Taking sides: The Political Economy of Solon's Law for Civil Wars*

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This version: 18 June 2019

Abstract

In 594 BCE the Athenian statesman Solon defused a grave social crisis by introducing wide-ranging constitutional, political and economic reforms which granted various rights to a nascent 'middle class' and reduced the power of the wealthy birth aristocracy. Solon's reforms included a law which perhaps counter-intuitively banned citizens from staying neutral in cases of civil conflict. After reviewing aspects of the law against neutrality debated by historians, the present paper employs the methodology of the economics of conflict to investigate the implications of the law for the stability of the constitutional order initiated by Solon. We examine a stylised model of three social classes, Rich, Middle and Poor, where the former two compete for control of the government, and the Poor may decide to stay neutral or side with either the Middle or the Rich. By solving the model we identify conditions for the Rich to accept the Solonian order or reject it and mount a coup.

Key words: Ancient Athens; Solon; social conflict; neutrality; social stability; constitutional choice.

JEL Classification: D7: Analysis of Collective Decision making; D72: Political Processes
D74: Conflict; Conflict Resolution; Alliances; N4: Economic History–
Government; N93 - Europe: Pre-1913 Regional and Urban History

1 Introduction

In 594 BCE the Athenian lawgiver Solon introduced a new constitutional and political dispensation for the city-state granting a number of political, economic and civil rights to previously disenfranchised Athenians. One of Solon's laws made compulsory that in the event of a civil conflict (stasis) every citizen must take sides. Aristotle, in the Athenian Constitution, 8.5, describes this obligation as follows.

> "Seeing that the city was often in a state of strife, and that some of the citizens through apathy accepted whatever might happen, he [=Solon] enacted a special law to deal with them, that if the city was torn by strife anyone should refuse to place his arms at the disposal of either side should be outlawed and have no share in the city".

This is one of the earliest examples of the dictum "you're either with us, or against us" pronounced in polarised circumstances to force non-participants to abandon neutrality in favour of one's side. What was Solon's rationale for compelling citizens to take sides? Mandating citizens to do so in a situation of conflict swells the numbers of potential combatants increasing the risk of violence and the size of losses from violence. Calling citizens to participate in the conflict alludes to a deliberate suspension of the Weberian notion of the state as an agent with the legitimate monopoly use of power. Staying neutral and abstaining from conflict would have been thought as the rational response in a situation of heightened risk to individual life and property. Why did a "sage" like Solon (one of the so-called "seven wise men" of ancient Greece) mandate non-neutrality? This paper offers a rational choice answer to this question.

The following section sketches the historical background to the law and surveys the modern literature inspired by it. Section 3 builds a model of conflict among the rich and the middle income class where the former seek to overthrow the 'Solonian order' and establish a 'tyranny' which repeals the rights of the middle income earners, while a third class, the poor may join or stay out of the conflict.

Section 4 discusses the intuition offered by the model. Section 5 concludes.

2 Historical background and review of scholarship

The first historical information we have about ancient Athens relates to events in the final third of the seventh century BCE when Athens was governed by a birth aristocracy of rich

landowners. In 632/1 Cylon, an aristocrat mounted a failed bid for power. In 622 Draco compiled a written code of harsh laws that seemed to have confirmed the hold of the landed aristocracy on power.[1] Following continued conflict between the aristocracy and the common people and an economic crisis, Solon, an aristocrat and poet, was called to resolve the crisis.[2] Solon confronted a breakdown of the social order and must have worried about the opportunities that such anarchy offered to ambitious rich individuals to impose their will as 'tyrants'[3] on an apathetic public and the consequent instability.

To resolve the crisis Solon introduced significant constitutional, political and economic reforms aiming to ameliorate the economic hardship which had hit poorer sections of the population and offer them a partial say in the government of Athens. His reforms granted standing rights to previously disenfranchised Athenians, allowing them to appeal against the decisions of magistrates, and empowered any third uninjured party to take legal action on behalf of an injured person for a crime. The reforms founded a new political dispensation which replaced government by a birth aristocracy with a timocracy, that is, one based on wealth.[4] His economic reforms emancipated poor Athenians indebted to the rich into what was effectively a form of serfdom, and turned them to small landholders. Further, by granting all Athenians the right to participate in the assembly, and establishing the right to appeal against the decisions of the government (archontes) in front of the court, Solon set in motion a sequence that culminated with the establishment of democracy after the reforms of Cleisthenes in 508/7.

Ancient authors did not doubt the existence of the law but, with Solon lamenting stasis and recommending restraint in his poetry, they found it rather baffling. Writing about Solon in the first century CE, Plutarch (Lives, Solon, XX) reiterates the existence of the law against neutrality and offers a normative justification for it:

> "Among his other laws there is a very peculiar and surprising one which ordains
> that he shall be disfranchised who, in time of faction, takes neither side. He
> wishes, probably, that a man should not be insensible or indifferent to the
> common weal, arranging his private affairs securely and glorying in the fact that

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[1] Of those only the law on unpremeditated homicide survives (henceforth, the term 'draconian').

[2] Appointment of a 'wise man' as a lawgiver, or arbitrator, during periods of civil strife with the mandate to institute political reforms was a practice common in the archaic Greek city-states of the seventh and sixth century, see Wallace (2007) for details.

[3] A tyrant was a ruler who had taken control of the government through extra-constitutional means, like a coup, rather than an oppressor.

[4] For extensive descriptions and critical discussions of the reforms of Solon see amongst others Hansen (1999, 29-32 and 43-46), Wallace (2007), Lyttkens (2013, 54-57 and 72-79), Ober (2015, 148-152) and Patriquin (2015, 11-16).

he has no share in the distempers and distresses of his country, but should rather espouse promptly the better and more righteous cause, share its perils and give it his aid, instead of waiting in safety to see which cause prevails".

Starting from Grote in the mid-nineteenth century, several scholars considered that the law aimed to defend Solon's constitution and deter potential tyrants from overthrowing it. However, modern historians have debated the existence of such a law and mulled over its exact meaning (see Goušchin 2016 for a recent survey of the literature). Hignett (1967, 26-17) rejected the existence of the law on neutrality on a variety of grounds, arguing that it was a priori improbable, inconsistent with Solon's views in favour of moderation, and it was not explicitly referred to by the orator Lysias in the "Against Philon" (speech 31), a forensic speech seeking to convict Philon for not taking sides during the 403 rise of the democrats that brought down the rule of the Thirty Tyrants (see Manville, 1980, for additional references). This view, however, was discarded by Goldstein (1972) who examined carefully the circumstances of Lysias speech and language of his text and concluded that Lysias did not need to echo Solon, as the law was not applicable. Philon was located away from the place of fighting against the Thirty Tyrants, while a recent amnesty had made difficult to accuse men for violating laws dating back before the end of the civil war. Similarly, according to Bers (1975) referring to the non neutrality law in the Philon speech might have raised fears for prosecutions of other Athenians who were inactive during the civil war against the Thirty Tyrants leading to cycles of retaliation and instability. For Bers, the law was authentic; Solon's motivation was to force his supporters to actively back his constitutional dispensation.

In the wording of Solon's law that an Athenian who shies away from taking sides is declared atimos which is translated as an 'outlaw'. The constitutional order launched by Solon granted rights to the non-elite masses elevating their public status. For Manville (1980), the law against neutrality fitted the historical setting of Solon's reforms, since "with new public rights came new public obligations. Those who failed to support the revolution would lose their share in it and consequently, for the first time, atimia entailed loss of a share in the polis. The atimos was now a different kind of outlaw: one with no claim to the citizenship which implied rights in the assembly and protection of suit and appeal" (pp.218-219).[5] For Wallace (2007) the law against neutrality is consistent with the rationale of the reforms initiated by Solon

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[5] Manville (1980) examines the co-evolution of the notions of citizenship and *atimia* and corresponding penalties from the archaic time of Draco to fifth century Classical Athens. Hansen (1979) has shown that during the Classical Period (510-322) the punishment for an *atimos* was the loss of various privileges enjoyed by citizens; they included withdrawal of the right to introduce decrees, participate in the assembly, serve as a juror, bring public or private suits, give evidence in court, becoming a magistrate, and enter sanctuaries or the agora.

where "every citizen, rich or poor, was expected to involve himself in public affairs ... not least in times of civic strife ... or be expelled from the civic body" (p.61).

Van 't Wout (2010) focuses on the language of the law as given in the Athenian Constitution and argues that Solon's "law on stasis does not penalize neutrality at all, but on the contrary advocates it ... the law should be read as one that requires citizens to play an active role in the resolution of a conflict" (p.290).

Teegarden (2014) discusses several arguments against the authenticity of the law, including that it was contrary to Solon's revulsion of stasis and the problem of how to enforce it. He rejects these arguments reasoning that the law would deter a coup attempt and thence prevent stasis; Solon wanted all Athenians to be actively involved in the life of their polis and difficulties of enforcement do not deny its authenticity. Nevertheless, Teegarden rejects the authenticity of the law on different grounds. If one believes that the purpose of the law was that whatever type of regime ruled Athens it must enjoy majority support, the law would have allowed the overthrow of the very constitutional order introduced by Solon.[6] However, he argues that Solon could not have consented to the unravelling of his constitution; henceforth, Solon would not have passed a law against neutrality.

Noting that there already was a law against tyranny in Athens, and in particular a law against tyranny enacted by Solon, one wonders why the law against neutrality was necessary. Goušchin (2016) answers this question by arguing that the law against neutrality was genuine and that it aimed to "close the path to tyranny by awakening the citizens' activism and consequently preventing stasis" (p.107).

An additional indication favouring Solon's authorship of the law against neutrality is the aforementioned part of the Solonian legislation granting "permission for anyone who wished to seek retribution for those who were wronged" (Athenian Constitution, 9.1). In common with the law against neutrality, we see that Solon again provided for a third player to intervene in a contest between two parties, the injurer, typically the mighty ruler, and the injured.

