A Theory of Multiplexity: Sustaining Cooperation with Multiple Relationships

[Preliminary; Comments Welcome]

Chen Cheng, Wei Huang and Yiqing Xing *

June 18, 2020

Abstract

People are embedded in multiple social relations. These relationships are not isolated from each other. This paper provides a framework to analyze the multiplex of networks. We present a model in which each pair of agents may form more than one relationship. Each relationship is captured by an infinitely repeated prisoner's dilemma with variable stakes of cooperation. We show that multiplexity, i.e. having more than one relationship on a link, boosters incentives as different relationships serve as social collateral for each other. We then endogenize the network formation and ask: when an agent has a new link to add, will she multiplex with a current neighbor, or link with a stranger? We find the following: (1) There is a strong tendency to multiplex, and "multiplexity trap" can occur. That is, agents may keep adding relationships with current neighbor(s), even if it is more compatible to cooperate with a stranger. (2) Individuals tend to multiplex when the current network (a) has a low degree dispersion (i.e., all individuals have similar numbers of friends), or (b) is positively assortative. We also find that when relationships differ in their importance, agents tend to multiplex when the new relationship is less important, and link with a stranger when it's more important. Lastly, we find empirical evidence that supports our theoretical findings.

Keywords: Multiplex, Networks, Cooperation, Network formation, Degree dispersion, Assortativity

JEL Classification Numbers: C73, D85, O17, Z13

---

*Cheng and Xing: Johns Hopkins University, Carey Business School, Baltimore, USA. Huang: National University of Singapore, Strategy and Policy, Real Estate, Singapore. Emails: chen_cheng@jhu.edu, bizhwei@nus.edu.sg and xingyq@jhu.edu. We thank Abhijit Banerjee, Bob Gibbons, Ben Golub, Sanjeev Goyal, Matt Jackson, David Miller, Kaivan Munshi, Nicola Persico, Alex Wolitzky for extremely helpful comments. We also thank seminars participants at Johns Hopkins, MSU, Michigan, MIT, PKU, Stanford, NSF Network Conference, Barcelona GSE Summer Forum and NBER OE Conference for their suggestions. All errors remain our own.

1 Introduction

People are embedded in multiple social relations. We dine out together, collaborate on research, borrow and lend money, etc. These relationships are not isolated from each other: the existence of one relationship is likely to affect the formation and the incentives in another. Natural questions arise: What is the relationship of multiple social relations? Do different networks overlap, when and why? Without understanding above questions, our analysis about network formation might be fundamentally flawed.[1]

However, despite a few empirical and case studies in the anthropology and sociology literature (e.g., Kaplan et al. (1985), Uzzi (1997)), the theoretical probing of the interdependence of multiple relations is still in its early stages. As McPherson, Smith-Lovin and Cook (2001) put it: "It is striking that 20 years after Fischer (1982) classic study of networks in North California communities, so few large-scale studies investigate the multiple, overlapping networks of different types of relationships that his research so admirably chronicled."

This paper studies the interaction of multiple social relations. We treat the formation of multiple relations as strategic decisions. And we ask the following question: given the existing relations, when a new relationship arises, will people link with a current friend (i.e., current neighbor in the network), or a stranger? In the paper, we call the tendency to link with a friend *multiplexity*.

Specifically, we model relationship as repeated prisoner's dilemma with variable stakes. In each relationship, cooperation opportunities arise sequentially over time with Poisson arrival rate λ, independent across relationships. In each cooperation, a pair of agents first choose the stakes of cooperation, say the quality of the co-authored paper, and the minimum of which will be enforced; they then choose to cooperate or defect. We examine equilibria with the maximal stakes of cooperation, which gives each agent their highest payoffs, and always exist in our structure. We compare the maximal stakes of cooperation across different network structures.

To see how multiplexity affects people's incentives, we first fix network structure, and find the following: Multiplexity enhances cooperation because different relationships serve as social collateral for each other. By having additional relationship with the same people, incentives on every existing relations get improved. This *multiplex effect* provides incentive spillover across different relations, and, without any assumed interdependence ex ante, makes different relations complementary to each other. Due to the large benefits of multiplexity in sustaining cooperation, sometimes "multiplexity trap" can occur. That is, agents may keep adding relationships with current friends, even when it is more efficient to link with a

[1] As Atkisson et al. (2019) write: "There has been relatively little work on multilayer and multiplex networks to date, $\cdots$, without a method $\cdots$, we are unlikely to recover the true effect of each network on the outcome of interest, possibly leading to incorrect conclusions."

stranger.

We also compare multiplexity with another more examined way of enforcing cooperation – community enforcement. We show that in a complete network in which everyone is connected, multiplexity and community enforcement have the same effect on cooperating incentives. However, since cooperation size is determined by the incentive-weak link, community enforcement relies more on the rest of the network structure than multiplexity.

We then proceed to endogenize network formation. When a new relationship arises, the agent can choose to link with current friend or a stranger.[2] The agent form the relationship to maximize her equilibrium payoff in the resulting network. Agents are myopic in the sense that they ignore future link dynamics.[3] Such tractable network allows us to explore the following question: When do people multiplex, when not? How network patterns of the existing networks will affect this choice?

We find the following: Individuals tend to multiplex either when the current network has low degree dispersion (i.e., all individuals have similar numbers of friends), or when the network has a large degree dispersion, but exhibits positive assortativity (i.e., agents are linked with those who have similar number of friends). In other words, agents tend to link with a stranger in networks that exhibit negative assortativity.

When networks exhibit low degree dispersion or positive assortativity, the multiplex effect dominates; whereas in negative assortative networks, the size of cooperation in the new link with a stranger could be pretty large, which may dominate the multiplex effect. The lesson is that, asymmetry in degree not only in society at large, but among neighbors, is key for agents to jump out of the multiplexity trap, and link with strangers.

Based on our theoretical analysis, we also conduct empirical analysis using the Indian Village Survey Data collected by Banerjee et al. (2013). The dataset contains network information of multiple types of relationships for each of the 75 Indian rural villages. Our empirical analysis shows strong evidence of multiplexity, and is consistent with our predictions regarding how network patterns affect the multiplexity choice. Specifically, we present evidence that supports the following testable hypotheses:

- Multiplexity prevails in networks (Hypothesis 1)
- Multiplexity is more likely to prevail in societies that have
    - ◇ low degree dispersion (Hypothesis 2a)
    - ◇ positive assortativity (Hypothesis 2b)

[2]The recipient of the relationship always accepts, because in our setting, having additional relationship is always beneficial. We can add linking costs and the analysis is essentially the same.

[3]We also discuss the relax of the assumption in Section 5.

Given the importance of caste system in Indian culture (e.g., Hsu (1963)), we provide a robustness check utilizing the caste information in the dataset. Specifically, we assume that the cooperation for villagers in the same subcastes is mainly driven by unmodelled factors such as religion, but incentives matter more for villagers that are not in the same subcastes. This shall imply that our predictions applies more to villagers in different subcastes, but should be insignificant for those who are in the same subcastes. The empirical result also supports above hypothesis.

We also extend our theory in the various directions. Most importantly, by allowing the importance of different relationships to vary, we could explore the impact of asymmetric relationships on multiplexity. And we find that people tend to link with current friends when the new relationship is less important, but link with a stranger when the new relationship is more important.[4] This echoes the old adage that "do not borrow money from your friends". And we provide a new rationale: when the new relationship is more important, the existing ones become less important, which renders the multiplex effect to be smaller, hence agents are more willing to link with a stranger rather than to multiplex with current friends.

The rest of the paper is organized as follows. We review the literature below. Section 2 sets up the baseline model which focuses on the incentives to cooperate given the network structure: we analyze the incentive effects of multiplexity in Section 2.2, and compare it with community enforcement in Section 2.3. Section 3 presents a simple model of network formation and explores our main question: "when to multiplex, when not"; we show multiplexity trap could occur in Section 3.1. Section 4 explores how network features affect the multiplexity choice. Section 6 conducts empirical analysis. Section 5 extends our theory in various directions. And Section 7 concludes.

1.1 Literature Review

Anthropologists and sociologists have long recognized the importance of the multilayer or multiplex networks. For example, some argue that failing to find reciprocal food sharing means reciprocity could span domains (Kaplan et al. (1985)). Uzzi (1997) finds that embedded ties (i.e., long term partners) primarily develop out of previous personal relations, "embedding the economic exchange in a multiplex relationship made up of economic investments, friendship, and altruistic attachments." In economic literature, Bernheim and Whinston (1990) provide a nice benchmark that shows in a multimarket context, firms' cooperative behavior will be improved only when those markets are not identical. Li and Powell (2017) show that when the environment is non-deterministic, even identical multimar-

[4] This extension also answers that question that how multiplexity is different from increasing the intensity of one link. Only by examining multiple relations can we explore the asymmetry of different relations.

ket context can lead to better cooperation.[5] Chen, Zenou and Zhou (2018) examine games in social network when agents exert effort in two activities. They show how own productivity affects equilibrium efforts and how network density impacts equilibrium outcomes.

However, none of the above work emphasizes the processes or mechanisms that lead to the multiplex of networks. That is, we emphasize the network formation process that gives rise to multiplexity, whereas previously mentioned work does not.

More recently, some theories about network structuring process that lead to multiplexity are developed (see Atkisson et al. (2019) for an overview). Most of them either posits that it?s the same feature that attracts people to build ties in several domains, or there is time (opportunity, etc.) constraint that make expanding the set of people from one domain to another costly. We assume none of the above, and the multiplexity decision is mainly driven by the incentive spillover provided by multiple relations.

Banerjee et al. (2018) use also the Indian Village Survey Data,[6] they find that the introduction of microfinance not only erodes the relationship among those who are more likely to get loans, but those who are less likely get loans as well. This potentially suggests correlation among different relationships, and they use the strategic complementarity across different relations to explain the empirical finding. We focus more on the question whether people interact with the same set of people across different relations, i.e., the specific cause of the correlation among relations. More importantly, neither Banerjee et al. (2018) nor work that we mentioned in the previous paragraph could explain how network features of the existing network affect the multiplexity decision, as we establish both theoretically and empirically.

