

Optimal Payment Contracts in Trade Relationships*

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February 2020

Abstract

Trade credit is one of the most important sources of short-term finance in buyer-seller transactions. This paper studies a seller's trade credit provision decision in a situation of repeated contracting with incomplete information over the buyer's ability and willingness of payment compliance when the enforceability of formal contracts is uncertain. We show that selecting the payment terms of a transaction corresponds to managing an inter-temporal trade-off between improving the quality of information acquisition and mitigating relationship breakdown risks. The dynamically optimal sequence of payment contracts can be uniquely determined provided that the quality of contract enforcement institutions is sufficiently low.

Keywords: Payment contracts, Trade credit, Trade dynamics, Relational contracts, Contract enforcement

JEL Classification: L14, F34, G32, D83

**Acknowledgments:* The author is grateful to Hartmut Egger and Jens Suedekum for invaluable advice. The author would also like to thank Julia Cajal Grossi, Fabrice Defever, Jonathan Eaton, Peter Egger, Matthias Fahn, Paul Heidhues, David Hémous, John Morrow, Marc Muendler, Peter Neary, Hans-Theo Normann, Ferdinand Rauch, Philip Sauré, Tim Schmidt-Eisenlohr, Nathan Sussman, and audiences at the TRISTAN Workshop Bayreuth, Düsseldorf University, ETSG Florence, the Geneva Graduate Institute, JGU Mainz, and FIW Vienna for helpful comments and suggestions. Financial support of the Joachim Herz Foundation is gratefully acknowledged.

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1 Introduction

Trade credit is one of the most important sources of short-term finance in buyer-seller transactions worldwide.¹ The theoretical literature on trade credit offers a whole set of explanations for its prevalence as opposed to the usage of credit provided by specialized financial institutions such as banks. These range from financing advantages of sellers over traditional lenders, over introducing a possibility of price discrimination to transaction cost reductions (cf. [Petersen and Rajan, 1997](#)). At the same time, a comprehensive understanding of firms' provision rationales also requires to acknowledge that trade credit exposes its issuer to uncertainty over the receiver's ability and willingness to repay as well as over the legal enforceability of repayment claims (cf. [Schmidt-Eisenlohr, 2013](#)).

Which are the mechanisms determining a firm's decision to provide trade credit in the presence of payment uncertainty? While a broad economic literature suggests that uncertainty in buyer-seller transactions requires to think of inter-firm cooperation from a dynamic, relational perspective little is known about the provision rationales of trade credit in such environments.² An understanding of trade credit in the dynamic context is important as a substantial share of buyer-seller transactions is part of long-termed trade relationships. Moreover, information is usually revealed and uncertainty dissolved in such relationships over time, suggesting that a seller's propensity to provide trade credit is endogenous to the evolution of the relationship.³

This paper studies the optimal provision of trade credit by a seller ("he") who repeatedly markets a product through a buyer ("she") to final consumers. For every transaction, the seller proposes a spot contract to the buyer specifying the volume of trade, a transfer from buyer to seller, and the payment contract of the transaction. Payment contracts define the point in time at which the transfer should be made and thereby determine whether or not the seller provides trade credit. The seller's provision decision is – in the absence of payment guarantees by banks and insurance firms – equivalent to a decision between a cash in advance and an open account payment contract. In the simplest scenario, while no trade credit

¹A broad literature documents that trade credit makes up for a major share of firms' current liabilities in many countries and industries. Using balance sheet data from a large set of European firms for the years 1993–1997, [Giannetti \(2003\)](#) shows that the share of trade credit in current liabilities ranged between 25 and 75 percent for firms across countries. Using data for the years 1993–2002, [Cuñat \(2007\)](#) documents for this measure a value of approx. 50 percent for the United Kingdom and the United States. [Garcia-Marin et al. \(2019\)](#) show that in 2017, non-financial U.S. firms had about \$3 trillion in trade credit outstanding equaling 20 percent of U.S. GDP.

²The existing literature on inter-firm cooperation in dynamic environments distinguishes two important sources of uncertainty, where the first relates to market conditions (see, e.g., [Green and Porter, 1984](#)), and the second to firm characteristics (see, e.g., [Hart and Tirole, 1988](#)). The model presented here features both types of uncertainty – firm level uncertainty with respect to the buyer's payment morale and market uncertainty regarding the reliability of contract enforcement institutions.

³The long-term orientation of supply relationships and its value-enhancing consequences have been documented in a plentitude of contexts. E.g., see [Egan and Mody \(1992\)](#), [Macchiavello and Morjaria \(2015\)](#), [Araujo et al. \(2016\)](#), and [Monarch and Schmidt-Eisenlohr \(2018\)](#).

is provided by the seller under cash in advance the situation is reversed under open account where the seller extends trade credit in the amount of the revenue that the transaction generates for him.⁴

Whether or not the buyer meets the payment obligations of a transaction depends on her *intrinsic willingness to comply* (i.e., her type), her *ability to comply* (i.e., her liquidity status), and the legal *enforceability* of the payment. Ex-ante, the seller faces uncertainty over either of these domains. However, while the buyer's liquidity status and the enforceability of contracts each follow a stochastic process that is orthogonal to the seller's contracting choices, the buyer's type is fixed and the seller is able to acquire new information on it as the relationship proceeds.

A first result of the paper is to show that payment contracts differ fundamentally in their capacity to reveal information about the buyer's type and in the respective risks of transaction failure. Payment contracts can be interpreted as *screening technologies* each exerting a distinct influence on the stability of buyer-supplier cooperation. Information acquisition is faster under cash in advance terms under which the seller optimally proposes a separating contract that only those buyers accept that are patient and liquid enough to comply. In contrast, the optimal spot contract under open account terms is always a pooling contract implying that type information is acquired gradually over time. A crucial assumption to obtain this result is that time elapses between the seller's investment in production and the buyer's revenue realization from the sale to final consumers, implying that financing trade is costly and that payment contracts allow to shift this burden between the buyer and the seller. Correspondingly, in the case of cash in advance the risk of transaction failure can be associated with the buyer not being able or willing to finance trade while in the case of open account to legal institutions not being able to enforce payment.

Acknowledging the risk and information management properties of payment contracts, we determine how the seller can use them to optimize his market outcome. We derive conditions that allow us to uniquely identify the dynamically optimal sequence of payment contracts (DOSPC) of a trade relationship. We find that whenever the seller is patient enough and the quality of contract enforcement institutions is sufficiently low the set of possible DOSPCs contains exactly three elements. While two of these sequences do not contain switches between payment terms, the third predicts a transition from cash in advance to open account terms on the equilibrium path. In this case, the seller exploits the buyer-separating nature of the cash in advance terms in the initial transaction and by changing to open account subsequently he can eliminate the risk of relationship breakdown due to buyer liquidity constraints in future transactions. Transitions in the reverse direction – i.e. from open account to cash in advance terms – are never optimal. Qualitatively, these predictions are broadly consistent with recent empirical findings

⁴To simplify the analysis, the paper abstracts from the possibility of partial cash in advance and other forms of step-wise payment.

from the trade finance literature. [Garcia-Marin et al. \(2019\)](#) show using representative customs data from Chile that the usage of open account (cash in advance) terms increases (decreases) significantly with the age of buyer-supplier relationships. Moreover, [Antràs and Foley \(2015\)](#) show using transaction-level data from a manufacturing U.S. exporter that transactions are less likely to occur on cash in advance terms as relationships develop. Also, this pattern is consistent with the work of [McMillan and Woodruff \(1999\)](#) who show using firm survey data from Vietnam that prior experience with business partners matters for the provision of trade credit and that trade relationships of longer duration can be associated with higher levels of trade credit provision.

Reducing the set of possibly optimal payment sequences to a tractable size makes the upper bound on contract enforcement institutions necessary. When enforcement institutions work well, open account terms guarantee comparably high stage payoffs from the very first transaction and the payoff-enhancing screening qualities of cash in advance are marginalized. In this scenario, the analysis of the optimal transition patterns becomes intractable as the payoff-relevance of learning for the seller is low. When presuming a sufficiently low level of contract enforceability, the model predicts that the seller will more likely extend trade credit to a buyer the higher the probability that she turns liquidity-constrained and the smaller the probability to (again) find a buyer of patient type, i.e. one who complies with the optimal contracting terms.⁵ In equilibrium, this pattern holds for both – new and established trade relationships.

Finally, the paper is extended to incorporate the possibility for the seller to obtain trade credit insurance from a perfectly competitive insurance market. In our model, the insurance generates value for the seller through the insurer's capability to support the seller in the screening of buyers. In this context, we show that the unique identification of the DOSPC remains possible. Moreover, given that the insurance is not too costly we show that there exist parameterizations of the model for which insurance is optimal in newly established trade relationships on the equilibrium path.

The remainder of the paper starts with a review of the related literature in Section 2. Section 3 introduces the model. Section 4 studies supply relationships under cash in advance and open account payment contracts when switches between payment terms are ruled out. Section 5 introduces this possibility and studies the seller's optimal choice of payment contracts over the course of the trade relationship. The impact of the availability of trade credit insurance and its effect on optimal payment contract choice are studied in Section 6. Section 7 concludes with a summary of our findings.

⁵Importantly, note that while we assume that liquidity-constrained buyers are unable to pay cash in advance they remain able to conduct a transaction on open account terms.

2 Related literature

This paper is related to two broad strands of literature where the first studies the financing terms of inter-firm trade. It builds on the interpretation of trade credit by [Smith \(1987\)](#) who first acknowledged its role as a screening device for sellers to elicit information about buyer characteristics. More generally, the paper is related to a literature that sees credit rationing as a screening device for creditors in markets with incomplete information (cf. [Stiglitz and Weiss, 1981](#)). Our model gives conditions under which, in equilibrium, trade credit is rationed either temporarily or permanently where in the former case this is due to screening considerations and in the latter case because financing trade is costly for the seller.

Most closely related to our work is a small set of papers that studies the provision of trade credit in settings with repeated buyer-seller interaction. Their results are complementary to ours. The setup of our model features similarities to that of [Antràs and Foley \(2015\)](#) who investigate the impact of a financial crisis in a dynamic model of payment contract choice. While they also study transitions between payment terms over time their model does not incorporate that the information acquisition process of sellers differs fundamentally between cash in advance and post shipment terms, inducing structural differences in the optimal growth patterns of transaction volumes and per-period payoffs. [Garcia-Marin et al. \(2019\)](#) derive conditions under which the provision of trade credit increases in attractiveness to sellers as their relationships with buyers mature. While in their model this prediction originates from a financing advantage for sellers under trade credit terms, in our setting it can be associated with the risk that the buyer faces a liquidity shock on the one side and to uncertainty over the buyer type in new relationships on the other side. Also related to us is [Troya-Martinez \(2017\)](#) who studies the optimal design of self-enforcing contracts under the requirement that a seller must provide trade credit to buyers.

While the main focus of this paper is on the self-financing of trade through the buyer and the seller, a large literature investigates the rationales of firms to use trade credit instead of credit provided by external financial institutions. [Burkart and Ellingsen \(2004\)](#) derive conditions under which trade and bank credit interact either as complements or substitutes with each other. [Demir and Javorcik \(2018\)](#) interpret trade credit provision as a margin of firm adjustment to competitive pressures arising from globalization. [Engemann et al. \(2014\)](#) understand trade credit as a quality signaling device that facilitates obtaining complementary bank credits. Moreover, our work is connected to a literature on payment guarantees in international trade finance through the discussion of trade credit insurance in Section 6. A concise summary of the most relevant work from this field was recently provided by [Foley and Manova \(2015\)](#).