Forsdyke (2005) observes that in addition to the elite versus non-elite conflict present in Athens before Solon, violent intra-elite competition for control of Athens was a fundamental problem. Intra-elite conflicts were characterised by violent clashes ending with the exile of the defeated side, only for the latter to regroup and return to the polis seeking revenge and

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[6] He offers three arguments why Solon would not have sanctioned, let alone facilitated the overthrow of his constitution: (a) he fought against Peisistratus despite the fact that he enjoyed popular support; (b) he promulgated a law against overthrowing his constitution; and (c) he had the Athenians swear an oath to uphold his constitution.

exile the former victors in a repeated cycle of instability. Fordsyke maintains that an essential part of Solon's reform programme was to resolve this intra-elite conflict and argues that it was for this reason that Solon initiated the on civil strife.[7] "By requiring non-elites to take sides in a situation of political conflict between elites, Solon hoped to prevent the violent and rapid transfer of power from one elite faction to another. In particular, elites would be deterred from seeking power violently, since they would now have to face the opposition of the entire citizen body and not just a narrow group of rival elites" (Fordsyke, 2005, p.99).

The law ultimately failed to protect the Solonian constitution. The Athenians split into three competing sides, the Men the Plain (around Athens), the men of the Coast, and the Men beyond the Mountains. Peisistratus, the leader of the latter, after two unsuccessful attempts in 561 and 555, eventually established himself as tyrant in 546 remaining in power until his death in 527. He was succeeded by his sons Hippias and Hipparchus until the overthrow of tyranny in 510.

In the present paper we follow the most commonly accepted interpretation that Solon's law was a law against neutrality. In closing this review, two observations are worth making. First, enforcing the law against neutrality may sound problematic (it bears noting that problems of enforcement do not necessarily negate the existence of the law). However, since the law did not mandate to defend a particular regime, enforcement in the sense of punishment had to be retrospective. That is, whoever emerged victorious from the stasis would administer the relevant sanction to those who had fallen foul of the law. Second, from a modern standpoint, an important consequemce of the law, overlooked by historians, is that it forces citizens to participate actively in political disputes. This way, citizens are more likely to take an active interest in what exactly is involved in the dispute and become more knowledgable; therefore it punishes rational ignorance.

3   Model

3.1   Structure of the Model

We consider a city-state (polis) consisting of three groups: the Poor (P), the Middle Class (M), and the Rich (R). The population shares of Poor, Middle Class and Rich are respectively ρ, μ, and $1 - \mu - \rho$, with $0 < \rho < 1/2 \leq \mu < 1 - \rho$. The members of each group are identical. Utility depends on consumption (γ). The Rich finance their consumption from their income

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from wealth denoted by ω, while the Middle Class and the Poor finance it from income from work denoted by ε_i for $i = P, M$ or transfers x_i. In what follows, the terms 'wealth' and 'tax on wealth' refer to the treatment of the Rich, and 'income' and 'tax on income' refer to the treatment of the Middle Classes and the Poor. Two stylised constitutional orders are compared, the Solonian regime and the tyranny. Under the Solonian order denoted by S, the Middle Class decides the fiscal variables (*tax rate on wealth*, *tax rate on income* and *transfer to the poor*),[8] whereas under a tyranny denoted by T they are decided by the Rich. Tyranny in the ancient Greek context meant a regime where a single man had taken control of the government arbitrarily in violation of the legal order; it should not be confused with the modern connotation of tyranny as an oppressive regime.

State revenue originates from taxation on the wealth of the rich, the incomes of the Middle Class and the Poor and exogenous earnings from state assets (like silver mines) G. We use the term "tax" revenue as a catch-all variable for the various financial obligations of the rich to pay taxes on their properties and contribute to public services (what in the classical times became known as liturgies). During Peisistratus' tyranny (546-510) a 1/10th tithe (*dekate*) tax on agricultural production was imposed with the proceeds going to the worship of goddess Athena. The latter tax was abolished by Cleisthenes with his democratic reforms of 508-507 (Harris, 2002), and only the rich were liable to a property tax introduced in 428/7 during the Peloponnesian war against Sparta.[9]

Public revenue finances the provision of a public good x_G, needed to ensure the smooth functioning of the state, whose level is fixed and exogenous, and an untaxed cash transfer x_T, which for simplicity but without loss of generality, is assumed to be paid only to the poor and, if at all, only when the Rich rule as in tyrannies. For notational clarity, we denote the taxes on income (of the Middle Class and the Poor) and on wealth (of the Rich) by t and k in the Solonian order and by τ and κ in a tyranny. All actors are assumed to vote for the policy outcome which maximises their expected payoffs.

In the Solonian order, a rich actor has an after tax payoff of

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[8] This is a simplification for analytical convenience. Solon did not introduce a democratic regime in Athens; his constitutional dispensation linked political participation of an individual to the level of his wealth, as measured by the size of agricultural output, a system termed *timocracy*. Nevertheless, his reforms broadened political participation (which was the exclusive domain of the aristocracy) to lower social groups by fully empowering those with middle size wealth lacking noble descent giving, and some standing rights to the poorer citizens below the middle. At the same time, the preserved laws also dealt with the threat that potential tyrants may seize power.

[9] Information on Athenian taxes during the Archaic Period, 700-500, is scarce. A lot more is known about the evolution of taxes and public revenues during the Classical Period; amongst others the reader may consult Andreades (1933), Kyriazis (2009), Lyttkens (2010) and van Wees (2013) and Tridimas (2014).

$$\pi_R = (1 - k)\omega \tag{1a}$$

In a tyranny his after tax payoff is

$$\pi_R = (1 - \kappa)\omega \tag{1b}$$

In a Solonian order, the payoffs of the Middle Class and the Poor are

$$\pi_j = \gamma_j = (1 - t)\varepsilon_j \quad \text{with } j = P, M. \tag{2a}$$

Finally their payoffs in a tyranny are

$$\pi_j = \gamma_j = (1 - \tau)\varepsilon_j + s_O \text{ with } j = P, M, s_O = 0 \text{ for j} = M \text{ and } s_O \geq 0 \text{ for } j = P. \tag{2b}$$

In this setting there is a potential conflict between the groups about the distribution of burdens and benefits from public spending. The structure of the conflict is illustrated in Figure 1.

Figure 1 here

Starting from the leftmost box, assume that, as in the Solonian order, the Middle Class decides the tax rate. If the Rich consider the taxes imposed as disproportionately burdensome, they may be better off by mounting a "coup" to overthrow the Solonian constitution and install a tyranny. If successful, they may abolish all taxes on their wealth and tax instead the Middle and Poor classes. Whether a tyranny is established depends on the reaction of the Middle Class. If they do not oppose the Rich, the society peacefully transforms into tyranny. If, instead, the Middle Class resist the Rich, a civil war breaks out. The civil conflict stays at a low level if the Poor do not back any group, but it escalates into a large scale civil war if they back either group.

The outcome of the conflict is modeled by a contest success function, where the probability that the constitutional order i prevails

$$\text{Probability} = \frac{e_i}{e_i + e_j} \quad \text{with} \quad i \neq j, \tag{3}$$

The probability depends on the conflict efforts e_i invested by alliance i (with $i = S, T$). We make the following assumptions for the effort of each class. The efforts of the Middle Class and the Poor are assumed to be equal to their numerical strength of each class, that is, $e_P = \rho$ and $e_M = \mu$, since the Middle Class and the Poor take up their own arms. On the other hand, the Rich are assumed to hire mercenaries and write $e_R = h$. Further,

$$e_i = \sum_k e_k \qquad \text{with} \quad i = S, T \quad \text{and} \quad k = P, M, R, \tag{4}$$

That is, if the Poor backs the Middle Class, the efforts spent to preserve the Solonian order are $e_s = e_M + e_P$, and efforts spent to establish tyranny are $e_O = e_R$ and so on. Given those considerations, we assume that the Middle Class' probability of winning the conflict and establishing a democracy is p if they are supported by the Poor, q if the Poor ally with the Rich, and n if the Poor do not participate in the conflict. Due to exclusion of a draw, $1 - z$ (with $z = p, q, n$) is the Rich's respective probability of repelling the Middle Class and establishing an oligarchy.

Participating in the conflict is costly for the members of all groups: The Poor and the Middle Class have to bear personal costs of $c_i > 0$ ($i = P, M$). The Rich has to bear the costs of the hired mercenaries $c_R = \sigma \cdot h$, with $\sigma > 0$ as the costs of one unit of mercenaries.

In addition, we assume that the winning coalition punishes its former opponent. According to archaic Greek practice, when the Rich lose, they are expelled and their wealth is confiscated. On the other hand, when the Rich win, they are only able to exclude their former opponents from the group-specific transfer x_T. Table 1 summarizes the notation used.

Table 1: Notation			
Variable	Description	Variable	Description
P, M, R	Players (Poor, Middle Class, Rich)	$\varepsilon_i = \gamma_i$	Income of $i = P, M$
μ, ρ	Share of M and P on society	S, T	Constitutional order, Solonian or Tyranny
ω	Wealth of a member of the Rich group	Ω	Rich group's total wealth, $=(1-\rho-\mu)\omega$
k, κ, t, τ	Taxes payed on wealth ω and on income ε_i	π_j^i	Payoff of player j with alliance I or in state i
x_G	Public good	e_i	Effort for conflict invested by alliance i = D, O
p, q, n	Middle Class' probability of winning	c_i	Costs of conflict of i = P, M, R
x_T	Transfer paid to the poor by the Tyranny	I	Total income; $= \mu \, \epsilon_M + \rho \, \epsilon_P$
h	Size of foreign mercenaries	σ	Costs of one unit of foreign mercenaries
G	Exogenous public revenue	$\overline{G} = x_G - G$	Difference between exogeneous public costs and revenues;
R_i^l	Reaction function of player i for alliance l	S, T, N	Alliance, i.e., who is supported by the Poor: Middle Class, Rich, none

We solve this game by using backwards induction and present the relevant results in the following. However, the complete solution can be found in the Appendix.

