Optimal Project Design

George Georgiadis (Northwestern Kellogg)

with D. Garrett (TSE), A. Smolin (TSE), and B. Szentes (LSE)

Motivation

- Rents due to agency problems is key determinant of economic welfare
- Determinants of these frictions are usually part of model description
  - In adverse selection models, distribution of types typically exogenous
  - In moral hazard models, production technology taken as given
- If an agent's payoff depends on agency frictions, then he is likely to take actions to generate these frictions optimally.

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Model

- *Players.* Risk-neutral principal & agent, and latter is cash-constrained
- *Timing.*
  i. Agent chooses a “project” $c : \Delta([0,1]) \to \mathbb{R}_+$; *i.e.*, a map from every output distribution with support on $[0,1]$ to a (nonnegative) cost.
  ii. Principal offers a wage scheme $w : [0,1] \to \mathbb{R}_+$
  iii. Agent chooses an “action” $F \in \Delta([0,1])$
  iv. Output $x \sim F$ and payoffs are realized
- *Payoffs.*
  - Agent: $\mathbb{E}_F[w(x)] - c(F)$
  - Principal: $\mathbb{E}_F[x - w(x)]$
  - Both players have outside option 0

Applications

- An entrepreneur (agent) seeks funding from a VC (principal)
- Before contracting, the entrepreneur must develop a business plan, specifying various aspects of his production function
- Conceivable he has at least some flexibility in choosing the biz plan.
- If VC has a lot of bargaining power, the entrepreneur benefits from putting forward a biz plan that exacerbates moral hazard problem.
- *Remark:* Abstract away from constraints in the agent's flexibility.
- More broadly, employees can often influence aspects of production function (*e.g.*, assignment of projects, goals, evaluation metrics, etc), which provides an opportunity to shape their production technology.

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Some Intuition

- *First Best.*
  - Agent sets $c(F) = 0$ for all F
  - Principal responds by offering wage 0 and implementing $F(x) = \mathbb{I}_{\{x=1\}}$
- Outcome is efficient but the agent is left with no rents!
- *Mechanism.* Agent chooses the project to make the moral hazard problem *severe*, which will enable him to extract rents.

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Problem Formulation

- *Principal.* Given project c, she solves:

$$\begin{aligned} \max_{w(\cdot),F} \quad & \mathbb{E}_F[x - w(x)] \\ \text{s.t.} \quad & \mathbb{E}_F[w(x)] - c(F) \geq \mathbb{E}_{\widetilde{F}}[w(x)] - c(\widetilde{F}) \quad \text{for all } \widetilde{F} \\ & w(x) \geq 0 \text{ for all } x \\ & F \in \Delta([0,1]) \end{aligned}$$

Denote the optimal contract by w^c and implemented action by F^c.

- *Agent.* Chooses the optimal project by solving:

$$\begin{aligned} \max \; & \mathbb{E}_{F^c}[w^c(x)] - c(F^c) \\ \text{s.t.} \; & c : \Delta([0,1]) \to \mathbb{R}_+ \end{aligned}$$

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Main Results

1. Optimal project is *coarse*: all feasible actions generate binary output
   - Binary projects effectively restrict the contracting space, making it more expensive for the principal to motivate the agent.
2. Action space is *rich*: Optimal (binary) project comprises
   - continuum of zero-cost actions where project succeeds with some prob.
   - a high cost action which guarantees success
   - a spectrum of actions in between.
3. *Inefficiency*: Maximal output realized in equilibrium at bloated costs
4. *Rents*: The agent extracts all rents
5. Characterization of payoff allocations for any production technology

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A Simple Example

- Suppose the agent is restricted to choosing a project comprising two actions, F_L and F_H, with binary output; *i.e.*, $supp(F_i) = \{0, 1\}$
- Easy to solve analytically and show that:
  - F_L costs 0 and leads to $x = 1$ with probability 1/2 (otherwise $x = 0$)
  - F_H costs 1/4 and leads to $x = 1$ with probability 1
  - Principal sets $w(0) = 0$ and $w(1) = 1/2$, implementing F_H
- *Remarks:*
  - Clearly, $c(F_L) = 0$: otherwise, agent can uniformly decrease costs
  - Cost $c(F_H) = 1/4$: just enough for principal to prefer to implement F_H
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Simple Example Continued

- Can the agent benefit from choosing a 3^{rd} action?