Joshi, Mahmud and Sarangi (2017) study how network feature of a network affect another network's formation. Our work differs in focus: while they emphasize the impact of an existing network on the new network's structure, we focus on the correlation of the two networks, that is, how network features affect the multiplexity decision. Moreover, we also reach very different conclusions. For example, they show that starting from a regular network, the new network is either empty or complete. While we show that the new network could never be complete (when the linking compatibility is the same across pairs), for agents will keep on multiplexing with friends rather than linking with strangers.

There is also a computer science literature (e.g., Kivelä et al. (2014)) that studies multiplexity and they focus on simulation and algorithms, which largely ignores strategic considerations.

Lastly, our paper contributes to the large literature on social capital and cooperation (e.g., Ghosh and Ray (1996), Karlan et al. (2009), Jackson, Rodriguez-Barraquer and Tan

[5] In our setting, even when relationships are identical, multiple relations could still enhance cooperation, and this is because cooperation opportunities arrive sequentially over time rather than simultaneously as in Bernheim and Whinston (1990) and Li and Powell (2017)

[6] Their dataset is panel while the one we use is cross-sectional.

(2012), Wolitzky (2012), Ali and Miller (2016)). Previous work mostly focus on community enforcement, and a key lesson there is that a more connected network is beneficial for cooperation. We focus on another way to enhance cooperation – multiplexity, and discuss the important difference and tradeoff between multiplexity and community enforcement when agents form a new link. And we show that the incentives to multiplex might hinder the completion of the network.

2 Baseline Model with Fixed Networks

Consider a society in which n agents $N = \{1, ..., n\}$ are interacting in K relationships. So essentially there are K networks, $G = (g^1, \ldots, g^K)$. All the K networks are undirected and unweighted, i.e. $g^k_{ij} = g^k_{ji} \in \{0, 1\}$, $\forall i, j, k$. In particular, $g^k_{ij} = 1$ means that agent i and j has a link in relationship k, and $g^k_{ij} = 0$ means otherwise. For the convenience of notation, we denote $ij \in G^k$ iff $g^k_{ij} = 1$.

Let $N^k_i(G) = \{j \neq i \mid ij \in G^k\}$ be the set of *neighbors* of agent i in relationship k. Let $d^k_i(G) = |N^k_i(G)|$ is i's *degree* (i.e., number of neighbors) in relationship k. We may simplify the notation as N^k_i and d^k_i, respectively, when there is no confusion. An agent i's *entire set of neighbors* (across all the relationships) is $N_i \equiv \bigcup_k N^k_i$, and her *total degree* is $d_i = \sum_k d^k_i$. We note that d_i is i's total number of links, which may not equal to the total number of her neighbors, since an agent can have more than one link with each neighbor under multiplexity. We do not distinguish link and relationship in the paper, and will use the terms interchangebly throughout.

We model each relationship as repeated prisoner's dilemma with endogenous stakes.[7] Specifically, an infinitely repeated game is played on each link/relationship. Cooperation opportunities (i.e., stage game) arrive randomly over time at a Poission rate $\lambda > 0$, independent across links/relationships. For example, in the co-authoring relationship, cooperation opportunities, such as a conference deadline, a revision request, etc., arrive randomly over time. When each cooperation opportunity arises, the ij pair plays the following extensive form stage game:

(i) First, agents simultaneously propose *the stakes of cooperation*, say the intended quality of the paper, $\phi^k_{ij,i}, \phi^k_{ij,j} \geq 0$; the minimum of the two, $\phi^k_{ij} = \min\{\phi^k_{ij,i}, \phi^k_{ij,j}\}$, is selected.

[7] This approach, following Ghosh and Ray (1996) and Ali and Miller (2016), allows for a transparent comparison across different equilibria with a fixed discount factor.

(ii) Then, they play the following prisoner's dilemma:

$$c_{ij}^k \begin{pmatrix} & \text{Cooperate} & \text{Defect} \\ \text{Cooperate} & \phi_{ij}^k, \phi_{ij}^k & -V(\phi_{ij}^k), T(\phi_{ij}^k) \\ \text{Defect} & T(\phi_{ij}^k), -V(\phi_{ij}^k) & 0, 0 \end{pmatrix}$$

in which $V(\phi) > 0$ and $T(\phi) > \phi, \forall \phi > 0$, and $T(0) = V(0) = 0$. Also $T'(0) = 1$ and $\lim_{\phi \to \infty} T'(\phi) = \infty$. This implies $T(\phi)/\phi$ increases from close to 1 when ϕ is small without bound. Throughout the paper, we assume that $V(\phi) = \phi$, and $T(\phi) = \phi + \phi^2$.

Agents are allowed to choose different stakes, ϕ, on different pairs ij and different relationship k. Nevertheless, to simplify notations we will omit the subscripts and superscript when there is no ambiguity.

In the above matrix, the parameter $c_{ij}^k > 0$ is called the "compatibility index". It captures the cases that some pairs can be more compatible than others for certain tasks, and/or different relationships can vary in their importance.

All agents discount the future with a common factor r. Therefore, the discounted payoff of future cooperation with stake ϕ between two agents in a single relationship is:

$$c\phi \int_0^\infty e^{-rt} \lambda \mathbf{d}t = \frac{c\phi\lambda}{r}$$

The society of the K networks and the compatibilities are common knowledge among all agents.

2.1 Equilibrium and Maximal Stakes of Cooperation (MSC)

We now define equilibria. To focus on network formation that will be discussed in later sections, we assume perfect monitoring across the entire society and relations, so that any deviation is detected immediately by the entire society. Consider the grim-trigger strategies: if agent i ever deviated in any relationship/link, all her neighbors punish her by not cooperating in every link/relationship with her, so that i's future payoff becomes zero.

In particular, for agent i to cooperate with j in relationship k, the next incentive constraints must hold:

$$c_{ij}^k \left(\phi_{ij}^k + (\phi_{ij}^k)^2 \right) \leq c_{ij}^k \phi_{ij}^k + \int_0^\infty e^{-rt} \lambda \mathbf{d}t \sum_{j',k'} c_{ij'}^{k'} \phi_{ij'}^{k'} 1_{\{ij' \in G^{k'}\}},$$

which simplifies as

$$c_{ij}^k (\phi_{ij}^k)^2 \leq \frac{\lambda}{r} \sum_{j',k'} c_{ij'}^{k'} \phi_{ij'}^{k'} g_{ij}^k \qquad (IC)$$

Definition 1 (Equilibrium). *Given the society (networks) G, an* equilibrium *(with grim-trigger strategies) is a profile of the stakes of cooperations, $\{\phi_{ij}^k\}_{g_{ij}^k=1}$, such that $\phi_{ij}^k = \phi_{ji}^k$ and (IC) holds for all i, j, k s.t. $g_{ij}^k = 1$.*

On equilibrium path, all agents on all relationships/links always choose to cooperate, and off-path, if any agent ever deviated in any relationship/link, all her neighbors punish her by not cooperating in every link/relationship with her.

It is easy to see that the grim-trigger strategies support the largest possible stakes of cooperation on equilibrium path, and therefore focusing on equilibria with such strategies characterizes the payoff frontier for the society.

Proposition **1** (Existence of equilibria)**.** *Equilibrium exists. In addition, the set of equilibria form a complete lattice. Consequently, there exists a largest equilibrium, in which all links/relationships achieve the highest stakes of cooperations across all equilibria.*

The key is that stakes of cooperations are complements: a larger stake on any relationship/link makes not cooperating on that link a larger threat to everyone, and therefore provides the society more incentives to cooperate on all links. Then the proposition holds by a standard application of Tarski's fixed point theorem.

This proposition illustrates the important observation that the largest equilibrium stakes of cooperation for all relationships/links can be achieved simultaneously. Such an equilibrium gives all agents their highest possible equilibrium payoffs.

Definition 2 (Maximal stakes of cooperation (MSC)). *The stakes of cooperation in the largest equilibrium, $\{\phi_{ij}^k\}_{g_{ij}^k=1}$, are called the* maximal stakes of cooperations*, or* MSC*.*

From now on, we will focus on equilibrium with maximal stakes of cooperation. We will first characterize the MSC's of a given network structure, then endogenize network formation and explore the impact of adding relationships/links on MSC.

The case of uniform compatibility.

In this case,$c_{ij}^k = \mathit{const} > 0, \forall i, j, k$. The compatibility index, as long as positive, does not affect the equilibrium stakes of cooperation and can be dropped. In addition, the stakes in all relationships of the same link are the same: $\phi_{ij}^k = \phi_{ij}^{k'}$, $\forall i, j, k, k'$ s.t. $g_{ij}^k = g_{ij}^{k'} = 1$. The networks G can be represented by a *weighted network*, such that $w_{ij} \equiv \sum_k g_{ij}^k$ counts the number of relationships between agents i and j.

The incentive constraints become

$$(\phi_{ij})^2 \leq \frac{\lambda}{r} \sum_{j' \neq i} w_{ij'} \phi_{ij'} \qquad (IC')$$

Throughout the paper, without further specification, we work with uniform compatibility so that we can focus on the implications of network structure.

A simple algorithm for identifying the maximal stakes of cooperation.

To simplify notations we present the algorithm for the case of uniform compatibility.

At step t of the algorithm, let $\phi_{ij,t}$ be the stake of cooperation in each relationship on pair ij. Initialize $\phi_{ij,0} = \min\{d_i, d_j\}\frac{\lambda}{r}$, which is an upper bound of the equilibrium stake.[8]

At step $t \geq 1$:

(i) For each ij s.t. $w_{ij} > 0$, define $\phi_{ij,t}$ be the supremum of ϕ_{ij} such that the incentive constraint (IC') holds for ij, given $\phi_{-ij,t-1}$. That is, $\phi_{ij,t}$ solves the following equation

$$(\phi_{ij,t})^2 = \frac{\lambda}{r}\left[\phi_{ij,t} + \min\{\sum_{j'\neq j} w_{ij'}\phi_{ij',t-1}, \sum_{i'\neq i} w_{i'j}\phi_{i'j,t-1}\}\right]$$

(ii) Terminate if $\phi_{ij,t} \equiv \phi_{ij,t-1}$ for all ij. Otherwise return to step (i).