3.2 The Poor

The Poor only have to decide if there is a civil war. Without any law against neutrality in a civil conflict, the Poor must decide first whether to side with another group or not and if they are backing someone, who. Given the possible alliances $(l = S, T, N)$ and regimes types $(i = S, T)$, and that the Rich only pay x_T if the Poor support them in society and their related benefits and sanctions, the Poor have expected payoffs of

$$\pi_P^S = p(1 - t_S)\varepsilon_P + (1 - p)(1 - \tau_S)\varepsilon_P - c_P \tag{5a}$$

$$\pi_P^T = q(1 - t_T)\varepsilon_P + (1 - q)[(1 - \tau_T)\varepsilon_P + x_T] - c_P \tag{5b}$$

$$\pi_P^N = n(1 - t_N)\varepsilon_P + (1 - n)(1 - \tau_N)\varepsilon_P \tag{5c}$$

It then follows that the Poor do not participate in a civil war between the Rich and the Middle Class if $\pi_P^N \geq \pi_P^S$ and $\pi_P^N \geq \pi_P^T$ hold true, yielding for the upper (ε_P^U) and lower (ε_P^L) boundaries of the Poor's income

$$\varepsilon_P \geq \varepsilon_P^L = \frac{(1-q)\ x_T - c_P}{\tau_T - \tau_N + n(\tau_N - t_N) + q(\tau_T - t_T)} \tag{6}$$

and

$$\varepsilon_P \leq \varepsilon_P^U = \frac{c_P}{n\ t_N - p\ t_S - (1-p)\tau_S + (1-\tau_N)n} \tag{7}$$

Accordingly, the Poor support the Middle Class for $\varepsilon_P > \varepsilon_P^U$, and the Rich for $\varepsilon_P < \varepsilon_P^L$. In contrast, without the possibility of remaining neutral in a conflict, the situation changes. The Poor still decide on their support in consideration of its expected costs and benefits but, now, the Poor support the Middle Class if $\pi_P^S \geq \pi_P^T$ holds true and *vice versa*, yielding

$$\varepsilon_P \geq \varepsilon_p^T = \frac{(1-q)\ x_T}{\tau_T - \tau_S + p(\tau_S - t_S) + q(\tau_T - t_T)} \tag{8}$$

as the threshold for the Poor's support. Hence, the Poor are backing the Middle Class for $\varepsilon_P \geq \varepsilon_p^T$ and the Rich for $\varepsilon_P < \varepsilon_p^T$. It is interesting to note that in this circumstance, the decision of the Poor regarding whom to support is not affected by the costs of conflict any longer.

3.3 The Middle Class

When the Rich mount a coup, the Middle Class receives respectively the following payoff depending on whether the Poor support the Middle Class, the Rich or remain neutral

$$\pi_M^S = p(1 - t_S)\varepsilon_M + (1 - p)(1 - \tau_S)\varepsilon_M - c_M \qquad \text{if } l = S \tag{9a}$$

$$\pi_M^T = q(1 - t_T)\varepsilon_M + (1 - q)(1 - \tau_T)\varepsilon_M - c_M \qquad \text{if } l = T \tag{9b}$$

$$\pi_M^N = n(1 - t_N)\varepsilon_M + (1 - n)(1 - \tau_N)\varepsilon_M - c_M \qquad \text{if } l = N \tag{9c}$$

If the Middle Class does not oppose the coup, but accepts the tyranny regime the payoff is

$$\hat{\pi}_M^l = (1 - \tau_l)\varepsilon_M \qquad \text{for} \quad l = S, T, N \tag{10}$$

Here the $\hat{}$ notation over a variable denotes tyranny. Comparing (9) and (10) we have $\pi_M^l > \hat{\pi}_M^l$, $l = S, T, N$, when the difference between the tax rates on the income of the Middle Class and the Poor under Solonian order and a tyranny exceeds the expression

$$\tau_l - t_l > \frac{c_M}{z\,\varepsilon_M} \quad \text{with } l = S, T, N \text{ and } z = p, q, n \tag{11}$$

It follows that by changing the probability of victory support by the Poor would motivate the Middle Class to oppose even small differences between the tax rate decided by the Rich and the Middle Class.

3.4   The Rich

The expected payoff of the Rich depending on whether the Poor support the Middle Class, the Rich or remain neutral respectively are, where h denotes the mercenaries recruited and $\Omega = (1 - \rho - \mu)\omega$ the Rich group's total wealth

$$\hat{\pi}_R^S = \frac{h}{h + \mu + \rho}(1 - \kappa_S)\,\Omega - \sigma h \tag{12a}$$

$$\hat{\pi}_R^T = \frac{h + \rho}{h + \mu + \rho}(1 - \kappa_T)\,\Omega - \sigma h \tag{12b}$$

$$\hat{\pi}_R^N = \frac{h}{h + \mu}(1 - \kappa_N)\,\Omega - \sigma h \tag{12c}$$

Maximisation of (12) with respect to h yields the first order conditions

$$h_S^* = \sqrt{\frac{(\mu + \rho)(1 - \kappa_S)\Omega}{\sigma}} - \mu - \rho, \qquad \text{if the Poor support the Middle Class} \tag{13a}$$

$$h_T^* = \sqrt{\frac{\mu(1 - \kappa_T)\Omega}{\sigma}} - \mu - \rho\,, \qquad \text{if the Poor support the Rich} \tag{13b}$$

$$h_N^* = \sqrt{\frac{\mu(1 - \kappa_N)\Omega}{\sigma}} - \mu, \qquad \text{if the Poor remain neutral} \tag{13c}$$

It is clear that if the Poor support the Rich the latter cut, *ceteris paribus*, their investments in mercenaries. At the same time, the Rich would only start a coup if their wealth (Ω) is large enough to justify the investments in mercenaries (i. e., $h > 0$). This consideration yields

$$\Omega > \overline{\Omega}_S \equiv \frac{\sigma(\mu+\rho)}{1-\kappa_S} \qquad\qquad \text{if the Poor support the Middle Class} \qquad (14a)$$

$$\Omega > \overline{\Omega}_T \equiv \frac{\sigma(\mu+\rho)^2}{(1-\kappa_T)\,\mu} \qquad\qquad \text{if the Poor support the Rich} \qquad (14b)$$

$$\Omega > \overline{\Omega}_N \equiv \frac{\sigma\mu}{1-\kappa_N} \qquad\qquad \text{if the Poor support the Rich} \qquad (14c)$$

Based on these considerations, the Rich would start a coup for

$$\pi_R^l = (1 - k_l)\,\Omega < [1 - z(h_l^*)]\,[(1 - \kappa_l)\,\Omega - \sigma\,h_l^*] = \hat{\pi}_R^l(h_l^*) \qquad (15)$$
$$\text{with} \quad z = p, q, n \quad \text{and} \quad l = S, T, N$$

$$k_l > K_l = \begin{cases} \kappa_l - \dfrac{\sigma(\mu+\rho)}{\Omega} + 2\sqrt{\dfrac{(1-\kappa_l)\sigma e_S}{\Omega}}, & \Omega > \overline{\Omega}_l \\[2ex] \dfrac{\mu+\rho\,\kappa_l}{\mu+\rho}, & \Omega \leq \overline{\Omega}_l \end{cases} \qquad (16)$$
$$\text{with } l = S, T, N \text{ and for } \rho = 0 \ \forall \ l = N \text{ and for } \kappa_l = 1 \ \forall \ l = S, T$$

Here, it is worth noting that prohibitive costs of conflict $\left(\Omega \leq \overline{\Omega}_T\right)$ do not impede coups when the Poor support the Rich because, then, $K_l < 1$ holds true for $h_l = 0$. In those situations, the Rich completely substitute mercenaries by the Poor. Without the latter's support, the Rich, however, would never start a coup for prohibitive costs of conflict as, then, $K_l = 1$ holds true. Accordingly, baning neutrality can facilitate coups if the potential putchists are willing to give the poor citizens a larger share in societal rents. At the same time, this also implies that such a law may prevent the Middle Class' to exceedingly burden rich citizens by making it cheaper for the latter to resist excessive taxation.

3.5   Societal Equilibrium & Decision on Taxes

Denoting public spending by Y_i, the revenue by D_i and the income of the Middle Class and the Poor by $I = \rho\varepsilon_P + \mu\varepsilon_M$ the government budget under political regime $i = S, T$ can be written as

$$Y_S = t_l\,I + k_l\,\Omega + G = x_G = D_S \qquad\qquad \text{under Solonian order} \qquad (17a)$$

$$Y_T = \tau_l\,I + \kappa_l\,\Omega + G = \rho\,x_O + x_G = D_T \qquad \text{in a tyranny.} \qquad (17b)$$

As already mentioned, under the Solonian regime it is the members of the Middle Class who decide policy by maximizing their payoffs, while under tyranny maximize it is the Rich who decide. Because neither faction receives the x_T transfer, their payoffs always monotonically

increase as the tax rates decrease and, thus, both factions search for the lowest possible tax rates. However, they also have to take into account the reactions of their opponents. In other words, because low own tax rates are tantamount to higher tax rates for the opponent, lowering one's own taxes could increase one's opponent's incentive to start a coup or countercoup. Put differently, both the Middle Class and the Rich actors seek the lowest possible tax rate, which simultaneously satisfies the budget constraint and stops the other side from overthrowing the system.

This necessitate small changes in the game's structure. The stylized game tree of the model definitively ends after any coup attempt, if the Rich mount a coup, or if the Rich decide not to challenge the status quo. If the game is solved according to this structure the Rich would anticipate that their preferred tyranny could never be challenged and, thus, would not have to worry about the reaction of the Middle Class to their chosen tax rates. Accordingly, the tax rates decided by the Rich in a tyranny would only be constrained by the budget constraint. Consequently, the model's result would exaggerate the attractiveness of coups because the Rich would be able to excessively burden the Middle Class. To avoid such bias, we model the Middle Class' and the Rich's decisions a bit differently. We assume that, even after a successful coup, the Rich are unable to eliminate the resistance of the Middle Class; hence, they must consider the possible reaction by the Middle Class. Now, the choices of tax rates by the Rich are not only affected by the budget constraint but also by the Middle Class reaction. That is, the Rich anticipate that the higher they tax the Middle Class after a successfull coup, the more likely that the Middle Class would in turn mount a countercoup after the Rich established a tyranny. More formally, we assume that the game does not end after a successful coup but that then the Middle Class have the opportunity to challenge the tyranny.

