describes the strategies for a period-1 hire (i.e., a stayer) and the bottom row demonstrates the strategies for a period-2 hire (i.e., a mover). Note that job assignments by new employers, e.g., $\tilde{J}_1(k)$, $\tilde{J}_2(k,J_2)$, $\tilde{J}_3^s(k,J_2,J_3^s)$, and $\tilde{J}_3^m(k,J_2,J_3^m)$, will not influence the market belief concerning abilities. This is because these jobs are assigned before new employers actually observe abilities, so they will only influence poaching wages in equilibrium, e.g., $v_1(k)$, $v_2(k,J_2)$, $v_3^s(k,J_2,J_3^s)$, and $v_3^m(k,J_2,J_3^m)$.

Take-away messages

The preliminary analysis delivers two main messages. First, we derive the condition for equilibrium regime. That is, firms will adopt the retention policy with intensive screening whenever the effective "output less payoff to exit" in period 3 outweighs that in period 2. Second, we can just focus on "output less poaching wage" to study the promotion policy (i.e., job assignment) in the game in Figure 2. Specifically, the preliminary analysis indicates that, in both regimes, the promotion policy in period 3 effectively maximizes "output less poaching wage" of period 3, while the promotion policy in period 2 effectively maximizes "output less poaching wage" of period 2 plus something in terms of "output less poaching wage" of period 3. This is because, with promotional signaling and asymmetric employer learning, the promotion policy will influence both outputs and poaching wages in equilibrium, while the retention policy will then be entirely governed by the condition for equilibrium regime.

5 The Equilibrium Analysis

With the preliminary analysis in hand, we now study the game in Figure 2, whereby the promotion policy signals abilities to the market.

5.1 Perfect Bayesian equilibrium

The equilibrium characterization

Table 1 displays a complete description of strategies for a worker of (θ,k) type. The equilibrium characterization is as follows. To ease expositions, as mentioned in Table 1, we will suppress job assignments by new employers. Also, we will suppress mobility decisions of workers (i.e., cutoff utility) since incumbent firms, as described in Section 4, will use the retention strategy (i.e., retention wages) accordingly to maximize expected profits.

In period 1, as no firm detects worker abilities, the worker will be placed as a labour if $p(h|k) < \hat{\theta}$—the posterior is below the threshold—and as a manager otherwise. Due to free entry, all firms offer the wage, $v_1(k)$, to earn an expected profit of zero.

In period 2, the incumbent firm places the worker to job, $J_2(\theta; k)$. Due to free entry, outside firms update believes and offer the poaching wage, $v_2(k, J_2)$, to earn an expected profit of zero. The incumbent firm then offers the ability-contingent retention wage, $w_2(\theta; k, J_2)$, to maximize the expected profit.

In period 3, given the history of job assignments and mobility, the incumbent firm places a period-2 hire (i.e., a mover) to job $J_3^m(\theta; k, J_2)$ and a period-1 hire (i.e., a stayer) to job $J_3^s(\theta; k, J_2)$. Due to free entry, outside firms update believes and offer the poaching wage, $v_3^m(k, J_2, J_3^m)$ to a mover and $v_3^s(k, J_2, J_3^s)$ to a stayer, to earn an expected profit of zero. The incumbent firm then offers the ability-contingent retention wage, $w_3^m(\theta; k, J_2, J_3^m)$ to a mover and $w_3^s(\theta; k, J_2, J_3^s)$ to a stayer, to maximize the expected profit.

Multiple equilibria and equilibrium selection

As mentioned above, in equilibrium the promotion policy will determines a worker's output and—as signals of abilities—will also influences the market belief and hence the poaching wage. From the preliminary analysis, to study the promotion policy we will confine to the changes in outputs and poaching wages for promoting a worker.

Note that the output specification in (1) (as depicted in Figure 1) indicates that output will increase for promoting an h type and will drop for promoting an l type. This suggests that, fixing the schooling level, the net gain from promoting an h type always exceeds that from promoting an l type, regardless of the poaching wage (i.e., the market belief concerning the promotion policy). Thus, an l type is promoted only if every h type is promoted in equilibrium and—depending on the market belief—there will be multiple equilibria of the game. We will use the intuitive criterion (Cho and Kreps, 1987) to select the most efficient equilibrium. Consequently,

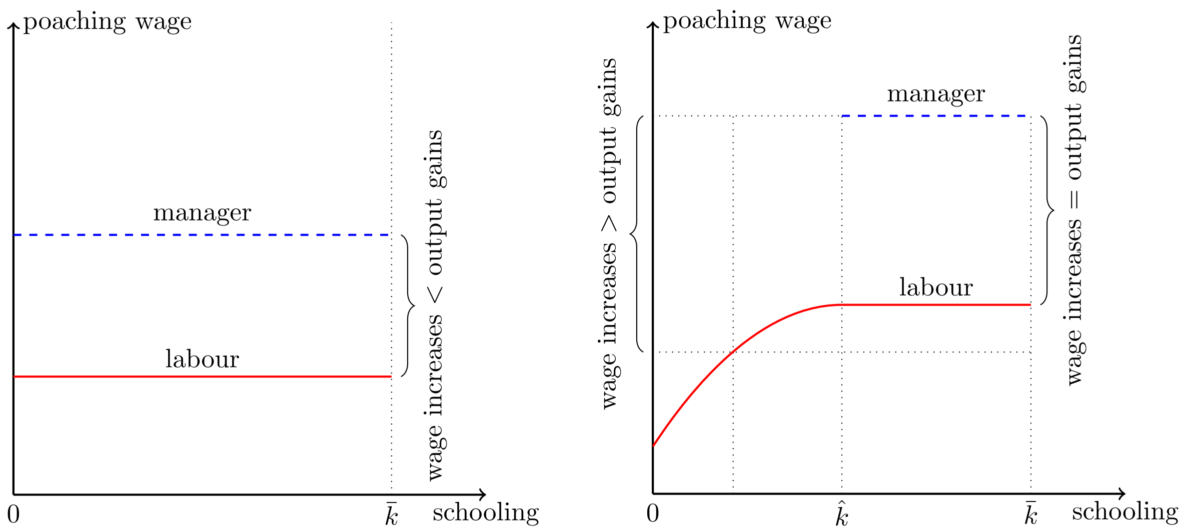
Claim 1. No l type will ever be placed as a manager.

Then, it suffices to consider the promotion policy concerning an h type.

5.2 The personnel policy in equilibrium

In what follows, we present the equilibrium results with a focus on the personnel policy (i.e., promotion and retention) of firms.

First, we show that the promotion policy is efficient in period 3. From the preliminary analysis, the promotion policy in period 3 effectively maximizes "output less poaching wage" of



Poaching wages of period 3 Poaching wages of period 2

Figure 4: Poaching wages in equilibrium.

period 3. When promoting an h type, output will increase and, due to promotional signaling, poaching wages will also increase.[15] We show that every h type will be promoted since output gains, $(1 + n_2)(\hat{\theta}^{-1} - 1)$, always outweigh increases in poaching wages, $\hat{\theta}^{-1} - 1$, where $\hat{\theta}^{-1}$ is the poaching wage to a manager and 1 is the poaching wage to a labour.

Claim 2. Every h type will be promoted in period 3.

The efficient promotion suggests that mobility history in the past and schooling will not affect the market belief. For instance, conditional on being placed on the same job in period 3, a stayer will receive the same poaching wage as a mover (even if they differ in the job placement in period 2 and schooling). As depicted in Figure 4, poaching wages will only depend on job assignments in period 3, where a manager receives a higher poaching wage than a labour.

Second, we show that the promotion policy is inefficient in period 2. From the preliminary analysis, given the efficient promotion policy in period 3, the promotion policy in period 2 effectively maximizes "output less poaching wage" of period 2. When promoting an h type, Assumption 1 indicates rents due to under-bidding are so large that increases in poaching wages, $\hat{\theta}^{-1} + \delta\phi(n_2\hat{\theta}^{-1}) - [1 + \delta\phi(n_2)]$—where $\hat{\theta}^{-1} + \delta\phi(n_2\hat{\theta}^{-1})$ is the poaching wage to a manager and $1 + \delta\phi(n_2)$ is the poaching wage to a labour—outweigh output gains, $(1 + n_2)(\hat{\theta}^{-1} - 1)$.[16] As a consequence, all l types are placed as labours, while not all h types are placed as managers, since

[15] Analogous to promotional signaling models (Waldman, 1984a; Bernhardt, 1995; Zábojník and Bernhardt, 2001; Golan, 2005; DeVaro and Waldman, 2012), we can interpret increases in poaching wages as informational losses since promotion, as signals of abilities, (partially) reveals worker abilities to the market.

[16] Define r as the value of "output less poaching wage", $\phi(r)$ captures rents due to under-bidding earned by incumbent firms. That is, with dispersed value of utility (of working), incumbent firms will under-bid the market in making retention offers to existing workers. Due to free entry, outside firms will need to concede all future rents to new hires, so poaching wages in period 2 will consist of the outputs in period 2 and rents in period 3.

an efficient promotion policy will entail increases in poaching wages greater than output gains.

Claim 3. Not every h type will be promoted in period 2.

As there are only two jobs, we can describe the promotion policy in period 2 as $f_2(\theta;k) \in [0,1]$—the probability that a (θ,k) type will be placed as a manager—with $f_2(h;k) \in [0,1)$ and $f_2(l;k) = 0$. Note that $f_2(h;k) \in (0,1)$ suggests that the promotion policy is in mixed strategies, so the incumbent firm must be indifferent between promoting and not promoting a (θ,k) type. Thus, a (θ,k) type is possibly (resp. never) promoted if the associated increases in poaching wages just equal (resp. outweigh) output gains.[17]

Third, we show that a threshold schooling level $\hat{k}$ exists such that an h type is promoted only if her schooling level is at least $\hat{k}$, i.e., $f_2(h;k) \in (0,1)$ if $k \geq \hat{k}$ and $f_2(h;k) = 0$ otherwise. For workers with schooling above the threshold, as no l type is ever promoted, the market will perceive anyone being promoted as an h type. Thus, as depicted in Figure 4, the poaching wage for a manager is invariant to schooling. This then suggests that, as increases in poaching wages just equal output gains, the poaching wage for a labor is also invariant to schooling. For workers with schooling below the threshold, the poaching wage increases with schooling since, with no prospect of being promoted, schooling will positively signal abilities to the market.

