

Chasing Lemons: Competition for Talent under Asymmetric Information*

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Abstract

We develop a model of competition for managerial talent in which firms asymmetrically learn about the ability of their managers. In equilibrium, firms poach talent from competitors, even in the absence of gains from trade. Our main result is that firms inefficiently chase lemons: some poached managers are less productive in their new jobs. Our model provides an equilibrium explanation for the apparent lack of portability of talent observed among some finance workers, such as security analysts and mutual fund managers. The model has predictions linking firm heterogeneity to managerial turnover, compensation, and the distribution of talent.

Keywords: Financial-Sector Labor Markets, Adverse Selection, Poaching

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1. Introduction

The main assets of financial services firms are organizational capital and human capital. It is thus unsurprising that such firms compete aggressively for talent. Indeed, talent poaching is widespread in the financial sector.¹ Despite this fact, evidence of the benefits from poaching finance workers is elusive. Berk, van Binsbergen, and Liu (2017) show that mutual fund managers who move to other firms are not as skilled as those who stay. The authors argue that firms have private information about the skill of their managers, implying that managers who are successfully poached are adversely selected. Groysberg, Lee, and Nanda (2008) show similar evidence in the context of security analysis: The performance of analysts who are successfully poached by competitors declines after analysts switch employers. The authors attribute this finding to the relevance of firm-specific skills.

While both asymmetric information and firm-specific skills can explain the apparent lack of talent portability, a puzzle remains: If talent is not very portable, why do firms poach employees from their competitors? Why would managers become less productive in their new jobs? In other words, why do firms “chase lemons”? Commenting on Groysberg, Lee, and Nanda (2008), Oyer and Schaefer (2011, p. 1804) describe this puzzle succinctly: “There may be substantial firm-specificity in analyst skills that is lost upon job mobility. It is also possible that this is evidence of a winner’s curse stemming from asymmetric learning. *It is not clear how this set of facts is consistent with equilibrium behavior by market participants.*”

In this paper, we develop a model of the market for knowledge workers (such as fund managers, security analysts, etc.), whom we call managers, for brevity. The model is built upon two main assumptions: (i) firm-specific skills are valuable and (ii) information about managerial ability is asymmetric. Empirically, such features appear to be relevant not only in the financial services industry, but also in other markets for high-skilled managers. Cziraki and Jenter (2020) show that more than 90% of all CEO hires are either insiders or connected outsiders. They conclude that “*firm-specific human capital and personal connec-*

¹For evidence of the importance of competition for talent and demand for managerial skills in the financial sector, see Philippon and Reshef (2012) and Célérier and Vallée (2018). For anecdotal evidence of the increasing incidence of talent poaching in the financial sector in recent years, see, e.g., Morrell (2018). More generally, Haltiwanger, Hyatt, and McEntarfer (2018) show that job-to-job transitions are an important empirical phenomenon, accounting for approximately half of all employee reallocations across a number of sectors.

tions determine CEO hiring.” Hacamo and Kleiner (2018) show that social networks can reduce informational asymmetries and improve a firm’s access to the managerial labor market. Consistent with employers having an informational advantage at discovering talent, Tate and Yang (2015) show evidence that firms use internal labor markets to allocate talent across divisions efficiently. Groen-Xu and Lü (2019) show evidence that boards use their private information about CEOs when setting compensation.²

The model is as follows. Managers have both general and firm-specific skills. Firms are heterogeneous and differ in quality (i.e., productivity or scale). Managerial talent and firm quality are technological complements.³ Firms and managers initially match randomly. Firms acquire private information about the general talent of their incumbent managers. Competing firms – those firms with vacancies – then try to poach those managers.

The model implies the existence of job ladders: Job-to-job flows are typically from lower quality firms to higher quality firms. Because lower quality firms retain their best managers, managers who move to higher quality firms are adversely selected. The main result in our model is that such adversely selected job-to-job flows can be inefficient. Our model displays an equilibrium in which firms rationally chase *lemons*, here defined as managers who become less productive after moving to higher quality firms.

The intuition for the main result is as follows. Firms retain their best managers because less-informed competitors are unable to separate top managers from mediocre ones. Because of firm-specific skills, even low-quality firms are able to retain some of their top talent. Because firms always retain the best managers, poached managers are adversely selected, that is, poached managers are mediocre managers. But why would high-quality firms poach mediocre managers from low-quality firms? In equilibrium, firms fire all managers with low general talent. By attempting to retain only those managers whose talent is above a certain

²There is an important empirical literature in labor economics on asymmetric employer learning. Gibbons and Katz (1991) provide empirical evidence compatible with the predictions of a model of layoffs with asymmetric employer learning. Schönberg (2007) finds evidence of asymmetric employer learning for college graduates. Pinkston (2009) constructs a model in which firms use bidding wars to compete for talent and finds empirical evidence of substantial asymmetric employer learning. Kahn (2013) tests two predictions of an asymmetric employer learning model: (i) the variance of wage changes is higher for stayers than for movers and (ii) an increase in the degree of informational asymmetry decreases the variance of wage changes more for movers than for stayers. She finds substantial evidence in favor of asymmetric learning.

³For evidence of heterogeneity in financial firms, see Hwang, Liberti, and Sturgess (2019). For evidence of strong complementarities between firm scale and talent in finance, see Célérier and Vallée (2018).

threshold, low-quality firms certify that such managers have above-average talent. Poachers are happy to hire mediocre managers, i.e., those who are above average but not stars, if the alternative is to hire an unproven manager. Mediocre managers are adversely selected with respect to the set of employed managers while, at the same time, being positively selected relative to the population as a whole.

A unique feature of our model, when compared to traditional asymmetric information models, is that there are job-to-job flows in equilibrium, even in the absence of gains from trade. Our results may appear surprising in light of the original analysis of markets with asymmetric information by Akerlof (1970). In a lemons market in which the seller of an asset has private information, there is typically little or no trade. By analogy, one would expect that a labor market in which the current employer knows more about the quality of its worker than a competitor is likely to generate too little “trade,” i.e., insufficient worker mobility. However, this analogy is imperfect. Workers are not like assets, which can be freely bought and sold. Assuming no slavery, a worker is free to work for the highest bidder. Some firms will then choose to specialize in discovering talent: They hire young workers, retain the best ones, and let the mediocre ones be poached by more productive firms. Because some of these mediocre workers would be better matched with less productive firms, in our model there is typically “too much trade,” in contrast with traditional lemons market models.

In addition to providing an explanation for the puzzling behavior of chasing lemons, our model is rich in empirical predictions. The model predicts that: (i) internal promotions are more common in large firms than in small firms, (ii) there is greater dispersion in managerial ability in low-quality firms than in high-quality firms, (iii) the quality of poached managers improves with firm heterogeneity, (iv) managerial turnover increases with the skewness of the distribution of talent, and (v) within-job compensation growth increases with firm heterogeneity. The model also implies the existence of job ladders in knowledge-based occupations: Job-to-job flows are typically from low-quality and low-wage firms to high-quality and high-wage firms.

Our paper adds to the growing theoretical literature on financial-sector labor markets. This literature has focused on issues such as the level and composition of pay, the allocation of talent, and market failures. Examples include Murphy, Shleifer, and Vishny (1991), Inderst

and Mueller (2010), Philippon (2010), Glode, Green and Lowery (2012), Thanassoulis (2012), Bond and Glode (2014), Axelson and Bond (2015), Biais and Landier (2015), Bolton, Santos, and Scheinkman (2016), Glode and Lowery (2016), Acharya, Pagano, and Volpin (2016), Bénabou and Tirole (2016), Van Wesepe and Waters (2018), Berk and van Binsbergen (2019), and Vladimirov (2020).

Our model differs from these previous works by focusing on the consequences of asymmetric learning about talent. In the literature of financial-sector labor markets, Strobl and Van Wesepe (2013) develop a dynamic asymmetric employer learning model in which some firms endogenously commit to reveal the ability of their workers to future potential employers. In their model, as in ours, low-quality firms specialize in discovering talent. Their model explains why firms may benefit from publicizing the performance of their workers. Our focus instead is on explaining why firms may chase lemons.^{4,5}

Our analysis also shares certain ideas with those found in models of executive markets. As in firm-CEO assignment models, managers and firms are heterogeneous (Edmans, Gabaix, and Landier, 2009; Eisfeldt and Kuhnen, 2013; Gabaix and Landier, 2008; Terviö, 2008). As in Frydman (2019), managers are endowed with both firm-specific and general skills. As in Edmans and Gabaix (2011), the process of matching managers with firms is distorted by informational frictions.

Our model has its origins in the asymmetric employer learning literature, which was initiated by Waldman (1984) and Greenwald (1986).⁶ In such models, the current employer learns about the talent of incumbent workers, while competing employers remain uninformed. Our model differs from the standard model in this literature because of the existence of adversely selected and inefficient job-to-job flows. This is made possible because of firm heterogene-

⁴Differences between the models also arise due to different assumptions concerning firms' abilities to disclose information.

⁵For an application of asymmetric learning in a different context, see Marquez (2002), who develops a model in which banks asymmetrically learn about the quality of their borrowers.

⁶The theoretical labor literature on asymmetric employer learning has focused on a number of different applications, such as the signaling effects of promotion and retention decisions (Waldman, 1984; Lazear, 1986; Milgrom and Oster, 1987; Ricart i Costa, 1988; Laing, 1994; Bernhardt and Scoones, 1993; Bernhardt, 1995; Golan, 2005; Li, 2013; Waldman and Zax, 2016), the optimal design of disclosure policies (Mukherjee, 2008), and investing in general and/or firm-specific skills (Waldman, 1990; Chang and Wang, 1996; Acemoglu and Pischke, 1998, 1999).

ity: Some firms are more productive than others.⁷ Differently from Waldman (1984) and Greenwald (1986), we show that, in a dynamic context, lower-quality firms specialize in discovering talent, which makes inefficient poaching possible. More generally, our paper is related to the literature on adverse selection in markets initiated by Akerlof (1970). This literature typically focuses on the impact of private information about the quality of a good on the occurrence of trade.⁸

There is also a literature on symmetric learning in labor economics. In this literature, the paper most closely related to ours is Terviö (2009), who also shows that competition for talent creates inefficiencies. In his model, a worker’s talent is revealed on the job, but – unlike in our model – this information is public. Terviö shows that, in a competitive labor market, firms invest too little in talent discovery and over-recruit workers with mediocre abilities. In contrast, we show that asymmetric information restores firms’ incentives to invest in talent discovery.

2. Model Setup and Timing

We first present a simple two-period version of the model, which we use to derive our main results. In Section 6, we present an overlapping-generations model, in which the two-period model of this section is repeated infinitely. The infinite-horizon model delivers similar predictions as the simpler two-period model. In addition, the infinite-horizon model rationalizes some of the assumptions that may appear less natural in the two-period version of the model.

The economy is populated with a continuum of firms and a continuum of agents (e.g., fund managers, security analysts, etc.), which for simplicity we refer to as *managers*, that

⁷Dispersion in productivity and profitability has been widely documented. A large body of strategy literature attributes profitability dispersion to monopoly profits, which are explained by barriers to entry or ownership of unique resources (McGahan and Porter, 1997; Rumelt, 1991). Even in industries with free entry, equilibrium (ex post) productivity dispersion can be explained by the accumulation of organizational capital (Atkeson and Kehoe, 2005). For a review of the literature on productivity dispersion, see Syverson (2011).

⁸For example, Ellingsen (1997) shows that there exists a separating equilibrium in which some trade of high-quality goods occurs in markets for lemons. Levin (2001) studies how the degree of information asymmetry affects trade. Adriani and Deidda (2009) consider a case in which a seller values a low-quality good more than the buyer does. Daley and Green (2012) and Fuchs and Skrzypacz (2019) develop dynamic models of adverse selection and its impact on trade. Bar-Isaac, Jewitt, and Leaver (2014) study how the degree of information asymmetry impacts efficiency when public and private information is multi-dimensional.

live for two periods, $t = 0, 1$. Firms can be of one of two types, L or H , representing both the type and the mass of firms of each type. We denote a firm of each type by $i \in \{l, h\}$. Firm i has *productivity parameter* θ_i . Low-quality firms – L firms – have parameter $\theta_l = 1$, and high-quality firms – H firms – have parameter $\theta_h = \theta$, where $\theta > 1$. Productivity differences are the only source of (exogenous) heterogeneity between firms. For each type $i \in \{l, h\}$, we use subscripts ji to denote a unique firm j of type i .