The second broad strand of related literature investigates the microeconomic aspects of learning and trade dynamics which, on the one side, considers applications to topics in international trade and, on

the other side, contains papers of a purely contract-theoretic nature. [Araujo et al. \(2016\)](#) study how contract enforcement and trade experience shape firm trade dynamics when information about buyers is incomplete. We share with their work the probabilistic approach to contract enforcement. The patterns of information acquisition and trade volume growth predicted by our model resemble the outcomes of their framework in the special situation when the seller continuously employs open account terms. [Rauch and Watson \(2003\)](#) study a matching problem between a buyer and a seller with one-sided incomplete information. They derive conditions under which starting a relationship with small trade volumes is preferable to starting with large transaction volumes from the very beginning. This pattern features a clear analogy to our model in which starting a relationship on open account terms corresponds to starting small, and on cash in advance terms to starting large. Extending beyond the scope of our analysis, [Ghosh and Ray \(1996\)](#) and [Watson \(1999, 2002\)](#) study agents' incentives to start small when information is incomplete on both sides of the market.⁶

Moreover, our work is related to a literature on self-enforcing relational contracts with incomplete information in the spirit of [Levin \(2003\)](#). Like us, [Sobel \(2006\)](#), [MacLeod \(2007\)](#), and [Kvaløy and Olsen \(2009\)](#) study the interaction of formal and self-enforcing contracts in repeated game models when legal contract enforcement is probabilistic. Most closely related to us is [Kvaløy and Olsen \(2009\)](#) who investigate a situation of repeated investment in a principal-agent setting with endogenous verifiability of the contracting terms. While in their setting verifiability is endogenized through the principal's investment in contract quality in our model the relevance of verifiability itself is endogenized through payment contract choice. The paper also adds to a growing literature on non-stationary relational contracts with adverse selection, in which contractual terms vary with relationship length. While in our paper learning about the buyer induces transitions between payment contract types, previous work has studied non-stationarities in different contexts.⁷

⁶Beyond the case of buyer-supplier transactions, relationship building has also been analyzed in the context of different applications. See, e.g., [Kranton \(1996\)](#) and [Halac \(2014\)](#).

⁷[Chassang \(2010\)](#) examines how agents with conflicting interests can develop successful cooperation when details about cooperation are not common knowledge. [Halac \(2012\)](#) studies optimal relational contracts when the value of a principal-agent relationship is not commonly known and, also, how information revelation affects the dynamics of the relationship. [Yang \(2013\)](#) investigates firm-internal wage dynamics when worker types are private information. [Board and Meyer-ter Vehn \(2015\)](#) analyze labor markets in which firms motivate their workers through relational contracts and study the effects of on-the-job search on employment contracts. Moreover, [Defever et al. \(2016\)](#) study buyer-supplier relationships in international trade in which new information can initiate a relational contract between parties.

3 The Model

The model considers the problem of a seller who markets a product through a buyer to final consumers. There exists a continuum of potential buyers with the ability to distribute the seller's product. The seller is a monopolist for the offered product and has constant marginal production costs $c > 0$. Selling $Q_t \geq 0$ units of the product to the final consumers in period t generates revenue $R(Q_t) = Q_t^{1-\alpha}/(1-\alpha)$, which is realized by the buyer. The revenue function is increasing and concave in the trade volume Q_t , where $\alpha \in (0, 1)$ determines the shape of the revenue function.⁸

We model the buyer-seller relationship as a repeated game, where in every period $t = 0, 1, 2, \dots$ a transaction is performed. The seller can engage in only one partnership at the same time. In every period, the seller first decides either to continue the relationship with his current buyer or to re-match and start a new partnership. He then proposes a spot contract $C_t = \{Q_t, T_t, F_t\}$ to the buyer specifying a trade volume $Q_t \geq 0$, a transfer payment T_t from the buyer to the seller, and a payment contract, $F_t \in \mathcal{F} = \{A, \Omega\}$, that determines the point in time at which the transfer T_t is made.⁹ Depending on the payment contract, the seller receives the transfer either before he produces and ships the goods (cash in advance terms, $F_t = A$) or after the buyer has sold them (open account terms, $F_t = \Omega$).

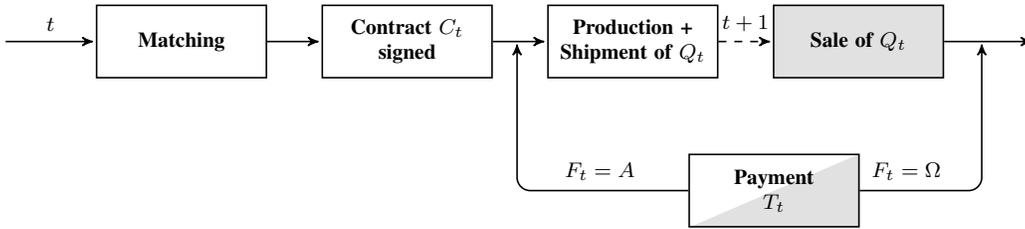


Figure 1: The spot contract C_t determines the timing of the stage game.

The timing of the transfer is payoff-relevant because shipment is time-consuming and players discount payoffs over time. Goods that are produced and shipped by the seller in period t can be sold to consumers only in the subsequent period $t + 1$. The corresponding discount factor of the seller is denoted by $\delta_S \in (0, 1)$. The buyer comes in one of two possible types, $j \in \{M, B\}$. Either she is fully myopic, $j = M$, with discount factor $\delta_M = 0$ and associates positive value only to payoffs of the current period. Alternatively, the buyer is patient, $j = B$, with discount factor $\delta_B \in (0, 1)$. The assumptions imply that by choosing open account terms the seller extends a *trade credit* to the buyer while this is not the case under cash in advance terms. A graphical summary of the stage game is given in Figure 1.

⁸Whether the concave shape of the revenue function stems from technology, preferences or market structure is not important for the analysis below.

⁹We assume that the seller can offer only one single contract to the buyer and rule out contract menus.

Whenever the seller decides to match with a new buyer he draws her type from an i.i.d. two-point distribution, where with probability $\hat{\theta} \in (0, 1)$ the buyer is myopic, and patient otherwise. We denote the seller's belief that the buyer is myopic in period t by θ_t and assume that the seller holds the belief $\theta_0 = \hat{\theta}$ at the beginning of the initial transaction with a new buyer.

Moreover, the seller faces uncertainty over the buyer's liquidity status. While any buyer is liquid ex-ante, she can become liquidity-constrained in any period with an i.i.d. probability of $1 - \gamma \in (0, 1)$. We assume that a liquidity-constrained buyer is permanently unable to deliver payment under cash in advance terms. At the beginning of the contracting stage, the buyer privately updates her liquidity status.

In every period, the contract C_t can be enforced with an i.i.d. probability $\lambda \in (0, 1)$. In our application, for the buyer this corresponds to the probability of not being able to deviate from making the prescribed transfer T_t and for the seller to the probability of being forced to produce and ship as agreed-upon.¹⁰

In the following, we summarize the stage game of period t which is repeated ad infinitum.

Stage game timing.

1. **Revenue realization.** The product shipped in the previous period generates revenue $R(Q_{t-1})$ to the buyer from the sale to final consumers.
2. **Payment (if $F_{t-1} = \Omega$).** The buyer decides whether to transfer T_{t-1} to the seller. She finds an opportunity not to pay with probability $1 - \lambda$. Upon non-payment the match is permanently dissolved.
3. **Matching.** Whenever unmatched, the seller starts a new partnership. Otherwise, the seller chooses either to stick to the current buyer or to re-match with a new one.
4. **Contracting.**
 - The seller decides whether to propose a one-period spot contract $C_t = \{Q_t, T_t, F_t\}$ to the buyer. The contract specifies a trade volume Q_t , a transfer T_t , and a payment contract F_t . Upon non-proposal, the match is permanently dissolved.
 - The buyer updates her liquidity status and decides either to accept or to reject C_t . Upon rejection, the match is permanently dissolved.

¹⁰This probabilistic approach to contract enforcement institutions was introduced in the context of payment contract choice by Schmidt-Eisenlohr (2013).

5. **Payment (if $F_t = A$).** The buyer decides whether to transfer T_t to the seller. She finds an opportunity not to pay with probability $1 - \lambda$. Upon non-payment the match is permanently dissolved.
6. **Production and Shipment.** The seller decides whether to produce and ship Q_t as specified in the contract. Upon non-shipment the match is permanently dissolved.

For the following, it will be helpful to define by $C = (C_t)_{t=0}^{\infty}$ the sequence of spot contracts offered by the seller over the course of the relationship. Moreover, we denote by $Q = (Q_t)_{t=0}^{\infty}$, $T = (T_t)_{t=0}^{\infty}$, and $F = (F_t)_{t=0}^{\infty}$ the corresponding sequences for trade volumes, transfer payments, and payment contracts, respectively.

4 Payment contracts in isolation

In this section, we study in isolation the two cases where the seller is restricted to choose either cash in advance or open account payment terms for all periods and rule out switches between payment terms over time. The latter possibility is introduced in the subsequent Section 5 in which the seller can freely choose the payment contract in the spot contract of any transaction. This expositional approach not only allows us to highlight the different screening properties of payment contract types but also requires us to derive two repeated game equilibria that are both relevant in our study of dynamic optimality.

We consider the following *strategy profile*. In both scenarios, the seller forms a new partnership whenever unmatched. He terminates an existing partnership if and only if the buyer defaults on the contract. In any period t , the seller chooses a trade volume Q_t and a transfer T_t that maximize his current period expected payoffs.¹¹ The buyer accepts the proposed contract C_t whenever participation promises her an expected payoff at least covering her outside option. The buyer's behavior with respect to an accepted contract is fully determined by her type. The myopic type will deviate from any accepted contract and not pay the transfer whenever it can not be enforced. In contrast, the patient buyer is patient enough to never default from an accepted contract (by assumption). The employed equilibrium concept is that of sequential equilibrium.¹²

Throughout, we assume that the transfer T_t is a share $s^i \in (0, 1)$, $i \in \mathcal{F}$, of the revenue generated by the current transaction, i.e. $T_t \equiv s^i R(Q_t)$. This specification allows the seller to set a transfer that can

¹¹Since we assume that only spot contracts are feasible and switching between payment contract types is ruled out for this section the maximization of the current period expected payoffs implies that the ex-ante expected payoffs are maximized simultaneously.

¹²For adverse selection scenarios as we study them here, sequential equilibrium is the relevant notion of equilibrium, see [Mailath and Samuelson \(2006\)](#), pp. 158–159.

be made specific to the type of the payment contract. Moreover, we normalize the outside options of all parties to zero.

4.1 Cash in advance terms

First, we study the case where the seller is restricted to write contracts on cash in advance terms (A -terms) only, i.e. in any trade relationship $F = (A, \dots)$. Under A -terms, buyer liquidity is essential for the success of a transaction. We introduce the possibility of a liquidity-constrained buyer by assuming that her *self-perceived discount factor* under A -terms drops to zero whenever she is liquidity-constrained. The participation constraint of a buyer of type $j \in \{M, B\}$ in period t is:

$$(\mathbb{1}_{j,t}\delta_j - s^A)R(Q_t) \geq 0, \quad (\text{PC}_{j,t}^A)$$

where $\mathbb{1}_{j,t}$ indicates the buyer's liquidity status in period t . The constraint states, that tomorrow's revenue $R(Q_t)$ realized from the sale of today's shipment Q_t must be larger than the share s^A of the revenue that the buyer has to transfer to the seller before shipment. Because goods can be sold to final consumers only in the period following t , the revenue is multiplied by the buyer's self-perceived discount factor $\mathbb{1}_{j,t}\delta_j$. Observe that because $\delta_M = 0$, the myopic buyer's participation constraint, $(\text{PC}_{M,t}^A)$, cannot be fulfilled for any $s^A > 0$. The same holds true for the patient buyer when she is liquidity-constrained, i.e. when $\mathbb{1}_{B,t}\delta_B = 0$. Consequently, the myopic buyer and the illiquid patient buyer will never accept any contract on A -terms.