Claim 4. A threshold schooling level, $\hat{k}$, exists such that an h type is promoted in period 2 only if his or her schooling level is at least $\hat{k}$, i.e., $f_2(h;k) \in (0,1)$ if $k \geq \hat{k}$ and $f_2(h;k) = 0$ otherwise.

For h types with non-zero prospect of being promoted (i.e., with schooling $k \geq \hat{k}$), Figure 4 also reflects the countervailing role of "promoted" and "schooling" as signals of abilities. Specifically, the poaching wage in period 2 to a labour with schooling $k \geq \hat{k}$ is invariant to schooling. This suggests that the negative signal of "not being promoted" should exactly offset the positive signal of "high schooling". Thus, as a worker becomes more educated, the likelihood of "being promoted" must increase (to balance the negative signal of "not being promoted" and the positive signal of "high schooling"). Put it differently, the promotion policy is biased towards more educated workers.

Claim 5. For an h type with schooling above the threshold, his or her promotion prospect increases in schooling, i.e., $f_2(h;k)$ increases in k for $k \geq \hat{k}$.

Finally, we show that firms will choose the retention policy with intensive screening. This is mainly because, due to free entry, outside firms need to concede all future rents to recruit a new hire. This leads to a high payoff to "exit" (i.e., moving to a new firm) in period 2, so the effective "output less payoff to exit" in period 3 will outweigh that in period 2. As a consequence, firms will use intensive screening to only retain workers with high utility.

[17] This stands in contrast to promotional signaling models with a continuum of abilities (Waldman, 1984a; Bernhardt, 1995; Zábojník and Bernhardt, 2001; Golan, 2005; DeVaro and Waldman, 2012), where only employees with abilities above a threshold level receive promotion in equilibrium. See Owan (2004) for a related model with binary abilities, where firms also adopt a similar promotion policy in mixed strategies.

Claim 6. For any worker, the retention policy conforms to the regime with intensive screening.

The next result summarizes the above findings concerning the personnel policy in equilibrium.

Proposition 1. *In equilibrium, the personnel policy of firms features that,*

(i) The promotion policy is efficient in period 3, whereby all l types are placed as labours and all h types are placed as managers.

(ii) The promotion policy is inefficient in period 2, whereby all l types are placed as labours and an h type is placed as managers only if his or her schooling is above the threshold level; moreover, the promotion policy favors an h type with more schooling.

(iii) For any worker, the retention policy conforms to the regime with intensive screening, whereby the cutoff for period 2 is strictly above the cutoff for period 3.

6 Nesting ILM and ELM Findings

In this section, we show that the equilibrium nests standard ILM findings (concerning the pay and promotion pattern inside firms) and standard ELM findings (concerning the pay and mobility pattern across firms). We also discuss the difference between our model and related models from the ILM/ELM literature. The next section establishes standard, yet unexplained, findings on the "interaction" of ILM and ELM (concerning effects of promotion on exit rates and effects of entry routes on career paths).

6.1 Standard ILM findings

From the biased promotion policy in period 2 as described in Proposition 1, the equilibrium of our model echoes empirical findings that highly educated workers are favored in the promotion process (Lluis, 2005; Belzil and Bognanno, 2010; DeVaro and Waldman, 2012; Bognanno and Melero, 2016).

Corollary 1. *(Promotion inside firms.) Controlling for seniority and tenure, the promotion prospect of a worker increases with his or her schooling.*

As the promotion policy is efficient in period 3, our model further predicts that this promotional favoritism towards more educated workers is most salient among junior/mid-aged workers since, with adequate history of job separations and assignments, schooling will no longer serve as signals of abilities among senior workers.

In addition, the equilibrium of our model captures two observations concerning the pay structure inside firms. That is: (a) there is a pay gap across jobs (Baker, Gibbs, and Holmström, 1994a,b; McCue, 1996); (b) there is a pay dispersion within jobs (Baker, Gibbs, and Holmström, 1994a,b; Dato et al., 2016).

Corollary 2. *(Pay inside firms.) Controlling for seniority, tenure, and schooling,*

(i) wages increase for workers being promoted, i.e., a pay gap across jobs;

(ii) wages vary among workers holding identical jobs, i.e., a pay dispersion within jobs.

Both results are driven by the hypothesis of asymmetric employer learning concerning abilities. Specifically, due to promotional signaling, poaching wages will increase for workers being promoted, which forces incumbent firms to also raise retention wages in equilibrium. Thus, there is a pay gap across jobs. In addition, retention wages will be ability-contingent such that h types will receive higher retention wages than l types. As not every h type is promoted in period 2, the labour position will consist of both h types and l types. This then indicates a pay dispersion within the labour position.[18]

Relation to existing ILM models

The logic above is analogous to standard promotional signaling models (Waldman, 1984a; Bernhardt, 1995; Zábojník and Bernhardt, 2001; Golan, 2005; DeVaro and Waldman, 2012). Note, however, that there are two notable differences. First, our model predicts that some workers change jobs across firms in equilibrium. This stands in sharp contrast to standard promotional signaling models where the equilibrium features no worker turnover. Second, our model predicts that some workers holding identical jobs will receive differential wages in equilibrium, whereas wages are attached to jobs, and hence identical within jobs, in standard promotional signaling models.

Specifically, in standard promotional signaling models incumbent firms can match (counteroffer) poaching wages, so outsiders can raid a worker only when the poaching wage exceeds the retention wage. This then leads to a winner's curse type result that outsiders will offer a pay equals the productivity at an outside firm of the worst worker with the same labor market signal. Due to the return to tenure, incumbent firms will always out-bid outside firms by simply matching poaching wages. Consequently, there will be no job separation in equilibrium. Moreover, wages are attached to jobs, so there will be a pay gap across jobs, yet the pay is equal within jobs.

In contrast, our model predicts that some workers, in order to resample utility of working, will separate from incumbent firms even though retention wages under-bid poaching wages. Thus, by considering utility of working, our model enriches standard promotional signaling models in two important aspects. That is, the equilibrium of our model not only captures a pay gap across jobs but also features the coexistence of worker turnover and a wage dispersion within jobs.

[18] There are many models, yet with a different explanation, which also feature a wage dispersion within jobs. For instance, Ricart-I-Costa (1988) and Golan (2009) consider settings where workers can offer contracts to firms or bargain wages with firms, so firms will offer higher wages to more productive workers as failing to retain such workers is more costly. With a different timing assumption that incumbent firms and outsider firms make offers simultaneously, Li (2013) shows that turnover and a wage dispersion within jobs can coexist whenever outsider firms randomize offers.

6.2 Standard ELM findings

The hypothesis of asymmetric employer learning also indicates an "adverse selection" effect in the labor market (Greenwald, 1986; Gibbons and Katz, 1991; Fan and DeVaro, 2015; Dato et al., 2016) that low-ability workers are more likely to exit than high-ability workers.

Corollary 3. *(The "adverse selection" effect.) Controlling for schooling and jobs, exit rates of period 2 decline with abilities.*

From the promotion policy in Proposition 1, the equilibrium of our model features reducing information asymmetry over time. This is because the promotion policy becomes efficient in period 3, so effectively there is no asymmetric information in period 3 since outsiders can perfectly infer worker abilities through the promotion policy. The promotional policy in period 2 is, however, inefficient. Consider an h type and an l type who are observationally equivalent to outside firms—they hold identical jobs and schooling—and will receive equal poaching wages. However, the h type will receive a higher retention wage and hence is less likely to exit than the l type in period 2.

From the retention policy in Proposition 1, we can also derive the dynamics of job changes across firms in equilibrium, which match stylized facts concerning the macroeconomic mobility pattern of workers (Mincer and Jovanovic, 1981; Farber, 1994, 1998, 1999; Fallick and Fleischman, 2004). That is, (a) exit rates decline with tenure; (b) exit rates decline with seniority; (c) many new jobs end early, and long-term employment relationships are common.

Corollary 4. *(The mobility pattern across firms.) Controlling for schooling and jobs,*

(i) incumbents are less likely to change jobs across firms than new hires in period 3;

(ii) some workers stay at the same firm in both period 2 and period 3;

(iii) workers are more likely to change jobs across firms in period 2 than in period 3.

For result (i) and (ii), incumbents are positively selected (concerning utility) and—even if they may resample utility in period 3—are, on average, less mobile than new hires; in particular, some will stay in both period 2 and period 3 if they do not resample utility in period 3. Result (iii) holds since workers hold a higher cutoff for period 2 and hence are more mobile than workers in period 3.

We also establish a "pay more for less" effect, with strong support in the empirical literature, for recruiting from outside. That is, new hires receive a pay increase for changing firms across firms (Bartel and Borjas, 1981; Mincer, 1986; Topel and Ward, 1992; Keith and McWilliams, 1999; Buchinsky et al., 2010), although their initial performances are poorer than incumbents (Baker, Gibbs, and Holmström, 1994a; Bidwell, 2011; Van der Klaauw and Da Silva, 2011; Kauhanen and Napari, 2012; Frederiksen, Halliday, and Koch, 2016).

