Authority and Information Acquisition in Cheap Talk with Informational Interdependence *

Daniel Habermacher †

April 2019 (PRELIMINARY AND INCOMPLETE)

Abstract

I study the allocation of decision rights in a two-dimensional cheap talk game with informational interdependence and imperfectly informed senders. The Principal allocates decision rights among all players including herself. Delegation is optimal when the expected informational gains outweigh the loss of control due to biased decisions. Delegating one decision leads to informational gains for the Principal when there are negative informational externalities (Levy and Razin, 2007). Partial delegation (of a controversial decision) is thus optimal when externalities are sufficiently strong. I characterize the maximum bias the Principal is willing to tolerate as a function of informational gains.

I also analyse agents' incentives for information acquisition. An agent invests in information when the expected utility gains from revealing it compensate its costs. Truthful communication is a necessary condition for information acquisition, but its influence on beliefs must also be sufficiently large. This implies centralization is always optimal when information costs are high. Endogenous information acquisition allows agents to specialize, which enhances communication incentives because it rules out contradictory information. Finally, I show that delegation leads to ex-post specialization: decision-makers typically receive more information about the more relevant state as compared to centralization.

Keywords: Multidimensional Cheap Talk, Industrial Organization, Delegation, Organizational Design.

JEL: D21, D83.

---

*I am grateful to my supervisors Marina Halac and Francesco Squintani for their invaluable support, to Robert Akerlof, Costas Cavounidis, Ayush Pant, Federico Trombetta, and participants at 2018 Warwick Economics PhD Conference, 2018 EEA-ESEM, and Warwick MTWIP seminar for helpful comments. All remaining errors are mine. Financial support from the ESRC is gratefully acknowledged.

†University of Warwick. Email: d.f.habermacher@warwick.ac.uk

1 Introduction

Consider a multinational firm to decide how much to invest on each of two projects. There is no competition for resources, so optimal investment depends on expected profitability. One of the projects consists of launching a new product whose profitability depends on commercially viable innovations from new technologies. The other is about upgrading a product the firm already commercializes, its profitability depends on increasing production efficiency on technologies the firm already uses. But projects are complementary up to some degree: development of new products can benefit from efficiency-gains in old technologies and upgrades can profit from technological breakthroughs. Information about both technologies is dispersed across the firm's subsidiaries, some of which are located close to technology centres and others close to manufacturing sites. As a consequence of location, subsidiaries in the first group favour the development of products based on new technologies, while those in the second group favour upgrades based on cost-efficient technologies. This preference divergence, in addition to informational interdependencies, can create conflict of interest resulting in poor communication.

Can the firm overcome these harmful effects of informational interdependencies? How do communication incentives change if decisions about new products were delegated to tech-leaning subsidiaries? How much information would a cost-leaning subsidiary acquire in such a case? What if these decisions were delegated to one of them? This paper studies how organizations cope with informational interdependencies using the allocation of decision-rights. Delegation can enhance communication between subsidiaries with similar preferences on the delegated decision(s), but incentives to do so also depends on how biased the headquarters expects those decisions to be. I characterize the relationship between informational gains and loss of control when decisions are interdependent and subsidiaries are imperfectly informed. I also show that the choice of organizational structure also affects incentives to acquire information.

The example reflects the challenges faced by technology-based Multinational Corporations in the organization of R&D (see Andersson and Forsgren, 2000; Gassmann and von Zedtwitz, 1999; Boutellier et al., 2008; Ecker et al., 2013 among others). But most environments with complex decision-making feature some degree of informational interdependence. Consider the case of public policies as an additional example. Problems that require public intervention are generally defined in terms of multiple determinants, and fine-tuning the associated decisions requires information about them. More often than not decisions vary in their effectiveness to address the determinants, so policies comprise many complementary decisions (policy measures) to address a given problem. In addition, the relevant information is dispersed among many public agencies, legislative committees, interest groups, and constituencies; each of whom have interests associated to decisions.

In a seminal work Baumgartner and Jones (2009) argue that institutional change can result from the interaction between multi-causal policy problems, the need for information, and informed parties' strong conflict of interests. Informational interdependence is implicit in their framework, and in this paper I make this relationship explicit by extending the tools from the legislative debate literature (see Gilligan and Krehbiel, 1987, 1989; Austen-Smith, 1993; Krishna and Morgan, 2001a; Battaglini, 2002; Dewan et al., 2015).

The model consists of a Principal (she), n agents, and two decisions. Optimal decisions depend on information about two state variables, such that information about each state affects both decisions.[1] Informational interdependence is linear, such that decisions are (informationally) correlated. Each agent (he) has access to two noisy signals, one for each state.[2] The Principal decides on the allocation of decision rights before the communication stage; she can retain authority over decisions (centralization), delegate one of them (partial delegation) or both (full delegation). Different organizational structures lead to very different incentives for communication. Under any form of delegation, incentives depend on decision-specific conflict of interest. Under centralization, the correlation between decisions affects the effective conflict of interest.

The optimal organizational structure resolves a trade-off between informational gains and loss of control. To see this consider a player with central preferences in one dimension —i.e. preferences that minimize the dimension-specific distance with the rest of the players. If the decision were delegated to this central player the number of agents revealing information to him would be maximal. When he is not the Principal the additional information are Direct Informational Gains; that is, he receives more information than when she decides on both dimensions. But the decision to delegate depends on his bias: whether the increased precision from better communication with other agents compensates the loss of control due to a biased decision. Differently from previous papers in the literature, the bias the Principal is willing to tolerate for delegation depends on the information the agent receives from others in equilibrium. I characterize the relationship between informational gains and loss of control in Proposition 3.

Informational gains from delegation can also refer to the Principal. In such a case, delegating one decision leads to more agents revealing information about the retained decision; these are Indirect Informational Gains. For these to take place the profile of preferences must include agents whose biases are small in one dimension and very large in the other, such that the latter impedes communication under centralization

[1]States can be interpreted as the determinants of the policy problem. For instance, the "Energy Problem" can be defined in terms of the need to guarantee stable supply over time, to minimize the cost for consumers and industry, and to reduce the impact on the environment. Alternative energy sources address each of these determinants with different degrees of success, such that any information related to the determinants will affect the relative viability of sources.

[2]In order to analyse incentives for communication I first assume he observes both signals (Section 2), and later relax this assumption to analyse endogenous information acquisition (Section 3), allowing each agent to decide which signal to observe (if any).

(negative informational externalities, see Levy and Razin, 2007). Delegating the controversial decision makes communication between these agents and the Principal depend on the small biases. As a consequence, sufficiently strong negative informational externalities lead to Partial Delegation.

I then introduce endogenous information acquisition (Section 3). Each agent decides which signal to observe (if any) at a given cost. Lemma 4 shows that agents only acquire information that is incentive compatible to reveal at the communication stage (similar to Di Pei, 2015). But investing in a signal must also be cost-effective in an ex-ante sense (Lemma 6): expected benefits from truthful communication must compensate the cost of that information. Because these benefits depend on the influence each signal has on both decisions, cost-effectiveness is decreasing in the number of other agents revealing the same information. In other words, information costs impose an upper bound on the number of agents investing in a given signal. This imposes restrictions on the benefits from delegation described above and, thus, have important implications on the optimality of different organizational structures. In particular, for sufficiently large costs centralization is always optimal (Corollary 4).

The possibility of acquiring information can lead to specialization. An agent invests in information about one state when his preferences are more aligned to the decision-maker's on the associated decision, or when it is too costly to acquire information about both states. Most importantly, specialization expands the possibilities of communication because it avoids contradictory information. When an agent observes information about two independent states, the influence of revealing both pieces of information can be smaller than revealing each of them individually.[3] This leads to a credibility loss because agents with such information have incentives to convey favourable information. But an agent acquiring information about one state only does not face this problem, and communication is incentive compatible for a larger set of biases.

Finally, I analyse the information flows under different organizational structures. For a fixed conflict of interest and negligible information costs, delegation results in decision-makers having more information about the more important state. Revealing information about a given state is more influential for on of the decisions,[4] then revealing the associated piece of information is incentive compatible for a larger set of biases than revealing the other. When the Principal does not know the exact profile of ideologies but expects to be evenly distributed within a given distance, she can expect to receive more balanced information under centralization than under delegation. The fact that delegation leads to ex-post specialization of decision-makers is thus another qualification of this organizational structure.

[3]In the case of positively correlated decisions, when information moves beliefs in opposite directions.

[4]This is the case for imperfect correlation, whereas the overall influence of a single state is normalized to one.

Related Literature (incomplete). Battaglini (2002, 2004) show that with multiple decisions the receiver can extract all the information from at least one sender in each dimension. But the result is not robust to restricted state spaces even with fully informed senders (Ambrus and Takahashi, 2008), nor imperfect information when decision are interdependent (Levy and Razin, 2007). The latter is particularly relevant in the present analysis, since Levy and Razin show that communication can break down completely if the conflict of interest in one dimension is sufficiently large.[5,6] They define this situation as a 'negative informational spillover'. My paper builds upon this intuition, showing that delegation can recover communication when these externalities are present under centralization. But unlike all these papers, my focus is on information aggregation.

The paper is also related to applications of communication models. Besides the references to the literature on legislative debate in the previous section, the notion of interdependent decisions also prevails among firms. Multi-divisional firms trade off the need for adaptation to local shocks with the need for coordination between divisions. Communication frictions may lead to inefficiencies in terms of giving-up benefits from specialization of production (Dessein and Santos, 2006) or the need for coordination through scheduled task instead of using communication (Dessein et al., 2016). When divisional managers' information is not verifiable the allocation of decision rights, along with interdependence and divisional conflict of interest, shape incentives for communication (Alonso et al., 2008, 2015). But the trade-off between adaptation and coordination may not be the relevant problem in many applications; in Multinational Corporations, for instance "R&D is torn between the pressures for scientific and commercial results. Control and coordination needed for the sake of internal consistency seems to apply little to research-based organization. Bureaucratic and hierarchical control as well as social control does not work as much as scientists feel more affiliated with their profession than to their employers." (Boutellier et al., 2008) In my framework interdependence between decisions is only informational, and the benefits of different organizational structures rely only on these.

The paper also contributes to the literature on delegation to motivate information acquisition. In their seminal paper, Aghion and Tirole show that delegation of formal authority can increase an agent's initiative on acquiring information, and the associated benefits for the Principal can outweigh the costs of a biased decision (loss of control). But the result can be somewhat different when communication is strategic. If the Principal can credibly punish the agent off-path (with the babbling equilibrium), she will be better-

[5] This could happen even if preferences are aligned in the other dimensions, configuring a negative informational spillover that harms communication.

[6] In a complementary paper, Chakraborty and Harbaugh (2007) show that, in two-player games, the sender can credibly communicate any ranking of the decision dimensions that reflects (at least partially) the order of the realization of the states across dimensions. Any of such rankings constitutes an influential equilibrium as long as some mild assumptions are met.

off retaining decision authority and delegating information acquisition –indeed, for some set of parameters, the agent will *over-invest* in information (Argenziano et al., 2016). More recently Liu and Migrow show that incentives to acquire information under different organizational structures may depend on the volatility about the salience of decisions (divisions' profits). In their model, the Principal is (ex-post) informed about the relative profitability of two divisions, and decisions are subject to the coordination-adaptation trade-off (Alonso et al., 2008). As long as there is uncertainty about the relative profitability, decentralization provide better incentives for info acquisition if coordination is sufficiently important. In my paper the informational interdependencies may increase incentives to invest in information, since any piece would affect more than one decision (provided the agent is willing to communicate it). This is more likely to take place under centralization, thus challenging the canonical result in Aghion and Tirole (1997).

The next section presents the Organizational Design Game, and the results on optimal allocation of decision-rights. In section 3 I integrate the allocation of decision-rights with endogenous information acquisition.

2 Organizational Structure

An organization is characterized by a Principal, n agents, and 2 decisions—$\mathbf{y} \in \Re^2$. Each decision has payoff-relevant consequences that depend on two states. Decision-specific uncertainties are defined by the vector δ in the following way:

$$\begin{bmatrix} \delta_1 \\ \delta_2 \end{bmatrix} \equiv \begin{bmatrix} w_{11}\,\theta_1 + w_{12}\,\theta_2 \\ w_{21}\,\theta_1 + w_{22}\,\theta_2 \end{bmatrix}$$

Where $\boldsymbol{\theta} = [\theta_1, \theta_2]$ represents the states. I assume θ_1 and θ_2 are uniformly distributed with support in the interval $[0, 1]$, and $\theta_1 \perp \theta_2$. Decision-specific uncertainties are correlated through the 2x2 matrix W.The elements of W are indexed by w_{dr} where d represents decisions and r states; that is, $y_d = \{y_1, y_2\}$ and $\theta_r = \{\theta_1, \theta_2\}$. I assume $w_{d1} + w_{d2} = 1$ and $w_{11}, w_{22} > \frac{1}{2}$, which amounts to say that information about θ_1 is more relevant for y_1 and that of θ_2 for y_2, and uncertainties are positively correlated. Preferences of a generic player are thus given by:

$$U^i(\boldsymbol{\theta}, \mathbf{y}, \mathbf{b}^i) = -\left(y_1 - \delta_1(\theta_1, \theta_2) - b_1^i\right)^2 - \left(y_1 - \delta_2(\theta_1, \theta_2) - b_2^i\right)^2$$

For $i = \{P, 1, \ldots, n\}$, where $\delta_d(\theta_1, \theta_2)$ is defined in terms of θ_1, θ_2 and $\mathbf{W}$, and b_d^i is the bias associated to y_d. The Principal's bias is normalized to zero ($\mathbf{b}^P = [0; 0]$) such that $\mathbf{b}^i$ captures the conflict of interest with agent i.

Agents have access to information about both states. In particular, each of them observes one signal associated to a different state (two signals in total); they are iid realizations of (independent) Bernoulli distributions associated to the states. Let $\mathbf{S}^i = [S_1^i; S_2^i]$ be agent i's signals and $\tilde{S}_r^i$ be the realization of component r, for $\theta_r = \{\theta_1, \theta_2\}$. The prior probability distribution for each signal is given by:

$$\Pr(\tilde{S}_1^i = 1) = \theta_1 \qquad \Pr(\tilde{S}_2^i = 1) = \theta_2$$

Because each signal is an i.i.d. Bernoulli trial, the associated message strategy is degenerated: each agent can either reveal or lie about that single signal (see Galeotti et al., 2013). The message space is still relatively complex though, but binary signals provide a convenient simplification for the characterization of communication incentives. In particular, the updated expectation and variance for each state depends on the number of agent revealing the corresponding signal truthfully, $k_r \leq n$, and the the number of those agents who report a 1, ℓ_r, for $\theta_r = \{\theta_1, \theta_2\}$.

$$E(\theta_r | k_r, \ell_r) = \frac{(\ell_r + 1)}{(k_r + 2)} \qquad \mathrm{Var}(\theta_r | k_r, \ell_r) = \frac{(\ell_r + 1)(k_r - \ell_r + 1)}{(k_r + 2)^2 (k_r + 3)}$$

Decision-rights are allocated before each agent learns his information. Formally, the Principal decides on a set of assignments that grants decision-making authority over the set of decisions, $\mathbf{y}$. The assignment grants complete jurisdiction over the delegated decision, such that authority over each decision is granted to a unique individual. Any player in charge of a given decision can also engage in private communication with each agent.