Given those considerations, an equilibrium consists of the combination of two equilibrium strategies t_l for the Middle Class and κ_l for the Rich under Solonian order and tyranny. Note that because of the tax revenue constraint described by equations (17a) and (17b), determining the income tax rate under the Solonian regime t_l and the wealth tax rate under tyranny κ_l uniquely identify k_l and τ_l respectively. Assuming for the time being that x_T is exogenous (an assumption relaxed in the next section); a strategy t_l^* is an equilibrium strategy of the Middle Class in case of alliance $l = S, T, N$ if

$$\pi_M^l(t_l^*, \kappa_l^*) \geq \pi_M^l(t_l, \kappa_l^*) \qquad \forall \; t_l, \kappa_l \in [0; 1] \tag{18}$$

$$\text{and if} \quad t_l^* = \frac{-k_l \, \Omega + \overline{G}}{I}$$

$$\text{and if} \quad \pi_R^l(k_l, t_l^*) \geq \hat{\pi}_R^l(\kappa_l^*, \tau_l)$$

hold true, whereby $\overline{G} = x_G - G$ is the difference between the exogeneous public revenues and costs. Analogously, κ_l^* is an equilibrium strategy of the Rich in case of alliance $l = S, T, N$ if

$$\pi_R^l(\kappa_l^*, t_l^*) \geq \pi_R^l(\kappa_l, t_l^*) \qquad \forall \ t_l, \kappa_l \in [0; 1] \tag{19}$$

$$\text{and if} \qquad \kappa_l^* = \frac{-\tau_l \, I + \rho \ s_O + \overline{G}}{\Omega}$$

$$\text{and if} \qquad \hat{\pi}_M^l(\tau_l, \kappa_l^*) \geq \pi_M^l(t_l^*, k_l)$$

hold true, whereby $\pi_M^l(t_l^*)$ is the Middle Class' expected payoff of a countercoup in the (fictive) case of a tyranny. A Solonian order such that the Middle Class does not oppose a coup by the Rich may then be characterised as always unstable. Based on this definition of a stable equilibrium, we are now able to deduce the condition for a stable Solonian order and an unstable tyranny. Both determine the area of possible taxes, which allow equilibria.

Lemma 1 *A Solonian order is always unstable for*

$$t_l^* > 1 - \frac{c_M \sqrt{\sigma \ e_D (\Omega + I - \overline{G} - \rho \ x_T)}}{\sigma \ e_S \ \varepsilon_M} \tag{20}$$

and

$$\kappa_l^* > \frac{\overline{G} + \rho \ x_T}{\Omega} - \frac{c_M \ I \left[c_M \ I + \sqrt{c_M^2 \ I^2 - 4 e_S \ \varepsilon_M^2 (\overline{G} + \rho \ x_T - \Omega)} \right]}{2 e_S \ \varepsilon_M^2 \ \sigma \ \Omega} \tag{21}$$

Proof. See Appendix A.1.

Lemma 1 reveals the intuitevely apealing result that the Solonian order is stable for as long as the tax rate for income under the Solonian order is between the upper and the lower limits implied by (21) and (22), that is, neither too high nor too low.

At the same time, tyrannies too may be unstable as Lemma 2 below shows.

Lemma 2 *The Rich will never attempt a coup for*

$$\kappa_l^* < \frac{\overline{G} + \rho \ x_T - I}{\Omega} \tag{22}$$

and for

$$t_l^* > \frac{\sigma(\mu + \rho) + \overline{G} - 2\sqrt{\sigma \ e_S \ \Omega}}{I} \qquad \text{with } \rho = 0 \ \forall \ l = N \tag{23}$$

Proof. See Appendix A.2.

Given those considerations regarding equilibria and stability, we are now able to derive the best-response functions of the actors.

Proposition 1 *Based on the conditions for equilibrium strategies in Equations (18) and (19) and the conditions for stability in Lemmas 1 and 2, the best-response function of the Middle Class is*

$$R_M^l = t_l^*(\kappa_l) = \begin{cases} \frac{\overline{G}+\sigma(\mu+\rho)-\kappa_l\,\Omega-2\sqrt{(1-\kappa_l)\Omega\,\sigma\,e_S}}{I}, & \text{for } \Omega > \overline{\Omega}_l \\ \frac{\overline{G}}{I} - \frac{\Omega_R(\mu+\kappa_l\,\rho)}{(\mu+\rho)I}, & \text{for } \Omega \le \overline{\Omega}_l \end{cases} \tag{24}$$

$$\text{with } \rho = 0 \ \forall \ l = N \quad \text{and} \quad \kappa_l = 1 \ \forall \ \left(l = N, M \mid \Omega_R \le \overline{\Omega}_R^l\right)$$

and the inverse best-response function of the Rich is

$$R_R^l = t_l(\kappa_l^*) = \begin{cases} \frac{\rho\,x_T+\overline{G}-\kappa_l^*\,\Omega}{I} - \frac{c_M\sqrt{(1-\kappa_l^*)\Omega\,\sigma\,e_S}}{\sigma\,\varepsilon_B\,e_S}, & \text{for } \Omega > \overline{\Omega}_l \\ \frac{\rho\,x_T+\overline{G}-\kappa_l^*\,\Omega}{I}, & \text{for } \Omega \le \overline{\Omega}_l \end{cases} \tag{25}$$

Proof. See Appendix A.3.

According to Proposition 1, the tax rate on income under Solonian order and the tax rate on wealth in tyrannies are strategic substitutes, i.e., increases in the opponent's parameter results in a lower optimal own parameter and *vice versa* (see also Figure 2). At the same time, there is a corner solution for $\Omega \le \overline{\Omega}_l$ yielding $h_l = 0$ and, thus, $k_l = 1$ for $l = S, N$ and $k_l < 1$ for $l = T$. Accordingly, t_l is determined only by the budget constraint in the former case and by the manpower of the Poor in the latter. As illustrated by the steeper fall of the best-response function of the Rich in Figure 2, those corner solutions reduce the bargaining power of the Rich. The actors' best response functions allow us to formulate the game's equilibria. In this regard, we have to take into account that there are several possible combinations of the parameters, which would not yield any intersection of the actors' best-response functions. Given the conditions for stability, those situations result, nonetheless, in stable equilibria with $k_l = 0$ or $k_l = 1$, depending on the relation of the functions.

Figure 2 here

Proposition 2 *Based on the conditions for equilibrium strategies in Equations (18) and (19), the conditions for stability in Lemmas 1 and 2 and the Best-Response-Functions in Proposition 1, the Rich choose the tax rate*

$$
\kappa_l^* = \begin{cases} 0, & R_M^l > R_R^l \ \ \forall \ \kappa_l \in [0;1] \\[2mm] 1 - \dfrac{\varepsilon_M^2 \, e_S \, \sigma [\rho \, x_T - (\mu+\overline{\rho})\sigma]^2}{(c_M \, I - 2\varepsilon_M \, e_S \, \sigma)^2 \Omega}, & \text{for } \Omega > \overline{\Omega}_l \\[3mm] 1 - \dfrac{c_M \, I (\mu+\overline{\rho})^2}{\varepsilon_M \, \Omega \, e_S^2} + \dfrac{\rho \, x_T (\mu+\overline{\rho})}{\Omega \, e_S}, & \text{for } \Omega \leq \overline{\Omega}_l \\[3mm] 1, & R_M^l < R_R^l \ \ \forall \ \kappa_l \in [0;1] \end{cases}
\tag{26}
$$

$$
\text{with } \overline{\rho} = \rho \ \ \forall \ l = S, T \quad \text{and} \quad \overline{\rho} = 0 \ \ \forall \ l = N
$$

and the Middle Class choose the tax rate

$$
t_l^* = \begin{cases} \dfrac{\overline{G} - \Omega + (\mu+\overline{\rho})\sigma}{I} + \dfrac{\varepsilon_M \, e_S \, \sigma}{I} \left(\dfrac{\varepsilon_M [\rho \, x_T - (\mu+\overline{\rho})\sigma]^2}{(c_M \, I - 2\varepsilon_M \, e_S \, \sigma)^2} - 2 \left| \dfrac{(\mu+\overline{\rho})\sigma - \rho \, x_T}{c_M \, I - 2\varepsilon_M \, e_S \, \sigma} \right| \right), & \text{for } \Omega > \overline{\Omega}_l \\[4mm] \dfrac{\overline{G} - \Omega}{I}, & \text{for } \Omega \leq \overline{\Omega}_l \mid l = N, S \\[3mm] \dfrac{\overline{G}}{I} + \dfrac{c_M \, I \, \rho(\mu+\rho) - \mu \, \varepsilon_M (\mu \, \Omega + \rho^2 \, x_T)}{I \, \varepsilon_M \, \sigma}, & \text{for } \Omega \leq \overline{\Omega}_l \mid l = T \end{cases}
\tag{27}
$$

Proof. See Appendix A.4.

Using Proposition 2, we are now able to analyze the behaviour of the Rich regarding their decision on x_T and its anticipation by the Middle Class. As argued before, an equilibrium $[t_l^*, \kappa_l^*(x_T)]$ can only be stable if the Rich have no incentive to revolt against the Solonian *status quo* in order to improve their welfare. Formally the Rich choose

$$
x_T^* = x_T \in \text{argmax} \ \pi_R^S(t_l^*, \kappa_l^*)
\tag{28}
$$

At the same time from (26) we have,

$$
\frac{\partial \kappa_l^*}{\partial x_T} > 0, \quad \forall \ x_T < \frac{\sigma(\mu+\overline{\rho})}{\rho} \quad \text{with } \overline{\rho} = 0 \ \forall \ l = N \text{ and } \overline{\rho} = \rho \ \forall \ l = S, T
\tag{29}
$$

This implies rising taxes on wealth in a tyranny and, as a corollary, rising (declining) taxes on wealth (income) under a Solonian order for an increasing x_T.[10] In other words, the Rich are forced to pay higher taxes themselves in a tyranny if they seek to expand the supply of x_T. This also implies that, generally, the Rich are not able to fully pass the costs for the increase to the Middle Class. At the same time, accepting higher taxes for themselves impairs the power of the Rich compared to that of the Middle Class, resulting in lower taxes on income under Solonian order. Based on those considerations, the Rich would only be willing to increase the transfer to the Poor if the latter would support the former in a potential conflict with the Middle Class, and if the Middle Class charge higher taxes under Solonian order.

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[10] See the Appendix for the detailed analysis.

4   Discussion

We now turn to discuss the implications of the results from our model for social stability in archaic Athens. We focus on two issues (i) the impact of the law against neutrality on the Middle Class and the decision of the Rich to levy taxes and (ii) its impact on the stability of the Solonian order.

4.1   Decision on Taxes and the Supply of Public Goods

As already mentioned before, the decision of the Rich on the tax rate and on the transfer to the Poor depends on the Poor's choice of alliance, which in turn depends on the fiscal instruments chosen by the Rich. In view of the complicated nature of the results, we show what is involved by means of graphical analysis.

Figure 3 illustrates the payoffs $\pi_i^l(t_l^*, \kappa_l^*)$ of the actors $i = P, M, R$ for x_T depending on the societal alliance $l = S, T, N$. The payoffs of the Rich are decreasing in x_T, since an increase in the latter results in an increase in the tax rate of wealth. On the other hand, the payoffs of the Middle Class and the Poor are increasing in x_T because the Rich are more willing to pay taxes themselves and, thus, the tax rate on income in tyrannies as well as under Solonian order decreases.