Acknowledging this, the seller offers a *separating contract* that only a liquid patient buyer accepts. He will do so by setting $s^A = \delta_B \equiv \tilde{s}^A$ and extract all rents from her. In this situation, the seller's stage payoff maximization problem under A -terms in period t is given as:

$$\max_{Q_t} \pi_t^A(Q_t) = \tilde{s}^A R(Q_t) - cQ_t,$$

i.e. he sets Q_t to maximize the difference between his revenue share and the production costs. Obviously, under A -terms the optimal trade volume is the same for all periods and given as:

$$Q^A \equiv \arg \max_{Q_t} \pi_t^A(Q_t) = \left(\frac{\delta_B}{c} \right)^{\frac{1}{\alpha}}, \quad \forall t \geq 0.$$

The corresponding stage payoffs, conditional on contract acceptance, are given as:

$$\bar{\pi}^A \equiv \pi_t^A(Q^A) = (\delta_B)^{\frac{1}{\alpha}} c^{\frac{\alpha-1}{\alpha}} \frac{\alpha}{1-\alpha}, \quad \forall t \geq 0.$$

In order to derive the seller's ex-ante expected payoffs, it is important to note that whenever a new

trade relationship survives the initial transaction the seller can be certain to be matched with a patient buyer. Correspondingly, his belief jumps from $\theta_0 = \hat{\theta}$ to $\theta_1 = 0$ right after the initial contract is accepted and remains at this level for all further transactions with that same buyer. Hence, the ex-ante expected payoffs from conducting an infinite sequence of transactions on A -terms can be derived from solving the following dynamic programming problem:

$$\begin{aligned} V_0^A &= \gamma(1 - \theta_0) [\bar{\pi}^A + \delta_S V_1^A] + (1 - \gamma(1 - \theta_0))\delta_S V_0^A, \\ V_1^A &= \gamma[\bar{\pi}^A + \delta_S V_1^A] + (1 - \gamma)\delta_S V_0^A. \end{aligned} \quad (1)$$

Note that a trade relationship with the same patient buyer is productive and continued only if this buyer remains liquid in the respective period, i.e. with probability γ . Solving the programming problem for V_0^A gives the seller's ex-ante expected payoffs under A -terms, Π^A . They are:

$$\Pi^A = \frac{\gamma(1 - \theta_0)\bar{\pi}^A}{(1 - \delta_S)(1 - \gamma\theta_0\delta_S)}.$$

Under A -terms, the buyer has to make the transfer before the seller's production and shipment decision. Consequently, the seller may have an incentive to deviate and not produce the output, seize the transfer, and re-match to a new buyer in the next period. The following Lemma 1 provides parameter restrictions that rule out any such deviation and guarantees equilibrium existence.¹³

Lemma 1. *Suppose that $\alpha > \tilde{\alpha} \in (0, 1)$. Then there exists a repeated game equilibrium that maximizes the seller's ex-ante expected payoffs under cash in advance terms, Π^A , for all $\delta_S \geq \tilde{\delta}_S \in (0, 1)$.*

Proof See Appendix.

Some remarks on Lemma 1 are in order. For an equilibrium of the repeated game to exist the revenue $R(Q^A)$ and therefore the stage payoffs generated from the sale of Q^A units of the product must be large enough, i.e. larger than some threshold level implied by $\tilde{\alpha}$ and satisfied for all $\alpha > \tilde{\alpha}$ (since $\partial\pi_t^A/\partial\alpha > 0$). Otherwise, a deviation by the seller cannot be ruled out since the transaction's profit margin becomes negligible and the deviation ensures the seller the full transfer at zero cost. Provided that $\alpha > \tilde{\alpha}$ holds there exist repeated game equilibria rationalizing the behavior prescribed by the strategy profile if the seller's valuation of the stream of transfers from the current buyer is high enough, as implied by the minimum discount factor $\tilde{\delta}_S$. Proposition 1 summarizes our key findings on the cash in advance equilibrium.

¹³To improve readability, the explicit statement and the derivations of all parameter thresholds of the paper are omitted in the main text and can be found in the Appendix.

Proposition 1. *Suppose that payment is only possible on A-terms and Lemma 1 holds. Then the seller proposes a separating contract C_t that only liquid patient buyers accept. In every period, the seller produces and ships the payoff-maximizing trade volume Q^A . The expected stage payoffs increase from $\gamma(1 - \theta_0)\bar{\pi}^A$ to $\gamma\bar{\pi}^A$ after the first transaction and stay at this level for the remainder of the trade relationship. The seller's ex-ante expected payoffs are Π^A .*

Proof Analysis in the text.

There are several points noteworthy about this equilibrium. First, profit maximization under cash in advance terms necessarily separates buyer types as these are very demanding for the buyer. This is demonstrated by the fact that A-terms exclude the myopic and illiquid patient buyers from cooperation altogether. For the seller, cash in advance terms have the advantage of excluding any risk of non-payment altogether and allow him to set a belief-free trade volume Q^A beginning with the first transaction. Moreover, all information about the buyer's type is acquired immediately with the acceptance or rejection of the initial contract C_0 .¹⁴

4.2 Open account terms

Let us now turn to the case where the seller is restricted to write contracts on open account terms (Ω -terms) only, i.e. in any trade relationship $F = (\Omega, \dots)$. Based on the strategy profile we can write the participation constraints of the two buyer types for a period t contract as:

$$(1 - s^\Omega)R(Q_t) \geq 0, \quad (\text{PC}_B^\Omega)$$

$$(1 - \lambda s^\Omega)R(Q_t) \geq 0, \quad (\text{PC}_M^\Omega)$$

where (PC_B^Ω) is the participation constraint of the patient buyer and (PC_M^Ω) that of the myopic buyer, respectively. A comparison reveals that under Ω -terms it is impossible to construct a separating contract that would guarantee to select only patient buyers. The reasons are twofold. First, myopic buyers anticipate to transfer a share of the generated revenue only if the seller can enforce the contract. This happens with probability λ and makes their PC more lenient compared to that of the patient type. Second, discounting and liquidity concerns do not affect the buyer's participation decision since both, revenue realization and payment for a period t contract happen in period $t + 1$. Consequently, since $s^\Omega \in (0, 1)$, any feasible transaction on open account terms involves a *pooling contract*.

¹⁴Note that the separation outcome under A-terms does not hinge on the assumption of a fully myopic buyer. Inspection of the buyer participation constraints shows that for any $\delta_M \in (0, 1)$, with $\delta_M < \delta_B$, a payoff-maximizing contract can be written that only the more patient type accepts.

Suppose now that buyers behave as prescribed by the strategy profile and consider the seller's belief on the buyer's type. Observe that patient buyers will never deviate and myopic types do so whenever possible (i.e. they do not make the transfer when contracts can not be enforced). Hence, if no deviation occurs up to the t th transaction with the same buyer, the seller's belief of facing a myopic type in period t is given by Bayes' rule as:

$$\theta_t^\Omega = \frac{\hat{\theta}\lambda^t}{1 - \hat{\theta}(1 - \lambda^t)}. \quad (2)$$

Using equation (2), the payment probability in period t of a relationship can be written as $\Lambda(t, \hat{\theta}, \lambda) = 1 - \theta_t^\Omega(1 - \lambda) = [1 - \hat{\theta}(1 - \lambda^{t+1})]/[1 - \hat{\theta}(1 - \lambda^t)] \equiv \Lambda_t$. Note that $\lim_{t \rightarrow \infty} \theta_t^\Omega = 0$ and $\lim_{t \rightarrow \infty} \Lambda_t = 1$, i.e. as the relationship with a buyer continues the seller's belief of being matched with a myopic type converges to zero while the associated payment probability converges to one. In the following, we will refer to this limiting situation in which the seller is sure to be matched with a patient buyer as the *full information limit*.

Equipped with this notion of belief formation and updating, the optimal trade volume Q_t^Ω in period t can be derived from maximizing the seller's stage game payoffs:

$$Q_t^\Omega \equiv \arg \max_{Q_t} \delta_S \Lambda_t s^\Omega R(Q_t) - cQ_t.$$

While the seller has to bear the costs of production cQ_t^Ω already in period t , he receives the expected transfer $\Lambda_t s^\Omega R(Q_t^\Omega)$ only in the following period which is therefore discounted by δ_S .

Under open account terms, when deciding on the transfer T_t it is not enough to merely account for the buyer's participation constraint to guarantee that the patient buyer does not deviate from the contract. Her granted revenue share must be large enough such that she does not seize the period's entire revenue and accepts being re-matched. The following Lemma 2 gives a simple condition that ensures buyer behavior as prescribed by the strategy profile, while maximizing the seller's stage game payoffs.

Lemma 2. *Under open account terms, the seller sets $\tilde{s}^\Omega = \delta_B$. He thereby makes the patient buyer indifferent between paying and not paying the agreed-upon transfer and maximizes his own payoffs.*

Proof See Appendix.

An immediate corollary of Lemma 2 is that the equilibrium transfer to the seller is the same revenue share under both payment contract types. To simplify notation, we define the equilibrium revenue share as $\tilde{s} \equiv \tilde{s}^\Omega = \tilde{s}^A$ for the following.

Using (2), the optimal trade volume Q_t^Ω and the corresponding stage game payoff $\pi^\Omega(Q_t^\Omega)$ in the t th

transaction with a buyer on open account terms can be calculated as:

$$Q_t^\Omega = \left(\frac{\delta_S \delta_B \Lambda_t}{c} \right)^{\frac{1}{\alpha}}, \quad \pi^\Omega(Q_t^\Omega) = (\delta_S \delta_B \Lambda_t)^{\frac{1}{\alpha}} c^{\frac{\alpha-1}{\alpha}} \frac{\alpha}{1-\alpha}.$$

We define the trade volume and stage payoffs at the full information limit as:

$$Q^\Omega \equiv \lim_{t \rightarrow \infty} Q_t^\Omega = \left(\frac{\delta_S \delta_B}{c} \right)^{\frac{1}{\alpha}}, \quad \bar{\pi}^\Omega \equiv \lim_{t \rightarrow \infty} \pi^\Omega(Q_t^\Omega) = (\delta_S \delta_B)^{\frac{1}{\alpha}} c^{\frac{\alpha-1}{\alpha}} \frac{\alpha}{1-\alpha}.^{15}$$

The seller's ex-ante expected payoff from a trade relationship on open account terms, Π^Ω , can be obtained from solving the following dynamic programming problem for V_0^Ω :

$$\forall t \geq 0: \quad V_t^\Omega = \pi^\Omega(Q_t^\Omega) + \delta_S (\Lambda_t V_{t+1}^\Omega + (1 - \Lambda_t) V_0^\Omega). \quad (3)$$

In the Appendix, we derive the following solution to this problem:

$$\Pi^\Omega = \frac{1 - \delta_S \lambda}{1 - \delta_S \lambda - \delta_S \theta_0 (1 - \lambda)} \bar{\pi}^\Omega \sum_{t=0}^{\infty} \delta_S^t \Lambda_t^{\frac{1}{\alpha}} (1 - \theta_0 (1 - \lambda^t)).$$

We summarize our findings on the open account equilibrium in Proposition 2.

Proposition 2. *Suppose that payments are only possible on Ω -terms. Then the seller proposes a pooling contract to the buyer and updates his belief as prescribed by θ_t^Ω as the relationship proceeds. Based on this belief, the trade volume Q_t^Ω (the expected stage payoffs π_t^Ω) increase gradually with the age of the relationship and converge to the full information level Q^Ω ($\bar{\pi}^\Omega$). The ex-ante expected payoffs of the seller are Π^Ω .*

Proof Analysis in the text.

4.3 Discussion

A comparison of the results of Sections 4.1 and 4.2 reveals important differences between cash in advance and open account payment terms. On the one side, they can be summarized as features related to the *learning process* about the buyer, and to the *risks of relationship breakdown* on the other side.

