

Monitoring with Career Concerns*

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Abstract

We study monitoring and manipulation in a dynamic career concerns model. An agent “manipulates” for a private benefit and is punished when a monitor detects the manipulation. The monitor’s detection ability is uncertain and requires investment to maintain. By manipulating, the agent experiments about the monitor’s ability and this experimentation motive encourages manipulation. Absent detection, the belief about the monitor’s ability decreases, which increases the agent’s willingness to manipulate, but discourages the monitor from investing in her ability. In equilibrium, the monitor lets her ability decay, even though she could prevent manipulation forever. Surprisingly, the monitor’s investment encourages manipulation. The relationship is generally inefficient and there are multiple equilibria in which the monitor over-invests. Term limits reduce manipulation by curbing the agent’s experimentation motive and long-serving monitors start accepting bribes to hide detections. The optimal organizational design exploits externalities between multiple manipulating agents.

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“It’s a little difficult to say when the fraud started [...] so I’m not sure at which point we crossed the line. [...] Now, all of the deals were technically approved by our attorneys and accountants, so if that’s your definition of fraud then there was no fraud.”

— Andrew Fastow, former Enron CFO.¹

1 Introduction

Misbehavior is an insidious process. When organizations finally discover misbehavior, they typically trace its origins to the distant past. Enron’s bankruptcy in 2001 has shed light on a fraudulent accounting scheme reaching back several years, throughout which both internal and external monitors have failed to act. The manipulation of emissions tests at Volkswagen might have been known to executives as early as 2007, but was only uncovered in 2015.² More starkly, problems with the component causing the *Challenger* disaster have been known nine years prior to the incident, but NASA continually shifted its own internal standards rather than censure the contractor responsible (Vaughan (1996)).³

This temporal dimension of manipulation has important implications for incentives. Agents who repeatedly “get away” with manipulation start to believe that their monitors are ineffective and that future manipulation is unlikely to be punished. Thus, they are more encouraged to manipulate in the future. Since manipulation reveals information about the monitor, agents effectively engage in experimentation (Bolton and Harris (1999); Keller et al. (2005)) and manipulate to gain information about the monitor’s effectiveness. In other words, agents manipulate to “test” the monitor.

Monitors, on the other hand, are tasked with detecting manipulation, which requires specialized skills. Investing in these skills is necessary for the monitor to be effective, while failing to do so leads to a deterioration of the monitor skills and allows the agent to manipulate at a low risk. Indeed, monitors’ skills becoming outdated is a central concern among practitioners and academics⁴ and is recognized as a key driver of managerial entrenchment. Once a monitor is behind evaluating complex manipulation schemes may be either impossible

¹See “Numbers manipulator describes Enron’s descent.” *Fraud Magazine*, March 2016. <https://www.fraud-magazine.com/article.aspx?id=4294991880>. Last accessed Aug. 22, 2019.

²See “Olaf Lies wirft Volkswagen-Managern kriminelles Verhalten vor” [Olaf Lies Accuses Volkswagen Managers of Criminal Behaviour], *Der Spiegel* (in German), September 30, 2015, <https://www.spiegel.de/wirtschaft/unternehmen/volkswagen-aufsichtsrat-lies-macht-managern-vorwuerfe-a-1055381.html>. Last accessed Aug. 22, 2019.

³Consistent with these anecdotes, a report by the *Association of Certified Fraud Examiners*, ACFE (2018), finds that the average corporate fraud scheme lasts for over a year before being uncovered.

⁴See Canavan et al. (2004) for boards, Miller (1991) for managers, and Singer and Zhang (2017) for auditors.

or prohibitively costly. This explains why monitors caught up in ongoing schemes, such as the ones above, fail over extended periods of time. When manipulation goes on undetected, the monitor is discouraged and the agent becomes entrenched and is able to manipulate at low risk.

In this paper, we provide a novel framework to address these issues, which is based on career concerns (Holmström (1999)) and experimentation (Keller et al. (2005)). While the standard paradigm of monitoring (Becker (1968)) ignores uncertainty about detection and learning, our theory emphasizes the agent’s learning about the monitor and its dynamic interaction with the monitor’s incentives.

We cast our model in continuous time. A monitor of unknown ability tries to detect an agent’s manipulation, in which case she receives a reward. For example, a regulator may monitor firms’ compliance, a manager may supervise an employee, a board member may try to prevent CEO’s empire building, and an auditor may try to find violations of accounting standards. The agent obtains private benefits from manipulating, but is penalized once the manipulation is detected. The monitor’s ability is unknown to both monitor and agent (Holmström (1999)). There are two types of monitor, good and bad, and only a good monitor can detect manipulation. The monitor’s type may represent match-specific ability or human capital (Jovanovic (1979)) that deteriorates over time if the agent does not invest to keep up. The monitor can either shirk or invest to preserve her ability. When the monitor shirks, with some probability her ability is destroyed.⁵ For example, a director may be uncertain whether she understands enough about the firm to monitor the CEO, who has an inherent information advantage. To effectively monitor, the director has to keep up with new developments in the firm, to avoid “falling behind.”⁶

Both monitor and agent learn about the monitor’s type over time based on the history of detections. Since only a good monitor can detect manipulation, detecting is good news about the monitor’s ability while failing to detect is bad news. This provides an experimentation motive for the agent. By manipulating, the agent speeds up learning about the monitor’s type. Moreover, undetected manipulation makes the monitor pessimistic about her own ability, which eventually leads her to stop exerting effort. This is beneficial for the agent, since once the monitor’s ability decays, the agent can manipulate without fear of being detected.

We consider Markov Perfect equilibria in which the belief about the monitoring ability

⁵This is a convenient modeling choice which allows us to capture the monitor’s incentives to preserve her ability without introducing another state variable. We relax this and other assumptions in Section 7 and show that our results go through in more general settings.

⁶Indeed, directors failing to maintain their skills is a central topic in the debate surrounding corporate boards. See our discussion in Section 2.1 for details.

is the only state variable. We can interpret this belief as the monitor’s reputation. We focus attention on equilibria with monotone manipulation strategies, in which the agent manipulates whenever the probability of detection is below a threshold. The monitor’s strategy is generally non-monotone, with the monitor shirking for low and high beliefs while exerting effort at intermediate ones. For high beliefs, the agent does not manipulate as the risk of detection is too high. In this region, a higher reputation is counterproductive to the monitor: the monitor can’t obtain rewards because her reputation preempts manipulation. The monitor thus shirks, and her ability deteriorates. This process drives her reputation down over time until the agent finds it optimal to start manipulating. At that point the monitor starts exerting effort to preserve her ability. However, in the absence of detection, her reputation continues to deteriorate. As the monitor becomes increasingly pessimistic about her own ability, she stops exerting effort altogether. These equilibrium dynamics illustrate the shortcomings of career concerns as an incentive mechanism. In our model, the monitor could deter manipulation and maintain her ability, but it is not optimal for her to do so. Instead, the monitor becomes ineffective over time.

Surprisingly, the agent’s manipulation incentive is stronger when the monitor invests. If the monitor shirks, the agent has an incentive to delay manipulating until the monitor’s ability is likely to have depreciated. By contrast, if the monitor is investing, waiting is less valuable to the agent, because the monitor type is not deteriorating. This creates a complementarity between the monitor’s investment and the agent’s manipulation. We prove that such complementarity is a source of multiplicity and there is a continuum of equilibria which differ in terms of the prevalence of manipulation. The *worst equilibrium* features too much monitor investment and generates excessive manipulation. Vice-versa, the *best equilibrium* features less monitor investment and also less manipulation. Thus, contrary to common wisdom, the monitor is prone to over-investing in detection and a “light touch” is more beneficial.

We use our model to evaluate commonly debated policies aimed at mitigating manipulation. Our prediction that the monitor becomes ineffective over time is consistent with a large academic literature on entrenchment.⁷ To solve the entrenchment problem, term limits are a popular solution.⁸ In the UK and France, board members must be rotated out after terms of 9 and 12 years, respectively. Some companies (e.g. Deloitte) have instituted term limits for CEOs. Mandatory auditor rotation was recently introduced in the E.U. in 2014.

⁷See e.g. [Hermalin and Weisbach \(1988\)](#), [Shleifer and Vishny \(1989\)](#), [Zwiebel \(1996\)](#), [Vafeas \(2003\)](#), [Bebchuk and Fried \(2006\)](#), [Garrett and Pavan \(2012\)](#), [Bonini et al. \(2017\)](#), and [Huang and Hilary \(2018\)](#).

⁸For example, Institutional Shareholder Services (ISS) started to include director tenure in its company governance ratings. ISS views tenure of more than nine years as excessive by virtue of potentially compromising a director’s independence (see [Huang and Hilary \(2018\)](#)).

Finally, of course, politicians, who monitor the executive branch, regularly face term limits.

In Section 4, we provide a new rationale for term limits: imposing a term limit lowers manipulation, because it decreases the agent’s value of learning. Intuitively, the agent’s benefit from manipulating comes from driving down the monitor reputation, which discourages her from investing. But if the monitor is replaced before this happens, the agent is deprived of his benefit. This learning channel appears to be absent from the discussion on term limits and entrenchment.

Term limits may also help prevent corruption. Over time, monitors may start colluding with agents and protect them instead of revealing misdeeds (Vafeas (2003)). In Section 5, we allow the agent to bribe the monitor to hide detections from the public. Consistent with the entrenchment literature, we find that longer serving monitors, who failed to detect manipulations, start accepting bribes and “look the other way,” while newer ones reject them. These dynamics are driven by changes to the joint surplus of monitor and agent, which we show is decreasing in monitor reputation.

Finally, we consider an extension with multiple agents, and show how the optimal organizational design harnesses externalities between the agents. In many settings, monitors supervise multiple agents. For example, managers oversee multiple employees, directors often sit on the boards of multiple companies, auditors have different clients, etc. When multiple agents manipulate, they jointly learn about the monitor’s ability. They are hence engaged in a game of strategic experimentation (Bolton and Harris (1999), Keller et al. (2005)). An organizational designer can then harness externalities between the agents to reduce manipulation. The optimal organization structure depends on the availability of information. With *individual punishments*, the monitor can detect each individual agent’s manipulation and punish him directly. In that case, overseeing multiple agents is optimal. Compared to our main model, agents free-ride on information generated by others, which then reduces the incentive to learn about the monitor and to manipulate. With *collective punishments*, the monitor does not know which agent has manipulated and can only provide group punishments for the entire team. In this case, overseeing multiple agents leads to more manipulation, because agents free-ride on punishments. Each agent manipulates more, knowing others will face part of the brunt.

Technical Contribution Experimentation models maintain tractability by exploiting symmetry, smooth pasting, or closed-form solutions (see Keller et al. (2005); Keller and Rady (2010, 2015)). In our model, the monitor and agent solve fundamentally different problems, so we cannot appeal to symmetry. Because of the complementarity between effort and manipulation, we generally cannot use smooth pasting either. Whenever the monitor’s

or agent’s strategies change, the other player’s value function exhibits a kink. Because of this, the marginal values of effort and manipulation are discontinuous in the belief. To characterize any equilibrium, we must characterize these jumps. While the value functions in our model admit closed form solutions, the particular form depends on the action of the other player. To characterize equilibria by using a combination of closed forms (as in Keller et al. (2005)) and monotonicity arguments for ODEs. The key step in our argument is to characterize the agent’s incentives in the upper region on which the monitor shirks. The left boundary of this region, which we will call p_h , is indeterminate because of the complementarity. Nonetheless, we completely characterize the set of equilibria and show that in any equilibrium, p_h belongs to an interval. We do this by characterizing the agent’s incentives *across equilibria* and show that they are monotone.

Literature Our paper contributes to the experimentation literature with exponential bandits.⁹ In Keller et al. (2005), the arrival process is a Poisson process with positive lump-sums occurring only if the arm is good. This is also the case in our model. Keller and Rady (2015) solve the opposite case with bad news. Keller and Rady (2010) consider the case in which breakthroughs are not conclusive. Grenadier et al. (2014), Dong (2016), and Thomas (2019) study signaling games in which the experimenter knows his type. In our model, the agent experiments, but he does not have private information. Bonatti and Hörner (2017) and Halac and Kremer (2018) study the experimenter’s career concerns. In our setting, the career concerns are on the side of the monitor, who does not experiment. Unlike all these papers, our model is not a pure experimentation setting. While the agent experiments, the monitor decides how much to effort to exert to retain her human capital. Thus, the monitor and agent face fundamentally different types of problems.

In seminal and closely related work, Halac and Prat (2016) study a setting where a myopic agent’s incentive to work depends on a forward-looking principal being attentive to the agent’s performance, because only when the principal is attentive the agent is rewarded for his effort. Thus, monitoring is a way to reward the agent’s effort (i.e., contrary to our setting, the agent wants to be monitored).

Contrary to Halac and Prat (2016), we assume that both the monitor and the agent are forward-looking. This is a key part of our contribution, since we are interested in the experimentation incentives of manipulating agents and in how these incentives shape the monitor’s behavior. Our results on the experimentation motive encouraging manipulation, on the presence of multiple equilibria and the inherent inefficiency of the monitoring relationship, on the optimality of term limits, and on the use of organizational design to harness

⁹See Bolton and Harris (1999) for the Brownian case.

externalities between agents all rely on the agent being forward-looking. They are unique to our paper and absent from Halac and Prat (2016).

Corona and Randhawa (2010) is also closely related. They consider a two-period setting where both the monitor detection ability and the agent’s manipulation propensity are unknown. The agent may test the waters in the first period by manipulating with some small probability and escalate in the second period if the first period manipulation goes unnoticed. Interestingly, they show that if the agent reputation is high, a dishonest monitor may choose to silence his findings in the second period, for this would reveal to the market he failed to detect manipulation in the first period.

Our paper is also related to an extensive work on monitoring, that goes back to Becker (1968). Recently, Dilmé and Garrett (2018) study a dynamic model, where the monitor faces fixed switching costs, from re-starting its monitoring activity. They argue that this switching costs lead to cycles of enforcement. In Kolb and Madsen (2019), the agent can undermine the principal, who may detect this and who chooses how much information about an underlying state to reveal. In Varas et al. (2017), a monitor chooses the detection intensity, and Orlov (2018) studies the effect of monitoring in a dynamic contracting model. In a previous paper, Marinovic and Szydlowski (2018), we consider monitoring with a long-term monitor who privately knows his type and a myopic agent. Relative to that paper, our key innovation is to consider a forward-looking agent. The results on experimentation motives and term limits, among others, cannot be obtained in our previous paper.

2 Model

We study a dynamic game in continuous time between two forward-looking players, referred to as monitor (she) and agent (he). We defer all proofs to the Appendix.

Monitor and Agent The agent takes an action $m_t \in [0, 1]$ that we label “manipulation,” while under the scrutiny of the monitor. Manipulation generates a flow benefit Bm_t for the agent, but, if detected by the monitor, generates a penalty K . The monitor can detect the agent’s manipulation, in which case she receives a reward R . The monitor’s ability to detect manipulation is uncertain and subject to shocks. The monitor has career concerns (Holmström (1999)) and her ability is unknown to both the agent and herself. She is either a good type, who detects manipulation at a Poisson rate λm_t , or a bad type who cannot detect manipulation. The good type’s ability may depreciate, and she may become the bad type. Once she is bad, she stays bad.¹⁰ To maintain her ability, the monitor can exert effort

¹⁰We generalize this feature in Section 7.2 and provide conditions such that our results go through.

$e_t \in [0, 1]$ at cost ce_t . This effort represents an investment in human capital. Formally, the monitor’s ability depreciates with Poisson rate $\gamma(1 - e_t)$ if her type is good and by choosing $e_t = 1$, the monitor can prevent her ability from decaying. Both the monitor’s and the agent’s actions are observable, but not contractible.¹¹ We assume that the relationship ends when the monitor detects manipulation. This terminal assumption is natural in settings where career concerns are significant and the power of pay for performance is limited (Dewatripont et al. (1999)).¹²

Beliefs Detections are publicly observable.¹³ Both monitor and agent learn from observing detection as well as from observing its absence. The common prior belief that the monitor is good is $p_0 \in (0, 1)$. Let p_t be the time- t equilibrium belief, which we can understand as the monitor’s reputation. Since only the good monitor can detect manipulation, detection fully reveals that the monitor is good, and the belief jumps to $p_t = 1$.¹⁴ Generally, the belief follows

$$dp_t = -(\lambda p_t(1 - p_t)m_t + \gamma p_t(1 - e_t))dt + (1 - p_t)m_t dN_t.$$

Here, N_t is the Poisson process which marks detection and which satisfies $E(dN_t) = \lambda p_t dt$.

Absent detection, the belief p_t drifts down. Since only the good monitor detects manipulation, a failure to detect increases the likelihood that the monitor is bad. The belief decreases faster when the monitor exerts less effort (e_t is small), because then her ability depreciates faster on average. Thus, the longer the monitor shirks, the more pessimistic she becomes about her ability. The belief also decreases faster when the agent manipulates more (m_t is large). The latter effect shows the experimentation role of manipulation: by manipulating, the agent learns about the monitor’s type. By manipulating more, the agent “speeds up” his learning about the monitor. Then, if the agent is not caught, both monitor and agent revise their belief downward more. As we will show, learning about the monitor is valuable for the agent and diminishes the monitor’s incentives to exert effort. Naturally, the learning depends on whether the agent is expected to be manipulating. Without manipulation, no detection is uninformative, since there is nothing to detect.

¹¹Otherwise, deviating from the equilibrium would create persistent private information. Intuitively, the monitor may know the agent is shirking, but must uncover evidence to justify penalizing the agent. The monitor’s ability to generate this evidence is uncertain.

¹²In some cases, it is more natural to assume that the monitor’s horizon goes beyond the first detection, in which case the monitor’s continuation value upon detection is given $V(1)$. We discuss this possibility in Section 7.4, where we provide conditions such that our results go through.

¹³In the baseline model, we assume that the monitor has no discretion to withhold detection to avoid the agent having to pay the penalty. In Section 5, we generalize this aspect and allow for collusion between monitor and agent.

¹⁴We consider the effect of imperfect good news and the case of bad news, where undetected manipulation is revealed by an external party, in Section 7.

Equilibrium Concept We restrict attention to Markov Perfect Equilibria (MPE) in which (m_t, e_t) depend on p_t alone. Furthermore, we focus on equilibria in which the agent uses a threshold strategy, i.e., he manipulates whenever the belief is below a threshold.¹⁵ We impose no such restriction on the monitor's strategy, which generally will be non-monotone. Following Keller and Rady (2015), we also require that the agent's and monitor's strategies are piecewise Lipschitz continuous in the belief. In equilibrium, the monitor's and agent's strategies will be piecewise constant.

Payoffs The monitor's and agent's continuation values at time t can be written as

$$V(p_t) = E_t \left[e^{-r(\tau-t)} R - \int_t^\tau e^{-r(s-t)} c e_s ds \right] \quad (1)$$

and

$$W(p_t) = E_t \left[\int_t^\tau e^{-r(s-t)} B m_s ds - e^{-r(\tau-t)} K \right], \quad (2)$$

where τ is the time of detection.

The continuation values solve the following Hamilton-Jacobi-Bellman (HJB) equations:

$$rV(p_t) = \max_{e_t \in [0,1]} -c e_t + \dot{p}_t V'(p_t) + \lambda p_t m_t (R - V(p_t)) \quad (3)$$

and

$$rW(p_t) = \max_{m_t \in [0,1]} B m_t + \dot{p}_t W'(p_t) - \lambda p_t m_t (K + W(p_t)), \quad (4)$$

where $\dot{p}_t = -\lambda p_t (1 - p_t) m_t - \gamma p_t (1 - e_t)$.

The monitor payoffs are straightforward. The monitor bears flow cost $c e_t$ arising from the effort required to maintain her ability e_t . In return, she obtains a lump-sum gain $R - V(p)$ if she detects manipulation. The likelihood that this happens depends on the belief p and the intensity of manipulation m_t . The agent enjoys a flow benefit B from manipulation, but if detected he pays a penalty K and loses his ability to extract further private benefits. If the belief decreases, the agent's value increases because the expected present value of future penalties decreases.

To measure the monitor and agent's incentives at a given point in time, we define the

¹⁵This restriction is natural. As we show below, since the agent's manipulation strategy is a threshold both in the myopic case and when the monitor's effort is constant (see Proposition 1). Without the restriction, the complementarity between monitor effort and manipulation generates an infinite number of non-monotone equilibria. These equilibria, however, do not have any additional economic content, and we choose not to focus on them.

instantaneous marginal benefit of effort as

$$v(p) \equiv \gamma p V'(p) - c,$$

and the instantaneous marginal benefit of manipulation as

$$\omega(p) \equiv B - \lambda p(1-p)W'(p) - \lambda p(K + W(p)).$$

These functions are the key objects of our study, since they determine when it is optimal for the monitor to exert effort or for the agent to manipulate.

Finally, we impose the following parametric restriction, which ensures that the agent only manipulates when the belief is sufficiently low.

Assumption 1. $B < \lambda K$.

Without this condition, the agent would always manipulate, even if he knows he is facing the good monitor.

2.1 Discussion

Here, we discuss our modeling assumptions and how they map to the literature and to concrete applications.

Career Concerns We assume that both agent and monitor are uncertain about the monitor's ability. This makes sense if the ability is match-specific (e.g. [Jovanovic \(1979\)](#)). To be effective, monitors such as regulators, directors, banks, or auditors often require firm-specific knowledge, which is independent of their education or acquired track record.¹⁶ Upon entering a new monitoring relationship, the monitor is thus uncertain whether she understands the firm enough to provide effective oversight.

Consistent with the career concerns literature, we do not consider optimal contracts for the monitor or the agent. Regulators, corporate directors, banks, or auditing firms face significant reputational incentives, which is well documented empirically.¹⁷ In many organizations, incentives for managers come mainly from promotions (e.g. [Lazear and Rosen \(1981\)](#)), which is consistent with the reward structure in our model. In reality, CEOs and employees who are caught committing fraud usually face termination. This is consistent with our assumption that the relationship ends after manipulation is detected. In Section

¹⁶See e.g. [Diamond \(1984\)](#) (for banks) or [Bonini et al. \(2017\)](#) (for directors).

¹⁷See e.g., [Lucca et al. \(2014\)](#) for regulators, [Fang \(2005\)](#) for banks, [Firth \(1990\)](#) for auditors, and [Fich and Shivdasani \(2007\)](#) for boards.

7.4, we consider the case when the relationship continues instead, and we provide conditions under which the equilibrium of Proposition 3 survives.

In Section 7.1, we consider more general payoff structures for the monitor, e.g. when she receives a flow wage which depends on her reputation. We provide sufficient conditions such that our results go through.

Human Capital Our modeling of the monitor’s ability captures concerns about monitors falling behind new developments or gradually losing their independence. Such concerns are prominent for corporate boards. For example, [Canavan et al. \(2004\)](#) features a round table discussion on board tenure, in which participants note that “the more accustomed you are to the procedures and approaches the company takes [...] the more you lose your ability to be critical of what management is doing.” These concerns have been echoed by shareholder activists, regulators, proxy advisory firms, and the press.¹⁸ Similar concerns arise in the literature on CEOs (e.g. [Miller \(1991\)](#)) and, discussing CEO tenure, the *Financial Times* pointedly notes that “[t]he risk is evident; the question is how long it really takes for atrophy to set in.”¹⁹ There is also a large debate on “auditor rotation” ([Singer and Zhang \(2017\)](#)), which features similar arguments against long auditor tenures.

In Section 7.2, we relax our assumption and allow the monitor’s effort to improve her ability. We provide sufficient conditions such that all our results go through.

Observability We assume that the monitor’s effort and the agent’s manipulation are observable, but not contractible.²⁰ This assumption captures the common situation where agents must submit their work to monitors, who can thus detect whether something “unusual” is happening, but who might require special expertise to detect actual violations. Often, agents keep close tabs on monitors and can observe whether a monitor exerts the required effort to keep up with ongoing manipulation schemes.