When the algorithm terminates, it finds the maximal stakes of cooperations in the largest equilibrium.

2.2 Multiplexity as Social Collateral

Before we endogenize network formation, we first examine how multiplexity, i.e., having more than one relationship between two agents, affects cooperating incentives. We show that multiplexity supports larger stakes of cooperation (compared to the single relationship case), as different relationships serve as social collateral for each other. Having additional relationship enhances incentives on not only the new relation, but reinforces incentives on *every* existing relationships. In this sense, having multiple relationships provides incentive spillover, and they work as complements to each other in boosting incentives.

We then compare multiplexity with another more examined way of enforcing cooperation, community enforcement. Both mechanisms, multiplexity and community enforcement, enhances cooperation. In this sense, they are only different kinds of social collateral. However, the two differ significantly in that community enforcement relies more heavily on the rest of the network structure, whereas multiplexity does not.

We illustrate above ideas via the following examples.

Example 1 (One pair of agents, single relationship). *Consider a two-agent society with a single link* 12*, i.e., only one relationship. In this case, the maximal stake of cooperation, MSC, is*

$$\phi = \frac{\lambda}{r}$$

[8] This upper bound is directly implied by the incentive constraints (IC).

Figure 1: One pair, single relationship

We note that in this single link/relation example, the compatibility index c has no effect on the stake of cooperation. As we will see later (e.g., in Section 3.1), it may shape agents' incentives when more than one link or relation are involved. Throughout this section, we assume $c = 1$ on all relationships/links, so that we can focus on the differences of the network structures.

Now we illustrates the forces of multiplexity: having more than one relationships between a pair of agents.

Example **2** (Multiplexity on a single pair)**.** *Consider a two-agent society with not only one, but $k > 1$ relationships. In this case, the maximal stake of cooperation, MSC, is*

$$\phi = k\frac{\lambda}{r}.$$

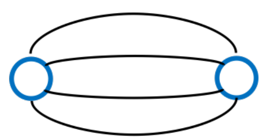


Figure 2: Multiplexity between one pair, $k = 4$

We see that having k relationships increases maximal stake of cooperation by k times compared to the one relationship case. Multiplexity boosters cooperating incentives because k relationships serve as social collateral for each other – once an agent deviates on one relationship, he loses all k relationships.

Moreover, the above MSC is the stake of cooperation on *every* relationship. So multiplexity with k relationships makes the maximal stake on every relationship k-times larger. As a result, the equilibrium payoff with k relationships is k^2 times bigger than it was in the single relationship case. In this sense, multiplexity provides incentive spillover: not only the additional relationship benefits, but all existing ones get enhanced. We call this reinforcing effect the *multiplex effect*.

2.3 Multiplexity vs. Community Enforcement

We just illustrated how multiplexity affects cooperating incentives. Another more studied mechanism to enforce cooperation is community enforcement. We show below that in a complete network, when all agents have the same degree k, community enforcement and multiplexity have the same effect in bolstering incentives.

EXAMPLE **3** (Community enforcement in a complete network)**.** *Consider a complete network with $k+1$ agents, with one relationship on each link. Each agent has a total degree of $d_i \equiv k$. Since all agents have the same (total) degree, it can be shown that MSC's are the same on all links, and the MSC's supported by community enforcement is*

$$\phi = k\frac{\lambda}{r}$$

It follows from the comparison between Examples 2 and 3 that multiplexity provides the same strength of incentives as the community enforcement does, given the same total degree. In this sense, multiplexity and community enforcement both enhances cooperation, and they are simply different types of social collateral.

However, as we will see in the next example, the effectiveness of community enforcement relies more heavily on the rest of the network, whereas multiplexity relies less.

EXAMPLE **4** (Network structure matters for community enforcement)**.** *Consider a star network with $k+1$ agents, such that the central agent has one relationship with each of the rest k agents, and there is no relationship among the peripheral agents.*

In this case, although the central agent has strong(er) incentives to cooperate, the peripheral agents' incentives are more binding and determine the maximal stakes on every relationship. It turns out the MSC in this example on every relationship/link is

$$\phi = \frac{\lambda}{r}.$$

So the MSC in the star network equals to that in the single link, single relationship case as shown in Example 1 .

We use the following figure to conclude this part. From the left to the right, the subfigures represent Example 2 – Example 4 respectively. The key messages conveyed here are that multiplexity (1) provides incentive spillover: it reinforces incentives on every existing relationship and (2) compared to community enforcement, it relies less on the rest of the network structure.

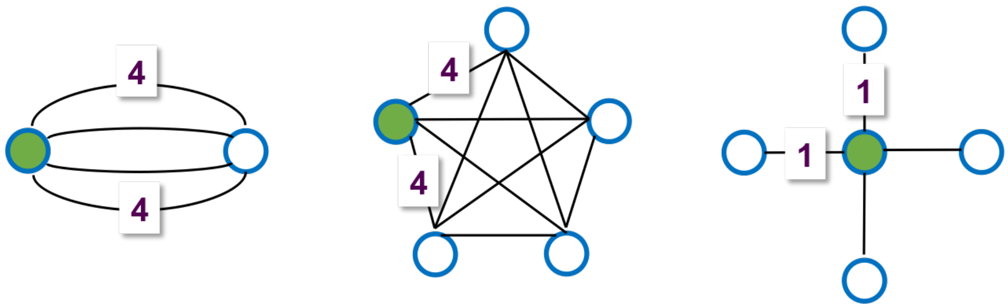


Figure 3: Maximal stakes of cooperation (MSC's) in some basic examples.

Notes: Each subgraph depicts the network structure as well as the maximal stake of cooperation (MSC) per link (the numbers, on representative links), with $\lambda/r = 1$ and uniform compatibility. Multiplexity with 4 relationships and community enforcement with a clique of size 5 (so each agent has a degree of 4) provide the same level of cooperation. Community enforcement does rely on the rest of networks, as shown in the third (star) network: although the central, green agent has 4 links, the stakes of cooperation are determined by the peripheral agents whose incentives are more binding. See Examples 2 - 4 for more details.

Now that we see two ways to enforce cooperation, i.e., multiplexity and community enforcement, when making linking decisions, which way shall an agent make use of? This provides the basic tradeoff when an agent chooses to link with a friend or a stranger, and we are now ready to examine network formation in the next section.

3 Multiplex or Link with a Stranger? A Framework of Network Formation

This section introduces a simple framework of network formation. We ask the following question: when a new relationship arises, will an agent add it on an existing link (i.e., with a neighbor/friend), or establish the new link with a stranger? When the agent chooses to link with a current neighbor, we say she chooses to *multiplex*.

To answer this question, consider the following network formation process. Start from a given society G, imagine that there is one agent who has a new relationship to add.[9] She can propose to establish the relationship with any other agent in the society.

We note that a more connected network always benefits everyone, as illustrated in the following lemma. It follows from it that the agent, when having an opportunity, always

[9]For example, consider an agent being an economist and she has a new research idea. She can choose to coauthor with a current neighbor, who may be working on other projects with her or having other types of relationsihps, or to coauthor with a "stranger".

wants to add a relationship to someone, and the chosen receipient would always accept that new link. This observation allows us to focus on whom to link with".

Lemma 1 (Monotonicity). *Consider two societies $G_2 \supset G_1$, then the "super" network G_2 have (weakly) larger maximal stakes of cooperation:*

$$\phi_{ij}^k(G_2) \geq \phi_{ij}^k(G_1),\ \forall i, j, k.$$

Since $G_2 \supset G_1$, incentive constraints (IC) for all i, j, k are more restrictive under G_1. Then $\vec{\phi}(G_2)$ would violate some incentive constraints under G_1. Actually, one can identify $\vec{\phi}(G_1)$ by running the algorithm in Section 2.1 (for G_1) and Initializing $\phi_{ij,0} = \phi_{ij}(G_2)$.

We assume the agent is sophisticated in that she chooses with whom to link to maximize her equilibrium payoff in the resulting network structure. But she is myopic in the sense that she only considers one-step-ahead in network formation, i.e., the consequences of adding the new link, but does not consider further links that might be added later (by other agents or herself). This myopic assumption applies when there is large uncertainty regarding link dynamics, or when the advent of a new relationship is rare.[10]

We start with simple network structures and show that the *multiplex effect* is so strong that may lead to "multiplexity trap". That is, agents prefer to link with a current friend even when its more efficient to link with a stranger. We then explore more complicated network structures, and see how patterns of existing networks affect the choice between friend vs. stranger (or multiplexity or linking with a stranger).

3.1 Multiplexity trap

As a start, in this subsection we illustrate that the insight that the incentives for multiplexity can be very strong. Consider a simple example with three agents. As shown in Figure 4, agent 1 and 2 already has one relationship at stake, and agent 3 does not link with anyone in the society. Agent 1 now has new relationship to add, say she has a new research idea, will she write the paper with her friend agent 2, or a stranger agent 3?

[10] For more justifications for the myopic assumption, we refer the readers to Jackson (2005) for example. However, agents need not be completely myopic for our results to hold. We relax this assumption in Section 5.

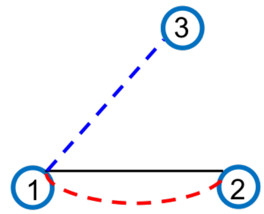


Figure 4: Multiplexity vs. stranger: a three-agent example

Notes: Agent 1 and 2 already has one relationship (black line). When agent 1 has a new relationship to add, will she link with a current friend 2 (red dashed line), or with a stranger 3 (blue dashed line)?

As it turns out in this simple example, agent 1 will strictly prefer to multiplex with agent 2 rather than link with agent 3. If she link with agent 2, all of the two relationships between them will get enhanced – the multiplex effect. Specifically, the maximal stake of cooperation between agent 1 and 2, ϕ_{12}, will become $2\lambda/r$. Since there are two relationships at hand, agent 1's total payoff when she chooses to multiplex is $4\lambda/r$.