YES!

- In the optimal project:
  - F_i leads to $x = 1$ w.p p_i, where $p_L < p_M < p_H$ & $c(F_L) < c(F_M) < c(F_H)$
  - Principal implements F_H, wherein $x = 1$ with probability 1
- Conditional on implementing F_H, intermediate action F_M is useful for the agent because it determines the optimal bonus.
- F_L determines if implementing F_H is optimal for principal.
  - Absent this action, F_M would be implementable with bonus $= c(F_M)$, which could be preferable for the principal (reducing rents to 0).
- Actions *support each other*, enabling agent to extract rents

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Plan of Attack

- *Theorem 1:* Show it suffices to restrict attention to binary projects
  - Given an arbitrary project, we construct a new project such that $c(F) < 1$ iff $supp(F) = \{0, 1\}$, and the agent is (weakly) better off.
- This dramatically reduces the dimensionality of the problem so that:
  - In Stage 1, the agent assigns a cost $C(p) \geq 0$ to each $p = \Pr\{x = 1\}$
  - In Stage 2, the principal offers a bonus contract $w(x) = b\mathbb{I}_{\{x=1\}}$
  - In Stage 3, agent chooses p at a cost $C(p)$
- *Theorem 2:* Characterize the optimal project (in closed form)

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Properties of an Optimal Project

Theorem 1.

- For any project c, there exists another project, $\widetilde{c}$, such that
  - i. $\widetilde{c}(F) < 1$ if and only if $supp(F) = \{0, 1\}$ (*i.e.*, output is binary), and
  - ii. the principal optimally implements $F(x) = \mathbb{I}_{\{x=1\}}$ (*i.e.*, $x = 1$ w.p 1),

  which gives the agent a (weakly) larger expected payoff.

- The principal optimally rewards those outputs which are indicative of the target action, and punishes those indicative of a deviation.
- Binary projects restrict the contracting space, limiting the principal's *screening* ability, and increasing the expected payment to the agent.

Binary Projects: Proof

- Fix a c & suppose principal offers w^*, implementing F^* (w/ mean μ^*)
- Construct a new project $\widetilde{c}$: For each $\mu \in [0,1]$, define

$$B_\mu = (1-\mu) + \mu \mathbb{I}_{\{x=1\}} \text{ and}$$
$$\widetilde{c}(B_\mu) = \inf\{c(F) \ : \ \mathbb{E}_F[x] = \mu\}$$

  i.e., B_μ is a distribution with support $\{0,1\}$ and mean μ, and we assign it the cost of the cheapest distribution in c with same mean.

- Given $\widetilde{c}$, wolog, the principal offers a *bonus contract* $w(x) = b\mathbb{I}_{\{x=1\}}$, or equivalently, a *linear contract* $w(x) = bx$.

Binary Projects: Proof

- Consider the problem of implementing any action at max profit

$$\Pi(F) = \sup_{w(\cdot)\geq 0} \{\mathbb{E}_F[x - w(x)] \ : \ F \text{ is IC}\} \text{ , and}$$

$$\widetilde{\Pi}(B_\mu) = \sup_{b\in[0,1)} \{(1-b)\mu \ : \ B_\mu \text{ is IC}\},$$

in the original and the new project, c and $\widetilde{c}$, respectively.

- *Lemma 1:* For any F such that $\mathbb{E}_F[x] = \mu$, $\widetilde{\Pi}(B_\mu) \leq \Pi(F)$.

  i.e., implementing B_μ is less profitable than an F with same mean.