First, consider the learning process about the buyer in a new relationship. Under cash in advance terms, the seller optimally offers a separating stage contract that immediately reveals the buyer's type. In contrast, immediate separation is not possible under Ω -terms where the payoff-maximizing stage contract

¹⁵For later use, note that the expected stage payoffs under belief θ_t^Ω can be rewritten as an expression that is proportional to the stage payoffs at the full information limit, i.e. $\pi^\Omega(Q_t^\Omega) = \Lambda(t, \hat{\theta}, \lambda)^{\frac{1}{\alpha}} \bar{\pi}^\Omega$.

necessarily features the pooling of buyer types. In this case, type information is acquired only gradually over time through the Bayesian updating process (see equation 2). Type separation under A -terms translates into a belief-free trade volume Q^A from the very first transaction while trade volumes under Ω -terms grow over time and converge to the belief-free level Q^Ω as the relationship matures. This has immediate repercussions on the development of the expected stage payoffs over the course of a trade relationship. While under A -terms the expected stage payoffs jump immediately after the first successful transaction from $\gamma(1 - \theta_0)\bar{\pi}^A$ to $\gamma\bar{\pi}^A$ and remain at this level for all following periods with the same buyer they increase at a strictly slower rate under Ω -terms as determined by the Bayesian updating process up to the level $\bar{\pi}^\Omega$.¹⁶

Second, let us compare the risks of transaction failure across payment terms. Under the considered strategy profile, transaction failure directly corresponds to the breakdown of the trade relationship with a buyer. It turns out that while under A -terms transaction failure can be exclusively triggered by buyer characteristics (i.e., her type and/or liquidity status) under Ω -terms the institutional environment in which the transaction takes place is decisive. Under the latter, a transaction can be unsuccessful only if contracts cannot be enforced which induces the *non-payment* of the transfer T_t in a match with a myopic buyer. In contrast, A -terms do not involve any payment risk since the transfer is made already before production and shipment. However, the latter can still result in an unsuccessful transaction in case of a match with a myopic or liquidity-constrained patient buyer, both of which leads to buyer *non-participation*. Ex-ante to contracting, the probability of transaction failure in period t is given for payment contract type $i \in \mathcal{F}$ and belief θ_t as $P_t^A = 1 - \gamma(1 - \theta_t)$ and $P_t^\Omega = \theta_t(1 - \lambda)$, respectively. Evidently, it holds that $P_t^\Omega < P_t^A$ and, moreover, the seller can benefit from a smaller risk of transaction failure under Ω -terms the stronger contracting institutions are.¹⁷

As a consequence, when deciding whether or not to provide trade credit to a new buyer (i.e., whether or not to offer payment on Ω -terms) the seller has to weigh the relationship stability-enhancing advantages of trade credit with the associated, comparably slow learning process about the buyer and the corresponding moderate growth of stage payoffs on the equilibrium path. In the following section, we study how the seller can manage this *trade-off between relationship stability and stage payoff growth* efficiently. We will determine the conditions under which the provision on trade credit through the seller maximizes his payoffs from the trade relationship and when it does not. In doing so we will allow for switches between payment contract types over time and derive conditions under which these are payoff-enhancing.

¹⁶Evidently, the expected stage payoffs at the full information limit may differ between cash in advance and open account terms. In section 5, we show that the optimal equilibrium can be characterized also for the special case where they are identical.

¹⁷Note that $P_t^\Omega < P_t^A$ holds irrespective of the probability $1 - \gamma$ with which the buyer becomes liquidity-constrained.

As we will see, the two payment contract sequences (A, \dots) and (Ω, \dots) for which we solved the payoff-maximizing equilibrium in this section turn out to be relevant cases also for the dynamically optimal choice of payment contracts.

5 Dynamically optimal payment contracts

In this section, we study the seller's optimal choice of payment contracts when he can freely decide between A - and Ω -terms in every period of the repeated game, i.e. $F_t \in \mathcal{F}$ for all $t \geq 0$. This will give us an understanding of how the inter-temporal trade-off outlined in the previous section determines payment contracts choice in the dynamic context.

Definition The sequence F that maximizes the seller's ex-ante expected payoffs from the trade relationship is called the *dynamically optimal sequence of payment contracts* (DOSPC).

5.1 Main results

Determining the DOSPC from a direct comparison of all available sequences is impossible since this set contains infinitely many elements when the time horizon is infinite. However, simple parameter refinements allow us to narrow down the set of possibly optimal sequences to three elements while maintaining the presence of the inter-temporal trade-off outlined in Section 4.3.

Proposition 3. *For all parameterizations of the model that satisfy the constraint*

$$\lambda < \bar{\lambda} \in (0, 1) \tag{4}$$

and, moreover, when $\alpha > \underline{\alpha} \in (0, 1)$ holds there exists $\underline{\delta}_S \in (0, 1)$ such that for all $\delta_S > \underline{\delta}_S$ we have $F \in \{(A, \dots), (\Omega, \dots), (A, \Omega, \Omega, \dots)\} \equiv \mathcal{F}^D$ as the DOSPC.

Proof See Appendix.

For understanding the reduction of the set of possible DOSPCs to \mathcal{F}^D the upper bound on the quality of contracting institutions in expression (4) is of central importance. It ensures that in trade relationships that are initiated on Ω -terms it is optimal to stick to these terms also in all subsequent transactions. For any belief θ_t^Ω , information about the buyer type is revealed at a faster rate under Ω -terms when contracting institutions are weak (i.e., when λ is small).¹⁸ Consequently, conditional on the usage of Ω -terms in the

¹⁸To see this formally, let us define by $\Theta_t \equiv (\theta_t^\Omega - \theta_{t+1}^\Omega)/\theta_t^\Omega$ the share of myopic buyers that is filtered out by the belief updating process in t , provided that the transaction in that period is successful. It is easily shown that $\partial\Theta_t/\partial\lambda < 0$, i.e. the share of myopic types that can be filtered out in any period on Ω -terms is smaller when the quality of contracting institutions is higher.

initial transaction, and hence conditional on starting trade with a comparably small volume Q_0^Ω , when (4) holds the growth of trade volumes under Ω -terms is sufficiently fast to rule out switches to A -terms in later periods. Conversely, when information is acquired slowly and (4) does not hold, the seller may be tempted to switch to the learning-efficient A -terms once the probability of buyer non-participation is reduced sufficiently. In this situation, the payoff-relevance of the trade-off outlined above is marginalized as in market environments with strong contracting institutions payment under Ω -terms can be enforced irrespectively of the buyer's type. As a consequence, the analysis of optimal payment contract choice in our model turns intractable when $\lambda > \bar{\lambda}$. Figure 2 illustrates how the growth of transaction volumes under the payment contract sequence (Ω, \dots) translates into the evolution of expected stage payoffs in a trade relationship for different levels of the contracting institutions parameter λ .

Moreover, any relationship that starts on A -terms reaches the full information limit after the first successful transaction. Consequently, given that the first transaction is conducted on A -terms, either the sequence (A, \dots) or $(A, \Omega, \Omega, \dots)$ must be optimal. Besides, it should be noted that the major role of the two further parameter restrictions on α and δ_S in Proposition 3 is to ensure that non-shipment deviations of the seller are ruled out for the latter two sequences.

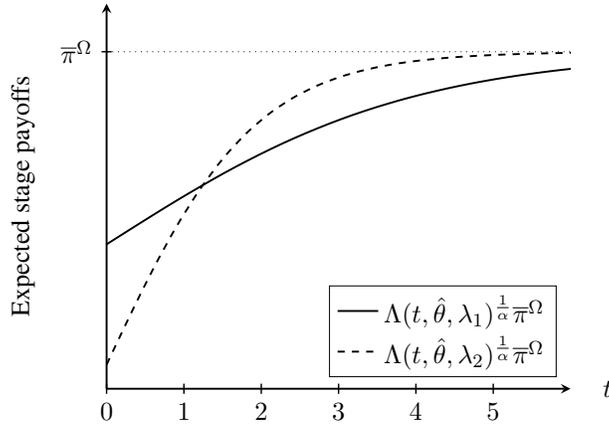


Figure 2: The evolution of the expected stage payoffs for $F = (\Omega, \dots)$ when $\lambda_1 > \lambda_2$.

Building on Proposition 3, we next investigate how the seller can optimally manage the trade-off between relationship stability and stage payoff growth over the course of a trade relationship. The following Corollary 1 determines the parameter conditions under which the respective elements of \mathcal{F}^D are dynamically optimal. It turns out that all $F \in \mathcal{F}^D$ can be optimal on the equilibrium path. Imposing mild refinements on the parameter requirements of Proposition 3 allows us to uniquely identify the DOSPC for all permissible model configurations.¹⁹

¹⁹The threshold refinements are sufficient conditions to ensure that Π^Ω is a strictly concave and monotonically decreasing

Corollary 1. Suppose that $\lambda < \bar{\lambda}' \in (0, \bar{\lambda}]$, $\alpha > \underline{\alpha}' \in [\underline{\alpha}, 1)$, and $\delta_S > \underline{\delta}_S$. Then for any tuple (γ, θ_0) the DOSPC can be uniquely determined. Three cases must be distinguished:

- When the probability that the buyer turns liquidity-constrained is **small**, i.e. when $\gamma \geq \bar{\gamma} \in (0, 1)$, then there exists a unique belief threshold $\bar{\theta}_0^A \in (0, 1)$, such that:

$$F = \begin{cases} (A, \dots) & \text{for } \theta_0 \leq \bar{\theta}_0^A, \\ (\Omega, \dots) & \text{for } \bar{\theta}_0^A \leq \theta_0. \end{cases}$$

- When the probability that the buyer turns liquidity-constrained is **moderate**, i.e. when $\gamma \in [\underline{\gamma}, \bar{\gamma}]$, then there exist unique belief thresholds $\underline{\theta}_0 \in (0, 1)$ and $\bar{\theta}_0^{A\Omega} \in (0, 1)$ with $\bar{\theta}_0^{A\Omega} > \underline{\theta}_0$, such that:

$$F = \begin{cases} (A, \dots) & \text{for } \theta_0 \leq \underline{\theta}_0, \\ (A, \Omega, \Omega, \dots) & \text{for } \underline{\theta}_0 \leq \theta_0 \leq \bar{\theta}_0^{A\Omega}, \\ (\Omega, \dots) & \text{for } \bar{\theta}_0^{A\Omega} \leq \theta_0. \end{cases}$$

- When the probability that the buyer turns liquidity-constrained is **high**, i.e. when $\gamma \leq \underline{\gamma} \in (0, 1)$, then $F = (\Omega, \dots)$.

Proof See Appendix.

Figure 3 provides a graphical summary of the results of Corollary 1. It shows the DOSPC for any combination of the seller's initial belief θ_0 , and the probability γ that the buyer remains liquid from one transaction to the next. Overall, the corollary demonstrates that for both – new and established relationships – the usage of Ω -terms and therefore the provision of trade credit by the seller is more likely optimal the higher θ_0 and the lower γ .²⁰ We elaborate on the reasons for this pattern in the following.

Consider first the situation in a newly matched buyer-seller relationship. Given the content of \mathcal{F}^D , exclusively the design of C_0 determines how the inter-temporal trade-off between relationship stability and payoff growth is resolved optimally. Corollary 1 shows that the mitigation of relationship breakdown risks is more likely prioritized to acquiring new information about the buyer the higher the probability θ_0 of drawing a myopic buyer and the smaller the probability γ that the buyer remains liquid. If θ_0 is large then conducting an initial transaction on A -terms is unlikely successful since only a small share of patient buyers – who moreover have to be liquid – will accept a contract on these terms of payment. This reduces the ex-ante expected payoffs associated with the sequences (A, \dots) and $(A, \Omega, \Omega, \dots)$, making the

function in θ_0 . This allows us to establish the uniqueness of the results in Corollary 1.

²⁰Analytically, this is implied by the fact that the threshold functions $\underline{\theta}_0$, $\bar{\theta}_0^A$, and $\bar{\theta}_0^{A\Omega}$ are all monotonically increasing in γ . See the proof of Corollary 1.

optimality of their usage less likely. Similarly, if γ is low, a buyer suffers from liquidity constraints with a high probability. Since these are never problematic under Ω -terms this makes using them during the phase of information acquisition more attractive. In the situation where $\gamma < \underline{\gamma}$ holds Ω -terms turn out to be optimal for any initial belief level.