However, when agent 1 chooses to add the new relationship with agent 3, although she still has two links at hand compared to the multiplex case, agent 2 and 3 only has one relationship each, so their incentives will be binding rather than agent 1's. Therefore, the maximal stakes of cooperation on each link, ϕ_{12} and ϕ_{13}, will be determined by agent 2 and 3's incentive constraint respectively. Since agent 2 and 3 each only has one relationship, we have $\phi_{12} = \phi_{13} = \lambda/r$, and agent 1's total payoff in this case is $2\lambda/r$.

Efficient vs. Equilibrium Network Formation In the above example, it's equally compatible for agent 1 to link with 2 and 3. That is, the stage game payoff when both agents cooperate is the same for the 12 pair and 13 pair. In this sense, the efficient network and equilibrium network formation coincide – it's both an efficient and equilibrium choice for agent 1 to link with agent 3.

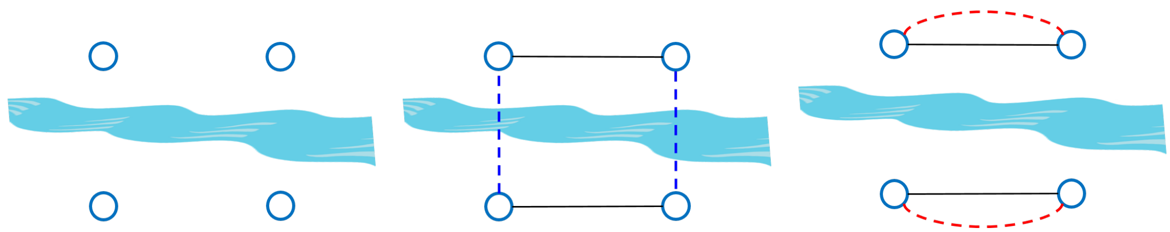
In the following, we show that when we allow the "compatibility index", c's, to vary across pairs and relationships, efficient network and equilibrium network could differ. We call this the "multiplex trap" – agents keeps on multiplexing with each other even when it's more efficient to link with strangers. More specifically, see the next example.

EXAMPLE **5.** *A river runs through a village. Each side has two agents (see Figure 5a). Each agent has two types of relationships that need to establish: babysit and trade. Each agent needs to find one partner for each relationship. Naturally, it's more convenient to babysit on the same side of the river, whereas the gain from trade is larger across the river. For the compatibility index, let* $c^{baby}_{same} = c^{trade}_{cross} = h$*, whereas* $c^{baby}_{cross} = c^{trade}_{same} = l$*, with* $h > l > 0$*.*

Suppose each agent already has the babysit relationship established, and the optimal partnerships are formed such that agents link with the partner on the same side of the river. This is shown by black solid lines in Figure 5b.

How would agents form the new trade relationship?

The efficient linking decision for trade is across the river (as shown by Figure 5b). However, in equilibrium, every agent strictly prefers to multiplex with the partner on the same side rather than linking across the river (as shown by Figure 5c), as long as $\frac{h}{l} < 9/4 = 2.25$. *This threshold is calculated in the proof of Proposition 2.*



(a) Two villages (b) Efficient network (c) Equilibrium network (when $\frac{h}{l} < 2.25$)

Figure 5: Efficient vs. Equilibrium Networks.

Notes: Blue band represents the river. Solid lines represent relationship 1 (babysitting), and dashed lines represent relationship 2 (trade).

Source of inefficiency. The source of inefficiency comes from agents' myopia/uncertainty toward how others will form links. If the villager knew for sure that the villager in the other village will form the trade relationship efficiently and immediately, he would make the efficient choice as well. However, since agents cannot foresee future link dynamics, either it's because of large network or the advent of new relationship is too rare, the multiplex effect would dominate the benefits of linking with a stranger, as long as the gap of compatibility is not too large. We discuss the relax of the myopia assumption in Section 5.[11]

The above observation can be extended to a society consisted of isolated pair.

PROPOSITION **2** (Multiplexity Trap: Isolated Pairs)**.** *Starting from a society G_0 of (isolated) pairs such that each pair has at least one existing relation with h. Every agent strictly prefers to multiplex, and the network will remain forever as a series of isolated pairs, as long as the compatibilities do not differ too much, such that*

$$\frac{\max c_{ij}^k}{\min c_{ij}^k} < 9/4 = 2.25.$$

[11] We show there that even when agents can perfectly foresee future link dynamics, as long as links are realized sequentially, multliplexity trap can still occur when agents have high discounting.

Proof of Proposition 2. The calculate the threshold, if sufficies to have two levels of compatibilities, $c_H > c_L = 1$. Consider an example of $n = 4$ agents, such that $G^1 = \{12, 34\}$, and $G^2 = \emptyset$. Suppose agent 1 is to add a link in G^2, either 12 or 13. The tendency of multiplexing is minimized when $c^1_{12} = c^1_{34} = c_L$, $c^2_{12} = c_L$, and $c^2_{13} = c_H$; that is, both existing relationships are of low compatibility, and the compatibility of the potential new relationship is high if 1 multiplex, and low if 1 links to the stranger (agent 3).

If 1 adds link 12: $\phi^1_{34} = 1$, and $\phi^1_{12} = \phi^2_{12} = 2$. So $\pi_1(G + 12) = \phi^1_{12} + \phi^2_{12} = 4$.

If 1 adds link 13: $\phi^1_{12} = \phi^1_{34} = 1$ (determined by the incentives of agents 2 and 4), and $\phi^2_{13} = \frac{1}{2} + \sqrt{\frac{1}{4} + \frac{1}{c_H}}$, which solves $c_H\phi^2 = c_H\phi + 1$. So $\pi_1(G + 13) = \phi^1_{12} + c_H\phi^2_{12} = 1 + c_H(\frac{1}{2} + \sqrt{\frac{1}{4} + \frac{1}{c_H}})$, which increases in c_H.

Let $\overline{c_H}$ be the threshold such that $\pi_1(G + 12) = \pi_1(G + 13)$; that is, $\overline{c_H}$ solves $4 = 1 + c_H(\frac{1}{2} + \sqrt{\frac{1}{4} + \frac{1}{c_H}})$. Then $\overline{c_H} = 9/4 = 2.25$.

Therefore, agent 1 strictly prefers to multiplex (on 12) if and only if $c_H < \bar{c} = 2.25$.

To complete the proof, we observe that having more relationships on isolated pairs only make everyone (weakly) more willing to multiplex. In particular, suppose there are $k > 1$ relationships (with low compatibility $c_L = 1$) to begin with on both pairs 12 and 34. Using a similar argument as in above, adding link 13 leads to a stake of $\phi_{13} = \frac{1}{2} + \sqrt{\frac{1}{4} + \frac{k}{c_H}}$, and the threshold $\overline{c_H}$ at which agent 1 is indifferent between adding 12 and 13 solves $2k + 1 = c_H(\frac{1}{2} + \sqrt{\frac{1}{4} + \frac{k}{c_H}})$. Then $\overline{c_H} = \frac{(2k+1)^2}{3k+1}$. Easy to see $\overline{c_H}$ increases in k. ∎

When agents form new links, there is a tradeoff between strong incentive provision (by multiplexity) and high compatibility. The above proposition highlights the fact that the incentive benefits can be large enough so that agents keep on multiplexing, even when the current neighbor is not suitable for any of the new relation(s). This effect can be fairly strong: agents prefer to multiplex and ignore the (in)compatibility as long as the compatibility with existing neighbor is at least about 38% of that from the most compatible strangers!

A pair of agents have incentives to add more relations between the two of them, to utilize their existing relations. Such incentives become even stronger as the pair of agents have more relations. This is the reason we call it a "multiplex trap": once the process of multiplexity starts, agents have more incentives to do so and can hardly escape from it.

However, not all societies start from isolated pairs. How would agents choose between multiplexing and linking with a stranger in more complicated network structures? We explore this question in the next section.

4 Multiplex or Not: General Network Structures

Now we extend our analysis to more complicated network structures. Again we are after the key question: When agents have a new relationship to add, will they multiplex or link with a stranger? If they choose to multiplex, they benefit from the multiplex effect so that every existing relationships get enhanced; if they link with a stranger, they could potentially make use of community enforcement to enhance cooperation. We show next that such a tradeoff is heavily affect by two network features: degree dispersion and assortativity.

4.1 Multiplexity Dominants When Degree Dispersion is Low

We first note that degree plays a prominent role in shaping agents' cooperation incentives.

Lemma **2** (MSC in regular networks)**.** *Consider any society G_0 that is regular with degree d, that is, every agent's total degree $d_i \equiv d > 0$. Then the maximal stake of cooperation (MSC) on every link/relationship is:*

$$\phi(d) = d\frac{\lambda}{r}.$$

Proof of Lemma 2. Again we normalize $\frac{\lambda}{r} = 1$ to simplify notations.

Recall the incentive constraint (IC') is

$$(\phi_{ij})^2 \leq \sum_{j'} w_{ij'}\phi_{ij'},$$

in which the weights $\sum_{j'} w_{ij'} \equiv d_i = d, \forall i$ by assumption. Therefore, the largest equilibrium features the same stake on every link $\phi_{ij} \equiv \phi(d)$, which solves $\phi(d)^2 \leq d\phi(d)\frac{\lambda}{r}$. Then $\phi(d) = d\frac{\lambda}{r}$. ∎

In regular networks, every agent, if deviates, is punished by the same number of link/relationships and hence have the same level of incentives to cooperate. MSC becomes $\phi = d\frac{\lambda}{r}$, which is one's total degree times the MSC supported by one single link/relation.

Now we show that starting with any regular network, when an agent is to add a new link, she strictly prefers to multiplex.

Proposition **3** (Multiplexity dominates in regular networks)**.** *Starting with any society G_0 that is regular with $d_i = d > 0$ for all i, when a new relationship arises, every agent strictly prefers to multiplex.*

Proof of Proposition 3. (To simplify notations we assume $\frac{\lambda}{r} = 1$ so the term can be dropped.) Consider agent 1's decision of adding a new relationship to an agent 2. Suppose $w_{12}(G_0) = k$. In particular, 2 is a neighbor of 1 in G_0 if $k > 0$, and a stranger if $k = 0$.