Corollary 5. *(The "pay more for less" effect.) Controlling for seniority, schooling, and jobs, new hires initially produce lower outputs than incumbents, yet receive a pay increase for changing jobs across firms.*

Our model provides two explanations for under-performing new hires. First, some l-type new hires, who have not received promotion at incumbent firms, may be misplaced as managers at new firms in period 2—since new firms place new hires based upon the market belief—and hence will under-perform incumbents. Second, even new firms make correct placement of new hires, incumbents will still out-perform new hires due to the return to tenure.

The wage increase for changing jobs across firms is driven by both intensive screening and under-bidding. On one hand, intensive screening indicates that incumbent firms lower retention wages in period 2 to weed out the workers with low utility and lower retention wages in period 3 to exploit information rents from the workers with high utility. On the other hand, with disperse values of utility, how much a worker likes a job (i.e., non-financial aspects of working) becomes more important relative to the pay (i.e., financial aspects of working) in the mobility decision. As a consequence, incumbent firms will under-bid outside firms by offering retention wages below poaching wages.

Relation to existing ELM models

The analysis of job changes across firms is remarkably fruitful in macro-labor models such as job search (Mortensen, 1986; Mortensen and Pissarides, 1999; Rogerson and Shimer, 2011) and job matching (Jovanovic, 1979). In particular, standard job-search models consider jobs as "inspection" goods, where workers sequentially sample wage offers from a given distribution. In contrast, standard job-matching model consider jobs as "experience" goods, where workers sample productivity-related match qualities and firms gradually detect match qualities of workers from their output performances.

Overall, macro-labor models can nicely capture the macroeconomic pattern concerning job changes across firms in Corollary 4. Our model differs from standard macro-labor models in two aspects. First, utility of working is an "experience" good at an outside firm and an "inspection" good at the incumbent firm. Second, utility of working is privately experienced by workers, so firms use dynamic screening to infer the match quality of utility. As a consequence, the sharing rule between firms and workers—the retention policy in equilibrium—is endogenous and non-fixed over time. In this regard, our model enriches standard macro-labor models, whereby job-search models abstract away from the retention policy and job-matching models presume some fixed sharing rules.[19]

[19] In a survey paper, Lazear and Oyer (2013) point out that the personnel policy of firms is often treated as a black box in standard macro-labor models.

The logic behind Corollary 4 is also closely related to the switching-cost approach (Blumen, Kogan, and McCarthy, 1955; Farber, 1994), whereby employees each endows some idiosyncratic cost for changing jobs across firms. As new hires are over-represented by those with relatively low switching costs, employees who have repeatedly changed firms will have a greater propensity to quit again in the future. In contrast, our model considers retention wages as dynamic screening mechanisms to weed out workers with low utility of working. Furthermore, utility of working is a firm-specific match quality, as opposed to an individual-specific parameter concerning the switching cost.

As to Corollary 5, the impact of job mobility on earnings growth is the subject of many macro-labor models. For instance, in Postel-Vinay and Robin (2002) and Bagger et al. (2014) firms could renegotiate piece-rate wages with employees who obtain more attractive poaching offers. Since more productive firms end with retaining existing workers or poaching new workers, employees will capture a wage growth in equilibrium for changing jobs across firms. In contrast, the pay increase for changing jobs across firms in our model does not hinge upon heterogeneity of production technology across firms. Rather, even with completely homogeneous firms, the retention strategy of incumbent firms—intensive screening and under-bidding—can rationalize the earning growth for job movers.

7 "Interaction" Findings

We have seen that the equilibrium of our model nests standard ILM and ELM findings. In this section, we further explore how the equilibrium of our model speakers to two sets of "interaction" findings with strong support in the empirical literature. One concerns effects of promotion on exit rates, and the other is about effects of entry routes on career paths and workforce compositions.

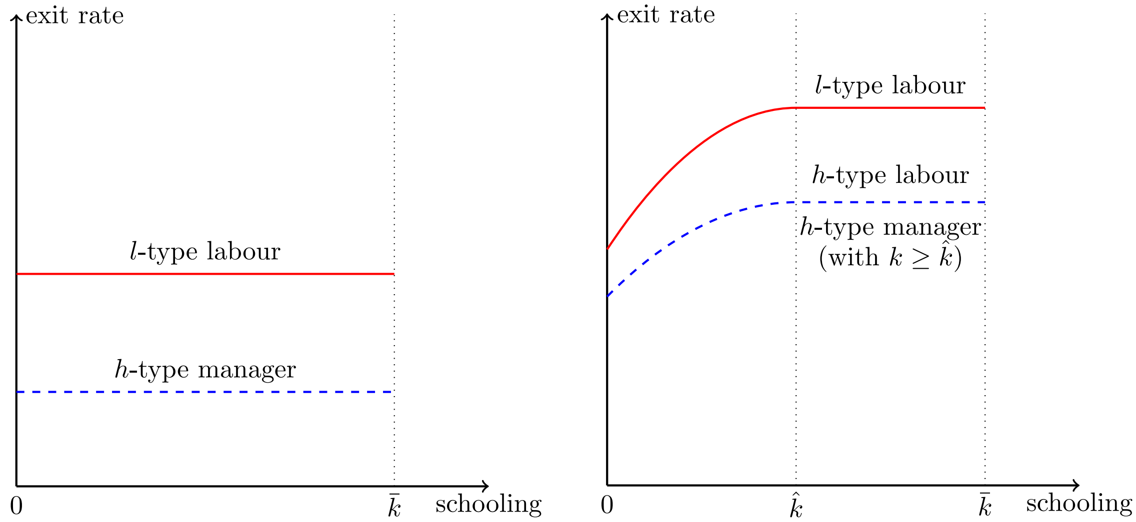
Effects of promotion on exit rates

To begin with, we examine effects of promotion on exit rates.

We find a negative relationship between promotion and exit rates (Carson et al., 1994; Trevor, Gerhart, and Boudreau, 1997; Saporta and Farjoun, 2003).

Corollary 6. *(Effect of promotion on exit rates.) Controlling for tenure, seniority, and schooling, workers becomes less likely to exit after being promoted.*

This result directly follows from the "adverse selection" effect in Corollary 3. That is, workers being promoted are purely h types, who will be less mobile relative to those not being promoted. As depicted in Figure 5, the efficient promotion policy in period 3 suggests that all h types are placed as managers in period 3, and thus will be less mobile than all l types being placed as labours. Figure 5 also indicates that l types are more mobile than h types in period 2. Thus,



Exit rates of period 3   Exit rates of period 2

Figure 5: Exit rates in equilibrium.

despite the inefficient promotion policy in period 2, labours—consisting of both h types and l types—will be on average more mobile than managers.

We also establish a "fast-track exit" effect (Baker, Gibbs, and Holmström, 1994a; Treble et al., 2001; Van der Klaauw and Da Silva, 2011). That is, those being promoted sooner are more mobile than those being promoted later.[20]

Corollary 7. *(The "fast-track exit" effect.) Controlling for schooling, a worker being promoted in period 2 is more likely to exit than a worker being promoted in period 3.*

This result is driven by intensive screening, where the cutoff for period 2 is higher than that for period 3. Specifically, Figure 5 suggests that—as a consequence of the promotion policy in mixed strategies as described in Proposition 1—the exit rate of an h-type manager equals that of an h-type labour in period 2. As exit rates decline with seniority (due to intensive screening as described in Corollary 4), an h type being promoted in period 3 will be less inclined to changes jobs across firms than an h type being promoted in period 2.

Effects of entry routes on career paths and workforce compositions

We then examine effects of entry routes on career paths.

[20] It is worth noting that Baker, Gibbs, and Holmström (1994a,b) also document a "fast-track promotion" effect that workers being promoted early are more likely to be promoted later. One can extend our model to consider three job levels (e.g., labour, mid manager, top manager). Although we have not formally proved it, we conjecture that such extension can generate the "fast-track promotion" effect due to the positive selection (in terms of abilities) of workers over time.

Corollary 8. *(Effects of entry routes on career paths.) Controlling for seniority, schooling, and jobs,*

(i) new hires are subsequently more likely to exit than incumbents;

(ii) new hires are subsequently less likely to be promoted than incumbents.

Result (i) reiterates the mobility pattern across firms in Corollary 4, where new hires are (subsequently) more likely to exit than incumbents. This finding also matches the documented positive serial correlation in job-separation rates (Baker, Gibbs, and Holmström, 1994a; Treble et al., 2001; Bidwell, 2011; Van der Klaauw and Da Silva, 2011).

Result (ii) is due to the "adverse selection" effect in Corollary 3. Specifically, among workers not being promoted in period 2, new hires have a lower promotion prospect in period 3 than incumbents (Baker, Gibbs, and Holmström, 1994a; Treble et al., 2001; Chan, 2006; Oyer, 2007; Bidwell, 2011; Kauhanen and Napari, 2012) because the former is more represented by l types than the latter.

Note that our model does not address the recruitment choice between external candidates and incumbents, which is the main focus of many ILM studies. For instance, Malcomson (1984) and Chan (1996) consider models where—because opening up a position to external candidates weakens the promotion prospect and hence the motives of incumbents—employers handicap external hiring so that insiders are favored for promotion.[21] In contrast, our model abstracts away from the motivational role of promotion. Rather, due to the "adverse selection" effect, job movers consist of more l types than job stayers, so from an efficient-sorting (selection) perspective, more job stayers will receive promotion than job movers.

Finally, we establish that new hires are in general observationally stronger (e.g., being more educated) than incumbents (Baker, Gibbs, and Holmström, 1994a; Bidwell, 2011; Kauhanen and Napari, 2012; DeVaro, Kauhanen, and Valmari, 2019).