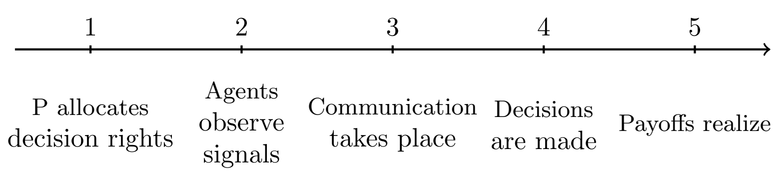


Figure 1: Timing of the Org. Structure game

Different allocations of decision-rights lead to different organizational structures, which can be grouped into three categories. Centralization refers to the case in which the Principal decides on both issues and engages in cheap talk communication with the informed agents. I call Full Delegation when the Principal grants authority to two different agents, each of them assigned to a different decision, each of them talking privately with other agents. Finally, in Partial Delegation the Principal retains authority over one issue and delegates the other to an agent.

At this point two clarifications are necessary. First, I rule out the case in which the Principal delegates both decisions to a single agent. This is without loss since incentives for communication would be similarly defined with a different 'normalization' of the conflict of interest. In both cases communication will depend on a specific measure of centrality of preferences (different from the cases involving delegation). The second clarification relates to the distinction between delegation of decision authority and decentralization of information processing / acquisition. In my framework authority can be centralized or decentralized, the latter is called delegation throughout the paper. Information, on the other hand, is always decentralized in the sense that it is dispersed among many agents.

The equilibrium concept is pure-strategies Perfect Bayesian Equilibria. Similar to Galeotti et al. (2013), a full characterization including mixed strategies is cumbersome and does not provide intuitions beyond the pure strategies case. In an online appendix I show that this decision is with little loss of generality.

Let me also introduce some notation before I define equilibrium strategies. First, I adopt the convention of indexing agents with i and decision-makers with j. Let $\mathbf{m}_j^i\left(\mathbf{S}^i, b_d^i, b_d^j\right) \in M$ denote i's message to a generic decision-maker j in charge of (generic) decision y_d. In addition, let $\mathbf{m}_j = \{..., \mathbf{m}_j^i, ...\}$ be the vector containing all the messages j receives from agents (including himself if applicable). Now, let $k_r^*(j) \equiv k_r(\mathbf{m}_j^*)$ denote the number of truthful messages that j receives in equilibrium and $\ell_r^*(j)$ the number of "ones" among those messages and let k_r^j be agent i's conjecture about the number of other agents revealing signal S_r to j in equilibrium.[7] Lastly, in order to keep track of who decides what, let $j', j'' \in \{P, 1, ..., n\}$ be the decision-makers for y_1 and y_2, resp. (the number of apostrophes coincide with the index of the decision allocated to each player).

A Perfect Bayesian Equilibrium of this game is then characterized by the triple $\left(\mathbf{y}^*, \mathbf{m}_{j'}^*, \mathbf{m}_{j''}^*\right)$ which represents the vector of decisions, vector of message strategies to j' and j'' (resp.), and the vector of information acquisition strategies. Optimal actions satisfy:

$$y_1^* = w_{11}\frac{(\ell_1^*(j')+1)}{(k_1^*(j')+2)} + w_{12}\frac{(\ell_2^*(j')+1)}{(k_2^*(j')+2)} + b_1' \qquad y_2^* = w_{21}\frac{(\ell_1^*(j'')+1)}{(k_1^*(j'')+2)} + w_{22}\frac{(\ell_2^*(j'')+1)}{(k_2^*(j'')+2)} + b_2''$$

[7] Note that i's conjecture will be correct in equilibrium, and whenever his message strategy involves revealing the corresponding signal then $k_r^*(j) = k_r(j) + 1$, otherwise $k_r^*(j) = k_r(j)$.

Allocation of decision-rights has two main effects. On the one hand, delegation implies a biased agent decides on behalf of the Principal, for which the decision will be itself biased. On the other hand, incentives for communication depend on the conflict of interest between the decision-maker(s) and the agent in question, which will be different depending on the organizational structure.

Agent i's optimal message strategy to decision-maker j satisfies:

$$\mathbf{m}_j^{i*}(\mathbf{S}^i, \mathbf{b}^i, b_1', b_2'') = \arg\max_{\mathbf{m}_j^i} \left\{ E\Big[-\left(y_1\left(m_{j'}^i, \mathbf{m}_{j'}^{\text{-}i}\right) - \delta_1 - b_1^i\right)^2 - \left(y_2\left(m_{j''}^i, \mathbf{m}_{j''}^{\text{-}i}\right) - \delta_2 - b_2^i\right)^2 \Big] \Big| \mathbf{S}^i \right\}$$

Where b_1' represents the bias of decision-maker j' with respect to y_1, and b_2'' that of j'' with respect to y_2. Under centralization $j' = j'' = P$ and both biases are equal to zero. From the Principal's perspective delegation then introduces an inefficiency due to the loss of control (Dessein, 2002).

As mentioned earlier the message strategies with respect to each signal is degenerated, agents can reveal it or not. But the set of message strategies is not just based on babbling or revealing information on separate dimensions independently. An agent could, for instance, fully reveal his information when the realization he observes coincide and play a different strategy when they do not. This possibility arises because states are orthogonal and, thus, any information about one does not reveal information about the other. I call these strategies 'dimensional non-separable' (DNS).

Before I characterize communication incentives, I introduce the equilibrium selection criterion and some notational conventions that will facilitate the analysis. Communication equilibria will be selected taking into account the decision-maker ex-ante expected utility, which constitutes the natural extension of criteria traditionally used in the cheap talk literature.

Definition 1. *Agent i's message strategy m_j^i is* ***more informative*** *than $\tilde{m}_j^i$ if the decision-maker prefers (ex-ante) the former over the latter, given the profile of preferences.*

Definition 2. *Let $k_r^{\text{C}} \equiv \{k_r(j) | \, j' = j'' = P\}$ denote the number of truthful messages about θ_r the Principal receives under centralization; let $k_r^{\text{P1}} \equiv \{k_r(P) | \, j' = P\}$ those she receives when decides on y_1 only; and $k_r^{\text{P2}} \equiv \{k_r(P) | \, j'' = P\}$ when she decides on y_2 only. For when P does not decide at all, let $k_r' \equiv k_r^*(j')$ and $k_r'' \equiv k_r^*(j'')$ refer to the number of truthful messages for decision maker of y_1 and y_2, respectively; while keep $k_r^j \equiv k_r^*(j)$ for a generic decision-maker.*

Incentives for communication. I start by describing the incentives of a generic agent i to reveal his information to decision-maker j in charge of y_d only. This is the case when the Principal delegates at least one decision. Under delegation each agent engages in private communication with each decision-maker; communication thus depend on decision-specific conflict of interest. In addition, binary signals imply i does not control the influence of the message associated to each signal; as a consequence, communication depends on how many other agents are expected to reveal the same information in equilibrium. These two determinants of communication have already been studied in the literature (Austen-Smith and Riker, 1987; Krishna and Morgan, 2001b; Galeotti et al., 2013), but in this framework there is a third determinant: the possibility of 'contradictory information.'

Signals are associated to two independent states, both of which affect each decision. When an agents observes $(\tilde{S}_1^i, \tilde{S}_2^i) = \{(0,1),(1,0)\}$, fully revealing would have a smaller influence on decisions than revealing any of these realizations individually because signals would compensate each other. If i were to reveal one signal only, he would be tempted to report the realization that moves the decision in the direction of his bias. This creates a loss of credibility that rules out the possibility of revealing only one signal under delegation. Similar intuitions can lead to dimensional non-separable message strategies. The Lemma below summarizes the result.

Lemma 1 (Incentive Compatibility of Communication on y_d.)**.** *Consider the equilibrium* $(\mathbf{y}^*, \mathbf{m}^*)$ *in which the Principal delegates* y_d *to agent* j*, who does not decide on the other decision.*[8] *Then, revealing either* S_r^i *or both signals is incentive compatible for agent* i *if:*

$$|b_d^i - b_d^j| \leq \frac{1}{2} \left| \frac{w_{d1}}{(k_1^j + 3)} - \frac{w_{d2}}{(k_2^j + 3)} \right| \tag{1}$$

And revealing both signals when they coincide [$(0,0)$ *or* $(1,1)$*] and babbling otherwise if:*

$$|b_d^i - b_d^j| \leq \frac{1}{4} \left[\frac{w_{d1}}{(k_1^j + 3)} + \frac{w_{d2}}{(k_2^j + 3)} \right] \tag{2}$$

Where $y_d = \{y_1, y_2\}$.

Proof. See Appendix B.3 □

Equation (1) bear some similarities to equation (3) in Galeotti et al. (2013).[9] The left-hand side captures the conflict of interest between i and j: incentive compatibility requires the expected utility gains from

[8] By assumption, communication between decision-makers only involves own signals (not information transmitted by other agents). For an analysis on hierarchies as information intermediation see Migrow (2017).

[9] Were y_d on information about one state only,[10] the right-hand side would be exactly the same as in the paper by Galeotti et al.. In such a case the whole problem boils down to one with two independent-decisions in which the set of message strategies is dimensional separable. But in the present framework both states affect each decision: the influence of the most important state is not 1 and communication incentives also depend on information about the 'less important' state.

increasing j's precision on y_d to compensate the conflict of interest. To see this, suppose for a moment that $w_{d1} > 1/2$. The right-hand side in (1) decreases as k_1^j increases: if i's expects many other agents to reveal information on θ_1, his incentives to that information decreases. These are the congestion effects affecting communication incentives (see Galeotti et al., 2013). But in my framework the congestion effects are more complex due to i having information about two states.

Consider the case in which i is expected to reveal one signal. Whatever he says about *the other state* is not believed on the equilibrium path and, as a consequence, he cannot transmit favourable information about this state.[11] If information about the other state –the one is is believed– is unfavourable, he will be tempted to lie because is convinced that both decisions should be moved towards his bias (at least to some extent). This is the reason why incentives to reveal one signal also depend on information the decision-maker has about the other –equation (1). In a companion paper I discuss how such incentives can lead to congestion effects that benefit the decision-maker (see Habermacher, 2018). The loss of credibility due to contradictory information rules out messages strategies in which the agent reveals only one signal when j has authority on one decision.[12]

Incentives to reveal both signals depend on how 'balanced' is the information j receives in equilibrium. Here the notion of balanced information relates to both the number of truthful messages she receives and the relative importance of the corresponding state (k_r and w_{dr}, resp.). If $\frac{w_{11}}{(k_1^j+3)} > \frac{w_{12}}{(k_2^j+3)}$ the decision-maker is not receiving enough information about θ_1 relative to how important it is for y_1 (recall that $w_{11} > w_{12}$). In other words, $\frac{w_{11}}{(k_1^j+3)} = \frac{w_{12}}{(k_2^j+3)}$ means that j is receiving more signals about θ_1 in a proportion that matches the higher importance of that state on y_1.

Equation (2) reflects the possibility of dimensional non-separable messages in this case. The only of such strategies emerging in equilibrium has i fully revealing his signals when they coincide and babbling otherwise. The overall expected influence is lower in equation (2) than in (1) because there exists an additional deviation (babbling) and the agent can expect to be influential for half of the possible realizations of signals. As a consequence the IC constraint is generically more restrictive than (1) and holds when the right-hand side in (1) is close to zero.[13] In other words, full revelation constitutes the substantive non-babbling message strategy in this case.

Communication incentives depend on the information j receives from other agents in equilibrium. What

[11] Here the notion of favourable (unfavourable) information is that of signals that would move decisions towards (against) his bias.

[12] Formally, the IC constraints for any message strategy in which i reveals one signals coincides with that of full revelation such that the latter will always be selected by the decision maker.

[13] In Appendix B.3 I show that the RHS of (2) is larger than that of (1) if $\frac{3w_{d\tilde{r}}}{w_{dr}} \geq \frac{(k_{\tilde{r}}+3)}{(k_r+3)}$.

i says to j affects y_d but not the other decision, meaning that there is no informational interdependencies across decisions. The organizational structure can then be used by the Principal to *take advantage*- or *get rid of* the correlation between decisions. The proposition below summarizes the equilibrium communication for one decision and two states.

Proposition 1 (Equilibrium Communication for y_d)**.** *Let agent j be the decision-maker of y_d. In the most-informative equilibrium $(\mathbf{y}^*, \mathbf{m}^*_{j'}, \mathbf{m}^*_{j''})$ between agents i and j, i fully reveals his information if and only if condition (1) hold. If the right-hand side of (2) is larger than that of (1), then agents with $|b^i_d - b^j_d|$ within these two values reveal both signals when they are either $(0,0)$ or $(1,1)$ and send babbling messages otherwise. Agents with conflict of interest beyond these two cases play babbling message strategies.*

Proof. See appendix B.4. □

Equilibrium communication in the case of one decision is characterized by the IC constraint in Lemma 1. As already discussed, full revelation dominates message strategies in which i reveals one signal —the IC constraints are the same and the decision-maker always prefer the former. Dimensional non-separable strategies arise only under restrictive conditions and can be overlooked with little loss of generality. Whenever any of such strategies is IC under delegation, revealing both signals is also IC but the decision-maker strictly prefers the latter over any form of dimensional non-separable strategy that includes babbling. Now is time to analyse communication between i and the Principal when she retains authority over both decisions.

Under centralization any information transmitted to the Principal affects both decisions. Now, the exact effect on decisions depends on the information being revealed because states' influence on decisions is well-defined by $\mathbf{W}$ (and k_rs in the equilibrium under consideration). Different message strategies have different measures for the conflict of interest because, for instance, revealing S^i_1 weighs more b^i_1 than b^i_2.[14] When i reveals both signals the overall influence is balanced due to the normalization. The lemma below summarizes the incentive compatibility constraints for a generic agent under centralization.

Lemma 2 (Incentive Compatibility of Communication under Centralization)**.** *Consider the equilibrium $(\mathbf{y}^*, \mathbf{m}^*)$, truthful communication will be incentive compatible for agent i in the following cases:*

- *Revealing S^i_r, if:*

$$|\beta_r| \leq \frac{(w_{1r})^2 + (w_{2r})^2}{2} \left[\frac{1}{(k^C_r + 3)} - \frac{C_r}{(k^C_{\tilde{r}} + 3)} \right] \tag{3}$$

[14] Recall that θ_1 is more important for y_1 than for y_2.