In order to understand the rationale for varying payment terms over time it is necessary and sufficient under the restrictions of Proposition 3 to focus on the situation where A -terms have been used initially. This is because the only sequence in \mathcal{F}^D that contains switches between payment terms over time is $F = (A, \Omega, \Omega, \dots)$. While, for any $\gamma > \underline{\gamma} = \delta \frac{1}{S}$, the expected stage payoffs in a non-initial transaction are larger under A -terms (i.e., $\gamma \bar{\pi}^A > \bar{\pi}^\Omega$) continuing the relationship on A -terms retains carrying the risk of loosing a certainly patient buyer due to newly arising liquidity problems.²¹ For this additional trade-off, Corollary 1 predicts that as long as the liquidity risks are in the moderate range (i.e., when $\gamma \in (\underline{\gamma}, \bar{\gamma})$) switching to Ω -terms and thereby eliminating the remaining breakdown risks can be preferable to obtaining a high level of stage payoffs under full information. More precisely, when the probability of finding a patient buyer upon relationship breakdown is comparably low (i.e. when $\theta_0 \in (\underline{\theta}_0, \bar{\theta}_0^{A\Omega})$) loosing the current buyer is a threat of high economic relevance to the seller and, as a consequence, he rather accepts lower stage payoffs instead of risking to loose the patient buyer that he is currently matched with. Conversely, when the probability of finding a patient buyer upon relationship breakdown is high (i.e. when $\theta_0 < \underline{\theta}_0$) the seller does not find it threatful to loose his current buyer and continues business on A -terms, i.e. employs $F = (A, \dots)$. Moreover, when liquidity risks are low (i.e. when $\gamma > \bar{\gamma}$) switching to Ω -terms after an initial transaction on A -terms is never optimal.

5.2 Further discussion

Our model shows that payment contracts can be interpreted as a device to select different *contract enforcement technologies*. While under Ω -terms, enforcement is ensured by publicly available institutions under A -terms it is ensured privately through the design of the contract terms which are only acceptable to those buyers that are factually able and willing to comply. Consider the situation in a newly matched buyer-supplier relationship. While under Ω -terms, the buyer's transfer payment is ensured through formal enforcement institutions under A -terms the optimal spot contract immediately separates buyer types and makes formal enforcement redundant since no myopic or illiquid patient buyer will accept such a contract. Consequently, when the share of myopic buyers is large (or the probability of facing a liquidity-constrained buyer is high) relying entirely on buyer selection to ensure payment becomes inefficient as

²¹Note that $\gamma > \underline{\gamma}$ is also a necessary condition for (Ω, \dots) not to be the payoff-dominant payment contract sequence.

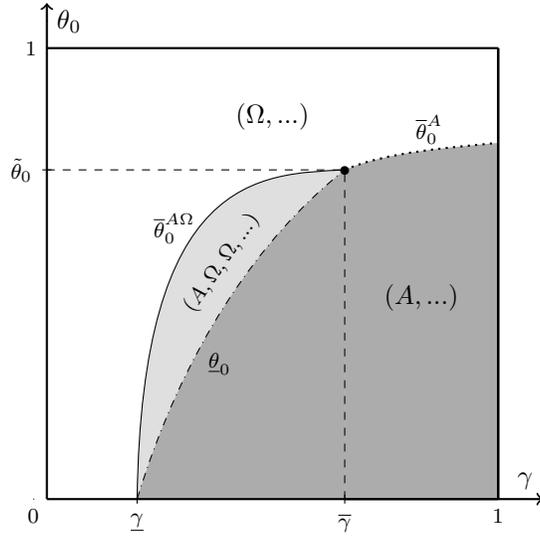


Figure 3: Dynamically optimal sequences of payment contracts in (γ, θ_0) -space.

any relationship with a buyer that is not simultaneously patient and liquid fails immediately under A -terms. In contrast, the “softer” screening under Ω -terms also allows these buyers to take up possibly productive trade relationships which has a stabilizing effect on the expected payoff stream of the seller.

6 Trade credit insurance

In international trade, the provision of trade finance through banks and insurance firms is an essential driver for the growth of firms’ trade volumes (cf. [Amiti and Weinstein, 2011](#)). As an example of external trade finance, we discuss the impact of the availability of trade credit insurance on dynamically optimal payment contract choice in this section.

Instead of taking the risk of buyer non-payment in an open account transaction in period t himself, the seller can rule it out by employing a trade credit insurance ($F_t = I$). We assume that such an insurance is available to the seller from a perfectly competitive insurance market and that the insurance fee I_t for the transaction in period t can be separated into a fixed and a variable component which is given by:

$$I_t = m + \delta_S(1 - \Lambda_t^I)T_t,$$

where the fixed (and time-invariant) component $m > 0$ covers setup and monitoring costs that the insurer incurs for managing the transaction. The second addend represents the variable component that depends on the size of the insured transfer, T_t . It is weighted by the probability of nonpayment $1 - \Lambda_t^I$, where Λ_t^I denotes the payment probability when in the t th transaction of a trade relationship is conducted under

insurance. Moreover, because potential payment default occurs only in $t + 1$ the variable component is discounted. For analytical simplicity we assume the insurer's discount factor is equal to that of the seller, δ_S . Finally, because the insurer has a vital interest that the buyer does not default on the contract it will engage in buyer screening itself before granting a credit insurance.²² We model this aspect by assuming that initially using a trade credit insurance reduces the proportion of myopic types in the population to $\hat{\theta}^I = \phi\hat{\theta}$, where $\phi \in (0, 1)$ is an inverse measure of the insurer's ability to screen out myopic types. Hence, the seller's belief to face a myopic buyer in the t th transaction on insurance terms is determined via Bayes' rule as $\theta_t^I = \hat{\theta}^I \lambda^t / [1 - \hat{\theta}^I(1 - \lambda^t)]$, and the probability of payment in t is given as $\Lambda_t^I = [1 - \hat{\theta}^I(1 - \lambda^{t+1})] / [1 - \hat{\theta}^I(1 - \lambda^t)]$.

6.1 The optimal spot contract with insurance

Employing the same strategy profile as before, the participation constraints of the two buyer types under insurance are the same as in the open account scenario, (PC_B^Ω) and (PC_M^Ω) , respectively. Also, the incentive constraint for the patient buyer to conduct payment is the same as under open account leading the seller to request the same revenue share \tilde{s} from the buyer. The optimal trade volume in period t , Q_t^I , is hence determined by maximizing the following stage payoff function:

$$Q_t^I \equiv \arg \max_{Q_t} \delta_S \tilde{s} R(Q_t) - cQ_t - I_t = \arg \max_{Q_t} \delta_S \delta_B \Lambda_t^I R(Q_t) - cQ_t - m.$$

Observe that even though the insurance eliminates the risk of non-payment, the probability of payment Λ_t^I still indirectly affects the seller's maximization problem through the variable fee component. The optimal trade volume Q_t^I and the corresponding stage payoffs $\pi^I(Q_t^I)$ are:

$$Q_t^I = \left(\frac{\delta_S \delta_B \Lambda_t^I}{c} \right)^{\frac{1}{\alpha}}, \quad \pi^I(Q_t^I) = (\delta_S \delta_B \Lambda_t^I)^{\frac{1}{\alpha}} c^{\frac{\alpha-1}{\alpha}} \frac{\alpha}{1-\alpha} - m.$$

6.2 Dynamically optimal payment contracts with insurance

In any period t the seller can now freely choose not only between cash in advance and open account terms but can alternatively decide to use a trade credit insurance, i.e. $F_t \in \mathcal{F}^+ \equiv \{A, \Omega, I\}$. In the following, we study how the availability of insurance affects the set of feasible dynamically optimal payment contract

²²This assumption is endorsed by the fact that trade credit insurers such as Euler Hermes and AIG advertise their insurance services with their expertise in monitoring the reliability of transaction counterparts.

Overall, our specification of the insurance fee follows the formalization of the letter of credit contract by [Niepmann and Schmidt-Eisenlohr \(2017\)](#). Since the introduction of banks as additional strategic players would render our dynamic model intractable we refrain from discussing the details of other forms of trade finance such as documentary collections and letters of credit in this paper and focus our study on the impact of the insurance on the seller's payment contract choices.

sequences. In fact, simple observations together with the parameter restrictions of Proposition 3 allow to establish that the set of possible DOSPCs is extended by one unique element in the presence of insurance terms.

Corollary 2. *Let $F_t \in \mathcal{F}^+$ for all $t \geq 0$. Suppose that the parameter constraints of Proposition 3 are satisfied. Then $F \in \mathcal{F}^D \cup (I, \Omega, \Omega, \dots) \equiv \mathcal{F}^{D+}$. Moreover, the seller's ex-ante expected payoffs for the sequence $F = (I, \Omega, \Omega, \dots)$ are given by:*

$$\Pi^{I\Omega} = \frac{1 - \delta_S \lambda}{1 - \delta_S \lambda - \delta_S \theta_0^I (1 - \lambda)} \left[-m + \bar{\pi}^\Omega \sum_{t=0}^{\infty} \delta_S^t (\Lambda_t^I)^{\frac{1}{\alpha}} (1 - \theta_0^I (1 - \lambda^t)) \right].$$

Proof See Appendix.

The proof of Corollary 2 establishes that $F = (I, \Omega, \Omega, \dots)$ is the only additional sequence that can become dynamically optimal. This is because, first, I -terms are payoff-dominated by Ω -terms at the full information limit and after the initial play of I -terms and, second, the informational benefit from insurer screening is largest in the initial period. In addition, the proof shows that the parameter thresholds of Proposition 3 are sufficient to establish that \mathcal{F}^{D+} is the full set of feasible DOSPCs when insurance becomes available. Acknowledging that some $F \in \mathcal{F}^{D+}$ must be optimal, the following Corollary 3 shows under which conditions there exist model parameterizations for which insuring the initial open account transaction maximizes the seller's ex-ante expected payoffs.

Corollary 3. *Suppose that the parameter constraints of Corollary 1 are satisfied. Then for any level of insurer screening efficiency $\phi \in (0, 1)$ there exist unique levels $\bar{m} > 0$ and $\hat{\theta}_0 \in (0, 1)$ such that for all $m < \bar{m}$ and all $\theta_0 > \hat{\theta}_0$ the sequence $F = (I, \Omega, \Omega, \dots)$ is the DOSPC. If $m > \bar{m}$, then $F \in \mathcal{F}^D$.*

Proof See Appendix.

Corollary 3 shows that no matter how efficient the insurer is in screening the population of buyers there always exists an upper bound of insurance fixed costs $\bar{m} > 0$ below which the seller finds it optimal to use $F = (I, \Omega, \Omega, \dots)$, provided that the marginal impact of the insurer's screening activity is high enough (i.e. the share of myopic buyers in the population is large enough). Conversely, when the fixed costs of the insurer are too large (i.e., when $m > \bar{m}$) insurance is never optimal for the seller and the set of possible DOSPCs reduces to \mathcal{F}^D .

7 Conclusion

In this paper, we have developed a theory of the optimal usage of payment contracts and the provision of trade credit in buyer-supplier relationships. A key feature of our theory is that the buyer's payment compliance is uncertain which generates transaction and relationship inefficiencies. We have shown that the seller can employ his decision whether or not to provide trade credit to the buyer strategically and thereby manage these inefficiencies. We have determined how the seller can optimally structure the usage payment contracts over time to maximize his payoffs from trade relationships. Importantly, we find that in doing so the seller is required to prioritize between the stability and the profitability of the exchange with his buyer. Moreover, the dynamically optimal choice of payment contracts can require switches between payment terms over time. For the situation where contract enforcement institutions are sufficiently weak the model predicts that whenever trade credit is offered to the buyer its optimal provision level is increasing with relationship age – a prediction that is, as we have argued, consistent with empirical findings in the trade finance literature.

While for the largest part of this paper the analysis has focused on the non-intermediated payment modes of cash in advance and open account, trade finance products provided by banks and insurance firms are of substantial practical relevance (cf. [Niepmann and Schmidt-Eisenlohr, 2017](#)). The paper incorporates external forms of trade finance into the discussion by analyzing and identifying the impact of trade credit insurance on the dynamically optimal choice of payment contracts. A promising avenue for future research is to further explore the micro-foundations of other relevant types of external trade finance such as letters of credit and documentary collections in a dynamic contracting framework.