Managers are endowed with general (i.e., portable) talent τ distributed according to a differentiable cumulative distribution function (c.d.f.) $F(\cdot)$ with support $[0, \bar{\tau}]$ and mean μ . A firm of type i that employs a manager with talent τ produces revenue $\theta_i\tau$ if the manager has already worked for the firm in a previous period and $\gamma\theta_i\tau$ if the manager is newly hired.⁹ Parameter $\gamma \in (0, 1)$ represents the loss in firm-specific skills that results when an outsider replaces an incumbent manager. Higher levels of γ indicate that firm-specific skills are less important.

At $t = 0$, a mass $M \gg H + L$ of managers enters the labor market. Each firm (of either type, L or H) hires one manager from the pool of available managers. Firm j of type i offers wage w_{ji}^y to a young manager. Because all managers are ex ante observationally identical, the initial pairing of firms and managers is random. Since jobs are in short supply, some managers remain unemployed.

At $t = 1$, each firm learns the talent τ of its incumbent manager. Managers do not observe τ . Our interpretation of this assumption is that a firm has a better signal of its manager's portable ability than the manager herself. For example, τ could represent skills specific to the industry, and an experienced executive might be better at assessing the value of such skills than a manager in the early stages of their career.¹⁰ This assumption excludes the possibility of managers signaling their types to potential employers. It also rules out the possibility of potential employers screening managers through a menu of contracts. We choose to rule out these possibilities in order to focus on the role of asymmetric information among employers. Our approach has the advantage of making clear precisely what informational assumptions

⁹For other models of multiplicative production functions, see Baker and Hall (2004) and Edmans, Gabaix, and Landier (2009).

¹⁰We assume that managers do not learn anything about τ for simplicity only. The important assumption here is that a manager learns less about τ than does the incumbent firm. An interpretation of this assumption is that the manager observes some measure of their talent but cannot evaluate how portable their talent is.

are required for the results. In contrast, the literature on employer learning typically adopts a different approach that imposes exogenous restrictions on actions – and sometimes on the space of contracts – to eliminate screening and employee signaling.

We also assume that a firm’s payoff is not directly observable and thus remains private information to the firm. This information cannot be credibly disclosed to outsiders. One interpretation is that performance is observed only with noise, which could occur for a number of reasons, such as insufficient disclosure, imperfect measurement of the performance of complex tasks, difficulties in measuring a manager’s individual contribution to the output of a team, or any other similar confounding effects. In all such cases, the firm could have an informational advantage over outsiders when estimating the performance of managers because the firm can directly observe a manager’s actions.

At the beginning of $t = 1$, all players face the following timing, summarized in Figure 1:

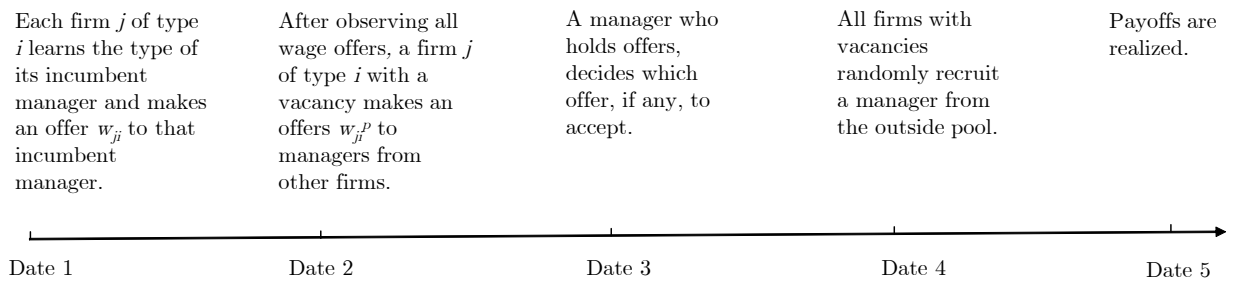


Figure 1. Time line

Date 1. Each firm j of type i learns the type $\tau_{ji} \in [0, \bar{\tau}]$ of its incumbent manager and independently commits to a wage offer $w_{ji} \in \mathbb{R}$ to this incumbent manager. We permit strictly negative wage offers. As these offers will not be accepted, a negative wage offer is equivalent to dismissing the incumbent manager. When firms offer a negative wage, managers quit immediately, thus creating vacancies. A vacancy is created, for example, if $\tau < \gamma\mu$. In this case, a firm prefers a randomly selected unemployed manager over its incumbent manager.¹¹

Date 2. After observing all wage offers made by all firms in the sector, a firm j of type i with a vacancy makes offers w_{ji}^p to managers from other firms; all firms act simultaneously.

¹¹All of our results remain unchanged if the vacancies are created for any other reasons, such as firm expansion, manager retirements, or exogenous separations.

Importantly, firms making poaching offers do not observe the incumbent managers' types. Instead, they form beliefs regarding these types after observing the set of all wage offers made by incumbent firms.

Date 3. A manager who holds offers decides which offer, if any, to accept. Managers always agree to work for the maximum non-negative wage offered to them:

Assumption A1 A manager who holds an offer w_{ji} accepts all poaching offers where $w_{ji}^p > w_{ji}$ and rejects all poaching offers where $w_{ji}^p \leq w_{ji}$.

In other words, if indifferent, a manager stays with their current employer, which is a standard assumption in the literature (see, e.g., Waldman, 1984). However, this assumption entails some loss of generality because it eliminates a number of equilibria in mixed strategies. Thus, we consider (A1) an equilibrium selection criterion with intuitive properties: Managers may have a small bias against changing jobs because of unmodeled costs.¹²

Date 4. All firms with vacancies at this date randomly recruit one manager from the *outside pool*, which is defined as the set of unemployed managers available for hire. The outside pool exclusively comprises managers not employed at $t = 0$ (this is without loss of generality; in equilibrium, a firm with a vacancy would never hire a manager who was dismissed by another firm).¹³ The outside option of an unemployed manager is normalized to zero.

Date 5. Payoffs are realized.

The timing assumes that firms with vacancies move after offers have been made to incumbent managers. Alternatively, there could also be multiple rounds of offers and counter-offers by incumbent firms and firms with vacancies. We assume a single round of offers as a simple way of introducing costs of delayed negotiations.¹⁴ However, our main results are robust to

¹²Relaxing this assumption makes mixed-strategy equilibria possible. A complete characterization and discussion of mixed-strategy equilibria can be found in the Internet Appendix.

¹³The implication of this assumption is that the distribution of talent in the outside pool is characterized by $F(\cdot)$. If fired managers cannot be distinguished from never-employed agents, then the unconditional c.d.f. of the agents in the outside pool is $\tilde{F}(\cdot) \neq F(\cdot)$. Nothing important changes in the model.

¹⁴We could also consider a situation in which there are potentially infinite rounds of offers and counter-offers, in which each additional round introduces a cost c paid by the incumbent firm (equivalently, the incumbent firm discounts the future). Firms with vacancies are competitive, thus the incumbent firm may face a different bidder for its manager in each round. In this modified game, the incumbent firm would immediately offer either the wage that would retain the manager or any wage that would not lead to retention.

assumptions about the timing of offers, as we show in Section 5.

We assume away bonding contracts: A manager is free to work for the highest bidder and the current employer receives no compensation if the manager is poached by another firm. There are no other contractual restrictions.¹⁵

To better understand the role of our assumptions for the implications of the model, in Section 5, we also consider the problem of a social planner who faces no exogenous restrictions on the set of mechanisms that can be chosen. We show that the main properties of the equilibrium do not depend on our assumptions on the contractual environment, timing of actions, structure of competition, and equilibrium selection.

3. Equilibrium

We now solve for the equilibrium. We focus on characterizing the equilibrium only at $t = 1$ because wage determination at $t = 0$ is a trivial problem. If there are no binding constraints on transfers from managers, firms will choose a negative $t = 0$ wage to extract all future expected surpluses from managers. If, instead, such constraints exist, $t = 0$ wages will be set at the lowest level compatible with these constraints. In Section 6, we solve an infinite-horizon version of the model in which, among other things, we characterize wages at all periods.

We make the following simplifying assumption:

Assumption A2 $\frac{H}{L} > \frac{1-F(\gamma\mu)}{2F(\gamma\mu)-1}$.

This assumption is sufficient – but not necessary – to guarantee that poachable managers are always in short supply relative to the vacancies created in H firms, which is the most interesting case to analyze.¹⁶

Without loss of generality, we restrict the analysis to the case in which only H firms make poaching offers. This restriction is not binding in equilibrium because, for the same

¹⁵In the Internet Appendix, we present a setting in which a firm commits in $t = 0$ to a deferred compensation contract in which a manager is paid only at the end of the game. We show that such contracts, even when feasible, may not be voluntarily adopted by firms.

¹⁶The condition is obtained by considering that vacancies in H firms, which are at least $F(\gamma\mu)H$, exceed the poachable managers in L and H firms, who are at most $(H + L)(1 - F(\gamma\mu))$.

manager, H firms would always make better offers than L firms.

We call an H firm with a vacancy at Date 2 a *poacher*. Poachers compete à la Bertrand; thus, their profits from poaching a manager must equal their outside payoff, $\gamma\theta\mu$.

3.1. Symmetric Information

In this subsection, we discuss the benchmark case of symmetric information, in which, at Date 1 of $t = 1$, all firms learn about managers' talent. We then show that the allocation of talent obtained in a market equilibrium with symmetric information is efficient.

The next proposition characterizes the equilibrium.

Proposition 1 *A unique equilibrium exists where*

1. L firms fire all manager types lower than $\gamma\mu$ and retain all manager types in $[\gamma\mu, \tau^\#]$,

where

$$\tau^\# = \begin{cases} \bar{\tau} & \text{if } \gamma\theta \leq 1 \\ \min\{(\theta - 1)\gamma\mu/(\theta\gamma - 1), \bar{\tau}\} & \text{if } \gamma\theta > 1. \end{cases} \quad (1)$$

2. H firms fire all types lower than $\gamma\mu$ and retain all types in $[\gamma\mu, \bar{\tau}]$.

3. In L firms, incumbent managers with types higher than $\tau^\#$ are poached by H firms.

Proof. See the Appendix. ■

The equilibrium is such that there is a *critical type* $\tau^\#$ above which all manager types initially assigned to L firms move to H firms. All firms fire all managers below threshold $\gamma\mu$. H firms retain all managers above this threshold, while L firms retain only *mediocre managers*, that is, managers in $[\gamma\mu, \tau^\#]$.

In equilibrium, managers who move up the job ladder are the most talented ones. If initially allocated to low-quality firms, such managers eventually move to high-quality firms and earn higher wages. To verify whether the equilibrium outcome is efficient, we consider what a social planner would choose. Because of firm-specific skills, it is never efficient to reallocate managers from one firm to another when both firms are of the same type. Similarly, transferring managers from H firms to L firms is always inefficient. Thus, the planner needs to consider only the possibility of transferring managers from L firms to H firms.

To simplify the exposition, we refer to an L firm with an incumbent manager at the beginning of $t = 1$ as an *incumbent firm*. The net surplus created by a manager of talent τ who is assigned to an incumbent firm is $\tau - \gamma\mu$. Similarly, the net surplus created by a manager of talent τ assigned to a poacher is $\gamma\theta\tau - \gamma\theta\mu$. A social planner who wants to maximize social surplus should: (i) replace all managers such that $\tau \leq \gamma\mu$ with a random replacement from the outside pool and (ii) assign manager $\tau \geq \gamma\mu$ to a poacher if and only if

$$\gamma\theta\tau - \gamma\theta\mu \geq \tau - \gamma\mu. \quad (2)$$

In other words, manager τ should be matched with a poacher when the incremental surplus to the poacher is larger than the net loss to the incumbent firm. Condition (2) implies that poaching should occur only if $\tau \geq \tau^\#$. We thus conclude that the decentralized equilibrium with symmetric information implements the efficient allocation of talent (i.e., the first-best allocation).