This interpretation maps closely to the interaction between a CEO and a board of directors. For example, in its investigation of the Enron scandal, the U.S. Senate ([U.S. Senate \(2002\)](#)) notes that “much of what was wrong at Enron was not concealed from its Board of Directors. High risk accounting practices [...] were known to and authorized by the Board,” since “[management provided the board] numerous Deal Approval Sheets (DASHs) for approval of major transactions.”²¹

¹⁸See e.g. “US Board Composition: Male, Stale and Frail?,” *Financial Times*, August 15 2016, <https://ig.ft.com/sites/us-board-diversity/> (last accessed 02/18/2020).

¹⁹See “The Perils of the Chief Who Stays Too Long at the Top,” *Financial Times*, April 16 2014, <https://www.ft.com/content/aee47536-c48f-11e3-8dd4-00144feabdc0> (last accessed 02/18/2020).

²⁰As we discussed, the assumption guarantees that the model features no persistent private information.

²¹Moreover, the senate committee identified “more than a dozen incidents over 3 years that should have

Likewise, CEOs routinely sit on board meetings and closely interact with individual board members. This is thus easy for them to observe whether board members are attempting to stay abreast of new developments. The same investigation into Enron revealed that “[t]he Board did not have a practice of meeting without Enron management present,” and that “[o]utside of formal [...] meetings, the Enron Directors describe very little interaction or communication [...] among board members.”²²

Experimentation Our modeling of the information arrival process is standard in the literature on exponential bandits. The same process appears in Keller et al. (2005), Grenadier et al. (2014), Bonatti and Hörner (2017), and Halac and Kremer (2018). Learning via good news makes sense in the context of our applications. In reality, manipulation only becomes known if it is detected by a manager, director, auditor, etc. If no detection is reported, we, by definition, cannot observe information about the agent or the monitor. Indeed, as Dyck et al. (2013) document, detecting manipulation is rare. They find that in a given year, fraud is detected in only 4% of companies. Thus, the average company goes long stretches without detection, just as in our model. Moreover, Dyck et al. (2013) find that monitors fail to detect a significant portion of manipulation, around 70%, which is also consistent with our assumptions. We generalize the information technology in Section 7. In Section 7.3, we study the case with good and bad news. Bad news arise when the monitor misses manipulation which then becomes public through other channels. In Section 7.5, we consider perfect bad news only and show that most of our results go through.²³

raised Board concerns about the activities of the company.” For example, “the Board was asked to and approved an unprecedented arrangement allowing Enrons CFO to set up private equity funds [...] for the purpose of improving Enrons financial statements. [...] Committee and Board presentations throughout 1999, 2000, and 2001 chronicled the companys foray into more and more off-the-books activity.”

²²See also Breeden (2003) for similar findings from an investigation of the WorldCom scandal. The report notes the board’s failure to keep up with developments: “As WorldCom did acquisition after acquisition, its risk posture grew more complicated and required much more careful attention. [...] There is no indication that the board analyzed how the enormous debts being accumulated by the Company in acquisitions would be carried and ultimately retired.” For the board to be effective, the report recommends that “[b]oard members need to be involved in the life of the company, visiting major facilities regularly, interacting with a range of senior and mid-level staff, and also discussing issues with major shareholders periodically.” From the absence of such activities, the CEO can easily detect whether the board is shirking in their duty to stay current.

²³We have also numerically solved a model with *imperfect* good news in which the bad monitor can detect manipulation at a lower but positive rate. We can find parameters such that the equilibrium of Proposition 3 survives.

3 Analysis

3.1 Benchmarks

A myopic agent's value of manipulation is $B - \lambda p_t K$. Thus, a myopic agent manipulates whenever the belief is below a threshold p_m , which satisfies

$$p_m = \frac{B}{\lambda K}.$$

To understand the agent's incentives, we first study two benchmarks that fix the monitor's effort strategy at two polar levels: *i*) when the monitor always exerts effort and *ii*) when the monitor always shirks. We then compare the agent's forward-looking behavior versus that of a myopic agent.

Proposition 1 (Complementarity). *If the monitor never exerts effort, there is a threshold $p_{nm} \in (0, 1)$, such that the agent manipulates if and only if $p < p_{nm}$. If the monitor always exerts effort, there is a threshold $p_{mon} \in (p_m, 1)$, such that the agent manipulates if and only if $p < p_{mon}$. We have $p_{nm} < p_{mon}$. If $(r + \gamma)B - \lambda rK < 0$, then $p_{nm} > p_m$, otherwise $p_{nm} < p_m$.*

In the benchmark where the monitor always exerts effort, the agent has strong incentives to manipulate, that is $p_{mon} > p_m$. This reflects the agent's experimentation motive: the agent manipulates, in part, to learn the monitor's type, which—given the monitor's effort—is very persistent.

Indeed, consider the agent's behavior when the belief hits the myopic threshold p_m . At that point by definition the agent obtains zero flow profit from manipulating. However, as the proposition shows, the agent still manipulates, precisely because his value of learning about the monitor is positive.

Following Keller et al. (2005), the agent's value of experimentation can be written as

$$-\lambda p(1 - p)W'(p) - \lambda pW(p).$$

The agent's value is decreasing in the belief, i.e. $W'(p) < 0$. This slope is sufficiently large, so that the agent's value from experimenting is positive.

This result is reminiscent of the idea that manipulation is a *slippery slope*: Once the agent manipulates and is not detected, he realizes that he is likely to get away with it in the future. The more he manipulates, the lower is his assessment of the risk of detection, which leads to a self-reinforcing spiral. In fact, absent detection the incentive to manipulate $\omega(p)$ grows

over time, which captures the notion that manipulation today makes future manipulation more appealing.

Consider, as a second benchmark, the case when the monitor always shirks. In this case the agent faces countervailing incentives. Though manipulation continues to yield experimentation benefits, the agent faces an incentive to wait rather than manipulate, to take advantage of the fact that the monitor’s ability deteriorates over time. If the monitor shirks, her type eventually goes bad, in which case the risk of detection vanishes. Because of this, the agent’s incentive to manipulate is smaller, and we have $p_{nm} < p_{mon}$. This incentive to wait may even dominate the experimentation incentive, and lead the agent to manipulate less (i.e., wait more) than a myopic agent. For example, this is the case when the rate of decay γ of the monitor’s ability is very large so that waiting has significant value.

Our analysis thus far suggest that there is complementarity between the monitor’s effort and manipulation: the agent manipulates more with monitor effort than without it, that is $p_{mon} > p_{nm}$.²⁴ This complementarity also affects the monitor’s incentives. The monitor only receives the reward when she detects manipulation. But if the agent does not manipulate, there is nothing to detect. As a result, the monitor has no more incentive to preserve her ability, since it does not generate any rewards for her.²⁵

Lemma 2. *In any MPE, there is no nonempty interval (\underline{p}, \bar{p}) such that $m(p) = 0$ and $e(p) = 1$ for all $p \in (\underline{p}, \bar{p})$.*

To understand this result, recall that effort is an investment that preserves the monitor’s ability to detect manipulation. One would think that if manipulation is likely to take place in the future, then the monitor would have an incentive to exert effort today, even if the agent is not manipulating, to ensure she detects the agent’s future manipulation. However, if there was a region with positive effort and no manipulation, then the belief would remain stuck in that region. Indeed, Equation (2) implies that $dp_t = 0$ in that case. Then, monitor would never reap any rewards, as the belief would never reach the region where the agent manipulates.

Taken together, Proposition 1 and Lemma 2 explain why monitoring incentives can be weak. When the belief is sufficiently high, the agent does not manipulate, and, consequently, the monitor shirks. Thus, even though the monitor could prevent her ability from deteriorating forever, it is not optimal for her to do so. Instead, her expected type monotonically

²⁴A similar result is found in Halac and Prat (2016) but for different reason. In that paper the agent’s payoff increases in the probability of monitoring. The key behind our complementarity result is the presence of stronger experimentation incentives when the monitor is exerting effort.

²⁵As mentioned before, we generalize this feature in Section 7.1, where we expand the model to incorporate belief-dependent flow payoffs for the monitor. Unless the monitor’s flow payoff is very steep with respect to the belief, all results below go through.



Figure 1: Benchmarks. A myopic agent manipulates whenever the belief is below p_m . When the monitor exerts effort, a forward-looking agent manipulates below p_{mon} , while when the monitor shirks, the agent manipulates below p_{nm} .

decreases on path, until eventually the monitor becomes ineffective. As we show next, this is exactly what happens in equilibrium.

3.2 Equilibrium

In our model, the monitor’s reputation is transitory; eventually the monitor’s ability and her reputation are destroyed. This is because *i*) the monitor shirks for high beliefs, so that her type deteriorates, and *ii*) because the agent manipulates for low beliefs. Undetected manipulation erodes the monitor’s belief in her own ability and eventually leads her to stop exerting effort. In a sense, the agent’s manipulation leads the monitor to “give up” over time, because she concludes her type is low.

We now show this formally. The main challenge of this problem is that both monitor effort and manipulation are jointly determined as part of the players’ dynamic optimization problem. Because players are forward-looking, the complementarity we have characterized in Proposition 1 becomes a dynamic one. If the agent does not expect the monitor to exert effort in the future, this changes her incentive to manipulate today. Indeed, the complementarity between effort and manipulation leads to multiple equilibria.

Proposition 3. *Suppose that the monitor reward R satisfies*

$$R > c \frac{r + \gamma}{r\gamma} \frac{B + rK}{B}. \quad (5)$$

Then, any equilibrium is characterized by two cutoffs $p_l < p_h$ such that

$$m(p) = \begin{cases} 1 & \text{if } p < p_h \\ 0 & \text{if } p \geq p_h \end{cases}$$

and

$$e(p) = \begin{cases} 0 & \text{if } p < p_l \\ 1 & \text{if } p \in [p_l, p_h) \\ 0 & \text{if } p \geq p_h. \end{cases}$$

There exist two values \underline{p}_h and \bar{p}_h , such that $1 > \bar{p}_h > \underline{p}_h > p_l$. The pair $\{p_l, p_h\}$ constitutes an equilibrium if and only if $p_h \in [\underline{p}_h, \bar{p}_h]$. In particular, any equilibrium features the same

p_l .

The equilibrium is predicated on the reward, R , being large relative to the cost, or else the monitor would never monitor, as in the no effort benchmark.

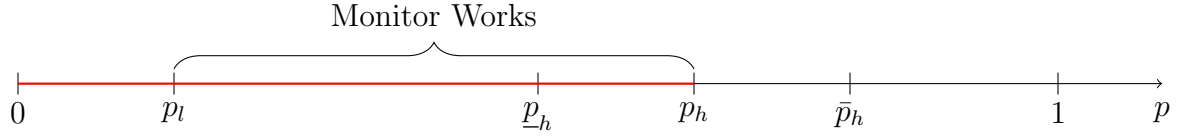


Figure 2: Equilibrium. The agent manipulates on the red region, when the belief is below p_h .

Any equilibrium consists of three regions (see Figure 2). For high beliefs, namely for $p_t > p_h$, the agent does not manipulate because the risk of detection is too high relative to the benefit of manipulation B . At the same time, the monitor shirks, because she cannot receive any rewards given that the agent is not manipulating. Lemma 2 applies to this region and our argument above applies as well. If the monitor were to exert effort, the belief would stay constant forever. But then, she would never receive the reward.

Consistent with this intuition, Figure 3 shows that the monitor value decreases in the belief beyond p_h . The non-monotonicity of $V(\cdot)$ and the fact that V decreases to the right of p_h is explained again by the observation that high beliefs deter manipulation, destroying the monitor’s ability to “cash-in.” The larger the belief, the longer the monitor has to wait till she is able to cash-in.

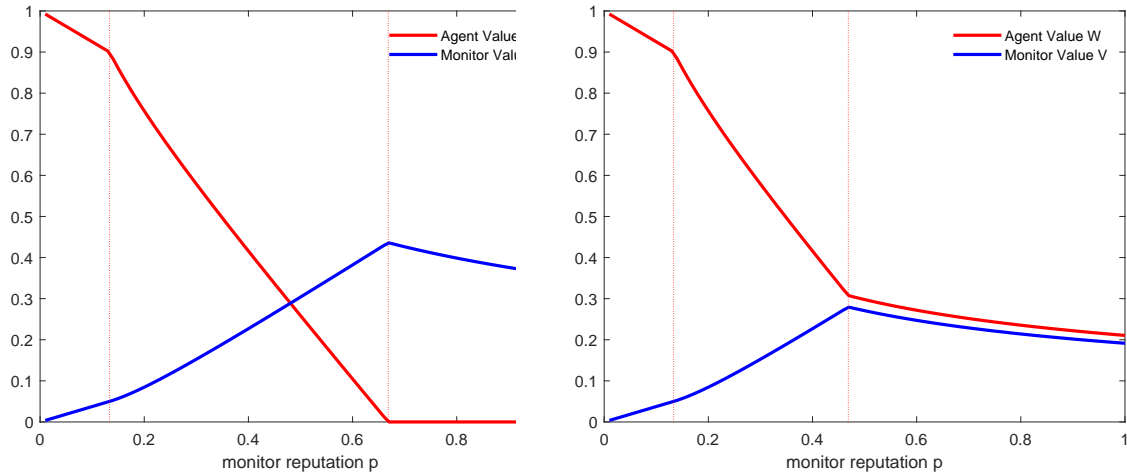


Figure 3: Equilibria. The left panel depicts the value functions when $p_h = \underline{p}_h$ and the right panel depicts the value functions when $p_h = \bar{p}_h$. The vertical dashed lines represent p_l and p_h . The monitor value is non monotonic in the belief, since above p_h the monitor needs to wait until the belief deteriorates to be able to cash in any rewards.

By shirking in the top region, the belief deteriorates, eventually reaching the intermediate region, $[p_l, p_h]$. There, the agent manipulates, encouraged by the lower probability of

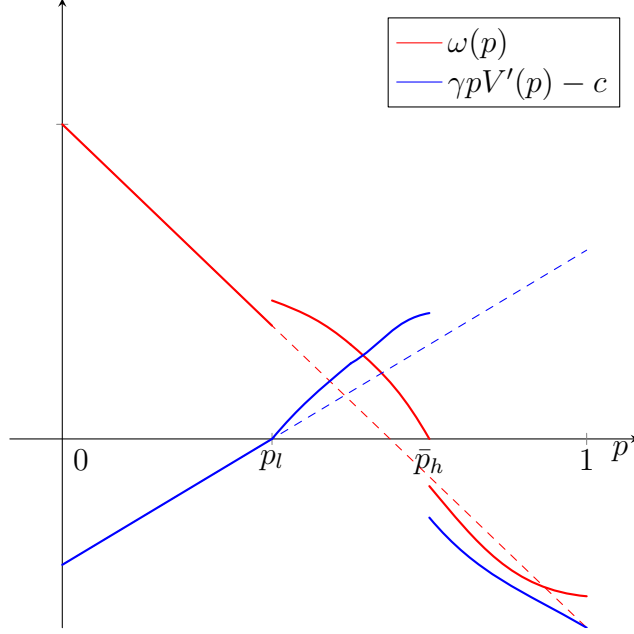


Figure 4: Incentives and reputation. The marginal incentives to manipulate (red) decrease in the belief, except at p_l where the monitor switches from shirking to exerting effort, thus boosting the incentive to manipulate.

detection. On the other hand, the monitor exerts effort because her incentives jump in the presence of manipulation: here she obtains a reward from detecting the agent's manipulation. Since her continuation value is relatively large, shirking is also costly. By putting in effort she preserves her ability. However, absent detection the belief continues to deteriorate on path. No detection is interpreted as a negative signal of monitor ability and, by manipulating, the agent drives down the belief about the monitor's ability. The longer the time elapsed without detection, the lower is the monitor reputation.

This progressive deterioration eventually leads the players into the (third) bottom region, where $p_t < p_l$. In this region the monitor stops exerting effort, discouraged by the low perceived value of her ability. On the other hand, the incentive of the agent to manipulate strengthens, as he is even more likely to get away with manipulation. Detection is still possible as long as $p_t > 0$, but that possibility becomes decreasingly likely over time and eventually vanishes.

In conclusion, our analysis suggests that the monitor's incentives are non-monotone, being stronger (weak) for intermediate (extreme) beliefs. It also shows that the monitor's reputation is transitory. Although the monitor could ensure that she keeps her reputation forever, it is not optimal for her to do so. Thus, the monitor eventually becomes ineffective.

There is a continuum of equilibria that differ in terms of the amount of manipulation they induce. Specifically, we find that the upper end of the manipulation region, p_h , belongs to

an interval $p_h \in [\underline{p}_h, \bar{p}_h]$ (while the lower end p_l is fixed). Any p_h in this interval is part of an equilibrium. The threshold \bar{p}_h captures the equilibrium with the most manipulation, which we refer to as the *worst equilibrium*. On the other extreme, \underline{p}_h captures the equilibrium with the least manipulation, which we label the *best equilibrium*.

This multiplicity of equilibria reflects the strategic complementarity between monitor effort and manipulation. The agent's manipulation incentive is stronger when he exerts effort (and effort incentives are stronger when the agent manipulates). The equilibria with more monitor effort also feature more manipulation. The multiplicity contrasts with the results in the experimentation literature (see Keller et al. (2005); Keller and Rady (2010)). We characterize the thresholds \underline{p}_h and \bar{p}_h in the following lemma.

Lemma 4. *Let $W(p)$ denote the solution to the agent's HJB Equation with $m(p) = e(p) = 1$, which satisfies the value matching condition at p_l . Define \underline{p}_h as*

$$\lim_{p \downarrow \underline{p}_h} \omega(p) \equiv B + \frac{\lambda r}{\gamma} (1 - \underline{p}_h) W(\underline{p}_h) - \lambda \underline{p}_h (K + W(\underline{p}_h)) = 0 \quad (6)$$

and define \bar{p}_h as

$$W(\bar{p}_h) = 0.$$

Then, $\underline{p}_h > p_l$, $1 > \bar{p}_h > p_m$, and $\underline{p}_h < \bar{p}_h$. For any $p_h \in [\underline{p}_h, \bar{p}_h]$, $m(p) = e(p) = 1$ are optimal on $[p_l, p_h]$ and no $p_h \notin [\underline{p}_h, \bar{p}_h]$ can be part of an equilibrium.

At the worst equilibrium threshold \bar{p}_h , the agent is indifferent between manipulating or not, provided that the monitor exerts effort. No $p_h > \bar{p}_h$ can be part of an equilibrium, because then the agent does not find it optimal to manipulate at p_h .

Because of the complementarity, we can have $p_h < \bar{p}_h$. That is, if the monitor stops exerting effort at p_h , the agent stops manipulating as well, even though continuing for both would constitute another equilibrium. For p_h to be part of an equilibrium, it must be optimal for the agent to not manipulate to the right, i.e. we must have $\lim_{p \downarrow p_h} \omega(p) \leq 0$. The best equilibrium threshold \underline{p}_h is the one for which the agent is just indifferent.²⁶ For any lower threshold p_h , the agent would prefer to manipulate to the right.

To conclude this section and for completeness we present the case when detection rewards are small.

Proposition 5. *Suppose that*

$$R \leq c \frac{r + \gamma}{r\gamma} \frac{B + rK}{B}. \quad (7)$$

²⁶Generally, $\omega(p)$ is discontinuous at p_h , so that its right and left limits do not coincide.

Then, there is a unique equilibrium. The monitor never exerts effort and the agent manipulates whenever $p < p_{nm}$, where p_{nm} is given by Proposition 1.

4 Term Limits and Turnover

In our model, the monitor's expected ability decreases over time. Eventually, for $p < p_l$, the monitor stops exerting effort and the agent is unlikely to be caught. The agent's benefit of experimentation comes partially from reaching this region. This suggests a simple policy intervention. We can impose a term limit on the monitor and replace her with a new one before p_l is reached. As we show in Proposition 6 below, this reduces the agent's incentive to manipulate.

Proposition 6. *Consider a time T_D at which the monitor is replaced with a new one if she fails to detect manipulation. Let $p_D = p_{T_D}$. If $p_D < p_0$ and $p_D < \bar{p}_h$, then \bar{p}_h decreases.*

Replacing the monitor reduces the value of learning the monitor's type, thus discouraging manipulation. The value of experimentation increases in the length of the relationship and the persistence of the monitor type; if the monitor is rotated frequently, the information the agent learns from manipulating is shorter lived and thus less valuable. This is why imposing term limits on the agent-monitor relationship can mitigate manipulation.

The monitor, of course, anticipates that she might be replaced. If her outside value is lower, being replaced is more costly, and she has a stronger incentive to exert effort.

Proposition 7. *Suppose that when the monitor is replaced, the monitor and the agent receive lump-sum payments V_D^0 and W_D^0 . Suppose that $p_D < p_l$, that $W_D^0 = W(p_D)$, and that $V_D^0 \leq V(p_D)$. Then, as V_D^0 decreases, p_l and \bar{p}_h both decrease.*

Here, V_D^0 is the monitor's outside value upon replacement. As being replaced becomes more costly, the monitor is more willing to exert effort to prevent the belief from decreasing. Thus, p_l decreases. Reaching low beliefs is less beneficial for the agent, because the monitor still exerts effort. Therefore, the agent manipulates less.

The agent faces similar incentives. If dissolving the relationship, e.g., via firing both monitor and agent or via shutting down the firm, manipulation becomes more costly for the agent, he manipulates less.

Proposition 8. *Fix $V_D = V(p_D)$, i.e., assume that at p_D the monitor receives a payment equal to her equilibrium continuation payoff. Then, as W_D^0 decreases, \bar{p}_h decreases as well. If $p_m < p_D < \frac{r}{r+\gamma}$ and W_D^0 is sufficiently close to zero, then the unique equilibrium features no manipulation.*

We now briefly discuss concrete applications of these results and their relation to the existing literature.

Board Entrenchment Board members oversee the actions of a firm’s CEO. As The Economist put it “Their job is to police the relationship between shareholders who own companies and managers who run them.”²⁷ Board members themselves face career concerns, since being perceived as competent allows them to obtain additional board seats.²⁸ Director entrenchment is a significant concern for regulators, investors, and companies (see e.g. Bacon and Brown (1973), Vance (1983), Hermalin and Weisbach (1988), Vafeas (2003), Bonini et al. (2017), and Huang and Hilary (2018)). The literature has recognized that the skills of long-serving directors may become outdated, which renders them ineffective. To prevent this, regulators in the UK and France have imposed term limits on directors.

Our analysis underpins this reasoning. Although the monitor can exert effort to keep her skills current, she fails to do so in equilibrium. As a result, the monitor becomes ineffective over time. Imposing a term limit is effective, partially because it provides incentives for the monitor and partially because it reduces the agent’s incentives to manipulate. The agent’s learning motive is new and, to our knowledge, missing from the literature on boards.

Managerial Turnover Managers supervise their direct reports. A large literature has identified the threat of turnover as an important incentive device.²⁹ Absent turnover, however, managers may become entrenched and cease being effective (see e.g. Shleifer and Vishny (1989), Zwiebel (1996), and Bebchuk and Fried (2006)). Our results establish a new channel for turnover being beneficial. Independently of the manager’s incentives to exert effort, turnover lowers the agent’s value from testing the manager’s ability. To the best of our knowledge, this channel has been absent from the turnover literature.

Auditor Rotation CEOs face an incentive to manipulate financial statements to boost the stock price or increase their own compensation. In turn, auditors monitor the integrity of the financial reporting process. Auditor reputation is a key asset that determines the auditor’s demand (see e.g., Firth (1990); Gipper et al. (2017)). The dynamics of auditor incentives and the need to rotate monitors is a key regulatory issue³⁰ and longer serving

²⁷See “Replacing the board”, August 16, 2014.