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Appendix

Proof of Lemma 1

At the Production and Shipment stage (6) of any period the seller will not deviate from the contract if and only if:

$$-cQ^A + \delta_S V_1^A \geq (1 - \lambda)(\delta_S V_0^A) + \lambda(-cQ^A + \delta_S V_1^A) \quad (5)$$

Equation (5) states that making the effort to produce the contracted output plus the continuation payoff from the current relationship with a patient buyer must result in a higher payoff than deviating by not producing and shipping the agreed quantity Q^A . In this latter case the current relationship breaks down and one with a new buyer is started in the following period. Note that deviation is possible only if contracts cannot be enforced which happens with probability $1 - \lambda$. Plugging explicit values for V_0^A and V_1^A into (5) and simplifying gives:

$$-cQ^A + \delta_S \frac{\gamma(1 - \delta_S \theta_0) \bar{\pi}^A}{(1 - \delta_S)(1 - \gamma \theta_0 \delta_S)} \geq \delta_S \frac{\gamma(1 - \theta_0) \bar{\pi}^A}{(1 - \delta_S)(1 - \gamma \theta_0 \delta_S)} \quad (6)$$

Observing that $cQ^A = \bar{\pi}^A(1 - \alpha)/\alpha$ we can simplify (6) to:

$$\delta_S \geq \frac{1 - \alpha}{\gamma \theta_0} \equiv \tilde{\delta}_S \quad (7)$$

For an equilibrium to exist we need to ensure that $\tilde{\delta}_S < 1$. This is the case whenever $\alpha > 1 - \gamma \theta_0 \equiv \tilde{\alpha} \in (0, 1)$ holds. In this situation, the non-production deviation of the seller can be ruled if he is patient enough, i.e. when $\delta_S \geq \tilde{\delta}_S$ holds. ■

Proof of Lemma 2

At the Payment stage (2) of period t of a trade relationship the patient buyer will decide not to deviate from the agreed payment if paying plus the expected continuation payoff from the relationship compensates at least her outside option (which we normalized to zero):

$$\begin{aligned} -s^\Omega R(Q_t^\Omega) + (1 - s^\Omega) \sum_{i=t+1}^{\infty} \delta_B^{i-t} R(Q_i^\Omega) &\geq 0 \\ \Leftrightarrow (1 - s^\Omega) \sum_{i=t+1}^{\infty} \delta_B^{i-t} R(Q_i^\Omega) &\geq s^\Omega R(Q_t^\Omega) \end{aligned} \quad (8)$$

We would like to obtain a value of s^Ω that allows inequality (8) to hold for any belief θ_t^Ω . Observing that $\frac{\partial R(Q_t^\Omega)}{\partial t} > 0$ and $\frac{\partial R(Q_t^\Omega)^2}{\partial t^2} < 0$ it is clear that if (8) holds for the limit belief (i.e. for $\lim_{t \rightarrow \infty} \theta_t^\Omega = 0$) it also holds for any other belief. Denoting the trade quantity at this limit by Q^Ω this implies:

$$(1 - s^\Omega) \sum_{i=1}^{\infty} \delta_B^i R(Q^\Omega) \geq s^\Omega R(Q^\Omega).$$

Simplifying and rearranging for s^Ω gives:

$$s^\Omega \leq \delta_B \equiv \tilde{s}^\Omega.$$

The seller will set $s^\Omega = \tilde{s}^\Omega$ which is the maximal transfer to the seller that the buyer will accept for any belief θ_t . ■

Derivation of the ex-ante expected payoffs Π^Ω

This appendix complements the analysis of the main text by providing a non-recursive expression of the seller's ex-ante expected payoffs under open account terms. We proceed in two steps. First, we rewrite the period t -version of equation (3) by repeatedly substituting in the value functions of all subsequent periods. Second, we solve the resulting equation for period $t = 0$. By substituting in, we can rewrite (3) to:

$$V_t^\Omega = \bar{\pi}^\Omega \left[\Lambda_t^{\frac{1}{\alpha}} + \sum_{i=t+1}^{\infty} \delta_S^{i-t} \Lambda_i^{\frac{1}{\alpha}} \prod_{j=t}^{i-1} \Lambda_j \right] + V_0^\Omega \left[\delta_S(1 - \Lambda_t) + \sum_{i=t}^{\infty} \delta_S^{i-t+2} (1 - \Lambda_{i+1}) \prod_{j=t}^i \Lambda_j \right] \quad (9)$$

Observing that $\prod_{j=t}^i \Lambda_j = (1 - \theta_0(1 - \lambda^{i+1})) / (1 - \theta_0(1 - \lambda^t))$, we can simplify (9) to:

$$V_t^\Omega = \frac{1}{1 - \theta_0(1 - \lambda^t)} \left[\bar{\pi}^\Omega \sum_{i=t}^{\infty} \delta_S^{i-t} \Lambda_i^{\frac{1}{\alpha}} (1 - \theta_0(1 - \lambda^i)) + \delta_S V_0^\Omega \left(\frac{\theta_0 \lambda^t (1 - \lambda)}{1 - \lambda \delta_S} \right) \right] \quad (10)$$

Now suppose that $t = 0$. Solving the resulting version of (10) for V_0^Ω gives:

$$\Pi^\Omega = \frac{1 - \lambda \delta_S}{1 - \delta_S(\theta_0 + (1 - \theta_0)\lambda)} \bar{\pi}^\Omega \sum_{t=0}^{\infty} \delta_S^t \Lambda_t^{\frac{1}{\alpha}} (1 - \theta_0(1 - \lambda^t)).$$

Proof of Proposition 3

The proof is conducted using the following steps:

1. We derive conditions ($\delta_S \geq \delta_S^*$, $\alpha > \alpha^*$ and $\lambda < \bar{\lambda}$) which guarantee that whenever choosing Ω -terms until period t is optimal, it is never optimal to switch to A -terms in period $t + 1$. This immediately implies that $F \in \mathcal{F}^D$.
2. For the sequence (A, Ω, \dots) we derive the ex-ante expected payoffs $\Pi^{A\Omega}$ and conditions equivalent to Lemma 1 to rule out a non-shipment deviation by the seller in the initial transaction.
3. Derive $\underline{\delta}_S$ and $\underline{\alpha}$ by combining the results from Step 1, Step 2, and Lemma 1.

Step 1. Suppose that in all periods $\{0, 1, \dots, t\}$ playing Ω -terms is optimal. For any t , we derive conditions which ensure that playing Ω -terms in $t + 1$ is also optimal. From equations (1) and (3) the value function V_t^j in period t for payment contract type $j \in \mathcal{F}$ and belief θ_t^Ω (which applies when Ω -terms were used in all periods prior to t) can, respectively, be rewritten as:

$$V_t^A = \underbrace{\gamma(1 - \theta_t^\Omega) \bar{\pi}^A}_{\equiv W_F^A} + \underbrace{\delta_S [\gamma(1 - \theta_t^\Omega) V_1^A + (1 - \gamma(1 - \theta_t^\Omega)) V_0]}_{\equiv W_C^A} \quad (1')$$

$$V_t^\Omega = \underbrace{(\delta_S \Lambda_t)^{\frac{1}{\alpha}} \bar{\pi}^A}_{\equiv W_F^\Omega} + \underbrace{\delta_S [\Lambda_t V_{t+1}^\Omega + (1 - \Lambda_t) V_0]}_{\equiv W_C^\Omega} \quad (3')$$

We proceed by induction. Note that we have $V_t^\Omega > V_t^A$ by assumption. Upon moving to period $t + 1$ on

Ω -terms, in order to guarantee that $V_{t+1}^\Omega > V_{t+1}^A$ holds for any belief θ_t^Ω , it is sufficient to ensure:

$$\frac{\partial (\Lambda_t \delta_S)^\frac{1}{\alpha}}{\partial t} > \frac{\partial \gamma(1 - \theta_t^\Omega)}{\partial t}, \quad (11)$$

$$\text{and } \frac{\partial \Lambda_t}{\partial t} > \frac{\partial \gamma(1 - \theta_t^\Omega)}{\partial t}. \quad (12)$$

Condition (11) guarantees that by moving from period t to period $t+1$ on open account terms (and thereby decreasing the belief to θ_{t+1}^Ω) increases the expected stage payoffs under open account terms, W_F^Ω , by more than those for cash in advance terms, W_F^A . Condition (12) ensures the same for the continuation payoffs W_C^Ω and W_C^A , respectively. To see that the conditions are sufficient note that while V_1^A and V_0 are independent of t the value of V_{t+1}^Ω is increasing in t since (3') has the same functional structure in all periods (only the belief θ_t^Ω varies and decreases with t).

We derive conditions for both, (11) and (12), to hold. We get:

$$\frac{\partial \Lambda_t}{\partial t} > \frac{\partial \gamma(1 - \theta_t^\Omega)}{\partial t} \Leftrightarrow \lambda < 1 - \gamma \equiv \bar{\lambda}, \quad \text{and} \quad (13)$$

$$\frac{\partial (\Lambda_t \delta_S)^\frac{1}{\alpha}}{\partial t} > \frac{\partial \gamma(1 - \theta_t^\Omega)}{\partial t} \Leftrightarrow \delta_S > \alpha^\alpha \left(\frac{\gamma}{1 - \lambda} \right)^\alpha \Lambda_t^{\alpha-1} \equiv \delta'_S.$$

For a solution to Step 1 of the proof to exist, we must ensure that $\delta'_S \in (0, 1)$. To do so, first note that $\lim_{\alpha \rightarrow 0} \delta'_S = 1/\Lambda_t > 1$ and $\lim_{\alpha \rightarrow 1} \delta'_S = \gamma/(1 - \lambda) < 1$ by condition (13). Moreover, note that δ'_S is strictly convex in α since:

$$\frac{\partial^2 \delta'_S}{\partial \alpha^2} = \left(\frac{\alpha \gamma}{1 - \lambda} \right)^\alpha \Lambda_t^{\alpha-1} \left(\left(\ln \left(\frac{\alpha \gamma}{1 - \lambda} \right) + \ln \Lambda_t \right) \left(\ln \left(\frac{\alpha \gamma}{1 - \lambda} \right) + \ln(\Lambda_t) + 2 \right) + \frac{1}{\alpha} + 1 \right) > 0.$$

This shows that there exists a unique $\alpha' \in (0, 1)$ such that $\delta'_S \in (0, 1)$ for all $\alpha > \alpha'$.

Since δ_S is fixed while δ'_S varies with t , we must reduce δ'_S to a threshold that is sufficient for all t . Observing that:

$$\frac{\partial \delta'_S}{\partial t} = \frac{(1 - \alpha)^\alpha \alpha^\alpha \left(\frac{\gamma}{1 - \lambda} \right)^\alpha (1 - \lambda) \lambda^t (1 - \theta_0) \theta_0 \Lambda_t^\alpha \ln(\lambda)}{(1 - \theta_0 (1 - \lambda^{1+t}))^2} < 0$$

establishes that:

$$\delta_S > \alpha^\alpha \left(\frac{\gamma}{1 - \lambda} \right)^\alpha \Lambda_0^{\alpha-1} \equiv \delta_S^*$$

is sufficient for all periods. We denote by:

$$\alpha^* \equiv \alpha'|_{t=0} = \frac{\ln \Lambda_0}{W \left(\frac{\gamma}{1 - \lambda} \Lambda_0 \ln \Lambda_0 \right)} \in (0, 1)$$

the corresponding lower bound of the revenue concavity parameter, where α^* can be expressed explicitly using the Lambert W function. Hence, equivalently to above there exists a unique $\alpha^* \in (0, 1)$ such that $\delta_S^* \in (0, 1)$ for all $\alpha > \alpha^*$ provided that $\lambda < \bar{\lambda}$ holds.

Summing up, we have established that whenever $\delta_S \geq \delta_S^*$, $\alpha > \alpha^*$, and $\lambda < \bar{\lambda}$, then $V_t^\Omega > V_t^A \Rightarrow V_{t'}^\Omega > V_{t'}^A$ for any $t' > t$. Consequently, whenever A -terms are part of a dynamically optimal sequence of payment contracts they will be used in the initial period. From the analysis in Section 4.1 we know that A -terms separate buyer types and, when used initially, $\theta_t = 0$ for all $t > 0$. In this situation, the

model reaches an absorbing state (full information) in which either A -terms or Ω -terms will be used in all periods $t > 0$. Hence, the dynamically optimal sequence of payment contracts must be an element of \mathcal{F}^D .