3.2. Asymmetric Information

3.2.1. Equilibrium: Assumptions and Definition

We now define the equilibrium conditions under asymmetric information. We first define the strategies for incumbents (i.e., firms at Date 1 of $t = 1$) and poachers (i.e., H firms with vacancies at Date 2 of $t = 1$). We denote an incumbent firm's strategy by $w_{ji} \in \mathbb{R}$. For simplicity, assume that an incumbent would never offer a positive wage if it is weakly dominated by offering a negative wage:

Assumption E1 Incumbent ji offers $w_{ji} \geq 0$ only if $\theta_i\tau_{ji} - w_{ji} \geq \gamma\theta_i\mu$.

The only action of poacher jh (i.e., an H firm with a vacancy at Date 2 of $t = 1$) is to offer a poaching wage w_{jh}^p . When a poacher observes an offer w made to a manager, the poacher believes that the manager's talent τ is distributed according to $F^W(\tau | w, i)$, where i is the type of the incumbent firm that made the offer, and W is the set of all offers made by all incumbent firms. We represent poachers' strategies by a function, $w_{jh}^p(w, i, W)$.¹⁷

¹⁷For notational simplicity, we assume that the identity of an incumbent affects the poaching wage only through its type i .

Because poachers compete among themselves in Bertrand fashion, no poacher can have a payoff larger than the outside payoff $\gamma\theta\mu$. A poacher thus offers

$$w_{jh}^p(w, i, W) = \theta\gamma \left(\int_0^{\bar{\tau}} \tau dF^W(\tau | w, i) - \mu \right) \quad (3)$$

to all managers who hold offers w from incumbent firms of type i .¹⁸ If $w_{jh}^p(w, i, W) < 0$, the offer is not accepted, implying that a negative poaching wage offer is equivalent to no offer. Because the right-hand side of (3) does not depend on jh , for simplicity, we now omit this subscript from function w^p .

We use Perfect Bayesian Equilibrium (PBE) as the equilibrium concept, augmented by some additional restrictions on beliefs. As usual in PBE definitions with many players, we assume that all poachers hold identical beliefs $F^W(\tau | w, i)$, both on and off the equilibrium path. Beliefs must be consistent with Bayes's rule on the equilibrium path. We also assume that poachers believe that the incumbent firms behave independently of one another, specifically implying that, if $ji \neq j'i$, $F^W(\tau_{ji}, \tau_{j'i} | w_{ji}, w_{j'i}, i) = F^W(\tau_{ji} | w_{ji}, i) \cdot F^W(\tau_{j'i} | w_{j'i}, i)$ for all W . We do not need to characterize managers' beliefs because such beliefs do not influence equilibrium outcomes.

Finally, we also assume the following:

Assumption E2 (*Divinity*) After observing an off-the-equilibrium-path wage w' , poachers believe that the probability that an incumbent firm with a manager of type $\tau' \geq \frac{w'}{\theta_i} + \gamma\mu$ offers wage w' is no less than the probability that a firm with a manager of type $\tau'' > \tau'$ offers w' .

(E2) is a technical assumption that restricts the set of admissible off-the-equilibrium-path beliefs. This assumption is an adaptation to our setup of the divinity criterion of Banks and Sobel (1987).¹⁹

¹⁸It is easy to see from (3) that, if L firms were allowed to make offers, these offers would be dominated by the offers made by H firms.

¹⁹The intuition for (E2) is as follows. For concreteness, suppose that type τ'' is retained by an L firm in an equilibrium with wage w'' , while type $\tau' \in [w' + \gamma\mu, \tau'']$ is not retained (the intuition for the other cases is analogous to this example). An incumbent with a manager of type τ'' that deviates and offers this type wage w' can benefit from the deviation only if poachers offer $w^p(w') \leq w'$. However, for this set of poaching wages, type τ' would also benefit from a deviation. Conversely, type τ'' would be worse off if $w^p(w') > w'$,

The role of (E1) and (E2) is to restrict the set of equilibria; thus, they can be interpreted as equilibrium selection criteria. They simplify the analysis significantly, although they do not eliminate equilibrium multiplicity. In Section 5, we show that our main results do not depend on any equilibrium selection assumptions, including (E1) and (E2).

3.2.2. Equilibrium: Characterization

We start by proving some preliminary results:

Lemma 1 *A firm offers the same wage to all manager types retained in equilibrium.*

This important result has a very simple proof. Suppose that there are two types, τ and τ' , where $\tau' > \tau$. Suppose that the incumbent firm wishes to retain both types. Suppose also that $w' > w$ (the argument is analogous if $w' < w$). This situation cannot be an equilibrium because there is a profitable deviation for an incumbent firm with manager τ' : The incumbent prefers to offer w to a manager of type τ' . Such a manager would nonetheless be retained, although at a lower wage.

Lemma 2 *Any equilibrium must have a threshold property: If an incumbent firm retains a manager of type τ , the firm also retains any manager of type $\tau' > \tau$.*

This result is again easily proven: For a given retention wage, w , if it is optimal to retain τ (that is, if $\theta_i\tau - w \geq \gamma\theta_i\mu$), then it is also optimal to retain any τ' such that $\tau' \geq \tau$.

The next proposition shows that, in equilibrium, incumbent managers will find themselves in one of the following three situations: unemployed, employed by their incumbent firm, or employed by a high-quality poacher. Because of Lemma 2, the very best managers will typically be retained by the incumbent firm, which implies that, if managers are retained at all, they must be the best managers. In equilibrium, incumbent firms never retain types $\tau < \gamma\mu$ because the unemployment replacement value is higher. Some mediocre types not retained by an incumbent will be either fired or poached. The following proposition provides a complete characterization of the equilibrium.²⁰

whereas type τ' would not be worse off. Thus, the logic of Banks and Sobel's divinity criterion requires that the probability of τ' deviating should be no less than that of τ'' deviating.

²⁰In what follows, for simplicity, we define all equilibrium sets of types as closed intervals. That is, we refrain from specifying what happens in equilibrium in the knife-edge cases in which an incumbent is

Proposition 2 *An equilibrium exists. All equilibria have the following properties:*

1. *There is a unique $\tilde{\tau}_i \in [\gamma\mu, \bar{\tau}]$ such that, for each firm type $i \in \{l, h\}$, all manager types $\tau \geq \tilde{\tau}_i$ are retained. Threshold $\tilde{\tau}_i$ is the same for all equilibria and is either $\bar{\tau}$ or the least element of the set of fixed points of*

$$G_i(x) \equiv \frac{w^*(x)}{\theta_i} + \gamma\mu, \quad (4)$$

where

$$w^*(x) = \gamma\theta \left(\int_x^{\bar{\tau}} \tau dF(\tau | \tau \geq x) - \mu \right) \quad (5)$$

is the wage offered (by both poachers and incumbents) to retained managers whose types are greater than x .

2. *All types $\tau \in [0, \gamma\mu]$ are fired in equilibrium (wages are negative).*
3. *There is a subset of manager types $P_i \subseteq [\gamma\mu, \tilde{\tau}_i]$, such that $\gamma\theta \left(\int_0^{\bar{\tau}} \tau dF(\tau | \tau \in P_i) - \mu \right) > 0$, who are poached in equilibrium, and a subset of manager types $S_i \subseteq [\gamma\mu, \tilde{\tau}_i]$ who are fired in equilibrium (wages are negative), with $S_i \cup P_i = [\gamma\mu, \tilde{\tau}_i]$.*
4. *If $\tau \in P_i$, then the incumbent firm offers any $w'_i \in [0, w^p(w'_i, i, W))$, where*

$$w^p(w'_i, i, W) = \gamma\theta \left(\int_0^{\bar{\tau}} \tau dF^W(\tau | w'_i, i) - \mu \right) \quad (6)$$

and $F^W(\tau | w'_i, i) = F(\tau | \tau \in P_i)$.

Proof. See the Appendix. ■

To illustrate the intuition behind this proposition, consider a firm that wants to retain a manager. The firm knows the manager's general ability. In contrast, competing firms observe the wage offered by the incumbent employer but not the manager's ability. A high wage is interpreted as a signal of high ability. To prevent the manager from being poached, the incumbent employer must offer a sufficiently high wage to the manager but will do so indifferent between retaining or not retaining a type. The equilibrium is unaffected by what happens in these cases.

only if the manager is indeed very talented. Therefore, only the very best managers are retained.

Because incumbent firms cannot retain manager types in $[\gamma\mu, \tilde{\tau}_i]$, such managers are either fired or poached. As before, we call these managers mediocre managers, although, in some cases, this interval also includes the very best managers (e.g., if $\tilde{\tau}_i$ is close to or equal to $\bar{\tau}$). An equilibrium with poaching (i.e., P_i is non-empty) exists if $\tilde{\tau}_i > \mu$. It is rational for H firms with vacancies to poach managers with types greater than μ because these managers are better than the average unemployed agents. Firms that poach managers are not fooled in equilibrium and have correct beliefs about the abilities of the managers that they hire.

Proposition 2 also reveals that equilibria differ from one another (meaningfully) only because the sets P_i and S_i can differ.²¹ In the infinite-horizon version of the model in Section 6, sets P_i and S_i are uniquely pinned down. However, in the current, simplified, two-period version, we require some additional equilibrium selection criteria to discuss the efficiency properties of the equilibrium. In this case, it is natural to select the most efficient equilibrium as the focal equilibrium:

Corollary 1 *There is a most efficient equilibrium in which $P_i = [\mu, \tilde{\tau}_i]$ and $S_i = [\gamma\mu, \mu]$.*

We prove the existence of this equilibrium in the proof of Proposition 2. In this equilibrium, firms perform the role of talent discoverers: by attempting to retain types in $P_i = [\mu, \tilde{\tau}_i]$, firms reveal that such managers are better than the average agent in the outside pool. In this one-shot version of the model, firms are not compensated for their talent discovery role. However, in the infinite-horizon version of the model, firms benefit from choosing some P_i that is more attractive to poachers than the outside pool. Thus, the infinite-horizon version of the model provides a micro-foundation for the selection of P_i .

²¹There are multiple combinations of sets P_i and S_i that constitute different equilibria, but the set of P_i subsets is restricted by condition $\theta\gamma \left(\int_0^{\bar{\tau}} \tau dF(\tau | \tau \in P_i) - \mu \right) > 0$. Two observationally equivalent equilibria with the same P_i and S_i can also differ from one another because they are sustained by different beliefs off the equilibrium path and can display different wages offered by incumbent firms for types in P_i .

3.2.3. Equilibrium: Efficiency

Proposition 2 implies that job-to-job flows are adversely selected. Here we consider the efficiency properties of the equilibrium.

The most-efficient equilibrium implies that managers with type $\tau \in [\tilde{\tau}_l, \bar{\tau}]$ are retained by low-quality firms, and managers with type $\tau \in [\mu, \tilde{\tau}_l]$ move up the job ladder to high-quality firms. We then have the following result:

Corollary 2 *In equilibrium, types $\tau_l \in [\mu, \min\{\tau^\#, \tilde{\tau}_l\}]$ are poached but should have been retained. That is, some managers become less productive after moving to higher-quality firms.*

This corollary implies that high-quality firms poach managers from low-quality firms, even in the absence of gains from trade. This is the main result of the paper. This inefficiency is solely a consequence of information asymmetries and firm heterogeneity, and not of the other assumptions of our model, as we show in Section 5.

Inefficient poaching also arises because some high-quality firms poach managers from other high-quality firms: Types $\tau_h \in [\mu, \tilde{\tau}_h]$ are poached but should have been retained. This result is, unlike Corollary 2, sensitive to assumptions on the timing of offers, as we show in Section 3 of the Internet Appendix.

In the most-efficient equilibrium, there are two additional distortions relative to the first-best scenario. The first distortion is excessive firing: Types $\tau \in [\gamma\mu, \mu]$ are fired but should have been retained. Firing these types is inefficient because valuable firm-specific skills are lost. The second distortion is excessive retention of high types: Types $\tau_l \in [\max\{\tau^\#, \tilde{\tau}_l\}, \bar{\tau}]$ are retained but should have been poached. Retaining these types is inefficient because they should instead be matched with better firms.