By assumption, after the newly added link agent 1 and either 2 will both have a degree of $d+1$, and all the other agents still have a degree of d. Therefore, agents other than 1 and 2 have binding incentive constraints on all links/relationships, whose equilibrium stakes remain the same as $\phi_{ij} = d, (\forall ij \neq 12)$.

Now we calculate the stake on link 12. $w_{12}(G_0 + 12) = k + 1$. Then all relationships on pair 12 will have a stake of $\phi_1 2$ which solves

$$(\phi_{12})^2 = (k+1)\phi_{12} + (d-k) \times d.$$

Easy to see $\phi_{12} > d$, and it increases in k: intuitively, there are more relationships between the two high-degree agents (1 and 2) and these high-powered relationships support each other. The more of them, larger the stake on each.

In addition, it follows from the fact $\phi_{12} > d$ that an increase in k also directly benefits agent 1 by having more high-stake links – this is reflected in her payoff function

$$\pi_1(G_0 + 12) = (k+1)\phi_{12} + (d-k) \times d.$$

In sum, $\pi_1(G_0 + 12)$ increases in k, the number of existing relationships on pair 12. The same calculation applies to adding any other link. Therefore, agent 1's best choice is to add a link to a neighbor that has most links with her in G_0. ∎

When a new link ij is added, the two agents i and j each have a total degree of $d+1$, and hence a larger size of cooperation can be supported between them. If i and j are already linked (i.e., ij is a multiplex link), such an increase in incentives benefit *all* relations between i and j: not only the newly added one, but also the existing ones. In contrast, if i and j were strangers and have no relationship to start with, the strong incentive only benefits one relationship (the newly added one). More importantly, since agents have the same degree before adding the new link, the size of cooperation on the new link between the stranger-pair will not be large enough to outperform the multiplex effect between a friend-pair.

As an example, consider the following regular network with $N = 4$ and $d = 2$ (Figure 6). Before a new relationship arises, everyone in the network has degree of 2. According to Lemma 2, the MSC on each link is $2\frac{\lambda}{r}$. When the green agent has a new link to add, he can either choose to multiplex, represented by the red dashed line, or link with a stranger, represented by the blue dashed line. Either way, the MSC on the new link will be larger than the $2\frac{\lambda}{r}$. If the green agent links with a stranger, the new link will be supported by two links with MSC of $2\frac{\lambda}{r}$ – the other two agents in the society are the incentive short points. Whereas if the green agent multiplexes, the red link will be supported by only one link with MSC equals $2\frac{\lambda}{r}$ and one with MSC larger than $2\frac{\lambda}{r}$. It's not hard to see that the MSC on the red link with be larger than that on the blue link.

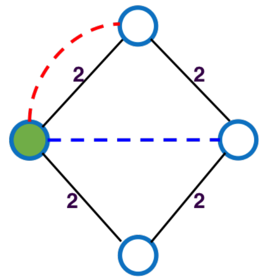


Figure 6: Multiplexity vs. Stranger in a Regular Network

Notes: This is a regular network with $d = 2$. The MSC on each link in this network is $2\frac{\lambda}{r}$. We ignore $\frac{\lambda}{r}$ in the graph. The green agent has a new link to add. He can either choose to link with a current friend, represented by the red dashed line, or with a stranger, represented by the blue dashed line.

Regular network is an extreme version of low degree dispersion – there is no degree dispersion at all. We can further generalize the above intuition to networks with low degree dispersion. When agents in a society have similar degrees, the benefits of linking with a stranger would not be that much most of the cases, but linking with a friend could reinforce all existing relationships.

4.2 Multiplexity and Assortativity

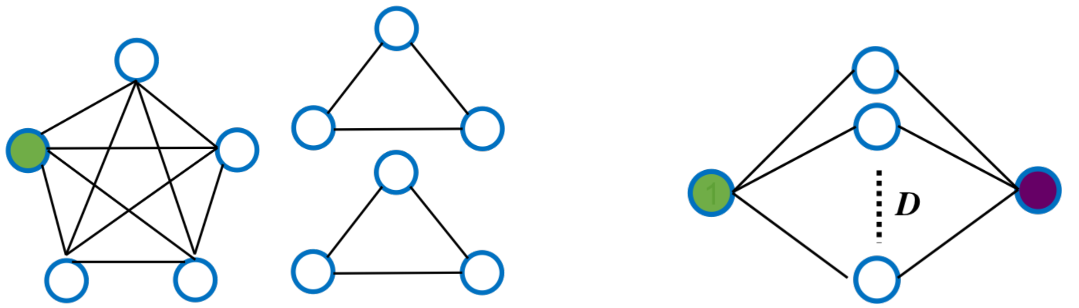
We have seen that agents tend to multiplex in networks with low degree dispersion. How about networks with large degree dispersion? We show below that when agents have very different degrees, their multiplex incentives depend on whether people with similar degrees are linked together. In other words, in networks with large degree dispersion, the multiplex incentives depend on the assortativity of a network.

For illustrative purposes, consider networks in which agents have two levels of total degrees. That is, $d_i = d$ or D $\forall i$, with $D > d$. In addition, suppose the fraction of agents having degree d is $D/(D + d)$, so that it is feasible for agents to only match with others of different degrees. There are two (extreme) cases:

(a) Positively assortative: all links are between agents with the same degree;

(b) Negatively assortative: all links are between agents with different degrees.

An example is depicted in Figure 7.



(a) Positive assortativity (b) Negative assortativity

Figure 7: Networks with different assortativity

Notes: This figure plots two networks with different assortativity: positive assortativity in panel 7a, and negative assortativity in panel 7b. In both networks, an agent has either high degree D or low degree $d = 2$. When an agent (colored in green) has a new relationship to add, multiplexity dominates in panel 7a, whereas she prefers to link with a stranger (colored in purple) in panel 7a when D is large enough (≥ 6).

We begin with the positive assortativity case, as in Figure 7a. We show below that agents strictly prefer to multiplex when networks exhibit positive assortativity.

Proposition 4 (Multiplexity dominates in positive assortative networks). *Starting with any (extremely) positive assortative society G_0, every agent strictly prefers to multiplex.*

This proposition is one step further from Proposition 3: an extremely positive assortative network G can be viewed as having two separate components G_d and G_D, in which all agents in each subcomponent have the same degree d and D, respectively. Proposition 3 suggests that every agent prefers to multiplex than to link with a stranger within the same (degree) group. The proposition further states that agents also prefer to multiplex than to link with someone in the different group.

It's important to note that the agents with a low degree d also strictly prefers to multiplex rather than linking with someone with a higher degree. To see this, suppose a degree-d agent establishes a new link with a degree-D agent. Adding this link is inferior to multiplexing due to two reasons: (1) the MSC on the new link is limited by the lower-degree agent, whose incentives are more binding; and (2) MSC's on all existing links do not benefit from the newly added link. This serves as an interesting separating point between our paper and the literature which usually predicts that people always prefer to link with agents with large/higher degree (e.g., see Joshi, Mahmud and Sarangi (2017)).

One implication of the above proposition is that *multiplexity and assortativity reinforce each other*: multiplexity is very likely in a (positively) assortative network, so more relationships are added between agents with similar degrees, and hence the network becomes more assortative.

However, when networks exhibit negative assortativity, there are situations in which agents are willing to link with a stranger rather than multiplexing, as we see in the next example.

Proposition 5. *Consider the network depicted in Figure 7b. The green agent, who has a current degree of D, only have low degree ($d = 2$) neighbors. When she has a new relationship to add, she prefers to link to a stranger with degree-D rather than to multiplex, if $D \geq 6$.*

Proof of Proposition 5. Denote the current society G_0. Let the green agent be agent 1, one of her neighbors (white) be agent 2, and the purple agent be agent 3. We compare 1's payoff between adding a relationship to link 12 (to multiplex) and adding link 13.

Adding 12: The network becomes $G_0 + 12$, in which $d_2 = w_{12} + w_{13} = 2 + 1 = 3$, and agents $d_1, d_3 \geq 3$. Therefore, agent 2's incentives determines the stakes on pairs 12 and 13. Then $\phi_{12} = \phi_{13} = 3$. All other links have a stake of 2 in the society. Thus agent 1's payoff $\pi_1(G_0 + 12) = 2\phi_{12} + 2(D - 1) = 2D + 4$.

Adding 13: The network becomes $G_0 + 13$. All links other than 13 have a stake of 2 in the society. The stake on the new link 13, ϕ_{13}, solves the incentive constraint $\phi^2 = \phi + 2D$. So $\phi_{13} = 0.5 + 0.5\sqrt{1 + 8D}$. Thus agent 1's payoff $\pi_1(G_0 + 13) = \phi_{13} + 2D = 0.5 + 0.5\sqrt{1 + 8D} + 2D$.

The threshold $\bar{D}$ that makes 1 indifferent between 12 and 13 solves $2D + 4 = 0.5 + 0.5\sqrt{1 + 8D} + 2D$, hence $\bar{D} = 6$.

In sum, agent 1 prefers to link with a stranger (adding 13) if $D \geq \bar{D} = 6$. ∎

In this society which is negatively assortative, all the current neighbors of the high-degree agents have a lower degree (of 2), and hence can support relatively small stakes of cooperation. Although multiplexity does help increase the stake of cooperation, a new link with stranger who has a high degree can be very powerful in providing a high MSC. When D is large enough ($\geq$ 6 in this case), having one large-stake link is more beneficial than multiplexity.[12]

Summary. In this section, we explored more complicated network structures and find two network features that determine the multiplexity-incentive: degree dispersion and assortativity. Multiplexity is more preferred when all agents have similar degrees (low dispersion), or when agents of similar degrees mostly link with each other (positive assortativity). The key

[12] It's worth noting that in the above example, it's not essential for the low-degree agents to be common friends with the two high-degree agents. And the incentives of the high-degree agents to link with each other is not driven by completing the triangle. This is because information is complete in our setting, so there is no issue of information transmission.

lesson is that for agents to jump out of the multiplexity trap, asymmetry in degree among neighbors, not only in the society at a whole, is important.