Figure 3 here

Regarding the Rich, it is interesting to see how they benefit from the support by the Poor. Even if the Rich do not invest in mercenaries, the probability of winning against the Middle Class is still positive due to the Poor's manpower. Without the latter's support (i.e., if the Poor are neutral or support the Middle Class), the Rich have lost all their bargaining chips and must accept a tax rate on wealth of 1. In other words, due to the larger benefits from the Poor's support, the Rich are more willing to pay in order to secure that support.

Figure 4 illustrates this point. The additional support by the Poor enhances the conflict capabilities of the Rich, which results in lower taxes for similar levels of x_T. In addition, the Rich also forgo the deployment of mercenaries and rely on the Poor to fight against the Middle Class.

Figure 4 here

From a societal point of view, these incentives have some benefits. Instead of bringing foreign mercenaries to the polis and deploy them there, the conflict remains between the citizens of

the polis; thus, balancing the interests should be easier. Hence, peace agreements should be more stable without foreign mercenaries because all parties to the conflict have an interest in ending the conflict. In addition and irrespective of who won, after a the end of a civil conflict it may be hard to convince the mercenaries to leave, if the latter are an important fighting force, which could also bring the polis under their control.

The consequences of a ban on neutrality are also shown by Figure 3. Due to the ban, the attractive (for the Poor) and cheap (for the Rich) option of "staying calm and carrying on" disappears. Baning neutrality now results in the need for the Poor to take one side and, for the Rich, in higher costs of persuading the Poor not to take the Middle Class' side. As we can also see from comparing the payoffs with and without the ban in Figure 3, the Rich now must pay a higher transfer x_T in order to prevent the Poor form supporting the Middle Class. Before the ban, the Rich would have to transfer Y_S (Figure 3 - middle) to the Poor. With a ban on neutrality, this transfer increases to X (Figure 3 - middle). At the same time, the Rich must pay a smaller transfer to receive the support of the Poor (Point Y_T with the ban, Point X without in Figure 3 - middle). Given those changes, the Rich now benefit from paying the transfer (Point C in Figure 3 – right), unlike the situation without (Point B and Point A in Figure 3 – right). Without the ban on neutrality, the Rich would have a payoff shown by Point B if the Poor support them but a (higher) payoff shown by Point A if the Poor are neutral. An active ban on neutrality changes those conditions. Now, the Rich have a payoff shown by Point C if they are supported by the Poor. Obviously, the payoffs of the Rich are always lower if the Poor support the Middle Class and, consequently, the Rich would pay the transfer.

A ban on neutrality could thus improve the bargaining position of the Rich if it enables them to persuade the Poor to support a coup to establish tyranny. A ban on neutrality turns out to improve the welfare of all groups. On the one hand, it may result in a higher willingness of the Rich to pay taxes under Solonian order, which results in the Middle Class and the Poor paying lower taxes. On the other hand, it also prevents an excessive exploitation of the Rich because, by forcing participation of the Poor, the Rich may be more willing to start a coup if the taxes are too high. It follows that the ban on neutrality limits the power of the more powerful groups (as the case may be).

4.2   Stability of the Solonian order

According to Lemma 1, the Solonian order can become unstable (that is, vulnerable to a coup that establishes a tyranny acceptable by the Middle Class) for too-high taxes on income and so can tyrannies for too-high taxes on wealth. The first condition is self-evident as it implies an underfunding of the public administration under the Solonian *status quo*. Such a

development could cause the breakdown of the Solonian order when it reduces the gap between the tax rates on income under Solonian order and tyranny. If the gap becomes close enough, a tyranny loses its relative disadvantage of too-high taxes and the Middle Class' resistance to tyrannies vanishes. More concisely, due to the inherent upper level of taxation, i.e., $\tau = 1$, and as a consequence of $t_l < \tau_l$, the Middle Class lifts its opposition to tyranny when τ_l reaches 1 but $t_l < 1$ could still rise because, the difference between the tax rates, i.e., the possible gain from a countercoup, does not justify a revolt anymore. Substituting Equation (27) in Equation (20), yields the following the public-revenue threshold for an unstable Solonian order

$$\overline{G} > \overline{G}_{\text{Under}} = I + \Omega - \rho\, x_T - \frac{e_S\, \varepsilon_M^2\, \sigma[\rho\, x_T - \sigma(\mu + \overline{\rho})]^2}{(c_M\, I - 2e_S\, \varepsilon_M\, \sigma)^2} \tag{30}$$

Here, this threshold is affected in two opposite ways by banning neutrality in civil war. First, if the ban results in a higher x_T, the threshold increases, but at the same time, the threshold decreases due to the increase of $\overline{\rho}$. Accordingly, the ban stabilizes the Solonian order if the former effect exceed the latter, i.e. if the increase of x_T is sufficiently large.

The second condition for a stable Solonian order, too-high taxes on wealth in tyrannies, implying too low taxes on income under Solonian order, is instead counterintuitive because it implies a revenue in excess of that required for balancing the budget. Here, we can observe the reverse of the first condition. For too-low public expenditures $\overline{G} = x_G - G$, the tax rate on income under Solonian order could reach the inherent lower level of taxation (i.e., $t = 0$) and, like in the case of insufficient revenue, the difference between the tax rates on income under Solonian order and in tyrannies becomes small enough that the tyranny loses its relative disadvantage. As a result, the Middle Class would not oppose a coup to establish a tyranny. Substituting Equation (27) in Equation (21), yields the public-revenues threshold for an unstable Solonian order

$$\overline{G} < \overline{G}_{Over} = \Omega - \rho\, x_T - [\sigma(\mu + \overline{\rho}) - \rho\, x_T]\frac{(c_M\, I - e_S\, \varepsilon_M\, \sigma)^2 + e_S\, \varepsilon_M^2\, \sigma[\sigma(\mu + \overline{\rho}) - \rho\, x_T - e_S\, \sigma]}{(c_M\, I - 2e_S\, \varepsilon_M\, \sigma)^2} \tag{31}$$

Baning neutrality affects the threshold, again, in two ways. If x_T increases due to the ban, the Solonian order is stabilized because

$$\frac{\partial\, \overline{G}_{Over}}{\partial\, x_T} < 0 \qquad \forall\ x_T > 0\ \text{ and }\ \sigma > \frac{c_M\, I}{\varepsilon_M(\mu + \overline{\rho})} \tag{32}$$

holds true for sufficiently large costs of mercenaries and an increasing x_T results in a lower threshold. At the same time, the threshold is also affected by the variations of $\overline{\rho}$ and if

applicable by e_S. Here, the total effect is unequivocal because the effects of all parameters changes work in the same direction and, thus, banning neutrality decreases the threshold and, accordingly, stabilizes the Solonian order. This is even true if the Poor support the Rich.

5 Conclusion

In 594 BCE Solon introduced a constitutional dispensation granting various political rights to the middle income earners and restricting the hold of aristocracy on the Athenian government. To protect the new constitutional order he also banned citizens from staying neutral in case of civil conflict. We have explored the implications of banning neutrality in a highly stylized model of a society of three classes, Rich, Middle and Poor, where the former two compete to determine the fiscal variables and the Poor may stay neutral or join either of the other two.

A ban on neutrality could thus improve the bargaining position of the Rich if it enables them to persuade the Poor to support a coup to establish tyranny. A ban on neutrality turns out to improve the welfare of all groups. On the one hand, it may result in a higher willingness of the Rich to pay taxes under Solonian order, which results in the Middle Class and the Poor paying lower taxes. In addition, the Rich also forgo the deployment of mercenaries and rely on the Poor to fight against the Middle Class. On the other hand, it also prevents an excessive exploitation of the Rich because, by forcing participation of the Poor, the Rich may be willing to start a coup if the taxes are too high. It follows that the ban on neutrality limits the power of the more powerful groups (as the case may be).

Appendix

A.1 Proof of Lemma 1

As defined before, the Solonian order is unstable if the difference between the tax rates on income under Solonian order and a tyranny is small enough so that the relative loss of the Middle Class in a tyranny would not justify a countercoup. According to Equation (11),[11] the Middle Class do oppose a coup if

$$\tau_l - t_l > \frac{c_M}{z \, \varepsilon_M} \quad \text{with } l = S, T, N \text{ and } z = p, q, n \tag{11}$$

holds true. Substituting Equation (13) into (3) yields the equilibrium probability that the Middle Class wins the conflict

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[11] For convenience, formulas, already appearing in this paper's main body, retain their numbering in the Appendix.

$$\text{Probability} = \sqrt{\frac{e_S\,\sigma}{(1-\kappa_l)\Omega}} \quad \text{with } l = S, T, N \tag{33}$$

for optimal behavior h_l^* of the Rich. At the same time, the difference between the tax rates on income under a Solonian order and a tyranny can only become sufficiently small if the Rich are not able to further increase the tax rate on income in a tyranny, i.e., if $\tau_l = 1$. Based on these considerations, the Solonian order becomes unstable for $\tau_l = 1$ and, according to the tyranny budget constraint in Equation (17), for

$$\kappa_l = \frac{\overline{G} + \rho\,x_T - I}{\Omega} \tag{34}$$

Substituting Equations (33), (34), and $\tau_l = 1$ in Equation (11) yields Equation (20), i.e., the first threshold,

$$t_l^* \leq 1 - \frac{c_M \sqrt{\sigma\,e_S\left(\Omega + I - \overline{G} - \rho\,x_T\right)}}{\sigma\,e_S\,\varepsilon_M} \tag{20}$$

of Lemma 1.

The second threshold of Lemma 1 is based on similar considerations. The Solonian order is also unstable if the Middle Class is not able to lower the tax rate on incomes under Solonian order anymore, i.e., if $t_l^* = 0$. Substituting $t_l^* = 0$ and τ_l from the budget constraint in a tyranny, i.e., Equation (17), in Equation (11) yields Equation (21), i.e., the second threshold,

$$\kappa_l^* \leq \frac{\overline{G} + \rho\,x_T}{\Omega} - \frac{c_B\,I\left[c_M\,I + \sqrt{c_M^2\,I^2 - 4e_S\,\varepsilon_M^2\left(\overline{G} + \rho\,x_T - \Omega\right)}\right]}{2e_S\,\varepsilon_M^2\,\sigma\,\Omega} \tag{21}$$

of Lemma 1.