4. Model Implications and Applications

Here, we discuss some of the empirical implications of the model. Our main result is as follows:

Prediction 1 *Firms poach managers who become less productive after switching jobs.*

This prediction follows from Corollary 2, which shows that there is typically excessive poaching of mediocre types. Consistent with this prediction, Groysberg, Lee, and Nanda (2008) find that the performance of security analysts who are successfully poached by competitors declines after switching employers. The decline in performance is more pronounced for managers who switch to firms with similar capabilities, as predicted by our model.

Prediction 2 *Firms poach adversely-selected managers.*

That is, managers who are retained by their firms are more talented than managers who are poached by other firms. Testing this prediction is difficult because of the need for a measure of skill that is observed by the econometrician but not by outside employers. In the context of mutual fund managers, Berk and van Binsbergen (2015) propose a measure of fund manager skill based on returns, fees and assets under management. This measure can be observed ex post but not ex ante. Using such a measure, Berk, van Binsbergen, and Liu (2017) find that mutual fund firms are able to identify their best managers, who are then retained. In contrast, managers who move up the job ladder to larger mutual fund firms are not as skilled as those who stay. Our model provides a possible explanation for the most puzzling aspect of this evidence, which is that manager flows between mutual fund firms are adversely selected.

Prediction 3 *Internal promotions are more frequent in high-quality firms than in low-quality firms.*

This prediction follows from the fact that (4) and (5) imply $\tilde{\tau}_l > \tilde{\tau}_h$. Consistent with this prediction, Cziraki and Jenter (2020) show evidence that large firms are more likely than small firms to promote insiders to the CEO post.

The fact that $\tilde{\tau}_l > \tilde{\tau}_h$ also implies the following two predictions:

Prediction 4 *Managers who stay with low-quality firms are on average better than managers who stay with high-quality firms.*

Prediction 5 *Managers who leave low-quality firms are on average better than managers who leave high-quality firms.*

Intuitively, low-quality firms are more concerned about the threat of poaching because they are competing with firms that value manager talent more and offer higher wages. Thus, low-quality firms are willing to compete only for the very best managers; consequently, more of their managers leave. These two predictions jointly imply that low-quality firms exhibit greater dispersion in managerial ability than high-quality firms do.

To perform comparative statics, we need to assume the existence of an interior solution. The following condition guarantees an interior solution (i.e., $\tilde{\tau}_l < \bar{\tau}$):²²

Condition G $\max_{x \in [0, \bar{\tau}]} x - G_l(x) > 0$.

For the comparative statics, we initially focus on $\Delta \equiv \frac{\theta_h}{\theta_l}$ (θ_l is normalized to 1 in the model, for simplicity), which could be interpreted as the (cross-sectional) measure of firm heterogeneity. It is immediate from (4) and (5) that Δ has no effect on $\tilde{\tau}_h$. However, Δ does affect $\tilde{\tau}_l$. By the implicit function theorem, we find the following:²³

$$\frac{d\tilde{\tau}_l}{d\Delta} = \gamma \frac{\int_{\tilde{\tau}_l}^{\bar{\tau}} \tau f(\tau) d\tau - [1 - F(\tilde{\tau}_l)] \mu}{[1 - F(\tilde{\tau}_l)][1 - G'_l(\tilde{\tau}_l)]} > 0. \quad (7)$$

That is, the retention threshold for L firms increases with firm heterogeneity Δ . Intuitively, as L and H firms become more heterogeneous, L firms find it increasingly difficult to retain managers and are thus able to retain only the very best managers. We then have the following prediction:

Prediction 6 *The quality of poached managers improves with firm heterogeneity.*

Our model also has predictions for managerial compensation. Consider, for example, w_l^* , which is the wage paid to managers retained by l firms. From (5) and (7) we have

$$\frac{dw_l^*}{d\theta_l} = -\frac{\Delta \gamma \theta_h}{\theta_l^2} \frac{dE(\tau \mid \tau \geq \tilde{\tau}_l)}{d\tilde{\tau}_l} \frac{d\tilde{\tau}_l}{d\Delta} < 0. \quad (8)$$

$$\frac{dw_l^*}{d\theta_h} = \gamma (E(\tau \mid \tau \geq \tilde{\tau}_l) - \mu) + \frac{\Delta \gamma}{\theta_l} \frac{dE(\tau \mid \tau \geq \tilde{\tau}_l)}{d\tilde{\tau}_l} \frac{d\tilde{\tau}_l}{d\Delta} > 0. \quad (9)$$

²²The condition is defined for L firms only because it always holds for H firms. Condition G always holds for any set of parameters if $\bar{\tau} \rightarrow \infty$.

²³Under Condition G, $\tilde{\tau}_l$ is the least fixed point of $G_l(x)$. Because $G_l(0) < 0$, it follows that $1 - G'_l(\tilde{\tau}_l) > 0$.

Prediction 7 *Compensation for retained managers increases with firm heterogeneity.*

In our model, wages increase with Δ for two reasons. First, an increase in θ_h makes managers more valuable to H firms; thus, H firms are willing to pay more for a manager with a given ability τ . To prevent poaching, incumbent firms then offer higher retention wages. Such forces are also present in competitive assignment models with symmetric information (e.g., Gabaix and Landier (2008) and Terviö (2008)). Second, an increase in Δ changes the retention threshold for L firms (see (7)). As the average retained type increases, the retention wage also increases. This effect is unique to our model.

If a manager is first hired with a zero wage (as would happen if, for example, they could not be paid negative wages), then the retention wage measures the increase in earnings for those managers who are retained by their firms. Thus, we obtain the following result:

Prediction 8 *Within-job wage growth in low-quality firms increases with firm heterogeneity.*

In the context of knowledge workers, Andersson et al. (2009) study compensation patterns in a number of sectors of the software industry. They find that sectors in which there is greater dispersion in potential payoffs (e.g., differences in productivity) offer higher earnings growth for employees who are retained by their firms.

It is also interesting to study the impact of changes in the distribution of talent on job mobility. In particular, we consider the effect of the skewness of the distribution of talent on mobility. One possible measure of skewness is

$$\eta(x) \equiv E[\tau \mid \tau \geq x] - \mu, \text{ for } x > \mu. \quad (10)$$

To see this, suppose that the distribution of talent changes in a way that keeps the mean μ constant but increases $E[\tau \mid \tau \geq x]$ for all $x \geq \mu$. This could happen, for example, if $\bar{\tau}$ increases, while some density weight from the right of the mean is shifted to the left of the mean (to keep the mean constant). Thus, the distribution of talent becomes more positively skewed. Skewness in talent and compensation is associated with existence of superstars (Rosen, 1981). The increasing importance of superstar managers can thus be modeled as an increase in η : A large η indicates the existence of very few managers with talent much above

the average. We then have

$$\frac{d\tilde{\tau}_i}{d\eta} = \frac{\theta}{\theta_i} \frac{\gamma}{1 - G'_i(\tilde{\tau}_i)} > 0, \quad (11)$$

where $d\eta$ is an informal notation for an increase in $\eta(x)$ for all $x \geq \mu$ while keeping μ constant. As the “right-tail dispersion” of talent increases, it becomes more expensive to retain the best managers; consequently, fewer of them are retained in equilibrium.

Prediction 9 *Managerial turnover increases with the skewness of the distribution of talent.*

Consider now the additional result:

Corollary 3 *H firms pay higher wages on average than L firms.*

Proof. See the Appendix. ■

That is, in equilibrium, different types of firms pay different average wages such that high (low)-quality firms are also high (low)-wage firms, leading to the following prediction:

Prediction 10 *Job-to-job flows are typically from (i) low-quality firms to high-quality firms and (ii) low-wage firms to high-wage firms.*

Part (i) follows from the fact that, in equilibrium, only H firms can be successful poachers. Part (ii) then follows from Corollary 3. This prediction implies the existence of productivity and wage job ladders. This is not a unique prediction of our model; for example, Strobl and Van Wesep’s (2013) model also implies the existence of such job ladders. However, the nature of the job ladder is different in their model: large firms poach only those managers who have been promoted to the top jobs in their original firms. In contrast, in our model, large firms poach only those who have been kept but not promoted by small firms. Cziraki and Jenter (2020) show evidence of job ladders in the market for CEOs: job-to-job flows are typically from small firms to large firms. They also show that large firms typically poach executives below CEO level; CEO-to-CEO movements are quite rare.²⁴

²⁴See also Haltiwanger, Hyatt, and McEntarfer (2018) and Haltiwanger, Hyatt, Kahn, and McEntarfer (2018) for evidence of job ladders using cross-industry data.

5. The Planner's Problem

Our main theoretical result is the existence of job-to-job flows in the absence of gains from trade. In this section, we show that this result does not depend on assumptions on the contractual environment, timing of actions, structure of competition, and equilibrium selection. To do so, here we consider the problem of a social planner who faces no exogenous restrictions on the set of mechanisms that can be chosen. We show that the social planner generally cannot achieve the first best allocation and that, in any allocation in which job-to-job flows exist, some managers are inefficiently poached.

As in the decentralized case, at $t = 0$ there is no meaningful decision problem; each firm should hire one manager from the outside pool. At $t = 1$, because of firm-specific skills, it is never efficient to reallocate managers from one firm to another when both firms are of the same type. Similarly, transferring managers from H firms to L firms is always inefficient. Thus, the planner needs to consider only the possibility of transferring managers from L firms to H firms.

The timing of decisions in $t = 1$ is significantly simplified. First, the planner offers (and commits to) a mechanism (i.e., a contract) to each incumbent firm. Second, each incumbent firm sends a message $\tau^m \in [0, \bar{\tau}]$.²⁵ Third, the allocation is implemented.

The planner's problem is to assign incumbent managers to one of three possible sets: P denotes the set of managers who are assigned to a poacher, R denotes the set of managers who remain with the incumbent firm, and S denotes the set of managers who are unassigned (i.e., they are "sacked").

For expositional simplicity, we restrict the analysis to the case in which, for a given $\hat{\tau} \in [0, \bar{\tau}]$, all managers with type $\tau < \hat{\tau}$ are fired (i.e., they are assigned to S) and all managers with type $\tau \geq \hat{\tau}$ are either retained (i.e., assigned to R) or poached (i.e., assigned to P). Although such a constraint substantially simplifies the presentation, it has no implications for the analysis, because this constraint is not binding in the optimal solution.

Definition 1 An *allocation* is a function $p(\tau | \hat{\tau}) : [\hat{\tau}, \bar{\tau}] \rightarrow [0, 1]$ where, for a given $\hat{\tau}$, $p(\tau | \hat{\tau})$ is the probability that a manager with type τ is assigned to set P .

²⁵Note that by appealing to the revelation principle, we can restrict the set of messages to the set of types.

In other words, we define an allocation as a stochastic assignment rule. The allocation function determines which types of incumbent managers are allocated to L firms, to H firms, or to no firm.²⁶

From Proposition 1, we know that the first-best allocation is

$$p^{FB}(\tau \mid \hat{\tau} = \gamma\mu) = \begin{cases} 1 & \text{if } \tau \in [\tau^\#, \bar{\tau}] \\ 0 & \text{if } \tau \in [\gamma\mu, \tau^\#] \end{cases}. \quad (12)$$

To make information asymmetries relevant, we maintain the assumption that outsiders (including the planner) cannot observe performance outcomes. We assume that the planner can force firms and managers to participate in any mechanism, and also that the planner can assign managers to firms in any way she chooses.²⁷ Similarly, we assume that the planner faces no constraints on the transfers she can impose on players, e.g., there are no liquidity or budget-balance constraints. Our planner is thus completely unconstrained in her choices and actions; the only friction the planner faces is incomplete information about the types of incumbent managers.

Because of (A2), the planner wants to make sure that no H firm with $\tau \geq \gamma\mu$ dismisses its manager, which can be easily accomplished by setting the maximum payoff for H firms who dismiss managers at $\theta\gamma\mu$. Thus, the planner needs to consider as potential poachers only the set of H firms with managers with talent below $\gamma\mu$.