²⁸See e.g., “Should a Board Have a Reputation?”, Harvard Law School Forum on Corporate Governance, August 9, 2018.

²⁹See e.g. Spear and Wang (2005), Clementi and Hopenhayn (2006), DeMarzo and Sannikov (2006), DeMarzo and Fishman (2007), Biais et al. (2007), He (2009), Garrett and Pavan (2012), and DeMarzo and Sannikov (2016).

³⁰“It was the year of the first X-ray, the first fatal car accident and the premiere of La Bohème. And 1896 was also the year that Barclays, a British bank, chose an ancestor of PwC as its auditor, a relationship

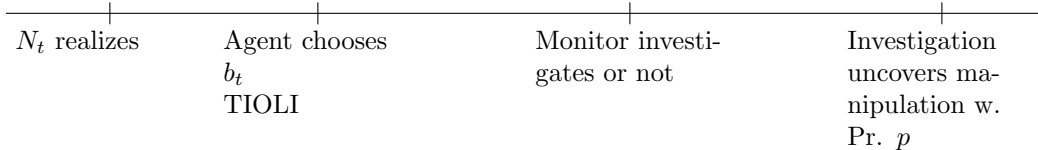


Figure 5: Timeline with bribes on a small interval $[t, t + h]$.

auditors appear to be less effective (see [Singer and Zhang \(2017\)](#)). In the US, the Sarbanes-Oxley Act mandates rotating audit partners after five years. Similarly, the European Union has mandated rotating audit firms after ten years. This is consistent with our results.

5 Bribes

Detection may cause a large loss to the agent K , relative to the reward obtained by the monitor R . This fact may induce the agent to bribe the monitor, as a renegotiation device, to avoid paying the penalty K . To study this possibility, we consider the following extension.

The agent and monitor observe when the manipulation shock N_t arrives (but the “public” does not observe the realization). We interpret this realization as news of potential manipulation or that some evidence is potentially available to incriminate the agent. Upon seeing the signal, the monitor can decide whether to investigate or not. The investigation is successful only if the monitor is good, with probability p_t . Without investigation, the monitor finds nothing. Thus, so far, the framework is the same as in our main model.

Here is the key change. Upon observing N_t , the agent can choose whether to offer a bribe b_t to the monitor, in exchange for the monitor “looking the other way”. (Formally, whether the monitor investigates is observable by the agent and the bribe can be made contingent on the decision to investigate.) For simplicity, we assume that the bribe is a take-it-or-leave-it offer. A heuristic timeline is given in Figure 5.

When the monitor receives the bribe, he does not know whether her investigation will be successful, since she does not know her type. Upon accepting the bribe, the change in the monitor’s value is simply b_t , since the belief stays the same, i.e. $p_t = p_{t-}$. If the monitor rejects the bribe and investigate, her expected payoff is $p(R - V(p))$, since the investigation succeeds only if the monitor is good. Similarly, the agent’s value changes by $-b_t$ if the monitor accepts the bribe and by $-p(K + W(p))$ if she declines the bribe. Thus,

unbroken to this day. Fidelity is the norm in auditing. GE, Procter & Gamble and Dow Chemical have also clocked up centuries with their auditors. The average tenure for an auditor of a British FTSE 100 company is 48 years. Two-thirds of Germany’s DAX 30 have had their auditors for over 20 years.” Excerpt from “Musical Chairs,” *The Economist*, September 2011.

the optimal bribe (the minimal bribe the monitor will accept) satisfies

$$b_t = p_t (R - V(p_t)).$$

On the other hand, offering a bribe is optimal to the agent whenever

$$b_t \leq p_t (K + W(p_t)).$$

Taken together, these conditions imply that the monitor is bribed when

$$R - K \leq V(p_t) + W(p_t). \tag{8}$$

The RHS is the joint value to monitor and agent from continuing the relationship at belief p_t , while the LHS is the joint value from investigating. Given that the agent’s offer is TIOLI, it is not surprising that the decision whether to make a bribe ends up maximizing the “social value” between monitor and agent. As we said before, bribes here play the role of a renegotiation device.

Given Equation (8), if the joint value $V + W$ is increasing in the belief, then bribes will be accepted only when the belief is high. Conversely, if the joint value is decreasing, monitors with low perceived ability will be bribed. Proposition 12 confirms that the latter is the case.

The mechanic behind this result is as follows. Below p_h , $V(p)$ is increasing in p . Thus, for higher p , the monitor can offer a lower bribe since the monitor is eager to continue. Intuitively, if the belief is high, the monitor is confident that he can catch the agent “next time” if he does not accept the bribe. So the value of uncovering the manipulation, $R - V(p)$, is relatively low. Conversely, if the belief is low, the monitor knows that if he does not accept the bribe, he is unlikely to ever catch the agent again. So he needs a relatively higher bribe to accept (note that p also enters the equation, which makes things a bit more complicated).

However, the agent’s incentives are opposite. If p is high, the agent’s value from continuing the relationship is low. He knows he’s likely going to get caught soon. So bribes are less valuable. When p is low, $W(p)$ is high, so the agent is more willing to offer bribes, but now he has to offer a relatively high one to make the monitor accept. These countervailing forces are summarized in Equation (8).

Proposition 9. *Suppose that c is sufficiently small and that Condition (5) holds. When bribes are possible, equilibria have the same structure as in Proposition 3. If*

$$\frac{B - c}{r} + K < R < \frac{B}{r} + K, \tag{9}$$

the monitor accepts bribes whenever the belief is below a threshold $0 < p_B \leq p_h$. If

$$R < \frac{B - c}{r} + K, \quad (10)$$

the monitor always accepts bribes, while if

$$R > \frac{B}{r} + K, \quad (11)$$

the monitor never accepts bribes. Whenever the monitor accepts bribes, the worst equilibrium features more manipulation than in the case without bribes.

The low c requirement ensures that Condition (5) in Proposition 3 is satisfied, so the monitor exerts effort in equilibrium. The proposition shows that when career concerns are strong (high R) monitors are never bribed. However, the “amount of bribes” is not continuous in R . That is, a small increase in R means that we go from bribes for low reputation monitors to no bribes at all. For intermediate career concerns, only the low reputation monitors accept bribes. Essentially, the value of offering bribes is decreasing when monitor reputation is high. Longer serving monitors become “entrenched” and start accepting bribes. After they’ve been accepting bribes for a while, they also start shirking. Naturally, for low career concerns (small R) the monitor always accept bribes.

6 Organizational Design

In many applications, monitors oversee multiple agents. For example, directors often sit in on multiple boards, credit agencies cover different firms, and managers supervise multiple employees.

We now study whether such an organizational structure is optimal. That is, should the monitor oversee multiple agents or just one? The answer depends on the available information. With *individual punishments*, the monitor can detect and punish each individual agent’s manipulation. With *collective punishments*, all agents are punished symmetrically if the monitor detects manipulation, regardless of who manipulated. We can think of this as a situation where manipulations cannot be attributed to an individual agent but are a collective failure of the team.

The optimal organizational structure harnesses the externalities between agents. With individual punishments, agents free-ride on experimentation, which reduces their willingness to manipulate. Thus, overseeing multiple agents reduces manipulation, compared to our

baseline model.³¹ With collective punishments, however, agents free-ride on punishments, which increases manipulation. Then, having a single monitor and agent is optimal.

The setup is as follows. Each agent $n \in \{1, \dots, N\}$ chooses manipulation $m_n \in [0, 1]$ and has value function W_n . The total manipulation of the team is given by $M = \sum_{n=1}^N m_n$. Throughout, we focus on symmetric monotone-manipulation equilibria which are Markovian in the public belief. That is, each agent n plays a symmetric, monotonically increasing manipulation strategy $m_n(p) = m(p)$.

We set up this extension so that the cooperative solution of the team problem is the same as that of the main model of Section 2.³² Thus, our main model serves as a benchmark under both individual and collective punishments, if there are no coordination frictions among the agents. Specifically, we scale the detection rate by N , so that the new detection rate is λ/N . This means that if all the agents manipulate, then the arrival rate is the same as that in the main model. Then, the monitor's value is given by

$$\begin{aligned} rV(p) = & -ce(p) - \left(\frac{\lambda}{N}p(1-p)M(p) + \gamma(1-e(p)) \right) V'(p) \\ & + \frac{\lambda}{N}pM(p)(R - V(p)). \end{aligned} \quad (12)$$

If for all p , $M(p)/N$ equals the solution of the single-agent model, then the solution to the monitor's HJB equation is also the same and her best effort strategy is the same as well.

Individual Punishments Each agent operates an independent Poisson process with arrival rate $\lambda/N \cdot m_n$. If agent n 's manipulation is detected, then only that agent is punished. Punishments are publicly observable, so that all agents learn that the monitor is good. To ensure that the cooperative solution is the same as the solution in the main model, we scale the punishment to $K \cdot N$. Each agent's individual value function is now

$$\begin{aligned} rW_n(p) = & \max_{m_n \in [0,1]} \left(B - \lambda p K - \frac{\lambda}{N}p(1-p)W'_n(p) - \frac{\lambda}{N}pW_n(p) \right) m_n \\ & - \left(\frac{\lambda}{N}p(1-p)W'_n(p) + \frac{\lambda}{N}pW_n(p) \right) M_{-n}(p) \\ & - \gamma p(1-e(p))W'_n(p). \end{aligned} \quad (13)$$

³¹This is opposite to the encouragement effect found in the experimentation literature, e.g. [Bolton and Harris \(1999\)](#) and [Keller and Rady \(2010\)](#). Our results differ because our benchmarks differ. The experimentation literature compares the non-cooperative equilibrium to the single-agent case. On our setting, the "single-agent case" refers to our baseline model, and we scale the model parameters so that the cooperative solution with N agents is the same as the one in [Proposition 3](#). This allows us to have a common and, we believe, sensible benchmark across the different informational regimes.

³²The cooperative solution is the one which maximizes the average value of each agent. See e.g. [Equation \(42\)](#) below.

Each agent prefers to manipulate whenever

$$\omega_n(p) = B - \lambda p K - \frac{\lambda}{N} p (1-p) W'_n(p) - \frac{\lambda}{N} p W_n(p) \geq 0. \quad (14)$$

Having multiple agents generates free-riding on experimentation. Each individual agent internalizes his own cost of being detected, but does not internalize the benefit his experimentation creates for other agents, by uncovering information about the monitor. Formally, this can be seen by comparing Equation (14) to Equation (2), its analog in the baseline model. With multiple agents, the value of experimentation is scaled by λ/N , while with a single agent, it is scaled by λ . This free-riding is beneficial, because it reduces manipulation.

Proposition 10. *Suppose that $R > \bar{R}$, so that $\omega_{n-}(p_l) > 0$, and that K is sufficiently small. With multiple agents and individual punishments, the worst equilibrium features less manipulation than with a single agent.*

The condition $R > \bar{R}$ is an analog of Condition (5). It guarantees that there is monitor effort in equilibrium. Unlike in Keller and Rady (2010), agents stop experimenting at a cutoff instead of choosing interior experimentation over a region in which they are indifferent. This is driven by the strategic complementarity between effort and manipulation, which is absent from Keller and Rady (2010), and by our assumption that K is small.

Collective punishments Suppose that the monitor punishes all agents equally whenever she detects manipulation, either because she can only see if *some* agent has manipulated, but not which one, or because she cannot target individual agents with punishments.

Now, there is free-riding on punishments, because agents do not internalize the cost of being detected. This effect increases manipulation compared to the baseline model.

Specifically, each agent's individual value is

$$\begin{aligned} rW_n(p) &= \max_{m_n \in [0,1]} \left(B - \frac{\lambda}{N} p K - \frac{\lambda}{N} p (1-p) W'_n(p) - \frac{\lambda}{N} p W_n(p) \right) m_n \\ &\quad - \left(\frac{\lambda}{N} p K + \frac{\lambda}{N} p (1-p) W'_n(p) + \frac{\lambda}{N} p W_n(p) \right) M_{-n}(p) \\ &\quad - \gamma p (1-e(p)) W'_n(p). \end{aligned} \quad (15)$$

Each agent prefers to manipulate whenever

$$\omega_n(p) = B - \frac{\lambda}{N} p K - \frac{\lambda}{N} p ((1-p) W'_n(p) + W_n(p)) \geq 0. \quad (16)$$

Comparing this expression with Equation (2), we see that each agent's expected punish-

ment is given by $\lambda pK/N$, as opposed to λpK in the baseline model. This is a consequence of the free-riding effect we have described.

Just as with individual punishments, agents have an incentive to free-ride on experimentation. However, free-riding on punishments is stronger and manipulation increases overall.

Proposition 11. *Suppose that Condition (5) holds, that $B < \frac{\lambda}{N}K$, and that K is sufficiently small. Then, the worst equilibrium with multiple agents and collective punishments features more manipulation than the single-agent case.*

Busy Directors vs. Overwhelmed Managers Our results on the organization of monitoring yield concrete implications. A prominent literature studies “busy directors,” who sit on multiple boards, and, as a result, may be less effective at monitoring each individual firm (see e.g. Ferris et al. (2003), Fich and Shivdasani (2006), and Field et al. (2013)). Our individual punishments case applies to this setting, because directors can attribute failures to a particular firm. Contrary to common wisdom, Proposition 10 shows that busy directors can be more effective.

By contrast, managers inside a firm often cannot determine which employee is responsible for failures. This setting corresponds to our collective punishments case and Proposition 11 shows that managers who oversee multiple employees are less effective. Indeed, a long line of literature, going back to Williamson (1967) and Calvo and Wellisz (1978), highlights managers’ “loss of control” as a limit to firm size.³³ Our results expand on this literature by incorporating learning about the manager’s effectiveness.

7 Robustness and Extensions

7.1 General Payoffs

Monitor Rewards So far we have assumed the monitor reward is independent of the belief. Suppose now that the monitor faces a market, which reward the monitor with $R(p)$ once she detects manipulation. We assume that $R(p)$ is increasing and differentiable. Below, we show that there is an equilibrium with the same structure as that in the baseline model, which is characterized by two thresholds p_l and p_h .

We can also study how the strength of career concerns, as measured by the slope of $R(\cdot)$, affects monitor effort and manipulation. We show that as rewards become more sensitive to the belief, the monitor exerts more effort and the agent manipulates less.

³³See also Qian (1994) and Faure-Grimaud et al. (2003).

Consider two alternative reward functions $\hat{R}(p), R(p)$ where the former is steeper and assume these functions cross at a single point \hat{p} . Let p_l denote the lower threshold of the monitoring region under $R(p)$. Also, denote by \bar{p}_h and \hat{p}_h the upper thresholds, in the worst equilibrium, under $R(p)$ and $\hat{R}(p)$ respectively. The following result can be established.

Proposition 12. *Suppose that r and K are sufficiently small and that $R(p)$ and $\hat{R}(p)$ are sufficiently large. Then, all equilibria have the same structure as in Proposition 3. If $p_l < \hat{p}$ is sufficiently large, then $\hat{p}_h < \bar{p}_h$.*

The logic of this result is similar to that of deadlines. If $R(p)$ is steeper, reaching lower beliefs leads to lower expected rewards. This provides incentives to exert more effort, so that p_l decreases. In response, the agent manipulates less.

The assumption that $R(p)$ is large is the analog of Condition (5). It ensures that the monitor exerts effort in equilibrium. We assume that the monitor is sufficiently patient, i.e. r is small, to guarantee that the monitor's incentive to exert effort is increasing in the belief on the region $[p_l, p_h]$.³⁴

Flow Payoffs In many applications, monitor flow payoffs depend on reputation as well. For example, auditor fees arguably depend on their reputation (see Firth (1990)). We now show that whenever the flow payoffs are sufficiently sensitive to reputation, the upper shirking region may disappear.

Specifically, suppose that the monitor receives a wage $w(p)$, which is strictly increasing and continuously differentiable in p , in addition to the reward $R(p)$. Given the non-linearity of $w(p)$, we make the following assumptions to ensure tractability. We assume that $w'(p)/w(p) > 1/p$ and that $\lim_{p \downarrow 0} w(p)/p$ is bounded. Additionally, we assume that r and c are sufficiently small.

The first assumption, together with the monitor being patient (r small), guarantees that the monitor's value is increasing in the belief. The second one is a technical condition which rules out a singularity at $p = 0$. The assumption that c is small plays a similar role as Condition (5). It ensures that the monitor exerts effort in equilibrium.

³⁴Intuitively, a relatively impatient monitor puts a high weight on her expected reward $\lambda p R(p)$ for beliefs which are close by. Since $R(p)$ is generally nonlinear, the monitor's incentive to exert effort, $\gamma p V'(p)$, may then be non-monotone. By contrast, a patient monitor knows that without effort, she will eventually reach very low beliefs, at which the expected rewards $\lambda p R(p)$ vanish. Also, we need p_l to be sufficiently large, because under $\hat{R}(p)$ the monitor's incentives are not uniformly stronger, compared to $R(p)$. For low beliefs, $\hat{R}(p)$ is below $R(p)$ and the monitor may have less incentive to exert effort. However, there exists a region of beliefs at which $\hat{R}(p) < R(p)$, i.e., the rewards are lower, but the incentives are stronger. If p_l lies in this region, then the monitor exerts more effort under $\hat{R}(p)$.

Proposition 13. *Suppose that the assumptions above hold. If*

$$w'(p) < \frac{rc}{\gamma p} \text{ and } w(p) < c,$$

then all equilibria have the same structure as in Proposition 3. If instead

$$w'(p) \geq \frac{rc}{\gamma p} \text{ or } w(p) > c,$$

then there exist equilibria for which $m(p) = 0$ and $e(p) = 1$ for $p \geq p_h$.

Thus, whenever the monitor's flow payoff is sufficiently sensitive to the belief, the upper shirking region disappears. Intuitively, when $w(p)$ is sufficiently steep, the monitor prefers to keep the belief high, irrespectively of whether the agent manipulates. Similarly, whenever $w(p)$ is sufficiently large the monitor prefers to exert effort even when the agent does not manipulate.

7.2 Effort Improves Monitor Type

In our main model, we assumed that the monitor's type can only deteriorate, but never improve. We now relax this assumption and provide conditions for which our results go through.

As before, suppose that if the monitor is good, she turns bad at rate $\gamma(1 - e_t)$. Additionally, when the monitor is bad, she turns good with rate ϕe_t . Thus, monitor effort can now improve her type.

In this case, absent detection, the belief evolves as

$$\frac{dp_t}{dt} = -\lambda p_t(1 - p_t)m_t + \phi(1 - p_t)e_t - \gamma p_t(1 - e_t).$$

We have the following result.

Proposition 14. *For ϕ sufficiently small, Proposition 3 characterizes all equilibria.*

7.3 Good and Bad News

So far, we have assumed that detecting manipulation is good news about the monitor's ability. We now relax this assumption and provide conditions for which the equilibrium in Proposition 3 survives.

Suppose that if the monitor is bad, manipulation causes a public loss (e.g. a loan default or a scandal), which arrives with Poisson rate λ_B . That is, the model now features both

good and bad news. On path, the belief evolves as

$$\frac{dp_t}{dt} = -(\lambda - \lambda_B) p_t (1 - p_t) m_t - \gamma p_t (1 - e_t).$$

For simplicity, assume that when the loss realizes the project is terminated. The monitor receives no reward and the agent bears no punishment.³⁵ Intuitively, the monitor must collect evidence that the agent manipulates to be able to punish him and the bad monitor fails to do so. A bad shock destroys the firm and the agent escapes punishment.

Proposition 15. *For γ_B sufficiently small, Proposition 3 characterizes all equilibria.*

7.4 Relationship Continues After Detection

If the relationship continues after detection, the analysis becomes significantly more complicated. After detection, everyone learns that the monitor is good, i.e. $p = 1$, and the monitor's and agent's continuation values are $V(1)$ and $W(1)$. These values are endogenous and depend on the values and strategies for other beliefs. Thus, we resort to fixed-point arguments.

While we cannot characterize the full set of equilibria in this case, we can show that the equilibrium of Proposition 3 survives.

Proposition 16. *Under the assumptions of Proposition 3, there exists an equilibrium such that $m(p) = 1$ if $p < p_h$ and $m(p) = 0$ if $p \geq p_h$ and $e(p) = 1$ if $p \in [p_l, p_h)$ and $e(p) = 0$ otherwise.*

7.5 Perfect Bad News

We now contrast our model with the perfect bad news case (e.g. Keller and Rady (2015)). We show that our main results survive: the agent's experimentation motive still encourages manipulation and term limits, this time for the agent, are still beneficial. Interestingly, the multiplicity we have found in Proposition 3 disappears.

Now, if the monitor is bad, a signal realizes with arrival rate λ . If the monitor is good, there is no realization. To make monitoring and manipulation meaningful in this setting, and to keep the model tractable, we make the following changes to our assumptions.

First, the monitor receives a flow reward R before the first occurrence of bad news and nothing afterwards. This means that avoiding bad news serves as an incentive for the monitor. Second, the agent now has a flow cost of manipulation, κm_t , and he receives a

³⁵That is, the agent "gets away" with manipulation.

private benefit B only if the bad news realizes. Third, after bad news realizes, the monitor is removed and the agent may continue manipulating.

Thus, we can interpret the bad news as the agent successfully stealing from the firm while concealing his identity, in which case the monitor gets fired. We can easily change the continuation game. For example, we could assume that the agent is matched with a new firm with a less effective monitor.

Finally, we reverse the monitor's human capital technology. If the monitor is bad, she becomes good at rate γ if she exerts effort. Otherwise, she stays good. This assumption, together with having perfect bad news, implies that the belief p_t is monotonically increasing on path, which is the opposite of our main model:

$$\frac{dp_t}{dt} = \lambda p_t (1 - p_t) m_t - p_t m_t dN_t + \gamma (1 - p_t) e_t.$$

The monitor's and agent's HJB equations are now

$$\begin{aligned} rV(p) &= R - ce(p) + (\lambda p(1-p)m(p) + \gamma(1-p)e(p))V'(p) \\ &\quad - \lambda(1-p)m(p)V(p) \end{aligned}$$

and

$$\begin{aligned} rW(p) &= -\kappa m(p) + (\lambda p(1-p)m(p) + \gamma(1-p)e(p))W'(p) \\ &\quad + \lambda(1-p)m(p)(B + W(0) - W(p)). \end{aligned}$$

In the myopic benchmark, the agent manipulates whenever the belief is sufficiently low, just as in our baseline model. We assume

$$B\lambda > \kappa,$$

which is the analog of Assumption (1) and which ensures that the myopic agent prefers to manipulate when the belief is low.³⁶ Then, a myopic agent manipulates whenever

$$p \leq p_m = \frac{\lambda B - \kappa}{\lambda B}.$$

The following Lemma, which is the analog of Proposition 1, shows that a forward-looking agent still has an experimentation motive, which leads him to manipulate more than a myopic

³⁶His value is

$$W(0) = \frac{\lambda B - \kappa}{r}.$$

Without this assumption, the agent would never manipulate.

agent.

Lemma 17. *If the monitor never exerts effort, the agent manipulates for $p \leq p_{nm}$ and does not manipulate for $p > p_{nm}$. If the monitor always exerts effort, the agent manipulates whenever $p \leq p_{mon}$. We have $p_m < p_{mon} < p_{nm}$.*

Unlike in our baseline model, when the monitor exerts effort, the agent manipulates *less*. Intuitively, when the monitor exerts effort, the belief increases faster. Being at a lower belief, where it is likely that bad news realizes and the agent gets the reward, is less valuable, because the time spent there is shorter. Thus, $W'(p)$ decreases and so does the agent’s value of experimentation. In our main model, the effect was opposite. There, if the monitor exerts effort, the belief changes less over time, which increases the agent’s incentive to manipulate.

Thus, the complementarity between monitor effort and manipulation disappears and the equilibrium is unique. As we show in the Proposition below, our result that term limits lower manipulation survives.