- *Revealing both when $\tilde{\mathbf{S}}^i = [0,0]$ and $\tilde{\mathbf{S}}^i = [1,1]$, if:*

$$\frac{(w_{11})^2 + (w_{21})^2}{(k_1^{\mathrm{C}}+3)} \left[\frac{1}{(k_1^{\mathrm{C}}+3)} + \frac{C_1}{(k_2^{\mathrm{C}}+3)} \pm 2\beta_1 \right] + \frac{(w_{12})^2 + (w_{22})^2}{(k_2^{\mathrm{C}}+3)} \left[\frac{1}{(k_2^{\mathrm{C}}+3)} + \frac{C_2}{(k_1^{\mathrm{C}}+3)} \pm 2\beta_2 \right] \geq 0 \tag{4}$$

- *Revealing both when $\tilde{\mathbf{S}}^i = [0,1]$ and $\tilde{\mathbf{S}}^i = [1,0]$, if:*

$$\frac{(w_{11})^2 + (w_{21})^2}{(k_1^{\mathrm{C}}+3)} \left[\frac{1}{(k_1^{\mathrm{C}}+3)} - \frac{C_1}{(k_2^{\mathrm{C}}+3)} \pm 2\beta_1 \right] + \frac{(w_{12})^2 + (w_{22})^2}{(k_2^{\mathrm{C}}+3)} \left[\frac{1}{(k_2^{\mathrm{C}}+3)} - \frac{C_2}{(k_1^{\mathrm{C}}+3)} \mp 2\beta_2 \right] \geq 0 \tag{5}$$

Where $\beta_r = b_1^i w_{1r} + b_2^i w_{2r}$, $C_r = \frac{w_{11}w_{12}+w_{21}w_{22}}{w_{1r}^2+w_{2r}^2} \in [0,1]$, and $\pm$ meaning that the condition must hold for the most restrictive of these operations, given the sign of the corresponding β.

Proof. See Appendix in Habermacher (2018). □

Lemma 2 shows how communication incentives depend on different measures of conflict of interest and how correlation determines the way decision-specific biases aggregate. Communication is incentive compatible if the (expected) utility gains from increasing the decision-maker's precision compensate the relevant aggregate conflict of interest, depending on β_r^i for some state θ_r. Agent i could indeed be willing to play dimensional non-separable message strategies under centralization; that is, he would reveal some information when observes some realizations and send babbling messages when observing others. The corresponding IC constraints can be found in Appendix A.

There are two main differences with the one-decision case. First, i could find incentive compatible to reveal only one signal. Equation (3) shows the IC constraint for revealing S_r^i. The aggregate conflict of interest in the left-hand side consists of a weighted average of the biases, where the weights relate to the relative importance of the state on each decision. The right-hand side shows the overall influence of revealing that signal, which depends on how it affects both decisions and how informed is the Principal about both states in the equilibrium under consideration. The latter depends negatively on the number of other agents revealing the same information and positively on those revealing information about the other state (loss of credibility due to contradictory information).

The other difference with the one-decision case relates to contradictory information. Revealing both signals would have a balanced overall influence on decisions. For i were to reveal contradictory information he would move both decisions in opposite directions, which opens the possibility of dimensional non-separable strategies involving full revelation of contradictory signals and babbling otherwise. Incentive compatibility of such strategies are capture in equation (17) in Appendix A. The same Appendix present the complete characterization of the most-informative equilibrium under centralization. I now focus on the analysis of the optimal organizational structure—for a more thorough discussion of communication incentives under centralization see Habermacher (2018).

Optimal Organizational Structure. Because each agent's incentives depend on his bias and the information transmitted by other agents, the profile of preferences $\mathbf{b}$ determines the communication equilibrium. The Principal thus affects the equilibrium by choosing the allocation of decision rights, given the different equilibria induced by $\mathbf{b}$. She is thus effectively choosing among these equilibria and the chosen one would maximize her ex-ante expected utility. There are two main determinants for that utility: how informed and how biased each decision will be. The result below shows the expression of the ex-ante expected utility for the Principal, for two generic decision-makers j' and j''.

Lemma 3 (Principal's ex-ante expected utility)**.** *Consider the equilibrium* $(\mathbf{y}, \mathbf{m})$ *in which* $j', j'' = \{P, 1, ..., n\}$ *decide on* y_1 *and* y_2*, respectively. The equilibrium is characterized by the number of agents reporting truthfully to each decision-maker,* k_1'*,* k_2'*,* k_1''*, and* k_2''*. Then, the Principal ex-ante expected utility is given by:*

$$\hat{U}^P(\mathbf{b}, j', j'') = -\left[(b_1')^2 + \frac{(w_{11})^2}{6\big(k_1'+2\big)} + \frac{(w_{12})^2}{6\big(k_2'+2\big)}\right] - \left[(b_2'')^2 + \frac{(w_{21})^2}{6\big(k_1''+2\big)} + \frac{(w_{22})^2}{6\big(k_2''+2\big)}\right] \tag{6}$$

Proof. See Appendix B.2 □

The terms in brackets correspond to the expected utility associated to each decision, comprising the sum of the ex-ante variances in the equilibrium in which j' and j'' decide, given $\mathbf{b}$ and the influence of each state on each decision.The variance is decreasing on k_1', k_2', k_1'', and k_2'' such that the Principal has incentives to allocate decision-rights to players with central preferences on the corresponding dimension. But if she delegates the expected utility involves a loss of control because the decision will be biased. The 'traditional' argument for delegation focuses on this channel: informational gains must compensate the Principal for the loss of control.

There could be additional utility gains for the Principal in this framework. If she delegates y_1 only, agents still have to send messages to her regarding y_2 and their incentives for communication will only depend on b_2^i (see equations in Lemma 1 for $b_2^j = b_2^P = 0$). This could alter agents' incentives as compared to centralization. In particular, the presence of negative informational externalities related to y_1 will lead to an increase in the number of agents revealing information when she delegates that decision: $k_1^{\text{P}\tilde{d}} > k_1^{\text{C}}$ and/or $k_2^{\text{P}\tilde{d}} > k_2^{\text{C}}$. The intuition is straightforward and relates to the very notion of informational externalities. There are agents with $b_2^i \simeq 0$ who are not willing to reveal much information under centralization because $|b_1|$ is large. Under partial delegation their communication with the Principal only affects y_2 and, thus, they are willing to reveal more information. The following observation summarizes the result.

Corollary 1. *When the Principal delegates y_d to agent j and retain decision power on $y_{\tilde{d}}$, utility gains derived from better information (if any) consist of:*

- ***Direct Informational Gains**: increased precision in the delegated decision. Relates to j's preferences being 'more central' than the Principal's, whose utility gains materialize if:*

$$DIG_j(y_d) \equiv \frac{(w_{d1})^2}{6}\left[\frac{1}{\left(k_1^{\text{C}}+2\right)} - \frac{1}{\left(k_1^{j}+2\right)}\right] + \frac{(w_{d2})^2}{6}\left[\frac{1}{\left(k_2^{\text{C}}+2\right)} - \frac{1}{\left(k_2^{j}+2\right)}\right] \geq (b_d^j)^2$$

- ***Indirect Informational Gains**: increased precision on the retained decision. Relate to partial delegation breaking informational interdependencies. Utility gains for the Principal take place if:*

$$IIG(y_{\tilde{d}}) \equiv \frac{(w_{\tilde{d}1})^2}{6}\left[\frac{1}{\left(k_1^{\text{C}}+2\right)} - \frac{1}{\left(k_1^{\text{P}\tilde{d}}+2\right)}\right] + \frac{(w_{\tilde{d}2})^2}{6}\left[\frac{1}{\left(k_2^{\text{C}}+2\right)} - \frac{1}{\left(k_2^{\text{P}\tilde{d}}+2\right)}\right] \geq 0$$

where y_d, $y_{\tilde{d}} = \{y_1, y_2\}$, $y_d \neq y_{\tilde{d}}$, and $k^{\text{P}d} = \{k^{\text{P}1}, k^{\text{P}2}\}$.

Direct Informational Gains (DIG, onwards) require that at least $k_1^j > k_1^{\text{C}}$ or $k_2^j > k_2^{\text{C}}$. Similarly, Indirect Informational Gains (IIG onwards) require that either $k_1^{\text{P}\tilde{d}} > k_1^{\text{C}}$ or $k_2^{\text{P}\tilde{d}} > k_2^{\text{C}}$ (or both). Now, k_1^{C} and k_2^{C} are determined by the system of equations defined in Lemma 2 and relate to the distribution of preferences *when decisions are interdependent*. Similarly k_r^j and $k_r^{\text{P}\tilde{d}}$ are related to equations in Lemma 1, where informational interdependencies are not present. The following result summarizes the necessary conditions for existence of informational gains.

Proposition 2. *Let $h, i, j \in N$, such that $h, i \neq j$; consider the case in which the Principal delegates y_1 to agent j and retain decision power on y_2. Then, Direct Informational Gains arise if and only if for every h such that b_1^h satisfies* (3) *there exist an agent i such that b_1^i satisfies* (1)*. In addition, there must exist an agent such that:*

$$|b_1^i - b_1^j| < \left|b_1^i + b_2^i \frac{w_{21}}{w_{11}}\right| \tag{7}$$

And <u>*Indirect Informational Gains*</u> *arise if and only if for every* h *such that* $b^h_{\tilde{d}}$ *satisfies* (3) *there exists an* $i \in N$ *such that* $b^i_{\tilde{d}}$ *satisfies* (1). *In addition, there must exist an agent such that:*

$$|b^i_2| < \left| b^i_1 \frac{w_{12}}{w_{22}} + b^i_2 \right| \tag{8}$$

Proof. See Appendix B.5 □

First, note that the ratios $\frac{w_{21}}{w_{11}}$ and $\frac{w_{12}}{w_{22}}$ are both lower than one. This means the condition for DIG puts more weight on the bias for the delegated decision, while that for IIG does the same for the retained. According to the lemma, then, DIG arise whenever the new decision-maker has more central preferences in the delegated decision than the Principal under centralization. For DIG to arise there must be at least as many agents revealing to him as those revealing the same signal to the Principal under centralization (first part of the statement). But there must also be agents who do not reveal that signal to her in the latter case—j's preferences are strictly more central than the Principal's on that dimension. The left-hand side in (7) must be sufficiently small.

The argument is similar to Dewan et al. (2015) in that informational gains are crucial determinants of allocation of authority. In their paper there are many decisions depending on a common state; under private communication, then, authority over all decisions should be allocated to the *most moderate* agent (Proposition 1 in that paper). In the present analysis, the agent's preferences should be central related to the delegated dimension. There could also be a role for negative informational externalities, since lack of communication under centralization may be due to high conflict of interest on the retained decision (second term in the right-hand side of (7)). As shown below this argument is more important for IIG.

The second part of Proposition 2 presents the most interesting result: Indirect Informational Gains arise if and only if there are negative informational externalities under centralization. As defined in Levy and Razin (2007), these externalities take place when i's bias with respect to y_d is so large that he would not reveal any information under centralization, even though $|b^i_{\tilde{d}}|$ is very small. When the Principal delegates the polemical decision then any information transmitted by i affects only y_2 and his incentives for communication increase. In this way she breaks the informational interdependence and gets rid of the negative externalities, improving upon centralization if IIG are sufficiently large.

Full Delegation is optimal when there are two different agents with central preferences and no informational externalities associated to retaining authority. The following observation summarizes the intuition behind Full Delegation.

Observation 1. *When the Principal delegates both decisions, it is due to Direct Informational Gains in both —that is,* $k_1^{'} > k_1^{\text{C}}$ *and* $k_2^{''} > k_2^{\text{C}}$.

The allocation of decision rights depends then on the different communication equilibria induced by the profile of preferences. From the discussion above it is clear that the argument can be reduced to whether the direct and/or indirect informational gains are sufficiently large, where sufficiency involves the loss of control from delegation. Lemma 10 in Appendix A provides a precise formulation of this argument, and serves as a formal counterpart of the discussion below.

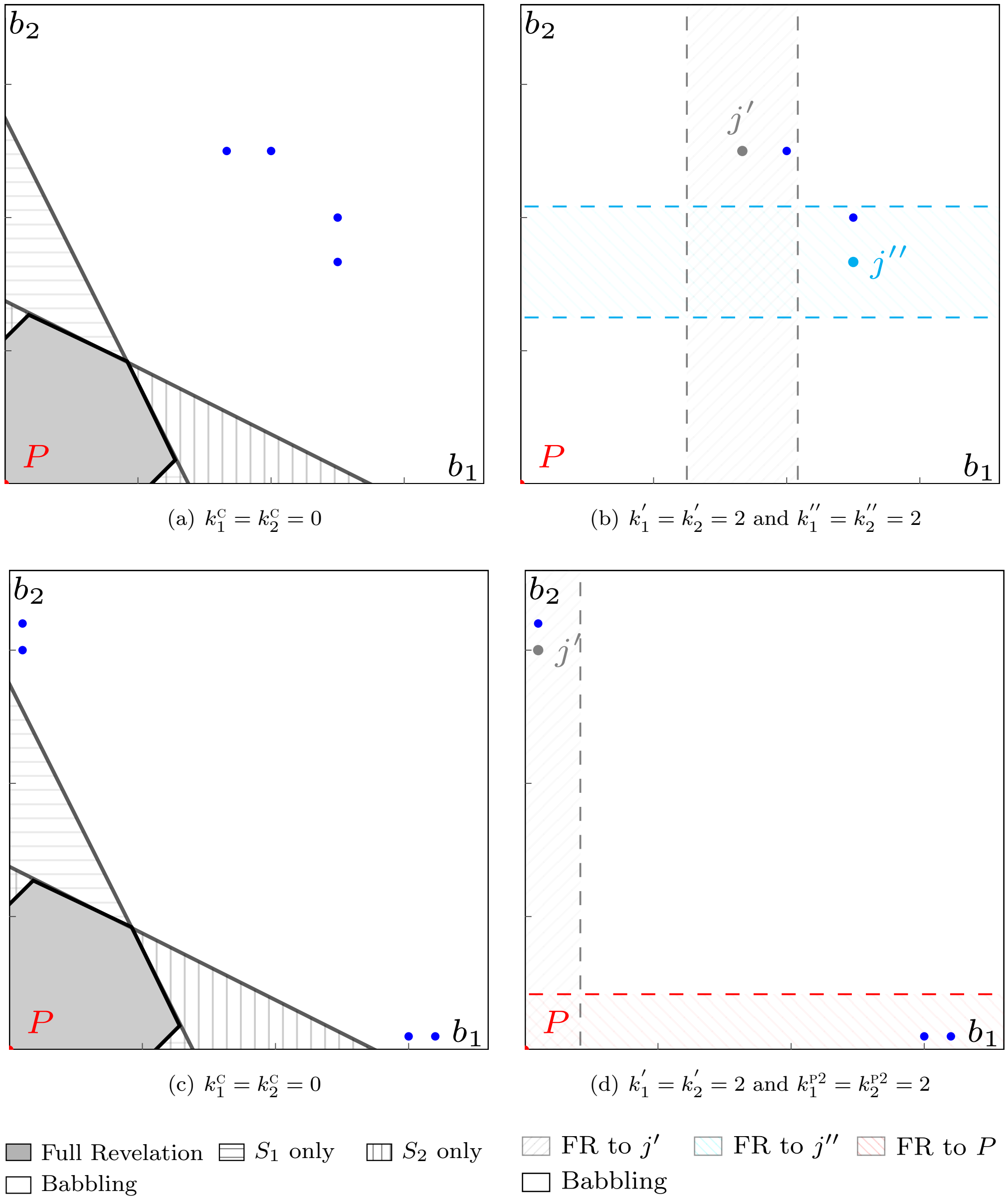
The Principal finds optimal to delegate both decisions when there are two agents with central preferences on the corresponding dimension. Again, because delegation breaks interdependence, the conflict of interest is measured by the decision-specific bias. But informational gains must at least compensate the Principal's utility losses from biased decisions, as well as the potential indirect informational gains if she retained any single decision. Full delegation is then optimal if either there are no indirect informational gains or they are small relative to the direct gains on the corresponding decision.