A *mechanism* $\langle p, t \rangle$ is an allocation rule $p(\tau^m \mid \hat{\tau})$ and a *transfer function* $t(\tau^m)$, where τ^m is a message sent by an L firm. We consider only symmetric mechanisms where the planner offers the same contract to all L firms. Thus, to simplify notation, we omit firm subscripts.

Let $U(\tau, \tau^m \mid p, t)$ denote the payoff of an incumbent firm with type τ from reporting τ^m under mechanism $\langle p, t \rangle$. An allocation p is *implementable* if there exists at least one transfer

²⁶Our definition of allocation does not consider feasibility. An allocation $p(\tau \mid \hat{\tau})$ must meet some market clearing conditions in order for it to be feasible. (A2) is a sufficient condition that guarantees that all allocations as defined above are feasible.

²⁷In other words, we do not require the mechanisms to satisfy individual rationality constraints. Our goal in this section is to show that incentive compatibility constraints are the main reason for our results.

function t such that

$$\tau \in \arg \max_{\tau^m \in [0, \bar{\tau}]} U(\tau, \tau^m \mid p, t). \quad (13)$$

In other words, p is implementable if there exists at least one transfer function such that truth-telling is incentive compatible.

Proposition 3 *For any implementable allocation p , if $p(\tau') > p(\tau'')$ for some $\tau', \tau'' \in [\hat{\tau}, \bar{\tau}]$, then it must be that $\tau' < \tau''$.*

Proposition 3 shows that incentive compatibility implies that any implementable allocation in which there are job-to-job flows, these flows are adversely selected. Proposition 3 has a straightforward corollary:

Corollary 4 *There is no mechanism that implements the first-best allocation.*

Intuitively, Corollary 4 holds because, under the first-best allocation, the planner has to compensate a firm that risks losing a high-ability manager with a high monetary transfer to induce this firm to truthfully reveal the manager's type. However, if the planner takes this approach, then a firm with a low-ability manager would prefer to pretend to have a high-ability manager in order to receive a higher transfer.²⁸

Remark 1. Because manager flows between two firms of the same type are always inefficient, the social planner, being unconstrained, can easily prevent such inefficiency. Thus, for an equilibrium with inefficient job-to-job flows to exist, we need firms to be heterogeneous. Proposition 3 implies that the social planner faces a trade-off: the planner can mitigate the problem of inefficient retention only by exacerbating the problem of inefficient poaching. How the planner will resolve this tension depends on her objective function. In the Internet Appendix, we show that a planner who maximizes social surplus will always choose a mechanism with a threshold property (as in Lemma 2). Furthermore, such a planner may optimally choose allocations with inefficient job-to-job flows.

Remark 2. Proposition 3, and its Corollary 4, imply that the inefficiency of job-to-job flows is a robust property of our setup. That is, we cannot restore efficiency by changing

²⁸Formally, Corollary 4 holds because the first best allocation violates the typical monotonicity requirement for implementable decisions (here, for simplicity, we call a decision an allocation) under incomplete information (see, e.g., Fudenberg and Tirole, 1991, p. 260).

assumptions regarding the structure of competition or equilibrium refinements. In particular, the timing of actions as described in Section 2 is not necessary for our results. As an example of this point, in the Internet Appendix, we show a complete analysis of the case in which incumbents move last.²⁹

6. An Infinite-Horizon Model

We now develop an infinite-horizon version of the model. This version delivers two new results. First, discovering talent is a real option available to firms: Firms hire young managers hoping to retain them once their talent is revealed. Second, firms benefit from their role as talent discoverers, because they can now extract some of the surplus that accrues to managers who are poached.

The economy is populated with many infinitely lived firms. Again, firms can be of one of two types, L or H , representing both the type and the mass of firms of each type. Managers live for two periods: young age and old age. Firms and managers are risk-neutral and share a common discount factor $\delta \in [0, 1)$. At each period t ($t = 0, 1, 2, \dots$), a mass M of young managers enter the labor market. For brevity, we do not present the benchmark case of symmetric information; a full analysis of this case can be found in the Internet Appendix.

At the beginning of a period, a firm can be in one of the following states:

- (i) The firm has a vacant position because its manager retired at the end of the previous period (that is, the manager was old).
- (ii) The firm does not have a vacant position because its manager was young in the previous period.

Both types of firms can have incumbent managers and can also become poachers. In each period t , the timing of actions for a firm with an incumbent manager is exactly as described in Section 2. At Date 2 in period t , a type- h firm can attempt to poach a manager from a type- l firm or from another type- h firm. In general, we also allow type- l firms to make poaching offers. However, for simplicity, we (implicitly) restrict our analysis to a set

²⁹For a model in which incumbents and poachers move simultaneously, see Li (2013).

of parameters for which, in equilibrium, managers would strictly prefer poaching offers from type- h firms. Thus, without loss of generality, we assume that type- l firms cannot poach managers.

As above, there could be a subset P_i of types poached in equilibrium and a subset S_i of types fired in equilibrium. For simplicity, we focus only on cases in which both P_i and S_i are convex sets; that is, they are intervals, which means that, if type τ is poached, then type $\tau' \in P_i$ and $\tau' > \tau$ is also poached. Similarly, if type τ is fired, then type $\tau' \in S_i$ and $\tau' < \tau$ is also fired. We call an equilibrium with this property a *monotonic equilibrium*.

In a monotonic equilibrium, in each period we need to find two types of thresholds. As discussed above, $\tilde{\tau}_i$, $i \in \{l, h\}$, denotes the threshold such that all types $\tau \geq \tilde{\tau}_i$ are retained. We define $\hat{\tau}_i$ as the threshold for which all types $\tau \leq \hat{\tau}_i$ are fired. Each monotonic equilibrium has a unique sequence of thresholds $\{\tilde{\tau}_l, \tilde{\tau}_h, \hat{\tau}_l, \hat{\tau}_h\}_t$, $t = 0, 1, \dots, \infty$. For simplicity, we focus only on equilibria in which these thresholds are time-invariant. Thus, we can omit the time subscript from the analysis that follows.

Now, at Date 4 in each period t , firms with vacancies offer wage w_i^y , $i \in \{l, h\}$, to unemployed young managers. Thus, we also need to determine such wages in equilibrium. We assume that firms can offer any wage that they want, including negative wages. Managers may accept negative wages when young if, by working for the firm, they can earn higher wages when old. Later, we briefly discuss the effects of relaxing this assumption. To select among possible equilibria, we assume that, at Date 4, firms publicly announce a threshold $\hat{\tau}_i$. We assume that all players (i.e., firms and managers) share the same beliefs on and off the equilibrium path, and beliefs are such that players expect incumbent firms to use threshold $\hat{\tau}_i$ if this threshold is announced (that is, we select truth-telling as an equilibrium refinement). This belief is rational because incumbent firms are indifferent with respect to which threshold $\hat{\tau}_i$ they use after the announcement.

Proposition 4 *A unique monotonic equilibrium with time-invariant thresholds $\{\tilde{\tau}_l, \tilde{\tau}_h, \hat{\tau}_l, \hat{\tau}_h\}$ and wages $\{w_l^y, w_h^y, w_l^{**}, w_h^{**}, w^*(\tilde{\tau}_l), w^*(\tilde{\tau}_h)\}$ exists and has the following properties:³⁰*

1. *For any given pair $(\hat{\tau}_l, \hat{\tau}_h)$, there is a unique $\tilde{\tau}_i$ such that, for each firm type $i \in \{l, h\}$,*

³⁰We consider uniqueness in the generic sense: Multiple equilibrium values could still arise for a set of parameters with measure zero.

all manager types $\tau \geq \tilde{\tau}_i$ are retained. Threshold $\tilde{\tau}_i$ is either $\bar{\tau}$ or the least element of the set of fixed points of

$$G_i(x) \equiv \frac{w^*(x) - w_i^y - \delta \theta_i \int_x^{\bar{\tau}} (\tau - \gamma \mu) dF(\tau)}{\theta_i [1 + \delta(1 - F(x))]} + \gamma \mu. \quad (14)$$

2. For any given pair $(\hat{\tau}_l, \hat{\tau}_h)$, equilibrium wages are such that all retained managers are offered

$$w^*(x) = \max \left\{ \gamma \theta \left(\int_x^{\bar{\tau}} \tau dF(\tau \mid \tau \geq x) - \mu \right) + w_h^y - \frac{\delta \int_{\tilde{\tau}_h}^{\bar{\tau}} (\theta \tau - w^*(\tilde{\tau}_h) - \gamma \theta \mu + w_h^y) f(\tau) d\tau}{[1 + \delta(1 - F(\tilde{\tau}_h))]}, 0 \right\}, \quad (15)$$

all managers who are poached (if any) are paid

$$w_i^{**} = \gamma \theta \left(\int_{\hat{\tau}_i}^{\tilde{\tau}_i} \frac{\tau f(\tau) d\tau}{F(\tilde{\tau}_i) - F(\hat{\tau}_i)} - \mu \right) + w_h^y - \frac{\delta \int_{\tilde{\tau}_h}^{\bar{\tau}} (\theta \tau - w^*(\tilde{\tau}_h) - \gamma \theta \mu + w_h^y) f(\tau) d\tau}{1 + \delta(1 - F(\tilde{\tau}_h))}, \quad (16)$$

and all young managers who agree to work for a type- i firm are offered wage

$$w_i^y = -\delta(1 - F(\tilde{\tau}_i))w^*(\tilde{\tau}_i) - \delta(F(\tilde{\tau}_i) - F(\hat{\tau}_i)) \max \{w_i^{**}, 0\}. \quad (17)$$

3. At Date 4, type- i firms with vacancies announce the threshold $\hat{\tau}_i$ that maximizes the present value of their expected profits given (14), (15), (16) and (17).

4. All types $\tau_i \in [0, \hat{\tau}_i]$ are fired in equilibrium (wages are negative).

Proof. See the Appendix. ■

From this proposition we conclude that the equilibrium displays the same type of talent misallocation as in the two-period model: The best types $[\tilde{\tau}_i, \bar{\tau}]$ are retained and the mediocre types $P_i = [\hat{\tau}_i, \tilde{\tau}_i]$ are poached. Thus, our main conclusions continue to hold in the infinite-horizon model.

The infinite-horizon version of the model differs from the two-period model in two important ways. First, hiring a young manager is a real option for the firm. When a firm

hires a young manager in period t , it will learn the type of this manager in period $t + 1$. Because learning is asymmetric, the incumbent benefits from its informational advantage. This option value reduces firms' incentives to become poachers.

Second, in the infinite-horizon model, unlike the two-period model, firms benefit from their role as talent discoverers: incumbent firms can now extract some of the surplus that accrues to managers who are poached when they are old by offering such managers less than their outside wage when they are young.³¹ Thus, when hiring young managers, firms choose the threshold $\hat{\tau}_i$ that maximizes the surplus that they can extract from managers; the solution to this maximization problem is unique (which uniquely pins down sets P_i and S_i).

The main implication of the combination of these two new features is that, as long as the discount factor δ is sufficiently low, any equilibrium must involve poaching. This contrasts with the two-period case, in which an equilibrium with poaching is one of many possible equilibria. In the infinite-horizon case, poaching will necessarily occur in equilibrium even when there is an exogenous lower bound on wages (such as a limited liability constraint), provided that this bound is not too high.

7. Final Remarks

In knowledge-based industries and sectors, firms play an important role as talent discoverers. Competition for talent implies that firms may not capture most of the value that they help create. In our model, firms asymmetrically learn about the abilities of their managers. This knowledge gives firms informational rents, helping to explain firms' incentives to invest in talent discovery. Because of their informational advantage, firms that invest in talent discovery are able to retain their best managers. In equilibrium, firms specialize in either discovering talent or poaching talent from other firms. Poachers hire mediocre managers, i.e., those who are above average but not stars. In equilibrium, poachers chase lemons: Poached

³¹Similar to our model, Strobl and Van Wesep (2013) develop a model of asymmetric learning in which low-quality firms specialize in discovering talent and benefit from such a role. The reputation-building equilibrium in their model cannot be sustained in our setup because of the assumption that firms cannot credibly reveal the types of their managers.

managers are adversely selected with respect to the set of employed managers. However, because talent-discovering firms act as certifiers of talent, poached managers are positively selected relative to the population of managers.

