Investment and information acquisition*

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Abstract

We study the interaction between productive investment and persuasion activities in a principal-agent setting with strategic disclosure. In an attempt to persuade the principal, the agent diverts substantial resources from productive activities to information acquisition for persuasion, even though productive activities are more efficient and raise the chances of success in persuasion. We show that a higher cost of an investment project results in a lower productive investment. We further demonstrate how a commitment by the agent to disclose all acquired information, or a commitment by the principal to a decision rule, curtail the inefficiency and stimulate productive investment.

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1 Introduction

Information is a valuable good that fosters the allocation of limited resources towards their best use. But gathering information is not costless. Rather, it requires expending resources taken away from other uses. If the party that gathers information does not appropriate the returns to information perfectly, information can be under-supplied. At the same time, information is also a strategic instrument that agents use to persuade other parties to make choices desirable for the former. It is therefore natural to inquire whether persuasion motives cause the information to be produced efficiently, to be overproduced or to remain under-supplied.[1]

Particularly, in this paper we study a novel tradeoff in resource allocation between investment in productivity and acquiring information used for persuasion. To this end we construct a simple principal-agent model where an agent (he) wants to persuade a principal (she) to approve a project of a certain cost and uncertain return. The principal approves the project only if the expected return exceeds the cost. The agent is endowed with a fixed budget that he can allocate between *productive investment* that improves the project return stochastically, and *information acquisition* that generates verifiable signals about the project return. The principal observes neither the budget allocation, nor the realization of the signals. The agent chooses which signals to disclose to the principal, who then decides whether to approve or reject the project.

To keep the model tractable, we consider a binary signal structure. Each signal is either a "success" or a "failure" where a success signals a higher project return, and a failure signals the opposite.[2] In our setup, the social value from investing a unit of the budget in information acquisition and obtaining an additional signal is positive but below the value of allocating the same unit into productive investment. This feature of the model allows us to identify and highlight the effect of the persuasion motive in information acquisition without confounding it with other motives.

We first show that in the equilibrium of our disclosure game the agent invests substantial resources in acquiring information. This stands in contrast with the first-best outcome in which the entire budget is allocated to productive investment, provided that the project cost is not too high. Intuitively, the agent faces the following tradeoff in allocating the resources between productive investment and information acquisition. As productive investment stochastically improves the distribution of the project return, it also increases the probability that any given signal is a success. In contrast, the distribution of the project return is not affected by information acquisition. However, acquiring more signals increases the chances that the agent obtains

[1]This question is relevant given the importance of persuasion activities in modern economies. Donald McCloskey and Arjo Klamer (1995) show that a substantial fraction of the US GDP is spent on persuasion activities. The more recent study of Gerry Antioch et al. (2013) largely confirms these findings.

[2]There are many contexts where information comes in binary form. For example, the technology can either work or not, a certain task can be completed or left unfinished or a test can be failed or passed. Often, it is too costly or infeasible to observe or assess the "intermediate" values of partial success or failure.

a sufficient number of successful ones to persuade the principal to implement the project.[3] This motivates the agent to shift resources towards information acquisition.

Moreover, our key result shows that as the project cost increases, the agent allocates more resources towards information acquisition and away from productive investment. To understand why this is so, consider the following thought experiment. Start with an equilibrium allocation under a given principal's cost of implementing the project and suppose that this cost goes up. This leads the principal to revise her criterion for the project approval: she now needs either more successful signals or a higher productive investment or both. But the only instrument at her disposal is a threshold number of successful signals required for the project approval, and in deciding how to change it the principal has to consider the agent's equilibrium response.

If the principal increases her requirement on the minimal number of successes, the agent optimally shifts more resources to information acquisition away from productive investment. But because productive investment increases the chances that any given signal is a success, the rate at which the agent reduces his productive investment in response to a higher signal success requirement is less than 1-to-1. This turns out to be both a blessing and a curse. It is a blessing because productive investment does not fall too much in response to the principal's increase in the number of successes required for the project approval. But it is a also a curse: since productive investment responds sluggishly, the principal finds it optimal to raise her signal threshold requirement after a project cost increase. In equilibrium the project is approved under a higher cost only when the sum of equilibrium investment level and the number of successful signals is sufficiently high, but the equilibrium investment goes down as the project cost increases.

We then ask whether the inefficiency of the resource allocation can be alleviated if one or both parties have the ability to commit, at least partially, to their strategy choices. Particularly, we consider the agent's commitment to full disclosure of acquired information, and also the principal commitment to a project approval rule.

First, by committing to full signal disclosure the agent ties his own hands as selective disclosure becomes impossible. So by obtaining a small number of signals the agent can now convey to the principal that he has allocated the rest of his budget to productive investment. However, there is one caveat to this rule that implies that the principal does not simply deduct the number of observed signals from the agent's budget to compute the productive investment. If the principal did that, then the agent would shirk and save the cost associated with productive investment that is small but positive in our model.[4] So, in order to be certain that the agent had invested the remainder of his budget productively and to approve the project, the principal

[3]In the Appendix we show that if with some probability productive investment does not have any effect on the project return, the first-best allocation features an interior solution for the budget allocation.

[4]We assume that the agent bears a small fixed cost of productive investment. This assumption is plausible as productive investment typically requires not only a budget allocation but also additional activities such as management, monitoring, etc.

requires at least one successful signal. Faced with this requirement, the agent nevertheless cannot reallocate a significant budget to information acquisition, as such reallocation would be detectable and cause the principal to lower her expectation of the project return. In equilibrium, the agent acquires such number of signals that the remaining productive investment, together with one successful signal, are sufficient to push the project's expected return above the cost. Therefore, as the project cost increases the agent acquires less information and increases her productive investment. In fact, under a high project cost the budget allocation is close to the efficient one as the agent acquires only a small number of signals. This result stands in a sharp contrast to the equilibrium without commitment where productive investment decreases in the project cost.

The principal's commitment to a decision rule results in an intermediate level of productive investment: below its level under the agent's commitment, but above its level in the disclosure game. The principal finds it optimal to commit to a minimal threshold: depending on the parameters, she requires either one or two successful signals for the project approval. Such low threshold shifts the agent's incentives towards more productive investment as the latter increases the probability that any given signal is a success. However, in order to maximize his chances of delivering a successful signal, the agent still allocates substantial resources to information acquisition. Also, since the principal's threshold requirement does not change with the project cost, the agent's investment in productive activities does not change with that cost either.

Finally, if the agent can commit to full signal disclosure, the principal does not derive any additional benefit from her ability to commit to an approval threshold. In fact, the principal's additional commitment to a disclosure threshold would be counterproductive as it would undermine the effect of the agent's commitment. Under the latter, the agent makes a large productive investment because the alternative – a high level of information acquisition – becomes observable and causes the principal to increase the threshold number of successful signals required for project approval. This effect would disappear if the principal committed to the threshold number of signals required for approval.

Our model has several applications. First, consider a manager who supervises the initial stage of product development. She can allocate the resources given to her between improving the technology and early testing of the product. At this stage, the work on the technology improvement is more efficient. However, the shareholders of the firm do not observe this work. Therefore, they require the manager to show some positive test results. Although improving the technology makes positive test results more likely, our results imply that the manager overinvests in testing. This inefficiency can be curtailed if either the manager can commit to disclose all test results, or if the shareholders can commit to approve the project after a small number of positive test results.

Second, consider a manager who decides over the promotion of an employee. Prior to the promotion decision, the employee allocates her time between investing in her fundamental skills that raise her productivity and self-promotion activities that signal her productivity. The latter involves participating in conferences, making presentations or preparing industry publications. Investing resources in the fundamental skills may be more efficient for the employee's development. However, the management does not observe such investment and so its decision is affected by the employee's self-promotion successes. Although self-promotion is more successful if the employee invests more in her fundamental skills, the employee overinvests in self-promotion activities.

Third, consider a local government that decides on funding a non-profit organization involved in socially beneficial projects, such as training and education. The non-profit can allocate its resources between developing the relevant skills and supporting the job-search of its clients. Although the investment in skills improves the chances of employment, the non-profit skews its resource allocation towards supporting the job-search activities in order to increase the number of positive outcomes and use this evidence to support its request for continued funding.

The unifying feature of the above examples is the unobservability of productive investment. It results in a substantial inefficiency of the budget allocation skewed in favor of information acquisition, provided that no party can commit either to a disclose policy or to a decision rule.

Related Literature: There is a large literature on disclosure games starting with Sanford J Grossman (1981) and Paul R Milgrom (1981) where an agent holds verifiable information and reveals it strategically.[5] However, to the best of our knowledge this literature does not study the question of strategic disclosure in the context of an allocation conflict between productive and persuasion-related activities.

The interaction between disclosure and investment is studied in A Beyer and Ilan Guttman (2012). In their paper a manager privately observes the value of the firm and undertakes productive investment. The manager can then disclose the investment level to the market. They show that the manager undertakes a suboptimal investment that she then publicly discloses in an attempt to distort the market's beliefs about the firm's value. In contrast to Beyer and Guttman (2012), in our paper the agent faces a different tradeoff of allocating limited resources between investment and information acquisition used for persuasion.

Elchanan Ben-Porath, Eddie Dekel and Barton L Lipman (2017) and Peter M DeMarzo, Ilan Kremer and Andrzej Skrzypacz (2018) study a disclosure setting where an agent chooses a distribution over outcomes and then decides whether to disclose the outcome to an outside observer. Ben-Porath, Dekel and Lipman (2017) show how the agent's control of information

[5] Early papers in this literature focused on the "unraveling" result where all types of a sender are revealed to an observer in equilibrium. Our paper relates to the subsequent literature that studies situations without full unraveling. For earlier papers see, e.g., Ronald A Dye (1985); Michael J Fishman and Kathleen M Hagerty (1990); Woon-Oh Jung and Young K Kwon (1988); Masahiro Okuno-Fujiwara, Andrew Postlewaite and Kotaro Suzumura (1990); Robert E Verrecchia (1983). More recent contributions include Viral V Acharya, Peter DeMarzo and Ilan Kremer (2011); Jacob Glazer and Ariel Rubinstein (2004, 2006); Ilan Guttman, Ilan Kremer and Andrzej Skrzypacz (2014); Barton L Lipman and Duane J Seppi (1995).

leads to inefficient risk-taking. In an attempt to impress an outside observer, the agent has an incentive to choose a risky project even if a safe alternative has a higher expected value. They show that the agent only discloses sufficiently good outcomes, and otherwise pretends to be uninformed. DeMarzo, Kremer and Skrzypacz (2018) study a setting where a seller chooses a test for a product of unknown quality, and then decides whether to disclose the test result to a buyer. Similar to Ben-Porath, Dekel and Lipman (2017) they show that the seller has an incentive to run an inefficient test.

While in Ben-Porath, Dekel and Lipman (2017) and DeMarzo, Kremer and Skrzypacz (2018) the agent chooses a distribution over observable outcomes, in our setup the agent effectively chooses two interdependent distributions. His investment decision influences the unobserved distribution of the project returns, and both the investment decision and the information acquisition decision determine the distribution of the number of successful signals.

A related literature (Glazer and Rubinstein, 2004, 2006; Sergiu Hart, Ilan Kremer and Motty Perry, 2017) studies informational efficiency of disclosure strategies when the agent is endowed with verifiable information and decides which information to disclose to affect the principal's decision. This literature shows that outcomes attained via optimal mechanism design can also be supported as equilibria of the disclosure game in which no commitments are made. In particular, Glazer and Rubinstein (2006) demonstrate this in a setup where the principal's action is binary. Itai Sher (2011) establishes this result for general action sets of the principal, and shows that it holds as long as the principal's payoff is concave. In our setup where the principal's set of action is binary, and the agent has state-independent preferences and possesses verifiable information, the outcome under commitment is preferred by the principal to the outcome of the disclosure game. The major difference between our paper and this literature is that the agent's type/project return is endogenous and his budget allocation choice affects both this type, and the information about it.

The rest of the paper is organized as follows. Section 2 presents the model. Section 3 contains the analysis and our main results. Section 4 concludes, while all proofs are presented in the Appendix.

2 Setup

A principal (she) possesses a project with a known cost $c \in [0, 1]$ and an unknown return θ. The principal's payoff is $\theta - c$ if the project is implemented, and 0 otherwise. Before deciding whether to approve the implementation of the project or not, the principal can hire an agent (he) to develop it, which includes investing to improve the project's return and/or acquiring verifiable information about this return. The agent is endowed with a fixed budget of size $n \geq 2$ that he can allocate between such investment and information acquisition as specified below. The budget allocation is unobserved by the principal. The agent receives a payoff 1 if the project is ultimately implemented and 0 otherwise. Thus, the agent always wants the project to be implemented.

Budget allocation: Productive investment and information acquisition. The agent can allocate up to n discrete units of his budget to productive investment in the project and spend the rest on information acquisition about the project return. We assume the size of the budget to be exogenously fixed. What we have in mind is a situation in which the agent has a fixed capacity for performing his tasks, and our main focus falls on the allocation of such capacity between investment and information acquisition, when the latter is verifiable and can be used for persuasion purposes. Intuitively, the agent may have a limited amount of time and/or money available. Equivalently, the principal may have a fixed budget that she can endow the agent with. An alternative assumption would be that the agent bears a cost for each unit of budget that he spends. Then the optimal budget would be determined by the tradeoff between the cost and benefits of an extra budget unit, which would make the model more notationally complex but would not change anything as far as the main tradeoff in the budget allocation is concerned. Likewise, the decision whether to implement the project or not will not be affected by this cost as it would be sunk by the time the implementation decision is made.