It could also be the case that the *DIG* associated to one decision are so large that the Principal is willing to tolerate some loss of information on the other decision (with respect to centralization). In such a case, delegation will dominate partial delegation only if the 'informational loss' is lower when delegating the corresponding decision (as compared to retaining authority). The informational gains should compensate for any 'informational loss' in the other decision (either delegated or retained).

In order to see this, Figure 2 compares centralization (left panels) with alternative organizational structures. Panels (a) and (c) show the equilibrium message strategies under centralization for two different distribution of agents' preferences, while panels (b) and (d) show the corresponding equilibrium strategies for full and partial delegation, respectively. Agents ideal decisions are represented as blue dots. In panel (a) agents are not revealing any information under centralization. When the Principal delegates y_1 to agent j' and y_2 to j'' –panel (b)– each agent has himself a signal and one more agent revealing all his information. These informational gains more than compensate the loss of control associated to their biases. Note that j' and j'' are the agents whose bliss points are closer to the Principal's in the corresponding decisions, such that the utility loss due to the bias is also minimal among all agents.

Panels (c) and (d) shows a case when Partial Delegation is optimal. Again, under centralization all agents play babbling message strategies. By delegating y_1 to agent j' the Principal now receive information from two agents. The fact that the same agents are babbling when both issues are decided jointly illustrates the notion of negative informational externalities. Delegation of y_1 to j' breaks this interdependency and allows

Figure 2: Optimal Organizational Structure: delegation



(a) $k_1^{\text{C}} = k_2^{\text{C}} = 0$

(b) $k_1^{'} = k_2^{'} = 2$ and $k_1^{''} = k_2^{''} = 2$

(c) $k_1^{\text{C}} = k_2^{\text{C}} = 0$

(d) $k_1^{'} = k_2^{'} = 2$ and $k_1^{\text{P2}} = k_2^{\text{P2}} = 2$

Note: $w_{11} = w_{22} = \frac{2}{3}$

the Principal to receive information associated to, at least, one decision.[15]

Finally, centralization is ex-ante optimal when the informational gains from delegation are insufficient (if any) relative to the loss of control. In other words, the Principal's preferences are sufficiently central in

[15]It is straightforward to note that the Principal could have obtained the same ex-ante welfare by delegating y_2 to the leftmost agent with preferences in the bottom right corner. Note that the optimality of partial delegation does not depends on asymmetries on agents' preferences as in Rantakari (2008).

both decisions given the correlation between them. This has to be true even in the presence of negative externalities, meaning that the utility loss from delegation is larger than the informational gains derived from breaking those externalities.

Because the choice among organizational structure depends on the trade-off between informational gains and loss of control, in the following subsection I explore this trade-off in more depth.

2.1 Relationship between informational gains and loss of control

Informational gains from delegation are incomplete in this framework. Unlike Gilligan and Krehbiel (1987, 1989); Dessein (2002), agents with decision rights have imperfect information in equilibrium. Indeed, the Principal will consider delegation if informational gains are sufficiently large given the agent's bias. The larger his bias the larger the informational gains in order to compensate the Principal for the loss of control. This defines a relationship between minimum informational gains and loss of control—or, equivalently, informational gains and maximum admissible loss of control. The following result presents this relationship, which depends on the Principal's ex-ante expected utility under different organizational structures.

Proposition 3 (Maximum Admissible Loss of Control). *Let $w_{11} = w_{22} = w$ and let players $j', j'' = \{P, 1, 2, ...\}$ be the decision-makers for y_1 and y_2 (respectively), such that $j' \neq j''$. In addition, let $\hat{\mathbf{b}}^{\text{D}} = [b_1'; b_2'']$ be the decision-makers' biases on the corresponding decision and $k_1^{\text{C}} = k_2^{\text{C}} = 0$, such that informational gains are non-negative. Then, the maximum $\hat{\mathbf{b}}^{\text{D}}$ for which the Principal is willing to delegate at least one decision is given by:*

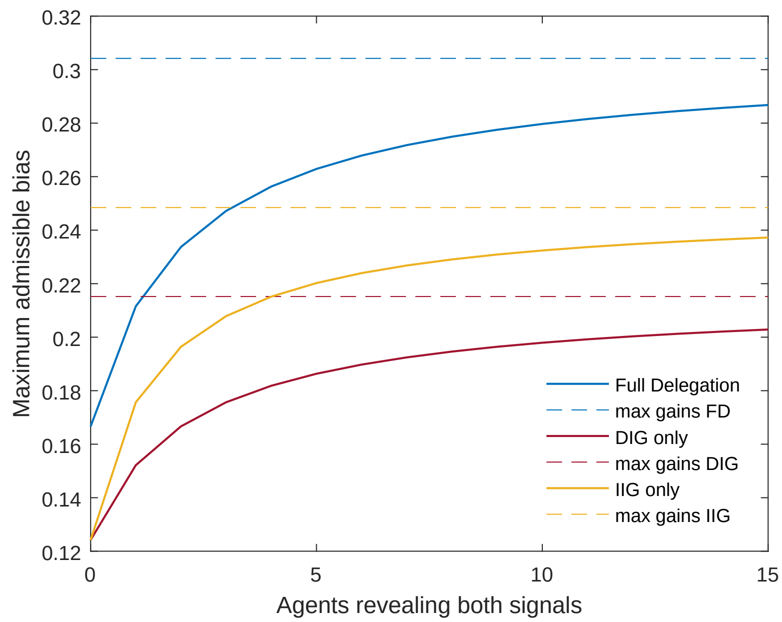
$$||\hat{\mathbf{b}}^{\text{D}}|| \equiv \left[\frac{w^2}{6}\left[1 - \left[\frac{1}{(k_1'+2)} + \frac{1}{(k_2''+2)}\right]\right] + \frac{(1-w)^2}{6}\left[1 - \left[\frac{1}{(k_2'+2)} + \frac{1}{(k_1''+2)}\right]\right]\right]^{\frac{1}{2}}$$

Proof. See Appendix B.6 ☐

The assumptions in Proposition 3 are without loss of generality. Non-negative informational gains allow me focus on the benefits from delegation (getting rid of $\frac{1}{(k^{\text{C}}+2)}$ from the expressions above), while 'symmetry' allows to group agents' equilibrium information by state.

The maximum bias the Principal is willing to tolerate is positively associated to informational gains from delegation. In the equation above, the left-hand side represents the relevant measure for the loss of control: how far are both decisions to the Principal's bliss point for any realization of the states. Under partial delegation one of the elements in $\hat{\mathbf{b}}^{\text{D}}$ is zero, and the possibility of breaking negative externalities could lead to delegation even if the new decision-maker is uninformed in equilibrium. The righ-hand side represents the informational gains. In the first term the informational gains are associated to the most important state for each decision; that is, how much information j' receives on θ_1 and j'' on θ_2. Improving information transmission through these channels give the most significant informational gains from delegation.

Figure 3: Admissible loss of control as a function of informational gains



Note: $w_{11} = w_{22} = \frac{2}{3}$, $k^{\text{C}} = 0$

Figure 3 shows $||\mathbf{b}^{\text{D}}||$ as a function of the number of agents revealing the corresponding signals, under the assumptions of Proposition 3. In particular, I assume the Principal does not receive any truthful message under centralization (a restriction on the distribution of agents' preferences). The blue lines represent the relationship between informational gains and bias under Full Delegation. Minimal informational gains in this case means decision-makers only have their own signals in equilibrium, and the Principal will tolerate up to $||\mathbf{b}^{\text{D}}|| = 0.176$. But this figure increases dramatically as long as both decision-makers start receiving information from other agents. In particular, the first 5 agents revealing both signals explain 85% of the maximum admissible bias—corresponding to the hypothetical case of both decision-makers being perfectly informed ex-post (dashed blue line).[16] Note that the horizontal axis in this case represents the number of agents revealing both signals to each decision-maker.

The red lines represent the case of direct informational gains only: the loss of control the Principal

[16]The maximum admissible bias with $k'_1 = k'_2 = k''_1 = k''_2 = \infty$ is $||\mathbf{b}^{\text{D}}|| = 0.3042$.

tolerates as a function of $k^j_1 = k^j_2$ when there are no indirect informational gains. A pattern similar to the full delegation case appears.[17] Note that 70% of the maximum admissible loss under Full Delegation is explained by informational gains in one decision only. Large informational gains in one dimension can thus justify delegation to biased agent, and in such a case the Principal would not tolerate much more loss of control on the other. In other words, the presence of a biased agent with sufficiently central preferences in one dimension can lead to partial delegation, mainly because it blocks delegation of the other decision to more moderate agents with less central preferences.

Finally, the yellow lines represent the case of partial delegation when only the Principal gets informational gains on the retained decision. This is a measure of how important the negative externalities can be. It measures the loss of control she is willing to tolerate in order to recover communication on the decision affected by externalities. The maximum bias when indirect informational gains are maximal ($k^{\text{P}d} = \infty$) is $||\mathbf{b}^{\text{D}}|| = 0.2484$. She would tolerate some loss even when not receiving any information in equilibrium, because j observes himself two signals. This relationship is increasing in the information held by j.[18]

In summary, the Principal is willing pay a utility cost in order to improve the precision of decisions. The increased precision could arise because there are agents whose preferences are more central given the profile of biases, or because of negative informational externalities that impede information transmission from some agents. In both cases delegation leads to informational gains. The costs, on the other hand, relate to the amount of bias she would allow on the delegated decision(s), which is increasing on the informational gains just described. To my knowledge this is the first characterization of such relationship between informational gains and loss of control as a fundamental driver of organizational structure.

3 Endogenous Information Acquisition

In this section I analyse agents' incentives to acquire information before the communication stage, but after decision-rights have been allocated. Agents could in principle decide on how much information to have about each state, involving both intensive and extensive margins of information acquisition. Here, however, I only allow each agent to acquire at most one binary signal per state (extensive margin), which is the simpler version of the problem because agents do not decide on their influence on beliefs. In a future paper I will address incentives to acquire information as way to 'buy influence' on a given set of decisions.

Each agent has thus access to one signal per state and decides which realizations to observe (if any).

[17] When j' is fully informed in equilibrium the Principal will tolerate a bias of $|0.2152|$.

[18] Its image is the set of biases between the red and blue dashed-lines when j is perfectly informed about both states.

Formally, let $\mathfrak{s}^i \in \{\{\emptyset\}, \{\tilde{S}_1\}, \{\tilde{S}_2\}, \{\tilde{S}_1, \tilde{S}_2\}\}$ be agent i's information-acquisition decision.[19] With this formulation i's type is given by the realizations of both signals, but he decides the extent to which he observes his type.

Definition 3. *The information structure for agent i in the Information Acquisition game consists of the following elements:* $\mathbf{S}^i = [S_1^i; S_2^i]$ *are the signals available to him,* $\tilde{\mathbf{S}}^i = [\tilde{S}_1^i; \tilde{S}_2^i]$ *the realization of the corresponding signals (his type), and* $\mathfrak{s}^i \in \{\{\emptyset\}, \{\tilde{S}_1\}, \{\tilde{S}_2\}, \{\tilde{S}_1, \tilde{S}_2\}\}$ *the information he actually decides to observe.*

The costs of different information structures are captured by the function $C(\circ)$, such that $C\left(\{\tilde{S}_1, \tilde{S}_2\}\right) > C\left(\{\tilde{S}_1\}\right) = C\left(\{\tilde{S}_2\}\right) > C(\emptyset) = 0$. The Principal has no direct access to information.[20] For all $i = \{1, \ldots, n\}$, agent i's pay-off is given by:

$$U^i(\boldsymbol{\theta}, \mathbf{x}, \mathbf{b}^i, \mathfrak{s}^i) = -\sum_{y_d=\{y_1,y_2\}} \left(y_d - \delta_d(\theta_1, \theta_2) - b_d^i\right)^2 - C(\mathfrak{s}^i)$$

Incentives to acquire information depend on each agent's cost-benefit analysis. Information costs were defined above, while the benefits depend on the influence on decisions. But for information to be influential the agent in question must truthfully reveal it to at least one decision-maker. As a consequence, the agent only acquires information he is willing to reveal —a conclusion that is similar to Di Pei (2015). Acquiring information only about θ_1 gives him more influence on y_1 because of the imperfect correlation, and similarly for θ_2 and y_2. This brings up the notion of specialization in the present framework: I define a *specialist* as an agent who is informed about one signal only.

Figure 4 shows the timing of the game. The allocation of decision rights is observed by all agents. Knowing who decides what, each agent chooses the information he will observe (if any). His investment (but not his information) is common knowledge and communication takes place as in the previous section. In this line, let i be a generic agent and j a generic decision-maker, let $j', j'' \in \{P, 1, ..., n\}$ be the decision-makers for y_1 and y_2, respectively, such that $i = \{1, ...n\}$ and $j = \{j', j''\}$. Let $k_r^j \equiv k_r^*(\mathbf{m}_j^*(\mathfrak{s}^*))$ be the number of truthful messages decision-maker j receives in equilibrium, and $k_r' \equiv k_r^*(j')$ and $k_r'' \equiv k_r^*(j'')$ refer to the number of truthful messages for decision maker of y_1 and y_2, respectively.

A PBE of this game (equilibrium henceforth) is then defined by the decision vector, $\mathbf{y}_d^*$, and collections of message and acquisition strategies for each agent and decision-maker j, $\mathbf{m}_j^* = \{\ldots, \mathbf{m}_j^{i*}, \ldots\}$ and $\mathfrak{s}^* =$

[19] Consisting on observing no signal, only that related to θ_1, that related to θ_2, or both signals, respectively.

[20] The Principal's preferences are captured by $U^P(\boldsymbol{\theta}, \mathbf{x}) = -\left(y_1 - \delta_1(\theta_1, \theta_2)\right)^2 - \left(y_2 - \delta_2(\theta_1, \theta_2)\right)^2$.