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A. Appendix: Proofs

Proposition 1.

Proof. Suppose an H firm has a vacancy at Date 2. Because of (A2), poachers are in excess supply, thus poachers compete à la Bertrand and their profits from poaching a manager with talent τ must be equal to their outside payoff. The poaching wage offered to type τ is given by

$$w^{pS}(\tau) = \theta\gamma(\tau - \mu), \quad (18)$$

where the superscript S denotes symmetric information.³²

In a subgame perfect equilibrium, incumbent firm ji solves $\max_{w \in \mathbb{R}} \pi_{ji}(w)$, where

$$\pi_{ji}(w) = \begin{cases} \tau_{ji} - w & \text{if } w \geq \max\{\theta\gamma(\tau_{ji} - \mu), 0\} \\ \theta_i\gamma\mu & \text{otherwise} \end{cases}. \quad (19)$$

Suppose first that $\tau_{ji} \leq \mu$. In this case, the firm does not have to worry about poaching and will pay $w_{ji} = 0$ if $\tau_{ji} \in [\gamma\mu, \mu]$ and some $w_{ji} < 0$ if $\tau_{ji} < \gamma\mu$ (in other words, it dismisses the manager).

If instead $\tau_{ji} > \mu$ and the firm wants to retain the manager, then it must offer at least as much as a poacher, that is, w_{ji} must be equal to or greater than $\theta\gamma(\tau_{ji} - \mu) > 0$. Then, ji 's payoff is $\pi_{ji} = \theta_i\tau_{ji} - \theta\gamma(\tau_{ji} - \mu)$, which implies that retaining the manager is an optimal choice if and only if $\theta_i\tau_{ji} - \theta\gamma(\tau_{ji} - \mu) \geq \theta_i\gamma\mu$. If $i = h$, this condition holds always, thus implying that, in equilibrium, no manager is poached from an H firm. An H firm's optimal

³²We drop the subscript ji for the poaching wage, since we only consider poaching offers from H firms and the poaching offer is independent of the poacher's identity j .

strategy regarding its incumbent manager is summarized by:³³

$$w_{jh}^S = \begin{cases} \text{any } w < 0 & \text{if } \tau_{jh} \leq \gamma\mu \\ 0 & \text{if } \tau_{jh} \in [\gamma\mu, \mu] \\ \theta\gamma(\tau_{jh} - \mu) & \text{if } \tau_{jh} \in [\mu, \bar{\tau}] \end{cases} . \quad (20)$$

Now the analysis that follows refers to L firms only. If $\theta\gamma \leq 1$, condition $\tau_{jl} - \theta\gamma(\tau_{jl} - \mu) \geq \gamma\mu$ is true for any $\tau_{jl} > \mu$ (recall that $\theta > 1$). If $\theta\gamma > 1$, this condition holds for any $\tau_{jl} \leq (\theta - 1)\gamma\mu / (\theta\gamma - 1)$. This reasoning implies that an L firm's optimal strategy is to offer

$$w_{jl}^S = \begin{cases} \text{any } w < 0 & \text{if } \tau_{jl} \leq \gamma\mu \\ 0 & \text{if } \tau_{jl} \in [\gamma\mu, \mu] \\ \theta\gamma(\tau_{jl} - \mu) & \text{if } \tau_{jl} \in [\mu, \tau^\#] \\ \text{any } w < w^{pS}(\tau_{jl}) & \text{if } \tau_{jl} \geq \tau^\# \end{cases} , \quad (21)$$

where

$$\tau^\# = \begin{cases} \bar{\tau} & \text{if } \theta\gamma \leq 1 \\ \min\{(\theta - 1)\gamma\mu / (\theta\gamma - 1), \bar{\tau}\} & \text{if } \theta\gamma > 1. \end{cases} \quad (22)$$

■

Proposition 2.

Proof. *Part 1:* From Lemma 2, we know that an equilibrium must have a threshold $\tilde{\tau}_i$ above which all manager types are retained by incumbent firms of type i . Here we want to find the value for $\tilde{\tau}_i$.

From Lemma 1 we know that all types τ_i in $[\tilde{\tau}_i, \bar{\tau}]$ are paid the same wage; let w^* denote such a wage. To retain such managers, an incumbent firm must offer $w^* \geq w^p(w^*, i, W)$, where function w^p denotes the wage offered by poachers when they observe an incumbent firm of type i that offers a wage w^* when the set of all equilibrium wage offers is W . Upon observing w^* , beliefs must be $F(\tau \mid \tau \geq \tilde{\tau}_i)$, which implies that the poaching wage is given by (here we use (A2) and Bertrand competition among poachers):

$$w^p(w^*, i, W) = \theta\gamma \left(\int_{\tilde{\tau}_i}^{\bar{\tau}} \tau dF(\tau \mid \tau \geq \tilde{\tau}_i) - \mu \right). \quad (23)$$

³³Recall that, for simplicity, we always use closed intervals to denote the equilibrium sets of types.

Consider an incumbent firm of type i with a manager of type $\tau_i \in [\tilde{\tau}_i, \bar{\tau}]$. For w^* to be an equilibrium wage offer, the incumbent firm must be better off by retaining the manager at this wage rather than hiring a new manager from the outside pool:

$$\theta_i \tau_i - w^* \geq \theta_i \gamma \mu, \quad (24)$$

which implies

$$\tilde{\tau}_i \geq \frac{w^*}{\theta_i} + \gamma \mu. \quad (25)$$

If the inequality above is strict, then there exists $\tau' < \tilde{\tau}$ such that $\tau' > \frac{w^*}{\theta_i} + \gamma \mu$ such that the incumbent firm would like to retain the manager at wage w^* , which contradicts the assumption that $\tilde{\tau}_i$ is an equilibrium threshold. Thus, it must be that

$$\tilde{\tau}_i = \frac{w^*}{\theta_i} + \gamma \mu. \quad (26)$$

We now show that $w^* = w^p(w^*, i, W)$. Suppose first that $w^* > w^p(w^*, i, W)$ and consider a deviation from an incumbent with a manager of type $\tau_i > \tilde{\tau}_i$ who chooses to offer $w^p(w^*, i, W)$ instead of w^* . For this not to constitute a profitable deviation, it must be that the manager rejects the incumbent firm's offer, that is the following condition needs to hold:

$$w^p(w^p(w^*, i, W), i, W) > w^p(w^*, i, W), \quad (27)$$

that is,

$$\gamma \theta \left(\int_0^{\bar{\tau}} \tau dF^W(\tau | w^p(w^*, i, W)) - \mu \right) > \gamma \theta \left(\int_{\tilde{\tau}_i}^{\bar{\tau}} \tau dF(\tau | \tau \geq \tilde{\tau}_i) - \mu \right). \quad (28)$$

This can only happen if distribution F^W puts more more weight on higher manager types than distribution $F(\tau | \tau \geq \tilde{\tau}_i)$. Formally, this requires that there exists at least one manager type $\tau'' > \tilde{\tau}_i \geq \frac{w^p(w^*, i, W)}{\theta_i} + \gamma \mu$ for which the probability of deviation of an incumbent firm is strictly greater than the probability of a deviation of an incumbent firm with a manager of type $\tau' \in (\tilde{\tau}_i, \tau'')$. However, this is ruled out by (E2). Thus, it must be that

$w^* = w^p(w^*, i, W)$, thus the equilibrium threshold must satisfy the following condition:

$$\tilde{\tau}_i = \frac{\theta\gamma}{\theta_i} \left(\int_{\tilde{\tau}_i}^{\bar{\tau}} \tau dF(\tau | \tau \geq \tilde{\tau}_i) - \mu \right) + \gamma\mu. \quad (29)$$

This condition is necessary, but not sufficient, and there may be multiple values of $\tilde{\tau}_i$ that solve this equation. Another necessary condition for an equilibrium is that

$$\tilde{\tau}_i + \varepsilon \geq \frac{\theta\gamma}{\theta_i} \left(\int_{\tilde{\tau}_i + \varepsilon}^{\bar{\tau}} \tau dF(\tau | \tau \geq \tilde{\tau}_i + \varepsilon) - \mu \right) + \gamma\mu \quad (30)$$

for $\varepsilon > 0$ arbitrarily small. To see this, suppose that

$$\tilde{\tau}_i + \varepsilon < \frac{\theta\gamma}{\theta_i} \left(\int_{\tilde{\tau}_i + \varepsilon}^{\bar{\tau}} \tau dF(\tau | \tau \geq \tilde{\tau}_i + \varepsilon) - \mu \right) + \gamma\mu \quad (31)$$

then the incumbent would be better off by not retaining types in the interval $[\tilde{\tau}_i, \tilde{\tau}_i + \varepsilon]$, which contradicts the assumption that $\tilde{\tau}_i$ is an equilibrium threshold.

Define the function

$$G_i(x) = \frac{\theta\gamma}{\theta_i} \left(\int_x^{\bar{\tau}} \tau dF(\tau | \tau \geq x) - \mu \right) + \gamma\mu. \quad (32)$$

Because $G_i(x)$ is continuous and $G_i(0) = \gamma\mu > 0$, at least one fixed point of G_i exists if and only if

$$\max_{x \in [0, \bar{\tau})} x - G_i(x) \geq 0. \quad (33)$$

This condition always holds if the incumbent is an H firm (i.e., $\theta_i = \theta$), but it may or may not hold if the incumbent is an L firm (i.e., $\theta_i = 1$). If (33) does not hold, the unique equilibrium displays no retention by L firms, that is, $\tilde{\tau}_l = \bar{\tau}$.

Assuming that (33) holds, we define the least element of the set of fixed points of $G_i(x)$:

$$\underline{x}_i = \min_{\{x: G_i(x)=x\}} x. \quad (34)$$

Since $G_i(x) \geq \gamma\mu$ for all $x \geq 0$, we have that $\underline{x}_i \geq \gamma\mu$.

We now show that \underline{x}_i is an equilibrium threshold. First, notice that setting $\tilde{\tau}_i = \underline{x}_i$

satisfies (29) because \underline{x}_i is a fixed point of $G_i(\cdot)$. Second, because $G_i(0) > 0$, $x - G_i(x)$ crosses zero from below at \underline{x}_i , which satisfies condition (30).

Now we show that no other fixed point of $G_i(x)$ that also satisfies (30) and such that $x > \underline{x}_i$ can be an equilibrium. Suppose that there is a candidate equilibrium threshold $x' > \underline{x}_i$ such that only types $\tau \geq x'$ are retained at wage

$$w' = \theta\gamma \left(\int_{x'}^{\bar{\tau}} \tau dF(\tau \mid \tau \geq x') - \mu \right). \quad (35)$$

Then, an incumbent firm with a manager of type $\underline{x}_i + \varepsilon$, with $\varepsilon > 0$ arbitrarily small, could deviate and offer $w_i^* < w'$, with

$$w_i^* = \theta\gamma \left(\int_{\underline{x}}^{\bar{\tau}} \tau dF(\tau \mid \tau \geq \underline{x}) - \mu \right). \quad (36)$$

If a manager of type $\underline{x}_i + \varepsilon$ is successfully retained at wage w_i^* , then the incumbent firm is strictly better off. For such a deviation not to be profitable, poachers' beliefs must be such that $w^p(w_i^*, i, W) > w_i^*$. This would occur if poachers believe that firms with managers with better types are more likely to deviate than those with worse types. Formally, this requires that there exists at least one manager type $\tau'' > \underline{x} \geq \frac{w_i^*}{\theta_i} + \gamma\mu$ for which the probability of deviation of an incumbent firm is strictly greater than the probability of a deviation of an incumbent firm with a manager of type $\tau' \in (\underline{x}_i, \tau'')$. However, this is ruled out by (E2). Thus, \underline{x}_i is the unique equilibrium threshold; i.e. $\tilde{\tau}_i = \underline{x}_i$. The unique retention wage is given by w_i^* as in (36).

Part 2. It follows trivially from (E1).