Corollary 9. *(Effects of entry routes on workforce composition.) Controlling for seniority and jobs, new hires are more educated than incumbents.*

This result can be viewed from exit rates as depicted in Figure 5. That is, controlling for jobs and abilities, exit rates of period 3 are invariant to schooling, whereas exit rates of period 2 (weakly) increase with schooling. Specifically, the efficient promotion policy in period 3 indicates that schooling will no longer signal abilities. Thus, regardless of schooling, workers holding identical jobs will receive the same poaching wage and hence are equally likely to exit in period 3. By contrast, due to the inefficient promotion policy in period 2, schooling will signal

[21] In the same spirit, DeVaro and Morita (2013) and Ke, Li, and Powell (2018) show that both firm sizes and spans of control can impact on the recruitment choice between new hires and incumbents and the extent of insider bias for promotion. Waldman (2003) points out that the dual role of promotion—incentives for lower-level workers versus sorting of high-ability workers to high-level jobs—poses a time-inconsistent problem, which can rationalize firms' commitment to internal promotion.

abilities of workers not being promoted (i.e., labours). Thus, labours who are more educated will receive higher poaching wages and hence are more likely to exit in period 2.

The observation that exit rates of period 2 (weakly) increase with schooling then suggest that new hires will be more educated than incumbents in period 2. Despite of the observation that exit rates of period 3 are invariant to schooling, new hires will still be more educated than incumbents in period 3. This is because, in period 2, job stayers are already over-represented by less educated workers than job movers. Then, in period 3, we know from the mobility pattern in Corollary 4 that many job stayers will also stay in period 3, which drags down the overall education attainments among incumbents.

Key messages

In short, we find that promotional signaling and intensive screening—which arise in equilibrium due to asymmetric employer learning of abilities and private information of utility—are crucial in driving the above two sets of "interaction" findings. As existing models are, however, silent to the "interaction" of the internal and external labor markets, we are the first to use an integrative approach to offer an explanation to these "interaction" findings.

Most importantly, our model can capture three broad sets of empirical findings in a single setting. That is, (1) standard ILM findings (concerning the pay and promotion pattern inside firms); (2) standard ELM findings (concerning the pay and mobility pattern across firms); (3) standard, yet unexplained, findings on the "interaction" of ILM and ELM (concerning effects of promotion on exit rates and effects of entry routes on future career paths and workforce compositions). In this regard, our model offers a micro-foundation for how the personnel policy of firms—promotion and retention—shapes worker careers with job changes both inside and across firms.

8 Conclusion

This paper takes a step towards bridging the theory of internal labor market (ILM) and external labor market (ELM) by studying an integrative framework with job changes both inside and across firms. In particular, we consider a dynamic model, whereby firms use retention and promotion as the personnel policy to positively select the workforce (workers) concerning both utility of working and abilities. The equilibrium captures three sets of empirical findings in a single setting. That is, (1) standard ILM findings (concerning the pay and promotion pattern inside firms); (2) standard ELM findings (concerning the pay and mobility pattern across firms); (3) standard, yet unexplained, findings on the "interaction" of ILM and ELM (concerning effects of promotion on exit rates and effects of entry routes on career paths and workforce compositions).

We hope to have shown how a partnership of utility of working and worker abilities can be used to obtain a deeper understanding of the personnel policy and labor market phenomena. Hopefully, more works will pursue this two-way interaction. There are several interesting directions for future research. First and foremost, we consider utility of working as how much a worker likes his or her job, which changes over time according to some exogenous stochastic process. That is, utility either remains unchanged with probability q or resamples with probability $1 - q$. One can develop more sophisticated models whereby the stochastic process will depend on the internal structure of firms and/or workforce compositions. Second, our analysis has confined to the simplest possible case where firms are completely homogeneous. One can enrich our model to allow heterogeneous spans of control and production technology. A promising direction is to enrich the market-based tournament (Zábojník and Bernhardt, 2001)—where firms use promotion to incentivize employees and the prize spread for promotion is market determined—with utility of working. Third, we only consider job changes along the job ladder inside single-divisional firms. One can adapt our model to also examine lateral moves inside firms, where employees may resample utility of working upon moving to a new division inside multi-divisional firms.

References

Acemoglu, Daron and Jörn-Steffen Pischke. 1998. "Why do firms train: theory and evidence." *Quarterly Journal of Economics* 113:79–119.

Agrawal, Anup, Charles Knoeber, and Theofanis Tsoulouhas. 2006. "Are outsiders handicapped in CEO successions?" *Journal of Corporate Finance* 12 (3):619–644.

Altonji, Joseph and Charles Pierret. 2001. "Employer learning and statistical discrimination." *Quarterly Journal of Economics* 116:313–350.

Bagger, Jesper, François Fontaine, Fabien Postel-Vinay, and Jean-Marc Robin. 2014. "Tenure, experience, human capital, and wages: A tractable equilibrium search model of wage dynamics." *American Economic Review* 104:1551–1596.

Baker, George, Michael Gibbs, and Bengt Holmström. 1994a. "The internal economics of the firm: evidence from personnel data." *Quarterly Journal of Economics* 109:881–919.

———. 1994b. "The wage policy of a firm." *Quarterly Journal of Economics* 109:921–955.

Barron, John, Mark Berger, and Dan Black. 2006. "Selective counteroffers." *Journal of Labor Economics* 24 (3):385–409.

Bartel, Ann. 1995. "Wage growth, and job performance: evidence from a company database." *Journal of Labor Economics* 13:401–425.

Bartel, Ann and George Borjas. 1981. "Wage growth and job turnover: An empirical analysis." In *Studies in Labor Markets*. University of Chicago Press, 65–90.

Belzil, Christian and Michael Bognanno. 2010. "The promotion dynamics of American executives." In *Jobs, Training, and Worker Well-Being*. Emerald Group Publishing Limited, 189–231.

Bernhardt, Dan. 1995. "Strategic promotion and compensation." *Review of Economic Studies* 62:315–339.

Bernhardt, Dan and David Scoones. 1993. "Promotion, turnover, and preemptive wage offers." *American Economic Review* 83:771–791.

Bidwell, Matthew. 2011. "Paying more to get less: specific skills, matching, and the effects of external hiring versus internal promotion." *Administrative Science Quarterly* 56:369–407.

Bjelland, Melissa, Bruce Fallick, John Haltiwanger, and Erika McEntarfer. 2011. "Employer-to-Employer Flows in the United States: Estimates Using Linked Employer-Employee Data." *Journal of Business & Economic Statistics* 29 (4):493–505.

Blumen, Isadore, Marvin Kogan, and Philip McCarthy. 1955. *The industrial mobility of labor as a probability process*, *Cornell Studies in Industrial and Labor Relation*, vol. 6. Ithaca, NY: Cornell University Press.

Bognanno, Michael and Eduardo Melero. 2016. "Promotion signals, experience, and education." *Journal of Economics & Management Strategy* 25:111–132.

Buchinsky, Moshe, Denis Fougere, Francis Kramarz, and Rusty Tchernis. 2010. "Inter-firm mobility, wages, and the returns to seniority and experience in the US." *Review of Economic Studies* 77:972–1001.

Carrillo-Tudela, Carlos and Leo Kaas. 2015. "Worker mobility in a search model with adverse selection." *Journal of Economic Theory* 160:340–386.

Carson, Paula, Kerry Carson, Rodger Griffeth, and Robert Steel. 1994. "Promotion and employee turnover: Critique, meta-analysis, and implications." *Journal of Business and Psychology* 8:455–466.

Chan, William. 1996. "External recruitment versus internal promotion." *Journal of Labor Economics* 14:555–570.

———. 2006. "External recruitment and intrafirm mobility." *Economic Inquiry* 44:169–184.

Chang, Chun and Yijiang Wang. 1996. "Human capital investment under asymmetric information: the Pigovian conjecture revisited." *Journal of Labor Economics* 14 (3):505–519.

Cho, In-Koo and David Kreps. 1987. "Signaling games and stable equilibria." *Quarterly Journal of Economics* 102:179–221.

Dato, Simon, Andreas Grunewald, Matthias Kräkel, and Daniel Müller. 2016. "Asymmetric employer information, promotions, and the wage policy of firms." *Games and Economic Behavior* 100:273–300.

Deb, Rahul and Colin Stewart. 2018. "Optimal adaptive testing: Informativeness and incentives." *Theoretical Economics* 13 (3):1233–1274.

Demougin, Dominique and Aloysius Siow. 1994. "Careers in ongoing hierarchies." *American Economic Review* 84:1261–1277.

Denis, David and Diane Denis. 1995. "Performance changes following top management dismissals." *Journal of Finance* 50 (4):1029–1057.

DeVaro, Jed, Antti Kauhanen, and Nelli Valmari. 2019. "Internal and external hiring: the role of prior job assignments." *Industrial and Labor Relations Review* 72 (4):981–1008. Working Paper.

DeVaro, Jed and Hodaka Morita. 2013. "Internal promotion and external recruitment: a theoretical and empirical analysis." *Journal of Labor Economics* 31:227–269.

DeVaro, Jed and Michael Waldman. 2012. "The signaling role of promotions: further theory and empirical evidence." *Journal of Labor Economics* 30:91–147.

Doeringer, Peter and Michael Piore. 1971. *Internal labor markets and manpower analysis*. Lexington, MA: D.C. Heath.

Fallick, Bruce C. and Charles A. Fleischman. 2004. "Employer-to-employer flows in the U.S. labor market: the complete picture of gross worker flows." Finance and Economics Discussion Series 2004-34, Board of Governors of the Federal Reserve System (US).

Fan, Xiaodong and Jed DeVaro. 2015. "Does job hopping help or hinder careers? The signaling role of work history." Working Paper.

Farber, Henry. 1994. "The analysis of interfirm worker mobility." *Journal of Labor Economics* 12:554–593.

———. 1998. "Are Lifetime Jobs Disappearing? Job Duration in the United States, 1973-1993." In *Labor statistics measurement issues*. University of Chicago Press, 157–206.