A.2 Proof of Lemma 2

The Rich will not attempt to establish a tyranny if the tax rate on wealth is too low to fund the public budget even for $\tau_l = 1$. Formally, substituting $\tau_l = 1$ in the budget constraint of a tyranny in Equation (17) yields Equation (21), i.e., the first threshold,

$$\kappa_l^* < \frac{\overline{G} + \rho\,x_T - I}{\Omega} \tag{22}$$

of Lemma 2. The second threshold results from similar considerations as for the second threshold of Lemma 1: If the tax rate on wealth under a Solonian order is not high enough to justify a coup in favour of tyranny, even for $\kappa_l = 0$, a tyranny is no threat to the Solonian

status quo. Substituting $\kappa_l = 0$ in Equation (16) yields the minimum tax rate on wealth under the Solonian order

$$k_l > \frac{2\sqrt{e_S \, \Omega \, \sigma} - \sigma(\mu + \overline{\rho})}{\Omega} \quad \text{with } \overline{\rho} = 0 \ \forall \ l = N \ \text{and} \ \overline{\rho} = \rho \ \forall \ l = S, T \tag{35}$$

which is sufficient to justify a coup to establish tyranny. Inserting this under Solonian budget constraint yields the second threshold, i.e., Equation (23),

$$t_l^* > \frac{\sigma(\mu + \overline{\rho}) + \overline{G} - 2\sqrt{\sigma \, e_S \, \Omega}}{I} \tag{23}$$
$$\text{with } \overline{\rho} = 0 \ \forall \ l = N \quad \text{and} \quad \overline{\rho} = \rho \ \forall \ l = S, T$$

of Lemma 2.

A.3 Proof of Proposition 1

A.3.1 General Best-Response-Functions for $h > 0$

As argued before, both, the Middle Class and the Rich, aim to set the lowest tax rates, which do not motivate their respective opponent to start a (counter-)coup and which, at the same time, satisfies the budget constraint. Accordingly, $\pi_R^l = \hat{\pi}_R^l$, defined in Equation (15), yields the maximum tax rate on wealth under Solonian order

$$k_l = \frac{\kappa_l \, \Omega - \sigma(\mu + \overline{\rho}) - 2\sqrt{(1 - \kappa_l)\Omega \, e_S \, \sigma}}{\Omega}, \tag{36}$$
$$\text{with } \overline{\rho} = 0 \ \forall \ l = N \quad \text{and} \quad \overline{\rho} = \rho \ \forall \ l = S, T$$

for which the Rich do not start a coup. By substituting this equation into the Solonian budget constraint, we identify the smallest budget-consistent tax rate on the income of the Middle Class and the Poor under Solonian order

$$t_l^* = \frac{\sigma(\mu + \overline{\rho}) - 2\sqrt{(1 - \kappa_l)\Omega \, e_S \, \sigma} + \overline{G} - \kappa_l \, \Omega}{I}, \tag{24}$$
$$\text{with } \overline{\rho} = 0 \ \forall \ l = N \quad \text{and} \quad \overline{\rho} = \rho \ \forall \ l = S, T$$

The latter depends on the tax rate on wealth κ_l, chosen by the Rich. This is the best response function of the Middle Class.

For the best-response functions of the Rich we apply a similar procedure; $\pi_M^l = \hat{\pi}_M^l$, defined by Equations (9) and (10), yields the maximum tax rate on income in a (fictive) tyranny for which the Middle Class would not start a countercoup

$$\tau_l = t_l + \frac{c_M \sqrt{(1 - \kappa_l)\Omega \, \sigma \, e_S}}{e_S \, \varepsilon_M \, \sigma}, \tag{37}$$

$$\text{with } \overline{\rho} = 0 \ \forall \ l = N \qquad \text{and} \qquad \overline{\rho} = \rho \ \forall \ l = S, T$$

Substituting this equation in the budget constraint of a tyranny yields the smallest budget-consistent tax rate on wealth in a (fictive) tyranny

$$\kappa_l = \frac{\overline{G} - t_l \, I + \rho \, x_T}{\Omega} - \frac{c_M^2 \, I^2 + c_M \, I \sqrt{c_M^2 \, I^2 + 4 e_S \, \varepsilon_M^2 \, \sigma (\Omega + t_l \, I - \overline{G} - \rho \, x_T)}}{2 e_S \, \varepsilon_M^2 \, \Omega \, \sigma} \tag{38}$$
$$\text{with } \overline{\rho} = 0 \ \forall \ l = N \qquad \text{and} \qquad \overline{\rho} = \rho \ \forall \ l = S, T$$

The latter depends on the tax rate on income t_l chosen by the Middle Class. For clarity, we use its inverse version

$$t_l = \frac{\rho \, x_T + \overline{G} - \kappa_l \, \Omega}{I} - \frac{c_M \sqrt{(1 - \kappa_l) \Omega \, \sigma \, e_S}}{\sigma \, \varepsilon_B \, e_S}, \tag{25}$$
$$\text{with } \overline{\rho} = 0 \ \forall \ l = N \qquad \text{and} \qquad \overline{\rho} = \rho \ \forall \ l = S, T.$$

A.3.2 Corner Solutions for $h = 0$

According to the threshold

$$\overline{\Omega}_l = \frac{\sigma (\mu + \overline{\rho})^2}{(1 - \kappa_l) e_S}, \qquad \text{with} \quad l = S, T, N \tag{14}$$

the Rich would not invest in mercenaries for $\Omega < \overline{\Omega}_l$. In other words, if the wealth of the Rich is not sufficient high $h_l^* = 0$ holds. At this point, we must differentiate between $l = S, N$ and $l = T$ because $h_l^* = 0$ implies $p, n = 1$ in the former but $q = \mu/(\mu + \rho)$ in the latter case. For $l = S, N$, it is obvious that the Middle Class can levy a tax rate of wealth $k_l = 1$ because the Rich are not able to fight and, thus, a Solonian alliance would always win a civil conflict. Accordingly, the tax rate on income under Solonian order is only restricted by the budget constraint for $k_l = 1$ and, thus,

$$t_l^* = \frac{\overline{G} - \Omega}{I} \tag{39a}$$

is the best-response function of the Middle Class and, as it is independent of k_l, it also represents the optimal tax rate of the Middle Class. If the Rich are supported by the Poor and, thus, $q = \mu/(\mu + \rho)$ although the Rich do not invest in mercenaries, the procedure is similar to the general solution. Again, $\pi_R^T = \pi_R^S$ for $h_l^* = 0$ but $q = \mu/(\mu + \rho)$ yields the maximum tax rate on wealth under Solonian order

$$k_l = \frac{\mu + \rho \, \kappa_l}{\mu + \rho} \tag{40}$$

and substituting those equations in the Solonian budget constraint results in the best-response function of the Middle Class

$$t_l^* = \frac{\overline{G}}{I} - \frac{\Omega(\mu + \kappa_l\,\rho)}{(\mu+\rho)I}.$$ (39b)

Proposition 1 merges the results of Equations (39a) and (39b) to the lower formula in (24).

The best-response function of the Rich is found analogously to the general solution in Appendix A.3.1. We only have to consider the differences for q for $l = S, N$ and $l = T$. Based on these considerations, we derive Equation (25),

$$t_l(\kappa_l^*) = \frac{\rho\,x_T + \overline{G} - \kappa_l^*\,\Omega}{I}.$$ (25)

A.4   Proof of Proposition 2

The equilibrium solution is at the interesction of the best-response functions (24) and (25) conditional on $\overline{\Omega}_l$. Equating the latter two yields the solutions (26) and (27). For circumstance where the Middle Class' best-response function is above or below the Rich's best-response function we have

$$R_R^l \neq R_M^l. \qquad \forall \quad \kappa_l, t_l \in \ [0; 1]$$ (41)

This is true for all

$$c_B \leq \frac{2e_S\,\varepsilon_M\,\sigma}{I}$$ (42)

and

$$x_T \neq \frac{\sigma(\mu+\overline{\rho})}{\rho} - \sqrt{\frac{(1-\kappa_l)\Omega(c_M\,I - 2\varepsilon_M\,\sigma\,e_S)^2}{\varepsilon_M^2\,\sigma\,\rho^2\,e_S}}$$ (43)

or for all

$$c_B > \frac{2e_S\,\varepsilon_M\,\sigma}{I}$$ (44)

and

$$x_T \neq \frac{\sigma(\mu+\overline{\rho})}{\rho} + \sqrt{\frac{(1-\kappa_l)\Omega(c_M\,I - 2\varepsilon_M\,\sigma\,e_S)^2}{\varepsilon_M^2\,\sigma\,\rho^2\,e_S}}.$$ (45)

In addition to the equilibria defined by the intersection of the best-response functions, there may also exist two corner solutions if the best-response functions do not intersect, namely $[R_M^l(\kappa_l = 0), \kappa_l = 0]$ and $[R_M^l(\kappa_l = 1), \kappa_l = 1]$. These equilibria are based on the assumption

that the Middle Class and the Rich wish to avoid situations where the Solonian order and the tyranny are both unstable (i.e., the Rich would mount a coup against the Solonian order but the Middle Class would also mount a coup against the subsequent tyranny resulting in an infinite iteration of coups and countercoups). This should be true in particular for the Rich, who after losing a civil conflict may be expelled and have their properties expropriated. Regarding the equilibria, the preferences of both actors conflict, as both prefer lower taxes for themselves implying higher taxes for the opponent. However, given the constraint of avoiding infinite iterations of coups and countercoups, the actors could accept high taxes for the sake of stability.

For those equilibria, the game also starts from the Solonian order where the Middle Class sets the tax rates. Let us call this Stage 1. As the Middle Class can decide, they determine the equilibrium upon taking into account the reaction of the Rich. Given the actors' ranking of equilibria, the Rich are always better in a Solonian order with the equilibrium $\left[R_M^l(\kappa_l = 0), \kappa_l = 0\right]$ because they pay lower taxes; hence, they always mount a coup in case of $t_l = R_M^l(\kappa_l = 1)$ – and k_l determined by the Solonian budget constraint – in order to establish a tyranny with $\kappa_l = 0$ – and τ_l determined by the tyranny budget constraint.