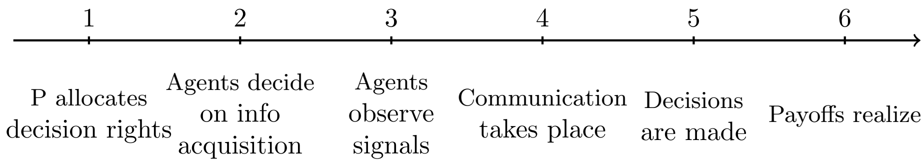


Figure 4: Timing of the Org. Structure / Info Acquisition game.

$\{\dots, \mathfrak{s}^{i*}, \dots\}$. The expressions for optimal actions and messages are similar to those of the previous section, noting that $k^*_r(\mathbf{m}^*(\mathfrak{s}^*))$, $y^*_d(\mathbf{m}^*_j(\mathfrak{s}^*))$, and $\mathbf{m}^{*i}_j\left(\mathbf{m}^{\text{-}i}(\mathfrak{s}^i, \mathfrak{s}^{\text{-}i})\right)$. Agent i's information acquisition strategy is given by:

$$\mathfrak{s}^{i*} = \arg\max_{\mathfrak{s}^i} \left\{ E\Big[-\left(y_1\left(\mathbf{m}^i_{j'}(\mathfrak{s}^i), \mathbf{m}^{\text{-}i}_{j'}\right) - \delta_1 - b^i_1\right)^2 - \left(y_2\left(\mathbf{m}^i_{j''}(\mathfrak{s}^i), \mathbf{m}^{\text{-}i}_{j''}\right) - \delta_2 - b^i_2\right)^2 \Big] - C(\mathfrak{s}^i) \right\}$$

Agent i's equilibrium message strategy depends on the information he acquired in an earlier stage of the game. Both his message and information acquisition decisions depend on beliefs about other agents' strategies. The fact that information acquisition is common knowledge at the communication stage (overt game) simplifies the beliefs space. In other words, each agent forms a conjecture about other agents' acquisition strategies when deciding about his own and another conjecture about others' message strategies in the communication stage, the latter depending on the information other agents have.

At the communication stage, incentive compatibility depends on the signals the agent possesses. If the agent acquired both signals the IC constraints for communication are those in Lemmas 3 and 4. But that is not longer true if the agent acquired only one signal because there is no contradictory information. Recall how communication incentives were affected when the agent has information about two independent states that affect the same decision: when each piece of information moves the decision in different directions he experiences stronger incentives to deviate from any influential equilibrium. This led to a credibility loss present in all message strategies. Acquiring only one signals kills such possibility, resulting in enhanced incentives for truthful communication. In particular, a specialist will be willing to reveal his information for a broader set of biases than when he observes both signals.

Now, costly information acquisition means that each agent will invest in a signal only if he expects to benefit from it; that is, revealing it truthfully. Incentive compatibility at the information acquisition stage then requires incentive compatibility at the communication stage in equilibrium.The Lemma below summarizes the result.

Lemma 4. *Let* $(\mathbf{y}^*, \mathbf{m}^*, \mathfrak{s}^*)$ *be equilibrium strategy profiles for the Principal and all agents. The equilibrium* *is characterized by the number of truthful messages decision-makers receive,* $k_1^j\left(\mathbf{m}_j^*(\mathfrak{s}^*)\right)$ *and* $k_2^j\left(\mathbf{m}_j^*(\mathfrak{s}^{i*})\right)$*, for* $j = \{j', j''\}$*. Then, agent* i *equilibrium information acquisition strategy,* $\mathfrak{s}^{i*}$*, satisfies:*

- $S_r \in \mathfrak{s}^{i*}$ *only if truthful revelation to* j *is incentive compatible, given* $k_r^j\left(\mathbf{m}_j^*(\mathfrak{s}^*)\right)$*;*
- $\{S_1, S_2\} \in \mathfrak{s}^{i*}$ *only if full revelation to* j *is incentive compatible, given* $k_r^j\left(\mathbf{m}_j^*(\mathfrak{s}^*)\right)$*.*

Proof. See Appendix C.1. □

The main implication of Lemma 4 is that the choice of organizational structure will affect agents' incentives for information acquisition, because it determines the relevant IC constraints. Incentives to invest in information depends on the possibility of being influential. But credibility hinges on both the conflict of interest and the number of other agents revealing similar information. The distribution of biases is common knowledge, so each agent form a conjecture about the number of other agents revealing each piece of information in order to decide what signal to invest on (if any). Then, it should be true that if agent i acquires a given signal in equilibrium he expects to reveal it at the communication stage, otherwise he would have been better-off had he not invested on it. It is not surprising that this conclusion is similar to Di Pei (2015): information structures available to agents and the cost function satisfy assumptions 1 and 2 of that paper ("Richness" and "Monotonicity"). Both seem rather natural assumptions in my framework: for a given information structure a 'coarser' alternative means investing in less signals, which will also be cheaper than the original choice.

Incentive compatibility means a given information structure leaves i ex-ante better-off than any alternative. Now, by Lemma 4 agent i acquires a signal that he is willing to reveal in the equilibrium under consideration, which is equivalent to i actually choosing between alternative message strategies. The comparison between different information structures thus involve different amounts of information transmitted to the Principal. In Appendix C.3, I derive the IC constraint for information acquisition. There I show that i's incentives depend on the expected change in the number of truthful messages the Principal receives in equilibrium (k_1 and k_2).[21] In other words, i's optimal information acquisition strategy balances the benefits from increased precision and the cost of that information.

Having more signals *per se* do not affect other agents' communication incentives: agent i only affects k_r^j by truthfully revealing the acquired signal.[22] So i acquires signals he is willing to reveal (Lemma 4), and if

[21] More precisely, on agent i's conjecture about k_1 and k_2.

[22] More formally, incentives for communication depend on having acquired the signal, on the agent's bias vector, and on k_1^j and k_2^j. Then, for i acquiring S_r^i *off-path* to change h's conjecture about k_r^j, b^i should be such he is willing to reveal that signal. In such a case, h (off-path) conjecture for k_r^j should be larger than the equilibrium value, but then i would be willing to reveal S_r^i in equilibrium and would have acquired it.

he fails to reveal any piece of information (on the equilibrium path) no other agent will change his previous equilibrium message strategy. The number of truthful messages does not change when i acquires a signal he does not reveal, but since he bears the costs will find optimal not to acquire it in the first place.

When i acquires only one signal he never observes contradictory information and, thus, incentives to reveal it are not affected by beliefs about the other state. This enhances communication incentives, as the lemma below shows.

Lemma 5. *Let $S_r \in \mathfrak{s}^{i*}$ and $S_{\tilde{r}} \notin \mathfrak{s}^{i*}$ for $\theta_r \neq \theta_{\tilde{r}}$, and $k_r^j\left(\mathbf{m}_j^*(\mathfrak{s}^*)\right)$ be i's conjecture about other agents revealing their information about θ_r to decision-maker j. Then, agent i's IC constraint for revealing S_r^i is:*

- *When $j = P$ decides on both issues (centralization),*

$$|\beta_r| \leq \frac{(w_{1r})^2 + (w_{2r})^2}{2\left(k_r^{\mathrm{C}} + 3\right)} \tag{9}$$

- *When j decides on y_d only,*

$$|b_d^i - b_d^j| \leq \frac{w_{dr}}{2\big(k_r^j + 3\big)} \tag{10}$$

Proof. See Appendix C.2 □

The main difference between the above expressions and their equivalents in Lemmas 1 and 2 lies on the right-hand sides; those in the above expressions are larger than when i observes both signals because there is no contradictory information when i observes only one signal. A direct implication is that the set of bias for which revealing S_r^i is incentive compatible is strictly larger when the agent only acquires that signal—the possibility of *not acquiring a signal* enhances communication prospects. In other words, for finite k_1^j and k_2^j there exists a non-empty set of bias vectors such that any agent with bias in that set would reveal one signal only if he does not have information about the other.

Another relevant implication of Lemma 5 is that it rules out dimensional non-separable message strategies under delegation. First note that the right-hand side of (10) is larger than that of (2), which means that the set of bias vectors for which the former holds includes that of the latter (given they are measured with respect to the same conflict of interest). More importantly, the decision-maker always prefer to receive information about one signal for sure than full revelation half of the time; that is, the first strategy dominates the second for the set of biases in which both IC constraints hold (see Appendix C.2).

But incentive compatibility of communication is only a necessary condition for information acquisition. Information costs are exogenous and similar across agents and signals. Expected benefits depend on the influence on decisions, which in turn depend on how many other agents reveal the same information to the same decision-maker (k_r^j). Agent i expects revealing S_r^i to have a small effect on beliefs when many other senders are revealing information about the same state. Given costs are strictly positive there exists a k_r^j for which acquiring S_r^i is not incentive compatible (despite revealing it could still be). The following result complete the characterization of i's incentives to acquire information –a condition I call *cost-effectiveness*– and the maximum number of agents for which acquiring that signal is indeed incentive compatible.

Lemma 6. *Let k_r^j denote agent i's conjecture about other agents revealing S_r^i truthfully to decision-maker j.*

Centralization: *acquiring signal S_r^i is cost-effective for i under centralization if:*

$$C(S_r^i) \leq \frac{(w_{1r})^2 + (w_{2r})^2}{6(k_r^C + 2)(k_r^C + 3)} \tag{11}$$

When willing to play DNS message strategies, cost effectiveness requires:

$$2\,C(S_1^i, S_2^i) \leq \frac{(w_{11})^2 + (w_{21})^2}{6(k_1^* + 2)(k_1^* + 3)} + \frac{(w_{12})^2 + (w_{22})^2}{6(k_2^* + 2)(k_2^* + 3)} \tag{12}$$

Delegation: *acquiring signal S_r^i is incentive compatible for agent i if for at least one decision, y_d, with the corresponding decision maker j is true that:*

$$C(S_r^i) \leq \frac{(w_{dr})^2}{6(k_r^j + 2)(k_r^j + 3)} \tag{13}$$

Proof. See Appendix C.3 and C.4. □

A given signal is worth acquiring if its influence on decision(s) is sufficiently large. The influence of a signal depends on how many other agents reveal their information truthfully, which in turn depends on the organizational structure. Under centralization, revealing a given signal influences both decisions. Under delegation the influence depends on whether i reveals the signal to one or both decision-makers. In the latter case the overall influence will be similar to centralization, but if he only reveals to one decision-maker he will affect the associated decision. As a consequence, the expected benefit of a given signal is weakly lower under delegation than centralization.

In this context there is also the possibility of dimensional non-separable strategies. Cost-effectiveness is more demanding though, because i expects to reveal information for half of the possible signal realizations.

The costs of acquiring both signals must then be sufficiently low such that full revelation half of the time compensates. But if revealing one signal is also incentive compatible for i, then this strategy is preferred to the previous by both the Principal and him.

The expected influence of truthful revelation is decreasing in the number of other agents revealing the same information. This imposes a limit on the number of agents for whom cost-effectiveness hold with respect to a given signal. The result below formalizes this intuition.

Corollary 2. *The maximum number of agents acquiring S_r in any equilibrium under centralization is given by:*

$$K_r^{\text{C}} = \left\lfloor \left[\frac{1}{4} + \frac{[(w_{1r})^2 + (w_{2r})^2]}{6\, C(S_r^i)} \right]^{1/2} - \frac{5}{2} \right\rfloor + 1 \tag{14}$$

And under delegation, the maximum number of agents acquiring S_r in any equilibrium will be:

$$K_r^{\text{D}} \in \left[\left\lfloor \left[\frac{1}{4} + \frac{(\hat{w}_{dr})^2}{6\, C(S_r)} \right]^{1/2} - \frac{5}{2} \right\rfloor + 1\, ;\, K_r^{\text{C}} \right] \tag{15}$$

Where $\hat{w}_{dr} \equiv \min\{w_{1r}, w_{2r}\}$

Expression (14) represents the maximum number of agents other than i for whom investing on S_r is cost-effective (and revealing it is IC), under centralization. This number is denoted by K_r^{C} and depends on the overall influence of θ_r and the cost of acquiring the associated signal. Under delegation, the same number (K_r^{D}) depends on how many agents are willing to reveal that information to both decision-makers. If the distribution of preferences is such that there are many of such agents, then $K_r^{\text{D}} = K_r^{\text{C}}$ because each agent affects both decisions. But the influence for agents revealing their signals to one decision-maker only is lower, and so the maximum number of those willing to acquire that signal.

Having defined the notion of incentive-compatibility for information acquisition I proceed to characterize agent i's equilibrium strategies.

Agents' equilibrium strategies. Now I combine the results in Lemma 4 and Lemma 6 to characterize agent i's equilibrium information acquisition and message strategies. I start with the case of centralization ($j' = j'' = P$) and then proceed to the case of delegated decisions. The following result summarizes the intuitions developed in the previous discussion, presenting the equilibrium acquisition and message strategies for a generic agent under centralization.

Proposition 4 (Equilibrium under Centralization)**.** *In the most informative equilibrium under centralization*

$(\mathbf{y}^*, \mathbf{m}^*, \mathfrak{s}^*)$, *agent i only acquires signals that are cost-effective and incentive compatible. In particular, i's equilibrium strategies are given by:*

Acquiring and revealing both signals: *if and only if conditions* (11) *and* (9) *hold for both signals, and* (5) *hold.*

Acquiring both signals and playing a dimensional non-separable strategy: *if condition* (12) *hold for both signals and* (9) *does not at all, in the following cases:*

- *Fully revealing both signals when they coincide and babbling otherwise, if condition* (16) *holds;*

- *Fully revealing both signals when they do not coincide and babbling otherwise, if condition* (17) *holds.*

Acquiring and revealing one signal only. *Agent i acquires and reveals S_1^i if* (9) *and* (11) *hold with respect to θ_1 and one of the following is true:*

- *Revealing S_2^i is not IC —i.e.* (9) *does not hold for θ_2; or*

- *Acquiring S_2^i is not CE —i.e.* (11) *does not hold for θ_2; or*

- *Acquiring S_2^i is CE and revealing it is IC, but revealing both signals is not IC —i.e.*(3) *and* (11) *hold for both signals, but* (5) *does not and* $\frac{(w_{1r})^2+(w_{2r})^2}{(k_r^*+2)(k_r^*+3)} \geq \frac{(w_{1\tilde{r}})^2+(w_{2\tilde{r}})^2}{(k_{\tilde{r}}^*+2)(k_{\tilde{r}}^*+3)}$

For $r \neq \tilde{r}$.