  - Suppose the principal were restricted to linear contracts in c. Then:

    $$\Pi_{lin}(F) = \widetilde{\Pi}(B_\mu) \quad \text{for all } F \text{ with mean } \mu.$$

  - Absent this restriction, her profit is weakly larger; *i.e.*, $\Pi(F) \geq \Pi_{lin}(F)$.

Binary Projects: Proof

- Define $B^* = B_{\mu^*}$ and $b^* = \mathbb{E}_{F^*}[w^*(x)]/\mu^* < 1$
- If $w(x) = b^* \mathbb{I}_{\{x=1\}}$ implements B^*, then:
  1. It makes the same expected payment to the agent as w^*.
  2. It generates profit equal to $\Pi(F^*)$ for the principal.
- If b^* does *not* implement B^*, adjust cost $\widetilde{c}(B^*) = \inf_\mu \{b^* \mu - c(B_\mu)\}$
- *Lemma 2:* Principal cannot implement B^* with any $b < b^*$.
  - Suppose B^* can be implemented by some $b < b^*$
  - If $\widetilde{c}(B^*)$ was adjusted, this contradicts the above definition of $\widetilde{c}(B^*)$.
  - If $\widetilde{c}(B^*)$ was not, then the premise contradicts Lemma 1.

Binary Projects: Proof

- By assumption, F^* is optimal in c; *i.e.*, $\Pi(F^*) \geq \Pi(F)$ for all F
- By Lemma 1, $\widetilde{\Pi}(B_\mu) \leq \Pi(F)$ for any F with mean μ
- By construction, $\widetilde{\Pi}(B^*) = \Pi(F^*)$, and therefore,

$$\widetilde{\Pi}(B^*) \geq \widetilde{\Pi}(B_\mu) \quad \text{for all } \mu$$

  i.e., the principal optimally implements B^* in $\widetilde{c}$.
- Also by construction, agent is weakly better off relative to $\{c, w^*\}$.
- If $\mu^* = 1$, then the proof is complete.

Binary Projects: Proof

- Suppose $\mu^* < 1$. Since b^* implements B^*, the following IC is satisfied

$$b^*\mu^* - \widetilde{c}(B^*) \geq b^*\mu - \widetilde{c}(B_\mu) \quad \text{for all } \mu.$$

- *Observation:* This constraint is slack for all $\mu > \mu^*$.
  - If not, b^* implements $B_{\mu'}$ for some $\mu' > \mu^*$ giving principal bigger profit
- Therefore, wolog, we can adjust $\widetilde{c}(B_\mu) = \infty$ for all $\mu > \mu^*$.
- Multiply bonus b^*, costs and success prob. $\Pr\{x = 1\}$ by $1/\mu^* > 1$.
  - Payoffs are scaled up and IC constraints are unchanged.
- **Summary:** New project comprises only actions with support $\{0, 1\}$, principal optimally implements $x = 1$ w.p. 1, and agent is better off.

Implication

- By Theorem 1, it suffices to restrict attention to:
  - Actions such that

$$x = \begin{cases} 1 & \text{with probability } p \\ 0 & \text{with probability } 1 - p \end{cases}$$

  - Cost function $C(p) \geq 0$ such that principal optimally implements $p = 1$
  - Bonus contracts $w(x) = b\mathbb{I}_{\{x=1\}}$ for some $b \geq 0$ to be chosen.
- We will solve the problem using backward induction

Heuristic Characterization – Stage 2

- Fix a cost function $C(\cdot)$. Then the principal solves

$$\begin{aligned} \max \ & p(1-b) \\ \text{s.t. } & pb - C(p) \geq \widetilde{p}b - C(\widetilde{p}) \quad \text{for all } \widetilde{p} \in [0,1] \\ & p \in [0,1] \quad \text{and} \quad b \geq 0 \end{aligned}$$

- Guess that C is twice differentiable and convex. Then we can replace the agent's IC constraint with its first-order condition:

$$b = C'(p)$$

and rewrite the principal's problem as

$$\pi := \max_p p\left[1 - C'(p)\right]$$

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Heuristic Characterization – Stage 1

- The agent solves

$$\max_{C(\cdot)\geq 0} \; p^* b - C(p^*)$$
$$\text{s.t } p^*\left[1 - C'(p^*)\right] \geq p\left[1 - C'(p)\right] \quad \text{for all } p \qquad (\mathrm{IC}_P)$$

where $p^* = 1$ by Theorem 1, and $b = C'(p^*)$ from the agent's FOC.