Proposition 18. *For r sufficiently small, there exists a unique MPE in which the agent manipulates if and only if $p \leq p_h$ and the monitor exerts effort if and only if $p \leq p_l \leq p_h$. A term limit after bad news has realized lowers p_h .*

A contingent term limit requires replacing the agent, e.g. via dissolving the firm, at some time T after bad news has been observed. This lowers the agent’s payoff from manipulating and decreases his incentive to manipulate.

The assumption that r is small helps ensure that the monitor’s incentive to exert effort is monotone. Intuitively, as p increases, effort becomes less effective, because it becomes less likely that the monitor is bad. A sufficiently patient monitor, however, will still want to exert effort decrease the chance that the bad news realizes.

Thus, in summary, (1) “manipulation as experimentation” survives, (2) term limits (sumtably modified) survive, and (3) the equilibrium is unique.

8 Conclusion

In this paper we develop a theory of monitoring and manipulation. We study a game between two forward-looking players, a monitor and an agent, when the monitor’s detection ability is uncertain. The agent receives private benefits from manipulating but bears a penalty if detected, in which case the monitor receives a reward. We analyze the effect that uncertainty about enforcement plays on the dynamics of monitoring and malfeasance.

We conclude with some thoughts about the practical relevance and limitations of our analysis. We strongly believe that the mechanism behind manipulations that we highlight in this paper is empirically relevant. In practice, manipulation provides an experimental benefit to manipulating agents, who are always uncertain about the level of enforcement they face (e.g., when they commit white collar crime, such as financial fraud, insider trading, or tax-avoidance). Agents learn about the level of enforcement they face precisely by engaging in manipulation. Such learning is valuable and may become a strong incentive to engage in manipulations, even when manipulation would not take place in the absence of such uncertainty. Our results thus suggest that regulators should try to reduce uncertainty about enforcement as a means of mitigating agents' propensity to manipulate, even if this leads some times to more manipulation (e.g., when enforcement is revealed to be weak). In other words, from a policy perspective, environments that feature more enforcement uncertainty are more prone to manipulation. Hence, policies aimed at reducing such uncertainty could play an important role in the battle against malfeasance. This is an interesting avenue for future research.

Studying the experimental nature of manipulation may help explain key facts about the dynamics of manipulation and monitoring. For example, manipulation is often characterized as a persistent habit that escalates over manipulating agents' life cycle.³⁷ Our model suggests that such persistence may arise as a result of the agent being uncertain about the monitor's effectiveness. Indeed, our model predicts that after the agent successfully manipulates for the first time, he never stops, until eventually detected. This persistent propensity to manipulate arises in our model not because of habit formation but because of learning. Second, our results suggest that the incentive to manipulate grows stronger over time (unless the agent is detected). If manipulation were a continuous choice, then it would start small and progressively snowball into something large, since successful past manipulation would lead the agent to grow (over)confident that the monitor is unable to detect his manipulation. Thus, as time goes by, manipulation would become more and more appealing to the agent. An interesting avenue for future research would be to explore, theoretically and empirically, whether experimentation incentives are able to explain a "power law" in the distribution of frauds.

³⁷This process is eloquently described by Bernard Madoff: "you know what happens is, it starts out with you taking a little bit, maybe a few hundred, a few thousand. You get comfortable with that, and before you know it, it snowballs into something big."

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A Proofs

A.1 Preliminaries

We begin with laying out definitions. On any nonempty interval of beliefs on which there is no manipulation or monitoring, i.e. $m(p) = e(p) = 0$, the monitor's HJB equation satisfies

$$rV = -\gamma p V', \quad (17)$$

while the agent's HJB equation satisfies

$$rW = -p\gamma W'. \quad (18)$$

For manipulation to be suboptimal, it must be the case that

$$\omega(p) = B - \lambda p(1-p)W'(p) - \lambda p(K + W(p)) \leq 0, \quad (19)$$

i.e., the value of manipulation to the agent is negative. Similarly, shirking is optimal for the monitor whenever

$$V'(p) \leq \frac{c}{\gamma p}. \quad (20)$$

On any nonempty interval on which the agent manipulates and the monitor shirks, i.e. $m(p) = 1$ and $e(p) = 0$, the monitor's value function satisfies

$$rV(p) = -(\lambda p(1-p) + \gamma p)V'(p) + \lambda p(R - V(p)), \quad (21)$$

while the agent's value function satisfies

$$rW(p) = B - (\lambda p(1-p) + \gamma p)W'(p) - \lambda p(K + W(p)). \quad (22)$$

Finally, on any nonempty interval on which the agent manipulates and the monitor exerts effort, i.e., $m(p) = e(p) = 1$, the monitor's value function solves

$$rV(p) = -c - \lambda p(1-p)V'(p) + \lambda p(R - V(p)) \quad (23)$$

and the agent's value solves

$$rW(p) = B - \lambda p(1-p)W'(p) - \lambda p(K + W(p)). \quad (24)$$

We next prove Lemma 2, which we restate below.

Lemma 19. *In any MPE, there is no nonempty interval (\underline{p}, \bar{p}) such that $m(p) = 0$ and $e(p) = 1$ for all $p \in (\underline{p}, \bar{p})$.*

Proof. Suppose that such an interval exists. On that interval, the monitor's HJB equation solves

$$rV(p) = -c$$

and thus $V'(p) = 0$. But then exerting effort cannot be optimal, since the condition $\gamma p V'(p) \geq c$ does not hold. ■

Lemma 2 implies that in any equilibrium, the HJB equations for monitor and agent (equations (3) and (4)) are non-degenerate at all $p > 0$. That is, in any equilibrium, $e(p) > 0$ and/or $m(p) > 0$ for all $p > 0$. Since we assumed that strategies are piecewise Lipschitz continuous in p , standard existence and uniqueness arguments apply to the HJB equations. Specifically, the monitor's HJB equation admits a unique solution for any candidate equilibrium strategy of the agent. This solution is continuous everywhere (in p) and continuously differentiable almost everywhere. The analog is true for the agent's HJB equation. See Davis (1993), Th. 41.4, p. 138. Throughout the proofs, we will make extensive use of value matching conditions, which exploit the continuity of the monitor's and agent's HJB equations.

Given the above properties of the HJB equations, standard verification arguments apply, which establish that the solution to the monitor's and agent's respective HJB equations equal their optimal values. See Davis (1993), Th. 42.8, p. 141.

In the proofs below, we will often consider different cases for possible equilibria. The Lemma above guarantees that we never have to consider the case where the monitor exerts effort but the agent does not manipulate. In the following, we skip this case without mention.

We first provide closed-form solutions to the monitor's and agent's HJB equations on regions where monitor effort and manipulation are constant. In the Lemma below, C_0^A and C_0^M denote generic constants. They differ from one equation to the other.

Lemma 20. *Consider a nonempty interval $(\underline{p}, \bar{p}) \subset [0, 1]$. The monitor's and agent's HJB equations have the following general solutions. If $m(p) = e(p) = 0$ for all $p \in (\underline{p}, \bar{p})$, then*

$$V(p) = C_0^M p^{-\frac{r}{\gamma}} \quad (25)$$

and

$$W(p) = C_0^A p^{-\frac{r}{\gamma}} \quad (26)$$

for two constants C_0^A and C_0^M . If $m(p) = e(p) = 1$ for all $p \in (\underline{p}, \bar{p})$, then

$$V(p) = \lambda p \frac{c + rR}{r(r + \lambda)} - \frac{c}{r} + C_0^M \frac{(1-p)^{\frac{r+\lambda}{\lambda}}}{p^{\frac{r}{\lambda}}} \quad (27)$$

and

$$W(p) = \frac{B}{r} - \lambda p \frac{B + rK}{r(r + \lambda)} + C_0^A \frac{(1-p)^{\frac{r+\lambda}{\lambda}}}{p^{\frac{r}{\lambda}}}. \quad (28)$$

If $m(p) = 1$ and $e(p) = 0$ for all $p \in (\underline{p}, \bar{p})$, then

$$V(p) = \frac{\lambda p R}{r + \lambda + \gamma} + C_0^M \frac{(\gamma + \lambda(1-p))^{\frac{r+\lambda+\gamma}{\lambda+\gamma}}}{p^{\frac{r}{\lambda+\gamma}}} \quad (29)$$

and

$$W(p) = \frac{B}{r} - \lambda p \frac{B + rK}{r(r + \lambda + \gamma)} + C_0^A \frac{(\gamma + \lambda(1-p))^{\frac{r+\lambda+\gamma}{\lambda+\gamma}}}{p^{\frac{r}{\lambda+\gamma}}}. \quad (30)$$

The monitor's and agent's HJB equations are linear first-order ODEs. General solutions to this class of equations can be found in [Polyanin and Zaitsev \(2002\)](#), Section 0.1.2-5. The expressions in the Lemma then follow after some algebra.³⁸

Finally, throughout the proofs we will abbreviate $f_+(p) = \lim_{p' \downarrow p} f(p')$ and $f_-(p) = \lim_{p' \uparrow p} f(p')$ for functions $f(p)$. The distinction is necessary because the derivatives of the monitor's and agent's value functions, and therefore their willingness to exert effort and to manipulate, respectively, are discontinuous in p .

A.2 Main Results

A.2.1 Proof of Proposition 1

If the monitor never exerts effort, then the agent's value admits the closed form solution

$$W(p) = \frac{B}{r} - \lambda p \frac{B + rK}{r(r + \lambda + \gamma)}$$

on the region $[0, p_{nm})$, which follows from Equation (30) and the boundary condition $W(0) = B/r$. This yields

$$\omega(p) = B - \lambda p \frac{r + \gamma}{r(r + \lambda + \gamma)} (B + rK). \quad (31)$$

We have $\omega(0) > 0 > \omega(1)$ and $\omega(p)$ is decreasing. This establishes the existence of a point $p_{nm} \in (0, 1)$, so that manipulating is optimal to the left of p_{nm} . To the right of p_{nm} , the agent's value satisfies the HJB Equation (18). Using this equation together with the HJB Equation (22), which holds to the left of p_{nm} and the value matching condition, implies that the agent's value satisfies smooth pasting at p_{nm} . Since on $[0, p_{nm})$, $\omega(p) = rW(p) + \gamma pW'(p) > 0$ and $W'(p) < 0$, we have $W(p) > 0$ for $p \in [0, p_{nm})$ and in particular $W(p_{nm}) > 0$. Since the agent's value satisfies the closed form solution in Equation (18) to the right of p_{nm} , we have $W'(p_{nm}) = -\frac{r}{\gamma p} W(p_{nm}) < 0$. Using Equation (18), we then have for $p > p_{nm}$,

$$\omega'(p) = \frac{\lambda r}{\gamma} (1 - p) W'(p) - \lambda (K + W(p)) < 0.$$

Thus, not manipulating is optimal to the right of p_{nm} .

To derive whether p_{nm} exceeds p_m , we can plug $B = \lambda pK$ into Equation (31). This implies that $\omega(p_m)$ is positive if and only if

$$(r + \gamma) B - \lambda r K < 0.$$

Thus, when the above inequality holds, $p_{nm} > p_m$ and otherwise $p_{nm} < p_m$.

If the monitor always exerts effort, the agent's value admits the closed form solution

$$W(p) = \frac{B}{r} - \lambda p \frac{B + rK}{r(r + \lambda)},$$

³⁸As we have shown in Lemma 2, the monitor shirks on any region where $m(p) = 0$. Thus, we do not need to consider the case where $m(p) = 0$ and $e(p) = 1$.

which follows from Equation (28) and the boundary condition $W(0) = B/r$. Since the agent's value satisfies the HJB Equation (24), we have $\omega(p) = rW(p)$, so that

$$\omega(p) = B - \lambda p \frac{B + rK}{r + \lambda}. \quad (32)$$

We can thus immediately see that $\omega(p)$ is decreasing and that $\omega(0) > 0 > \omega(1)$. This establishes existence of $p_{mon} \in (0, 1)$. That no manipulation is optimal to the right of p_{mon} follows from a similar argument as in the previous case. To show that $p_{mon} > p_m$, we plug $B = \lambda p K$ into the closed form of the agent's value. Using the condition $B < \lambda K$ and some algebra, we can then confirm that $\omega(p_m) > 0$.

Finally, to show that $p_{nm} < p_{mon}$, we can subtract Equation (32) from Equation (31). The two equations are equal at $p = 0$. For all $p > 0$, the difference is negative.

A.2.2 Proof of Proposition 3

The proof of Proposition 3 proceeds via a sequence of Lemmas. Because of Lemma 2, the belief has strictly negative drift. Because of this, the monitor's and agent's values at any given belief p are independent of the strategies played for all $p' > p$. This allows us to use a backward-induction type argument. We start constructing equilibria with the region $[0, p_l)$ and then characterize the adjacent region $[p_l, p_h)$. Finally, we construct the region $[p_h, 1]$.

Intuitively, we prove that on $[0, p_l)$, the incentives of the monitor to exert effort are increasing in the belief. We then define p_l as the belief at which the monitor is indifferent between shirking or not. On $[p_l, p_h)$, the monitor's incentives are still increasing in p , whereas the agent's incentive to manipulate is decreasing. The belief \bar{p}_h is the point at which the agent is indifferent between manipulating or not. To the right of \bar{p}_h , not manipulating and not exerting effort is optimal.

We start with characterizing the region $[0, p_l)$.

Lemma 21. *On $[0, p_l)$, the monitor's value is given by*

$$V(p) = \frac{\lambda p R}{r + \lambda + \gamma} \quad (33)$$

and the agent's value is given by

$$W(p) = \frac{B}{r} - \lambda p \frac{B + rK}{r(r + \lambda + \gamma)}. \quad (34)$$

The interval $[0, p_l)$ is part of an equilibrium in Proposition 3 only if

$$\frac{\gamma \lambda p_l R}{r + \lambda + \gamma} = c \quad (35)$$

and

$$p_l < \frac{B(r + \lambda + \gamma)}{(B + rK)\lambda(1 + \frac{\gamma}{r})}.$$

The last two equations are equivalent to Condition (5) in the statement of Proposition 3.

Proof. The expressions for the monitor's and agent's value functions follow from Lemma 20. The boundary conditions $W(0) = \frac{B}{r}$ and $V(0) = 0$ imply $C_0^A = C_0^M = 0$ in Equations (30) and (29), respectively.

Given the closed form in Equation (34), we have

$$\omega(p) = B - \lambda p \frac{(r + \gamma)(B + rK)}{r(r + \lambda + \gamma)}, \quad (36)$$

which is decreasing in p .

We now establish the necessary conditions for p_l in the statement of the Lemma. Suppose by way of contradiction that p_l is such that $\gamma p_l V'_-(p_l) < c$ and $\omega_-(p_l) > 0$. Suppose the equilibrium switches to $m(p) = 1$ and $e(p) = 1$ to the right of p_l . Then, using the monitor's HJB Equations (21) and (27) and the value matching condition at p_l , we can derive the following expression

$$\gamma p_l (V'_+(p_l) - V'_-(p_l)) = \gamma p_l V'_-(p_l) - c < 0.$$

Thus, $\gamma p_l V'_+(p_l) < c$, so that $e(p) = 1$ cannot be optimal to the right of p_l .

Suppose instead the equilibrium switches to $m(p) = e(p) = 0$. Then, using the agent's HJB Equations (22) and (18) and the value matching condition at p_l , we can similarly derive

$$\gamma p_l (W'_-(p_l) - W'_+(p_l)) = \omega_-(p_l) > 0.$$

Thus, $\omega_+(p_l) > \omega_-(p_l) > 0$, so $m(p) = 0$ cannot be optimal to the right of p_l , which is another contradiction. Therefore, we cannot have $\gamma p_l V'_-(p_l) < c$ and $\omega_-(p_l) > 0$.

Suppose now that $\gamma p_l V'_-(p_l) > c$. Then, there must exist a region left of p_l where the monitor does not exert effort, but it is optimal for her to do so. Thus, such a p_l cannot be part of an equilibrium. Similarly, if p_l is such that $\omega_-(p_l) < 0$, then the agent manipulates at some $p < p_l$ at which $\omega(p) < 0$, which is not optimal.

Thus, we have the following necessary conditions: (1) $\gamma p_l V'_-(p_l) = c$ when $\omega_-(p_l) > 0$, (2) $\gamma p_l V'_-(p_l) \leq c$, and (3) $\omega_-(p_l) \geq 0$.

We now consider the case when $\omega_-(p_l) = 0$ and $\gamma p_l V'_-(p_l) < c$. That is, at p_l , the monitor does not exert effort, but the agent weakly prefers to stop manipulating.

Suppose that right of p_l , the equilibrium features $m(p) = e(p) = 0$. Then, using again the agent's HJB Equations (22) and (18) and the value matching condition at p_l , we can show that $W'_-(p_l) = W'_+(p_l)$, i.e. smooth pasting at p_l holds for the agent. Right of p_l , the agent's value is given by Equation (18) and the agent prefers to not manipulate. This case does not have the same structure as the equilibrium in Proposition 3 which features a region with $m(p) = e(p) = 1$ to the right of p_l . Thus, a necessary condition for p_l to be part of the conjectured equilibrium is $\gamma p_l V'_-(p_l) = c$ and $\omega_-(p_l) > 0$.³⁹

Plugging in Equation (33) then yields condition (35). Using Equation (34) and some algebra, we can see that $\omega_-(p_l) > 0$ is equivalent to

$$p_l < \frac{r(r + \lambda + \gamma)B}{\lambda(r + \gamma)(B + rK)}. \quad (37)$$

³⁹In other words, the conditions $\gamma p_l V'_-(p_l) = c$ and $\omega_-(p_l) > 0$ rule out (among others) the case when there is no monitoring and manipulation to the right of p_l .

Note that this condition is equivalent to Equation (5) in Proposition 3. This concludes the proof. ■

We now construct the region $[p_l, p_h)$, on which exerting effort and manipulating are optimal. To do so, we must characterize the jumps in the agent's incentives at p_l and we must verify that to the right of p_l , the agent indeed prefers to manipulate and that the monitor prefers to exert effort.

Lemma 22. *If Equation (5) holds, then there exists a nonempty interval $[p_l, p_h)$, such that $m(p) = 1$ and $e(p) = 1$ are optimal for all $p \in [p_l, p_h)$.*

Proof. We first show that it is optimal for the agent to manipulate. Using the agent's HJB Equations (30) and (28) and the value matching condition at p_l yields

$$\begin{aligned} W'_+(p_l) &= \frac{B - \lambda p_l K - (r + \lambda p_l) W(p_l)}{\lambda p_l (1 - p_l)} \\ &< \frac{B - \lambda p_l K - (r + \lambda p_l) W(p_l)}{\lambda p_l (1 - p_l) + \gamma p_l} = W'_-(p_l). \end{aligned}$$

This implies that $\omega_+(p_l) > \omega_-(p_l) > 0$. Since the agent's HJB Equation (24) is linear and has continuous coefficients, the agent's value function is continuously differentiable on any interval $[p_l, p_h)$ on which the monitor's effort stays constant. Thus, $\omega(p)$ is continuous to the right of p_l and there exists a region on which $\omega(p) > 0$, i.e. manipulation is indeed optimal.

At p_l , the monitor's value satisfies $\gamma p_l V'_-(p_l) = c$. Using this condition together with the value matching condition and the monitor's HJB Equations (21) and (24) yields $V'_-(p_l) = V'_+(p_l)$ after some algebra. That is, the monitor's value satisfies smooth pasting at p_l . Since $V'_-(p_l) > 0$ and since $V'(p)$ is continuous on any interval $[p_l, p_h)$ on which the agent's manipulation is constant, there exists a $p_h > p_l$ such that $\gamma p V'(p) > c$ on $[p_l, p_h)$. That is, exerting effort is indeed optimal. ■

The following Lemma implies that the agent's value of manipulation $\omega(p)$ is decreasing in p on $[p_l, p_h)$, while the monitor's value of effort, $\gamma p V'(p) - c$, is increasing.

Lemma 23. *On any interval $[p_l, p_h)$ such that $m(p) = 1$ and $e(p) = 1$ are optimal for all $p \in [p_l, p_h)$, we have $W'(p) < 0$, $V'(p) > 0$, and $V''(p) > 0$.*

Proof. Using the agent's HJB Equation (24), we can calculate the second derivative of the agent's value as

$$W''(p) = \frac{1}{\lambda p (1 - p)} \frac{1}{p} (rW(p) - B - rpW'(p)).$$

If $W'(p)$ is zero, then $W''(p)$ is strictly negative, because $W(p) < B/r$ for all $p > 0$. Thus, $W'(p)$ cannot cross zero from below. Since $W'_+(p_l) < W'_-(p_l) < 0$, this establishes that $W'(p)$ is negative for all $p \in [p_l, p_h)$.

To show that $V''(p) > 0$, we can use the closed-form solution to the monitor's value in Equation (27) together with the closed-form solution on $[0, p_l)$, in Equation (33), the value matching condition at p_l , and the condition $\gamma p_l V'_-(p_l) = c$ to show that the constant C_0^M in Equation (27) is positive. Then, we can directly calculate $V''(p)$ from the closed-form solution, which is strictly positive on $[p_l, p_h)$. Since $V'(p_l) > 0$ this also implies that $V'(p) > 0$ for all $p \in [p_l, p_h)$. ■

We now characterize the equilibria with the least and most manipulation.

Lemma 24. Let $W(p)$ denote the solution to the agent's HJB Equation (24), i.e., when $m(p) = e(p) = 1$, which satisfies the value matching condition at p_l . There exist two unique thresholds \underline{p}_h and \bar{p}_h , such that

$$B + \frac{\lambda r}{\gamma} (1 - \underline{p}_h) W(\underline{p}_h) - \lambda \underline{p}_h (K + W(\underline{p}_h)) = 0 \quad (38)$$

and

$$W(\bar{p}_h) = 0.$$

Moreover, $\underline{p}_h > p_l$, $1 > \bar{p}_h > p_m$, and $\underline{p}_h < \bar{p}_h$. The threshold p_h is part of an equilibrium only if $p_h \in [\underline{p}_h, \bar{p}_h]$. For any such p_h , $m(p) = e(p) = 1$ are optimal on $[p_l, p_h]$.

Proof. Suppose the equilibrium switches to no manipulation and shirking at some $p_h > p_l$, so that $m(p) = e(p) = 0$ are optimal for all $p \geq p_h$. At p_h , we must have $\omega_-(p_h) \geq 0$ and $\omega_+(p_h) \leq 0$, otherwise the agent's strategy is not optimal. Equation (38) defines \underline{p}_h via $\omega_+(\underline{p}_h) = 0$.

We first consider the bound \underline{p}_h . We show that \underline{p}_h exists, is unique, and that we have $\underline{p}_h \in (p_l, 1)$. The argument to establish that $\underline{p}_h < 1$ is as follows. For $p \geq p_l$, the agent's value satisfies the closed form solution in Equation (28). This solution implies that if p approaches one, then $W(p) < 0$. Thus, for $p > p_m$ and p sufficiently large, we have

$$\omega_+(p) = B + \frac{\lambda r}{\gamma} (1 - p) W(p) - \lambda p (K + W(p)) < 0.$$

Thus, if \underline{p}_h exists, it must be strictly below one.

Since $W(p)$ is differentiable on $[p_l, p_h]$, $\omega_+(p)$ is continuous and differentiable. The function $\omega_+(p)$ is strictly decreasing in the belief. To show this, we can plug in the HJB Equation (18) into Equation (38). The derivative of the left-hand side of Equation (38) with respect to p is

$$-\frac{\lambda}{\gamma^2 p} W(p) (r^2 (1 - p) + \gamma^2 p) - \lambda K < 0,$$

which follows after some algebra.