Productive investment can represent money, time, effort, attention or any other productive resource that positively affects the project return. In addition, we assume that the agent incurs a small fixed cost $b > 0$ when he chooses a positive level of investment.[6]

Without any investment, the project return θ is determined by a draw from the uniform prior $U[0, 1]$. Each unit of investment results in an additional draw from $U[0, 1]$, and the realized project return θ is the maximum of these $k + 1$ draws. For instance, one could think of each draw as an experiment or development work that produces an alternative technology, so that the highest realization of $k + 1$ draws represents the best available technology. Thus, with k units of investment, the project return is distributed according to the cdf $F_k(\theta) = \theta^{k+1}$.[7] The players do not observe θ until the payoffs are realized.

If the agent spends k units of budget on investment, he can spend the rest on information acquisition to obtain $r \leq n - k$ privately observed but verifiable signals about the project return. Thus, one budget unit pays for one signal. Each signal s is binary and can either be a "success" ($s = 1$) or a "failure" ($s = 0$), with probability of success conditional on θ, $Pr(s = 1|\theta)$, equal to θ.

Thus, we can represent the agent's budget allocation strategy by a pair $(k, r) \in \{0, ..., n\}^2$ s.t. $k + r \leq n$. The obtained r signals constitute hard evidence set $S_r := \{s_1, .., s_r\}$ that is not

[6] This assumption reflects that investment is arguably a more complex activity than information acquisition, as the former requires more management and monitoring than the latter. Technically, it allows us to rule out uninteresting equilibria.

[7] Thus, the project return distribution with an additional unit of productive investment first-order stochastically dominates the project return distribution without an additional investment unit. In the Appendix we relax this assumption, explicitly allowing for the possibility that investment might not change the prior distribution of the project return.

observed by the principal.

Disclosure: As signals are verifiable, the agent can only disclose or not disclose each signal to the principal, but he cannot forge them. To formalize the disclosure process, let $j(S_r)$ be the number of successes in the evidence set S_r: $j(S_r) = \sum_{i=1}^{r} s_i$. The agent's disclosure to the principal can be represented by a message that consists of two numbers: the first is the number of disclosed signals h', and the second is the number of successes j' among the disclosed signals. We use the notation $m_{h'}^{j'}$ to denote such message. Given the evidence set S_r, the set of feasible messages is $M(S_r) = \{(h', j') | h' \leq r, h' - r + j(S_r) \leq j' \leq j(S_r)\}$. The lower bound on j' stems from the fact that the agent cannot disclose more failures than their actual number in the set S_r, that is $r - j(S_r)$.

Timing: The timing of the game is as follows. First, the agent chooses how to allocate the budget between investment and information acquisition. Second, the return θ is determined. Then the signals are realized and are privately observed by the agent. After this the agent makes the disclosure decision. Following this, the principal decides whether to approve the implementation of the project or not, and then the payoffs are realized. This timing is depicted in Figure 1.

Equilibrium: We use the standard notion of perfect Bayesian equilibrium and focus on equilibria in pure strategies. The agent's resource allocation and disclosure strategies must be sequentially rational given the principal's belief and her approval strategy. The principal's approval strategy must be sequentially rational given the agent's strategy and the principal's beliefs, that are represented by a mapping from the set of disclosures into the set of probability distributions over $[0, 1] \times \{0, ..., n\}^3$, the product of the set of possible project returns, the number of invested units k, the number of signals r and the number of successes j. The beliefs will be denoted by $\mu(m_{r'}^{j'})$ where $m_{r'}^{j'}$ stands for the agent's disclosure including r' signals and j' successes. The beliefs must be consistent with the agent's resource allocation and disclosure strategy i.e., derived from those by Bayes rule on the equilibrium path. On an off equilibrium path, these beliefs must satisfy the restriction that $\mu(m_{r'}^{j'})$ puts a positive probability only on 4-tuples (θ, k, r, j) such that $k + r \leq n$, $r \geq r'$ and $j \geq j'$.

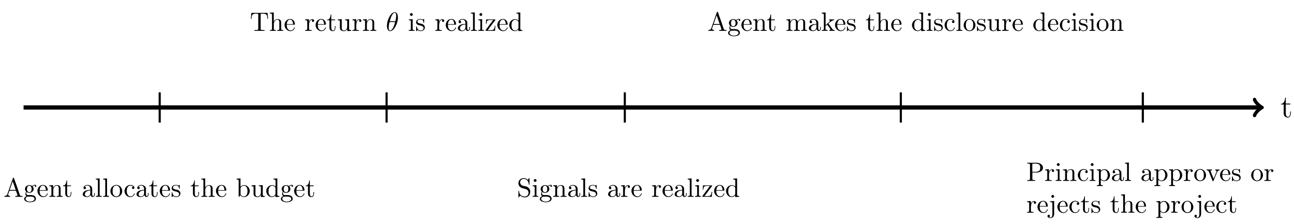


Figure 1: The Timing of the Game

3 Analysis and Main Results

3.1 Principal's beliefs and approval strategy

The principal approves the project if $E_P(\theta|\mu(m_{r'}^{j'})) \geq c$, and rejects it otherwise, where $E_P(\theta|\mu)$ is the principal's expectation of θ given beliefs μ.

In particular, if the principal puts probability 1 on the agent choosing k units of investment, and acquiring r signals of which j are successes, then her posterior beliefs θ are characterized by the probability distribution:

$$f(\theta|k,r,j) = \theta^{k+j}(1-\theta)^{r-j}\frac{(k+r+1)!}{(k+j)!(r-j)!}. \tag{1}$$

In turn, the principal's expectation of θ in this case is given by:

$$E_P(\theta|k,r,j) = \frac{k+j+1}{r+k+2}. \tag{2}$$

Both equations are derived in the Appendix.

We now introduce some intuitive restrictions on the equilibrium strategies and beliefs. First, it is without loss of generality to focus on the agent's investment and information acquisition strategies (k,r) such that $k+r=n$. In words, the agent always exhausts his budget. Recall that feasibility requires that $k+r \leq n$. However, acquiring $r < n-k$ signals is weakly dominated by acquiring $n-k$ signals because: (i) the principal does not observe the number of acquired signals and so this number does not affect her beliefs; (ii) any disclosure that is feasible with r signals is also feasible with a larger number of signals. So the agent can never be worse off by acquiring less than the maximal possible number of signals.

Thus, the equilibrium resource allocation strategy of the agent can be represented by a pair $(k^*, n-k^*)$.

Next, we will restrict the principal's beliefs to have the following properties.

Property 1. *(Uncontroverted k^* and Skepticism) Suppose that the agent discloses r' signals satisfying $r' \leq n-k^*$. Then the principal's beliefs put probability 1 on the event that the agent has invested k^* units, has acquired $n-k^*$ signals and has failed to disclose $n-k^*-r'$ signals* *all of which are failures.*

Property 2. *Suppose that the agent discloses r' signals and j' successes where r' satisfies $r' > n - k^*$. Then the principal's beliefs put probability 1 on the event that the agent has invested $n - r'$ or less units, and has disclosed all successes. In particular, the principal's beliefs about the number of invested units are characterized by probability distribution $\mu_w(r', j')$ that is invariant to j' and satisfies $\sum_{i=0}^{n-r'} \mu_w^i(r', j') = 1$ where $\mu_w^i(r', j') \geq 0$ is the probability that i units were invested.*

Property 1 implies that the agent cannot convince the principal that he has invested more than the equilibrium level k^* by disclosing r' signals s.t. $r' \leq n - k^*$. Disclosing such lower number of signals r' is still consistent with k^* units being invested. So, in such case, the principal maintains her beliefs that with probability 1 the agent has invested k^* units and adopts a skeptical point of view that the "missing," undisclosed, $n - k^* - r'$ signals are all failures.

On the other hand, if the agent discloses a "large" number of signals $r' > n - k^*$ that makes it impossible that k^* units were invested, the principal becomes convinced that a deviation from k^* units to a lower investment has occurred. Yet, she is still skeptical with regards to the number of successful signals and believes that all successes have been disclosed.

Thus, Properties 1 and 2 imply that the principal believes that the agent never conceals successful signals. Given these properties, we can now describe the principal's equilibrium posterior expectation of θ and her decision rule.

Lemma 1. *Suppose that the principal's beliefs satisfy Properties 1 and 2, and the agent discloses r' signals among which there are j' successes.*

1. *If $r' \leq n - k^*$, then the principal's posterior expectation of θ is given by $\frac{k^*+j'+1}{n+2}$, and so she approves the project if and only if $\frac{1+j'+k^*}{n+2} \geq c$.*
2. *If $r' > n - k^*$, then the principal's posterior expectation of θ is bounded above by $\frac{n-r'+j'+1}{n+2}$, and so she approves the project only if $\frac{n-r'+j'+1}{n+2} \geq c$.*

Lemma 1 implies that the principal's equilibrium threshold j^* i.e., the minimal number of successes that she requires to approve the project, satisfies:

$$j^* = \lceil c(n+2) \rceil - (k^* + 1), \tag{3}$$

where k^* is the equilibrium level of investment.

A useful insight from Lemma 1 is that a disclosed success is a perfect substitute for a unit of investment from the principal's perspective: each affects the principal's posterior beliefs in the same way.

3.2 Disclosure and budget allocation

In this subsection we first characterize the agent's optimal information disclosure strategy, given that the principal's beliefs satisfy Properties 1 and 2 and given her decision rule (3). Recall that the agent's equilibrium strategy is denoted by $(k^*, n-k^*)$ i.e., he invests k^* units and acquires $r^* = n - k^*$ signals.

Lemma 2. *Suppose that the agent obtains r' signals with j' successes.*

(i) If $r' \leq r^* = n - k^*$, then it is optimal for the agent to disclose all signals,

(ii) If $r' > r^*$, then it is optimal for the agent to disclose $r^* = n - k^*$ signals and $\min\{j', n - k^*\}$ successes.

After any optimal disclosure, the principal believes that the agent has invested k^* units with probability 1.

Next, let us consider the agent's budget allocation decision. To provide a benchmark case, we start with the first-best allocation attained when the principal can choose the budget allocation directly and is able to observe all signal realizations.

Lemma 3. *If the principal can choose a budget allocation, she would allocate the entire budget into productive investment, provided that $c \leq \frac{n+1}{n+2}$ and approve the project with probability 1. Otherwise, if $c > \frac{n+1}{n+2}$ the principal never approves any project and never undertakes any productive investment.*

To understand Lemma 3 note that each additional unit of investment increases the posterior expectation of θ. In contrast, a signal affects the distribution of the project return in the same way as one invested unit only if the signal realization is a success, which happens with a probability strictly less than 1. A failed signal shifts the posterior distribution of returns to the left. Hence, to maximize the posterior expectation of θ, the principal allocates the entire budget into productive investment.

This result comes with a qualifier that the project implementation cost c cannot be too large. In particular, c cannot exceed $\frac{n+1}{n+2}$. This is because with n invested units (or, alternatively, with $k' < n$ invested units and $n - k'$ successes), the posterior expectation of θ is equal to $\frac{n+1}{n+2}$, so the principal never approves the project if c exceeds this level.

While for $c \leq \frac{n+1}{n+2}$ the principal prefers to allocate all resources into productive investment, such allocation cannot arise in the disclosure game, as shown in the next lemma.

Lemma 4. *There is no equilibrium in which $k^* = n$.*

Were the principal to expect $k^* = n$ from the agent and therefore no signals, the agent would always deviate to $k = 0$ saving the fixed cost $b > 0$ of investment.

Turning to the agent's equilibrium budget allocation strategy, we first provide our central qualitative result for the disclosure game in the following Theorem:

Theorem 1. *Suppose that $n > 4$ and $c \in [\frac{n+4}{2(n+2)}, \frac{n}{n+2}]$. If the project cost increases, the equilibrium level of investment decreases. Thus, a higher project cost is associated with a lower productive investment when $c \geq \frac{1}{2}$ and n is large.*

Theorem 1 follows directly from Proposition 1 provided below. The Theorem highlights that the level of information acquisition increases at the expense of productive investment as the project cost c increases. This is inefficient since the first-best involves allocating the whole budget into investment (Lemma 3). Significantly, the extent of this inefficiency grows as the project cost increases.

In order to prove Theorem 1 and Proposition 1 below, we need to characterize the equilibrium allocation k^* for various values of the cost c. To do so, we first obtain the probability of having $j \in \{0, .., n-k\}$ successes, provided that the agent invests k units and obtains $n-k$ signals:

$$Pr(j|k, n) = \frac{(k+1)(j+k)!(n-k)!}{j!(n+1)!}, \tag{4}$$

The derivation of expression (4) is provided in the Appendix. It allows us to obtain the probability $Pr(j \geq j'|k, n)$ that the agent receives at least j' successes out of $n-k$ signals when he invests k units.

Lemma 5. *The probability $Pr(j \geq j'|k, n)$ is given by the following:*

$$Pr(j \geq j'|k, n) = 1 - \frac{(n-k)!(k+j')!}{(n+1)!(j'-1)!}. \tag{5}$$

The equilibrium requires that at the budget allocation stage the agent does not deviate from k^* to some $k' \in \{0, ..., n-j^*\}$ investment units where, according to (3), $j^* = \lceil c(n+2) \rceil - (k^*+1)$ is the minimal number of successes that the principal requires to approve the project based on her belief that the agent has chosen the level of investment k^*. Therefore, the agent's equilibrium strategy k^* must be a solution to the following maximization problem:

$$\max_{k' \in \{0,..,n-j^*\}} Pr(j \geq j^*|k', n) - \mathbf{1}_{k'>0} b \tag{6}$$

Naturally, if the fixed cost b of investment is large enough, the agent will never choose a positive level of investment in equilibrium. To focus on the interesting case where $k^* > 0$, we assume that b is sufficiently small:

Assumption 1. *The fixed cost of investment is sufficiently small and satisfies $b < \frac{n-2}{n(n+1)}$ for any $n \geq 3$.*

We can now characterize the equilibrium budget allocation.