- Using that $C'(1) = 1 - \pi$, we can rewrite this maximization program as

$$\max 1 - \pi - \int_0^1 C'(q)dq$$
$$\text{s.t. } C'(p) \geq 1 - \frac{\pi}{p} \quad \text{for all } p < 1$$
$$C(\cdot) \geq 0 \quad \text{and} \quad \pi \in [0, 1]$$

Heuristic Characterization – Stage 1 (Continued)

- *Step 1:* For (any) fixed π, we solve

$$\max_{C(\cdot)\geq 0} 1-\pi-\int_0^1 C'(p)dp$$
$$\text{s.t. } C'(p)\geq 1-\frac{\pi}{p} \quad \text{for all } p<1$$

- Objective decreases in $C'(p)$ and constraint imposes lower bound. So

$$C'(p)=\left[1-\frac{\pi}{p}\right]^+$$

- *Step 2:* Plugging $C'(\cdot)$ into the agent's objective, we solve

$$\max_{\pi\in[0,1]} \{-\pi\ln\pi\}=\frac{1}{e} \quad \text{and} \quad \pi^*=\frac{1}{e};$$

i.e., the principal's, as well as the agent's payoff is $1/e$.

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Characterization

Theorem 2. Optimal Project

- There exists an optimal project in which the agent chooses

$$C'(p) = \begin{cases} 0 & \text{if } p \leq 1/e \\ 1 - \frac{1}{pe} & \text{if } p > 1/e \end{cases}$$

- The principal offers bonus contract with $b = 1 - 1/e$
- Each player obtains payoff equal to $1/e$

- The agent chooses a convex cost function s.t any $p \leq 1/e$ is costless, while larger p's are progressively more expensive and the principal is is indifferent across any bonus contract with $b \in [0, 1 - 1/e]$.
- Principal's profit $\pi = 1/e$, and agent captures all rents for $p > 1/e$.

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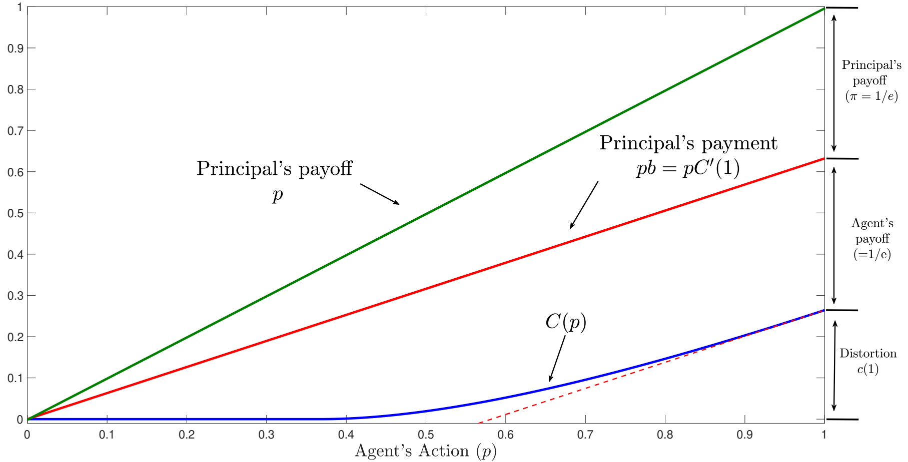

- To capture rents, agent commits to rent seeking activity costing $C(p)$.