———. 1999. *Mobility and stability: the dynamics of job change in labor markets*, chap. 37. Handbook of Labor Economics, 2439–2483.

Farber, Henry and Robert Gibbons. 1996. "Learning and wage dynamics." *Quarterly Journal of Economics* 111:1007–1047.

Ferreira, Daniel and Radoslawa Nikolowa. 2017. "Adverse selection and assortative matching in labor markets." Working Paper.

Frederiksen, Anders, Timothy Halliday, and Alexander Koch. 2016. "Within- and cross-firm mobility and earnings growth." *Industrial and Labor Relations Review* 69:320–353.

Friedrich, Benjamin. 2019. "Internal labor markets and the competition for managerial talent." Working Paper.

Fudenberg, Drew and J Miguel Villas-Boas. 2006. "Behavior-based price discrimination and customer recognition." *Handbook on Economics and Information Systems* 1:377–436.

Ghosh, Suman. 2007. "Job mobility and careers in firms." *Labour Economics* 14:603–621.

Gibbons, Robert and Lawrence Katz. 1991. "Layoffs and lemons." *Journal of Labor Economics* 9:351–380.

Gibbons, Robert and Michael Waldman. 1999a. *Careers in organizations: theory and evidence*, chap. 36. Handbook of Labor Economics, 2373–2437.

———. 1999b. "A theory of wage and promotion dynamics inside firms." *Quarterly Journal of Economics* 114:1321–1358.

———. 2006. "Enriching a theory of wage and promotion dynamics inside firms." *Journal of Labor Economic* 24:59–107.

Golan, Limor. 2005. "Counteroffers and efficiency in labor markets with asymmetric information." *Journal of Labor Economics* 23:373–393.

———. 2009. "Wage signaling: a dynamic model of intrafirm bargaining and asymmetric learning." *International Economic Review* 50:831–854.

Greenwald, Bruce. 1986. "Adverse selection in the labour market." *Review of Economic Studies* 53:325–347.

Harris, Milton and Bengt Holmström. 1982. "A theory of wage dynamics." *Review of Economic Studies* 49:315–333.

Jovanovic, Boyan. 1979. "Job matching and the theory of turnover." *Journal of Political Economy* 87:972–990.

Kahn, Lisa. 2013. "Asymmetric information between employers." *American Economic Journal: Applied Economics* 5:165–205.

Kauhanen, Antti and Sami Napari. 2012. *Career and wage dynamics: evidence from linked employer-employee data*. Research in Labor Economics, 35–76.

Ke, Rongzhu, Jin Li, and Michael Powell. 2018. "Managing careers in organizations." *Journal of Labor Economics* 36 (1):197–252.

Keith, Kristen and Abagail McWilliams. 1999. "The returns to mobility and job search by gender." *Industrial and Labor Relations Review* 52:460–477.

Laing, Derek. 1994. "Involuntary layoffs in a model with asymmetric information concerning worker ability." *Review of Economic Studies* 61:375–392.

Lazear, Edward. 1986. *Raids and offer matching*, chap. 35. Research in Labor Economics, 141–165.

Lazear, Edward and Paul Oyer. 2013. *Personnel Economics*, chap. 12. Handbook of Organizational Economics, 479–519.

Li, Jin. 2013. "Job mobility, wage dispersion, and technological change: an asymmetric information perspective." *European Economic Review* 60:105–26.

Lluis, Stephanie. 2005. "The role of comparative advantage and learning in wage dynamics and intrafirm mobility: Evidence from Germany." *Journal of Labor Economics* 23:725–767.

MacLeod, W. Bentley and James Malcomson. 1988. "Reputation and hierarchy in dynamic models of employment." *Journal of Political Economy* 96:832–854.

Malcomson, James. 1984. "Work incentives, hierarchy, and internal labor markets." *Journal of Political Economy* 92:486–507.

McCue, Kristin. 1996. "Promotions and wage growth." *Journal of Labor Economics* 14:175–209.

Milgrom, Paul and Sharon Oster. 1987. "Job discrimination, market forces, and the invisibility hypothesis." *Quarterly Journal of Economics* 102:453–476.

Mincer, Jacob. 1986. "Wage changes in job changes." *Research in Labor Economics* 8:171–197.

Mincer, Jacob and Boyan Jovanovic. 1981. "Labor Mobility and Wages." In *Studies in Labor Markets*. University of Chicago Press, 21–64.

Montgomery, James. 1991. "Social Networks and Labor-Market Outcomes: Toward an Economic Analysis." *American Economic Review* 81:1408–1418.

Mortensen, Dale. 1986. *Job search and labor market analysis*, vol. 2, chap. 15. Handbook of Labor Economics, 849–919.

Mortensen, Dale and Christopher Pissarides. 1999. *New developments in models of search in the labor market*, vol. 3, chap. 39. Handbook of Labor Economics, 2567–2627.

Ortega, Jaime. 2001. "Job rotation as a learning mechanism." *Management Science* 47 (10):1361–1370.

Owan, Hideo. 2004. "Promotion, turnover, earnings, and firm-sponsored training." *Journal of Labor Economic* 22:955–978.

Oyer, Paul. 2007. "Is there an insider advantage in getting tenure." *American Economic Review* 97:501–505.

Pastorino, Elena. 2015. "Job matching within and across firms." *International Economic Review* 56:647–671.

———. 2019. "Careers in firms: estimating a model of job assignment, learning, and human capital acquisition." Working Paper.

Pinkston, Joshua. 2009. "A model of asymmetric employer learning with testable implications." *Review of Economic Studies* 76:367–394.

Postel-Vinay, Fabien and Jean-Marc Robin. 2002. "Equilibrium wage dispersion with worker and employer heterogeneity." *Econometrica* 70:2295–2350.

Ricart-I-Costa, Joan. 1988. "Managerial task assignment and promotions." *Econometrica* 56 (2):449–466.

Rogerson, Richard and Robert Shimer. 2011. *Search in macroeconomic models of the labor market*, vol. 4a, chap. 7. Elsevier, 619–700.

Rosen, Sherwin. 1974. “Hedonic prices and implicit markets: product differentiation in pure competition.” *Journal of Political Economy* 82 (1):34–55.

———. 1982. “Authority, control, and the distribution of earnings.” *Bell Journal of Economics* 13:311–323.

Saporta, Ishak and Moshe Farjoun. 2003. “The relationship between actual promotion and turnover among professional and managerial-administrative occupational groups.” *Work and Occupations* 3:255–280.

Sattinger, Michael. 1975. “Comparative advantage and the distribution of earnings and abilities.” *Econometrica* 43:455–468.

Schönberg, Uta. 2007. “Testing for asymmetric employer learning.” *Journal of Labor Economics* 25 (4):651–691.

Spence, Michael. 1973. “Job market signaling.” *Quarterly Journal of Economics* 87:355–374.

Tirole, Jean. 2016. “From bottom of the barrel to cream of the crop: sequential screening with positive selection.” *Econometrica* 84:1291–1343.

Topel, Robert and Michael Ward. 1992. “Job mobility and the careers of young men.” *Quarterly Journal of Economics* 107:439–479.

Treble, John, Edwin van Gameren, Sarah Bridges, and Tim Barmby. 2001. “The internal economics of the firm: further evidence from personnel data.” *Labour Economics* 8 (5):531–552.

Trevor, Charlie, Barry Gerhart, and John Boudreau. 1997. “Voluntary turnover and job performance: Curvilinearity and the moderating influences of salary growth and promotions.” *Journal of Applied Psychology* 82:44–61.

Van der Klaauw, Bas and António Dias Da Silva. 2011. “Wage dynamics and promotions inside and between firms.” *Journal of Population Economics* 24:1513–1548.

Waldman, Michael. 1984a. “Job assignments, signalling, and efficiency.” *RAND Journal of Economics* 15:255–267.

———. 1984b. “Worker allocation, hierarchies, and the wage distribution.” *Review of Economic Studies* 51:95–109.

———. 1990. “Up-or-out contracts: a signaling perspective.” *Journal of Labor Economics* 8:230–250.

———. 2003. “Ex ante versus ex post optimal promotion rules: the case of internal promotion.” *Economic Inquiry* 41:27–41.

———. 2013. *Theory and evidence in internal labor markets*, chap. 13. Handbook of Organizational Economics, 520–571.

Zábojník, Ján and Dan Bernhardt. 2001. “Corporate tournaments, human capital acquisition, and the firm size-wage relation.” *Review of Economic Studies* 68:693–716.

A Technical Details for the Simplified Game

A.1 The retention strategy

We first state an auxiliary result. Consider an optimization problem, $\phi(r) = \max_\eta [1 - G(\eta)](r + \eta)$, where r is some constant. Denote the FOC as $\Phi(\hat{\eta}) \equiv \frac{1-G(\hat{\eta})}{g(\hat{\eta})} - \hat{\eta} = r$. Note from $d\frac{g(\eta)}{1-G(\eta)}/d\eta \geq 0$ (since G is log-concave) that the FOC is strictly decreasing, so $\hat{\eta} = \Phi^{-1}(r)$ is unique. The Inverse Function Theorem then suggests that $d\hat{\eta}/dr = d\Phi^{-1}(r)/dr = [d\Phi(\hat{\eta})/d\hat{\eta}]^{-1} < 0$.

Lemma A1. *$\hat{\eta} = \Phi^{-1}(r)$ is unique and decreases in r.*

In what follows, we formally study the retention strategy for a period-2 hire and a period-1 hire.