In Section 6, we will present a series of empirical results that support the above theoretical predictions.

5 Discussion and Extensions

In this section, we discuss several key assumptions and extensions of our paper.

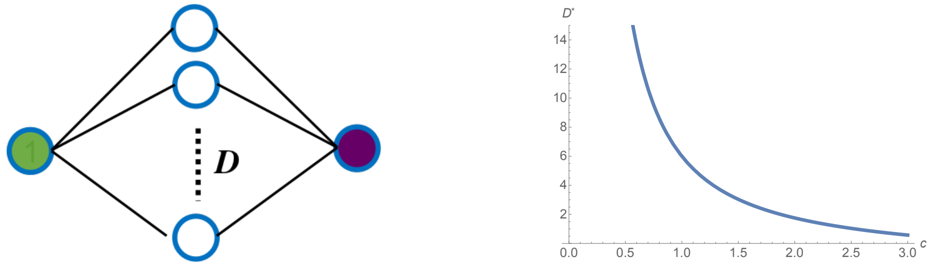
5.1 Relationships with Varying Importance

In this part, we explore the implications when different relationships vary in their importance. For instance, people might value friendship more than other relationships such as advice-giving. Once we allow for asymmetry among relationships, we could explore the following question: given existing relationships, will people link with current friends on more or less important relationships?

We model this by letting the compatibility index to vary across relationships. Specifically, recall agents' cooperation payoff is $c_{ij}^k \phi_{ij}$. Let c_{ij}^k change with relationship k, which shows that different relationships vary in their importance. When $c^x > c^y$, we say relationship x is more important than relationship y.

We have shown in Proposition 5 that agents prefer to linking with a stranger rather than multiplexing when networks exhibit negative assortativity, if the number of friends of the stranger exceed certain threshold $\bar{D}$. We show below that the threshold D decreases in the importance index c.

Proposition **6.** *Recall the negative assortativity example (Figure 8a). $\bar{D}$ represents the threshold beyond which linking with a stranger is preferred. When different relationships vary in their importance, i.e., the compatibility index c_{ij}^k changes for relationship k, the threshold $\bar{D}$ decreases with c^k (Figure 8b). That is, agents are more willing to link with a stranger on more important relationships.*



(a) Negative Assortative Network

(b) Threshold $\bar{D}$ decreases in c

Figure 8: Threshold of linking with a stranger decreasing in c, the compatibility (importance) of the new relationship.

Proof of Proposition 6. To simplify notations we again normalize $\frac{\lambda}{r} = 1$.

We repeat the proof of Proposition 5 by introducing the additional parameter c. The existing relationships in soceity G_0 all have a compatibility normalized to $c_0 = 1$. Again, let the green agent be agent 1, one of her neighbors (white) be agent 2, and the purple agent be agent 3. We compare 1's payoff between adding a relationship to link 12 (to multiplex) and adding link 13.

Adding 12: The network becomes $G_0 + 12$. Agent 2's incentives determines the stakes on pairs 12 and 13. Let ϕ_0 be the stake in each of 2 relationships with compatibility $c_0 = 1$ (those in G_0), and ϕ_c be the stake in the new relationship with compatibility $c = 1$. ϕ_0 and ϕ_c solve the following equations:

$$(\phi_0)^2 = 2\phi_0 + c\phi_c;$$

$$c(\phi_c)^2 = 2\phi_0 + c\phi_c;$$

Easy to see $\phi_0 = \sqrt{c}\phi_c$, so $\phi_0 = 2 + \sqrt{c}$ and $\phi_c = \frac{1}{\sqrt{c}} + 1$. All other links have a stake of 2 in the society.

Thus agent 1's payoff $\pi_1(G_0 + 12) = c\phi_c + \phi_0 + 2(D-1) = 2D + 1 + 2\sqrt{c}$.

Adding 13: The network becomes $G_0 + 13$. All links other than 13 have a stake of 2 in the society. The new link 13 has a compatibility of c, and its stake ϕ_{13} solves the incentive constraint $c\phi^2 = c\phi + 2D$. So $\phi_{13} = 0.5 + 0.5\sqrt{1 + 8D/c}$. Thus agent 1's payoff $\pi_1(G_0 + 13) = c\phi_{13} + 2D = 0.5c + 0.5c\sqrt{1 + 8D/c} + 2D$.

The threshold $\bar{D}$ that makes 1 indifferent between 12 and 13 solves $2D + 1 + 2\sqrt{c} = 0.5c + 0.5c\sqrt{1 + 8D/c} + 2D$, hence $\bar{D} = \frac{4+16\sqrt{c}+15c-2c\sqrt{c}}{8c}$. ∎

The above finding shows that the more important the new relationship is, the green agent is more willing to link with the purple agent (who is a stranger before the new link establishes). In other words, people tend to multiplex with friends on relatively less important relationships.

The rationale behind Proposition 6 is as following: since the benefits of multiplexity mainly come from the multiplex effect, i.e., all existing relationships get enhanced, then the more important the new relationship is, the less important the existing relationships are, the smaller the multiplex effect. As a result, agents are more willing to link with a stranger rather than multiplex when the new relationship is more important.

One might link the above finding with the old adage that "do not borrow money from your friends". The conventional wisdom for the above adage is that people do not want to risk losing friendship due to some irresistible noise in other relationships. Our finding shows that people may still avoid certain relationships with friends even if there is no noise.

5.2 Self-Enhencing Multiplex

Multiplex is self-enhencing: agents would continue to prefer to multiplex, in a more "multiplexed" network. This is illustrated in the following proposition.

PROPOSITION **7.** *Starting with any given society G, suppose an agent i prefers to multiplex by adding some relationship ij^k. Then in the resulting society $G + ij^k$, this agent i still prefers to multiplex than to link with a stranger.*

5.3 Myopic vs. Farsighted Agents

In our previous sections, we assume that agents are myopic in that they ignore future link dynamics. This myopia does not have to reflect agents' irrationality – it could completely come from physical constraints such as uncertainty in a large network. We showed in Section 3.1 that such myopia could cause inefficient network formation. However, for the inefficiency result to hold, we do not have to assume complete myopia. As long as new relationships arrive randomly over time, and agents are not sufficiently patient, network formation could still be inefficient, i.e., when it's socially efficient to link with a stranger, agents may still prefer to multiplex with current friends. Such inefficient patterns become more likely as the opportunities of new relationships become less frequent and/or agents are more impatient.

6 Empirical Analysis

So far, we have shown that multiplexity could enhance cooperation and the incentive benefits could be so large that we may see multiplexity trap. We further show that agents prefer to multiplex more likely in networks that exhibit low degree dispersion or positive assortativity. We summarize the above theoretical findings as the following three testable hypotheses:

Testable hypotheses

- Multiplexity prevails in networks (*Hypothesis 1*)
- Multiplexity is more likely in societies that have:
  - ◇ low degree dispersion (*Hypothesis 2a*)
  - ◇ positive assortativity (*Hypothesis 2b*)

Data description. The data we use is the Indian Village Survey data from Banerjee et al. (2013) and Jackson, Rodriguez-Barraquer and Tan (2012). It covers 75 rural villages in Karnataka, an area of southern India within a few hours of Bangalore. The survey contains information about multiple relationships among individuals and households in the same village, which serves as a good starting point to test our theory about multiplexity.

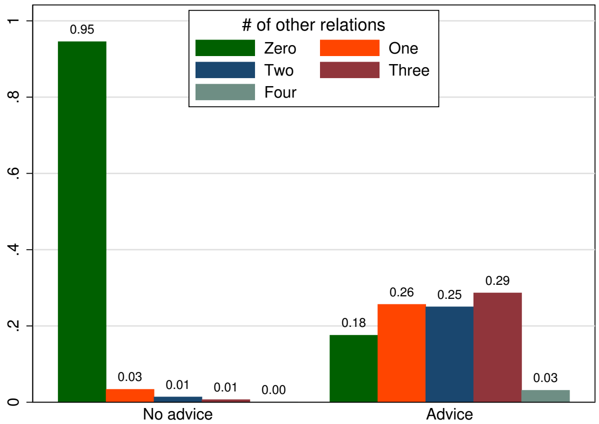
Specifically, we use five types of relationships among households in our study: (1) going to temple together; (2) visits during free time; (3) advice giving/receiving; (4) rice/kerosene borrowing/lending ; (5) money borrowing/lending. We treat households as nodes in the network.[13] We say a pair of households are connected, if at least one member in one household has any of the above relationships with any member in the other household. Accordingly, there are five types of connections. When two households have at least two different types of connections, we say they multiplex with each other.

Testing Hypothesis 1: The Prevalence of Multiplexity.

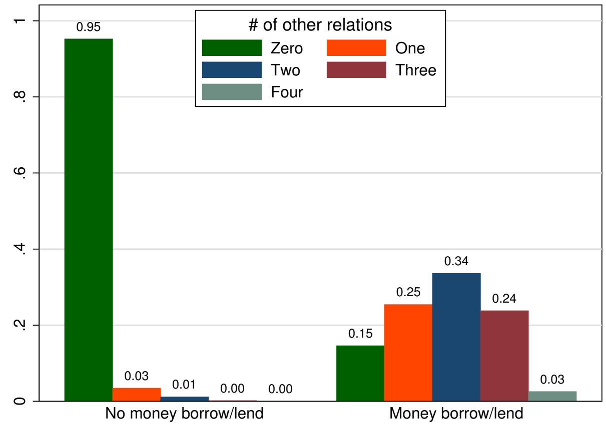
Our first, and the baseline, theoretical prediction is that multiplexity trap can occur. That is, agents tend to build new relationships on top of old ones, even when it's not efficient. One implication of this prediction is that multiplexity should be prevalent.

This prevalence can be illustrated in Figure 9, which compares the histogram of the number of other relationships conditional on having a given relationship k or not. For example, Panel 9a shows that conditional on the household pair ij do not have the relationship on advice, the probability that they do not have any other relationship is 95%; whereas this probability drops to 18% when the pair do have the relationship on advice.