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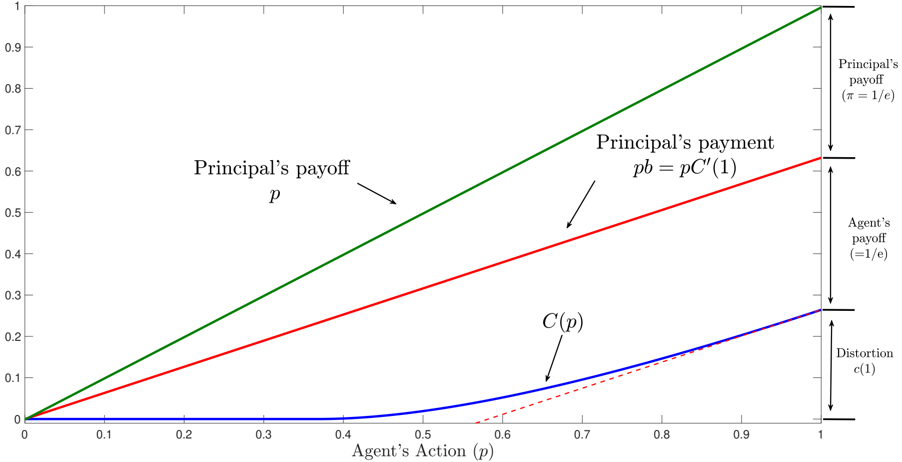

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Payoff pairs implementable by an arbitrary binary project

- Insofar, we have assumed the agent can choose *any* cost function

$$c : \Delta([0,1]) \to \mathbb{R}_+$$

- Suppose the agent is constrained and must choose among a subset of these cost functions.
- **Q:** Can we make any predictions regarding surplus allocation?
- Let $V(c) = \{\pi^*, U^*\}$ be the set of equilibrium payoffs for given c, and define the payoff possibility set:

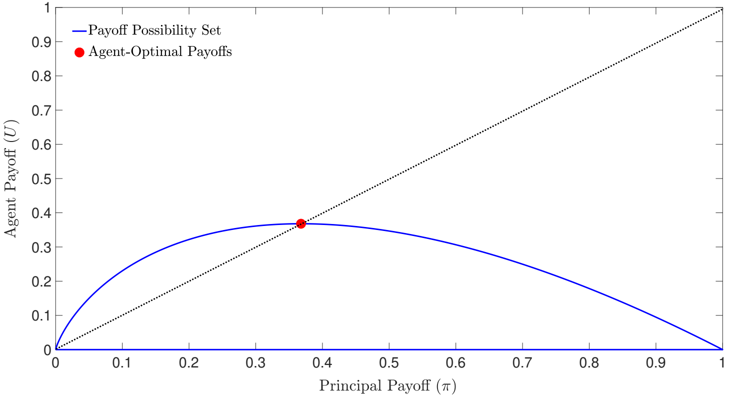
$$\mathcal{P} = \bigcup_{c : \Delta([0,1]) \to \mathbb{R}_+} V(c).$$

Payoff pairs implementable by an arbitrary binary project

Theorem 3. Payoff Possibility Set

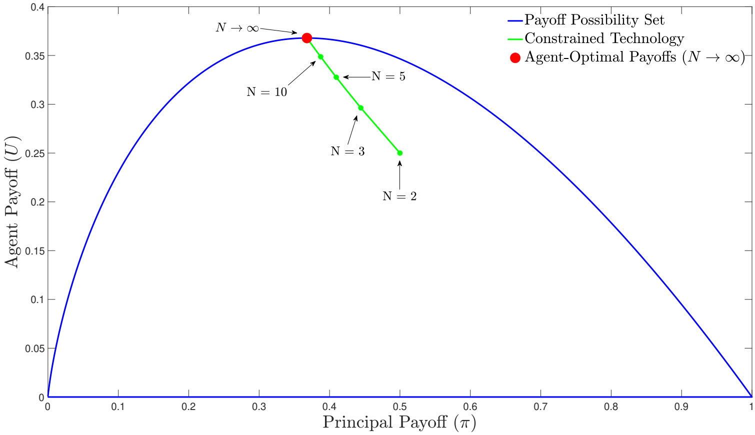
The payoff possibility set is

$$\mathcal{P} = \mathrm{co}\left(\{\pi, -\pi \log \pi\} : \pi \in [0,1]\right).$$