We now use this fact to show that $\omega_+(p_l) > 0$. Suppose by way of contradiction that $\omega_+(p_l) \leq 0$. This implies that there exists an equilibrium where $p_h = p_l$, so that there is manipulation but no monitor effort left of p_l , and no effort and no manipulation right of p_l . Then, the agent's value satisfies Equation (22) left of p_l and (18) right of p_l . These equations, together with the value matching condition, imply that $W'_-(p_l) > W'_+(p_l)$, so that $0 < \omega_-(p_l) < \omega_+(p_l)$, a contradiction. Therefore, we have $\omega_+(p_l) > 0$.

We have shown that $\omega_+(p_l) > 0 > \omega_+(1)$. Since $\omega_+(p)$ is strictly decreasing and continuous, there exists a unique $\underline{p}_h \in (p_l, 1)$, such that $\omega_+(\underline{p}_h) = 0$. This is what we set out to prove.

We now characterize \bar{p}_h . The closed form solution in Equation (28) and Assumption 1 imply that $W(1) < 0$. We have $W(p_l) > 0$ and we have shown in Lemma 23 that $W'(p) < 0$ on $[p_l, p_h]$. Thus, there exists a unique point $\bar{p}_h \in (p_l, 1)$ at which $W(p)$ crosses zero.

That $\bar{p}_h > p_m$ follows from $W(\bar{p}_h) = 0$ and the agent's HJB Equation (24), which implies

$$B - \lambda \bar{p}_h K = \lambda \bar{p}_h (1 - \bar{p}_h) W'_-(\bar{p}_h) < 0.$$

The inequality holds because $W'(p) < 0$ for $p \in [p_l, p_h]$.

Next, we show that $\underline{p}_h < \bar{p}_h$. Since $\bar{p}_h > p_m$ and $W(\bar{p}_h) = 0$, we have

$$\begin{aligned} & B + \frac{\lambda r}{\gamma} (1 - \bar{p}_h) W(\bar{p}_h) - \lambda \bar{p}_h (K + W(\bar{p}_h)) \\ = & B - \lambda \bar{p}_h K < 0. \end{aligned}$$

Because the left-and side of Equation (38) is decreasing, we have $\underline{p}_h < \bar{p}_h$.

We now show that p_h is part of an equilibrium only if $p_h \in [\underline{p}_h, \bar{p}_h]$. First, suppose that $p_h < \underline{p}_h$. By construction, \underline{p}_h is the lowest belief at which $\omega_+(p_h) \leq 0$. Thus, $p_h < \underline{p}_h$ cannot be part of an equilibrium, because then we have $\omega_+(p_h) > 0$, so not manipulating to the right of p_h is suboptimal. Similarly, $p_h > \bar{p}_h$ cannot be part of an equilibrium, because we would have $m(p) = 1$ and $\omega(p) < 0$ for $p \in (\bar{p}_h, p_h)$.⁴⁰

We close by showing that for any $p_h \in [\underline{p}_h, \bar{p}_h]$, $m(p) = e(p) = 1$ are optimal on $[p_l, p_h]$. Since we have shown in Lemma 23 that the monitor's incentive to exert effort is increasing in p on $[p_l, p_h]$, we only need to check that the agent prefers to manipulate. This is straightforward, since $\omega(p) = rW(p) \geq 0$ for all $p \leq p_h \leq \bar{p}_h$. ■

We finish our characterization by showing that there is indeed no manipulation or monitor effort to the right of p_h . Thus, any $p_h \in [\underline{p}_h, \bar{p}_h]$ constitutes an equilibrium.

Lemma 25. *For any $p_h \in [\underline{p}_h, \bar{p}_h]$, the equilibrium features no effort and manipulation on $[p_h, 1]$.*

Proof. We first show that the agent does not prefer to manipulate for $p \geq p_h$. By construction of p_h , we have $\omega_+(p_h) \leq 0$. For $p > p_h$, we have

$$\omega'(p) = -\lambda(1-p)W'(p) - \lambda p(1-p)W''(p) - \lambda(K+W(p)).$$

Taking the closed form for the agent's HJB Equation (26), we can express the second derivative as

$$W''(p) = -\frac{r+\gamma}{\gamma p} W'(p).$$

Thus, we have

$$\omega'(p) = \frac{\lambda r}{\gamma} (1-p)W'(p) - \lambda(K+W(p)).$$

Since $W'(p) \leq 0$, this expression is negative. Thus, $\omega(p) \leq 0$ for all $p > p_h$. Manipulating is not optimal for the agent.

It only remains to show that $\gamma p V'(p)$ stays below c on $[p_h, 1]$, so that exerting effort is not optimal. We have $V(p_l) > 0$ and by Lemma 23, $V(p)$ is increasing on $[p_l, p_h]$. Thus, we have $V(p_h) > 0$. On $[p_h, 1]$, the monitor's value admits the closed-form solution in Equation (25). The value matching condition at p_h and the fact that $V(p_h) > 0$, imply that the constant C_0^M in Equation (25) is positive. Then, Equation (25) implies that $V'(p)$ is negative and therefore $\gamma p V'(p) < 0 < c$ for $p > p_h$. ■

If Condition (5) does not hold, the equilibrium features no monitor effort. Intuitively, R is small and the monitor's incentives are weak. She is only willing to exert effort for very high beliefs, at which the agent does not manipulate anyway.

⁴⁰Recall that $\omega(p) = rW(p)$ on $[p_l, p_h]$.

Lemma 26. *Suppose that*

$$R \leq c \frac{r + \gamma}{r\gamma} \frac{B + rK}{B}. \quad (39)$$

Then, there is a unique equilibrium. The monitor never exerts effort and the agent manipulates whenever $p < p_{nm}$, where p_{nm} is given by Proposition 1.

Proof. We first prove that manipulating below p_{nm} and not exerting effort is an equilibrium. Pick

$$p_{nm} = \frac{r(r + \lambda + \gamma)B}{\lambda(r + \gamma)(B + rK)}.$$

This is the same value as in Proposition 1. Then, $\omega(p_{nm}) = 0$ and $\omega(p) > 0$ for all $p < p_{nm}$, which follows from the closed-form of the agent's value function in Equation (26).

Condition (39) implies that shirking is optimal for the monitor for all $p < p_{nm}$. Suppose that the equilibrium is such that $m(p) = e(p) = 0$ for all $p \geq p_{nm}$. Repeating a similar argument as in the proof of Lemma 25, establishes that not manipulating and shirking is optimal on $[p_{nm}, 1]$. Thus, $e(p) = 0$ for all p and $m(p) = 1$ if $p < p_{nm}$ and $m(p) = 0$ if $p \geq p_{nm}$ constitutes an equilibrium.

We now prove uniqueness. Suppose that the agent stops manipulating for some $\hat{p}_{nm} < p_{nm}$. Then, using the agent's HJB Equations (22) and (18) and the value matching condition at \hat{p}_{nm} yields

$$\gamma \hat{p}_{nm} (W'_-(\hat{p}_{nm}) - W'_+(\hat{p}_{nm})) = \omega_-(\hat{p}_{nm}) > 0$$

and therefore $\omega_+(\hat{p}_{nm}) > \omega_-(\hat{p}_{nm}) > 0$. Thus the agent strictly prefers to manipulate to the right of \hat{p}_{nm} and there can be no such equilibrium. Similarly, we cannot have $\hat{p}_{nm} > p_{nm}$, because then for $p \in (p_{nm}, \hat{p}_{nm})$, we have $\omega(p) < 0$, so manipulating cannot be optimal. ■

A.3 Proofs for Section 4

We prove the results in a different order, since this helps us minimize repetition.

A.3.1 Proof of Proposition 8

To prove the result, we need to consider different cases, depending on whether p_D is larger or smaller than p_l . Throughout, let $V_D(p)$ and $W_D(p)$ denote the monitor and agent's values for $p \geq p_D$ with the deadline at p_D . Since we are not changing the monitor's value at p_D , her incentives to monitor remain the same, as long as the agent manipulates.

Lemma 27. *Suppose that $p_D < p_l$ and that W_D^0 is sufficiently close to $W(p_D)$. Then, the agent manipulates on $[p_D, p_l)$ and \bar{p}_h decreases when W_D^0 decreases.*

Proof. Suppose that the agent manipulates on $[p_D, p_l)$. Then, the agent's value $W_D(p)$ satisfies the HJB Equation (22) with boundary condition $W_D(p_D) = W_D^0$. We can rewrite this equation as

$$W'_D(p) = \frac{B - \lambda p K - (r + \lambda p) W_D(p)}{\lambda p (1 - p) + \gamma p}.$$

$W_D(p)$ is uniformly continuous in W_D^0 on $[p_D, p_l)$ and the above equation shows that $W'_D(p)$ is uniformly continuous as well. This establishes that for W_D^0 sufficiently close to $W(p_D)$, we have $\omega_D(p) > 0$ for $p \in [p_D, p_l)$, i.e., manipulating remains optimal.

We next show that \bar{p}_h decreases. Consider two solutions to the agent's HJB Equation (22) with different initial condition W_D^0 . The equation above implies that these solutions can never cross, because when the values are equal, the derivative must be equal as well. Thus, as W_D^0 decreases, $W_D(p)$ decreases for all $p \in [p_D, p_l]$.

Right of p_l , the agent's value satisfies the HJB Equation (24). When W_D^0 decreases, $W_D(p_l)$, the value at the left boundary of this region, decreases, as we have just shown. We can rewrite the HJB equation as

$$W_D'(p) = \frac{B - \lambda p K - (r + \lambda p) W_D(p)}{\lambda p (1 - p)},$$

which again shows that two solutions with different initial conditions cannot cross on any interval $[p_l, p_h]$. Thus, as W_D^0 decreases so does $W_D(p)$ for all $p \geq p_l$. Since on that region, we have $rW_D(p) = \omega_D(p)$, \bar{p}_h must decrease as well. ■

Lemma 28. *Suppose that $p_D \geq p_l$ and that W_D^0 is sufficiently close to $W(p_D)$, so that the agent manipulates on a nonempty interval with p_D as its left boundary. Then, \bar{p}_h decreases when W_D^0 decreases.*

We omit the proof because it is similar to the proof of the preceding Lemma.

Lemma 29. *If $p_m < p_D < \frac{r}{r+\gamma}$ and W_D^0 is sufficiently close to zero, the unique equilibrium features no manipulation.*

Proof. We first show that there exists an equilibrium in which the agent does not manipulate and the monitor does not exert effort to the right of p_D . We start with characterizing the agent's incentives.

In this equilibrium, not manipulating on a small neighborhood to the right of p_D is optimal whenever W_D^0 is sufficiently small. To see this, we use the HJB Equation (18) to write

$$\omega_D(p_D) = B - \lambda p_D K + \frac{\lambda}{\gamma} (r - (r + \gamma) p_D) W_D^0,$$

which becomes negative as W_D^0 approaches zero. Thus, not manipulating close to p_D is indeed optimal for W_D^0 sufficiently small, because $\omega_D(p_D) < 0$.

To show that not manipulating is optimal for all $p > p_D$, we can differentiate $\omega_D(p)$ to get

$$\omega_D'(p) = -\lambda K + \frac{\lambda}{\gamma} (r - (r + \gamma) p) W_D'(p) - \frac{\lambda(r + \gamma)}{\gamma} W_D(p),$$

which is negative for $p \leq \frac{r}{r+\gamma}$. Thus, $\omega_D(p)$ cannot cross zero for $p \leq \frac{r}{r+\gamma}$. If $\omega_D(p) > 0$ for some $p > \frac{r}{r+\gamma}$, we have

$$\omega_D(p) = B - \lambda p K + \frac{\lambda}{\gamma} (r - (r + \gamma) p) W_D(p),$$

which is negative because $W_D(p)$ is positive. Thus, $\omega_D(p)$ cannot cross zero for $p > \frac{r}{r+\gamma}$ either. This establishes that $\omega_D(p) < 0$ for all $p \geq p_D$.

We next show that the monitor shirks to the right of p_D . Since the monitor's value satisfies Equation (25) for $p \geq p_D$ and since her value is positive at p_D , the constant in Equation (25) is positive. Then, Equation (25) implies that $V'(p) < 0$ for $p > p_D$, so the monitor indeed prefers to shirk.

The above arguments establish that there exists an equilibrium in which the agent does not manipulate and the monitor shirks for all $p \geq p_D$. We now show that this equilibrium is unique.

Monotonicity of $m(p)$ implies that there is no other equilibrium at which $m(p) = 1$ for $p > p_D$ if $m(p) = 0$ on a neighborhood of p_D . We thus only have to rule out equilibria where $m(p) = 1$ on a neighborhood of p_D .

First, suppose that $m(p) = e(p) = 1$ for all $p \in [p_D, p_D + \varepsilon)$ for some $\varepsilon > 0$. Then, $\omega_D(p) = rW_D(p)$ and in particular $\omega_D(p_D) = rW_D(p_D)$. The HJB Equation (24), implies that

$$W'(p_D) = \frac{B - \lambda p_D K - (r + \lambda p_D) W_D^0}{\lambda p_D (1 - p_D)}.$$

Since we assumed that $p_D > p_m$, the first term in the numerator is negative. Thus, as W_D^0 approaches zero, $W'_D(p_D)$ becomes negative. By continuity, there is a $p \in (p_D, p_D + \varepsilon)$ for which $\omega_D(p) = rW_D(p) < 0$. Thus, such an equilibrium cannot exist.

Now, suppose that $m(p) = 1$ and $e(p) = 0$. In this case, a similar argument shows that $W'_D(p_D) < 0$ whenever W_D^0 is sufficiently small. For $p > p_D$, we have $\omega_D(p) = rW_D(p) + \gamma p W'_D(p)$. By continuity, there is then a $p > p_D$ for which $W_D(p) < 0$ and $W'_D(p) < 0$, so that $\omega_D(p) < 0$. This rules out such an equilibrium. ■

A.3.2 Proof of Proposition 6

Proposition 6 is a straightforward corollary to Lemma 28. When the monitor is replaced with a new one, the agent's continuation value is $W_D^0 = W(p_0)$, which is strictly smaller than the agent's value without replacement.

A.3.3 Proof of Proposition 7

Before we prove the result, we first introduce notation. Let $V_D(p, V_D^0)$ and $W_D(p, V_D^0)$ denote the monitor and agent's value functions given the boundary condition at p_D , respectively. The agent's value only depends on V_D^0 through the monitor's effort. Similarly, let $p_l(V_D^0)$ denote the threshold below which the monitor shirks and let $\bar{p}_h(V_D^0)$ denote the threshold which characterizes the equilibrium with the most manipulation.

Lemma 30. $p_l(V_D^0)$ is decreasing.

Proof. Consider the region $[0, p_l(V_D^0))$. Using the monitor's HJB Equation (21) we can see that two solutions with different boundary conditions can never cross. Thus, as V_D^0 decreases, the value $V_D(p, V_D^0)$ decreases for all $p \in [p_D, p_l)$. Writing the monitor's HJB equation as

$$V'_D(p, V_D^0) = \frac{\lambda p R - (r + \lambda p) V_D(p, V_D^0)}{\lambda p (1 - p) + \gamma p}$$

we can see that if $V_D(p, V_D^0)$, then $V'_D(p, V_D^0)$ is larger. This implies that $p_l(V_D^0)$ is decreasing. Otherwise, we would have for $\hat{V}_D^0 < V_D^0$

$$\gamma p_l(\hat{V}_D^0) V'_D(p_l(\hat{V}_D^0), \hat{V}_D^0) > c = \gamma p_l(V_D^0) V'_D(p, V_D^0),$$

so that $p_l \left(\hat{V}_D^0 \right)$ cannot be an equilibrium threshold. ■

To establish that $\bar{p}_h \left(V_D^0 \right)$ is also decreasing, we need the following auxiliary result.

Lemma 31. *Let $\tilde{W} \left(p \right)$ be a decreasing solution to the HJB Equation (22), i.e., when $m \left(p \right) = 1$ and $e \left(p \right) = 0$, and let $\hat{W} \left(p \right)$ be the solution to the HJB Equation (24), i.e., when $m \left(p \right) = e \left(p \right) = 1$. Then, $\hat{W} \left(p \right)$ cannot cross $\tilde{W} \left(p \right)$ from below.*

Proof. Taking Equations (22) and (24), we can see that whenever $\tilde{W} \left(p \right) = \hat{W} \left(p \right)$, then

$$-\gamma p \tilde{W}' \left(p \right) = \lambda p \left(1 - p \right) \left(\tilde{W}' \left(p \right) - \hat{W}' \left(p \right) \right) > 0.$$

Thus, $\hat{W} \left(p \right)$ cannot cross $\tilde{W} \left(p \right)$ from below. ■

Lemma 32. *$\bar{p}_h \left(V_D^0 \right)$ is decreasing.*

Proof. We establish the result by showing that $W \left(p, \hat{V}_D^0 \right) < W \left(p, V_D^0 \right)$ for $p > p_l$ and $\hat{V}_D^0 < V_D^0$. Since for $p \in [p_l, p_h)$, we have $\omega_D \left(p \right) = rW \left(p, V_D^0 \right)$, this implies that $\bar{p}_h \left(V_D^0 \right)$ is decreasing.

Following a similar argument as in the proof of Lemma 23, we can show that

$$W'_{D+} \left(p_l \left(V_D^0 \right), V_D^0 \right) < W'_{D-} \left(p_l \left(V_D^0 \right), V_D^0 \right),$$

so that the agent still prefers to manipulate on some region right of $p_l \left(V_D^0 \right)$, for any V_D^0 .

Now, pick $\hat{V}_D^0 < V_D^0$. If $\bar{p}_h \left(\hat{V}_D^0 \right) < p_l \left(V_D^0 \right)$, then we are done. If not, then $p_l \left(V_D^0 \right) \in \left[p_l \left(\hat{V}_D^0 \right), \bar{p}_h \left(\hat{V}_D^0 \right) \right]$.⁴¹ We have

$$W \left(p_l \left(V_D^0 \right), \hat{V}_D^0 \right) < W \left(p_l \left(V_D^0 \right), V_D^0 \right), \quad (40)$$

that is, at $p_l \left(V_D^0 \right)$, the agent's value under \hat{V}_D^0 must be smaller. This follows from Lemma 31.

The argument is as follows. We know that $W \left(p, \hat{V}_D^0 \right) < W \left(p, V_D^0 \right)$ for $p \in [p_D, p_l \left(\hat{V}_D^0 \right)]$. On $\left[p_l \left(\hat{V}_D^0 \right), p_l \left(V_D^0 \right) \right]$, the value function $W \left(p, \hat{V}_D^0 \right)$ satisfies the HJB Equation (24), while $W \left(p, V_D^0 \right)$ admits the closed form in Equation (34), which is a decreasing solution to the HJB Equation (22). Lemma 31 then establishes that $W \left(p, \hat{V}_D^0 \right)$ cannot cross $W \left(p, V_D^0 \right)$ from below on $\left[p_l \left(\hat{V}_D^0 \right), p_l \left(V_D^0 \right) \right]$, which implies Inequality (40).

Now, we can compare the values $W \left(p, \hat{V}_D^0 \right)$ and $W \left(p, V_D^0 \right)$ on the interval $\left[p_l \left(V_D^0 \right), \bar{p}_h \left(\hat{V}_D^0 \right) \right]$. Since both satisfy the HJB Equation (24), since two solutions to that HJB equation cannot cross, and since $W \left(p_l \left(V_D^0 \right), \hat{V}_D^0 \right) < W \left(p_l \left(V_D^0 \right), V_D^0 \right)$, we have $W \left(p, \hat{V}_D^0 \right) < W \left(p, V_D^0 \right)$ for all $p \in \left[p_l \left(V_D^0 \right), \bar{p}_h \left(\hat{V}_D^0 \right) \right]$. This implies that $\bar{p}_h \left(\hat{V}_D^0 \right) < \bar{p}_h \left(V_D^0 \right)$. ■

⁴¹Lemma 30 establishes that $p_l \left(\hat{V}_D^0 \right) < p_l \left(V_D^0 \right)$.

A.4 Proofs for Section 6

A.4.1 Proof of Proposition 10

The precise expression for the bound \bar{R} in the proposition statement is

$$\bar{R} = \frac{c}{\gamma r B} ((r + \gamma)(B + rK) + (N - 1)r(r + \lambda + \gamma)K). \quad (41)$$

Condition (41) is the analog of Condition (5) in the single-agent case. It guarantees that the monitor's incentives are sufficiently strong. Otherwise, the equilibrium would feature no monitor effort. Condition (41) is stronger than Condition (5). Thus, under the assumptions of Proposition 10, Proposition 3 (for the single-agent case) holds as well.

The cooperative value function is given by $W_c = \frac{1}{N} \sum_{n=1}^N W_n$. It satisfies the HJB equation

$$\begin{aligned} rW_c(p) &= \max_{M \in [0, N]} B \frac{M}{N} - \lambda p K \frac{M}{N} - \left(\lambda \frac{M}{N} p (1 - p) + \gamma p (1 - e(p)) \right) W'_c(p) \\ &\quad - \lambda p \frac{M}{N} W_c(p). \end{aligned} \quad (42)$$

Since $M \in [0, N]$, we can see that this equation is identical to the single-agent HJB equation in the main model, Equation (4). If for all p , $M(p)/N$ equals the solution of the single-agent model, then the solution to the monitor's HJB equation is also the same and her best response is the same as well.

In a symmetric equilibrium, whenever $m(p) = e(p) = 1$, then each agent's HJB equation is given by Equation (24) and the monitor's HJB equation is given by Equation (27). The analog holds for the cases when $m(p) = 1$ and $e(p) = 0$ and when $m(p) = e(p) = 0$. We will exploit this fact throughout.

The proof proceeds in a sequence of Lemmas and mirrors the proof of Proposition 3. The key step is to show that on the region $[p_l, p_h]$, each agent's incentives to manipulate are smaller than in the single-agent case.

Lemma 33. *There exists a region $[0, p_l]$, such that $m(p) = 1$ and $e(p) = 0$ for all $p \in [0, p_l]$. At p_l , we have $\gamma p_l V'_-(p_l) = c$. The point p_l is the same point as in the single-agent case.*

Proof. We first show that shirking and manipulation are an equilibrium on $[0, p_l]$. Since by construction, the monitor's value function is the same as in the single-agent case, we focus on the agent's incentives.

Suppose that p_l is determined by the monitor's indifference condition $\gamma p_l V'_-(p_l) = c$. The assumption in Equation (41) implies that $\omega_n(p) > 0$ for all $p \leq p_l$. This follows from the closed form solutions in Equation (30) for the agent and Equation (29) for the monitor, which imply

$$\omega_n(p) = B - \lambda p K - \frac{\lambda}{N} p \left(\frac{B}{r} - \frac{\lambda(B + rK)}{r(r + \lambda + \gamma)} \right) \quad (43)$$

and

$$p_l = \frac{c(r + \lambda + \gamma)}{\gamma \lambda R}.$$

Condition (41) guarantees that $\omega_{n-}(p_l) > 0$. From Equation (43), we can see that $\omega_n(p)$ is strictly decreasing. Thus, $\omega_n(p) > 0$ for all $p \geq p_l$. It is optimal for the agent to manipulate on $[0, p_l]$.

Next, we show that no other p_l constitutes an equilibrium. Suppose by way of contradiction that the region $[0, p_l]$ instead has some value $\hat{p}_l \neq p_l$ as its right boundary. First, suppose that $\hat{p}_l < p_l$, such that $\gamma\hat{p}_l V'_-(\hat{p}_l) < c$ and $\omega_{n-}(\hat{p}_l) > 0$. If the equilibrium switches to $m(p) = e(p) = 1$ to the right of \hat{p}_l , then, using similar arguments as in the proof of Lemma 21, we can show that $V'_+(\hat{p}_l) < V'_-(\hat{p}_l)$. This implies that $\gamma\hat{p}_l V'_+(\hat{p}_l) < c$, which contradicts $e(p) = 1$ being optimal to the right of \hat{p}_l . If the equilibrium switches to $m(p) = e(p) = 0$, then we can use the agent's HJB Equations (22) and (18) to show that

$$\gamma\hat{p}_l (W'_-(\hat{p}_l) - W'_+(\hat{p}_l)) = \omega_-(\hat{p}_l).$$

For all $N \geq 1$, if Condition (41) holds, so does Condition (5). Thus, $\omega_-(p_l) > 0$ and $\omega(p) > 0$ for all $p \leq p_l$. Since we assumed that $\hat{p}_l < p_l$ and since $\omega(p)$ is decreasing, we have $\omega_-(\hat{p}_l) > 0$. This implies that $W_-(\hat{p}_l) > W_+(\hat{p}_l)$ and thus $\omega_{n+}(\hat{p}_l) > \omega_{n-}(\hat{p}_l) > 0$, i.e. not manipulating to the right of \hat{p}_l cannot be optimal.