Step 2. The ex-ante expected payoffs implied by the payment contract sequence (A, Ω, \dots) can be obtained from solving the following recursion for $V_0^{A\Omega}$:

$$V_0^{A\Omega} = \gamma(1 - \theta_0) [\bar{\pi}^A + \delta_S V_1^{A\Omega}] + (1 - \gamma(1 - \theta_0))\delta_S V_0^{A\Omega}, \quad V_1^{A\Omega} = \frac{\bar{\pi}^\Omega}{1 - \delta_S}.$$

The solution is:

$$\Pi^{A\Omega} = \frac{\gamma(1 - \theta_0)(\delta_S \bar{\pi}^\Omega + (1 - \delta_S)\bar{\pi}^A)}{(1 - \delta_S)(1 - \delta_S + \delta_S \gamma(1 - \theta_0))}. \quad (14)$$

We have to show that at the Production and Shipment stage of any period the seller will not deviate from the contract under the sequence $(A, \Omega, \Omega, \dots)$. Using the same logic as in the proof of Lemma 1 this is the case if and only if:

$$-cQ^A + \delta_S V_1^{A\Omega} \geq \delta_S V_0^{A\Omega} \quad \Leftrightarrow \quad \Gamma(\alpha, \gamma, \delta_S, \theta_0) \equiv \alpha + \delta_S - \alpha\delta_S(1 - \delta_S^{\frac{1}{\alpha}}) - \delta_S \gamma(1 - \theta_0) - 1 \geq 0.$$

Our aim is to show existence of $\delta_S'' \in (0, 1)$ such that for all $\delta_S \geq \delta_S''$ the non-shipment deviation is ruled out. To do so, it is necessary to show that $\partial\Gamma/\partial\delta_S > 0$. Note that:

$$\frac{\partial\Gamma}{\partial\delta_S} > 0 \quad \Leftrightarrow \quad \delta_S > \left(\frac{\gamma(1 - \theta_0) - 1 + \alpha}{1 + \alpha} \right)^\alpha \equiv \hat{\delta}_S.$$

Since the equilibria that we study are constrained to $\delta_S > \delta_S^*$, in order to show that $\partial\Gamma/\partial\delta_S > 0$ it is sufficient to ensure that $\delta_S^* > \hat{\delta}_S$ holds. Existence of $\hat{\delta}_S$ requires that $\alpha > 1 - \gamma(1 - \theta_0) \equiv \alpha'' \in (0, 1)$ and also implies that $\delta_S^* > \hat{\delta}_S$.

Provided that $\alpha > \alpha''$ holds, the equation $\Gamma(\alpha, \gamma, \delta_S, \theta_0) = 0$ implicitly determines the minimum patience level δ_S'' ensuring non-deviation for all $\delta_S > \delta_S''$. Note that $\delta_S'' < 1$ always holds since $\lim_{\delta_S \rightarrow 1} \Gamma = \alpha - \gamma(1 - \theta_0) > 0$ for all $\alpha > \alpha''$.

Step 3. It directly follows from Steps 1 and 2 of the proof and Lemma 1 that whenever $\alpha > \max\{\alpha^*, \alpha'', \tilde{\alpha}\} \equiv \underline{\alpha} \in (0, 1)$ and $\lambda < \bar{\lambda}$ hold there exists $\underline{\delta}_S \equiv \max\{\delta_S^*, \delta_S'', \tilde{\delta}_S\} \in (0, 1)$ such that for all $\delta_S > \underline{\delta}_S$ we have $F \in \mathcal{F}^D$ as the dynamically optimal sequence of payment contracts in the repeated game equilibrium. ■

Proof of Corollary 1

The proof is conducted using the following steps:

1. Show that Π^A and $\Pi^{A\Omega}$ are monotonically decreasing and strictly concave functions in θ_0 . Establish that the same is true for Π^Ω provided that $\lambda < \bar{\lambda}'$ and $\alpha > \underline{\alpha}'$ hold.
2. Show that for all $\gamma > \underline{\gamma} \in (0, 1)$ there exists a unique value $\bar{\theta}_0^{A\Omega} \in (0, 1)$ (respectively, $\bar{\theta}_0^A \in (0, 1)$) such that $\Pi^{A\Omega} > \Pi^\Omega$ (resp., $\Pi^A > \Pi^\Omega$) if and only if $\theta_0 < \bar{\theta}_0^{A\Omega}$ (resp., $\theta_0 < \bar{\theta}_0^A$) holds. Conversely, when $\gamma < \underline{\gamma}$ we get $\Pi^\Omega > \max\{\Pi^A, \Pi^{A\Omega}\}$ for all $\theta_0 \in (0, 1)$.
3. Show there exists a unique $\underline{\theta}_0$ such that $\Pi^A > \Pi^{A\Omega}$ if and only if $\theta_0 < \underline{\theta}_0$. Moreover, $\underline{\theta}_0 \in (0, 1)$ if and only if $\gamma \in (\underline{\gamma}, \tilde{\gamma})$ with $\tilde{\gamma} \in (0, 1)$.

4. Show existence of $\bar{\gamma} \in (\underline{\gamma}, \tilde{\gamma})$ which allows to establish the systematic characterization of all parameter combinations $\{\gamma, \theta_0\}$ into unique DOSPC as proposed by the Corollary.

Step 1. The desired properties are easily established for Π^A and $\Pi^{A\Omega}$ by observing that:

$$\begin{aligned} \frac{\partial \Pi^A}{\partial \theta_0} &= -\frac{\gamma(1-\delta_S\gamma)\bar{\pi}^A}{(1-\delta_S)(1-\delta_S\gamma\theta_0)^2} < 0, & \frac{\partial^2 \Pi^A}{\partial \theta_0^2} &= -\frac{2\delta_S\gamma^2(1-\delta_S\gamma)\bar{\pi}^A}{(1-\delta_S)(1-\delta_S\gamma\theta_0)^3} < 0, \\ \frac{\partial \Pi^{A\Omega}}{\partial \theta_0} &= -\frac{\gamma[(1-\delta_S)\bar{\pi}^A + \delta_S\bar{\pi}^{\Omega}]}{(1-\delta_S + \delta_S\gamma(1-\theta_0))^2} < 0, & \frac{\partial^2 \Pi^{A\Omega}}{\partial \theta_0^2} &= -\frac{2\delta_S\gamma^2[(1-\delta_S)\bar{\pi}^A + \delta_S\bar{\pi}^{\Omega}]}{(1-\delta_S + \delta_S\gamma(1-\theta_0))^3} < 0. \end{aligned}$$

The argument for Π^Ω is more subtle because the function contains a complicated infinite sequence. To proceed, let us define:

$$\Pi_t^\Omega \equiv \frac{(1-\lambda\delta_S)(1-\theta_0(1-\lambda^t))}{1-\delta_S(\theta_0+(1-\theta_0)\lambda)} \delta_S^t \Lambda_t^{\frac{1}{\alpha}} \bar{\pi}^\Omega,$$

where $\Pi^\Omega = \sum_{t=0}^{\infty} \Pi_t^\Omega$. Let us start by showing concavity. Our aim is to establish a condition under which, for all periods t , $\partial^2 \Pi_t^\Omega / \partial \theta_0^2 < 0$ holds which implies that $\partial^2 \Pi^\Omega / \partial \theta_0^2 < 0$ holds as well. We get:

$$\frac{\partial^2 \Pi_t^\Omega}{\partial \theta_0^2} < 0 \quad \Leftrightarrow \quad K(t, \delta_S, \lambda, \theta_0, \alpha) \equiv \frac{1-\alpha}{\alpha} \Delta(t, \delta_S, \lambda, \theta_0) - 2\delta_S(1-\lambda) [E(t, \delta_S, \lambda, \theta_0) + \alpha Z(t, \delta_S, \lambda)] < 0, \quad (15)$$

$$\begin{aligned} \text{where } \Delta(t, \delta_S, \lambda, \theta_0) &\equiv \frac{(1-\delta_S\lambda - \delta_S\theta_0(1-\lambda))^2(1-\lambda)\lambda^t}{(1-\theta_0(1-\lambda^{t+1}))^2(1-\theta_0(1-\lambda^t))} > 0, \\ E(t, \delta_S, \lambda, \theta_0) &\equiv \frac{(1-\delta_S\lambda - \delta_S\theta_0(1-\lambda))(1-\lambda)\lambda^t}{(1-\theta_0(1-\lambda^{t+1}))} > 0, \\ Z(t, \delta_S, \lambda) &\equiv 1 - \delta_S - \lambda^t(1-\delta_S\lambda). \end{aligned}$$

Let $H(t, \delta_S, \lambda, \theta_0) \equiv E(t, \delta_S, \lambda, \theta_0) + \alpha Z(t, \delta_S, \lambda)$. Observe that $H > 0$ for all $\alpha \in (0, 1)$ if we establish that $H|_{\alpha \rightarrow 1} > 0$ since Z is possibly negative, and $E > 0$. We get:

$$H|_{\alpha \rightarrow 1} > 0 \quad \Leftrightarrow \quad \xi(t, \delta_S, \lambda) \equiv 1 - \lambda^{t+1} - \delta_S(1 - \lambda^{t+2}) > 0.$$

Since $\partial \xi / \partial t > 0$, it is sufficient to check $\xi|_{t=0} > 0$. Rearranging the latter gives:

$$\lambda < \frac{1-\delta_S}{\delta_S} \equiv \hat{\lambda} > 0.$$

Consequently, under the assumption that $\lambda < \hat{\lambda}$, we have that K is decreasing in α and since $\lim_{\alpha \rightarrow 1} K = -2\delta_S(1-\lambda)H < 0$ and $\lim_{\alpha \rightarrow 0} K = \infty$ there must exist $\bar{\alpha} \in (0, 1)$ such that $K < 0$ for all $\alpha > \bar{\alpha}$. We therefore conclude that Π^Ω is concave in θ_0 for all $\alpha > \bar{\alpha}$ and all $\lambda < \hat{\lambda}$.

Remains to show that Π^Ω is decreasing in θ_0 . To do so we show that the parameter conditions that establish concavity are sufficient for $\partial \Pi_t^\Omega / \partial \theta_0 < 0$ to hold in all periods as well which implies that $\partial \Pi^\Omega / \partial \theta_0 < 0$ is true. We get:

$$\frac{\partial \Pi_t^\Omega}{\partial \theta_0} < 0 \quad \Leftrightarrow \quad H(t, \delta_S, \lambda, \theta_0) > 0.$$

Clearly, the same arguments as above establish that $\partial \Pi^\Omega / \partial \theta_0 < 0$ if $\lambda < \hat{\lambda}$. For further use we define $\underline{\alpha}' \equiv \max\{\underline{\alpha}, \bar{\alpha}\}$ and $\bar{\lambda}' \equiv \min\{\bar{\lambda}, \hat{\lambda}\}$.

Step 2. We proceed by studying the limit properties of the payoff functions in θ_0 . First, observe that

$\lim_{\theta_0 \rightarrow 1} \Pi^{A\Omega} = \lim_{\theta_0 \rightarrow 1} \Pi^A = 0 < \lim_{\theta_0 \rightarrow 1} \Pi^\Omega = \lambda^{\frac{1}{\alpha}} \bar{\pi}^\Omega / (1 - \delta_S)$. Moreover, we have:

$$\lim_{\theta_0 \rightarrow 0} \Pi^{A\Omega} = \frac{\gamma(\delta_S \bar{\pi}^\Omega + (1 - \delta_S) \bar{\pi}^A)}{(1 - \delta_S)(1 - \delta_S + \delta_S \gamma)}, \quad \lim_{\theta_0 \rightarrow 0} \Pi^A = \frac{\gamma \bar{\pi}^A}{1 - \delta_S}, \quad \lim_{\theta_0 \rightarrow 0} \Pi^\Omega = \frac{\bar{\pi}^\Omega}{1 - \delta_S}.$$

It is easily shown that $\lim_{\theta_0 \rightarrow 0} \Pi^A > \lim_{\theta_0 \rightarrow 0} \Pi^{A\Omega} > \lim_{\theta_0 \rightarrow 0} \Pi^\Omega$ if and only if $\gamma > \delta_S^{\frac{1}{\alpha}} \equiv \underline{\gamma} \in (0, 1)$ and $\lim_{\theta_0 \rightarrow 0} \Pi^A < \lim_{\theta_0 \rightarrow 0} \Pi^{A\Omega} < \lim_{\theta_0 \rightarrow 0} \Pi^\Omega$ otherwise.