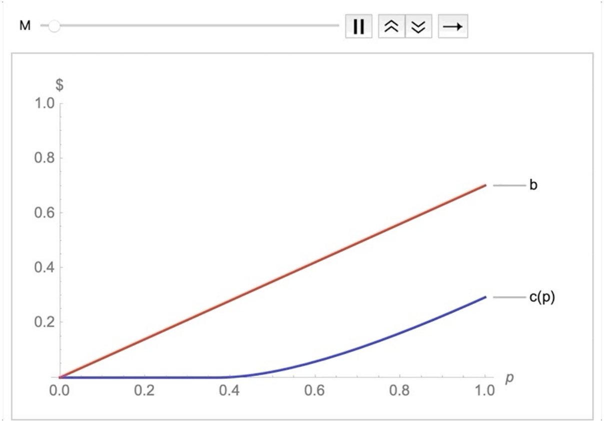
Bounded Project Complexity

- Suppose the agent can choose a project with at most N actions.
- By Theorem 1, wolog, he chooses $p_i \in [0, 1]$ and $C(p_i) \geq 0$ for each i



Negative Payoffs

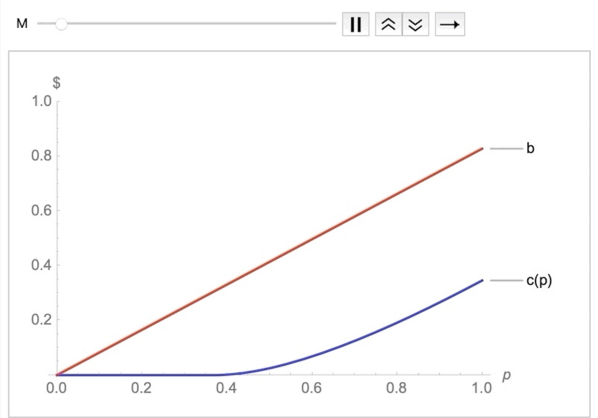
- Suppose agent can choose output distributions with support $[-M, 1]$.
- Suffices to focus on binary projects s.t $F(x) = \mathbb{I}_{\{x=1\}}$ is implemented.



- When $M = 0$, $C(\cdot)$ and b are given in Theorem 2.

Negative Payoffs

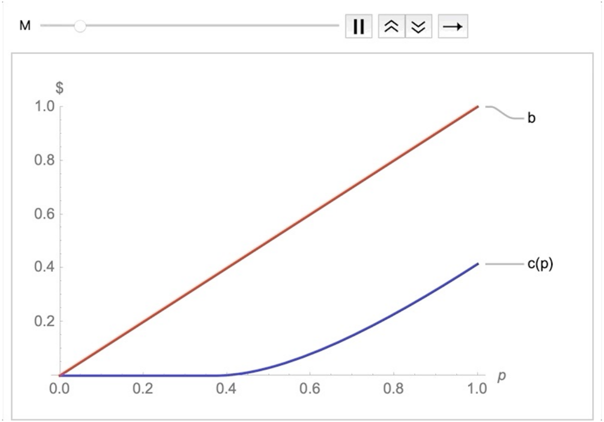
- Suppose agent can choose output distributions with support $[-M, 1]$.
- Suffices to focus on binary projects s.t $F(x) = \mathbb{I}_{\{x=1\}}$ is implemented.



- As $M \uparrow$, both $C(\cdot)$ and b are shifted upwards.