A.1.1 The retention strategy for a period-2 hire

For a period-2 hire, the retention strategy is as follows. In period 3, a period-2 hire is retained whenever her payoff to stay exceeds the payoff to exit, i.e., $w_3^m + \eta_3 \geq e_3^m$, so the cutoff is $\hat{\eta}_3^m = e_3^m - w_3^m$ for period 3. Note that, regardless of whether utility is resampled or not, utility distribution for a period-2 hire is G in period 3. That is, the probability of "move" is given by $\Pr\{\eta_3 \leq e_3^m - w_3^m\} = q\Pr\{\eta_3 = \eta_2 \leq \hat{\eta}_3^m\} + (1-q)\Pr\{\eta_3 \leq \hat{\eta}_3^m\} = G(\hat{\eta}_3^m)$. The firm offers a retention wage w_3^m to maximize the expected profit

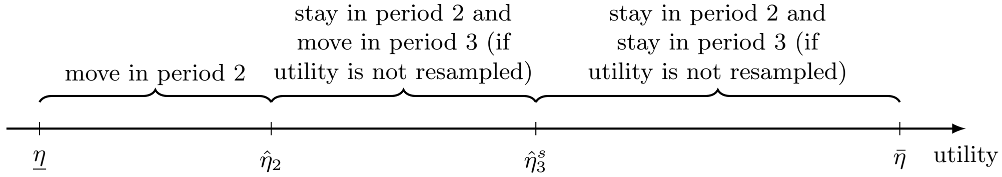
$$\pi_3^m = \max_{w_3^m} \Pr\{\eta_3 \geq e_3^m - w_3^m\}(y_3^{m,s} - w_3^m) = \max_{\hat{\eta}_3^m}[1 - G(\hat{\eta}_3^m)][y_3^{m,s} - (e_3^m - \hat{\eta}_3^m)],$$

so the cutoff and expected profit are, respectively, $\hat{\eta}_3^m = \Phi^{-1}(y_3^{m,s} - e_3^m)$ and $\pi_3^m = \phi(y_3^{m,s} - e_3^m)$.

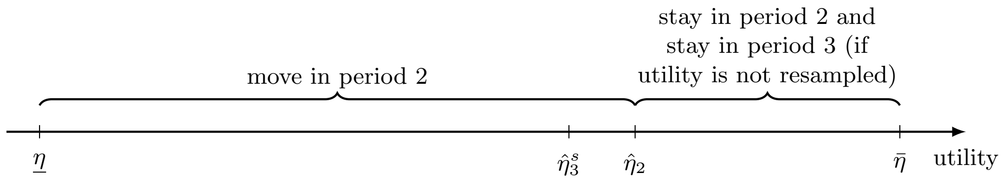
Lemma A2. *For a period-2 hire, the cutoff for period 3 is $\hat{\eta}_3^m = \Phi^{-1}(y_3^{m,s} - e_3^m)$.*

A.1.2 The retention strategy for a period-1 hire

For a period-1 hire, the retention strategy is as follows. Since the utility distribution for a stayer in period 2 is left truncated, there are two possible cases (regimes)—$\hat{\eta}_2 \leq \Phi^{-1}(y_3^{s,s} - e_3^s)$ or $\hat{\eta}_2 > \Phi^{-1}(y_3^{s,s} - e_3^s)$ where $\Phi^{-1}(y_3^{s,s} - e_3^s)$ is the benchmark cutoff—as depicted in the Figure below. Notice that the regime with intensive screening features a higher cutoff in both period 2 and period 3 than the regime with non-intensive screening.



Case with $\hat{\eta}_2 \leq \Phi^{-1}(y_3^{s,s} - e_3^s)$ (regime with non-intensive screening)



Case with $\hat{\eta}_2 > \Phi^{-1}(y_3^{s,s} - e_3^s)$ (regime with intensive screening)

Case with $\hat{\eta}_2 \leq \Phi^{-1}(y_3^{s,s} - e_3^s)$.

Solving the game backward, we first consider retention in period 3. The payoff to stay is $w_3^s + \eta_3$ and the payoff to exit is e_3^s, so the cutoff is $\hat{\eta}_3^s = e_3^s - w_3^s$ for period 3. The firm offers a retention wage w_3^s to maximize the expected profit

$$\begin{aligned}\pi_3^s = \max_{w_3^s} & \left[q \cdot \overbrace{\begin{matrix}\text{Prob. of "stay" in period 3} \\ \text{if utility is not resampled}\end{matrix}}^{\Pr\{\eta_3 = \eta_2 \geq e_3^s - w_3^s | \eta_2 \geq \hat{\eta}_2\} = \frac{1-G(\hat{\eta}_3^s)}{1-G(\hat{\eta}_2)}} + (1-q) \cdot \overbrace{\begin{matrix}\text{Prob. of "stay" in period 3} \\ \text{if utility is resampled}\end{matrix}}^{\Pr\{\eta_3 \geq e_3^s - w_3^s\} = 1-G(\hat{\eta}_3^s)} \right] (y_3^{s,s} - w_3^s) \\ &= \max_{\hat{\eta}_3^s} \left[\frac{q}{1-G(\hat{\eta}_2)} + (1-q) \right] [1 - G(\hat{\eta}_3^s)][y_3^{s,s} - (e_3^s - \hat{\eta}_3^s)].\end{aligned}$$

Thus, the firm will implement a cutoff coincidences with the benchmark cutoff in period 3, i.e., $\hat{\eta}_3^s = \Phi^{-1}(y_3^{s,s} - e_3^s)$, and the expected profit is $\pi_3^s = \left[\frac{q}{1-G(\hat{\eta}_2)} + (1-q)\right] \phi(y_3^{s,s} - e_3^s)$.

We then consider retention in period 2. Consider a worker with cutoff utility, $\hat{\eta}_2$, in period 2. If utility is not resampled, she will exit in period 3 since her utility is below the cutoff, i.e., $\eta_3 = \hat{\eta}_2 \leq \hat{\eta}_3^s = \Phi^{-1}(y_3^{s,s} - e_3^s)$, so her period-3 payoff is given by e_3^s; if utility is resampled, she will stay in period 3 whenever her resampled utility is above the cutoff, i.e., $\eta_3 \geq \hat{\eta}_3^s$, so her period-3 payoff is given by

$$w_3^s + \int_{\eta \geq \hat{\eta}_3^s} \eta dG(\eta) = w_3^s + \hat{\eta}_3^s + \int_{\eta \geq \hat{\eta}_3^s} \eta - \hat{\eta}_3^s dG(\eta) = e_3^s + \varphi(\hat{\eta}_3^s).$$

Thus, the expected payoff to stay is

$$\overbrace{\text{Period-2 payoff}}^{w_2+\eta_2}+\delta\left[q\cdot\overbrace{\begin{array}{c}\text{Period-3 payoff if utility}\\ \text{is not resampled}\end{array}}^{e_3^s}+(1-q)\cdot\overbrace{\begin{array}{c}\text{Period-3 payoff if utility}\\ \text{is resampled}\end{array}}^{e_3^s+\varphi(\hat{\eta}_3^s)}\right],$$

and the payoff to exit is e_2, so the cutoff is given by $\hat{\eta}_2 = e_2 - [w_2 + \delta e_3^s + \delta(1-q)\varphi(\hat{\eta}_3^s)]$ for period 2. The firm offers a retention wage w_2 to maximize the expected profit

$$\begin{aligned}\pi_2 &= \max_{w_2} \Pr\{\eta_2 \geq \hat{\eta}_2\}(y_2^s - w_2 + \delta\pi_3^s)\\ &= \max_{\hat{\eta}_2}[1-G(\hat{\eta}_2)]\left\{y_2^s - \underbrace{[e_2 - \delta e_3^s - \delta(1-q)\varphi(\hat{\eta}_3^s) - \hat{\eta}_2]}_{=w_2} + \delta(1-q)\phi(y_3^{s,s} - e_3^s)\right\} + \delta q\phi(y_3^{s,s} - e_3^s),\end{aligned}$$

so the cutoff and expected profit are, respectively,

$$\hat{\eta}_2 = \Phi^{-1}\left(y_2^s - e_2 + \delta e_3^s + \delta(1-q)[\varphi(\hat{\eta}_3^s) + \phi(y_3^{s,s} - e_3^s)]\right),$$
$$\pi_2 = \phi\left(y_2^s - e_2 + \delta e_3^s + \delta(1-q)[\varphi(\hat{\eta}_3^s) + \phi(y_3^{s,s} - e_3^s)]\right) + \delta q\phi(y_3^{s,s} - e_3^s).$$

Since $\hat{\eta}_2 \leq \Phi^{-1}(y_3^{s,s} - e_3^s)$, it must hold that $y_2^s - e_2 + \delta e_3^s + \delta(1-q)[\varphi(\hat{\eta}_3^s) + \phi(y_3^{s,s} - e_3^s)] \geq y_3^{s,s} - e_3^s$.

Case with $\hat{\eta}_2 > \Phi^{-1}(y_3^{s,s} - e_3^s)$.

The analysis is analogous to above case. In period 3, the firm offers a retention wage w_3^s to maximize the expected profit

$$\begin{aligned}\pi_3^s &= \max_{w_3^s}\left[q\cdot\overbrace{\begin{array}{c}\text{Prob. of "stay" in period 3}\\ \text{if utility is not resampled}\end{array}}^{\Pr\{\eta_3=\eta_2\geq e_3^s-w_3^s|\eta_2\geq\hat{\eta}_2\}=1}+(1-q)\cdot\overbrace{\begin{array}{c}\text{Prob. of "stay" in period 3}\\ \text{if utility is resampled}\end{array}}^{\Pr\{\eta_3\geq e_3^s-w_3^s\}=1-G(\hat{\eta}_3^s)}\right](y_3^{s,s}-w_3^s)\\ &= \max_{\hat{\eta}_3^s}[1-(1-q)G(\hat{\eta}_3^s)][y_3^{s,s} - (e_3^s - \hat{\eta}_3^s)].\end{aligned}$$

As the stayers of period 2 are positively selected (in terms of utility), the firm will implement a slightly higher cutoff than the benchmark cutoff in period 3, i.e., $\hat{\eta}_3^s = \Phi_q^{-1}(y_3^{s,s} - e_3^s) > \Phi^{-1}(y_3^{s,s} - e_3^s)$ and the expected profit is $\pi_3^s = \phi_q(y_3^{s,s} - e_3^s)$.