Acquiring no signal, *if only if any of the statements below is true:*

- *No signal is CE to acquire —i.e. condition* (11) *does not holds for any signal; and/or*

- *No signals is IC to reveal —i.e. condition* (9) *does not hold for any signal, nor* (5) *holds.*

Proof. See Appendix C.5 □

As discussed earlier, the number of agents revealing the same piece of information is an important determinant of cost-effectiveness. It could lead some agents to acquire less information than what each is willing to reveal, resulting in either specialization or non-investment. The possibility of abstaining to acquire a given signal can enhance incentives for communication (for the other signal) because it kills the effects of contradictory information.

Dimensional non-separable message strategies can arise under centralization. As in the pure communication game these strategies take the form of full revelation for some realizations and babbling for the rest. Because any of these involves acquiring both signals and revealing them half of the time, they arise when costs are sufficiently low and only if revealing one signal is not IC. Then, typically agent i prefers to reveal a single signal than play a DNS strategy, provided both are incentive compatible and cost-effective.[23]

Similar intuitions apply to the case of delegation. The result below characterizes equilibrium strategies for agent i and decisions y_d and $y_{\tilde{d}}$.

Proposition 5 (Equilibrium under Delegation)**.** *When the organizational structure involves more than one decision-maker, agent i only acquires signals that are cost-effective and for which communication is incentive compatible. In the most informative equilibrium ,$(\mathbf{y}^*, \mathbf{m}^*, \mathfrak{s}^*)$, i's equilibrium strategies are:*

Acquiring and revealing both signals: *if and only if conditions* (1) *and* (13) *hold for both signals and at least one decision-maker –and the associated decision.*

Acquiring and revealing S_1 only*, if acquiring this signal is both cost-effective and incentive compatible for agent i in the following cases:*

1. *Revealing S_2 is not IC for any decision —i.e. condition* (10) *does not hold for S_2^i; or*
2. *Acquiring S_2 is not CE for any decision —i.e. condition* (13) *does not hold for S_2^i for any decision; or*
3. *Both S_1 and S_2 are CE and IC, but revealing both is not IC with respect to any decision-maker —i.e. conditions* (10) *and* (13) *hold for both signals and at least one decision-maker, but* (1) *does not hold for any of them and $\frac{(w_{dr})^2}{(k_r^*+2)(k_r^*+3)} \geq \frac{(w_{d\tilde{r}})^2}{(k_{\tilde{r}}^*+2)(k_{\tilde{r}}^*+3)}$*

For $r \neq \tilde{r}$.

Acquiring no signal *if only if any of the statements below are true:*

1. *Condition* (10) *does not hold for any signal and any decision, nor* (5) *hold; and/or*
2. *Condition* (13) *does not holds for any signal, any decision.*

Proof. See Appendix C.5. □

In presence of two decision-makers agents acquire a signal if it is cost-effective and incentive compatible to reveal it to at least one of them. As discussed in Lemma 5, the possibility of not acquiring one of the signals makes credible the strategy of revealing only one signal. When such a strategy is incentive compatible it always refers to the state the decision-maker is less informed in equilibrium. In other words, the agent will acquire the signal for which the influence is larger, which in turn implies that DNS are dominated by this strategy (see discussion for the case of centralization). As a consequence, no dimensional non-separable strategy can emerge in equilibrium under delegation.

[23]For a formal discussion see Appendix C.3.

Specialization. Given the characterization of i's equilibrium strategies, I now analyse under which conditions he chooses to specialize. The set of bias vectors for which acquiring one signal is IC is larger than when agents observe both signals because there is no contradictory information when i observes one signal only. There are three generic cases in which specialization constitutes the optimal information acquisition strategy for agent i:

1. Specialization driven by preferences: when i is willing to reveal only one piece of information. Under centralization this happens if (9) holds for one signal only; under delegation if (10) does but (1) does not.

2. Specialization driven by influence: revealing one signal is incentive compatible when i acquired only that signal, but it is not when he has both signals (despite this may be cost-effective). Under centralization this takes place when condition (9) holds for both signals but (3) does not; while under delegation (10) holds for both signals and (1) does not. He then acquires and communicates the signal the Principal will be less informed in equilibrium, and if her information is expected to be balanced i will randomly select to observe one of the signals available to him.

3. Specialization driven by costs: in this case i is willing to reveal information about both states, but their cost is so large that it is optimal to invest in one of them.

I present the intuitions in an example with 2 agents. Let assume $w_{11} = w_{22} = w > \frac{1}{2}$ and $C(\mathfrak{s}^i) = c \times (\# \mathfrak{s}^i)$, and let A_1 and A_2 be the agents. I focus on the centralization equilibrium in which A_1 acquires S_1 while A_2 acquires S_2. The Principal thus receives two signals, $k_1^* = 1$ and $k_2^* = 1$, having ex-post more information than each of the agents. The paper by Alonso et al. (2015) analyses a similar situation in the form of generalist-specialist information structure, where each agent specializes in a different piece of information and fully transmit it to the Principal. The Corollary below formalizes the result and panel (a) in Figure 5 illustrates the set of biases for which A_1 acquires S_1 only.

Corollary 3 (Specialization under centralization)**.** *Suppose that there are only two agents, A_1 and A_2, and the marginal cost of each signal is linear and equal to c. The most-informative equilibrium under centralization, $(\mathbf{y}^*, \mathbf{m}^*, \mathfrak{s}^*)$, consist in A_1 acquiring and revealing information on θ_1 only, and A_2 acquiring and revealing information about θ_2 only, in the following cases:*[24]

[24]Formally, the equilibrium consists in $\mathfrak{s}^{1*} = \{\tilde{S}_1^1\}$ and $m^{1*} = \big\{\{(0,0),(0,1)\},\{(1,0),(1,1)\}\big\}$ for A_1, and $\mathfrak{s}^{2*} = \{\tilde{S}_2^2\}$ and $m^{2*} = \big\{\{(0,0),(1,0)\},\{(0,1),(1,1)\}\big\}$ for A_2.

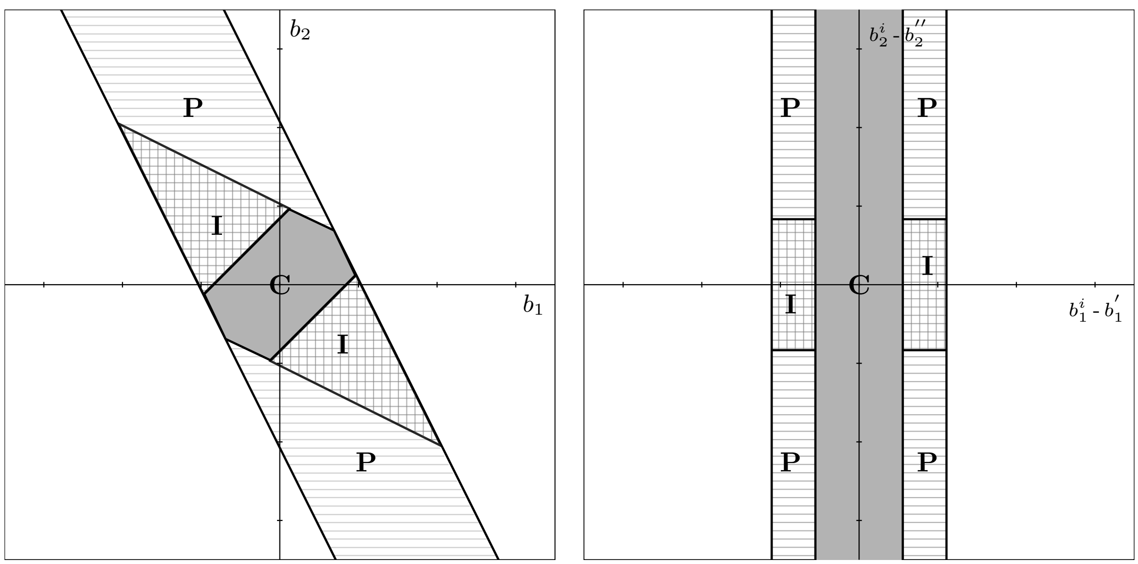
1. *For* $c \leq \frac{w^2+(1-w)^2}{72}$*, if and only if* A_1*'s preferences satisfy* (9) *for* S_1^1*,* A_2*'s satisfy that for* S_2^1*, and neither satisfies* (5)*;*

2. *For* $\frac{w^2+(1-w)^2}{72} < c \leq \frac{w^2+(1-w)^2}{36}$*, if and only if condition* (9) *holds for the corresponding signal for* A_1 *and* A_2*.*

Proof. See Appendix C.6 □

When the cost of a signal is close to zero, information acquisition does not impose restrictions on communication. Agents will then acquire signals that are willing to reveal and they specialize on different signals driven by their preferences, as shown by the striped region in panel (a) of Figure 5). Note that the set of biases spanned by this region is larger than the equivalent when agents have information about both states; A_1's decision of *not to observe* a signal boosts credibility and thus expands the possibilities of communication.

Figure 5: Specialization in the 2-agents model — By preferences (**P**), influence (**I**), and costs (**C**).



(a) A_1 specializes on θ_1 – Centralization (b) A_1 specializes on θ_1 – Delegation

Notes: $w = \frac{2}{3}$ and $k_1^* = k_2^* = 1$

Agent A_1 may find incentive compatible to reveal any signal individually but not together; such preferences are illustrated by the cross-hatched region in panel (a) of Figure 5. In the equilibrium under consideration A_1 conjectures that A_2 will acquire and reveal S_2, so he is not indifferent about which signal to acquire and reveal: the overall influence of acquiring S_1 is larger. He the decides to specialize on θ_1 driven by its *larger influence* on the Principal's beliefs, given A_2's equilibrium strategy. An alternative equilibrium exists when both agents' preferences lie in either of the cross-hatched regions. In such a case $\mathfrak{s}^{1*} = \{\tilde{S}_2^1\}$ and $\mathfrak{s}^{2*} = \{\tilde{S}_1^2\}$ can also be sustained, and agents face a coordination problem for which there is no clear selection criterion –i.e. the Principal is ex-ante (and ex-post) indifferent between any of these. Both equilibria involve specialization mainly because no agent is willing to reveal both signals.

When A_1 is willing to reveal both signals, specialization can only emerge because it is too costly to acquire them. The signal he actually ends up acquiring depends on what A_2 is expected to do; in other words, he will invest in information about the state the Principal is less informed in equilibrium. The solid gray region in panel (a) illustrates this case. Now I focus attention on specialization under delegation.

Panel (b) in Figure 5 illustrates the set of bias vectors for which A_1 specializes in θ_1 for the case of delegation. The figure is based on the same 2-agent example and equilibrium as before. Because $w_{11} = w_{22} > 1/2$, A_1's information is more useful for y_1 and his incentives for communication will thus be stronger with respect to that decision. Indeed, whenever A_1's preferences are close to j'' –with respect to y_2– the agent will prefer to acquire S_2 before S_1; this is why there is no specialization on θ_1 when the conflict of interest is large on y_1 and small on y_2.

As in the centralization case, A_1 specializes on θ_1 if he is willing to reveal only that signal to j' and it is cost-effective to do so. The area with vertical lines represents the set of vector biases for which the previous hold. Our agent could also be willing to reveal the most important signal to the decision-maker in charge of the corresponding decision. In this case he acquires S_1 when its influence in larger, given A_2 is specializing on θ_2 in equilibrium. Finally, when A_1 and j''s preferences are sufficiently close, specialization will only arise if it were too costly to acquire S_2 –as shown in the solid grey area.

Optimal Organizational Structure. As discussed in the previous section, the Principal will delegate a decision if the informational gains compensate the loss of control, and the optimal allocation of decision-rights maximizes her ex-ante expected utility. But in this section we learned that information costs impose restrictions on the number of agents willing to acquire a signal—Lemma 6 and Corollary 2. Whenever the cost of each signal is sufficiently low the allocation of decision-rights derived in the previous section applies. This is because K_r^{C} and K_r^{D} converge to infinity when costs go to zero, and Lemma 10 (Appendix A) holds. But high costs will affect the optimality of the different organizational structures. The following result show that information costs impose stronger restrictions to delegation than centralization.

Proposition 6. *Let κ be the maximum number of agents willing to reveal S_r to both decision-makers under* *delegation. Then, for any* $\kappa < n$

$$C(S_r) \in \left(\frac{(\hat{w}_{dr})^2}{6(\kappa+2)(\kappa+3)}, \frac{(w_{1r})^2 + (w_{2r})^2}{6(\kappa+3)(\kappa+4)} \right] \;\Rightarrow\; K_r^{\mathrm{C}} > K_r^{\mathrm{D}}$$

Where $\hat{w}_{dr} = \max\{w_{1r}, w_{2r}\}$.

Proof. See Appendix C.7. □

When information is costly, the maximum informational gains under delegation are weakly lower that those under centralization. Typically, agents will reveal their information to only one decision-maker under delegation; which is the reason why authority was given to that agent. Under centralization, on the contrary, any information transmitted affects both decision and agents for whom communication is incentive compatible will have higher overall influence. The utility gains for such an agent will then be weakly higher under centralization; the result follows given the cost for information are exogenous and common to all agents.

Proposition 6 does not mean that centralization is always optimal. If costs are not too large, there are profile of preferences, **b**, for which some form of delegation is preferred to centralization by the Principal. Conversely, if information costs are sufficiently large centralization always dominates. The following result shows a case in which this holds.

Corollary 4. *Let* $w \equiv w_{11} = w_{22}$*,* $\kappa = 1$*, and* $\frac{\hat{w}_{dr}^2}{72} < C(S_r) \leq \frac{[(w_{1r})^2+(w_{2r})^2]}{72}$ *for both* θ_r*.*

If there are no agents j' *and* j'' *whose preferences are such that* $||\mathbf{b}^d|| \leq 1-2w(1-w)$*, then the Principal strictly prefers centralization over any form of delegation. If there where such agents, the Principal still prefers centralization (strictly) as long as there is at least one agent* i *who fully reveals his information —i.e.* $\mathbf{b}^i$ *satisfy conditions* (3) *and* (5) *with respect to both signals.*

Proof. See Appendix C.8. □

Under the parameters of Corollary 4, costs are so high that the maximum number of truthful messages under delegation is zero. Having the chance to influence both decisions makes acquire one signal cost-effective for a single agent—provided communication is incentive compatible. The result also illustrates the restrictions imposed by information costs on the optimality of different organizational structures (Lemma 10). For sufficiently high costs the Principal always prefer to retain decision power over both issues, but restrictions weaken as the costs of acquiring a signal decreases. In the limit, when the costs tend to zero the framework converges to that of Section 2.

One last implication of Proposition 6 relates to the maximum bias the Principal will tolerate when delegating a decision: the relationship between loss of control and informational gains. In the previous section we learned that the maximum $||\hat{\mathbf{b}}^{\text{D}}||$ the Principal would tolerate is increasing in the informational gains from delegation. But in this section I showed that information costs impose limits on informational gains; so now I show how $||\hat{\mathbf{b}}^{\text{D}}||$ is affected by the costs. The maximum bias as a function of K^{D} (maximum informational gains) is then given by:

$$||\hat{\mathbf{b}}^{\text{D}}|| = \left[\frac{[w^2 + (1-w)^2]}{6}\left[1 - \frac{2}{(K^{\text{D}}+2)}\right]\right]^{\frac{1}{2}}$$

Which together with equation (15) in Corollary 2 leads to the following result.