Proposition 1. *Consider $c \geq \frac{1}{2}$. The equilibrium budget allocation k^* and the principal's evidence threshold j^* are given by the following:*

1. *If $c > \frac{n+1}{n+2}$, the agent does not undertake any investment, $k^* = 0$, and the principal never approves the project.*

   If $c \in (\frac{n}{n+2}, \frac{n+1}{n+2}]$, then $k^* = 0$ (the agent does not undertake any investment), and $j^* = n$.

2. *If c belongs to the interval $[\frac{1}{2}, \frac{n}{n+2}]$ and $n > 3$, then:*

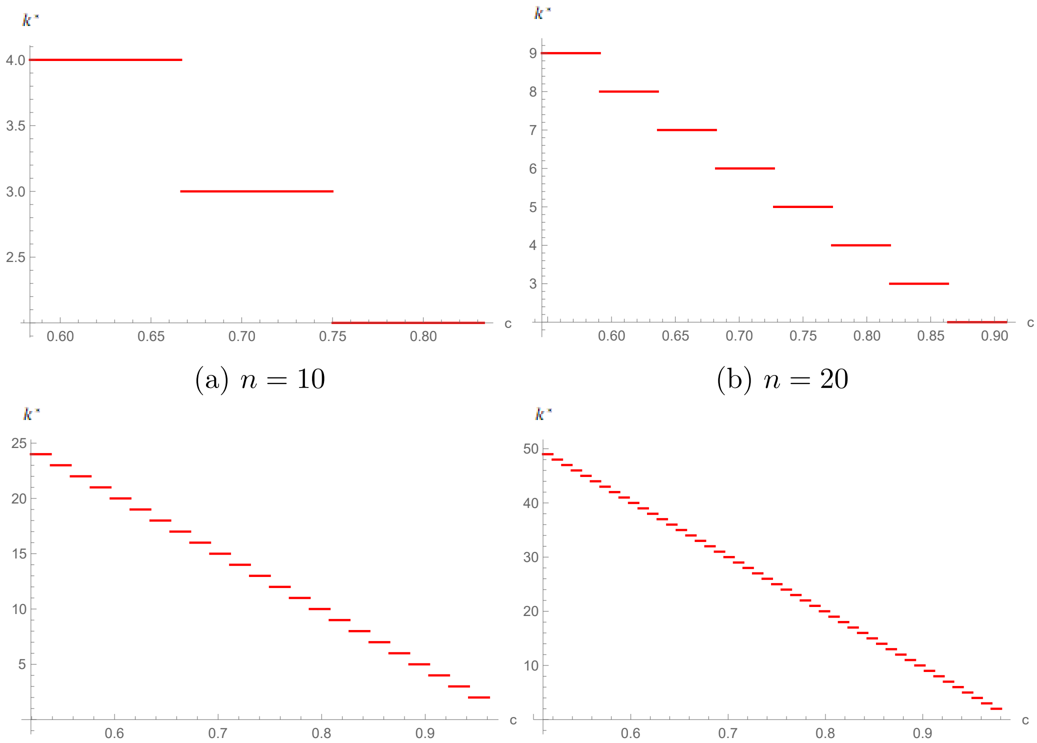
   (a) *If $c \in [\frac{n+4}{2(n+2)}, \frac{n}{n+2}]$, then $k^* = (n+2) - \lceil c(n+2) \rceil$ and $j^* = 2\lceil c(n+2) \rceil - (n+3)$,*

   (b) *If $c \in [\frac{n+3}{2(n+2)}, \frac{n+4}{2(n+2)})$, then $k^* = (n+1) - \lceil c(n+2) \rceil$ and $j^* = 2\lceil c(n+2) \rceil - (n+2)$,*

   (c) *If $c \in [\frac{1}{2}, \frac{n+3}{2(n+2)})$, then $k^* = n - \lceil c(n+2) \rceil$ and $j^* = 2\lceil c(n+2) \rceil - (n+1)$.*

3. *If $n = 2$, then $k^* = 0$ and $j^* = 2$ for $c \in [\frac{1}{2}, \frac{3}{4}]$.*

4. *If $n = 3$, then $k^* = 1$ and $j^* = 1$ for $c \in [\frac{1}{2}, \frac{3}{5}]$.*



(a) $n = 10$ (b) $n = 20$

(c) $n = 50$ (d) $n = 100$

Figure 2: k^* for $n = 10, 20, 50, 100$ in the interval $c \in [\frac{n+4}{2(n+2)}, \frac{n}{n+2}]$

The equilibrium allocation is illustrated in Figures 2 and 3. Figure 2 shows that for values of c close to $\frac{1}{2}$ the equilibrium investment is substantial. But as the cost c increases on the interval $[\frac{n+4}{2(n+2)}, \frac{n}{n+2}]$, the equilibrium investment k^* decreases.

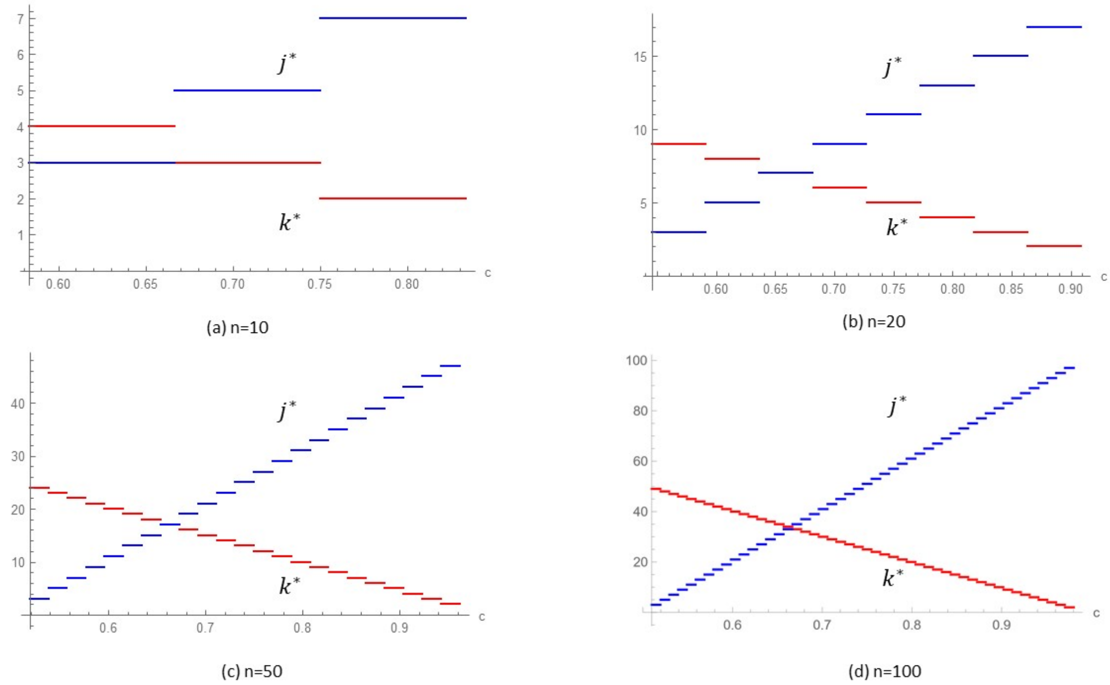


Figure 3: k^* decreasing in c, and j^* increasing in c, for $n = 10, 20, 50, 100$ and $c \in [\frac{n+4}{2(n+2)}, \frac{n}{n+2}]$

To understand why the investment decreases in the project cost, consider Figure 3. It shows that, as the project cost increases, j^* grows faster than k^* decreases. Intuitively, as c grows, the principal has to be more confident that the project return is sufficiently high. So she needs either more successful signals or a higher productive investment, or both. However, since the principal does not observe the investment, she can only change the threshold number of successful signals required for the project approval; and when she does so, she considers the agent's response. Particularly, when the successful signal threshold goes up, the agent shifts resources into information acquisition away from productive investment, but the rate at which the agent reduces his productive investment in response to a higher signal threshold is less than 1-to-1, because productive investment increases the chances that any given signal is a success. Since productive investment responds in this sluggish manner, the principal finds it optimal to raise her signal threshold requirement after a project cost increase. So, under a higher cost the project is approved only when the sum of equilibrium investment level and the number of successful signals is sufficiently high, but the equilibrium investment goes down as the project cost increases.

The above analysis focuses on the case $c \geq \frac{1}{2}$. To complete this section, we briefly highlight the outcomes for $c < \frac{1}{2}$.

First, if $c \leq \frac{1}{n+2}$, then the principal approves the project irrespective of her beliefs about the agent's actions and disclosure. This is so because in the worst case of zero investment and n failed signals, the principal's expectation of θ is $\frac{1}{n+2}$. Note that as n grows large, $\frac{1}{n+2}$ converges to 0, and so the project is approved automatically only when the cost c is very small.

For $c \in (\frac{1}{n+2}, \frac{1}{2}]$ there exists an "uninformative" equilibrium in which the principal approves the project with probability 1. This equilibrium is supported by the principal's beliefs that the agent does not undertake any investment and does not obtain any signals.[8]

At the same time, for $c < \frac{1}{2}$ an equilibrium supported by weakly skeptical beliefs fails to exist. To see why, note that the incentive constraint preventing a deviation to a higher investment under weakly skeptical beliefs is $k^* \geq n - \lceil c(n+2) \rceil$, while the equilibrium requirement $j^* \geq 1$ is equivalent to $k^* \leq \lceil c(n+2) \rceil - 2$. However, when $c < \frac{1}{2}$, we have $n - \lceil c(n+2) \rceil > \lceil c(n+2) \rceil - 2$, so both the feasibility constraint and the constraint preventing a deviation to a larger investment cannot hold simultaneously.

3.3 Commitment and its Effects

Given the inefficiency of the budget allocation in the disclosure game, we next consider whether such inefficiency can be mitigated when at least one of the parties has some commitment power. We start our analysis with the scenario in which the agent is able to commit to full signal disclosure. The outcome in this case is characterized in the following proposition.

Proposition 2. *Suppose that the agent commits to reveal all signal realizations. Then, for* $c \in [\frac{1}{2}, \frac{n+1}{n+2}]$, *in equilibrium the agent choose investment level* $k^* = \lceil c(n+2) \rceil - 2$ *and obtains* $n - k^*$ *signals, and on the equilibrium path the principal approves the project if the signals include at least one success i.e.,* $j^* = 1$.

The key aspect of the agent's commitment is that it allows him to convey the information about his productive investment to the principal. By revealing that he has acquired only a few signals the agent "indicates" that he spent the rest on productive investment. There is one caveat to the credibility of such message: the principal has to be sure that the agent did not shirk and invested nothing. For this reason, the principal requires that the agent delivers at least one successful signal. Otherwise, if no successes are required for the approval, the agent would not undertake any investment in order to save the fixed cost b. At the same time, the agent does not want to induce the principal to require strictly more than one success, which would make approval more difficult. So, in equilibrium the agent chooses a budget allocation with sufficiently few signals which, given the principal's belief that the rest of the budget is productively invested, keeps the principal's optimal threshold at $j^* = 1$.

Proposition 2 also establishes that the agent's investment increases in the project cost. This is because approval under a higher cost requires either a larger investment or a higher number of successful signals, or both. So, in order to keep the principal's approval threshold at exactly one success, the agent increases his productive investment and decreases information acquisition as the project cost goes up.

[8] Note that this equilibrium belongs to a different class of equilibria compared to the ones studied above, because it relies on a different kind of principal's beliefs that do not involve skepticism.

Next, suppose that only the principal has the ability to commit. In particular, assume that she can commit to a decision rule and choose an evidence threshold j^c for approval before the agent makes his budget allocation choice. The principal's decision rule must have a threshold nature, as otherwise the agent would simply withhold successful signals.

As we show in the proof of Proposition 3, the agent's best response to the principal's commitment to the threshold j^c is to invest the amount $k(j^c) = \lfloor \frac{n-j^c+1}{2} \rfloor$ and to allocate the rest of the budget to information acquisition. So the principal's optimal commitment decision rule j^c solves the following program:

$$\max_{j' \in \{1,..,n-1\}} \sum_{j=j'}^{n-k(j')} Pr(j|k(j'), n)\Big(\frac{k(j') + j + 1}{n+2} - c\Big), \tag{7}$$

The solution to (7) is characterized in the following proposition.

Proposition 3. *Suppose that the principal can commit to approval threshold, that $n \geq 6$ and $c \in [\frac{1}{2}, \frac{n-1}{n+2}]$.*

1. *If n is even, then the principal's equilibrium commitment threshold is $j^c = 1$. The agent's best response is to invest $k^* = \frac{n}{2}$ units.*

2. *If n is odd, then the principal's equilibrium commitment threshold is $j^c = 2$. The agent's best response is to invest $k^* = \frac{n-1}{2}$ units.*

To understand this proposition, note that the agent's best response investment is decreasing in the principal's approval threshold. So by committing to a low approval threshold the principal induces the agent to undertake substantial productive investment, even though it sometimes hurts the principal ex-post i.e., when the number of realized signal successes is low. Still, the principal does not want to commit to approve the project without any successes, as such policy would cause the agent to shirk and invest nothing. The reason why the principal's approval threshold varies between 1 (for even n) and 2 (for odd n) successful signals has to do with the discreteness of our model. In particular, when n is odd, both thresholds $j = 1$ and $j = 2$ induce the agent to undertake the same investment, and the principal chooses $j = 2$ to reduce the risk of approving an unprofitable project. However, when n is even, switching to threshold $j = 2$ from threshold $j = 1$ causes the investment to go down by one unit, so the principal chooses $j = 1$ because she prefers higher investment.

However, whether n is odd or even, the main message of Proposition 3 is the same: the principal commits to a low approval threshold in order to stimulate investment. At the same time, it shows that the power of the principal's commitment is limited: she cannot induce the agent to spend more than half of his budget on investment. So, there remains a considerable gap between the outcome attained under the principal's commitment and the first-best allocation that requires allocating all budget to investment.