We then consider retention in period 2. Consider a worker with cutoff utility, $\hat{\eta}_2$, in period 2. If utility is not resampled, she will stay in period 3 since her utility is above the cutoff, i.e., $\eta_3 = \hat{\eta}_2 > \hat{\eta}_3^s = \Phi_q^{-1}(y_3^{s,s} - e_3^s)$, so her period-3 payoff is given by $w_3^s + \eta_2 = e_3^s - \hat{\eta}_3^s + \eta_2$; if utility is resampled, she will stay in period 3 whenever her resampled utility is above the cutoff, i.e., $\eta_3 \geq \hat{\eta}_3^s$, so her period-3 payoff is given by $e_3^s + \varphi(\hat{\eta}_3^s)$. Thus, the expected payoff to stay is

$$\overbrace{\text{Period-2 payoff}}^{w_2+\eta_2} + \delta \left[q \cdot \overbrace{\begin{array}{c}\text{Period-3 payoff if utility}\\ \text{is not resampled}\end{array}}^{e_3^s - \hat{\eta}_3^s + \eta_2} + (1-q) \cdot \overbrace{\begin{array}{c}\text{Period-3 payoff if utility}\\ \text{is resampled}\end{array}}^{e_3^s + \varphi(\hat{\eta}_3^s)} \right],$$

and the payoff to exit is e_2, so the cutoff is given by $(1+\delta q)\hat{\eta}_2 = e_2 - \{w_2 + \delta[e_3^s - q\hat{\eta}_3^s + (1-q)\varphi(\hat{\eta}_3^s)]\}$ for period 2. The firm offers a retention wage w_2 to maximize the expected profit

$$\begin{aligned}\pi_2 &= \max_{w_2} \Pr\{\eta_2 \geq \hat{\eta}_2\}(y_2^s - w_2 + \delta\pi_3^s)\\ &= \max_{\hat{\eta}_2} [1 - G(\hat{\eta}_2)] \left\{ y_2^s - \underbrace{[e_2 - \delta e_3^s + \delta q \hat{\eta}_3^s - \delta(1-q)\varphi(\hat{\eta}_3^s) - (1+\delta q)\hat{\eta}_2]}_{=w_2} + \delta\phi_q(y_3^{s,s} - e_3^s) \right\},\end{aligned}$$

so the cutoff and expected profit are, respectively,

$$\begin{aligned}\hat{\eta}_2 &= \Phi^{-1}\left((1+\delta q)^{-1}\left[y_2^s - e_2 + \delta e_3^s - \delta q\hat{\eta}_3^s + \delta(1-q)\varphi(\hat{\eta}_3^s) + \delta\phi_q(y_3^{s,s} - e_3^s)\right]\right),\\ \pi_2 &= (1+\delta q)\phi\left((1+\delta q)^{-1}\left[y_2^s - e_2 + \delta e_3^s - \delta q\hat{\eta}_3^s + \delta(1-q)\varphi(\hat{\eta}_3^s) + \delta\phi_q(y_3^{s,s} - e_3^s)\right]\right).\end{aligned}$$

Since $\hat{\eta}_2 > \Phi^{-1}(y_3^{s,s} - e_3^s)$, it must hold that $(1+\delta q)^{-1}[y_2^s - e_2 + \delta e_3^s - \delta q\hat{\eta}_3^s + \delta(1-q)\varphi(\hat{\eta}_3^s) + \delta\phi_q(y_3^{s,s} - e_3^s)] < y_3^{s,s} - e_3^s$ or $y_2^s - e_2 + \delta e_3^s + \delta(1-q)\varphi(\hat{\eta}_3^s) + \delta[\phi_q(y_3^{s,s} - e_3^s) - q(y_3^{s,s} - e_3^s + \hat{\eta}_3^s)] < y_3^{s,s} - e_3^s$, which reduces to $y_2^s - e_2 + \delta e_3^s + \delta(1-q)[\varphi(\hat{\eta}_3^s) + \phi(y_3^{s,s} - e_3^s)] < y_3^{s,s} - e_3^s$ since

$$\begin{aligned}\phi_q(y_3^{s,s} - e_3^s) - q(y_3^{s,s} - e_3^s + \hat{\eta}_3^s) &= [1 - (1-q)G(\hat{\eta}_3^s)](y_3^{s,s} - e_3^s + \hat{\eta}_3^s) - q(y_3^{s,s} - e_3^s + \hat{\eta}_3^s)\\ &= (1-q)[1 - G(\hat{\eta}_3^s)](y_3^{s,s} - e_3^s + \hat{\eta}_3^s) < (1-q)\phi(y_3^{s,s} - e_3^s).\end{aligned}$$

The result below summarizes the retention strategy for a period-1 hire.

Lemma A3. *For a period-1 hire, there are two regimes. That is, if*

$$\underbrace{y_2^s + \delta(1-q)\phi(y_3^{s,s} - e_3^s)}_{\textit{effective output in period 2}} - \underbrace{\{e_2 - \delta[e_3^s + (1-q)\varphi(\hat{\eta}_3^s)\}}_{\textit{effective payoff to exit in period 2}} \geq y_3^{s,s} - e_3^s, \tag{2}$$

cutoffs for period 2 and period 3 are, respectively,

$$\hat{\eta}_2 = \Phi^{-1}\left(\begin{array}{c} y_2^s + \delta(1-q)\phi(y_3^{s,s} - e_3^s) - e_2 \\ +\delta[e_3^s + (1-q)\varphi(\hat{\eta}_3^s)] \end{array}\right) \leq \Phi^{-1}(y_3^{s,s} - e_3^s) = \hat{\eta}_3^s; \text{ (non-intensive screening)}$$

otherwise, cutoffs for period 2 and period 3 are, respectively,

$$\hat{\eta}_2 = \Phi^{-1}\left((1+\delta q)^{-1}\left[\begin{array}{c} y_2^s + \delta\phi_q(y_3^{s,s} - e_3^s) + e_2 \\ +\delta[e_3^s + (1-q)\varphi(\hat{\eta}_3^s) - q\hat{\eta}_3^s] \end{array}\right]\right) > \Phi_q^{-1}(y_3^{s,s} - e_3^s) = \hat{\eta}_3^s.$$

(intensive screening)

To understand the above result, consider the regime with non-intensive screening. For a worker with cutoff utility, $\hat{\eta}_2$, in period 2, she will stay in period 3 if her resampled utility is above the cutoff, $\hat{\eta}_3^s$, and she will move otherwise. Thus, the effective output in period 2 is $y_2^s + \delta(1-q)\phi(y_3^{s,s} - e_3^s)$, where $(1-q)\phi(y_3^{s,s} - e_3^s)$ reflects the expected profit in period 3. Likewise, the effective payoff to exit in period 2 is $e_2 - \delta[e_3^s + (1-q)\varphi(\hat{\eta}_3^s)]$, where $e_3^s + (1-q)\varphi(\hat{\eta}_3^s)$ captures the expected payoff in period 3 that the worker will forgo if she moves in period 2. Then, the condition in (2) suggests that the equilibrium regime features non-intensive screening if and only if the effective "output less payoff to exit" of period 2 outweighs that of period 3.

A.2 The equilibrium regime: intensive screening vs. non-intensive screening

Finally, we embed the above analysis in a labor market, where multiple firms—all being identical—compete for worker services. As a consequence, the effective payoff to exit is given by the expected payoff to "working at a new firm". Specifically, the effective payoff to exit in period 3 is $e_3^s = v_3^s$ for a period-1 hire (i.e., a stayer) and $e_3^m = v_3^m$ for a period-2 hire (i.e., a mover), while the effective payoff to exit in period 2 is $e_2 = v_2 + \delta[v_3^m + \varphi(\hat{\eta}_3^m)]$ for a period-1 hire, where $v_3^m + \varphi(\hat{\eta}_3^m)$ captures the expected payoff in period 3 that the worker will obtain if she moves in period 2. As in Lemma A2 and Lemma A3, the result below summarizes the retention strategy in equilibrium.

Lemma A4. *For a period-2 hire, the cutoff is $\hat{\eta}_3^m = \Phi^{-1}(y_3^{m,s} - v_3^m)$ for period 3. For a period-1 hire, there are two regimes. That is, if*

$$y_2^s + \delta(1-q)\phi(y_3^{s,s} - v_3^s) - \{v_2 + \delta[v_3^m + \varphi(\hat{\eta}_3^m)] - \delta[v_3^s + (1-q)\varphi(\hat{\eta}_3^s)]\} \geq y_3^{s,s} - v_3^s, \quad (3)$$

cutoffs for period 2 and period 3 are, respectively,

$$\hat{\eta}_2 = \Phi^{-1}\left(\begin{array}{c} y_2^s + \delta(1-q)\phi(y_3^{s,s} - v_3^s) - \{v_2 + \delta[v_3^m + \varphi(\hat{\eta}_3^m)]\} \\ +\delta[v_3^s + \delta(1-q)\varphi(\hat{\eta}_3^s)] \end{array}\right) \leq \Phi^{-1}(y_3^{s,s} - v_3^s) = \hat{\eta}_3^s;$$

otherwise, cutoffs for period 2 and period 3 are, respectively,

$$\hat{\eta}_2 = \Phi^{-1}\left((1+\delta q)^{-1}\left[\begin{array}{c} y_2^s + \delta\phi_q(y_3^{s,s} - v_3^s) - \{v_2 + \delta[v_3^m + \varphi(\hat{\eta}_3^m)]\} \\ +\delta[v_3^s + (1-q)\varphi(\hat{\eta}_3^s) - q\hat{\eta}_3^s] \end{array}\right]\right) > \Phi_q^{-1}(y_3^{s,s} - v_3^s) = \hat{\eta}_3^s.$$

By inspection of both regimes, we find that cutoffs will be determined by "output less poaching wage" in equilibrium. That is, $\hat{\eta}_2$, $\hat{\eta}_3^s$, and $\hat{\eta}_3^m$ are just functions of $y_2^s - v_2$, $y_3^{s,s} - v_3^s$, and $y_3^{m,s} - v_3^m$.