Part 3. Suppose that there is some type τ'_i in $[\gamma\mu, \tilde{\tau}_i]$ that is retained in equilibrium. Lemma 2 implies that all types in $[\tau'_i, \tilde{\tau}_i]$ are also retained, and Lemma 1 implies that all types in $[\tau'_i, \bar{\tau}]$ must be paid the same wage. However, because $\tau'_i \leq \tilde{\tau}_i$, then by the definition of $\tilde{\tau}_i$ in (34), we have $\tau'_i - G_i(\tau'_i) \leq 0$. Thus, type τ'_i cannot be profitably retained. Thus, all types in $[\gamma\mu, \tilde{\tau}_i]$ must be either poached (and thus included in set P_i) or fired (and thus included in set S_i). Since a manager only accepts an offer from a poacher if that offer is positive, for any set P_i it must be that $\theta\gamma \left(\int_0^{\bar{\tau}} x dF(\tau \mid \tau_i \in P_i) - \mu \right) > 0$ (at least one equilibrium with $P_i \neq \emptyset$ exists if $\tilde{\tau}_i > \mu$). Thus, if an equilibrium exists, Part 3 must hold.

Part 4. If $\tau_i \in P_i$, then the incumbent must offer the managers in this set some wage w'_i that is lower than the poaching wage $w^p(w'_i, i, W)$. Because poachers' beliefs must be Bayesian on the equilibrium path, then

$$w^p(w'_i, i, W) = \theta\gamma \left(\int_0^{\bar{\tau}} \tau dF^W(\tau | w'_i, i) - \mu \right), \quad (37)$$

and poachers' beliefs are given by $F^W(\tau | w'_i, i) = F(\tau | \tau_i \in P_i)$

To complete the proof, we only need to show that at least one equilibrium exists. Suppose first that $\max_{\tau_l \in [0, \bar{\tau}]} \tau_l - G_l(\tau_l) > 0$. In this case, we know that there exists a unique pair $\{\tilde{\tau}_l, \tilde{\tau}_h\} < \{\bar{\tau}, \bar{\tau}\}$. The following fully characterizes one possible equilibrium:

Consider the retention wages

$$w_i(\tau_i) = \begin{cases} w_i^* & \text{if } \tau_i \in [\tilde{\tau}_i, \bar{\tau}] \\ 0 & \text{if } \tau_i \in [\mu, \tilde{\tau}_i] \\ -1 & \text{if } \tau_i \in [0, \mu] \end{cases}, \quad (38)$$

the poaching wages on the equilibrium path

$$w^p(w_i) = \begin{cases} w_i^* & \text{if } w_i = w_i^* \\ \theta\gamma \left(\int_{\mu}^{\tilde{\tau}_i} \tau dF(\tau | \tau \in [\mu, \tilde{\tau}_i]) - \mu \right) & \text{if } w_i = 0 \\ -1 & \text{if } w_i = -1 \end{cases}, \quad (39)$$

and beliefs such that $F(\tau | \tau \geq \frac{w_i}{\theta_i} + \gamma\mu)$ for any w_i that is off the equilibrium path. In this equilibrium, $P_i = [\mu, \tilde{\tau}_i]$ and $S_i = [\gamma\mu, \mu]$.

If we have $\max_{\tau_l \in [0, \bar{\tau}]} \tau_l - G_l(\tau_l) \leq 0$, nothing is changed for H firms. For L firms, no type τ_l is retained, and an equilibrium in which all types $\tau_l \geq \mu$ are offered $w_l = 0$, and types below μ are fired, exists and is sustained by beliefs such that $F(\tau | \tau \geq w_l + \gamma\mu)$ for any w_l that is off the equilibrium path. This equilibrium implies $P_l = [\mu, \bar{\tau}]$ and $S_l = [\gamma\mu, \mu]$.

■

Corollary 3

Proof. Equilibrium average wages in L firms are given by

$$w_l^a = [1 - F(\tilde{\tau}_l)]\theta\gamma \left(\int_{\tilde{\tau}_l}^{\bar{\tau}} \frac{\tau f(\tau)}{1 - F(\tilde{\tau}_l)} d\tau - \mu \right), \quad (40)$$

and equilibrium average wages in those H firms that do not poach any manager are

$$w_h^a = [1 - F(\tilde{\tau}_h)]\theta\gamma \left(\int_{\tilde{\tau}_h}^{\bar{\tau}} \frac{\tau f(\tau)}{1 - F(\tilde{\tau}_h)} d\tau - \mu \right). \quad (41)$$

Since

$$\frac{\partial w_i^a}{\partial \tilde{\tau}_i} = (\mu - \tilde{\tau}_i) f(\tilde{\tau}_i) < 0, \quad (42)$$

and because $\tilde{\tau}_l > \tilde{\tau}_h$ (this is implied by (7)), $w_h^a > w_l^a$ for those H firms that do not poach managers. H firms that poach managers offer positive wages to those managers, which implies that their average wage is higher than w_h^a . ■

Proposition 3

Proof. The revelation principle implies that there is no loss of generality from focusing on truth-telling direct mechanisms. Define an incumbent firm's payoff function under mechanism $\langle p, t \rangle$ as

$$U(\tau, \tau^m | p, t) = \begin{cases} (1 - p(\tau^m))\tau + p(\tau^m)\gamma\mu + t(\tau^m) & \text{if } \tau^m \in [\hat{\tau}, \bar{\tau}] \\ \gamma\mu + t(\tau^m) & \text{if } \tau^m \in [0, \hat{\tau}) \end{cases}. \quad (43)$$

Note that an implicit assumption here is that a firm that loses its manager ends up employing a random manager from the outside pool. Suppose that an allocation p with $p(\tau') > p(\tau'')$ for some pair (τ', τ'') is implementable (i.e., it is incentive compatible for the firm to report $\tau^m = \tau$). Incentive compatibility requires

$$\begin{aligned} (1 - p(\tau'))\tau' + p(\tau')\gamma\mu + t(\tau') &\geq (1 - p(\tau''))\tau' + p(\tau'')\gamma\mu + t(\tau'') \\ t(\tau') - t(\tau'') &\geq [p(\tau') - p(\tau'')](\tau' - \gamma\mu) \end{aligned} \quad (44)$$

and

$$\begin{aligned} (1 - p(\tau'')) \tau'' + p(\tau'') \gamma \mu + t(\tau'') &\geq (1 - p(\tau')) \tau'' + p(\tau') \gamma \mu + t(\tau') \\ t(\tau'') - t(\tau') &\geq [p(\tau'') - p(\tau')] (\tau'' - \gamma \mu). \end{aligned} \quad (45)$$

Adding both sides of (44) and (45) yields

$$0 \geq [p(\tau') - p(\tau'')] (\tau' - \tau'') \quad (46)$$

which implies $\tau' < \tau''$ ³⁴. ■

Proposition 4

Proof. To prove Part 1, we need to find the unique pair $\{\tilde{\tau}_l, \tilde{\tau}_h\}$ conditional on a given pair of equilibrium thresholds $\{\hat{\tau}_l, \hat{\tau}_h\}$, which for now we take as givens. Because many of the steps are similar to those in the proof of Proposition 2, we refer the reader to that proof in some instances.

Lemma 2 implies that an equilibrium with retention must have a threshold $\tilde{\tau}_i$. Lemma 1 implies that all types in $[\tilde{\tau}_i, \bar{\tau}]$ are paid the same wage. To prevent poaching, this wage must be such that $w^*(\tilde{\tau}_i) \geq w^p(w^*(\tilde{\tau}_i))$, where $w^p(w^*(\tilde{\tau}_i))$ is the wage offered by poachers who observe $w^*(\tilde{\tau}_i)$ ($w^p(\cdot)$ will be derived below). Because poachers know that all types in $[\tilde{\tau}_i, \bar{\tau}]$ are offered $w^*(\tilde{\tau}_i)$, their beliefs must be given by $F(\tau | \tau \geq \tilde{\tau}_i)$ upon observing $w^*(\tilde{\tau}_i)$. The poaching wage offered by a type- h firm with a vacant position is implicitly determined by the following condition:

$$V_h^p(\tilde{\tau}_i) - V_h^y = 0, \quad (47)$$

where

$$V_h^p(\tilde{\tau}_i) = \theta \gamma \int_{\tilde{\tau}_i}^{\bar{\tau}} \frac{\tau f(\tau) d\tau}{1 - F(\tilde{\tau}_i)} - w^p(w^*(\tilde{\tau}_i)) + \delta V_h^y, \quad (48)$$

$$V_h^y = \theta \gamma \mu - w_h^y + \delta V_h^o, \quad (49)$$

and

$$V_h^o = F(\tilde{\tau}_h) V_h^y + (1 - F(\tilde{\tau}_h)) \left(\int_{\tilde{\tau}_h}^{\bar{\tau}} \frac{\theta \tau f(\tau)}{(1 - F(\tilde{\tau}_h))} d\tau - w^*(\tilde{\tau}_h) + \delta V_h^y \right). \quad (50)$$

³⁴We cannot have $p(\tau') > p(\tau'')$ for $\tau'' = \tau'$ because p must be a function.

From equations (49) and (50), we obtain:

$$V_h^o - V_h^y = \frac{\int_{\tilde{\tau}_h}^{\bar{\tau}} (\theta\tau - w^*(\tilde{\tau}_h) - \theta\gamma\mu + w_h^y) f(\tau) d\tau}{[1 + \delta(1 - F(\tilde{\tau}_h))]} \quad (51)$$

The poaching wage offered by a type- h firm upon observing $w^*(\tilde{\tau}_i)$ is

$$w^p(w^*(\tilde{\tau}_i)) = \theta\gamma \left(\int_{\tilde{\tau}_i}^{\bar{\tau}} \frac{\tau f(\tau) d\tau}{1 - F(\tilde{\tau}_i)} - \mu \right) + w_h^y - \frac{\delta \int_{\tilde{\tau}_h}^{\bar{\tau}} (\theta\tau - w^*(\tilde{\tau}_h) - \theta\gamma\mu + w_h^y) f(\tau) d\tau}{[1 + \delta(1 - F(\tilde{\tau}_h))]} \quad (52)$$

Using this poaching wage, we can now proceed exactly as in the proof of Proposition 2 to show that $w^*(\tilde{\tau}_i) = \max\{w^p(w^*(\tilde{\tau}_i)), 0\}$ if the equilibrium threshold is $\tilde{\tau}_i$ for $i \in \{l, h\}$.³⁵

Solving it for $w^*(\tilde{\tau}_h)$, we obtain (after some algebra)

$$w^*(\tilde{\tau}_h) = \max \left\{ \theta\gamma \left(\int_{\tilde{\tau}_h}^{\bar{\tau}} \frac{\tau f(\tau) d\tau}{1 - F(\tilde{\tau}_h)} - \mu \right) + w_h^y - \delta\theta \int_{\tilde{\tau}_h}^{\bar{\tau}} (1 - \gamma)\tau f(\tau) d\tau, 0 \right\}, \quad (53)$$

which can be plugged into (52) to find $w^*(\tilde{\tau}_l)$:

$$w^*(\tilde{\tau}_l) = \max \left\{ \theta\gamma \left(\int_{\tilde{\tau}_l}^{\bar{\tau}} \frac{\tau f(\tau) d\tau}{1 - F(\tilde{\tau}_l)} - \mu \right) + w_h^y - \delta\theta \int_{\tilde{\tau}_h}^{\bar{\tau}} (1 - \gamma)\tau f(\tau) d\tau, 0 \right\}. \quad (54)$$

Because $w^*(\tilde{\tau}_i) = \max\{w^p(w^*(\tilde{\tau}_i)), 0\}$, a necessary condition for an incumbent type- i firm with a manager with type $\tau \in [\tilde{\tau}_i, \bar{\tau}]$ not to deviate and fire the manager is:

$$V_i^o(\tilde{\tau}_i) \geq V_i^y, \quad (55)$$

where

$$V_i^o(\tilde{\tau}_i) = \theta_i \tilde{\tau}_i - w^*(\tilde{\tau}_i) + \delta V_i^y, \quad (56)$$

with

$$V_i^y = \theta_i \gamma \mu - w_i^y + \delta V_i^o, \quad (57)$$

³⁵Formally, we need to modify Assumption E2 slightly to fit the dynamic setup: After observing an off-the-equilibrium-path wage w'_i , poachers believe that the probability that type $\tau' \geq w'_i + \theta_i \gamma \mu - w_i^y + \delta(V_i^o - V_i^y)$ deviates is no less than the probability that type $\tau'' > \tau'$ deviates. The application of this equilibrium refinement thus depends on some other equilibrium values (w_i^y , V_i^o , and V_i^y); this creates no difficulties as the condition can always be checked for each candidate equilibrium.