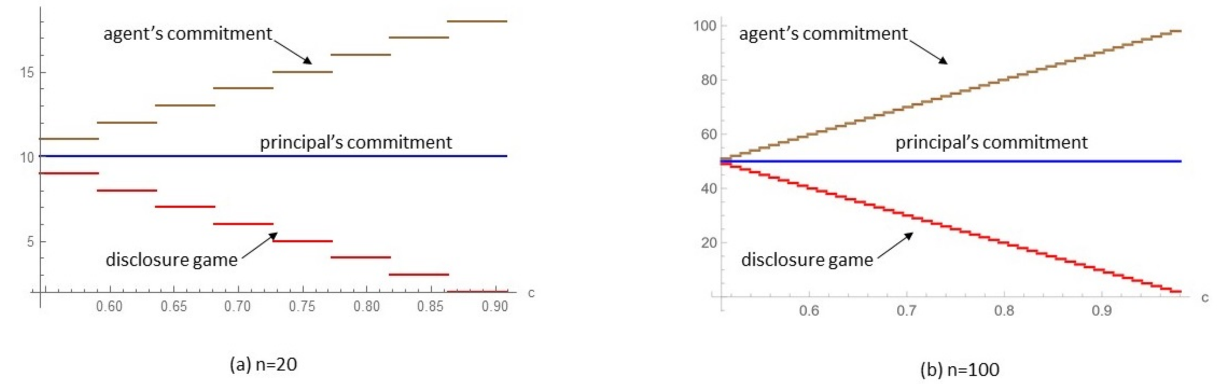


Figure 4: Productive investment k^* (vertical axis) for $n = 20$ and $n = 100$, $c \in [\frac{1}{2}, \frac{n-1}{n+2}]$.

Figure 4 illustrates the level of productive investment under the three scenarios that we have studied. Observe that the difference in the investment level under any commitment scenario and in the disclosure game gets larger as the project cost c increases. Eventually, under high project cost – but not prohibitively high to prevent any investment activities – the investment level under the agent's commitment gets close to the principal's first-best, whereas in the disclosure game the agent allocates almost the entire budget to information acquisition.

Figure 5 depicts the level of information acquisition under the three scenarios. It shows that amount of information acquisition is: (i) lowest and decreasing in cost under the agent's commitment; (ii) highest and increasing in cost in the equilibrium of the disclosure game. Under the principal's commitment to a decision rule the number of signals is intermediate and is independent of the project's cost.

Based on our previous results, we can now compare the principal's and the agent's payoffs under different commitment scenarios and in the disclosure game.

Proposition 4. *Assume that* $c \in (\frac{n+4}{2(n+2)}, \frac{n-1}{n+2}]$ *and* $n > 6$.

(i) The principal prefers agent's commitment to her own commitment, and her own commitment to the equilibrium of the disclosure game.

(ii) The agent prefers both principal's commitment and his own commitment to the equilibrium of the disclosure game.

To understand the principal's ranking of the three scenarios, note that the principal can at least replicate the outcome of the disclosure game by committing to the same decision rule that she uses in the equilibrium of the latter. So, the possibility to commit to a decision rule makes the principal better off than in the disclosure game. Also, a comparison of Propositions 2 and 3 shows that the agent's commitment to disclosure leads to a more efficient outcome – namely, a higher investment under the same approval threshold – than the principal's commitment to a decision rule. For this reason, the agent's commitment is more valuable for the principal than her own commitment to the decision rule.

On the other hand, the agent prefers a lower evidence threshold for the project approval.

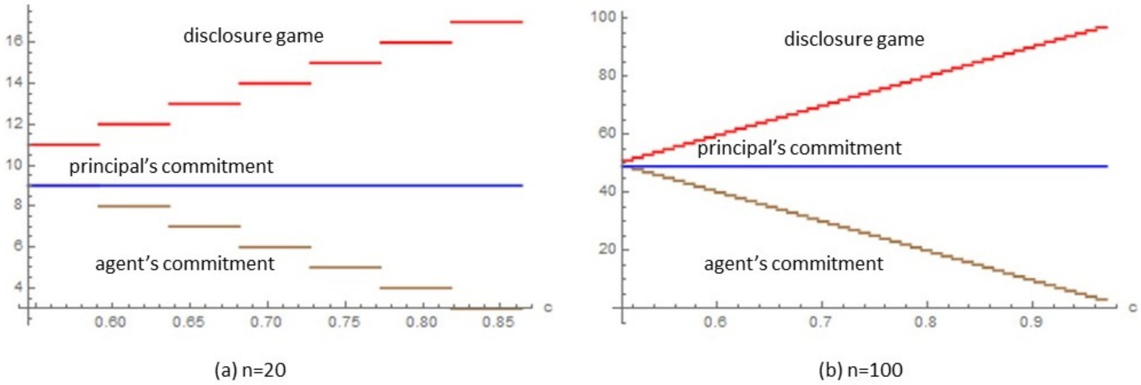


Figure 5: Number of signals acquired (vertical axis) for $n = 20$ and $n = 100$ for $c \in [\frac{n+4}{2(n+2)}, \frac{n-1}{n+2}]$.

That is why he is better off under both commitment scenarios than in the disclosure game. The only ranking not determined by Proposition 4 is the agent's ranking of the outcomes under his own and under the principal's commitment. This ranking remains ambiguous as it depends on the budget n and the corresponding evidence threshold j.

To conclude this section, we provide the following corollary of Propositions 2 and 3 showing that, when the agent can commit to reveal all signals, the principal does not get any additional benefit from her ability to commit to a decision rule.

Corollary 1. *Consider $c \in [\frac{1}{2}, \frac{n-1}{n+2}]$ and suppose that the agent is able to commit to full signal disclosure. Then the principal does not benefit from her additional ability to commit to a decision rule.*

The intuition for this corollary is simple. When the principal commits to an approval threshold, the agent's disclosure has no effect on the outcome. So the agent's budget allocation would be the same as under the principal's commitment only. Thus, the principal's additional commitment would only undermine strong investment incentives that the agent has under her own commitment to disclosure. Therefore, an additional commitment to an approval threshold does not benefit the principal, provided that the agent is committed to full signal disclosure.

4 Conclusions

In this paper, we have studied the interaction between productive investment and information acquisition used for persuasion. We have demonstrated that in the disclosure game, persuasion motives lead to a significant inefficiency and misallocation of resources away from productive investment and towards information acquisition. However, the agent's ability to commit to a full signal disclosure partially alleviates this inefficiency and leads to a significant increase in productive investment. In contrast, if only the principal can commit to a decision rule before the agent makes his choices, the allocation is less efficient than under the agent's commitment. Notably, under both commitment scenarios the resulting allocation remains below the first-best.

There are several promising avenues for future research. In particular, it would be interesting to study what would happen if the principal had access to additional instruments including transfers, allocation of decision rights or monitoring technologies. The analysis of such instruments would be relevant for understanding real-world organizations in which the management typically employs multiple tools to overcome informational problems as well as the conflicts of interest between various parties in an organization. It would also be interesting to consider how the tradeoff between productive investment and persuasion activities is resolved in a framework with soft information or when the agent could design the public information structure as in (Robert J Aumann and Michael Maschler (1995); Emir Kamenica and Matthew Gentzkow (2011)).

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5 Appendix

Derivation of expressions (1) and (2):

If the principal puts probability 1 on the agent investing k units, and acquiring r signals of which j are successes, then her beliefs about θ can be computed in two stages as follows. First, conditioning on k invested units, the density of the highest realization of θ is $f_k(\theta) = (k+1)\theta^k$. The probability of j successes in r independent trials given that the probability of success in each trial is θ, $Pr(j|r,\theta)$, is given by $\binom{r}{j}\theta^j(1-\theta)^{r-j}$. So, by Bayes rule, the posterior density of θ given a triple (k, r, j) is equal to:

$$f(\theta|k,r,j) = \frac{Pr(j|r,\theta)(k+1)\theta^k}{\int_0^1 Pr(j|r,\theta)(k+1)\theta^k d\theta} = \frac{\binom{r}{j}\theta^j(1-\theta)^{r-j}(k+1)\theta^k}{\int_0^1 \binom{r}{j}\theta^j(1-\theta)^{r-j}(k+1)\theta^k d\theta}$$

$$= \theta^{k+j}(1-\theta)^{r-j}\frac{(k+r+1)!}{(k+j)!(r-j)!}.$$

Note that the last equality in (1) relies on the identity $\int_0^1 \theta^{k+j}(1-\theta)^{r-j}d\theta = \frac{(k+j)!(r-j)!}{(r+k+1)!}$. Therefore,

$$E_P(\theta|k,r,j) = \int_0^1 \theta f(\theta|k,r,j)d\theta = \int_0^1 \theta^{k+j+1}(1-\theta)^{r-j}d\theta\frac{(r+k+1)!}{(k+j)!(r-j)!} = \frac{k+j+1}{r+k+2}.$$

Proof of Lemma 1 If $r' \leq n - k^*$, then the principal's posterior beliefs about θ are characterized by the probability density function:

$$f(\theta|k^*, n-k^*, j') = \frac{Pr(j'|n-k^*,\theta)(k^*+1)\theta^{k^*}}{\int_0^1 Pr(j'|n-k^*,\theta)(k^*+1)\theta^{k^*}d\theta}$$
$$= \frac{\binom{n-k^*}{j'}\theta^{j'}(1-\theta)^{n-k^*-j'}(k^*+1)\theta^{k^*}}{\int_0^1 \binom{n-k^*}{j'}\theta^{j'}(1-\theta)^{n-k^*-j'}(k^*+1)\theta^{k^*}d\theta}$$
$$= \theta^{k^*+j'}(1-\theta)^{n-k^*-j'}\frac{(n+1)!}{(k^*+j')!(n-k^*-j')!}. \tag{8}$$

Correspondingly, the principal's expectation of θ is given by:

$$E_P(\theta|k^*, n-k^*, j') = \int_0^1 \theta f(\theta|k^*, n-k^*, j')d\theta$$
$$= \int_0^1 \theta^{k^*+j'+1}(1-\theta)^{n-k^*-j'}d\theta\frac{(n+1)!}{(k^*+j')!(n-k^*-j')!} = \frac{k^*+j'+1}{n+2}. \tag{9}$$

If $r' > n - k^*$, then the principal's posterior beliefs about θ are characterized by the probability density function:

$$f(\theta|m_{r'}^{j'}) = \sum_{i=0}^{n-r'} \mu_w^i(r',j')\frac{Pr(j'|n-i,\theta)(i+1)\theta^i}{\int_0^1 Pr(j'|n-i,\theta)(i+1)\theta^i d\theta}$$
$$= \sum_{i=0}^{n-r'} \mu_w^i(r',j')\frac{\binom{n-i}{j'}\theta^{j'}(1-\theta)^{n-i-j'}(i+1)\theta^i}{\int_0^1 \binom{n-i}{j'}\theta^{j'}(1-\theta)^{n-i-j'}(i+1)\theta^i d\theta}$$
$$= \sum_{i=0}^{n-r'} \mu_w^i(r',j')\theta^{i+j'}(1-\theta)^{n-i-j'}\frac{(n+1)!}{(i+j')!(n-i-j')!}. \tag{10}$$

Correspondingly, the principal's expectation of θ satisfies:

$$E_P(\theta|m_{r'}^{j'}) = \sum_{i=0}^{n-r'} \mu_w^i(r',j') \int_0^1 \theta^{i+j'+1}(1-\theta)^{n-i-j'} d\theta \frac{(n+1)!}{(i+j')!(n-i-j')!}$$

$$= \sum_{i=0}^{n-r'} \mu_w^i(r',j') \frac{i+j'+1}{n+2} \leq \frac{n-r'+j'+1}{n+2}. \tag{11}$$

Q.E.D.

Proof of Lemma 2: First, note that the agent's optimal disclosure strategy maximizes the principal's posterior expectation of θ given by (9) if $r' \leq n-k^*$ or (11) if $r' > n-k^*$.

Let us start with case (i) in which the agent acquires (weakly) less than the equilibrium number of signals $r^* = n-k^*$. Then, no matter what he discloses, according to Property 1 the principal believes that with probability 1 the agent has invested k^* units.

So, if the agent discloses all signals, which includes j' successes, then according to (9) the principal's posterior expectation of θ is equal to $E_P(\theta|k^*, n-k^*, j') = \frac{k^*+j'+1}{n+2}$,

On the other hand, if the agent makes a disclosure $m_{r''}^{j''}$ that includes j'' successes and r'' signals where $j'' \leq j'$ and $r'' \leq r'$ then, again by (9), the principal's posterior expectation of θ is equal to $E_P(\theta|k^*, n-k^*, j'') = \frac{k^*+j''+1}{n+2}$. Since the latter is less than $E_P(\theta|k^*, n-k^*, j') = \frac{k^*+j'+1}{n+2}$ as $j'' \leq j'$, such deviation is not profitable.

Now consider case (ii) in which $r' > r^* = n-k^*$. If the agent chooses to disclose $r^* = n-k^*$ signals and $\min\{j, n-k^*\}$ successes, then according to Property 1 the principal believes that with probability 1 the agent has invested k^* units. So, by (9) the principal's posterior expectation of θ is equal to $E_P(\theta|k^*, n-k^*, j') = \frac{k^*+\min\{j',n-k^*\}+1}{n+2}$. Thus, to complete the proof we need to show that the principal's posterior expectation under any alternative disclosure strategy cannot exceed $\frac{k^*+\min\{j',n-k^*\}+1}{n+2}$.