Next, suppose that $\hat{p}_l > p_l$, which implies that $\gamma\hat{p}_l V'_-(\hat{p}_l) > c$, and also suppose that $\omega_{n-}(\hat{p}_l) \geq 0$. Then, there exists an interval left of \hat{p}_l on which the monitor does not exert effort even though it is optimal for her to do so. If $\hat{p}_l > p_l$ and $\omega_{n-}(\hat{p}_l) < 0$ then there exists a region on which both monitor and agent make suboptimal choices.

Thus, we must have $\hat{p}_l = p_l$ in equilibrium. Then, the monitor's value to the left of p_l is the same as in the single-agent case. This implies p_l is the same as in the single-agent case as well. ■

Lemma 34. *There exists a region $[p_l, p_h]$ such that $m(p) = e(p) = 1$ for all $p \in [p_l, p_h]$. On any such region, $V'(p) > 0$, $V''(p) > 0$, $W'_n(p) < 0$, and $\omega'_n(p) < 0$.*

Proof. On the conjectured region, each agent's value function solves the HJB Equation (24), i.e. the HJB equation in the single-agent case when $m(p) = e(p) = 1$. We can exploit this fact and use the same argument as in the proof of Lemma 22 to show that $W'_{n+}(p_l) < W'_{n-}(p_l)$, which in turn implies that $\omega_{n+}(p_l) > \omega_{n-}(p_l)$, so that manipulation is optimal on a nonempty interval to the right of p_l . As in the single-agent case, the monitor's value satisfies smooth pasting at p_l . These two facts establish that a nonempty region with $m(p) = e(p) = 1$ exists to the right of p_l . Analogous arguments as in Lemma 23 establish that $V'(p) > 0$, $V''(p) > 0$, and $W'_n(p) < 0$ on this region.

To show that $\omega'_n(p) < 0$, we can use the HJB Equation (24) to write

$$\omega_n(p) = \frac{N-1}{N} (B - \lambda p K) + \frac{r}{N} W_n(p), \quad (44)$$

which is decreasing in p . Finally, note that since $V'(p) > 0$ and $V''(p) > 0$, $\gamma p V'(p) - c$ is increasing in p . Thus, the monitor prefers to exert effort on $[p_l, p_h]$. ■

We now characterize the largest region on which manipulation and monitor effort are both optimal. We denote its right boundary with \hat{p}_h , which satisfies

$$\omega_{n-}(\hat{p}_h) = 0,$$

similarly to the single-agent case. As before, \bar{p}_h denotes the analogous boundary with a single agent, which solves $W(\bar{p}_h) = 0$.

Lemma 35. *For any region $[p_l, p_h)$ such that $m(p) = e(p) = 1$ for all $p \in [p_l, p_h)$, we have $p_h \leq \hat{p}_h$. Also, $\hat{p}_h < \bar{p}_h$.*

Proof. The preceding lemma implies that $\omega_n(p)$ is decreasing and $V'(p)$ is increasing on $[p_l, \hat{p}_h]$. Therefore, $m(p) = e(p) = 1$ is optimal on this region. Since $\omega_n(p)$ is strictly decreasing, $p_h > \hat{p}_h$ cannot be an equilibrium, because it would require that $m(p) = 1$ for some p for which $\omega_n(p) < 0$. Thus, in any equilibrium, $p_h \leq \hat{p}_h$.

To show that $\hat{p}_h < \bar{p}_h$, suppose by way of contradiction that $\hat{p}_h = \bar{p}_h$. Then, $W_n(p) = W(p)$ for $p \in [p_l, \bar{p}_h]$, i.e. each agent's value function equals the value function in the single-agent case. Then, we have

$$\begin{aligned} \omega_{n-}(\bar{p}_h) - \omega_-(\bar{p}_h) &= \frac{N-1}{N} (B - \lambda\bar{p}_h K - \omega_-(\bar{p}_h)) \\ &= \lambda\bar{p}_h (1 - \bar{p}_h) W'_-(\bar{p}_h) + \lambda\bar{p}_h W(\bar{p}_h) \\ &= \lambda\bar{p}_h (1 - \bar{p}_h) W'_-(\bar{p}_h) < 0. \end{aligned}$$

The last inequality uses the fact that $W(\bar{p}_h) = 0$ and that $W(p)$ is strictly decreasing on $[p_l, p_h)$. Since \bar{p}_h satisfies $\omega_-(\bar{p}_h) = 0$, the above equation implies that $\omega_{n-}(\bar{p}_h) < 0$. This is a contradiction, because \hat{p}_h must satisfy $\omega_{n-}(\hat{p}_h) = 0$. ■

To finish constructing the equilibrium, it remains to show that for $p > \hat{p}_h$ the equilibrium features no manipulation and no monitor effort.

Lemma 36. *Under the assumptions of Proposition 10, we have $m(p) = e(p) = 0$ for all $p > \hat{p}_h$.*

Proof. We prove the result by showing that $\omega_n(p) < 0$ for all $p \geq \hat{p}_h$. We first show that $\omega_{n+}(\hat{p}_h) < 0$. Using Equation (44), the condition $\omega_{n-}(\hat{p}_h) = 0$, the agent's HJB Equation (18), and the value matching condition at \hat{p}_h yields

$$\omega_{n+}(\hat{p}_h) = -\frac{r}{N-1} W_n(\hat{p}_h) + \frac{\lambda}{\gamma N} (r - (r + \gamma)\hat{p}_h) W_n(\hat{p}_h).$$

A sufficient condition for this expression to be negative is $W_n(\hat{p}_h) > 0$ and $\hat{p}_h > \frac{r}{r+\gamma}$. Substituting $\omega_{n-}(\hat{p}_h) = 0$ in Equation (44), we get

$$rW_n(\hat{p}_h) = -(N-1)(B - \lambda\hat{p}_h K). \quad (45)$$

Thus, $W_n(\hat{p}_h)$ is positive if and only if $\hat{p}_h > p_m$. We now show this is true.

Suppose not, i.e., $\hat{p}_h \leq p_m$. Then, $W_n(\hat{p}_h) < 0$. Now, since $\hat{p}_h < \bar{p}_h$, on $[0, \hat{p}_h]$, we have $W_n(p) = W(p)$, since both value functions satisfy the HJB Equation (24) with the same boundary condition at p_l . This implies that $W_n(\hat{p}_h) = W(\hat{p}_h) < 0$. Since, $W(p)$ is decreasing and $\bar{p}_h > \hat{p}_h$, we have $W(\bar{p}_h) < 0$. But this is impossible, since \bar{p}_h , by definition, satisfies $W(\bar{p}_h) = 0$. Thus, we must have $\hat{p}_h > p_m$. This implies that $W_n(\hat{p}_h) > 0$.

To show that $\omega_{n+}(\hat{p}_h) < 0$, it remains to prove that $\hat{p}_h > \frac{r}{r+\gamma}$. We now show that this is true whenever K is sufficiently small, which we have assumed in Proposition 10. Specifically, we show that as K decreases, \hat{p}_h increases and that as K becomes small, \hat{p}_h tends to one.

We know that \hat{p}_h satisfies Equation (45). Intuitively, the implicit function theorem yields

$$\frac{d\hat{p}_h}{dK} = -\frac{r \frac{\partial W_n(\hat{p}_h)}{\partial K} - (N-1) \lambda \hat{p}_h}{W'_{n-}(\hat{p}_h) - (N-1) \lambda K}.$$

Here, we denote $\lim_{p \uparrow \hat{p}_h} \frac{\partial}{\partial p} W_n(p) = W'_{n-}(\hat{p}_h)$. We suppress the dependence of W_n on K to keep the notation simple. Since $W'_{n-}(\hat{p}_h) < 0$, \hat{p}_h is increasing in K whenever $W_n(\hat{p}_h)$ is decreasing in K . We now show this is the case, via a sequence of value matching arguments. Of course, we must guarantee that W_n is differentiable in K to apply the argument. Using the closed-form in Equation (28), we see that this is the case.

As K increases, each agent's closed-form value in Equation (30) on $[0, p_l]$ decreases uniformly. Importantly, p_l does not depend on K , which we can see from Equation (35) in the single-agent case.⁴² Thus, at p_l , the boundary condition for each agent's HJB Equation (24) on $[p_l, \hat{p}_h]$ decreases.

Using the closed-form solution in Equation (28), we can calculate how the constant C_0^A must change when K changes at p_l . Specifically, the value matching condition at p_l is

$$\frac{B}{r} - \lambda p_l \frac{B + rK}{r(r + \lambda + \gamma)} = \frac{B}{r} - \lambda p_l \frac{B + rK}{r(r + \lambda)} + C_0^A \frac{(1 - p_l)^{\frac{r+\lambda}{\lambda}}}{p_l^{\frac{r}{\lambda}}},$$

where the left-hand side is obtained from the closed-form solution in Equation (34). After some algebra, we get

$$C_0^A \frac{(1 - p_l)^{\frac{r+\lambda}{\lambda}}}{p_l^{\frac{r}{\lambda}}} = \frac{\lambda \gamma p_l (B + rK)}{r(r + \lambda)(r + \lambda + \gamma)},$$

which shows that C_0^A is increasing in K . We also have

$$W'_n(p) = -\frac{\lambda}{r(r + \lambda)} (B + rK) - C_0^A \frac{\frac{r+\lambda}{\lambda} (1-p)^{\frac{r}{\lambda}} + \frac{r}{\lambda p} (1-p)^{\frac{r+\lambda}{\lambda}}}{p^{\frac{r}{\lambda}}}$$

for $p \geq p_l$. Since C_0^A is increasing in K , $W'_n(p)$ is decreasing in K for all $p \geq p_l$. Thus, as K increases, the agent's value function starts at a lower value at p_l and has a smaller slope. Thus, $W_n(p)$ is uniformly decreasing in K for $p \geq p_l$. In particular, $W_n(\hat{p}_h)$ is decreasing in K for a fixed \hat{p}_h . Then, applying the implicit function theorem to Equation (45) establishes that \hat{p}_h is decreasing in K .

We now show that as K becomes small, \hat{p}_h converges to one. Specifically, as K approaches $\frac{B}{\lambda}$ from above, p_m converges to one. Above, we have shown that $\hat{p}_h > p_m$. Therefore, \hat{p}_h converges to one as well so that, for K sufficiently small, we then have $\hat{p}_h > \frac{r}{r+\gamma}$.

We have now shown that $\omega_{n+}(\hat{p}_h) < 0$. To conclude the proof, we must show that $\omega_n(p) < 0$ for all $p > \hat{p}_h$. Using the closed-form in Equation (26), we get

$$\omega_n(p) = B - \lambda p K + \frac{\lambda}{\gamma N} (r - (r + \gamma)p) W_n(p),$$

which is decreasing, since $W'_n(p) < 0$ on that region. Thus, $\omega_n(p) < \omega_{n+}(\hat{p}_h)$ for all $p > \hat{p}_h$. That

⁴²Also, if Condition (14) holds for some K , it holds for all $\hat{K} < K$.

the monitor prefers to shirk follows from the same argument as in Lemma 25. ■

A.4.2 Proof of Proposition 11

The condition $B < \frac{\lambda}{N}K$ is the analog of Condition (1) in the single-agent case.⁴³ If it holds, then Condition (1) holds as well. Thus, under the assumptions of Proposition 11, Proposition 3, for the single-agent case, holds.

The cooperative value is

$$\begin{aligned} rW_c(p) &= \max_{M \in [0,1]} B \frac{M}{N} - \frac{\lambda}{N} p K M - \left(\frac{\lambda}{N} M p (1-p) + \gamma p (1-e(p)) \right) W'_c(p) \\ &\quad - \frac{\lambda}{N} p M W_c(p). \end{aligned} \quad (46)$$

As in the case with individual punishments, the cooperative value function is identical to the single-agent HJB equation in the main model, Equation (4). Also, in a symmetric equilibrium, each agent's value is given by Equation (24) whenever all agents choose $m(p) = 1$ and the monitor chooses, $e(p) = 1$. The analog holds for the cases when $m(p) = 1$ and $e(p) = 0$ and when $m(p) = e(p) = 0$.

The proof proceeds similarly to the proofs of Proposition 3 and Proposition 10. We therefore only provide a brief sketch. The main step is to show that on the region $[p_l, p_h)$, each agent's incentive to manipulate is stronger than in the single-agent case.

Proof. We denote with \tilde{p}_h the threshold in the equilibrium with the most manipulation. As before, \bar{p}_h is the analog in the single-agent case.

Using Equation (5), we can verify that the proof of Lemma 21 goes through, so p_l is characterized by $\gamma p_l V'_-(p_l) = c$ and is the same as in the single-agent case.

The proof of Lemma 22 goes through without modifications, so there exists a region $[p_l, p_h)$ on which $e(p) = 1$ and $m_n(p) = 1$ for all n is optimal. Using the same argument as in the proof of Lemma 23, yields $V'(p) > 0$ and $W'_n(p) < 0$ for $p \in [p_l, p_h)$. Using the HJB Equation (24), we can write

$$\omega_n(p) = \frac{N-1}{N} \left(B - \frac{\lambda}{N} p K \right) + \frac{1}{rN} W_n(p), \quad (47)$$

which shows that $\omega_n(p)$ is strictly decreasing on $[p_l, p_h)$.

We now show that $\tilde{p}_h > \bar{p}_h$. First, suppose that $p_h = \bar{p}_h$. We will show that this is an equilibrium, which implies that $\tilde{p}_h \geq \bar{p}_h$. On the interval $[p_l, \bar{p}_h]$, each agent's value function is the same as the value function in the single-agent case, in Equation (24).⁴⁴ We thus have $W_n(p_h) = 0$ and

$$\begin{aligned} 0 &= \omega_-(p_h) \\ &= B - \lambda p_h K - \lambda p_h \left((1-p_h) W'_-(p_h) + W(p_h) \right) \\ &< NB - \lambda p_h K - \lambda p_h \left((1-p_h) W'_{n-}(p_h) + W_n(p_h) \right) \\ &= N\omega_{n-}(p_h). \end{aligned}$$

⁴³In the case with individual punishments, we did not need to impose a similar condition, because each agent's instantaneous benefit from manipulating was $B - \lambda p K$, the same as in the single-agent case. See Equation (13).

⁴⁴Here, recall that because we assumed that Equation (5) holds, the threshold p_l is the same as in the single-agent case and the boundary condition for each agent's value is the same as well.

On the region $[p_h, 1]$, we have $m(p) = e(p) = 0$. To see this, we can use the HJB Equation (18) to calculate

$$\begin{aligned}\omega_{n+}(p_h) &= B - \frac{\lambda}{N}p_hK + \frac{\lambda}{\gamma N}(r - (r + \gamma)p_h)W_n(p_h) \\ &= B - \frac{\lambda}{N}p_hK.\end{aligned}$$

This expression is negative if $\bar{p}_h > \frac{BN}{\lambda K}$. In the proposition statement, we have assumed that $\frac{BN}{\lambda K} < 1$. As K approaches $\frac{B}{\lambda}$ from above, a similar argument as in the proof of Lemma 36 shows that \bar{p}_h converges to one. Thus, for K sufficiently small, the condition $\bar{p}_h > \frac{BN}{\lambda K}$ holds and $\omega_{n+}(p_h) < 0$. Using the value matching condition $W_n(p_h) = 0$, we can see that the constant C_0^A in Equation (26) is zero and thus $W_n(p) = 0$ for all $p > p_h$. Therefore, $\omega_n(p) < 0$ for $p \geq p_h$ and not manipulating is indeed optimal.⁴⁵

To show that $\tilde{p}_h > \bar{p}_h$, note that $\omega_{n+}(\bar{p}_h) < 0$ and that the constant C_0^A in Equation (26) depends continuously on the value matching condition. This implies that on $[p_h, 1]$, $W_n(p)$ is uniformly continuous with respect to the boundary condition at p_h . Thus, there exists a sufficiently small $p_h > \bar{p}_h$ such that $\omega_{n+}(p_h) < 0$ and $\omega_n(p) < 0$ for all $p > \bar{p}_h$. Thus, to the right of such a p_h , $m(p) = 0$ is optimal. That $e(p) = 0$ is optimal follows from essentially the same argument as in Lemma 25. This concludes our proof. ■

A.5 Bribes: Proof of Proposition 9

Consider first the case when R is intermediate, so that the monitor accepts bribes when $p < p_B$. Specifically, we assume that Condition (9) holds, i.e.,

$$\frac{B - c}{r} + K < R < \frac{B}{r} + K.$$

Following Proposition 3, we also assume that Condition (5) holds, i.e.,

$$R > c \frac{r + \gamma}{r\gamma} \frac{B + rK}{B}.$$

The two conditions are not mutually exclusive. Both hold simultaneously whenever

$$c < B \frac{\gamma}{r + \gamma},$$

i.e. c is not too large, which we have assumed in the statement of Proposition 9.

We now prove the result in the proposition. To do this, we construct the regions $[0, p_l)$, $[p_l, p_h)$, and $[p_h, 1]$, just as we have done in the proof of Proposition 3. Since parts of the previous analysis carry over, our exposition focuses on the differences to keep the repetition at a minimum.

The key technical difficulty is that with bribes, the agent's value function also depends on the monitor's value (see Equations (49) and (53) below). Thus, we must be careful in showing that the agent indeed wants to manipulate on $[0, p_l)$ and on $[p_l, p_h)$.

⁴⁵We skip the characterization of the monitor's incentives. It is similar to the single-agent case.

Additionally, we must verify that it is optimal to offer bribes whenever $p \leq p_B$. We do this by showing that the joint value $S(p)$ can cross $R - K$ at most once from above, so that Condition (8) can only hold for beliefs below a threshold. The analysis here is subtle, since on different regions $S(p)$ satisfies different ODEs (see Equations (52), (54), and (56)) and since we cannot rely on $S(p)$ being monotone.

Throughout the analysis, we will compare the monitor's and agent's value functions to the ones in the baseline model. To avoid confusion, we denote the value functions when bribes are possible as $W_B(p)$ and $V_B(p)$ and the agent's value of manipulation as $\omega_B(p)$. We continue denoting the corresponding functions without bribes as $W(p)$, $V(p)$, and $\omega(p)$.

We now start constructing the equilibrium with the region $[0, p_l]$. Since $W_B(0) = \frac{B}{r}$ and $V_B(0) = 0$, Condition (9) implies that offering a bribe at $p = 0$ is optimal. That is,

$$\begin{aligned} R - K &< S(0) \\ &= V_B(0) + W_B(0) \\ &= \frac{B}{r}. \end{aligned}$$

As in Proposition 3, there exists a region $[0, p_l]$ on which the agent manipulates and the monitor does not exert effort. We have $p_B > p_l$ and p_l is the same as in the case without bribes.

Lemma 37. *Let p_l be given by the monitor's indifference condition in Equation (35). We have $p_B > p_l$ and no other threshold $\tilde{p}_l \neq p_l$ so that $m(p) = 1$ and $e(p) = 0$ for $p < \tilde{p}_l$ can be part of a monotone-manipulation equilibrium.*

Proof. Since the optimal bribe is

$$b(p) = p(R - V_B(p)) \tag{48}$$

for $p < p_B$, the monitor's value function is exactly the same as in the case without bribes. That is, the monitor's value follows

$$\begin{aligned} rV_B(p) &= -(\lambda p(1-p) + \gamma p) V_B'(p) + \lambda b(p) \\ &= -(\lambda p(1-p) + \gamma p) V_B'(p) + \lambda p(R - V_B(p)), \end{aligned}$$

which is the same as in Equation (21). Thus, on $[0, \min\{p_B, p_l\}]$, we have $V_B(p) = V(p)$. In particular, the monitor's value admits the closed form solution in Equation (33). This implies that shirking is optimal for the monitor for $p \in [0, \min\{p_B, p_l\}]$.

On $[0, \min\{p_B, p_l\}]$, the agent's value satisfies

$$rW_B(p) = B - (\lambda p(1-p) + \gamma p) W_B'(p) - \lambda b(p).$$

Plugging in the optimal bribe in Equation (48) and the closed form of the monitor's value in Equation (33), we get

$$rW_B(p) = B - (\lambda p(1-p) + \gamma p) W_B'(p) - \lambda p \left(R - \lambda p \frac{R}{r + \lambda + \gamma} \right). \tag{49}$$

As we have done in Lemma 20, we can calculate a general solution to this ODE. It is given by

$$W_B(p) = \frac{B}{r} - \lambda p \frac{R}{r + \lambda + \gamma} + C_0^A \frac{(\gamma + \lambda(1-p))^{\frac{r}{\lambda+\gamma}}}{p^{\frac{r}{\lambda+\gamma}}}.$$

The only solution which satisfies the boundary condition $W_B(0) = \frac{B}{r}$ is the one with $C_0^A = 0$. Thus, the agent's value function admits the closed-form solution

$$W_B(p) = \frac{B}{r} - \lambda p \frac{R}{r + \lambda + \gamma}. \quad (50)$$

From this equation, we can compute the agent's incentive to manipulate, which is given by

$$\omega_B(p) = B - \lambda p \frac{(r + \gamma) R}{r + \lambda + \gamma}. \quad (51)$$

In the case without bribes, this equation has its analog in Equation (36), in the proof of Lemma 21. That is,

$$\omega(p) = B - \lambda p \frac{(r + \gamma)(B + rK)}{r(r + \lambda + \gamma)}.$$

Condition (9) implies that $\omega_B(p) > \omega(p)$. Thus, the agent is more willing to manipulate on $[0, \min\{p_B, p_l\})$ when he can offer bribes. Under Condition (5), manipulating is then optimal on $[0, \min\{p_B, p_l\})$.

The above arguments have established that for $p < [0, \min\{p_B, p_l\})$, the agent manipulates and the monitor shirks. We now show that $p_B > p_l$.

Suppose by way of contradiction that $p_B \leq p_l$. Then, on $[0, p_B]$, the joint value $S(p)$ is given by

$$\begin{aligned} S(p) &= V_B(p) + W_B(p) \\ &= \lambda p \frac{R}{r + \lambda + \gamma} + \frac{B}{r} - \lambda p \frac{R}{r + \lambda + \gamma} \\ &= \frac{B}{r} > 0. \end{aligned} \quad (52)$$

Here, we have used the closed form solutions for the agent's value in Equation (50) and the monitor's value in Equation (33). Condition (9) then implies that $S(p) > R - K$ for $p \leq p_B$, i.e., Condition (8) holds. In particular, $S(p_B) > R - K$, so not offering a bribe at p_B is suboptimal, which is a contradiction.

To conclude the argument, it only remains to show that no other threshold $\tilde{p}_l \neq p_l$ can constitute the right boundary of the region $[0, \tilde{p}_l)$ on which $m(p) = 1$ and $e(p) = 0$. The arguments are analogous to the ones in Lemma 21 and we omit them. ■

Now, suppose that $p_B \in [p_l, \bar{p}_h]$. Recall that \bar{p}_h is the threshold in Proposition 3, which characterizes the equilibrium with the most manipulation.

We consider the region $[p_l, p_B)$ and show that manipulating and exerting effort is optimal. The monitor prefers to exert effort, because her value is the same as in the case without bribes.