Corollary 5. *The effect of information costs on* $||\hat{\mathbf{b}}^{\text{D}}||$ *is given by:*

$$\frac{\partial||\hat{\mathbf{b}}^{\text{D}}||}{\partial C(S_r)} = \left[\frac{[w^2 + (1-w)^2]}{6}\left[1 - \frac{2}{(K^{\text{D}}+2)}\right]\right]^{-\frac{1}{2}} \frac{[w^2 + (1-w^2)]}{12}\frac{\partial K^{\text{D}}}{\partial C(S_r)} < 0$$

The overall maximum bias the Principal will tolerate decreases as the cost of information increases. Increasing costs reduce the number of agents willing to acquire any signal, lowering the upper bound of informational gains that can be achieved through delegation. This could be particularly relevant when the Principal is not sure about the exact profile of preferences and the cost of info acquisition is high. In such a case she can expect the benefits from delegation to be relatively low.

Another way to see this relates to informational gains. When the expected maximal informational gains from delegation are low (because of high costs), the Principal is willing to tolerate less bias on decisions. This shows a relationship between the Principal's benefits from delegation and the amount of information a given agent has in equilibrium. The latter depends on the centrality of his preferences in my model, but could also be related to the intensive margin of information acquisition—if information accumulation is too costly, the Principal's willingness to delegate will depend on the amount of information the agent can afford. Exploring the implications of this intuition will be subject of future work.

In the last section of the paper I analyse another consequences of delegation with costly information acquisition: decision-maker *ex-post specialization.*

3.1 Organizational structures and equilibrium information balance

The idea of this subsection is to analyse the information the Principal can expect to receive from agents randomly allocated in the bias space. With this I try to capture a notion of 'long-run informational effects' of different organizational structures—related to the idea that institutions have persistent effects on policy outcomes (see Baumgartner and Jones, 2009). In order to isolate the mechanism I assume $C(S_r) = 0$ for both signals, which means no restrictions on the informational gains from delegation but agents decide not to observe a given signal (or both). I focus the analysis on the relative amount of information each decision-maker expects to receive in equilibrium, under different organizational structures. I say that a given decision-maker receives 'balanced' information when $k_1^j \simeq k_2^j$. To do that I consider an arbitrarily large number of agents whose preferences represent the same conflict of interest with the Principal. I then analyse the expected pattern of information transmission resulting from those agents' incentives.

Note that if the conflict of interest is sufficiently low all agents reveal both signals. In such cases decision-makers can expect to receive the same number of signals for each state. The same argument applies to dimensional non-separable message strategies because agents reveal both signals for some realizations and nothing otherwise. The only way decision-makers can receive more information about one signal is when many agents reveal it and few reveal the other. I then focus on incentives to reveal one signal under different organizational structures.

First consider the case of centralization. Let $\varepsilon \in \Re_+$ and $N_\varepsilon = \{1, 2, ..., n_\varepsilon\}$ be a group of agents that satisfy the following property: for all $i \in N_\varepsilon$ then $||\mathbf{b}^i|| = \varepsilon$. Now, let $\lambda_r \equiv \left\{\mathbf{z} \in \Re^2 |\ \mathbf{z}'\,\mathbf{W}_r = 0\right\}$ be the locus with slope $-\frac{w_{1r}}{w_{2r}}$ related to θ_r. This locus represents the bias vectors for which incentives to reveal S_r to the Principal are maximal.The IC constraint for revealing one signal under centralization –equation (3)– can be expressed as:

$$||\mathbf{b}^i - \text{Proj}_{\lambda_r}(\mathbf{b}^i)|| \leq \frac{\left[(w_{1r})^2 + (w_{2r})^2\right]^{\frac{1}{2}}}{2\,(k_r^{\text{C}} + 3)}$$

Where $\text{Proj}_{\lambda_r}(\mathbf{b}^i)$ is the projection of i's bias vector onto the locus λ_r. The result below follows:

Lemma 7. *Let $w \equiv w_{11} = w_{22}$. Given $\varepsilon \in \Re_+$ and arbitrarily large n_ε, then for every integer κ and $i \in N_\varepsilon$ with $||\mathbf{b}^i - Proj_{\lambda_1}(\mathbf{b}^i)|| \leq \frac{\left[(w)^2+(1-w)^2\right]^{\frac{1}{2}}}{2\,(\kappa+3)}$, there exists a $j \in N_\varepsilon$ with $||\mathbf{b}^j - Proj_{\lambda_2}(\mathbf{b}^j)|| \leq \frac{\left[(1-w)^2+(w)^2\right]^{\frac{1}{2}}}{2\,(\kappa+3)}$.*

Proof. See appendix B.7 □

Under centralization the Principal can expect to have balanced information in any equilibrium.[25] The

[25] An implicit assumption for this to hold is that states are equally important across decisions –i.e. $w_{11} + w_{21} = w_{12} + w_{22}$.

expectation relates to the uncertainty about the profile of preferences, for a given magnitude of conflict of interest —i.e. $||\mathbf{b}^i|| = \varepsilon$ for all $i \in N_\varepsilon$.

The argument is straightforward. If the conflict of interest is large ($\gamma\varepsilon$, with $\gamma > 1$) agents will either play DNS strategies or reveal only one signal.[26] Incentives to reveal one signal for a generic agent depends on the distance between his bliss point and the loci λ_1 and λ_2. Lemma 7 says that the IC constraints for revealing one signal have the same 'width' with respect to the corresponding locus, when number of agents revealing each signal is fixed. In other words, for any given k_1^{C} the number of agents still willing to reveal information about θ_1 is always the same as those willing to reveal information about θ_2, for $k_1^{\mathrm{C}} = k_2^{\mathrm{C}}$. Panel (a) in Figure 6 illustrates this.

Now consider the case of delegation. Let $\lambda_r^d \equiv \{\mathbf{z} \in \Re^2 |\ \mathbf{z}'\,\mathbf{I}_d = 0\}$ be the locus of maximal incentives to reveal S_r when deciding on y_d.[27] The locus λ_r^d captures the fact that communication depends only on the conflict of interest associated to y_d, so coincides with either the vertical or the horizontal axis —for λ_r^1. and λ_r^2, respectively. Note that $\lambda_1^d = \lambda_2^d$ for any $y_d = \{y_1, y_2\}$. Condition (10) for communication with the Principal can be expressed as:

$$||\mathbf{b}^i - \mathrm{Proj}_{\lambda_r^d}(\mathbf{b}^i)|| \leq \frac{w_{dr}}{2\big(k_r^{\mathrm{P}d} + 3\big)}$$

Then, the equivalent of Lemma 7 in this case is:

Lemma 8. *Let $w \equiv w_{11} = w_{22}$. Given $\varepsilon \in \Re_+$ and an arbitrarily large n_ε, there exists an integer κ and $i \in N_\varepsilon$ with $||\mathbf{b}^i - Proj_{\lambda_r^d}(\mathbf{b}^i)|| \leq \frac{w_{dr}}{2(\kappa+3)}$ such that $||\mathbf{b}^i - Proj_{\lambda_{\tilde{r}}^d}(\mathbf{b}^i)|| > \frac{w_{d\tilde{r}}}{2(\kappa+3)}$.*

Moreover, this is true for the state associated to w_{dr} because $w_{dr} > w_{d\tilde{r}}$

Proof. Given that $\lambda_r^d = \lambda_{\tilde{r}}^d$ and $w_{dr} > w_{d\tilde{r}}$, the result holds for any $b_d^i \in \left(\frac{w_{d\tilde{r}}}{2(\kappa+3)}; \frac{w_{dr}}{2(\kappa+3)}\right]$ □

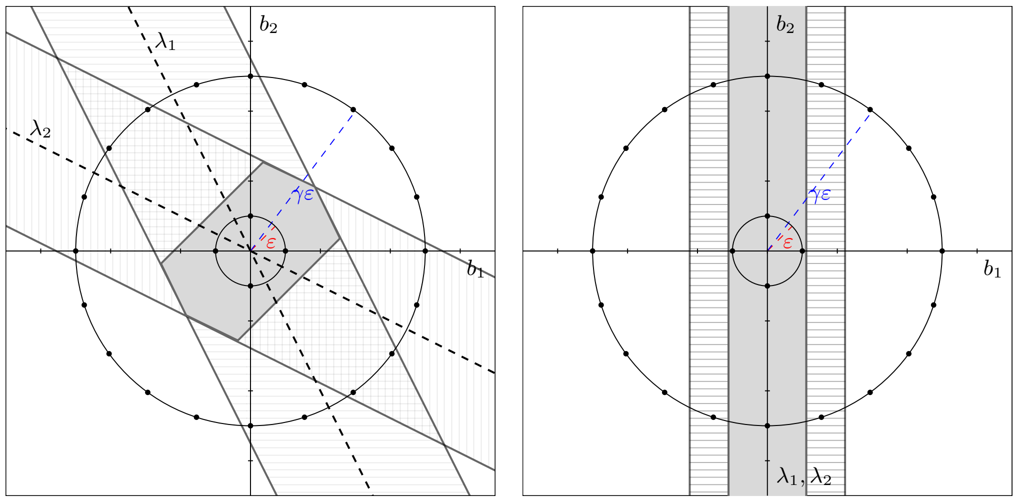
The above Lemma says that when the Principal decides over y_d only, agents with sufficiently large $|b_d|$ will only reveal S_r. As a consequence, she can expect to have more information about θ_r, which is the (more) relevant state for y_d. Then, under delegation the Principal can expect to receive more information about the associated state. Panel (b) in Figure 6 illustrate this when the Principal decides on y_1 only. Agents' incentives for communication then depend on b_1 and, given $w_{11} > 1/2$ information about θ_1 has a higher influence (for a given $k_1^{\mathrm{P}d} = k_2^{\mathrm{P}d}$). As a consequence, the IC constraints for revealing S_1 only will hold for a larger sets of biases than that of revealing both signals; which in turn implies there will be more agents, at

[26] If the fixed conflict of interest (ε) is sufficiently small there is a finite number of agents for which revealing both signals is incentive compatible, as shown in the inner circle in Figure 6.a. The Principal receives similar amount of information from these agents.

[27] Where $\mathbf{I}$ is the 2-by-2 identity matrix, and $\mathbf{I}_d$ is its dth column, which matches the index of the decision under consideration.

a given distance from the origin, willing to reveal S_1 than those willing to reveal both signals. The Principal then expects to have more information about θ_1 when she delegates y_2, becoming a sort of *ex-post specialist*.

Figure 6: Information transmission under different organizational structures



(a) Centralization (b) Delegation of y_2

Full Revelation | S_1 only | S_2 only | Babbling

Note: $w_{11} = w_{22} = \frac{2}{3}$

These result represents a qualification for the benefits of delegation. Breaking negative informational externalities may provide a just foundation for partial delegation when the Principal observes the preferences of a group of informed agents. But given decision-rights constitute one of the main institutions within an organization, she may have to consider the 'long run informational effects' of delegation; namely, losing control over payoff-relevant information. This qualification is even more relevant when the organizational structure affects agents' incentives to invest costly personal resources in the firm's favour.

Appendix A Complementary results

Lemma 9 (IC constraints for dimensional non-separable strategies under centralization)**.** *In any equilibrium* $(\mathbf{y}^*, \mathbf{m}^*)$ *in which* $\mathbf{m}^{i*}$ *includes a babbling strategy; then, i's incentives to reveal both signals against this deviation are characterized by:*

- *For* $\mathbf{S}^i = \{0,0\}$ *and* $\mathbf{S}^i = \{1,1\}$*, if:*

$$\left|\frac{\beta_1^i}{(k_1+3)} + \frac{\beta_2^i}{(k_2+3)}\right| \leq \frac{1}{4}\left[\frac{(w_{11})^2+(w_{21})^2}{(k_1+3)^2} + \frac{(w_{12})^2+(w_{22})^2}{(k_2+3)^2} + \frac{2\,[w_{11}\,w_{12}+w_{21}\,w_{22}]}{(k_1+3)(k_2+3)}\right] \tag{16}$$

- *For* $\mathbf{S}^i = \{0,1\}$ *and* $\mathbf{S}^i = \{1,0\}$*, if:*

$$\left|\frac{\beta_1^i}{(k_1+3)} - \frac{\beta_2^i}{(k_2+3)}\right| \leq \frac{1}{4}\left[\frac{(w_{11})^2+(w_{21})^2}{(k_1+3)^2} + \frac{(w_{12})^2+(w_{22})^2}{(k_2+3)^2} - \frac{2\,[w_{11}\,w_{12}+w_{21}\,w_{22}]}{(k_1+3)(k_2+3)}\right] \tag{17}$$

Proof. See Habermacher (2018) □

Proposition 7 (Characterization of P-Optimal equilibrium under centralization —Proposition 2 in Habermacher, 2018)**.** *The P-optimal Perfect Bayesian Equilibrium for sender i consists of the following strategies:*

1. ***Revealing both signals****, if* $\mathbf{b}^i$ *satisfies conditions* (4)*,* (5) *and* (9) *with respect to both states.*
2. ***Revealing one signal only****, if* $\mathbf{b}^i$ *satisfies condition* (3) *for* S_r^i *only.*
3. ***Dimensional non-separable message strategies*** *in the following cases:*
   (a) *Fully revealing* $\mathbf{S}^i = \{(0,0);(1,1)\}$ *if* $\mathbf{b}^i$ *satisfies condition* (16) *only;*
   (b) *Fully revealing* $\mathbf{S}^i = \{(0,1);(1,0)\}$ *if* $\mathbf{b}^i$ *satisfies condition* (17) *only.*
4. ***No communication*** *(babbling strategy), if none of the above holds.*[28]

Proof. See Habermacher (2018). □

Lemma 10 (Optimal Organizational Structure)**.** *Given the vector of preferences,* $\mathbf{b} = \{\mathbf{b}_1, \ldots, \mathbf{b}_n\}$*, and generic agents i,* j'*, and* j''*; the organizational structure that maximizes the Principal's ex-ante welfare is:*

[28]Where Full Revelation corresponds to the equilibrium message strategy $\mathbf{m}^i = \Big\{\{(0,0)\};\{(0,1)\};\{(1,0)\};\{(1,1)\}\Big\}$; revealing S_1^i or S_2^i only correspond to $\mathbf{m}^i = \Big\{\{(0,0);(0,1)\};\{(1,0);(1,1)\}\Big\}$ and $\mathbf{m}^i = \Big\{\{(0,0);(1,0)\};\{(0,1);(1,1)\}\Big\}$; DNS message strategies correspond to $\mathbf{m}^i = \Big\{\{(0,0)\};\{(1,1)\};\{(0,1);(1,0)\}\Big\}$ when $(0,0)$ and $(1,1)$ fully reveal their types, and $\mathbf{m}^i = \Big\{\{(0,0);(1,1)\};\{(0,1)\};\{(1,0)\}\Big\}$ the case in which $(0,0)$ and $(1,1)$ do so; and finally the babbling strategy $\mathbf{m}^i = \Big\{\{(0,0);(1,1);(0,1);(1,0)\}\Big\}$.