First, if $j' \geq n-k^*$, then by (9) disclosing $r^* = n-k^*$ signals and $n-k^*$ successes induces the principal's posterior expectation equal to $\frac{n+1}{n+2}$. On the other hand, if the agent follows any alternative disclosure strategy $m_{r''}^{j''}$ s.t. $r'' > n-k^*$, then according to (11) the principal's posterior expectation of θ does not exceed $\frac{n-r''+j''+1}{n+2}$. The latter expression does not exceed $\frac{n+1}{n+2}$ since $j'' \leq r''$. So deviating from disclosure $m_{n-k^*}^{n-k^*}$ is not profitable.

Next, suppose that $r' > n-k^*$ and $j' < n-k^*$. Then by making a disclosure $m_{r''}^{j''}$ where $r'' \leq n-k^*$ and $j'' \leq j'$, the agent induces the principal's posterior expectation equal to $\frac{k^*+j''+1}{n+2}$ by (9). The latter does not exceed $\frac{k^*+j'+1}{n+2}$, the principal's posterior expectation of θ when the agent discloses $n-k^*$ signals and j' successes. Therefore, such disclosure $m_{r''}^{j''}$ where $r'' \leq n-k^*$ and $j'' \leq j'$, is suboptimal.

Finally, suppose that the agent's disclosure $m_{r''}^{j''}$ is such that $r'' > n-k^*$ and $j'' \leq j'$. Then by (11) the principal's posterior expectation does not exceed $\frac{n-r''+j''+1}{n+2}$ which, in turn, is less than $\frac{k^*+j'+1}{n+2}$. The latter is the principal's posterior after disclosure $m_{n-k^*}^{j'}$. So we conclude that the disclosure $m_{r''}^{j''}$ is suboptimal.

Finally, since the agent's optimal disclosure is always such that $r'' \leq k^*$, it follows by Property 1 that the principal believes that the agent has invested k^* units with probability 1. *Q.E.D.*

Proof of Lemma 3: First, for $c > \frac{n+1}{n+2}$ there is no budget allocation resulting in $E(\theta|\cdot) > \frac{n+1}{n+2}$. So, the principal optimally chooses $k^* = 0$. Moreover, there is an equilibrium where she does not conduct any trials as no signal realization ever leads to the project approval since in the best possible scenario – when all trials are successes – the expected value of θ is $\frac{n+1}{n+2}$.

Consider now $c \leq \frac{n+1}{n+2}$. Suppose that the principal invests n units. Then, the expected payoff is given by $\frac{n+1}{n+2} - c \geq 0$.

Now, suppose that the principal chooses $k < n$ invested units instead. Then, her expected payoff is

$$\sum_{j=0}^{n-k} Pr(j|k,n) \max\{0, [E(\theta|k,j,n) - c]\} = \sum_{j=0}^{n-k} Pr(j|k,n) \max\left\{0, \left[\frac{j+k+1}{n+2} - c\right]\right\}$$

$$< \sum_{j=0}^{n-k} Pr(j|k,n) \max\left\{0, \left[\frac{n+1}{n+2} - c\right]\right\} \leq \frac{n+1}{n+2} - c$$

Thus, the principal maximizes her payoff by investing $k = n$ units.
Q.E.D.

Derivation of equation (4):

$$Pr(j|k,n) = \int_0^1 Pr(j|k,n,\theta)Pr(\theta|k)d\theta$$

$$= \int_0^1 \binom{n-k}{j} \theta^j (1-\theta)^{n-k-j}(k+1)\theta^k d\theta = \frac{(k+1)(j+k)!(n-k)!}{j!(n+1)!}.$$

Proof of Lemma 4: Suppose the agent invests n units. In this case, there are no resources available for signal acquisition and therefore $m = \emptyset$. If $c \leq \frac{n+1}{n+2}$ and the principal believes that $k^* = n$, she approves the project upon no disclosure. Anticipating the principal's best response, the agent deviates and chooses $k^* = 0$ to save on cost b, a contradiction. If $c > \frac{n+1}{n+2}$, there is no budget allocation resulting in $\mathbb{E}_P(\theta|k,n) > \frac{n+1}{n+2}$ and therefore the principal never approves a project. The agent's best response is to deviate and to choose $k^* = 0$ to save cost b, a contradiction. Therefore, there is no equilibrium with $k^* = n$.
Q.E.D.

Proof of Lemma 5: Using (4) we obtain:

$$Pr(j \geq j'|k,n) = \sum_{j=j'}^{n-k} \frac{(k+1)(j+k)!(n-k)!}{j!(n+1)!}$$

$$= 1 - \sum_{j=0}^{j'-1} \frac{(k+1)(j+k)!(n-k)!}{j!(n+1)!} = 1 - \frac{(k+1)(n-k)!}{(n+1)!} \sum_{j=0}^{j'-1} \frac{(j+k)!}{j!}. \tag{12}$$

Now let us apply a known result on the sum of partial factorials saying that:

$$\sum_{j=0}^{j'-1} \frac{(j+k)!}{j!} = \frac{(k+j')!}{(k+1)(j'-1)!}. \tag{13}$$

Substituting (13) into (12) yields:

$$Pr(j \geq j'|k,n) = 1 - \frac{(k+1)(n-k)!}{(n+1)!} \frac{(k+j')!}{(k+1)(j'-1)!} = 1 - \frac{(k+j')!(n-k)!}{(n+1)!(j'-1)!}.$$

Q.E.D.

Proof of Proposition 1:

The equilibrium level of investment k^* must be a solution to the program (6). To find k^* we, first, ignore the fixed cost b and derive the conditions under which

$$k^* \in \arg \max_{k' \in \{0,..,n-j^*\}} Pr(j \geq j^*|k',n). \tag{14}$$

This ensures that the agent would not deviate from k^* to any other positive number of invested units. Then we show that provided that Assumption 1 holds, a deviation to zero invested units is also unprofitable.

If k' units are invested, then the probability of getting at least j^* successes and hence getting the project approved is equal to

$$1 - \frac{(k'+j^*)!(n-k')!}{(j^*-1)!(n+1)!}. \tag{15}$$

Thus, k^* is optimal for the agent if and only if

$$k^* \in \arg \min_{k' \in \{0,...,n-j^*\}} (k'+j^*)!(n-k')! \tag{16}$$

In order to solve the problem (16), let us first allow a continuous choice k' i.e., $k' \in [0, n-j^*]$. Then the objective of (16) can be rewritten as

$$D(k',j^*,n) \equiv \Gamma(k'+j^*+1)\Gamma(n-k'+1) \equiv \int_0^\infty x^{k'+j^*} e^{-x} dx \times \int_0^\infty x^{n-k'} e^{-x} dx$$

Note that

$$\frac{dD(k',j^*,n)}{dk'} = \left(\log(k'+j^*) - \log(n-k')\right) \Gamma(k'+j^*+1)\Gamma(n-k'+1) \tag{17}$$

It follows from (17) that $\frac{dD(k',j^*,n)}{dk'} < 0$ ($\frac{dD(k',j^*,n)}{dk'} > 0$) if $k' < \frac{n-j^*}{2}$ ($k' > \frac{n-j^*}{2}$) and $\frac{dD(k',j^*,n)}{dk'} = 0$ if $k' = \frac{n-j^*}{2}$. Hence, $D(k', j^*, n)$ attains a unique minimum at $k' = \frac{n-j^*}{2}$. Also note that, for fixed (j^*, n), $D(k', j^*, n)$ is symmetric around $k' = \frac{n-j^*}{2}$. Therefore, k^* minimizing (16) over a discrete set $\{0, ..., n - j^*\}$ has to satisfy:

$$\frac{n - j^* - 1}{2} \leq k^* \leq \frac{n - j^* + 1}{2}. \tag{18}$$

Next, recall that by Lemma 1, $j^* := \lceil c(n + 2)\rceil - (k^* + 1)$. Substituting the latter into (18) yields:

$$n - \lceil c(n + 2)\rceil \leq k^* \leq n + 2 - \lceil c(n + 2)\rceil. \tag{19}$$

Note that a pair $k^* > 0$ and $j^* = 0$ cannot be part of an equilibrium because in this case the agent would deviate to save the fixed cost $b > 0$ of investment. So, k^* must also satisfy $j^* = \lceil c(n + 2)\rceil - (k^* + 1) \geq 1$. This equilibrium requirement can be rewritten as follows:

$$k^* \leq \lceil c(n + 2)\rceil - 2. \tag{20}$$

In the rest of the proof, we identify the solution over different parts of the relevant cost range $[\frac{1}{2}, 1]$.

Case 0. $c > \frac{n}{n+2}$**.**

First, if $c > \frac{n+1}{n+2}$, then (19) implies that $k^* = 0$. The project never gets approved, because $\frac{1+j^*+k^*}{n+2} < c$ i.e., the posterior expectation of θ is below c.

Second, consider $c \in (\frac{n}{n+2}, \frac{n+1}{n+2})$. In this range, the only solution to inequality (19) is $k = 1$, and so $j^* = \lceil c(n+2)\rceil - (k^* + 1) = n - 1$. Using (23), the probability of obtaining the evidence if the agent adheres to the candidate equilibrium strategy $k = 1$ and when he deviates to $k = 0$ is the same and equal to $1 - \frac{j^*}{n+1}$. Thus, there is no fixed cost $b > 0$ that can support an equilibrium with $k^* = 1$. We conclude that for $c > \frac{n}{n+2}$, $k^* = 0$.

For the following Cases 1-3 assume $n > 3$.

Case 1. $c \in [\frac{n+4}{2(n+2)}, \frac{n}{n+2}]$**.**

In this case, $n + 2 - \lceil c(n + 2)\rceil \leq \lceil c(n + 2)\rceil - 2$. So, the equilibrium k^* is determined only by (19). There are multiple solutions to (19). Focusing on a Pareto efficient equilibrium implies that we need to choose the largest possible k. Hence, in this case we have

$$k^* = (n + 2) - \lceil c(n + 2)\rceil.$$

Case 2. $c \in [\frac{n+3}{2(n+2)}, \frac{n+4}{2(n+2)})$**.**

In this case, $n + 1 - \lceil c(n + 2)\rceil \leq \lceil c(n + 2)\rceil - 2 < n + 2 - \lceil c(n + 2)\rceil$. So the largest k^*

that satisfies both (19) and (20) is

$$k^* = n + 1 - \lceil c(n+2) \rceil.$$

Case 3. $c \in [\frac{1}{2}, \frac{n+3}{2(n+2)})$**.**

In this case, $n - \lceil c(n+2) \rceil \leq \lceil c(n+2) \rceil - 2 < n + 1 - \lceil c(n+2) \rceil$. So the largest k^* that satisfies both (19) and (20) is

$$k^* = n - \lceil c(n+2) \rceil.$$

In the following we show that in Cases 1-3 the agent adheres to investing k^* units provided that Assumption 1 holds i.e., $b \leq \frac{n-2}{n(n+1)}$. In fact, we will show that the agent prefers to invest 1 unit rather than 0 units, which then implies that investing k^* units is better than investing zero units.

Note that the probability of obtaining at least j^* successes and hence getting the project approved when $k = 0$, is $1 - \frac{j^*}{n+1}$. Also, the probability of obtaining at least j^* successes when $k = 1$, is $1 - \frac{(j^*+1)j^*}{(n+1)n}$. The difference between these probabilities, which we denote by $DP(j^*, n)$, is given by:

$$DP(j^*, n) = 1 - \frac{(j^*+1)j^*}{(n+1)n} - \left(1 - \frac{j^*}{n+1}\right) = \frac{j^*(n - j^* - 1)}{n(n+1)}. \tag{21}$$

As a function of j^*, $DP(j^*, n)$ reaches its minimum both at $j^* = 1$ and at $j^* = n - 2$. So for $n > 2$, we have

$$\min_{j^*} DP(j^*, n) = DP(1, n) = DP(n-2, n) = \frac{n-2}{n(n+1)} > 0.$$

Thus, if $b < \frac{n-2}{n(n+1)}$ and $n > 3$, then the agent would not deviate to zero invested units, because she prefers to invest 1 unit to no investment.

Finally, we consider $n = 2$ and $n = 3$. For $n = 2$, the only candidate equilibrium with a positive number of invested units requires $k^* = 1$ and $j^* = 1$. But then, according to (21), $DP(1, 2) = 0$ i.e., the agent gets the same probability of approval by choosing no investment. Thus, there is no equilibrium with $k = 1$, and so for $c \geq \frac{1}{2}$ and $n = 2$, $k^* = 0$.

Next, consider $n = 3$. In this case, we must have $k^* < 2$, for otherwise we will have $j^* = 0$, which cannot be a part of an equilibrium. Let us now show that an equilibrium with $n = 3$ and $k^* = 1$ exists. Given $k^* = 1$, the principal's decision rules implies that $j^* = 1$. Next, we may compute:

$$Pr(j \geq j^* | k = 1, n = 3) - Pr(j \geq j^* | k = 0, n = 3) = \frac{1}{12}(4 - \lceil 5c \rceil)(\lceil 5c \rceil - 2).$$

Note that $\frac{1}{12}(4-\lceil 5c\rceil)(\lceil 5c\rceil-2)$ is equal to $\frac{1}{12}$ for $c\in[\frac{1}{2},\frac{3}{5}]$ and 0 for $c>\frac{3}{5}$.

By Assumption 1, $b<\frac{n-2}{n(n+1)}$, which is equal to $\frac{1}{12}$ when $n=3$. So, the agent does not have an incentive to deviate to no investment. Hence we conclude that for $n=3$ in equilibirum we have $k^*=1$ if $c\in[\frac{1}{2},\frac{3}{5}]$ and $k^*=0$ if $c>\frac{3}{5}$.
Q.E.D.

Proof of Proposition 2: First, let us establish which strategies cannot be supported in an equilibrium:
Claim 1: There is no equilibrium with $j^*=0$ and $k^*>0$.

Suppose to the contrary that there exist an equilibrium in which the agent chooses $k^*>0$ and obtains r^* signals and the principal uses threshold $j^*=0$. Then the agent has a profitable deviation: choose zero investment and obtain r^* signals in order to save the cost $b>0$. This establishes Claim 1.