Specifically, on $[p_l, p_B)$, her value satisfies the HJB Equation

$$\begin{aligned} rV_B(p) &= -c - \lambda p(1-p)V_B'(p) + \lambda b(p) \\ &= -c - \lambda p(1-p)V_B'(p) + \lambda p(R - V_B(p)), \end{aligned}$$

which is identical to Equation (23). Since on $[0, p_l)$, we have $V_B(p) = V(p)$, the boundary condition at p_l is the same as well. Thus, $V_B(p) = V(p)$ on $[p_l, p_B)$. Lemma 23 then goes through and establishes that the monitor prefers to exert effort on $[p_l, p_B)$.

The lemma below shows that the agent prefers to manipulate.

Lemma 38. *On $[p_l, p_B)$, we have $\omega_B(p) > 0$.*

Proof. We will show that $W_B(p) > W(p)$ on $[p_l, p_B)$. Since $\omega_B(p) = rW_B(p)$ and $\omega(p) = rW(p)$ on this region, this establishes the result.

The proof relies on a comparison argument. On $[p_l, p_B)$, the agent's value satisfies the following HJB equation,

$$rW_B(p) = B - \lambda p(1-p)W_B'(p) - \lambda p(R - V_B(p)), \quad (53)$$

while the monitor's value satisfies $V_B(p) = V(p)$.

At p_l , the boundary value of the agent's HJB equation is larger than in the case without bribes, i.e. $W_B(p_l) > W(p_l)$, because $W_B(p) > W(p)$ for $p < p_l$.⁴⁶

We now show that $W_B(p)$ and $W(p)$ cannot cross on $[p_l, p_B)$. Define

$$\tilde{p} = \inf \{p \in (p_l, p_B) : W_B(p) = W(p)\}.$$

Suppose by way of contradiction that \tilde{p} exists. Using the two HJB Equations and some algebra yields

$$\lambda p(1-p)(W'(\tilde{p}) - W_B'(\tilde{p})) = \lambda p(R - K - (V_B(\tilde{p}) + W_B(\tilde{p}))).$$

By construction, we have $R - V_B(p) < K + W_B(p)$ on $[p_l, p_B)$. Thus, the RHS in the above equation is negative, which implies that $W'(\tilde{p}) < W_B'(\tilde{p})$. Thus, $W(p)$ cannot cross $W_B(p)$ from below. But, since $W_B(p_l) > W(p_l)$, if \tilde{p} exists, we must have $W'(\tilde{p}) \geq W_B'(\tilde{p})$, a contradiction. Thus, we have $W_B(p) > W(p)$ for all $p \in [p_l, p_B)$.

Since $p_B \leq \bar{p}_h$, we have $\omega(p) \geq 0$ on $[p_l, p_B)$. But then

$$\omega_B(p) = rW_B(p) > rW(p) = \omega(p) \geq 0,$$

which is what we set out to prove. ■

We now consider the interval $[p_B, p_h)$. We confirm that on this region, the monitor prefers to exert effort and the agent prefers to manipulate.

Lemma 39. *For $p \in [p_B, p_h)$, we have $\omega_B(p) > \omega(p) \geq 0$ and $\gamma p V_B'(p) > c$.*

Proof. As in the previous cases, the monitor's value function is identical to the case without bribes, i.e., $V_B(p) = V(p)$ for $p \in [p_B, p_h)$, so all results about the monitor's value function in Lemma 23 apply. In particular, the monitor prefers to exert effort for $[p_B, p_h)$.

⁴⁶We have established this in Lemma 37.

The agent's value function satisfies the HJB Equation (24). The only difference is that at p_B , the boundary value is larger, i.e., $W_B(p_B) > W(p_B)$. A similar argument as in Lemma 38 above shows that $W_B(p)$ and $W(p)$ cannot cross on $[p_B, p_h]$, which implies that $\omega_B(p) > \omega(p) \geq 0$. Thus, manipulating is optimal for the agent. ■

To finish constructing equilibria, we must show that not manipulating and not exerting effort is optimal for $p \geq p_h$. The argument in Lemma 25 goes through, so we omit the proof. As in Lemma 24, we can characterize bounds \underline{p}_h^B and \bar{p}_h^B so that any equilibrium must have $p_h \in [\underline{p}_h^B, \bar{p}_h^B]$. We also omit these arguments since they are similar.

We now show that if $p_B \in [p_l, p_h)$, the joint value $S(p)$ is decreasing for $p \in [p_l, p_B)$ and stays below $R - K$ for $p \in (p_B, p_h)$. We do not need to consider $p \geq p_h$, because when the agent does not manipulate, N_t never realizes. Thus, no bribes can potentially be offered. In other words, we must have $p_B \leq p_h$ in any equilibrium.

Lemma 40. *If $p_B \in [p_l, p_h)$, then, on $[p_l, p_h)$, $S(p)$ crosses $R - K$ exactly once from above at p_B .*

Proof. We treat the regions $[p_l, p_B)$ and $[p_B, p_h)$ separately. We start with $[p_l, p_B)$.

Using the monitor's HJB Equation (23) and the agent's HJB Equation (53) yields

$$r(V_B(p) + W_B(p)) = B - c - \lambda p(1 - p)(V_B'(p) + W_B'(p)).$$

Thus, the joint value $S(p)$ satisfies the ODE

$$rS(p) = B - c - \lambda p(1 - p)S'(p). \quad (54)$$

This equation admits the general solution

$$S(p) = \frac{B - c}{r} + C_0^S \frac{(1 - p)^{\frac{r}{\lambda}}}{p^{\frac{r}{\lambda}}}. \quad (55)$$

We will show that $C_0^S > 0$, which implies $S'(p) < 0$. At p_l , have $S(p_l) = \frac{B}{r}$, which follows from Equation (52). Thus, the value matching condition is

$$\frac{B}{r} = \frac{B - c}{r} + C_0^S \frac{(1 - p_l)^{\frac{r}{\lambda}}}{p_l^{\frac{r}{\lambda}}}$$

or equivalently

$$C_0^S \frac{(1 - p_l)^{\frac{r}{\lambda}}}{p_l^{\frac{r}{\lambda}}} = \frac{c}{r} > 0.$$

This shows that $S'(p) < 0$ on $[p_l, p_B)$. At p_B , we must have $S(p_B) = R - K$. Plugging this into Equation (54) yields

$$rR - rK - B + c = -\lambda p_B(1 - p_B)S'(p_B).$$

Because of Condition (9), the LHS is positive and thus indeed $S'_-(p_B) < 0$.

Next, consider the interval $[p_B, p_h)$. We have

$$\begin{aligned} r(V_B(p) + W_B(p)) &= B - c - \lambda p(1-p)(V_B'(p) + W_B'(p)) \\ &\quad + \lambda p(R - K - V_B(p) - W_B(p)). \end{aligned}$$

Thus, $S(p)$ solves the ODE

$$\begin{aligned} rS(p) &= B - c - \lambda p(1-p)S'(p) \\ &\quad + \lambda p(R - K - S(p)). \end{aligned} \tag{56}$$

Using ODEs (54) and (56), the value matching condition, and the fact that $S(p_B) = R - K$ implies that $S'_-(p_B) = S'_+(p_B) < 0$. That is, smooth pasting holds at p_B and $S'_+(p_B)$ is negative.

The joint value $S(p)$ cannot cross $R - K$ again on the interval (p_B, p_h) . To see this, suppose that

$$\tilde{p} = \inf \{p \in (p_B, p_h) : S(p) = R - K\}$$

exists. At this point, we have

$$rR - rK - B + c = -\lambda\tilde{p}(1-\tilde{p})S'(\tilde{p}).$$

Condition (9) implies that $S'(\tilde{p}) < 0$. Since $S(p)$ must cross $R - K$ from below at \tilde{p} , this is a contradiction. Thus, $S(p) < R - K$ on (p_B, p_h) . ■

Since Condition (8) holds at $p = 0$, the above result establishes the existence of a point $p_B \leq p_h$, such that offering bribes is optimal for $p < p_B$.

It does not necessarily have to be the case that $S(p_B) = R - K$. In particular, we could have $S(p) > R - K$ for all $p \in [0, p_h)$. In this case, p_B is determined via $p_B = p_h$, since for $p > p_h$, there is no manipulation and therefore no possibility to pay bribes. If, on the other hand, $S(p_h) < R - K$, then $p_B < p_h$ and $S(p_B) = R - K$.

Let us now briefly sketch why this class of equilibria is unique. Because of Condition (9), offering bribes on $[0, p_l)$ is always optimal, as we have shown above. Suppose that $p_B \in [p_l, p_h)$, so that offering bribes is optimal for $p \leq p_B$, but there is another threshold $\tilde{p}_B > p_B$, so that offering bribes is optimal at \tilde{p}_B . On $[p_B, \tilde{p}_B)$, we must have $m(p) = e(p) = 1$. Therefore, the joint value satisfies Equation (56). At \tilde{p}_B , we must have $S(\tilde{p}_B) = R - K$. But Lemma 40 shows that this is impossible. Thus, no such point can exist. Any equilibrium must feature bribes if and only if p is below p_B .

We close our characterization by showing that with bribes, the agent is more willing to manipulate. This follows as a Corollary to Lemma 39.

Corollary 41. *Let \bar{p}_h^B be the threshold characterizing the equilibrium with the most manipulation when there are bribes, i.e., $\bar{p}_h^B > p_l$ and $W_B(\bar{p}_h^B) = 0$. We have $\bar{p}_h^B > \bar{p}_h$.*

Proof. In Lemma 39, we have shown that $\omega_B(p) > \omega(p)$ for $p \in [p_B, p_h)$. Picking $p = p_h = \bar{p}_h$ implies $\omega_B(\bar{p}_h) > \omega(\bar{p}_h) = 0$. Thus, $\bar{p}_h^B > \bar{p}_h$. ■

We now characterize the case when bribes are always offered. Since much of the argument is identical to the previous case, we only provide a brief sketch. The proof relies on deriving a contradiction when $p_B \in (p_l, p_h)$, which shows that $p_B = p_h$.

Suppose that Condition (10) and Condition (5) hold. As before, the two conditions hold simultaneously whenever c is sufficiently small. Condition (10) implies

$$R < \frac{B}{r} + K.$$

Because of this, the analysis in Lemmas 37 and 38 goes through. Thus, offering bribes must be strictly optimal for $p \in [0, p_l]$ and, by continuity of $S(p)$, on some nonempty interval $[p_l, p_B)$

Suppose now that $p_B \in (p_l, p_h)$, so that offering bribes is not optimal for $p > p_B$. We now derive our contradiction.

On $[p_l, p_B)$, the joint value satisfies Equation (54) in Lemma 40 with the same boundary condition, i.e. $S(p_l) = \frac{B}{r}$. Just as in Lemma 40, this implies that $S'(p) < 0$ for $p \in [p_l, p_B)$. At p_B , we must have $S(p_B) = R - K$. Plugging this into Equation (54) yields

$$\lambda p(1-p) S'_-(p_B) = B - c - rR + rK.$$

However, Condition (10) implies that the RHS is positive. Thus, $S'_-(p_B) > 0$. Since $S'(p)$ is continuous on (p_l, p_B) , this is a contradiction.⁴⁷ This shows that there cannot exist a $p_B \in (p_l, p_h)$. The only possibility is that $p_B = p_h$. That is, the agent always offers bribes.

Finally, we show that whenever R is large, no bribes are offered in equilibrium. The argument is similar to the previous case. We therefore only provide a brief sketch.

Assume that Condition (11) and Condition (5) hold. Offering bribes is strictly suboptimal on $[0, p_l]$. Here is the argument. Condition (11) implies that $S(0) < R - K$. Suppose by way of contradiction that it is optimal to offer bribes at $p_B < p_l$. Then, on $[0, p_B)$, we have

$$S(p) = \frac{B}{r} + \lambda p \frac{rR - B - rK}{r(r + \lambda + \gamma)},$$

which follows from Equations (34) and (33). Because of Condition (11), $S(p)$ is increasing. However, equating $S(p_B) = R - K$ yields $p_B > 1$, a contradiction. This shows that we have $S(p) < R - K$ on $[0, p_l]$, so offering bribes is strictly suboptimal on that region.

The interval $[0, p_l]$ is then the same as in Lemma 21. On this region, the monitor's and agent's values are the same as in the case without bribes.

By continuity of $S(p)$, there exists some interval $[p_l, p_B)$, so that offering bribes is strictly suboptimal. Suppose that offering bribes is optimal at belief $p_B \in (p_l, p_h]$. We will show that this yields a contradiction.

The monitor's and agent's values continue to be the same as in the case without bribes on $[p_l, p_B)$. In particular, Lemmas 22 and 23 continue to hold on that region. On $[p_l, p_B)$, $S(p)$ satisfies ODE (56), since offering bribes is not optimal on that region. Since $S(p)$ is continuous, we must have $S(p) < R - K$ for $p \in [p_l, p_B)$ and $S(p_B) = R - K$. However, at p_B , we have

$$\lambda p(1-p) S'_-(p) = B - c - rR + rK < 0,$$

because of Condition (11). This contradicts $S(p)$ crossing $R - K$ at p_B from below. Thus, we cannot have $p_B \in (p_l, p_h]$. In particular, since $p_B > p_l$, we cannot have $p_B \leq p_h$. Thus, in equilibrium, bribes are never offered.

⁴⁷Continuity of $S'(p)$ follows from the general solution in Equation (55).

A.6 Robustness

A.6.1 Proof of Proposition 12

We split Proposition 12 into two parts and prove them separately.

Proposition 42. *Suppose that $R(p)$ is strictly increasing and continuously differentiable. Then, for r sufficiently small and $R(p)$ sufficiently large, any monotone-manipulation equilibrium is characterized by two thresholds $p_l < p_h$, such that the monitor exerts effort if and only if $p \in [p_l, p_h)$ and the agent manipulates if and only if $p < p_h$.*

Proof. The proof is similar to the proof of Proposition 3. We therefore keep it brief.

The monitor's HJB equation is

$$\begin{aligned} rV(p) &= -ce(p) - (\lambda p(1-p)m(p) + \gamma p(1-e(p)))V'(p) \\ &\quad + \lambda pm(p)(R(p) - V(p)). \end{aligned} \quad (57)$$

We first construct the region $[0, p_l)$, on which $m(p) = 1$ and $e(p) = 0$. On this region, we have

$$pV'(p) = \frac{\lambda pR(p) - (r + \lambda p)V(p)}{\lambda(1-p) + \gamma}. \quad (58)$$

Recall that the monitor prefers to shirk whenever $\gamma pV'(p) < c$. This inequality holds whenever

$$\lambda p(R(p) - V(p)) - rV(p) < \frac{\lambda c}{\gamma}(1-p) + c.$$

Letting p approach zero, the inequality becomes

$$rV(0) > -c \frac{\lambda + \gamma}{\gamma}.$$

Since $V(0) = 0$, this condition holds. Thus, for p sufficiently close to zero, $\gamma pV'(p) < c$. The region $[0, p_l)$ exists.

On $[0, p_l)$, we have $V'(p) > 0$. Here is the argument. The monitor's value is nonnegative for all p , since for any strategy of the agent, the monitor can guarantee himself a nonnegative payoff by choosing $m(p) = 0$. This implies that $V'(0) > 0$. We can differentiate the monitor's HJB equation to yield

$$(\lambda p(1-p) + \gamma p)V''(p) = \frac{r}{p}(V(p) - pV'(p)) + \lambda pR'(p). \quad (59)$$

Thus, $V''(p) > 0$ whenever $V'(p) = 0$ so that $V'(p)$ cannot cross zero from above. Thus, we have $V'(p) > 0$ for all $p \in [0, p_l)$.

We now adapt the arguments of Lemma 21. Define p_l with $\gamma p_l V'_-(p_l) = c$. On this region, the agent's value function is given by Equation (34). Mirroring Condition (5), we have assumed that $R(p)$ is sufficiently large, so that $\omega(p) > 0$ for all $p < p_l$.

Suppose by way of contradiction that the region ends at a point $\hat{p}_l < p_l$, at which $\gamma \hat{p}_l V'_-(\hat{p}_l) < c$. Suppose first that to the right of \hat{p}_l , the equilibrium features $m(p) = e(p) = 1$. Then, using a similar argument as in the proof of Lemma 21, we can show that $V'_+(\hat{p}_l) < \hat{V}'_-(p_l)$, so that exerting

effort to the right of \hat{p}_l is not optimal. Specifically, the monitor's HJB equation to the left of \hat{p}_l is

$$rV(\hat{p}_l) = -(\lambda\hat{p}_l(1 - \hat{p}_l) + \gamma\hat{p}_l)V'_-(\hat{p}_l) + \lambda\hat{p}_l(R(\hat{p}_l) - V(\hat{p}_l))$$

and the HJB equation to the right of \hat{p}_l is

$$rV(\hat{p}_l) = -c - \lambda\hat{p}_l(1 - \hat{p}_l)V'_+(\hat{p}_l) + \lambda\hat{p}_l(R(\hat{p}_l) - V(\hat{p}_l)).$$

Thus,

$$\lambda\hat{p}_l(1 - \hat{p}_l)(V'_+(\hat{p}_l) - V'_-(\hat{p}_l)) = -c + \gamma\hat{p}_lV'_-(\hat{p}_l) < 0.$$

Suppose instead that to the right of \hat{p}_l , the equilibrium features $m(p) = e(p) = 0$. The same argument as in the proof of Lemma 21 then shows that $W'_+(\hat{p}_l) < W'_-(\hat{p}_l)$, so that $\omega_+(\hat{p}_l) > \omega_-(\hat{p}_l) > 0$. Not manipulating right of \hat{p}_l thus cannot be an equilibrium.

Finally, suppose that we have $\hat{p}_l > p_l$, so that $\gamma\hat{p}_lV'_-(\hat{p}_l) > c$. Then, there is a region on which the monitor does not exert effort, even though it would be optimal for him to do so. This establishes that the region $[0, p_l]$ is characterized by the indifference condition $\gamma p_l V'_-(p_l) = c$.

Now, consider the region $[p_l, p_h)$ on which the agent manipulates and the monitor exerts effort. The monitor's value function satisfies smooth pasting at p_l . Using a similar argument as above, we can show that $V'(p)$ cannot cross zero from above on $[p_l, p_h)$. Since $V'_-(p_l) = \frac{c}{\gamma p_l} > 0$, this, together with smooth pasting, establishes that $V'(p) > 0$ on $[p_l, p_h)$. The same arguments as in the proof of Lemma 23 yield $W'(p) < 0$ and $\omega'(p) < 0$.

It only remains to show that $\gamma p V'(p)$ is increasing, which guarantees that the monitor is willing to exert effort on $[p_l, p_h)$. Suppose that $V'(p) + pV''(p) = 0$ for some $p > p_l$. Differentiating the monitor's HJB equation at this belief yields

$$0 = \frac{1}{p} (rV(p) + c + p(\lambda - r - \lambda p)V'(p)) + \lambda p R'(p).$$

For any $p < 1$, there exists a sufficiently small r at which this equation cannot hold. This is because $V'(p) > 0$ and $\lambda - r - \lambda p > 0$ for $p < \frac{\lambda - r}{\lambda}$. Thus, $\gamma p V'(p) > c$ for all $p \in [p_l, p_h)$.

Since the agent's value function satisfies Equation (24) on $[p_l, p_h)$ and Equation (18) on $[p_h, 1]$, we can use the same arguments as in the proof of Lemmas 23 and 25 to construct the equilibrium to the right of p_h . In particular, we can again define \bar{p}_h as $\omega(\bar{p}_h) = 0$. On $[p_h, 1]$, the monitor's value function has the closed-form solution in Equation (25), i.e. the same as in the case with constant reward. The same argument as in the proof of Lemma 25 then establishes that $\gamma p V'(p) < c$ for $p > p_h$. ■

Proposition 43. *Suppose that $\hat{R}(p)$ and $R(p)$ are strictly increasing and continuously differentiable and that r and K are sufficiently small. Additionally, suppose that $\hat{R}'(p) > R'(p)$ for all p and that $\hat{R}(p)$ and $R(p)$ satisfy single crossing, i.e. $\hat{R}(p) < R(p)$ for $p < \hat{p}$ and $\hat{R}(p) > R(p)$ for $p > \hat{p}$. Let p_l be the threshold at which the monitor starts exerting effort under $R(p)$. Then, there exists a $\tilde{p}_l < \hat{p}$, so that if $p_l > \tilde{p}_l$ the worst equilibrium under $\hat{R}(p)$ features less manipulation than the worst equilibrium under $R(p)$.*

Proof. We denote with $\hat{V}(p)$ the monitor's value function given reward $\hat{R}(p)$ and with $V(p)$ the value given $R(p)$. Let p_l denote the threshold below which the monitor shirks under reward $R(p)$, i.e. the threshold defined via $\gamma p V'(p) = c$, and let \hat{p}_l denote its analog under $\hat{R}(p)$. We will show

that $\hat{V}'(p) > V'(p)$ for $p \geq \tilde{p}_l$ for some $\tilde{p}_l < \hat{p}$, which implies the desired result, i.e., $\hat{p}_l > p_l$ whenever p_l is sufficiently close to \hat{p} .

On the region $[0, p_l)$, the monitor's value satisfies

$$V'(p) = \frac{\lambda p R(p) - (r + \lambda p) V(p)}{\lambda p (1 - p) + \gamma p}, \quad (60)$$

which follows from the HJB Equation (57). The analog holds for $\hat{V}(p)$.

We now study solutions to the differential Equation (60) with the boundary conditions $V(0) = 0$ and $\hat{V}(0) = 0$, respectively, on the domain $[0, 1]$.⁴⁸

Using Equation (60) yields

$$(\lambda p (1 - p) + \gamma p) (\hat{V}'(p) - V'(p)) = (r + \lambda p) (V(p) - \hat{V}(p)) + \lambda p (\hat{R}(p) - R(p)). \quad (61)$$

This equation implies the following. First, we have $\hat{V}'(0) < V'(0)$, because $V(0) = \hat{V}(0) = 0$ and $\hat{R}(0) < R(0)$.

Second, let

$$\tilde{p} = \inf \left\{ p > 0 : \hat{V}'(p) = V'(p) \right\}.$$

This point exists. If it does not exist, then $\hat{V}'(p) < V'(p)$ for all $p > 0$ and therefore $\hat{V}(p) < V(p)$ for all $p > 0$. But then for $p > \hat{p}$, Equation (61) cannot hold, which is a contradiction.

We have $V(\tilde{p}) > \hat{V}(\tilde{p})$ by construction of \tilde{p} . Moreover, Equation (61) implies that $\hat{R}(\tilde{p}) < R(\tilde{p})$. Thus, we have $\tilde{p} < \hat{p}$.

Third, let

$$\tilde{\pi} = \inf \left\{ p > 0 : \hat{V}(p) = V(p) \right\}.$$

If $\tilde{\pi}$ does not exist, then $\hat{V}(p) < V(p)$ for all $p > 0$. In this case, Equation (61) implies that $\hat{V}'(p) > V'(p)$ for all $p \geq \hat{p}$, which is the property we are trying to prove. Suppose that $\tilde{\pi}$ exists. Then, $\hat{V}(p)$ must cross $V(p)$ from below at $\tilde{\pi}$. This implies that $\hat{V}'(\tilde{\pi}) \geq V'(\tilde{\pi})$. Given Equation (61), this inequality holds if and only if $\hat{R}(\tilde{\pi}) \geq R(\tilde{\pi})$. Thus, we must have $\tilde{\pi} \geq \hat{p}$. This implies that $\hat{V}(p) < V(p)$ for all $p < \hat{p}$.

Fourth, $\hat{V}(p)$ can cross $V(p)$ at most once. If there are multiple crossing points, there must exist a point to the right of $\tilde{\pi}$ at which $\hat{V}(p)$ crosses $V(p)$ from above. Since $\tilde{\pi} > \hat{p}$, that point must lie to the right of \hat{p} , and Equation (61) implies that $\hat{V}'(p) > V'(p)$, a contradiction.