Negative Payoffs

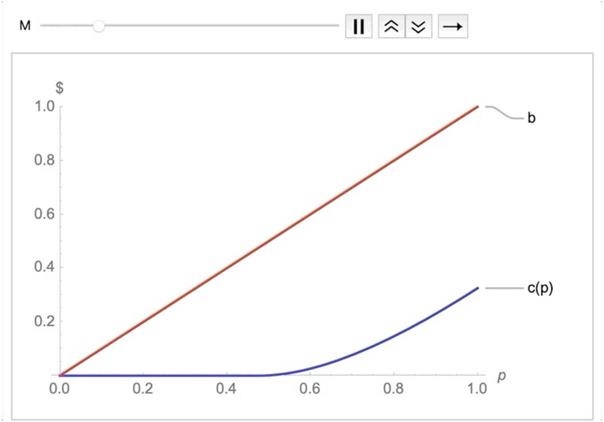
- Suppose agent can choose output distributions with support $[-M, 1]$.
- Suffices to focus on binary projects s.t $F(x) = \mathbb{I}_{\{x=1\}}$ is implemented.



- For M sufficiently large, $b = 1$, and agent extracts all surplus.

Negative Payoffs

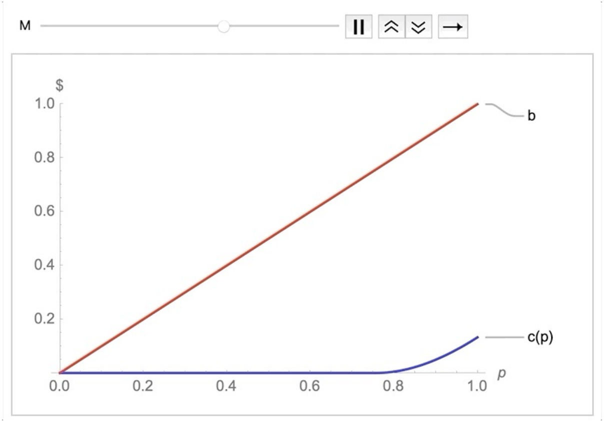
- Suppose agent can choose output distributions with support $[-M, 1]$.
- Suffices to focus on binary projects s.t $F(x) = \mathbb{I}_{\{x=1\}}$ is implemented.



- As $M \uparrow$ further, $C(\cdot)$ is shifted downwards, decreasing distortion.

Negative Payoffs

- Suppose agent can choose output distributions with support $[-M, 1]$.
- Suffices to focus on binary projects s.t $F(x) = \mathbb{I}_{\{x=1\}}$ is implemented.



- As $M \to \infty$, $b = 1$ and $C(\cdot) \to 0$ leading to efficiency.

Risk-averse Agent

- Theorem 1 holds if the agent is not *too risk-averse*.

Corollary 1. Risk-averse Agent

- Let $u_k(\cdot)$ be a sequence of functions satisfying $u_k'' < 0 < u_k'$ for each k, and $\lim_{k\to\infty} u_k(\omega) = \omega$ uniformly.
- There exists a K such that a binary project optimal whenever $k \geq K$.

- Theorem 2 — the characterization of the optimal binary project is straightforward for any concave utility function.

Related Literature (Incomplete List)

- *Principal-agent models:*
  - Mirrlees (1976), Holmström (1979), Innes (1990)
  - *Gaming / multitasking:* Carroll (2015), Barron et al. (2020)
  - *Endogenous monitoring technology:* Georgiadis and Szentes (2020)
- *Sequential mechanism design:*
  - Krähmer and Kovác (2016)
  - Bhaskar et al. (2019)
  - Condorelli and Szentes (2020)

Discussion

- We consider an agency model of moral hazard in which production technology is endogenous and chosen by the agent.
- The agent optimally designs a project with binary output such that the principal is indifferent between b^* and any smaller bonus, enabling him to extract all rents.
- *Potential implication.* Promoting more flexibility for workers to design their job as an alternative to regulation (*e.g.*, minimum wages)