Note a direct consequence of Claim 1 is that there is no equilibrium with $k^*=n$. It also implies that in an equilibrium we must have $j^*\geq 1$, for otherwise the probability of approval would be zero.

Taking as given that the principal's approval strategy must satisfy $j^*\geq 1$, we can now prove the following claim:
Claim 2: A strategy under which the agent chooses $k'>0$ and obtains $r'>0$ signals s.t. $k'+r'<n$ is dominated by a strategy of investing $n-r'$ units and obtaining r' signals.

Consider an agent's strategy that does not exhaust the budget i.e., under which the agent invests $k'>0$ units and obtains r' signals s.t. $r'<n-k'$. Then the agent has a profitable deviation to $n-r'>k'$ invested units as this would increase the likelihood of a successful signal, $Pr(s_i=1|\theta,k,n)$, and hence raise the probability of the project approval. This establishes Claim 2.

Claim 2 implies that, if there exists an equilibrium with a positive number of invested units k', then in this equilibrium the agent obtains $n-k'$ signals.

Next, to complete the proof that in equilibrium the agent invests $k^*=\lceil c(n+2)\rceil-2$ units and commits to disclosing $n-k^*$ signals, while the principal uses approval threshold $j^*=1$, we rule out two types of deviations. We first rule out the case of no positive investment. Then we show that the agent would not deviate to any other positive number of invested units and a complementary number of signals.
Claim 3: Deviation to no investment is unprofitable for the agent.

Assume, first, that the agent obtains $n-k^*+t$ signals with $t\geq 1$. Consider the most optimistic principal's belief: upon observing the signals she believes that k^*-t units are invested. Denote her new threshold by j' with $j'\geq 2$. Then, the agent expects the principal to approve the project with probability $1-\frac{(j')!(n)!}{(j'-1)!(n+1)!}$. Recall that on path, the probability of persuading the principal is $1-\frac{(k^*+1)!(n-k^*)!}{(n+1)!}$ where we use $j^*=1$.

In the following we show that for any $k^*(c,n)$

$$1-\frac{(k^*+1)!(n-k^*)!}{(n+1)!}-\Big(1-\frac{(j')!(n)!}{(j'-1)!(n+1)!}\Big)$$
$$=\frac{j'}{n+1}-\frac{\Gamma(k^*+2)\Gamma(n-k^*+1)}{\Gamma(n+2)}\geq\frac{n-2}{n(n+1)}. \tag{22}$$

Consider the following derivative:

$$\frac{\partial\frac{\Gamma(k+2)\Gamma(n-k+1)}{\Gamma(n+2)}}{\partial k}=\frac{\Gamma(k+2)\Gamma(n-k+1)(\psi(k+2)-\psi(n-k+1))}{\Gamma(n+2)}.$$

This derivative is monotonically increasing on $k\in\{1,..,n-1\}$ because

$$\frac{\partial^2\frac{\Gamma(k+2)\Gamma(n-k+1)}{\Gamma(n+2)}}{\partial k^2}=\frac{\Gamma(k+2)\Gamma(n-k+1)\left((H_{k+1}-H_{n-k})^2+\psi(n-k+1)+\psi(k+2)\right)}{\Gamma(n+2)}>0.$$

Given that $\psi(k+2)-\psi(n-k+1)<0$ for $k=1$, and $\psi(k+2)-\psi(n-k+1)>0$ for $k=n-1$, it must be that the function $\frac{\Gamma(k+2)\Gamma(n-k+1)}{\Gamma(n+2)}$, first, decreases in k, and then increases in k. Therefore the maximum of the function is attained either at $k=1$ or at $k=n-1$. Because the function $\frac{\Gamma(k+2)\Gamma(n-k+1)}{\Gamma(n+2)}$ is $\frac{2}{n(n+1)}$ at $k=1$, and is $\frac{1}{n+1}$ at $k=n-1$, and since $\frac{2}{n(n+1)}<\frac{1}{n+1}$ for $n\geq 3$, the maximum of the function $\frac{\Gamma(k+2)\Gamma(n-k+1)}{\Gamma(n+2)}$ for $n\geq 3$ is at $k=n-1$.

Recall that the minimal approval threshold used by the principal is $j'=2$. If we use $j'=2$ on the LHS of (22), together with $k^*=n-1$, the inequality is satisfied because

$$\frac{2}{n+1}-\frac{1}{n+1}=\frac{1}{n+1}>\frac{n-2}{n(n+1)},$$

and is therefore satisfied for any $k^*(c,n)$. Therefore, we conclude that the agent has no incentive to deviate to no investment and to obtain $n-k^*+t>n-k^*$ signals.

Second, suppose that the agent obtains $n-k^*+t<n-k^*$ signals i.e. that $t<0$. Assume that upon observing the signals, the principal believes that the agent invests at most k^*-1 units. But then, as the principal's decision rule satisfies $j'\geq 2$, above we have shown that the inequality (22) is satisfied for any $k^*(c,n)$. Therefore the agent has no incentive to deviate to no investment and to obtain $n-k^*+t<n-k^*$ signals.

Claim 4: A deviation to $k'\neq k^*=\lceil c(n+2)\rceil-2$ is unprofitable for the agent.

Note that by Claim 2 we only need to consider deviations that involve k' invested units and $n-k'$ signals, after which the principal following her optimal decision rule approves the project only if the number of successes j is such that $j=\max\{\lceil c(n+2)\rceil-k'-1,1\}$.

First, suppose that $k'\geq\lceil c(n+2)\rceil-1$ and so $j=1$. Since $c>\frac{1}{2}$, it follows that $k'>\frac{n}{2}$.

Then with $j = 1$ the probability that the project is approved is equal to:

$$1 - \frac{(k'+1)!(n-k')!}{(n+1)!}. \tag{23}$$

Consider now the agent deviating and choosing $k'' = k' - 1$ invested units. In this case, $k'' \geq \lceil c(n+2) \rceil - 2$, so the principal's approval threshold remains $j = 1$, and so the probability of the project approval is equal to $1 - \frac{k'!(n-k'+1)!}{(n+1)!}$. This probability is greater than the value of (23) because $k'!(n-k'+1)! < (k'+1)!(n-k')!$ The latter inequality holds because it is equivalent to $n - k' + 1 < k' + 1$ which holds because $k' > \frac{n}{2}$.

Thus, it follows that the agent is strictly better off choosing $k' = \lceil c(n+2) \rceil - 2$ than any larger number of invested units.

Next, let us show that the agent would not deviate to $k'' < \lceil c(n+2) \rceil - 2$. We prove this by showing that, when $k \leq \lceil c(n+2) \rceil - 2$, the agent's payoff when she chooses k is greater than the payoff that she gets by choosing $k-1$. First, when $k \leq \lceil c(n+2) \rceil - 2$ the principal uses threshold $j(k)$ s.t. $j(k) + k = \lceil c(n+2) \rceil - 1$. This implies that the agent's payoff when she chooses k is greater than the payoff that she gets by choosing $k-1$ iff

$$\frac{(n-k)!}{(j(k)-1)!} < \frac{(n-k+1)!}{j(k)!}.$$

The latter inequality is equivalent to $n + 1 > j(k) + k$, which holds because $j(k) + k = \lceil c(n+2) \rceil - 1 < n + 1$. The latter inequality follows from $c < \frac{n+1}{n+2}$.
Q.E.D.

Proof of Proposition 3: Let j^c denote the threshold that the principal's commits to. The proof of Theorem 3 establishes the following. If $c \in [\frac{1}{2}, \frac{n-1}{n+2}]$ and the principal uses threshold j' at the project approval stage, then the agent's best response is to invest $k(j')$ units such that $\frac{n-j'-1}{2} \leq k(j') \leq \frac{n-j'+1}{2}$. So, given the principal's commitment to use threshold j^c, we can take the agent's best response to be $k(j^c) = \lfloor \frac{n-j^c+1}{2} \rfloor$ i.e., $k(j^c) = \frac{n-j^c+1}{2}$ if $n - j^c$ is odd, and $k(j^c) = \frac{n-j^c}{2}$ if $n - j^c$ is even.

Below, we provide the argument for the two cases where n is even (Case 1) and n is odd (Case 2).

Case 1A: Suppose that n is even and j^c is restricted to be odd, so that $k(j^c) = \frac{n-j^c+1}{2}$. Let us show that in this case the optimal commitment strategy for the principal is to choose $j^c = 1$.

First, note that if the agent discloses j successes and the principal believes – as she does in equilibrium – that the agent has invested $k(j^c) = \frac{n-j^c+1}{2}$ units and disclosed all successes, then the principal's expected value of θ is $\frac{k(j^c)+j+1}{n+2} = \frac{3+2j-j^c+n}{4+2n}$. Therefore, the principal's optimal commitment threshold j^{c*} solves the following optimization problem:

$$\max_{j^c\in\{1,..,n-1\}} \sum_{j=j^c}^{n-k(j^c)} Pr(j|k(j^c),n)\Big(\frac{k(j^c)+j+1}{n+2}-c\Big)$$
$$= \max_{j^c\in\{1,..,n-1\}} \underbrace{\sum_{j=j^c}^{n-k(j^c)} \frac{(k(j^c)+1)(j+k(j^c))!(n-k(j^c))}{(j)!(n+1)!}\Big(\frac{3+2j-j^c+n}{4+2n}-c\Big)}_{\equiv T(c,n,j^c)},$$

Let us consider the objective of the above problem:

$$T(c,n,j^c) = \sum_{j=j^c}^{n-k(j^c)} Pr(j|k(j^c),n)\Big(\frac{k(j^c)+j+1}{n+2}-c\Big)$$
$$= \sum_{j=j^c}^{n-k(j^c)} \frac{(k(j^c)+1)(j+k(j^c))!(n-k(j^c))}{(j)!(n+1)!}\Big(\frac{k(j^c)+j+1}{n+2}-c\Big)$$
$$= -c\left(1-\frac{(n-k(j^c))!(k(j^c)+j^c)!}{(n+1)!(j^c-1)!}\right)+\sum_{j=j^c}^{n-k(j^c)} \frac{(k(j^c)+1)(j+k(j^c)+1)!(n-k(j^c))}{(j)!(n+2)!}$$
$$= \frac{k(j^c)+1}{k(j^c)+2}-c+\frac{(n-k(j^c))!(k(j^c)+j^c)!}{(n+1)!(j^c-1)!}\left(c-\frac{(k(j^c)+1+j^c)(k(j^c)+1)}{(n+2)(k(j^c)+2)}\right) \tag{24}$$

Using $k(j^c)=\frac{n-j^c+1}{2}$, we can rearrange (24) as follows:

$$T(c,n,j^c) = -c+c\frac{(j^c+n+1)\Gamma\left(\frac{1}{2}(j^c+n+1)\right)^2}{2\Gamma(j^c)\Gamma(n+2)}+$$
$$\frac{n-j^c+3}{n-j^c+5}-\frac{((n+3)^2-(j^c)^2)\Gamma(\frac{1}{2}(j^c+n+3))^2}{(n-j^c+5)(j^c+n+1)\Gamma(j^c)\Gamma(n+3)} \tag{25}$$

Next we will establish the following claims:

- Claim 1: $T(c,n,j^c)$ is decreasing in c and $|\frac{\partial T(c,n,j^c)}{\partial c}|$ is decreasing in j^c.
- Claim 2: Suppose that $c=\frac{n-1}{n+2}$, and $j^c\in\{1,2,3,..,n-1\}$. Then $T(c,n,j^c)$ reaches a maximum in j^c at $j^c=1$.

In combination Claims 1 and 2 imply that for n even and $j^c\in\{1,3,..,n-1\}$, $j^c=1$ maximizes $T(c,n,j^c)$ for all $c\in[\frac{n+4}{2(n+2)},\frac{n-1}{n+2}]$.

Proof of Claim 1: Differentiating (25) we get:

$$\frac{\partial T(c,n,j^c)}{\partial c} = \frac{(j^c+n+1)\Gamma\left(\frac{1}{2}(j^c+n+1)\right)^2}{2\Gamma(j^c)\Gamma(n+2)}-1<0 \tag{26}$$

Further, let j^c be a real number in $[1, n-1]$. Then differentiating (26) we get:

$$\frac{\partial^2 T(c,n,j^c)}{\partial c\, \partial j^c} = \frac{\Gamma\left(\frac{1}{2}(j^c+n+1)\right)^2 \left((j^c+n+1)[\psi\left(\frac{1}{2}(j^c+n+1)\right) - \psi(j^c)] + 1\right)}{2\Gamma(j^c)\Gamma(n+2)} > 0, \quad (27)$$

where $\psi(.) = \frac{\Gamma'(.)}{\Gamma(.)}$ is the digamma function. The sign of the inequality in (27) follows from the fact that $\psi'(x) > 0$ for $x > 0$.

Combining (26) and (27) establishes that $|\frac{\partial T(c,n,j^c)}{\partial c}|$ is decreasing in j^c, which completes the proof of Claim 1

Proof of Claim 2.