We now use the properties of $\tilde{\pi}$ we have just derived to show that $\hat{V}'(p) > V'(p)$ for all $p \geq \hat{p}$.⁴⁹ Using Equation (59), we can derive

$$\begin{aligned} (\lambda p (1 - p) + \gamma p) (\hat{V}''(p) - V''(p)) &= \frac{r}{p} \left(\hat{V}(p) - V(p) - p (\hat{V}'(p) - V'(p)) \right) \\ &\quad + \lambda p (\hat{R}'(p) - R'(p)). \end{aligned}$$

⁴⁸To save notation, we still use $V(p)$ and $\hat{V}(p)$ to denote these solutions.

⁴⁹As we have argued above, when $\tilde{\pi}$ does not exist, then $\hat{V}'(p) > V'(p)$ for $p \geq \hat{p}$.

We know that $\hat{V}'(\tilde{\pi}) > V'(\tilde{\pi})$. Define with

$$\pi' = \inf \left\{ p > \tilde{\pi} : \hat{V}'(p) = V'(p) \right\}.$$

If no such point exists, then $\hat{V}'(p) > V'(p)$ for all $p \geq \tilde{\pi}$. Suppose by way of contradiction that π' exists. Since $\hat{V}(p) > V(p)$ for $p > \tilde{\pi}$, the above equation implies that $\hat{V}''(\pi') > V''(\pi')$, which is a contradiction, since $\hat{V}'(p)$ must cross $V'(p)$ from above at π' . Thus, no such π' exists and $\hat{V}'(p) > V'(p)$ for $p \geq \tilde{\pi}$.

For $p \in [\hat{p}, \tilde{\pi})$, Equation (61) implies that $\hat{V}'(p) > V'(p)$, because $V(p) > \hat{V}(p)$ and $\hat{R}(p) \geq R(p)$. Thus, $\hat{V}'(p) > V'(p)$ for all $p \geq \hat{p}$. Since both $\hat{V}'(p)$ and $V'(p)$ are continuous, there exists a $\tilde{p}_l < \hat{p}$, such that $\hat{V}'(p) > V'(p)$ for all $p > \tilde{p}_l$.

Now, suppose that $p_l > \tilde{p}_l$, as in the statement of the Proposition. Then, we have $\hat{p}_l < p_l$. This result is immediate from our previous arguments, because they imply that $\hat{V}'(p_l) > V'(p_l)$.

To conclude our proof, we only have to show that with the lower p_l , \bar{p}_h decreases. The effect on the agent's incentives is similar to the case with deadlines, i.e. in Proposition 7. In fact, since the agent's incentives are unchanged except for the decrease in p_l , the argument in the proof of Lemma 32 goes through without modification. This establishes that \bar{p}_h decreases. ■

A.6.2 Proof of Proposition 13

The proof is similar to Proposition 42. We only consider the monitor's incentives. Similar arguments as in the proof of Proposition 3 guarantee that the agent manipulates for $p < p_h$ and does not manipulate for $p \geq p_h$.

We start with characterizing the region $[0, p_l)$. On $[0, p_l)$ the monitor's HJB equation is

$$\begin{aligned} rV(p) &= w(p) - (\lambda p(1-p) + \gamma p)V'(p) \\ &\quad + \lambda p(R(p) - V(p)), \end{aligned}$$

which implies

$$pV'(p) = \frac{w(p) + \lambda pR(p) - (r + \lambda p)V(p)}{\lambda(1-p) + \gamma}.$$

Thus, $\gamma pV'(p) < c$ whenever

$$w(p) + \lambda pR(p) - (r + \lambda p)V(p) < \frac{c}{\gamma}\lambda(1-p) + c.$$

This inequality holds at $p = 0$ whenever

$$w(0) < \frac{c}{\gamma}\lambda + c,$$

i.e., $w(0)$ is sufficiently small. This is true, since we assume that $\lim_{p \downarrow 0} w(p)/p$ is bounded, which implies that $w(0) = 0$. This implies that the region $[0, p_l)$ exists.

We next show that if

$$\frac{w'(p)}{w(p)} > \frac{1}{p},$$

we have $V'(p) > 0$ on $[0, p_l]$.

We can differentiate the monitor's HJB equation to get

$$\lambda p(1-p)V''(p) = w'(p) + \lambda p R'(p) + \frac{1}{p}(rV(p) - w(p)) - rV'(p). \quad (62)$$

Given the condition, whenever $V'(p) = 0$, we have $V''(p) > 0$, so $V'(p)$ cannot cross zero from above. Since $V(0) = 0$ and $V(p) > 0$ for $p > 0$, we then have $V'(0) > 0$, which establishes that $V'(p) > 0$ on $[0, p_l]$.

We next show that $V''(p) > 0$ on $[0, p_l]$, under the conditions that $\lim_{p \downarrow 0} \frac{w(p)}{p}$ is bounded and that r is sufficiently small. This follows from Equation (62) above. As $r \rightarrow 0$, the monitor's HJB equation approaches

$$0 = w(p) - (\lambda p(1-p) + \gamma p)V'(p) + \lambda p(R(p) - V(p)).$$

This equation has a closed form solution

$$V(p) = C_0(\gamma + \lambda(1-p)) + \gamma - \lambda p - \lambda \int_p^1 \frac{\lambda s R(s) + w(s)}{s(\gamma + \lambda(1-s))^2} ds.$$

Differentiating this equation implies that $V'(p)$ is bounded on $[0, p_l]$ if $\lim_{p \downarrow 0} \frac{w(p)}{p}$ is bounded. Thus, under the stated conditions, as $r \rightarrow 0$, the term $rV'(p)$ disappears in Equation (62). Thus, we have $V''(p) > 0$ on $[0, p_l]$. Since $V'(p) > 0$ and $V''(p) > 0$, $\gamma p V'(p)$ is strictly increasing.

As before, we define p_l via $\gamma p_l V'(p_l) = c$. This point exists whenever c is sufficiently small. The argument is analog to Condition (5) and Lemma 21. For c sufficiently small, we also have $\omega(p) > 0$ for $p \leq p_l$, so that the agent finds it optimal to manipulate on $[0, p_l]$.

On $[p_l, p_h]$, the monitor's HJB equation is given by

$$\begin{aligned} rV(p) &= w(p) - c - \lambda p(1-p)V'(p) \\ &\quad + \lambda p(R(p) - V(p)). \end{aligned}$$

Differentiating the HJB equation yields

$$\lambda p(1-p)V''(p) = w'(p) + \lambda p R'(p) + \frac{\lambda}{p}(rV(p) - w(p) + c) - rV'(p).$$

The same arguments as above establish that $V'(p) > 0$ and $V''(p) > 0$, so that $\gamma p V'(p)$ is increasing.

On $[p_h, 1)$, we have

$$rV(p) = w(p) - \gamma p V'(p)$$

and we need that $\gamma p V'(p) < c$, so that shirking is optimal. At $r = 0$, we have $\gamma p V'(p) = w(p)$. Thus, $\gamma p V'(p) < c$ if and only if $w(p) < c$. For r sufficiently small, $w(p) < c$ also implies that $\gamma p V'(p) < c$.

We now have constructed an equilibrium as in Proposition 3. The uniqueness arguments are similar and we skip them.

We close by constructing an equilibrium where $m(p) = 0$ and $e(p) = 1$ on $(p_h, 1)$, under the

condition

$$w'(p) > \frac{rc}{\gamma p}.$$

The monitor's HJB equation for $p > p_h$ is now

$$rV(p) = w(p),$$

so that $V'(p) = \frac{w(p)}{r}$ and

$$\gamma p V'(p) = \frac{\gamma p}{r} w'(p).$$

This expression exceeds c whenever

$$w'(p) > \frac{rc}{\gamma p},$$

which we have assumed. Thus, the monitor indeed prefers to exert effort. It only remains to check that the agent prefers to not manipulate. The agent's value for $p > p_h$ is given by $V(p) = 0$, since he does not manipulate and since the belief never changes. Thus, we have

$$\omega(p) = B - \lambda p K.$$

The agent prefers to not manipulate for $p > p_h$ whenever $p_h > p_m$. As we have shown in the proof of Proposition 3, we have $\bar{p}_h > p_m$, so equilibria with $p_h > p_m$ exist.

A.6.3 Proof of Proposition 14

The arguments are similar to the ones in the proof of Proposition 3. Therefore, we only provide a brief sketch.

Shirking is optimal whenever

$$(\phi(1-p) + \gamma p) V'(p) \leq c. \tag{63}$$

Consider the region $[0, p_l)$ and the closed-form solution in Equation (33). For ϕ sufficiently small, the LHS of Equation (63) is positive below a threshold p_l , which now satisfies

$$p_l = \frac{1}{\gamma - \phi} \left(\frac{c}{\lambda R} (r + \lambda + \gamma) - \phi \right).$$

We can confirm that under analog assumptions to Condition (5), the agent finds it optimal to manipulate to the left of p_l . On $[p_l, p_h)$, the monitor's value function satisfies the following HJB equation:

$$rV(p) = -c - (\lambda p(1-p) - \phi(1-p)) V'(p) + \lambda p (R - V(p)).$$

The solution to this equation is uniformly continuous with respect to ϕ , for p bounded away from zero. Thus, for ϕ sufficiently small, we have $V'(p) > 0$ for $p > p_l$. Lemmas 22, 23 and 24 then go through. For $p > p_h$, Lemma 25 applies without modification.

A.6.4 Proof of Proposition 15

We only sketch the result. The monitor's and agent's HJB equations are

$$\begin{aligned} rV(p) &= -ce(p) - ((\lambda - \lambda_B)p(1-p)m(p) + \gamma p(1-e(p)))V'(p) \\ &\quad + \lambda pm(p)(R - V(p)) - \lambda_B(1-p)m(p)V(p) \end{aligned}$$

and

$$\begin{aligned} rW(p) &= Bm(p) - ((\lambda - \lambda_B)p(1-p)m(p) + \gamma p(1-e(p)))W'(p) \\ &\quad - \lambda pm(p)(K + W(p)) - \lambda_B pm(p)W(p). \end{aligned}$$

For λ_B sufficiently small, beliefs are monotonically decreasing on path. There exists a region $[0, p_l)$ on which the monitor shirks and the agent manipulates. On that region, the monitor's and agent's value functions are uniformly continuous in λ_B . Thus, for λ_B sufficiently small, we have $\omega(p) > 0$ for $p < p_l$ and $\gamma pV'(p) \leq c$ for $p \leq p_l$. Likewise, picking λ_B small ensures that the region $[p_l, p_h)$ exists (Lemma 22), that the agent's incentive to manipulate is decreasing and the monitor's incentive to exert effort is increasing (Lemma 23), and that no effort and no manipulation is optimal for $p > p_h$ (Lemma 25). We can characterize the bounds \underline{p}_h and \bar{p}_h as in Lemma 24.

A.6.5 Proof of Proposition 16

The monitor's and agent's HJB equations are

$$\begin{aligned} rV(p) &= -ce(p) - (\lambda p(1-p)m(p) + \gamma p(1-e(p)))V'(p) \\ &\quad + \lambda pm(p)(V(1) + R - W(p)) \end{aligned}$$

and

$$\begin{aligned} rW(p) &= Bm(p) - (\lambda p(1-p)m(p) + \gamma p(1-e(p)))W'(p) \\ &\quad + \lambda pm(p)(W(1) - K - W(p)). \end{aligned}$$

Intuitively, we have the same HJB equation as in our main model, except that the reward is $\tilde{R} = R + V(1)$ and the punishment is $\tilde{K} = K - W(1)$. For fixed values (\tilde{R}, \tilde{K}) , the construction of equilibria is the same as before. In particular, if $\{p_l, p_h\}$ constitute an equilibrium when the relationship ends after detection (i.e. Condition (5) holds), then they also constitute an equilibrium when the relationship continues. This is because we have $\tilde{R} > R$ and $\tilde{K} < K$ which imply that if Condition (5) holds for R and K , it also holds for \tilde{R} and \tilde{K} . Specifically, $\tilde{R} > R$ implies that p_l is lower than in the main model. Together with $\tilde{K} < K$, this implies that $\omega_-(p_l) > 0$.

Thus, to construct equilibria it only remains to pin down the pair $(V(1), W(1))$ endogenously. On $[p_h, 1)$, we have $m(p) = 0$ so the value functions are the same as in the main model and are given by the closed form solutions in Equations (27) and (28). The closed forms imply that $W(1) = C_0^A$ and $V(1) = C_0^M$, where the constants are determined by value matching at p_h . We now construct an equilibrium with $p_h = \bar{p}_h$. In this equilibrium, we have $W(\bar{p}_h) = 0$, so that $C_0^A = 0$. The agent's value function is then the same as in the main model, i.e. $\tilde{R} = R$.⁵⁰ For the monitor, we can redo

⁵⁰Except that p_l will be different, which will affect the agent's incentives.

the construction of the value function on $[0, p_l)$ and $[p_l, \bar{p}_h)$ with \tilde{R} instead of R . All of our previous results go through. It then only remains to pin down C_0^M .

Let C_{00}^M be the constant in the main model. Let us pick an arbitrary $C_0^M \geq C_{00}^M$. Then for any such C_0^M , our equilibrium construction goes through and there exists an equilibrium which is characterized by $\{p_l(C_0^M), \bar{p}_h(C_0^M)\}$. We can show that p_l is decreasing in C_0^M and that \bar{p}_h is decreasing as well.⁵¹ The constant C_0^M must satisfy the value matching condition at \bar{p}_h , i.e.,

$$V(\bar{p}_h(C_0^M), C_0^M) = C_0^M \bar{p}_h(C_0^M)^{-\frac{\pi}{\gamma}}.$$

Here, we have written the monitor's value as $V(p, C_0^M)$ for $p \leq \bar{p}_h$ to highlight the dependence of this function on C_0^M . The monitor's value and agent's values are continuous in C_0^M for all p . This implies that $\bar{p}_h(C_0^M)$ is continuous as well.

Existence of a C_0^M which satisfies the value matching condition follows from the continuous mapping theorem. We only have to show that for C_0^M small,

$$V(\bar{p}_h(C_0^M), C_0^M) > C_0^M \bar{p}_h(C_0^M)^{-\frac{\pi}{\gamma}}$$

and that for C_0^M large, the opposite inequality holds. Suppose that $C_0^M = C_{00}^M$. Then, \bar{p}_h is the same as in the main model. We have

$$V(\bar{p}_h(C_{00}^M), C_{00}^M) > V(\bar{p}_h(C_{00}^M)) = C_{00}^M \bar{p}_h^{\frac{\pi}{\gamma}},$$

where $V(p)$ denotes the value in the main model, i.e. when the relationship ends after detection.

Conversely, if C_0^M becomes large, then p_l approaches zero and \bar{p}_h approaches p_{mon} . The closed form solution in Equation (27) implies that as C_0^M becomes large, the monitor's value approaches

$$\lambda p \frac{c + r(R + C_0^M)}{r(r + \lambda)} - \frac{c}{r}.$$

Thus, a sufficient condition for

$$V(\bar{p}_h(C_0^M), C_0^M) < C_0^M \bar{p}_h(C_0^M)^{-\frac{\pi}{\gamma}}$$

to hold is

$$\lambda p_{mon} \frac{c + r(R + C_0^M)}{r(r + \lambda)} - \frac{c}{r} < C_0^M p_{mon}^{-\frac{\pi}{\gamma}}.$$

As C_0^M becomes large, this inequality holds if

$$\frac{\lambda}{r + \lambda} < p_{mon}^{\frac{-\pi + \gamma}{\gamma}},$$

which is true because $p_{mon} < 1$. Thus, the continuous mapping theorem implies that there exists a C_0^M such that value matching holds. We have now constructed an equilibrium.

⁵¹The argument for \bar{p}_h being decreasing is the same as in Lemma 30. That p_l is decreasing follows from the closed form solution (33). Increasing C_0^M increases \tilde{R} , which then increases $V'(p)$ for all $p \in [0, p_l]$.

A.6.6 Proofs for Section 7.5

Proof of Lemma 17. Suppose first that the monitor never exerts effort. The agent's value function solves the HJB equation

$$\begin{aligned} rW(p) &= \lambda(1-p)B - \kappa + \lambda p(1-p)W'(p) \\ &\quad + \lambda(1-p)(W(0) - W(p)) \end{aligned}$$

and admits the closed form solution

$$W(p) = \frac{(B + W(0))\lambda(1-p)}{r + \lambda} - \frac{\kappa(r + \lambda p)}{r(r + \lambda)} + C_0^A \frac{p^{\frac{r+\lambda}{\lambda}}}{(1-p)^{\frac{r}{\lambda}}}. \quad (64)$$

The agent manipulates for $p \leq p_{nm}$, and the boundary condition is $W(p_{nm}) = 0$.

We now show that $p_{nm} > p_m$. The optimal threshold satisfies smooth pasting, i.e. $W'(p_{nm}) = 0$. Otherwise, if $W'(p_{nm}) < 0$, the agent can increase his value by continuing at p_{nm} , until he reaches a threshold $p'_{nm} > p_{nm}$ at which $W'(p_{nm}) = 0$. We have

$$\omega(p_{nm}) = B\lambda(1 - p_{nm}) - \kappa + \lambda p_{nm}(1 - p_{nm})W'(p_{nm}) + \lambda(1 - p_{nm})(W(0) - W(p_{nm})) = 0.$$

Since $W'(p_{nm}) = W(p_{nm}) = 0$ and $W(0) > 0$, the first two terms must be negative. Thus, $p_{nm} > p_m$.

The closed form in Equation (64) and the fact that $W(p_m) > 0$ then imply that $C_0^A > 0$, so that $W''(p) > 0$ for $p \leq p_{nm}$ and $W'(p) < 0$ for $p < p_{nm}$.

Consider the case when the monitor always exerts effort. We denote the agent's value with $\tilde{W}(p)$. The agent's value satisfies the HJB equation

$$\begin{aligned} r\tilde{W}(p) &= -\kappa + (\lambda p(1-p) + \gamma(1-p))\tilde{W}'(p) \\ &\quad + \lambda(1-p)(B + W(0) - \tilde{W}(p)) \end{aligned}$$

and admits the closed form solution

$$\tilde{W}(p) = \frac{(B + W(0))\lambda(1-p)}{r + \lambda + \gamma} - \frac{\kappa(r + \lambda p + \gamma)}{r(r + \lambda + \gamma)} + C_0^A \frac{(\gamma + \lambda p)^{\frac{r+\lambda+\gamma}{\lambda+\gamma}}}{(1-p)^{\frac{r}{\lambda+\gamma}}}. \quad (65)$$

The argument for establishing that $p_{mon} > p_m$ is exactly the same as in the previous case.

To show that $p_{mon} < p_{nm}$, we subtract the two HJB equations to get

$$(r + \lambda(1-p))(W(p) - \tilde{W}(p)) = \lambda p(1-p)(W'(p) - \tilde{W}'(p)) - \gamma(1-p)\tilde{W}'(p).$$

Suppose that $p_{mon} \geq p_{nm}$. Then, at p_{nm} , we have $\tilde{W}(p_{nm}) \geq W(p_{nm}) = 0$ and $\tilde{W}'(p_{nm}) \leq W'(p_{nm}) = 0$. However, since $\tilde{W}'(p) < 0$ for $p < p_{mon}$ (because $\tilde{W}(p)$ is convex and its derivative is zero at p_{mon}), this contradicts the equation above. Thus, we have $p_{mon} < p_{nm}$. ■

Proof of Proposition 18. Let p_h denote the threshold below which the agent manipulates. In any equilibrium, we must have $p_l \leq p_h$. Once the agent stops manipulating, there is no value for the

monitor to exert effort, so that $e(p) = 0$ for all $p \geq p_h$. Thus, $p_l \leq p_h$.

We first consider the case $p_l < p_h$. Consider the region $[p_l, p_h]$. On this region, the agent's value does not depend on actions to the left of p_l . Thus, it is the same as in Lemma 17 and p_h is determined the same way. The monitor's value is given by

$$V(p) = \frac{R(rp + \lambda)}{r(r + \lambda)} + C_0^M \frac{p^{\frac{r+\lambda}{\lambda}}}{(1-p)^{\frac{r}{\lambda}}},$$

with boundary condition $V(p_h) = \frac{R}{r}$. We denote the monitor's incentive to exert effort with

$$\omega_M(p) = -c + \gamma(1-p)V'(p).$$

By construction, we have $\omega_M(p_l) = 0$. We now show that $\omega_M(p) < 0$ on (p_l, p_h) .

The boundary condition implies that $C_0^M > 0$, so that $V(p)$ is convex on $[p_l, p_h]$. Specifically, we have

$$C_0^M = R \frac{1-p_h}{r+\lambda} \frac{(1-p_h)^{\frac{r}{\lambda}}}{p_h^{\frac{r+\lambda}{\lambda}}} \quad (66)$$

Differentiating the closed-form expression for the monitor's value yields

$$\begin{aligned} \omega_M(p) = & -c + \gamma(1-p) \frac{R}{r+\lambda} \\ & + \gamma(1-p) C_0^M \left(\frac{r+\lambda}{\lambda} \left(\frac{p}{1-p} \right)^{\frac{r}{\lambda}} + \frac{r}{\lambda} \left(\frac{p}{1-p} \right)^{\frac{r+\lambda}{\lambda}} \right). \end{aligned}$$

For r sufficiently small, we get $\omega'_M(p) < 0$. As $r \rightarrow 0$, C_0^M is bounded, which can be seen from Equation (66), and the last term in brackets vanishes.

We have shown that $\omega_M(p) < 0$ on $[p_l, p_h]$. Now, we can simply define p_l as the point at which $\omega_M(p)$ hits zero. It remains to verify that the agent indeed prefers to manipulate for $p < p_l$. On this region, we have

$$\omega(p) = rW(p) - \gamma(1-p)W'(p), \quad (67)$$

while for $p \in [p_l, p_h]$, we have $\omega(p) = rW(p)$. This implies that

$$\omega_-(p_l) - \omega_+(p_l) = -\gamma(1-p_l)W'_-(p_l). \quad (68)$$

The two closed-form expressions in Equations (64) and (65), together with the value matching condition at p_l , imply that the constant C_0^A in Equation (65) is positive, so that $W(p)$ is strictly convex on $[0, p_l]$. A similar argument as in the proof of Lemma 22 then establishes that

$$(\lambda p(1-p) + \gamma(1-p))(W'_-(p_l) - W'_+(p_l)) = -\gamma(1-p)W'_+(p_l) > 0,$$

because $W'_+(p_l) < 0$. Plugging in Equation (68) yields $\omega_-(p_l) > \omega_+(p_l)$. Thus, it is optimal for the agent to manipulate on some interval to the left of p_l .

Since $\omega_+(p_l) > 0$, Equation (68) implies that $W'_-(p_l) < 0$. Since $W(p)$ is convex on $[0, p_l]$, we

have $W'(p) < 0$ for any $p \in [0, p_l)$. Differentiating Equation (67), yields

$$\omega'(p) = (r + \gamma) W'(p) - \gamma(1 - p) W''(p) < 0,$$

since $W'(p) < 0$ and $W''(p) > 0$. Thus, $\omega(p) > 0$ for $p < p_l$ and the agent indeed prefers to manipulate on $[0, p_l)$.

We now consider the case $p_l = p_h$. In that case, the monitor exerts effort for all $p \leq p_h$, i.e. $\omega_M(p) > 0$ for all $p \leq p_h$. The agent's incentives are the same as in the benchmark case in Lemma 17 and p_h is the same as well.

Finally, we show that a term limit reduces manipulation. Specifically, suppose that, conditional on bad news occurring, the relationship is dissolved after a fixed time T . This lowers $W(0)$ and therefore p_h . ■