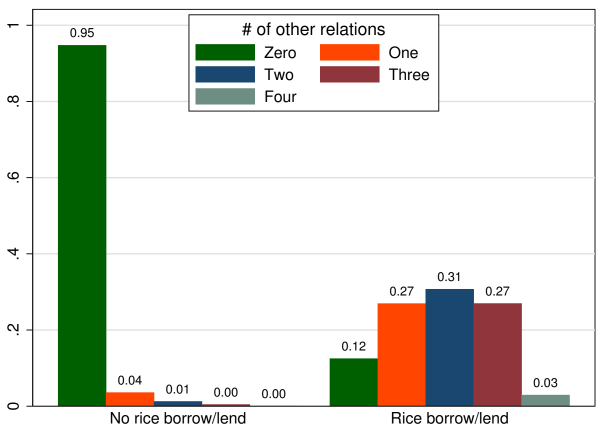
[13] The empirical results are the same if we use individuals as nodes.



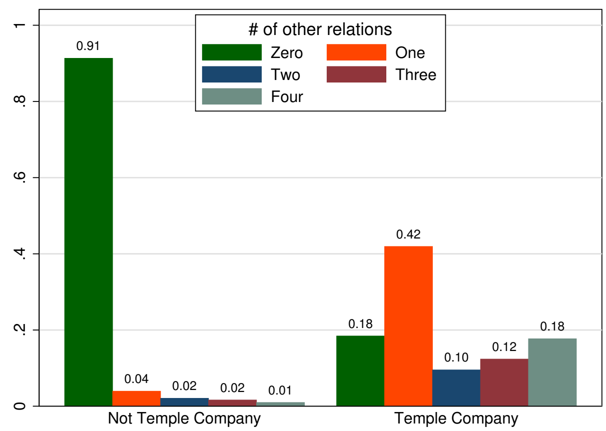
(a) $k = advice$



(b) $k = money$



(c) $k = rice$



(d) $k = temple$

Figure 9: Prevalence of Multiplexity

Notes: Each panel compares the histograms of the number of other relationships, conditional on having a given relationship k (left) or not (right). The panels share the same pattern: when a pair of households do not have the given relationship, it is highly likely (above 90% that they do not have any other relationships; while when they have the given relationship, it is highly likely (above 80%) that they also have at least one of the other relationships.

We use the following baseline regression to see how prevalent multiplexity is:

$$Relation_{ij}^{-k} = \alpha_0 + \alpha_1 Relation_{ij}^{k} + \epsilon_{ij} \tag{1}$$

The key independent variable $Relation_{ij}^{k}$ is an indicator variable whether there is a relationship k between the household i and household j in village v, $Relation_{ij}^{k} = 1$ if the answer is yes, and 0 otherwise. The dependent variable $Relation_{ij}^{-k}$ indicates whether there is any relationship other than k between the two households, $Relation_{ij}^{-k} = 1$ if the answer is yes, and 0 otherwise. There are five types relationships, and we let $k = temple, visit, advice, money, rice$. We can also let $Relation_{ij}^{-k}$ to represent the number of relationships between households i and j other than k, and the result is similar.

The following Table 1 summarizes our result.

	(1)	(2)	(3)	(4)	(5)
Relationship k	Temple	Visit	Advice	Rice	Money
Panel A: Dependent variable: Relationship $-k$ (Yes = 1)					
Relationship k	0.83***	0.71***	0.80***	0.83***	0.81***
(Yes = 1)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
Observations	718,951	718,951	718,951	718,951	718,951
R-squared	0.04	0.43	0.30	0.39	0.42
Panel B: Dependent variable is the number of relationship $-k$ (0-4)					
Relationship k	2.60***	1.42***	1.82***	1.74***	1.69***
(Yes = 1)	(0.05)	(0.03)	(0.03)	(0.03)	(0.02)
Observations	718,951	718,951	718,951	718,951	718,951
R-squared	0.067	0.46	0.39	0.46	0.51

Table 1: Prevalence of Multiplexity

We see that the existence of one relationship between households has a significant and positive impact on whether they have other relationships (Panel A) and the number of other relationships (Panel B).

The above provides baseline evidence for our *Hypothesis 1*: multiplexity is prevalent in networks. However, there might be many reasons for multiplexity. To further distinguish our theory from others, we need to test our *Hypotheses 2a* and *2b*. That is, how network patterns will affect the prevalence of multiplexity.

Testing Hypothesis 2: Determinants of Multiplexity. Our *Hypotheses 2a* and *2b* concern the determinants of multiplexity. Recall that we have the following theoretical predictions:

- Multiplexity is more likely to appear in societies that have:

2a. low degree dispersion;

2b. postive assortativity.

To test the above hypotheses, we treat each village as independent observation, and follow two steps. First, we measure the level of multiplexity for each village; then, we put the estimated multiplexity as dependent variable, and the network features (degree dispersion and assortativity) for each village as independent variables.

Specifically, we first run the following baseline regression at the village level:

$$Relation_{ij}^{-k} = \alpha_0^v + \alpha_1^v Relation_{ij}^k + \epsilon_{ij}^v \tag{2}$$

This results in a point estimation for α_1^v, $v = 1, ..., 75$, for each village. We use it as the measure for the level of multiplexity in village v.

We then provide measures for the key independent variables that we are interested in. The first variable is the degree dispersion. Specifically, for each village, we plot the distribution of the total degree for all the local households, and then we define the dispersion in degree of the network by using 75th percentile dividing by the 25th percentile of the degree. The higher the value is, the more dispersed the network is.

The second variable, assortativity, measures how assortative a village is in terms of degree. This variable is defined as the inverse of the average absolute difference between degrees of connected households in the village. The value of assortativity ranges from 0.1 to 0.3 in the data.

Then we conduct the following regression to investigate the relationship between multiplexity and degree dispersion and assortativity.

$$Multiplex_v = \beta_0 + \beta_1 D_v + \epsilon_v \tag{3}$$

The dependent variable is the multiplexity measure in village v, which is estimated α_1^v in previous equation. D_v denotes degree dispersion or assortativity measure discussed above. Since we control for the village specific dummies, the main effects of D_v have been absorbed. The coefficient, β_1, is of main interest because it captures how multiplexity is associated with degree dispersion or assortativity.

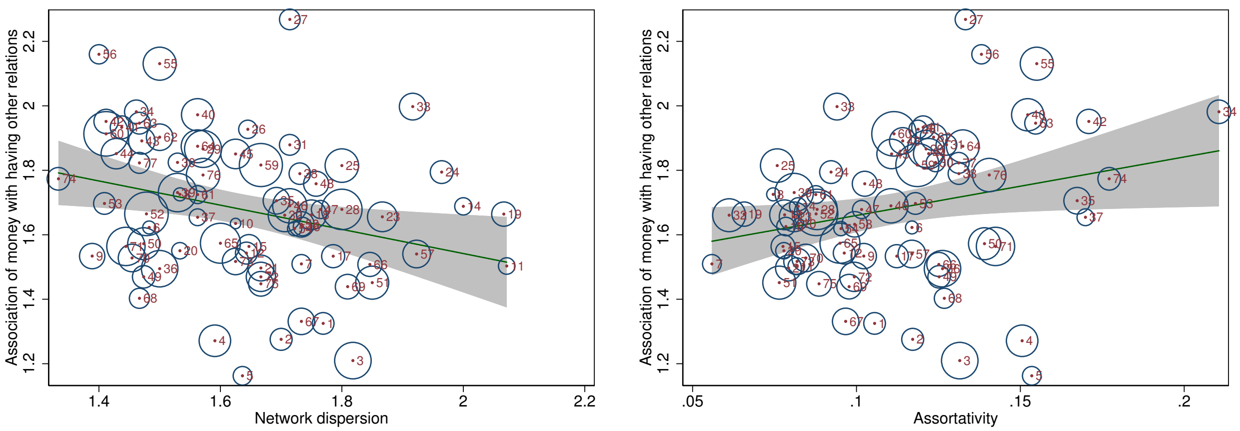
Panel A in Figure 10 reports the results.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Relationship k	Temple company		Visit		Advice		Rice		Money	
Panel A: Multiplexity based on all pairs										
Dispersion	-0.178**		-0.0694		-0.135***		-0.0570		-0.117***	
	(0.0760)		(0.0532)		(0.0435)		(0.0359)		(0.0353)	
Assortativity		0.849**		0.275		0.131		0.383*		0.459**
		(0.423)		(0.295)		(0.235)		(0.206)		(0.184)
Observations	75	75	75	75	75	75	75	75	75	75
R-squared	0.071	0.052	0.026	0.013	0.143	0.004	0.036	0.052	0.142	0.070
Panel B: Multiplexity based on same subcaste pairs										
Dispersion	-0.189*		-0.0788		-0.0566		-0.0450		-0.0287	
	(0.0980)		(0.0592)		(0.0396)		(0.0492)		(0.0371)	
Assortativity		0.661		0.345		0.0453		0.264		0.107
		(0.569)		(0.303)		(0.258)		(0.252)		(0.208)
Observations	75	75	75	75	75	75	75	75	75	75
R-squared	0.061	0.019	0.032	0.016	0.027	0.000	0.017	0.015	0.010	0.004
Panel C: Multiplexity based on different subcaste pairs										
Dispersion	-0.308***		-0.0621		-0.192***		-0.0628*		-0.162***	
	(0.105)		(0.0611)		(0.0607)		(0.0360)		(0.0420)	
Assortativity		0.569		0.209		0.209		0.445*		0.570***
		(0.524)		(0.322)		(0.317)		(0.231)		(0.213)
Observations	75	75	75	75	75	75	75	75	75	75
R-squared	0.147	0.018	0.018	0.007	0.148	0.006	0.036	0.065	0.191	0.085

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Figure 10

Consistent with our theoretical predictions, we find that degree dispersion is negatively associated with multiplexity, and assortativity is positively related with multiplexity at village level. The following Figure 11 show this more intuitively.