To establish this Claim, we show that $T(c = \frac{n-1}{n+2}, n, j^c = 1) - T(c = \frac{n-1}{n+2}, n, j^c = f) > 0$ where $f \in \{3, 5, n-1\}$. First, note that rearranging (25) yields:

$$\begin{aligned} &T(c,n,j^c) = \\ &-c + \frac{n - j^c + 3}{n - j^c + 5} - \frac{j^c\left(2c(n+2)(j^c - n - 5) - (j^c)^2 + (n+3)^2\right)\Gamma\left(\frac{1}{2}(j^c+n+3)\right)^2}{(n+5-j^c)(j^c+n+1)\Gamma(j^c+1)\Gamma(n+3)}. \end{aligned} \quad (28)$$

Note that, when $c = \frac{n-1}{n+2}$, then

$$j^c\left(2c(n+2)(j^c-n-5) - (j^c)^2 + (n+3)^2\right) = -j^c\left((j^c)^2 - 2j^c(n-1) + n(n+2) - 19\right).$$

Using this equality in the numerator of (28) we obtain:

$$\begin{aligned} T(c = \frac{n-1}{n+2}, n, j^c = f) &= -\frac{2}{n+5-f} - \frac{3}{n+2} - \\ &-\frac{f\left(f^2 - 2f(n-1) + n(n+2) - 19\right)\Gamma\left(\frac{1}{2}(f+n+1)\right)^2 (f+n+1)}{4(f-n-5)\Gamma(f+1)\Gamma(n+3)} \end{aligned}$$

In particular,

$$T(c = \frac{n-1}{n+2}, n, j^c = 1) = \frac{n+8}{n^2+6n+8} + \frac{(n-4)\Gamma\left(\frac{n}{2}+2\right)^2}{(n+2)\Gamma(n+3)}.$$

Combining the last two equalities yields:

$$\begin{aligned} &T(c = \frac{n-1}{n+2}, n, j^c = 1) - T(c = \frac{n-1}{n+2}, n, j^c = f) \\ &= \frac{\frac{\left(-2fn + f(f+2) + n^2 + 2n - 19\right)\Gamma\left(\frac{1}{2}(f+n+1)\right)^2 (f+n+1)}{4\Gamma(f)\Gamma(n+3)} + \frac{2-2f}{n+4}}{f-n-5} + \frac{(n-4)\Gamma\left(\frac{n}{2}+2\right)^2}{(n+2)\Gamma(n+3)}. \end{aligned} \quad (29)$$

Since $f - n - 5 < 0$, (29) is positive if and only if

$$(n+2)\left(\frac{(-2fn + f(f+2) + n^2 + 2n - 19)\,\Gamma\left(\frac{1}{2}(f+n+1)\right)^2 (f+n+1)}{4\Gamma(f)} - \frac{2(f-1)\Gamma(n+3)}{n+4}\right)$$
$$- (n-4)(n+5-f)\Gamma\left(\frac{n}{2}+2\right)^2 < 0. \tag{30}$$

The inequality (30) holds because both terms in it are negative. It is easy to see that its second term is negative because $f < n$. Additionally, in Appendix B we show that the first term in (30) is negative.

Case 1B: Now suppose that n is even and j^c is even, $j^c \in \{2, 4, ..., n\}$. Then $k(j^c) = \frac{n-j^c}{2}$. Thus, if the principal believes that the agent has invested $k(j^c)$ units and that the sum of the $n-k(j^c)$ signals is j, the principal's expected value of θ is $\frac{k(j^c)+j+1}{n+2} = \frac{2j-j^c+n+2}{2n+4}$. The principal's optimal threshold j^c solves the following optimization problem:

$$\arg\max_{j^c\in\{2,..,n-2\}} \sum_{j=j^c}^{n-k(j^c)} Pr(j|k(j^c),n)\Big(\frac{k(j^c)+j+1}{n+2} - c\Big)$$

$$= \arg\max_{j^c\in\{2,..,n-2\}} \underbrace{\sum_{j=j^c}^{n-k(j^c)} \frac{(k(j^c)+1)(j+k(j^c))!(n-k(j^c))}{(j)!(n+1)!}\Big(\frac{2j-j^c+n+2}{2n+4} - c\Big)}_{\equiv \hat{T}(c,n,j^c)},$$

where

$$\hat{T}(c,n,j^c) = 1 - c - \frac{2}{4-j^c+n} +$$
$$\frac{(2c(n+2)(j^c-n-4) - (j^c)^2 + (n+2)^2)\,\Gamma\left(\frac{1}{2}(j^c+n+2)\right)\Gamma\left(\frac{1}{2}(j^c+n+4)\right)}{(j^c-n-4)(j^c+n+2)\Gamma(j^c)\Gamma(n+3)}.$$

Following the same approach as in Case 1 A we establish the following Claims.

Claim 3: $\hat{T}(c,n,j^c)$ is decreasing in c and $|\frac{\partial \hat{T}(c,n,j^c)}{\partial c}|$ is decreasing in j^c.

Proof of Claim 3. First, we have:

$$\frac{\partial \hat{T}(c,n,j^c)}{\partial c} = \frac{\Gamma\left(\frac{1}{2}(j^c+n+2)\right)^2}{\Gamma(j^c)\Gamma(n+2)} - 1 < 0.$$

Second, consider a continuous variable $\hat{j}^c \in \mathbb{R}_+$. We have:

$$\frac{\partial \hat{T}(c,n,\hat{j}^c)}{\partial c\,\partial \hat{j}^c} = \frac{\Gamma\left(\frac{1}{2}(\hat{j}^c+n+2)\right)^2\left(\psi\left(\frac{1}{2}(\hat{j}^c+n+2)\right) - \psi(\hat{j}^c)\right)}{\Gamma(\hat{j}^c)\Gamma(n+2)} > 0.$$

So $\frac{\partial \hat{T}(c,n,j^c)}{\partial c}$ increases in j^c and since it is negative, we conclude that $|\frac{\partial \hat{T}(c,n,j^c)}{\partial c}|$ decreases in j^c

. This completes the proof of Claim 3.

Claim 4: Suppose that $c = \frac{n-1}{n+2}$, and $j^c \in \{2, 4, .., n\}$. Then $\hat{T}(c, n, j^c)$ reaches a maximum in j^c at $j^c = 2$.

Proof of Claim 4. Next, we show that at $c = \frac{n-1}{n+2}$, the maximum of $\hat{T}(c = \frac{n-1}{n+2}, n, j^c)$ over $\{2, 4, .., n-2\}$ is at $j^c = 2$. To see this, assume $f \geq 4$ and consider the difference

$$\hat{T}(c = \frac{n-1}{n+2}, n, j^c = 2) - \hat{T}(c = \frac{n-1}{n+2}, n, j^c = f)$$

$$= \frac{\frac{\left(f^2-2f(n-1)+n(n+2)-12\right)\Gamma\left(\frac{1}{2}(f+n+2)\right)^2}{(f-n-4)\Gamma(f)} + \frac{((n-2)n-4)\Gamma\left(\frac{n}{2}+2\right)^2}{n+2}}{2\Gamma(n+3)} + \frac{2(f-2)}{(n+2)(-f+n+4)}.$$

The inequality $\hat{T}(c = \frac{n-1}{n+2}, n, j^c = 2) - \hat{T}(c = \frac{n-1}{n+2}, n, j^c = f) > 0$ holds if and only if

$$((n-2)n-4)(-f+n+4)\Gamma\left(\frac{n}{2}+2\right)^2 + 4(f-2)\Gamma(n+3) -$$

$$\frac{(n+2)\left(-2fn+f(f+2)+n^2+2(n-6)\right)\Gamma\left(\frac{1}{2}(f+n+2)\right)^2}{\Gamma(f)} > 0$$

which is true since both

$$4(f-2)\Gamma(n+3) - \frac{(n+2)\left(-2fn+f(f+2)+n^2+2(n-6)\right)\Gamma\left(\frac{1}{2}(f+n+2)\right)^2}{\Gamma(f)} > 0$$

and

$$((n-2)n-4)(-f+n+4)\Gamma\left(\frac{n}{2}+2\right)^2 > 0.$$

So, we have established that $\hat{T}(c = \frac{n-1}{n+2}, n, j^c = 2) - \hat{T}(c = \frac{n-1}{n+2}, n, j^c = f') > 0$ for $f' = 4, 6, ..n$, completing the proof of Claim 4.

Given Claims 3 and 4, in order to complete Case 1B we only need to prove the following claim.

Claim 5: If n is even, then the principal gets a higher payoff under commitment threshold $j^c = 1$ than under commitment threshold $j^c = 2$

Proof of Claim 5. We have:

$$\hat{T}(c = \frac{n-1}{n+2}, n, j^c = 1) - \hat{T}(c = \frac{n-1}{n+2}, n, j^c = 2) =$$

$$\frac{\frac{n+8}{n+4} + \frac{(n-4)\Gamma\left(\frac{n}{2}+2\right)^2}{\Gamma(n+3)}}{n+2} - \frac{\frac{((n-2)n-4)\Gamma\left(\frac{n}{2}+2\right)^2}{\Gamma(n+3)} + 2}{2(n+2)} = \frac{\frac{8}{n+4} - \frac{(n-2)^2\Gamma\left(\frac{n}{2}+2\right)^2}{\Gamma(n+3)}}{2(n+2)} > 0.$$

This inequality establishes Claim 5.

We now consider **Case 2** where n is odd.

Case 2A: Suppose that n is odd and j^c is odd, so that $k(j^c) = \frac{n-j^c}{2}$. Let us show that $j^c = 1$ is the principal's optimal choice.

Given the results derived in Case 1B, namely that $\frac{\partial \hat{T}(c,n,j^c)}{\partial c} < 0$ and $\frac{\partial \hat{T}(c,n,j^c)}{\partial c \partial j^c} > 0$ where $\hat{T}(c, n, j^c)$ is the principal's expected payoff when $k(j^c) = \frac{n-j^c}{2}$, we only need to establish that at $c = \frac{n-1}{n+2}$, $\hat{T}(c, n, j^c = 1) - \hat{T}(c, n, j^c = f) > 0$ for $f = 3, 5, .., n - 1$.

Indeed, we have:

$$\begin{aligned} &\hat{T}(c, n, j^c = 1) - \hat{T}(c, n, j^c = f) \\ &= \frac{1}{2}\left(\frac{\frac{\left(-2fn+f(f+2)+n^2+2(n-6)\right)\Gamma\left(\frac{1}{2}(f+n+2)\right)^2}{\Gamma(f)\Gamma(n+3)} + \frac{4-4f}{n+3}}{f-n-4} + \frac{(n-3)\Gamma\left(\frac{n+3}{2}\right)^2}{\Gamma(n+3)} \right) \end{aligned} \tag{31}$$

Since $f - (n + 4) < 0$, the expression in (31) is positive if and only if

$$\Gamma(n+3)\left(\frac{\left(-2fn + f(f+2) + n^2 + 2(n-6)\right)\Gamma\left(\frac{1}{2}(f+n+2)\right)^2}{\Gamma(f)\Gamma(n+3)} + \frac{4-4f}{n+3} \right) -$$

$$(n-3)(n+4-f)\Gamma\left(\frac{n+3}{2}\right)^2 < 0$$

which is true since both

$$\frac{\left(-2fn + f(f+2) + n^2 + 2(n-6)\right)\Gamma\left(\frac{1}{2}(f+n+2)\right)^2}{\Gamma(f)\Gamma(n+3)} + \frac{4-4f}{n+3} < 0$$

and

$$-(n-3)(n+4-f)\Gamma\left(\frac{n+3}{2}\right)^2 < 0.$$

Thus, $\hat{T}(c, n, j^c = 1) - \hat{T}(c, n, j^c = f) > 0$ for any $f \geq 3$. This completes the proof for Case 2A.

Case 2B: Suppose that n is odd and j^c is restricted to be even, so that $k(j^c) = \frac{n-j^c+1}{2}$. Let us show that $j^c = 2$ is the principal's optimal choice in this case.

First, let us show that $T(c, n, j^c = 2) - T(c, n, j^c = f) > 0$ for $f \in \{4, 6, .., n\}$. Given the results in Case 1A, namely that $\frac{\partial T(c,n,j^c)}{\partial c} < 0$ and $\frac{\partial T(c,n,j^c)}{\partial c \partial j^c} > 0$ where $T(c, n, j^c)$ is the principal's expected payoff when the threshold is $k(j^c) = \frac{n-j^c+1}{2}$, to prove this claim we only need to establish that $T(c, n, j^c = 2) - T(c, n, j^c = f) > 0$ for $f \in \{4, 6, .., n\}$ at $c = \frac{n-1}{n+2}$. First, assuming $f \geq 4$ we obtain:

$$T(c = \frac{n-1}{n+2}, n, j^c = 2) - T(c = \frac{n-1}{n+2}, n, j^c = f)$$

$$= \frac{\frac{\left(-2fn+f(f+2)+n^2+2n-19\right)\Gamma\left(\frac{1}{2}(f+n+3)\right)^2}{(f+n+1)\Gamma(f)\Gamma(n+3)} + \frac{4-2f}{n+3}}{f-n-5} + \frac{((n-2)n-11)\Gamma\left(\frac{n+5}{2}\right)^2}{(n+3)\Gamma(n+4)}. \tag{32}$$

Because $f-(n+5)<0$, the expression in (32) is positive if and only if:

$$((n-2)n-11)(f-n-5)\Gamma\left(\frac{n+5}{2}\right)^2 -$$

$$\frac{(n+3)^2\left(-2fn+f(f+2)+n^2+2n-19\right)\Gamma\left(\frac{1}{2}(f+n+3)\right)^2}{(f+n+1)\Gamma(f)} - 2(f-2)\Gamma(n+4) < 0.$$

Note that the last inequality holds because

$$((n-2)n-11)(f-n-5)\Gamma\left(\frac{n+5}{2}\right)^2 < 0$$

and

$$-\frac{(n+3)^2\left(-2fn+f(f+2)+n^2+2n-19\right)\Gamma\left(\frac{1}{2}(f+n+3)\right)^2}{(f+n+1)\Gamma(f)} - 2(f-2)\Gamma(n+4) < 0.$$

To finalize the proof for Case 2 (n is odd), we need to establish the following claim:

Claim 6: Provided that n is odd, $T(c,n,j^c=2)-\hat{T}(c,n,j^c=1)>0$.
Proof of Claim 6: We have:

$$\begin{aligned}
&T(c,n,j^c=2)-\hat{T}(c,n,j^c=1)\\
&= \frac{\left(n^3-15n-22\right)\Gamma\left(\frac{n+5}{2}\right)^2+(n+5)\Gamma(n+4)}{(n+2)(n+3)\Gamma(n+4)} - \left(\frac{n+5}{n^2+5n+6} + \frac{(n-3)\Gamma\left(\frac{n+3}{2}\right)^2}{2\Gamma(n+3)}\right)\\
&= \frac{(n-5)(n+1)\Gamma(n+3)\Gamma\left(\frac{n+5}{2}\right)^2}{\Gamma(n+4)^2}.
\end{aligned} \tag{33}$$

Since $n-5>0$, the value of (33) is positive which establishes Claim 6. This completed the proof of Proposition 3.
Q.E.D.