From these observations and the properties of the payoff functions established in Step 1 it immediately follows that for all $\gamma > \underline{\gamma} \in (0, 1)$ there exists a unique value $\bar{\theta}_0^{A\Omega} \in (0, 1)$ (respectively, $\bar{\theta}_0^A \in (0, 1)$) such that $\Pi^{A\Omega} > \Pi^\Omega$ (resp., $\Pi^A > \Pi^\Omega$) if and only if $\theta_0 < \bar{\theta}_0^{A\Omega}$ (resp., $\theta_0 < \bar{\theta}_0^A$) holds. Moreover, since $\partial \Pi^A / \partial \gamma > 0$, $\partial \Pi^{A\Omega} / \partial \gamma > 0$, and $\partial \Pi^\Omega / \partial \gamma = 0$ it follows that $\partial \bar{\theta}_0^A / \partial \gamma > 0$ and $\partial \bar{\theta}_0^{A\Omega} / \partial \gamma > 0$.

By taking together the above arguments and by observing that $\lim_{\theta_0 \rightarrow 0} \Pi^A = \lim_{\theta_0 \rightarrow 0} \Pi^{A\Omega} = \lim_{\theta_0 \rightarrow 0} \Pi^\Omega$ at $\gamma = \underline{\gamma}$ it immediately follows that $\Pi^\Omega > \max\{\Pi^A, \Pi^{A\Omega}\}$ for all $\gamma < \underline{\gamma}$.

Step 3. Observe that:

$$\Pi^A \geq \Pi^{A\Omega} \quad \Leftrightarrow \quad \theta_0 \leq \frac{\gamma - \delta_S^{\frac{1}{\alpha}}}{\delta_S \gamma (1 - \delta_S^{\frac{1}{\alpha}})} \equiv \underline{\theta}_0.$$

Note that $\underline{\theta}_0 > 0$ if and only if $\gamma > \underline{\gamma}$. Clearly, $\underline{\theta}_0$ is monotonically increasing and strictly concave in γ and there exists $\tilde{\gamma} \equiv \delta_S^{\frac{1}{\alpha}} / (1 - \delta_S(1 - \delta_S^{\frac{1}{\alpha}})) \in (0, 1)$ such that $\underline{\theta}_0 \in (0, 1)$ for all $\gamma \in (\underline{\gamma}, \tilde{\gamma})$.

Step 4. Observe that $\underline{\theta}_0 = 1$ at $\gamma = \tilde{\gamma}$ and that $\bar{\theta}_0^A < 1$ and $\bar{\theta}_0^{A\Omega} < 1$ for all $\gamma \in (0, 1)$. Consequently since the thresholds $\bar{\theta}_0^A$, $\bar{\theta}_0^{A\Omega}$, and $\underline{\theta}_0$ are all increasing in γ and by definition of these thresholds there exists $\bar{\gamma} \in (\underline{\gamma}, \tilde{\gamma})$ and a corresponding belief level $\tilde{\theta}_0 \in (0, 1)$ at which $\bar{\theta}_0^A = \bar{\theta}_0^{A\Omega} = \underline{\theta}_0$ holds. From the properties of the payoff functions derived in Step 1 and 2 and the threshold definitions we have that $\bar{\theta}_0^{A\Omega} > \bar{\theta}_0^A > \underline{\theta}_0$ if and only if $\gamma \in (\underline{\gamma}, \bar{\gamma})$ and moreover that $\Pi^{A\Omega} > \max\{\Pi^A, \Pi^\Omega\}$ for all $\theta_0 \in (\underline{\theta}_0, \bar{\theta}_0^{A\Omega})$ in this γ -range. For the scenario where $\gamma \in (\underline{\gamma}, \bar{\gamma})$ it also follows from the threshold definitions that $\Pi^A > \max\{\Pi^{A\Omega}, \Pi^\Omega\}$ for all $\theta_0 \in (0, \underline{\theta}_0)$ and that $\Pi^\Omega > \max\{\Pi^{A\Omega}, \Pi^A\}$ for all $\theta_0 \in (\bar{\theta}_0^{A\Omega}, 1)$. When $\gamma > \bar{\gamma}$ we have that $\bar{\theta}_0^A > \bar{\theta}_0^{A\Omega}$ which together with the properties of the payoff functions derived in Step 1 and 2 implies that $\Pi^A > \max\{\Pi^{A\Omega}, \Pi^\Omega\}$ for all $\theta_0 \in (0, \bar{\theta}_0^A)$ and $\Pi^\Omega > \max\{\Pi^{A\Omega}, \Pi^A\}$ otherwise. ■

Proof of Corollary 2

First, note that I -terms cannot follow on A -terms because at the full information limit I -terms are dominated by Ω -terms. The reason is that with A -terms being used in the first period the game reaches the full information limit after the initial transaction and upon playing Ω -terms the seller can save the fixed costs of the insurance, m .

Second, note that I -terms cannot follow on Ω -terms. To see this, let us rewrite the belief under payment contract $j \in \{\Omega, I\}$ for period $t + 1$ as $\theta_{t+1}^j = \theta_t^j \lambda / (1 - \theta_t^j (1 - \lambda))$. Note that θ_{t+1}^j is an increasing and strictly convex function in θ_t^j . Consequently, the incentive to employ insurance is largest in the initial period since it implies the largest informational gain from the insurer's screening activity. Hence, whenever trade credit insurance is used it will be employed in the initial transaction.

Note also, that insurance will not be used for more than the initial period. The reason is that in any further transaction with the same buyer the seller can benefit from the insurer's screening technology

also under Ω -terms. However, by not using the insurance he can save the fixed insurance costs m in the subsequent periods.

To complete the proof, it remains to establish that A -terms cannot follow on an initial period on I -terms. To do so we can apply fully analogously the induction technique from the proof of Proposition 3. Assume that in the initial period I -terms are used and Ω -terms in all following transactions up to period $t - 1$. Then in period t the value functions under A -terms and Ω -terms respectively are:

$$V_t^A = \gamma(1 - \theta_t^I) \bar{\pi}^A + \delta_S [\gamma(1 - \theta_t^I) V_{t+1}^A + (1 - \gamma(1 - \theta_t^I)) V_0], \quad (1'')$$

$$V_t^\Omega = (\delta_S \Lambda_t^I)^{\frac{1}{\alpha}} \bar{\pi}^A + \delta_S [\Lambda_t^I V_{t+1}^\Omega + (1 - \Lambda_t^I) V_0]. \quad (3'')$$

Comparison of (1'') and (3'') with equations (1') and (3') shows that the only difference between the respective expressions is the belief on the buyer type, θ_t^I , which derives from the identical updating process as under Ω -terms. The only difference is that the initial belief under I -terms is shifted downwards to $\theta_0^\Omega \phi$. Acknowledging this, we can proceed with the identical steps as in the proof of Proposition 3 to establish that under the same parameter conditions $V_t^\Omega > V_t^A \Rightarrow V_{t+1}^\Omega > V_{t+1}^A$.

The ex-ante expected payoffs under the sequence $F = (I, \Omega, \Omega, \dots)$ can be obtained from the following program:

$$\begin{aligned} V_0^{I\Omega} &= \pi^I(Q_0^I) + \delta_S [\Lambda_0^I V_1^{I\Omega} + (1 - \Lambda_0^I) V_0^{I\Omega}], \\ \forall t > 0: \quad V_t^{I\Omega} &= \pi^\Omega(Q_t^I) + \delta_S [\Lambda_t^I V_{t+1}^{I\Omega} + (1 - \Lambda_t^I) V_0^{I\Omega}]. \end{aligned} \quad (16)$$

Solving (16) for $V_0^{I\Omega}$ by using the same steps as in the derivation of Π^Ω gives:

$$\Pi^{I\Omega} = \frac{1 - \delta_S \lambda}{1 - \delta_S \lambda - \delta_S \theta_0^I (1 - \lambda)} \left[-m + \bar{\pi}^\Omega \sum_{t=0}^{\infty} \delta_S^t (\Lambda_t^I)^{\frac{1}{\alpha}} (1 - \theta_0^I (1 - \lambda^t)) \right]. \quad \blacksquare$$

Proof of Corollary 3

We begin by showing that $\Pi^{I\Omega}$ is monotonically decreasing and strictly concave under the conditions of Corollary 1. Let us rearrange $\Pi^{I\Omega}$ as:

$$\Pi^{I\Omega} = M + \underbrace{\frac{1 - \delta_S \lambda}{1 - \delta_S \lambda - \delta_S \theta_0^I (1 - \lambda)} \bar{\pi}^\Omega \sum_{t=0}^{\infty} \delta_S^t (\Lambda_t^I)^{\frac{1}{\alpha}} (1 - \theta_0^I (1 - \lambda^t))}_{\equiv \Pi_-^{I\Omega}},$$

where $M \equiv -m(1 - \delta_S \lambda)/(1 - \delta_S \lambda - \delta_S \theta_0^I (1 - \lambda))$. First, note that $\partial M / \partial \theta_0 < 0$, and $\partial^2 M / \partial \theta_0^2 < 0$. Next, note that because $\theta_0^I = \phi \theta_0$, the exact same arguments as in the proof of Corollary 1 can be used to establish that $\Pi_-^{I\Omega}$ is a monotonically decreasing and strictly concave function in θ_0 . Taking this together with the functional properties of M derived above establishes that $\Pi^{I\Omega}$ is a monotonically decreasing and strictly concave function in θ_0 under the parameter conditions of Corollary 1. Note, that by the same line of arguments the same functional properties are obtained w.r.t. the insurance screening parameter ϕ .

We continue by comparing the limit properties of Π^Ω and $\Pi^{I\Omega}$ w.r.t. θ_0 . First, note that $\lim_{\theta_0 \rightarrow 0} \Pi^{I\Omega} = -m + \bar{\pi}^\Omega / (1 - \delta_S) < \lim_{\theta_0 \rightarrow 0} \Pi^\Omega$. Since both, Π^Ω and $\Pi^{I\Omega}$ are monotonically decreasing and strictly

concave in θ_0 , whenever:

$$\begin{aligned} & \lim_{\theta_0 \rightarrow 1} \Pi^{I\Omega} > \lim_{\theta_0 \rightarrow 1} \Pi^\Omega \\ \Leftrightarrow & m < \bar{\pi}^\Omega \left[\frac{\lambda^{\frac{1}{\alpha}}(1 - \delta_S \lambda - \delta_S \phi(1 - \lambda))}{(1 - \delta_S)(1 - \delta_S \lambda)} - \sum_{t=0}^{\infty} \delta_S^t \left(\frac{1 - \phi(1 - \lambda^{t+1})}{1 - \phi(1 - \lambda^t)} \right)^{\frac{1}{\alpha}} (1 - \phi(1 - \lambda^t)) \right] \equiv \bar{m} \end{aligned}$$

then there exists a unique $\hat{\theta}'_0 \in (0, 1)$ at which $\Pi^{I\Omega} = \Pi^\Omega$ and $\Pi^{I\Omega} > \Pi^\Omega$ if and only if $\theta_0 > \hat{\theta}'_0$. Noting from Corollary 1 that for $\theta_0 \rightarrow 1$ the sequence (Ω, \dots) payoff-dominates (A, \dots) and $(A, \Omega, \Omega, \dots)$, we can infer that there must exist $\hat{\theta}_0 \in [\hat{\theta}'_0, 1)$ such that for all $\theta_0 > \hat{\theta}_0$ we have that $\Pi^{I\Omega} > \max\{\Pi^\Omega, \Pi^{A\Omega}, \Pi^A\}$.

■