and

$$V_i^o = F(\tilde{\tau}_i)V_i^y + (1 - F(\tilde{\tau}_i)) \left(\int_{\tilde{\tau}_i}^{\bar{\tau}} \frac{\theta_i \tau f(\tau)}{(1 - F(\tilde{\tau}_i))} d\tau - w^*(\tilde{\tau}_i) + \delta V_i^y \right). \quad (58)$$

Hence, after some rearranging, condition (55) becomes:

$$\tilde{\tau}_i - \gamma\mu - \frac{w^*(\tilde{\tau}_i) - w_i^y + \delta\theta_i \int_{\tilde{\tau}_i}^{\bar{\tau}} (\tau - \gamma\mu) f(\tau) d\tau}{\theta_i[1 + \delta(1 - F(\tilde{\tau}_i))]} \geq 0. \quad (59)$$

The wage w_i^{**} offered by poachers (i.e. type- h firms) to managers from type- i firm with talent $\tau \in [\hat{\tau}_i, \tilde{\tau}_i]$ is determined by the following condition (from Bertrand competition):

$$\theta\gamma \int_{\hat{\tau}_i}^{\tilde{\tau}_i} \frac{\tau f(\tau) d\tau}{F(\tilde{\tau}_i) - F(\hat{\tau}_i)} - w_i^{**} + \delta V_h^y = V_h^y, \quad (60)$$

We use equations (49) and (50) to derive the wage for those managers who are poached (by h firms) in equilibrium:

$$w_i^{**} = \theta\gamma \left(\int_{\hat{\tau}_i}^{\tilde{\tau}_i} \frac{\tau f(\tau) d\tau}{F(\tilde{\tau}_i) - F(\hat{\tau}_i)} - \mu \right) + w_h^y - \delta\theta \int_{\tilde{\tau}_h}^{\bar{\tau}} (1 - \gamma)\tau f(\tau) d\tau. \quad (61)$$

From young managers' participation constraint, we obtain:

$$w_i^y = -\delta(1 - F(\tilde{\tau}_i))w^*(\tilde{\tau}_i) - \delta(F(\tilde{\tau}_i) - F(\hat{\tau}_i)) \max\{w_i^{**}, 0\}. \quad (62)$$

We now characterize the thresholds and wages offered by type- h firms only. From (59) and (53), the condition for a type- h firm becomes:

$$\tilde{\tau}_h - \gamma \frac{\int_{\tilde{\tau}_h}^{\bar{\tau}} \tau f(\tau) d\tau}{(1 - F(\tilde{\tau}_h))} \geq 0. \quad (63)$$

At $\tilde{\tau}_h = 0$, this condition does not hold. If $\tilde{\tau}_h = \bar{\tau}$, then we have $\bar{\tau} - \gamma\bar{\tau} > 0$ because $\gamma < 1$. Thus, by continuity, there is at least one threshold such this condition holds with equality. By the same arguments as in Proposition 2, the lowest of such thresholds is the unique equilibrium value for $\tilde{\tau}_h$. Note that $\tilde{\tau}_h$ is exactly the same as in the static case and depends only on γ and $F(\cdot)$. In particular, $\tilde{\tau}_h$ is independent of $\{\hat{\tau}_l, \hat{\tau}_h\}$.

We now characterize the wages offered by h -firms when there is strictly positive poaching $(w_h^y, w_h^{**}, w^*(\tilde{\tau}_h))$:

$$w_h^{**} = \theta\gamma \left(\int_{\hat{\tau}_h}^{\tilde{\tau}_h} \frac{\tau f(\tau) d\tau}{F(\tilde{\tau}_h) - F(\hat{\tau}_h)} - \mu \right) + w_h^y - \delta\theta \int_{\tilde{\tau}_h}^{\bar{\tau}} (1 - \gamma)\tau f(\tau) d\tau \quad (64)$$

$$w^*(\tilde{\tau}_h) = \theta\gamma \left(\int_{\tilde{\tau}_h}^{\bar{\tau}} \frac{\tau f(\tau) d\tau}{1 - F(\tilde{\tau}_h)} - \mu \right) + w_h^y - \delta\theta \int_{\tilde{\tau}_h}^{\bar{\tau}} (1 - \gamma)\tau f(\tau) d\tau \quad (65)$$

$$w_h^y = -\delta(1 - F(\tilde{\tau}_h))w^*(\tilde{\tau}_h) - \delta(F(\tilde{\tau}_h) - F(\hat{\tau}_h))w_h^{**}. \quad (66)$$

We can express w_h^y as a function of thresholds $\{\tilde{\tau}_h, \hat{\tau}_h\}$

$$w_h^y = \frac{-\delta\theta(1 - F(\hat{\tau}_h))}{1 + \delta(1 - F(\hat{\tau}_h))} \left(\gamma \int_{\hat{\tau}_h}^{\bar{\tau}} \frac{(\tau - \mu) f(\tau)}{(1 - F(\hat{\tau}_h))} d\tau - \delta \int_{\tilde{\tau}_h}^{\bar{\tau}} (1 - \gamma)\tau f(\tau) d\tau \right), \quad (67)$$

which can be plugged into (64) and (65) to obtain w_h^{**} and $w^*(\tilde{\tau}_h)$ as a functions of $\tilde{\tau}_h$ and $\hat{\tau}_h$ only. At Date 4, a type- h firm with a vacancy has expected profit

$$V_h^y = \theta\gamma\mu - w_h^y + \delta V_h^o, \quad (68)$$

where V_h^o is given by (50). Solving for V_h^y , (after some algebra) we get

$$V_h^y = \frac{\theta\gamma\mu - w_h^y}{1 - \delta} + \frac{\delta}{1 - \delta} \int_{\tilde{\tau}_h}^{\bar{\tau}} \theta(1 - \gamma)\tau f(\tau) d\tau. \quad (69)$$

A type- h firm with a vacancy announces threshold $\hat{\tau}_h$; we assume that all players (i.e., firms and managers) share the same beliefs, on and off the equilibrium path, and beliefs are such that players expect incumbent firms to use threshold $\hat{\tau}_h$ if this threshold is announced. Given such beliefs, the announcement of $\hat{\tau}_h$ pins down w_h^y as given by (67) (recall that $\tilde{\tau}_h$ is uniquely determined by (63)). Note that a firm that announces $\hat{\tau}_h$ at period t has no incentives to deviate and play a different threshold $\hat{\tau}'_h \neq \hat{\tau}_h$ at period $t + 1$, because at $t + 1$ the firm is unable to retain any type below $\tilde{\tau}_h$ and thus the firm is indifferent between any two thresholds $\hat{\tau}'_h$ and $\hat{\tau}_h$.

A type- h firm chooses $\hat{\tau}_h \in [0, \tilde{\tau}_h]$ to maximize its expected profit (69). A solution exists because of continuity and the fact that $[0, \tilde{\tau}_h]$ is a closed interval. The solution $\hat{\tau}_h$ is

(generically) unique because the expected profit is differentiable with respect to $\hat{\tau}_h$ in the interior of $[0, \tilde{\tau}_h]$.

Now that we have determined a (generically) unique set of equilibrium thresholds for h firms $\{\hat{\tau}_h, \tilde{\tau}_h\}$, we can find the equilibrium thresholds for l firms. For each $\hat{\tau}_l$, define the function:

$$G_l(\tau; \hat{\tau}_l) = \gamma\mu + \frac{w^*(\tau) - w_l^y + \delta \int_{\tau}^{\bar{\tau}} (x - \gamma\mu) f(x) dx}{1 + \delta(1 - F(\tau))}, \quad (70)$$

with domain over $\tau \in [\hat{\tau}_l, \bar{\tau}]$, where

$$w^*(\tau) = \max \left\{ \theta\gamma \left(\int_{\tau}^{\bar{\tau}} \frac{xf(x)dx}{1 - F(\tau)} - \mu \right) - \delta\theta \frac{\gamma \int_{\hat{\tau}_h}^{\bar{\tau}} (x - \mu) f(x)dx + \int_{\hat{\tau}_h}^{\bar{\tau}} (1 - \gamma)xf(x)dx}{1 + \delta(1 - F(\hat{\tau}_h))}, 0 \right\} \quad (71)$$

$$w_l^{**} = \theta\gamma \left(\int_{\hat{\tau}_l}^{\tau} \frac{xf(x)dx}{F(\tau) - F(\hat{\tau}_l)} - \mu \right) - \delta\theta \frac{\gamma \int_{\hat{\tau}_h}^{\bar{\tau}} (x - \mu) f(x)dx + \int_{\hat{\tau}_h}^{\bar{\tau}} (1 - \gamma)xf(x)dx}{1 + \delta(1 - F(\hat{\tau}_h))} \quad (72)$$

$$w_l^y = -\delta(1 - F(\tau))w^*(\tau) - \delta(F(\tau) - F(\hat{\tau}_l)) \max \{w_l^{**}, 0\}. \quad (73)$$

The existence of an equilibrium with retention for a given $\hat{\tau}_l$ requires $\tau - G_l(\tau; \hat{\tau}_l)$ to be non-negative for some τ . Because, $G_l(\tau; \hat{\tau}_l)$ is continuous and $G_l(0; 0) = \gamma\mu + \frac{\delta\mu(1-\gamma)}{1+\delta} > 0$, at least one fixed point exists if and only if $\max_{\tau \in [0, \bar{\tau}]} \tau - G_l(\tau; \hat{\tau}_l) \geq 0$. As before, if this latter condition does not hold, then no type is retained by firm l in equilibrium, i.e., $\tilde{\tau}_l = \bar{\tau}$. If $\max_{\tau \in [0, \bar{\tau}]} \tau - G_l(\tau) \geq 0$, this proves the existence of at least one threshold τ' such that $\tau' = G_l(\tau'; \hat{\tau}_l)$. Among all such τ' , we define $\tilde{\tau}_l(\hat{\tau}_l)$ as the lowest one. To show that this threshold is part of an equilibrium, notice that because $G_l(0; 0) > 0$, unless $\tilde{\tau}_l = \bar{\tau}$, $\tau - G_l(\tau; \hat{\tau}_l)$ crosses zero from below at $\tilde{\tau}_l$, which is also a necessary condition for an equilibrium. To show that no other $\tau' > \tilde{\tau}_l$ can be an equilibrium, we use the same argument as in the the proof of Proposition 2. Thus, $\tilde{\tau}_l(\hat{\tau}_l)$ is uniquely determined given $\hat{\tau}_l$.

The final step is to determine $\hat{\tau}_l$. By announcing $\hat{\tau}_l$, under the assumption that players believe the announcement, a type- l firm determines a unique equilibrium retention threshold $\tilde{\tau}_l(\hat{\tau}_l)$. Firm l thus chooses $\hat{\tau}_l$ to maximize its expected profit and then the optimal $\hat{\tau}_l$ is given by

$$\hat{\tau}_l \in \arg \max_{x \in [0, \bar{\tau}]} V_l^y(x) = \frac{\gamma\mu - w_l^y + \delta \int_{\tilde{\tau}_l}^{\bar{\tau}} (\tau - w^*(\tilde{\tau}_l)) f(\tau) d\tau}{(1 - \delta) [1 + \delta(1 - F(\tilde{\tau}_l))]}, \quad (74)$$

subject to $\tilde{\tau}_l(x)$ and

$$w_l^y = -\delta\theta \left[\gamma \int_x^{\bar{\tau}} (\tau - \mu) f(\tau) d\tau - \delta(1 - F(x)) \frac{\gamma \int_{\hat{\tau}_h}^{\bar{\tau}} (\tau - \mu) f(\tau) d\tau + \int_{\hat{\tau}_h}^{\bar{\tau}} (1 - \gamma)\tau f(\tau) d\tau}{1 + \delta(1 - F(\hat{\tau}_h))} \right]. \quad (75)$$

From continuity, the solution to this problem is (generically) unique. This completes the characterization of the equilibrium. ■