B Technical Details for the (Full) Game

B.1 The parametric restriction

Define r as the value of "output less poaching wage", $\phi(r)$ will capture the rents to incumbent firms due to under-bidding. Assumption 1 then amounts to

$$n_1(\hat{\theta}^{-1}-1) < \delta[\phi(n_2\hat{\theta}^{-1}) - \phi(n_2)] < n_2(\hat{\theta}^{-1}-1),$$

where the first inequality holds whenever G has a large support (and δ is not prohibitively small) and the second inequality always holds (since the Envelope Theorem indicates that $d[r-\delta\phi(r)]/dr = 1-\delta[1-G(\Phi^{-1}(r))] > 0$).

B.2 Proof of Proposition 1

From the claims in the main text, we can just focus on the promotion policy in period 2 concerning an h type since no l type will ever be promoted and the promotion policy in period 3 is efficient.

We first establish the relationships of promotion policy, turnover rates, and market believes in period 2. Since no l type will ever be promoted (i.e., $f_2(l;k)=0$), the market belief is updated such that the expected ability of a new hire, with schooling k, who has been placed on job J_2 is

$$\lambda_2(k,J_2) = \begin{cases} \frac{p(h|k)f_2(h;k)M_2(h;k,M)}{p(h|k)f_2(h;k)M_2(h;k,M)+p(l|k)f_2(l;k)M_2(l;k,M)} = 1 & \text{if } J_2 = M, \\ \frac{p(h|k)[1-f_2(h;k)]M_2(h;k,L)}{p(h|k)[1-f_2(h;k)]M_2(h;k,L)+p(l|k)[1-f_2(l;k)]M_2(l;k,L)} & \\ = \frac{p(h|k)[1-f_2(h;k)]M_2(h;k,L)}{p(h|k)[1-f_2(h;k)]M_2(h;k,L)+p(l|k)M_2(l;k,L)} < 1 & \text{if } J_2 = L, \end{cases} \tag{4}$$

where $M_2(\theta;k,J_2)$ denotes the exit rate of this worker if his or her ability is θ.

This then suggests that a new hire, who has been placed as a manager (i.e., $J_2 = M$), is perceived as an h type since $\lambda_2(k, J_2 = M) = 1$, so the new firm will also place her as a manager (i.e., $\tilde{J}_2(k,M) = M$). Due to free entry, the poaching wage to a manager is given by

$$v_2(k,M) = \hat{\theta}^{-1} + \delta\phi(n_2\hat{\theta}^{-1}), \tag{5}$$

i.e., the sum of expected output in period 2 and expected rents in period 3. Likewise, a new hire, who has been placed as a labour (i.e., $J_2 = L$), is perceived as an h type with probability $\lambda_2(k,L) < 1$, so the new firm will place her as a manager (i.e., $\tilde{J}_2(k,L) = M$) if $\lambda_2(k,L) \geq \hat{\theta}$ and as a labour (i.e., $\tilde{J}_2(k,L) = L$) otherwise. Due to free entry, the poaching wage to a labour is given by

$$v_2(k,L) = \begin{cases} \lambda_2(k,L)\hat{\theta}^{-1} + \delta\{\phi(n_2) + \lambda_2(k,L)[\phi(n_2\hat{\theta}^{-1}) - \phi(n_2)]\} & \text{if } \lambda_2(k,L) \geq \hat{\theta}, \\ 1 + \delta\{\phi(n_2) + \lambda_2(k,L)[\phi(n_2\hat{\theta}^{-1}) - \phi(n_2)]\} & \text{otherwise,} \end{cases} \tag{6}$$

i.e., the sum of expected output in period 2 and expected rents in period 3.

We can then use (4), (5), and (6) to analyze the promotion policy. Consider an (h,k) type with non-zero probability of being promoted, i.e., $f_2(h;k) \in (0,1)$. Since the promotion policy is in mixed strategies, incumbent firms must be indifferent between placing an h type as a manager and as a labour. This then indicates that output gains must equal increases in poaching wages, i.e., $(1+n_1)[x(h;M) - x(h;L)] = v_2(k,M) - v_2(k,L)$, so the poaching wage to a labour is given by

$$v_2(k,L) = v_2(k,M) - (1+n_1)(\hat{\theta}^{-1} - 1). \tag{7}$$

Note from (5) and (7) that poaching wages, $v_2(k,L)$ and $v_2(k,M)$, are both invariant to schooling, so exit rates, $M_2(l;k,L)$, $M_2(h;k,L)$, and $M_2(h;k,M)$ must be invariant to schooling (as depicted in Figure 5). Moreover, the ratio of exit rates, $\frac{M_2(l;k,L)}{M_2(h;k,L)}$, must be a constant.

By inspection of (6), the poaching wage to a labour, $v_2(k,L)$, is continuous in the belief, $\lambda_2(k,L)$. Since $v_2(k,L)$ is invariant to schooling, $\lambda_2(k,L)$ must also be invariant to schooling. That is $\lambda_2(k',L) = \lambda_2(k'',L)$ for $\forall k'' > k'$ or

$$\frac{p(l|k'')}{p(h|k'')} \frac{M_2(l;k'',L)}{M_2(h;k'',L)} \frac{1}{1-f_2(h;k'')} = \frac{p(l|k')}{p(h|k')} \frac{M_2(l;k',L)}{M_2(h;k',L)} \frac{1}{1-f_2(h;k')},$$

which rewrites to

$$\frac{1-f_2(h;k'')}{1-f_2(h;k')} \underbrace{\frac{M_2(l;k',L)/M_2(h;k',L)}{M_2(l;k'',L)/M_2(h;k'',L)}}_{=1} = \underbrace{\frac{p(l|k'')/p(h|k'')}{p(l|k')/p(h|k')}}_{<1 \text{ (due to MLRP)}} \Rightarrow \frac{1-f_2(h;k'')}{1-f_2(h;k')} < 1.$$

Consequently, the promotion policy is biased toward more educated h types, i.e., $f_2(h;k'') > f_2(h;k') > 0$ for $\forall k'' > k'$. Furthermore, the MLRP (i.e., $d[p(h|k)/p(l|k)]/dk > 0$) and $p(h|k = 0) = 0$ suggest that there exists a threshold schooling level, $\hat{k}$, such that $f_2(h;k) > 0$ if $\forall k \geq \hat{k}$ and $f_2(h;k) = 0$ otherwise, i.e., the probability of being promoted is non-zero for an h type if and only if his or her schooling level is at least $\hat{k}$.

Finally, we verify that intensive screening is the retention policy in equilibrium. For the simplified game in Figure 3, the equilibrium regime features intensive screening whenever the condition in (3) holds, i.e.,

$$\begin{aligned} y_3^{s,s} - v_3^s > y_2^s &+ \delta(1-q)\phi(y_3^{s,s} - v_3^s) - \{v_2 + \delta[v_3^m + \varphi(\hat{\eta}_3^m)] - \delta[v_3^s + (1-q)\varphi(\hat{\eta}_3^s)]\} \\ &= y_2^s + \delta(1-q)\phi(y_3^{s,s} - v_3^s) - \{v_2 + \delta[v_3^m - v_3^s + \varphi(\hat{\eta}_3^m) - (1-q)\varphi(\hat{\eta}_3^s)]\}. \end{aligned}$$

Note that the efficient promotion policy in period 3 indicates that $v_3^m = v_3^s$, i.e., job movers will receive equal poaching wages as job stayers since outside firms can perfectly infer abilities through the efficient promotion policy in period 3. Moreover intensive screening (as described in

Lemma A4) suggests that the cutoff for job stayers is greater than that for job movers, $\hat{\eta}_3^s > \hat{\eta}_3^m$, so it holds that $\varphi(\hat{\eta}_3^s) < \varphi(\hat{\eta}_3^m) \Rightarrow (1-q)\varphi(\hat{\eta}_3^s) < \varphi(\hat{\eta}_3^m)$, i.e., the option value of "move" exceeds the option value of "stay". Thus, it suffices to verify that $y_3^{s,s} - v_3^s > y_2^s - v_2 + \delta(1-q)\phi(y_3^{s,s} - v_3^s)$ holds for every worker type in equilibrium.

Due to free entry, poaching wages are given by $v_2 = y_2^m + \delta\phi(y_3^{m,s} - v_3^m)$, $v_3^s = y_3^{s,m}$, and $v_3^m = y_3^{m,m}$, so the condition, $y_3^{s,s} - v_3^s > y_2^s - v_2 + \delta(1-q)\phi(y_3^{s,s} - v_3^s)$, is equivalent to

$$y_3^{s,s} - y_3^{s,m} > y_2^s - y_2^m - \delta\phi(y_3^{m,s} - y_3^{m,m}) + \delta(1-q)\phi(y_3^{s,s} - y_3^{s,m}).$$

Note that $y_3^{s,s} - y_3^{s,m} > y_2^s - y_2^m$ since outputs will increase over time whenever h types are promoted and the return to tenure increases with seniority, while $y_3^{m,s} - y_3^{m,m} = y_3^{s,s} - y_3^{s,m}$ suggests that $\phi(y_3^{m,s} - y_3^{m,m}) > (1-q)\phi(y_3^{s,s} - y_3^{s,m})$. Thus, the above condition for intensive screening will hold in equilibrium for every worker type. □