Proof of Proposition 4: Recall from Proposition 3 that the level of investment under principal's commitment is $k_p \in \{\frac{n}{2}, \frac{n-1}{2}\}$, depending on whether n is even or odd. On the other hand, the level of investment under the agent's commitment is $k_a := \lceil c(n+2)\rceil - 2$. Consider the following expected payoffs:

1. First, consider the case under the principal's commitment with n as an even integer. In

this case, $j^c = 1$ and $k(j^c) = \frac{n}{2}$. Then, we can directly calculate the following objects

$$Pr(j|k(j^c), n) = \frac{\Gamma\left(\frac{n}{2}+2\right)\Gamma\left(j+\frac{n}{2}+1\right)}{\Gamma(j+1)\Gamma(n+2)}$$

and $E(\theta|k(j^c), n) = \frac{1}{2} + \frac{j}{n+2}$. Therefore the principal's expected payoff is

$$\sum_{j=1}^{n-k(j^c)} \left(\frac{\Gamma\left(\frac{n}{2}+2\right)\Gamma\left(j+\frac{n}{2}+1\right)}{\Gamma(j+1)\Gamma(n+2)} - c\right) = \frac{(2c-1)\Gamma\left(\frac{n}{2}+2\right)^2}{\Gamma(n+3)} - c + \frac{n+2}{n+4}. \tag{34}$$

2. Second, consider the case under the principal's commitment with n as an odd integer. In this case, $j^c = 2$. Thus, we know that for $j^c = 1$ the principal's payoff would be lower than for $j^c = 2$. Consider $j^c = 1$: then, the principal's expected payoff is the same as calculated above,

$$\frac{(2c-1)\Gamma\left(\frac{n}{2}+2\right)^2}{\Gamma(n+3)} - c + \frac{n+2}{n+4}.$$

3. Third, consider the principal's expected payoff under the agent's commitment. Here, $j^c = 1$ and $k_a(j^c = 1) = \lceil c(n+2) \rceil - 2$. Consider a change of variables, and take $\hat{k} = c(n+2) - 2$. Then, $E(\theta|\hat{k}, n) = c + \frac{j-1}{n+2}$, and

$$Pr(j|\hat{k}, n) = \frac{(c(n+2)-1)\Gamma(n-c(n+2)+3)\Gamma(j+c(n+2)-1)}{\Gamma(j+1)\Gamma(n+2)}.$$

Therefore, the principal's expected payoff under the agent's commitment is

$$\sum_{j=1}^{n-\hat{k}} \frac{(c(n+2)-1)\Gamma(n-c(n+2)+3)\Gamma(j+c(n+2)-1)}{\Gamma(j+1)\Gamma(n+2)} \frac{j-1}{n+2}$$
$$= -\frac{1}{c(n+2)} + \frac{\Gamma(c(n+2))\Gamma(n-c(n+2)+3)}{\Gamma(n+3)} - c + 1. \tag{35}$$

Now, we want to show that the difference (35) − (34) > 0. The difference (35) − (34) is

$$-\frac{1}{c(n+2)} - \frac{(2c-1)\Gamma\left(\frac{n}{2}+2\right)^2}{\Gamma(n+3)} + \frac{\Gamma(c(n+2))\Gamma(n-c(n+2)+3)}{\Gamma(n+3)} - \frac{n+2}{n+4} + 1. \tag{36}$$

Consider the following derivatives:

$$\frac{\partial \; -\frac{1}{c(n+2)} - \frac{(2c-1)\Gamma\left(\frac{n}{2}+2\right)^2}{\Gamma(n+3)} - \frac{n+2}{n+4} + 1}{\partial c} = \frac{1}{c^2(n+2)} - \frac{2\Gamma\left(\frac{n}{2}+2\right)^2}{\Gamma(n+3)} \tag{37}$$

that itself decreases in c, and the derivative

$$\frac{\partial \frac{\Gamma(c(n+2))\Gamma(n-c(n+2)+3)}{\Gamma(n+3)}}{\partial c}$$
$$= \frac{(n+2)\Gamma(c(n+2))\Gamma(n-c(n+2)+3)(\psi(c(n+2))-\psi(n-c(n+2)+3))}{\Gamma(n+3)} > 0 \quad (38)$$

since $\psi(c(n+2)) > \psi(n-c(n+2)+3)$. We want to show that (37) > (38). To see this, note that the derivative of (38) with respect to c is positive:

$$\frac{\partial \frac{(n+2)\Gamma(c(n+2))\Gamma(n-c(n+2)+3)(\psi(c(n+2))-\psi(n-c(n+2)+3))}{\Gamma(n+3)}}{\partial c} =$$

$$\begin{aligned}&(n+2)^2\Gamma(c(n+2))\Gamma(n-c(n+2)+3)\times\\ &\frac{\left((\psi^{(0)}(c(n+2))-\psi^{(0)}(n-c(n+2)+3))^2+\psi^{(1)}(c(n+2))+\psi^{(1)}(n-c(n+2)+3)\right)}{\Gamma(n+3)} > 0.\end{aligned}$$

Then, using $c' := \frac{n-1}{n+2}$, the difference (37) − (38) may be computed as

$$\frac{n+2}{(n-1)^2} - \frac{2\Gamma\left(\frac{n}{2}+2\right)^2}{\Gamma(n+3)} - \frac{6(n+2)\Gamma(n-1)\left(\psi^{(0)}(n-1)+\gamma-\frac{11}{6}\right)}{\Gamma(n+3)} \quad (39)$$

and (39) > 0 can be expressed as

$$-2(n-1)^2\Gamma\left(\frac{n}{2}+2\right)^2+(n+2)\Gamma(n+3)-6(n+2)(n-1)^2\Gamma(n-1)\left(\psi^{(0)}(n-1)+\gamma^{EM}-\frac{11}{6}\right) > 0.$$

where γ^{EM} is the Euler-Masceroni constant.

In the next step, consider $\hat{c} = \frac{n+4}{2(n+2)}$. Then

$$-\frac{1}{\hat{c}(n+2)} - \frac{(2\hat{c}-1)\Gamma\left(\frac{n}{2}+2\right)^2}{\Gamma(n+3)} - \frac{n+2}{n+4} + 1 = \frac{\Gamma(\hat{c}(n+2))\Gamma(n-\hat{c}(n+2)+3)}{\Gamma(n+3)}$$

and therefore we conclude that (36) > 0. Thus, we conclude that the principal's expected payoff under the agent's commitment is (weakly) higher than the principal's payoff under the principal's commitment.

Note further that the principal's expected payoff under her commitment is better than under the equilibrium of the disclosure game, because the principal can always replicate the outcome of the disclosure game under commitment.

Next, consider the agent. We first show that the agent prefers his commitment to full disclosure to his equilibrium payoff in the disclosure game. We maintain the assumption that the agent wants to invest at least one unit, and therefore we need to compare the probabilities of the project approval in the two environments. First, under the commitment to full signal disclosure the probability of the project approval that is, provided that $j^* = 1$ and $k^* = \lceil c(n+2) \rceil - 2$:

$$1 - \frac{\Gamma(n - \lceil c(n+2) \rceil + 3)\Gamma(\lceil c(n+2) \rceil)}{\Gamma(n+2)}.$$

In the equilibrium of the disclosure game, provided that $c \in \left(\frac{n+4}{2(n+2),\frac{n}{n+2}}\right]$, the probability of the project approval is:

$$1 - \frac{\Gamma(\lceil c(n+2) \rceil - 1)\Gamma(\lceil c(n+2) \rceil)}{\Gamma(n+2)\Gamma(-n + 2\lceil c(n+2) \rceil - 3)}.$$

Then, the agent (strictly) prefers the commitment to full signal disclosure if

$$\frac{\Gamma(\lceil c(n+2) \rceil - 1)}{\Gamma(n - \lceil c(n+2) \rceil + 3)\Gamma(-n + 2\lceil c(n+2) \rceil - 3)} > 1. \tag{40}$$

Due to the properties of the ceiling-function, we have:

$$\frac{\Gamma(\lceil c(n+2) \rceil - 1)}{\Gamma(n - \lceil c(n+2) \rceil + 3)\Gamma(-n + 2\lceil c(n+2) \rceil - 3)} \geq \frac{\Gamma(c(n+2) - 1)}{\Gamma(n - c(n+2) + 3)\Gamma(-n + 2c(n+2) - 2)}. \tag{41}$$

Now, one may show that

$$\frac{\partial \frac{\Gamma(c(n+2)-1)}{\Gamma(n-c(n+2)+3)\Gamma(-n+2c(n+2)-2)}}{\partial c} =$$

$$\frac{(n+2)\Gamma(c(n+2)-1)(-2\psi((2c-1)(n+2)) + \psi(n - c(n+2) + 3) + \psi(c(n+2)-1))}{\Gamma((2c-1)(n+2))\Gamma(n - c(n+2) + 3)},$$

and that it is positive below some interior c within the cost interval specified in the lemma, and negative otherwise. Thus, the RHS of (41) is, first, increasing and then decreasing on the given cost interval.

First, consider the realization of the RHS of (41) at $c = \frac{n+4}{2(n+2)}$, that is 1. Second, consider the realization of the RHS of (41) at $c = \frac{n-1}{n+2}$, that is $\frac{1}{6}(n-4)(n-3) > 1$ for $n > 6$. Given that the RHS of (41) increases in c at $c = \frac{n+4}{2(n+2)}$, it must be the case that the inequality (40) is satisfied on $c \in \left(\frac{n+4}{2(n+2)}, \frac{n-1}{n+2}\right]$, and therefore the agent prefers commitment to full signal disclosure to the equilibrium of the disclosure game.

Next, we show that the agent prefers principal's commitment to the outcome of the disclosure game. To obtain this result, consider, first, the agent's expected payoffs under the principal's commitment:

1. Consider n even; $j^c = 1$ and $k^* = \frac{n}{2}$. In this case the agent's expected payoff is (i.e. by

omitting the fixed cost of investment it is just the probability of persuading the principal)

$$1-\frac{\Gamma\left(\frac{n+3}{2}\right)\Gamma\left(\frac{n+4}{2}\right)}{\Gamma(n+2)},$$

2. Consider n odd; $j^c=2$ and $k^*=\frac{n-1}{2}$. In this case the agent's expected payoff is

$$1-\frac{\Gamma\left(\frac{n+3}{2}\right)\Gamma\left(\frac{n+5}{2}\right)}{\Gamma(n+2)}.$$

As we know from the previous part of the proof, in the disclosure game the agent's expected payoff is:

$$1-\frac{\Gamma(\lceil c(n+2)\rceil-1)\Gamma(\lceil c(n+2)\rceil)}{\Gamma(n+2)\Gamma(2\lceil c(n+2)\rceil-(n+3))} \tag{42}$$

that is (weakly) decreasing in c. At the lower bound of the cost interval, $c=\frac{n+4}{2(n+2)}$, the expected payoff (42) becomes

$$1-\frac{\Gamma\left(\left\lceil\frac{n}{2}\right\rceil+1\right)\Gamma\left(\left\lceil\frac{n}{2}\right\rceil+2\right)}{\Gamma(n+2)\Gamma\left(-n+2\left\lceil\frac{n}{2}\right\rceil+1\right)}. \tag{43}$$

Consider n even. Then, since $\lceil\frac{n}{2}\rceil=\frac{n}{2}$, (43) can be expressed as

$$1-\frac{\Gamma\left(\frac{n+3}{2}\right)\Gamma\left(\frac{n+4}{2}\right)}{\Gamma(n+2)\Gamma\left(-n+2\frac{n}{2}+1\right)}=1-\frac{\Gamma\left(\frac{n+3}{2}\right)\Gamma\left(\frac{n+4}{2}\right)}{\Gamma(n+2)}$$

which is exactly the same as the payoff under the principal's commitment with n even. Since the agent's payoff in the game decreases in c, the agent prefers principal's commitment to the outcome of the game.

Consider n odd. Then, since $\lceil\frac{n}{2}\rceil=\frac{n+1}{2}$, (43) can be expressed as

$$1-\frac{\Gamma\left(\frac{n+3}{2}\right)\Gamma\left(\frac{n+5}{2}\right)}{\Gamma(n+2)\Gamma\left(-n+2\frac{n+1}{2}+1\right)}=1-\frac{\Gamma\left(\frac{n+3}{2}\right)\Gamma\left(\frac{n+5}{2}\right)}{\Gamma(n+2)}$$

where the latter equality is satisfied since $\Gamma(2)=1$. The above payoff is the same as the payoff under the principal's commitment with n odd. And again, since the agent's payoff in the game decreases in c, the agent prefers principal's commitment to the outcome of the game.
Q.E.D.

Proof of Corollary 1:

Suppose that the agent is committed to full signal disclosure and the principal is committed to some approval threshold j^c. Then the agent's disclosure has no effect on approval of the project. Therefore, the agent's best response budget allocation would be the same as the one

characterized in Proposition 3. So the equilibrium outcome would be the same as the one that emerges when only the principal can commit and which is characterized in Proposition 3. But by Proposition 4, the principal prefers the equilibrium outcome under the agent's commitment to the outcome under her own commitment. So, the principal would be better off not to use any commitment when the agent is committed to disclosure.
Q.E.D.