Based on this consideration, the Middle Class would anticipate this reaction and may settle for the equilibrium $\left[R_M^l(\kappa_l = 0), \kappa_l = 0\right]$ (or for the related tax rates under Solonian order, i.e., $t_l = R_M^l(\kappa_l = 0)$ and k_l determined by the budget constraint). However, as explained at the beginning of Section 3.5, the Rich too must consider the reaction of the Middle Class following the coup. Accordingly, the Rich at this Stage 2 must take into account that the Middle Class may revolt against the then-established tyranny, mounting a countercoup, and trying to establish a new Solonian order (at what is effectively Stage 3).

Given all those considerations, we assume that the Rich would only mount a coup in case of the equilibrium $\left[R_M^l(\kappa_l = 1), \kappa_l = 1\right]$ (or the related tax rates under Solonian order) if they anticipate that the Middle Class would not mount a countercoup in the then-established tyranny. Going now back to Stage 1, the Middle class anticipates those reactions and counter-reactions. The Middle Class then settles for equilibrium $\left[R_M^l(\kappa_l = 0), \kappa_l = 0\right]$ (or the related tax rates under Solonian order) if they anticipate a subsequent coup by the Rich in case of $\left[R_M^l(\kappa_l = 1), \kappa_l = 1\right]$, and settle for $\left[R_M^l(\kappa_l = 1), \kappa_l = 1\right]$ if they anticipate that the Rich would not mount a coup because the latter anticipates a subsequent countercoup by the Middle Class. In the following, we examine this formally by checking whether at Stage 3 the Middle Class would mount a countercoup in case of a tyranny with $\kappa_l = 0$ and τ_l determined by the tyranny budget constraint or not, and by doing so, whether the Rich at Stage 2 would mount a coup at all in case of a Solonian order with $\left[R_M^l(\kappa_l = 1), \kappa_l = 1\right]$. If the Rich do not mount a

coup, at Stage 1 the Middle Class chooses equilibrium $\left[R_M^l(\kappa_l = 1), \kappa_l = 1\right]$ and, otherwise, $\left[R_M^l(\kappa_l = 0), \kappa_l = 0\right]$. Cutting a long story short, given the above considerations, we only have to check whether the Middle Class would accept a tyranny with $\kappa_l = 0$ and τ_l. If they do the only stable corner solution is $\left[R_M^l(\kappa_l = 0), \kappa_l = 0\right]$ and if they would start a countercoup in such a tyranny the only stable cornersolution is $\left[R_M^l(\kappa_l = 1), \kappa_l = 1\right]$.

Starting with $R_R^l > R_M^l$ and, thus, with $x_T > \frac{\sigma(\mu + \overline{\rho})}{\rho} + \sqrt{\frac{(1-\kappa_l)\Omega(c_M\,I - 2\varepsilon_M\,\sigma\,e_S)^2}{\varepsilon_M^2\,\sigma\,\rho^2\,e_S}}$ and $c_B > \frac{2e_S\,\varepsilon_M\,\sigma}{I}$ or with $x_T > \frac{\sigma(\mu + \overline{\rho})}{\rho} - \sqrt{\frac{(1-\kappa_l)\Omega(c_M\,I - 2\varepsilon_M\,\sigma\,e_S)^2}{\varepsilon_M^2\,\sigma\,\rho^2\,e_S}}$ and $c_B \leq \frac{2e_S\,\varepsilon_M\,\sigma}{I}$. The Middle Class would accept a tyranny with $\kappa_l = 0$ if the corresponding tax rate for the Middle Class τ_l, needed to satisfy the budget constraint, is low enough that that Middle Class would not mount a countercoup aiming at establishing a Solonian order with $t_l = R_M^l(\kappa_l = 0)$, i.e., with the equilibrium $\left[R_M^l(\kappa_l = 0), \kappa_l = 0\right]$ but under Solonian order. By using the upper Equation in (24), we can calculate

$$t_l = R_M^l(\kappa_l) = \frac{\sigma(\beta + \overline{\rho}) + \overline{G} - 2\sqrt{e_D\,\Omega\,\sigma}}{I} \tag{46}$$

as the Middle Class optimal reaction to $\kappa_l = 0$. At the same time, substituting Equations (33) and the budget constraint of a tyranny for $\kappa_l = 0$ in Equation (11) yields

$$t_l > \bar{t}_l = \frac{\overline{G} + \rho\,s_O}{I} - \frac{c_M\,\Omega}{\varepsilon_M\sqrt{e_D\,\Omega\,\sigma}} \tag{47}$$

as the threshold for a stable tyranny. Given the conditions in Equations (43) and (45), the tax rate chosen by the Middle Class is not large enough, i.e., $t_l = R_M^l(\kappa_l = 0) < \bar{t}_l$ in order for the tyranny be stable. In other words, in such a tyranny, the Middle Class would always mount a coup, aiming at establishing a Solonian order with the same equilibrium $\left[R_M^l(\kappa_l = 0), \kappa_l = 0\right]$. Accordingly, the Rich must anticipate that every coup would provoke a countercoup and, hence, they must accept the Solonian order because, otherwise, the Middle Class would then constantly try to overthrow the tyranny until they are successful and the Rich are expelled.

For $R_R^l < R_M^l$ and, thus, for $x_T < \frac{\sigma(\mu + \overline{\rho})}{\rho} + \sqrt{\frac{(1-\kappa_l)\Omega(c_M\,I - 2\varepsilon_M\,\sigma\,e_S)^2}{\varepsilon_M^2\,\sigma\,\rho^2\,e_S}}$ and $c_B > \frac{2e_S\,\varepsilon_M\,\sigma}{I}$ or for $x_T < \frac{\sigma(\mu + \overline{\rho})}{\rho} - \sqrt{\frac{(1-\kappa_l)\Omega(c_M\,I - 2\varepsilon_M\,\sigma\,e_S)^2}{\varepsilon_M^2\,\sigma\,\rho^2\,e_S}}$ and $c_B \leq \frac{2e_S\,\varepsilon_M\,\sigma}{I}$, the situation reverses. Now, there exists a stable equilibrium for $\left[\kappa_l = 0,\ t_l = R_M^l(\kappa_l = 0)\right]$, because $t_l = R_M^l(\kappa_l = 0) > \bar{t}_l$ holds true and, hence, the Middle Class would not start a coutercoup for $\kappa_l = 0$. Accordingly, by anticipating this stable tyranny, the Rich would always start a coup and, consequently, the Middle Class must accept this situation, by reducing the taxes on wealth under Solonian order.

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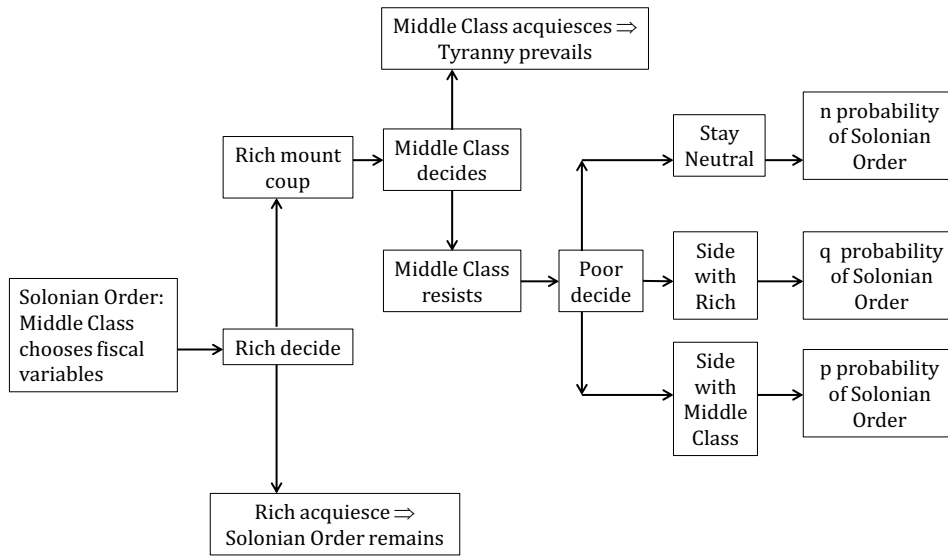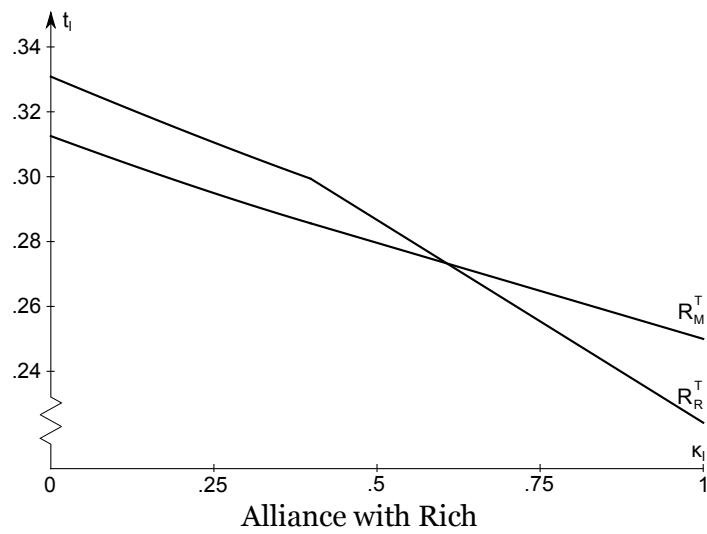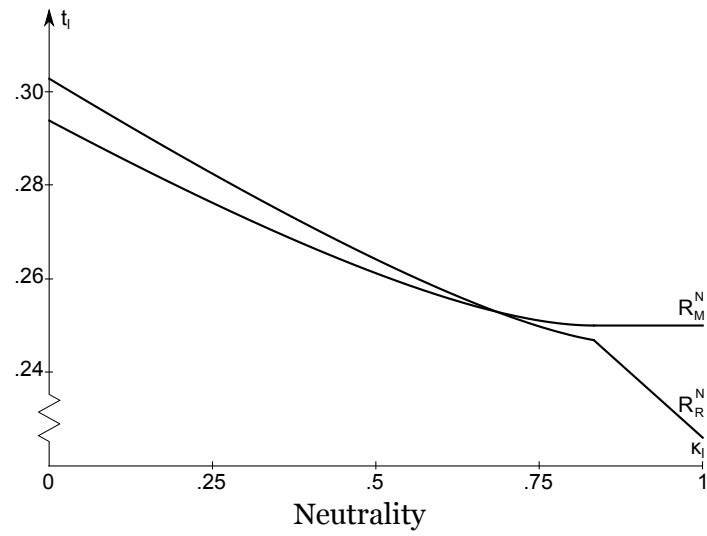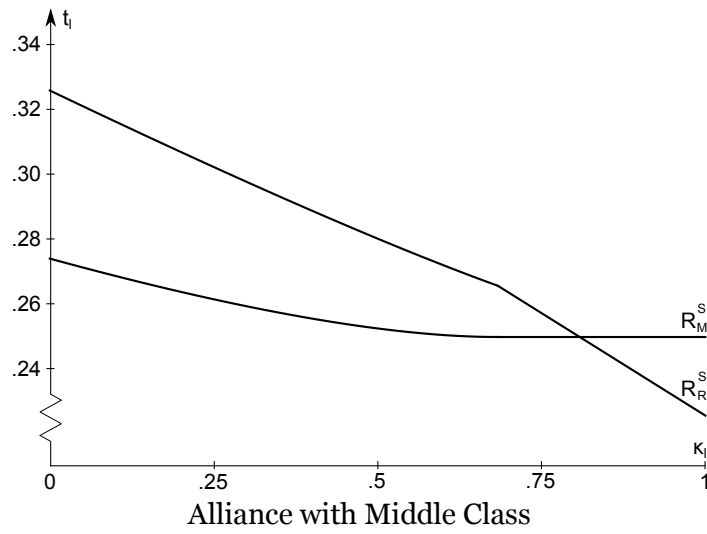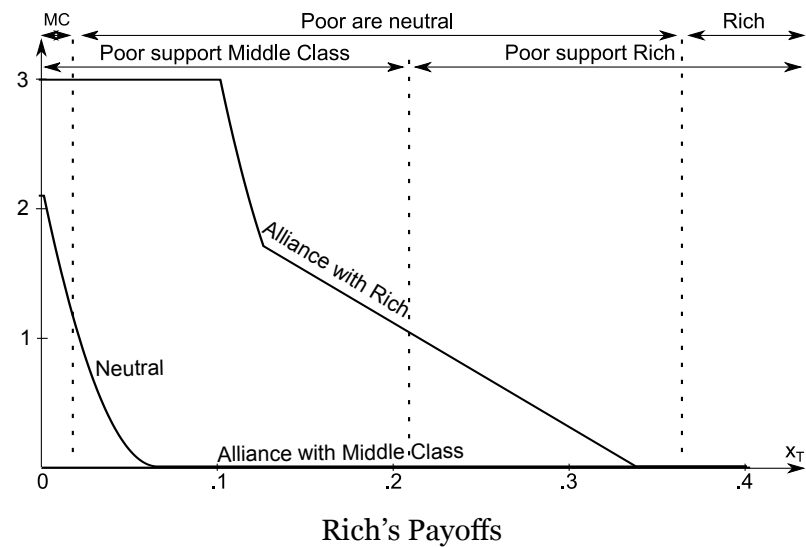**Figure 1:** The Solonian Order – Tyranny game with and without neutrality