(a) Degree dispersion

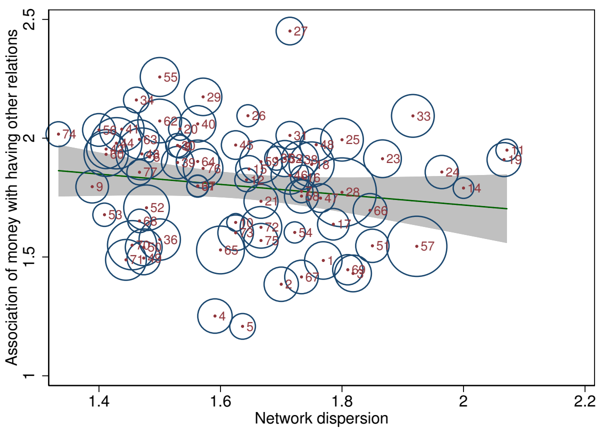
(b) Assortativity

Figure 11: Determinants of multiplexity, money as the given relationship

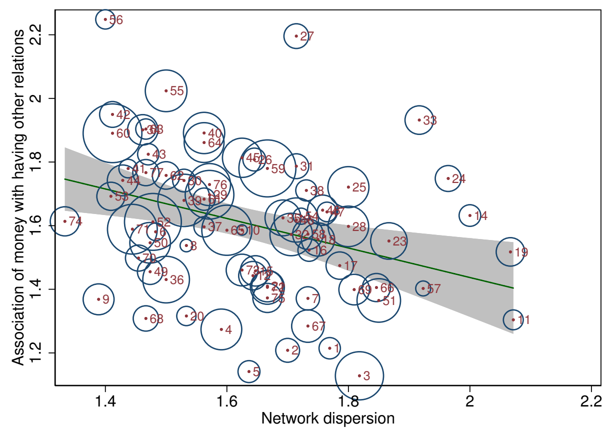
Robustness Check. Given the importance of caste in Indian society, we conduct the parallel investigation for same-subcaste pairs and different-subcaste pairs seperately. We conjecture that, if cooperation is mostly driven by unmodeled factors among same-subcaste pairs, our theory applies more to different-subcaste pairs because incentive issue matters more here. This leads to the following robustness check:

The effects of degree dispersion and assortativity are stronger (and more significant) for different-subcaste pairs, and are weaker (and less significant) for same-subcaste pairs (Hypothesis 3).

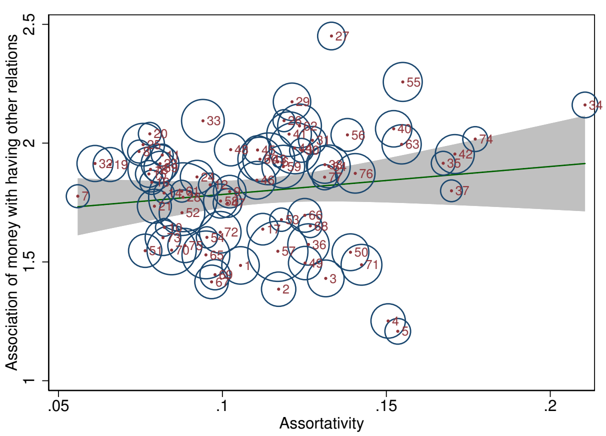
Figure 12 provides evidence that supports *Hypothesis 3*. Panels 12a and 12c are for same-subcaste pairs and Panels 12b and 12d are for different-subcaste pairs. The associations between multiplexity and degree dispersion/assortativity are only significant among different-subcaste pairs. Regression results are reported in Panel B and C in Figure 10 respectively.



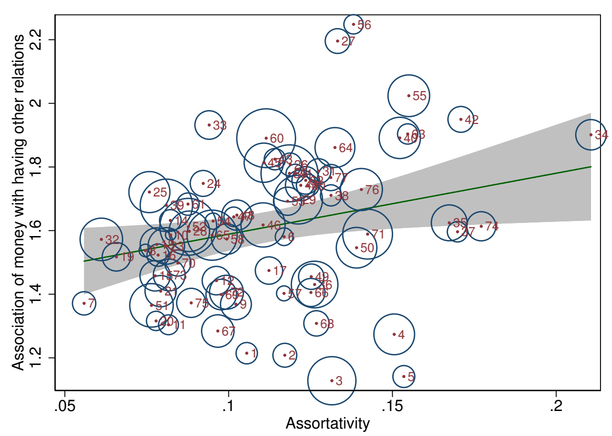
(a) Degree dispersion, same-subcaste



(b) Degree dispersion, different-subcaste



(c) Assortativity, same-subcaste



(d) Assortativity, different-subcaste

Figure 12: Determinants of multiplexity, for same-subcaste or different-subcaste pairs

7 Conclusion

In this paper, we explore the interdependence among multiple relations, and examine when do people interact with the same set of people across different relations. We contribute to the literature by endogenizing the network formation process when agents form multiple relations, and shows how network features of the existing network affects the multiplexity decision.

We find that multiple relations could enhance cooperation because multiplexity provides incentive spillover across relations. But also due to this large benefits of multiplexity, sometimes agents fail to link with a stranger even when it's more efficient – the so-called multiplexity trap could occur. Mostly importantly, we find that agents tend to multiplex when networks have low degree-dispersion, or when networks exhibit positive assortativity. This finding distinguishes our theory of multiplexity from other theories, for previous work does not speak to how network patterns affect the multiplexity choice. Using the Indian Village Survey Data collected by Banerjee et al. (2013), we are able to test our theories and find supportive evidence.

Our work is a preliminary step for us to understand why in some societies, people mainly build up their relations in the familiar-circle, while in others, people could expand their circle across different domains. The factor that we emphasize is the existing network features. But definitely there are many other factors that are important and play a role, such as the formal institutions. The interaction of formal institution and informal ones, such as multiplexity and community enforcement, will be worth exploring in future work.

References

Acemoglu, Daron, and Alexander Wolitzky. (forthcoming). "Sustaining cooperation: Community enforcement vs. specialized enforcement." *Journal of the European Economic Association.*

Ali, S Nageeb, and David A Miller. 2016. "Ostracism and forgiveness." *The American Economic Review*, 106(8): 2329–2348.

Atkisson, Curtis, Piotr J Górski, Matthew O Jackson, Janusz A Hołyst, and Raissa M D'Souza. 2019. "Why understanding multiplex social network structuring processes will help us better understand the evolution of human behavior." *arXiv preprint arXiv:1903.11183.*

Banerjee, Abhijit, Arun G Chandrasekhar, Esther Duflo, and Matthew O Jackson. 2013. "The diffusion of microfinance." *Science*, 341(6144): 1236498.

Banerjee, Abhijit V, Arun G Chandrasekhar, Esther Duflo, and Matthew O Jackson. 2018. "Changes in social network structure in response to exposure to formal credit markets." *Available at SSRN 3245656.*

Bernheim, B Douglas, and Michael D Whinston. 1990. "Multimarket contact and collusive behavior." *The RAND Journal of Economics*, 1–26.

Chen, Ying-Ju, Yves Zenou, and Junjie Zhou. 2018. "Multiple activities in networks." *American Economic Journal: Microeconomics*, 10(3): 34–85.

Fischer, Claude S. 1982. *To dwell among friends: Personal networks in town and city.* University of chicago Press.

Gagnon, Julien, and Sanjeev Goyal. 2017. "Networks, markets, and inequality." *The American Economic Review*, 107(1): 1–30.

Ghosh, Parikshit, and Debraj Ray. 1996. "Cooperation in community interaction without information flows." *The Review of Economic Studies*, 63(3): 491–519.

Hsu, Francis LK. 1963. "Clan, caste, and club."

Jackson, Matthew. (forthcoming). "The past and future of network analysis in economics." In *The Oxford Handbook of the Economics of Networks.*

Jackson, Matthew O. 2005. "A survey of network formation models: stability and efficiency." *Group formation in economics: Networks, clubs, and coalitions*, 664: 11–49.

Jackson, Matthew O, and Brian W Rogers. 2007. "Meeting strangers and friends of friends: How random are social networks?" *The American economic review*, 97(3): 890–915.

Jackson, Matthew O, and Yves Zenou. 2014. "Games on Networks." *Handbook of Game Theory*, 95.

Jackson, Matthew O, Brian W Rogers, and Yves Zenou. 2016. "Networks: An Economic Perspective." *arXiv preprint arXiv:1608.07901.*

Jackson, Matthew O, Brian W Rogers, and Yves Zenou. 2017. "The economic consequences of social-network structure." *Journal of Economic Literature*, 55(1): 49–95.

Jackson, Matthew O, Tomas Rodriguez-Barraquer, and Xu Tan. 2012. "Social capital and social quilts: Network patterns of favor exchange." *The American Economic Review*, 102(5): 1857–1897.

Joshi, Sumit, Ahmed Saber Mahmud, and Sudipta Sarangi. 2017. "Network formation with multigraphs and strategic complementarities."

Kaplan, Hillard, Kim Hill, Rowe V Cadeliña, Brian Hayden, David Charles Hyndman, Richard J Preston, Eric Alden Smith, David E Stuart, and David R Yesner. 1985. "Food sharing among ache foragers: Tests of explanatory hypotheses [and comments and reply]." *Current anthropology*, 26(2): 223–246.

Karlan, Dean, Markus Mobius, Tanya Rosenblat, and Adam Szeidl. 2009. "Trust and social collateral." *The Quarterly Journal of Economics*, 124(3): 1307–1361.

Kivelä, Mikko, Alex Arenas, Marc Barthelemy, James P Gleeson, Yamir Moreno, and Mason A Porter. 2014. "Multilayer networks." *Journal of complex networks*, 2(3): 203–271.

Li, Jin, and Michael Powell. 2017. "Multilateral Interactions Improve Cooperation Under Random Fluctuations." Northwestern University Working paper.

McPherson, Miller, Lynn Smith-Lovin, and James M Cook. 2001. "Birds of a feather: Homophily in social networks." *Annual review of sociology*, 27(1): 415–444.

Uzzi, Brian. 1997. "Social structure and competition in interfirm networks: The paradox of embeddedness." *Administrative science quarterly*, 35–67.

Wolitzky, Alexander. 2012. "Cooperation with network monitoring." *Review of Economic Studies*, 80(1): 395–427.