Figure 2: Best-Response-Function R_i and Equilibria for Different Alliances

Alliance with Middle Class

Neutrality

Alliance with Rich
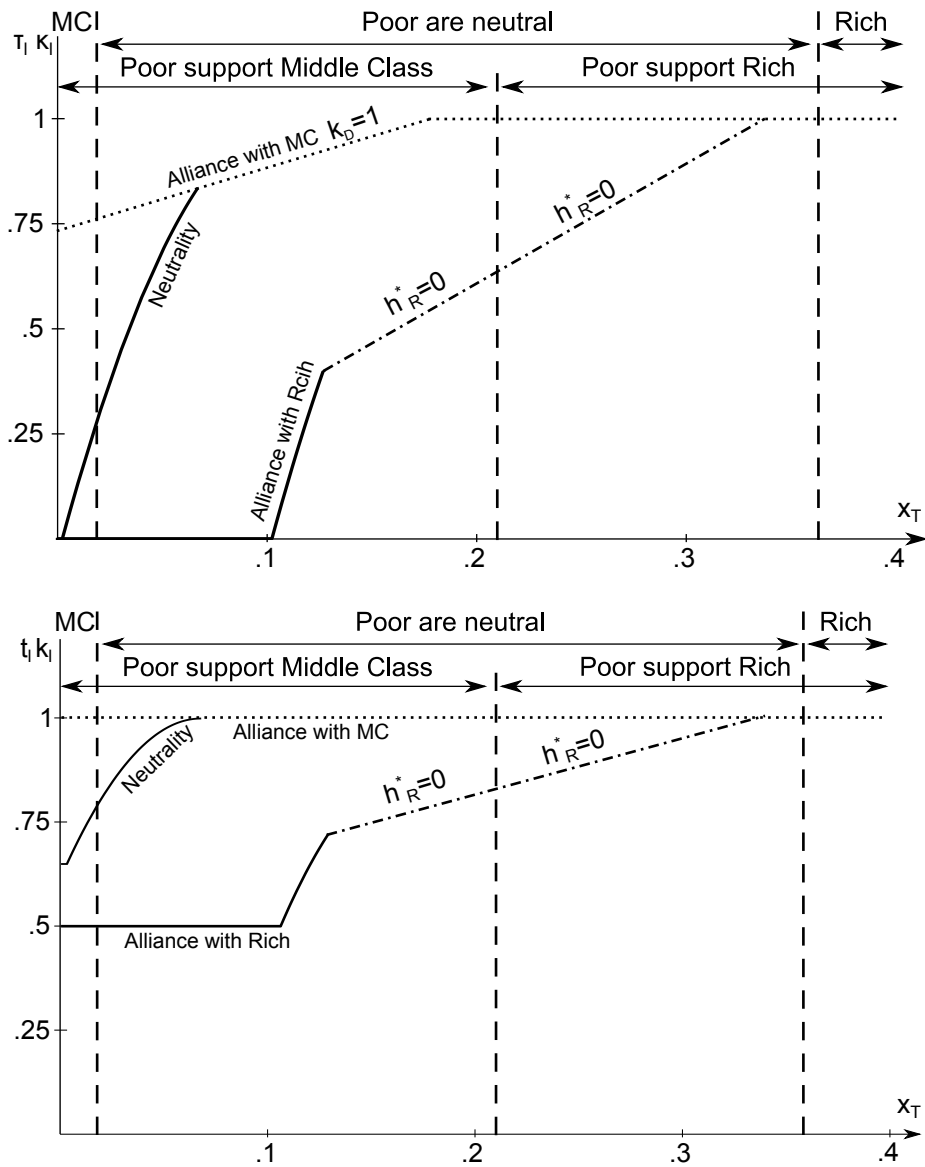
Numerical evaluation with $c_M = 0.1, \varepsilon_p = 1, \varepsilon_M = 5, \omega_R = 8, G = 0.5, s_G = 1, \sigma = 0.1,$ $\mu = 0.5,$ and $\rho = 0.45.$

Figure 3: Payoffs Depending on Alliance

Middle Class' Payoffs

Poor's Payoffs

Rich's Payoffs
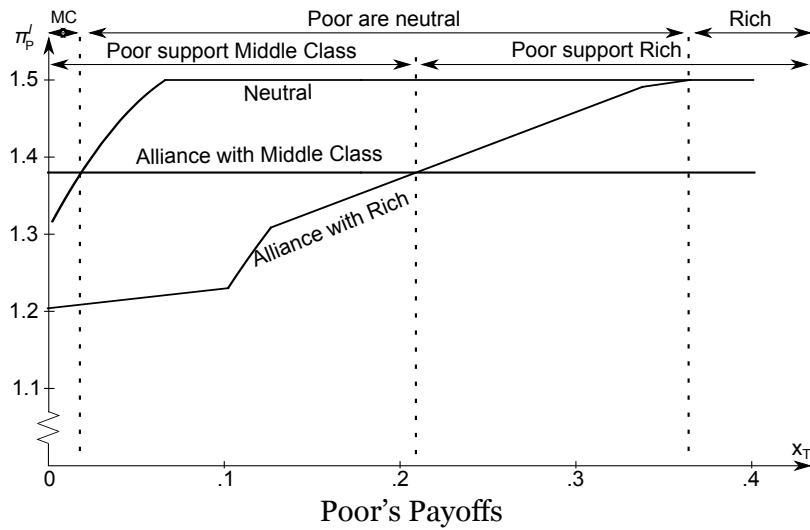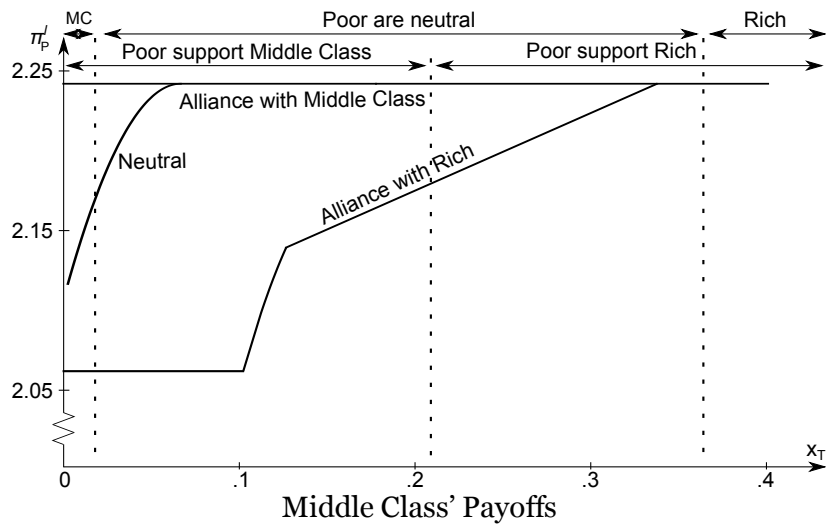
Numerical evaluation with $c_P = 0.12, c_M = 0.1, \varepsilon_p = 2, \varepsilon_M = 3, \omega = 6, G = 0.1, s_G = 1, \sigma = 0.1, \mu = 0.5,$ and $\rho = 0.45$.

Figure 4: Tax Rates on Wealth in Tyranny (upper) and under Solonian Order (lower)

Numerical evaluation with $c_P = 0.12, c_M = 0.1, \varepsilon_p = 2, \varepsilon_M = 3, \omega = 6, G = 0.1, s_G = 1, \sigma = 0.1, \mu = 0.5,$ and $\rho = 0.45.$