Full Delegation. *That is, agents j' and j'' decide on y_1 and y_2 (resp.) if and only if:*

1. $DIG_{j'}(y_1) - (b'_1)^2 > \max\Big\{DIG_i(y_1) - (b^i_1)^2\,,\, IIG(y_1)\,,\, -\{DIG_{j''}(y_2) - (b''_2)^2\}\Big\}$ *for any* $i \neq j'$; *and*

2. $DIG_{j''}(y_2) - (b''_2)^2 > \max\Big\{DIG_i(y_2) - (b^i_2)^2\,,\, IIG(y_2)\,,\, -\{DIG_{j'}(y_1) - (b'_1)^2\}\Big\}$ *for any* $i \neq j''$.

Partial Delegation. *That is, agent j decides on y_d and the Principal retains decision authority over $y_{\tilde{d}}$; if and only if there exist both Direct and Indirect informational gains such that:*

1. $DIG_j(y_d) - (b^j_d)^2 > \max\Big\{DIG_i(y_d) - (b^i_d)^2\,,\, IIG(y_d)\,,\, -IIG(y_{\tilde{d}})\Big\}$ *for any* $i \neq j$; *and*

2. $IIG(y_{\tilde{d}}) > \max\Big\{DIG_i(y_{\tilde{d}}) - (b^i_{\tilde{d}})^2\,,\, -\{DIG_j(y_d) - (b^j_d)^2\}\Big\}$ *for any* $i \neq j$.

Centralization. *That is, the Principal decides on both issues, if and only if there are no agent i and j such that:*

1. $DIG_j(y_d) - (b^j_d)^2 + IIG(y_{\tilde{d}}) > 0$; *nor*

2. $DIG_{j'}(y_1) - (b'_1)^2 + DIG_{j''}(y_2) - (b''_2)^2 > 0$

Proof. The proof is constructive. The optimal organizational structure maximizes the Principal's ex-ante expected utility. Given that in the worst-case scenario she can retain authority over both issues and decide without information, delegation will necessarily involve informational gains of some sort.

Full Delegation is then optimal if there exists two agents j' and j'' who decide on y_1 and y_2, respectively; such that informational gains associated to each agent-decision pair more than compensate the decision-maker's bias. In addition, these gains are maximal among all agents, and larger than the indirect informational gains the Principal would have obtained had she retained the corresponding decision. Finally, the gains should also compensate any loss of information that may take place — recall that these are measured with respect to centralization.

Partial Delegation can be optimal in either of two cases (non-exclusive). First, it could be that the intuitions on the previous paragraph applies to one agent-decision pair, but the Principal is better-off retaining authority over the other decision. This may be true even when she experiences informational losses on the retained decision, if the direct informational gains are sufficiently large.

Second, and more interestingly, it could be that indirect informational gains are large. As we learned from Proposition 2 this happens when there are negative informational externalities under centralization, such that delegating y_d breaks the interdependence between decisions and unlocks communication to the Principal on the retained decision. This may hold even if there are no informational gains in the delegated decision, as long as the indirect ones are sufficiently large.

Finally, Centralization is optimal when any potential informational gain is small, such that it does not compensate the loss of control on the delegated decision(s). □

Appendix B Organizational Structure and Communication

B.1 Generic IC constraints for communication

Proof. Let $j', j'' \in \{P, 1, ..., n\}$ be the decision-makers for y_1 and y_2, respectively; and $i \in \{1, ..., n\}$ be a generic sender. Let $\mathbf{m}^{i*}_j$ denote i's equilibrium message strategy with respect to j, and $\hat{\mathbf{m}}^i_j$ an alternative message strategy (deviations to be considered in each case). Then, $y_d(\mathbf{m}^i_j, \mathbf{m}^{-i}_j)$ represents the action decision-maker j will take when i is expected to play $\mathbf{m}^{i*}_j$ and other senders are playing $\mathbf{m}^{-i}_j$. Given that i takes other equilibrium strategies as given, I can simplify notation in the following way: $y_d(\mathbf{m}^{i*}_j(\mathbf{S}^i), \mathbf{m}^{-i}_j) = y_d(\mathbf{m}^{i*}_j)$ and $y_d(\hat{\mathbf{m}}^i_j(\mathbf{S}^i), \mathbf{m}^{-i}_j) = y_d(\hat{\mathbf{m}}^i_j)$.

Strategy $\mathbf{m}^{i*}$ is then incentive compatible for agent i if and only if for any alternative $\hat{\mathbf{m}}^i$:

$$-\int_0^1\int_0^1\Bigg[\Big[\big(\delta_1+b_1^i-y_1(\mathbf{m}_{j'}^{i*})\big)^2+\big(\delta_2+b_2^i-y_2(\mathbf{m}_{j''}^{i*})\big)^2\Big]- \\ -\Big[\big(\delta_1+b_1^i-y_1(\hat{\mathbf{m}}_{j'}^{i})\big)^2+\big(\delta_2+b_2^i-y_2(\hat{\mathbf{m}}_{j''}^{i})\big)^2\Big]\Bigg]f(\theta_1,\mathbf{m}^{\text{-}i}|\mathbf{S}^i)\,f(\theta_2,\mathbf{m}^{\text{-}i}|\mathbf{S}^i)\,d\theta_1\,d\theta_2\geq 0$$

By operating inside the square brackets with the identity $a^2-b^2=(a+b)(a-b)$, by definition of optimal decisions, $y_d^*=E(\delta_d|\mathbf{m}_j)+b_d^j$, and by denoting:

$$\Delta(\delta_1)=E(\delta_1|\mathbf{m}_{j'}^{i*},\mathbf{m}_{j'}^{\text{-}i})-E(\delta_1|\hat{\mathbf{m}}_{j'}^{i},\mathbf{m}_{j'}^{\text{-}i})$$
$$\Delta(\delta_2)=E(\delta_2|\mathbf{m}_{j''}^{i*},\mathbf{m}_{j''}^{\text{-}i})-E(\delta_2|\hat{\mathbf{m}}_{j''}^{i},\mathbf{m}_{j''}^{\text{-}i})$$

I get:[29]

$$-\int_0^1\int_0^1\Bigg[\Big[\frac{E(\delta_1|\mathbf{m}_{j'}^{i*},\mathbf{m}_{j'}^{\text{-}i})+E(\delta_1|\hat{\mathbf{m}}_{j'}^{i},\mathbf{m}_{j'}^{\text{-}i})}{2}-\delta_1-(b_1^i-b_1')\Big]\Delta(\delta_1)+ \\ +\Big[\frac{E(\delta_2|\mathbf{m}_{j''}^{i*},\mathbf{m}_{j''}^{\text{-}i})+E(\delta_2|\hat{\mathbf{m}}_{j''}^{i},\mathbf{m}_{j''}^{\text{-}i})}{2}-\delta_2-(b_2^i-b_2'')\Big]\Delta(\delta_2)\Bigg] \\ f(\theta_1|\mathbf{m}^{\text{-}i},S_1^i)\,f(\theta_2|\mathbf{m}^{\text{-}i},S_2^i)\,P(\mathbf{m}^{\text{-}i}|S_1^i)\,P(\mathbf{m}^{\text{-}i}|S_2^i)\,d\theta_1\,d\theta_2\geq 0$$

Given that the equilibrium message strategies for players other than i, $\mathbf{m}^{-i}$, are independent of i's actual signal realizations, the expressions $P(\mathbf{m}^{-i}|S_1^i)$ and $P(\mathbf{m}^{-i}|S_2^i)$ can be taken out the double-integral. Then, by definition of $\delta_d=w_{d1}\theta_1+w_{d2}\theta_2$, I get the generic IC constraint:

$$-\Big[-\frac{\Delta(\delta_1)}{2}-(b_1^i-b_1')\Big]\,\Delta(\delta_1)-\Big[-\frac{\Delta(\delta_2)}{2}-(b_2^i-b_2'')\Big]\,\Delta(\delta_2)\geq 0 \tag{18}$$

Note that under Centralization $b_1'=b_2''=0$ and we are back to the IC constraints in Habermacher (2018). When there are more than one decision-makers, on the contrary, the message strategy to j' only affects y_1 and the second term in the LHS of (18) is zero —and the same happens with the first term for message strategy to j''. As a consequence, the IC constraint under delegation consists only on the term corresponding to the decision under consideration. ☐

B.2 Proof of equation (6).

Proof. Under centralization, the Principal's ex-ante expected utility in equilibrium $(\mathbf{y},\mathbf{m})$ is given by:

$$E\left[U^{\mathrm{C}}(\boldsymbol{\delta},\mathbf{b});\mathbf{m}\right]=-E\left[(y_1-\delta_1)^2;\mathbf{m}\right]-E\left[(y_2-\delta_2)^2;\mathbf{m}\right]$$

Which, by definitions of y_d and δ_d yield:

$$E\left[U^{\mathrm{C}}(\boldsymbol{\delta},\mathbf{b});\mathbf{m}\right]=-\sum_{d=\{1,2\}}E\left[\Big(w_{d1}\big(E(\theta_1|\mathbf{m})-\theta_1\big)+w_{d2}\big(E(\theta_2|\mathbf{m})-\theta_2\big)\Big)^2\right]+E\left[E(\delta_d|\mathbf{m})-\delta_d\right]$$

[29] Note that $f(\theta_1,\mathbf{m}^{\text{-}i}|\mathbf{S}^i)=f(\theta_1|\mathbf{m}^{\text{-}i},S_1^i)\,P(\mathbf{m}^{\text{-}i}|S_1^i)$ and that $f(\theta_2,\mathbf{m}^{\text{-}i}|\mathbf{S}^i)=f(\theta_2|\mathbf{m}^{\text{-}i},S_2^i)\,P(\mathbf{m}^{\text{-}i}|S_2^i)$

With some rearrangement and given the last term equals zero, I have:

$$E\left[U^{\mathrm{C}}(\boldsymbol{\delta},\mathbf{b});\mathbf{m}\right] = -\left[(w_{11})^2+(w_{21})^2\right] E\left[\left(E(\theta_1|\mathbf{m})-\theta_1\right)^2;\mathbf{m}\right] - \left[(w_{12})^2+(w_{22})^2\right] E\left[\left(E(\theta_2|\mathbf{m})-\theta_2\right)^2;\mathbf{m}\right] \tag{19}$$

Now, the expectation of the squared deviation for each state is given by:

$$\begin{aligned} E\left[\left(E(\theta_r|\mathbf{m})-\theta_r\right)^2;\mathbf{m}\right] &= \int_0^1 \sum_{\ell_r=0}^{k_r^{\mathrm{C}}} \Big(E(\theta_r|\mathbf{m})-\theta_r\Big)^2 f(\ell_r|k_r,\theta_r)\,d\theta_r \\ &= \int_0^1 \sum_{\ell_r=0}^{k_r^{\mathrm{C}}} \Big(E(\theta_r|\mathbf{m})-\theta_r\Big)^2 \frac{h(\theta_r|\ell_r,k_r)}{(k_r^{\mathrm{C}}+1)}\,d\theta_r \\ &= \frac{1}{(k_r^{\mathrm{C}}+1)} \sum_{\ell_r=0}^{k_r^{\mathrm{C}}} \int_0^1 \Big(E(\theta_r|\mathbf{m})-\theta_r\Big)^2 h(\theta_r|\ell_r,k_r)\,d\theta_r \\ &= \frac{1}{(k_r^{\mathrm{C}}+1)} \sum_{\ell_r=0}^{k_r^{\mathrm{C}}} \mathrm{Var}(\theta_r|\ell_r,k_r) \\ &= \frac{1}{(k_r^{\mathrm{C}}+1)} \sum_{\ell_r=0}^{k_r^{\mathrm{C}}} \frac{(\ell_r+1)(k_r-\ell_r+1)}{(k_r+2)^2(k_r+3)} \end{aligned}$$

Solving the sum and plugging the above into (19) yields:

$$E\left[U^{\mathrm{C}}(\boldsymbol{\delta},\mathbf{b});\mathbf{m}\right] = -\frac{\left[(w_{11})^2+(w_{21})^2\right]}{6\,(k_1^{\mathrm{C}}+2)} - \frac{\left[(w_{12})^2+(w_{22})^2\right]}{6\,(k_2^{\mathrm{C}}+2)}$$

When agent j has authority over y_d his decision will be biased from the Principal's perspective, that is:

$$y_d^* = E\big(\delta_d|\mathbf{m}_j\big) + b_d^j$$

Then, the Principal's ex-ante expected utility will be given by (assuming she decides on $y_{\tilde{d}}$):

$$\begin{aligned} E\left[U^{\mathrm{P}d}(\boldsymbol{\delta},\mathbf{b},b_d^{'});\mathbf{m}\right] &= -E\left[(y_d-\delta_d+b_d^j)^2;\mathbf{m}\right] - E\left[(y_{\tilde{d}}-\delta_{\tilde{d}})^2;\mathbf{m}\right] \\ &= -(b_d^j)^2 - E\left[(y_d-\delta_d)^2;\mathbf{m}\right] - E\left[(y_{\tilde{d}}-\delta_{\tilde{d}})^2;\mathbf{m}\right] \end{aligned}$$

Which, following the same steps as the case of centralization yields the corresponding expression, while expression (6) follows from applying the same logic to the other decision as well. □

B.3 Proof of Lemma 1

Proof. To be updated. Consider the equilibrium $(\mathbf{y}^*,\mathbf{m}^*)$ in which the Principal delegates y_d to agent j, who does not decide on the other decision. By the assumption on private information with each decision-maker, i's messages to j only affect y_d. The IC constraint in (18) then becomes:

$$\Delta(\delta_d)\,(b_d^j-b_d^i) \le \frac{\big(\Delta(\delta_d)\big)^2}{2}$$

Suppose that i reveals S_r^i. Let k_r^j be other senders truthfully revealing their information about θ_r to j, and ℓ_r^j the number of ones among these. Then, from i's perspective the updated expectation for θ_r as a function of her signal's realization $\tilde{S}_r$ is given by:

$$E\left(\theta_r|\tilde{S}_r^i, \mathbf{m}_j^{\text{-}i}\right) = \frac{(\ell_r^j + 1 + \tilde{S}_r)}{(k_r^j + 3)}$$

Which, taking expectation over the realization of others' signals (by the Law of Iterated Expectations) becomes:

$$\begin{aligned} E\left[E\left(\theta_r|\ell_r, \tilde{S}_r^i, \mathbf{m}^{\text{-}i}\right)\right] &= \frac{1}{(k_r^j + 3)} \sum_{\ell_r=0}^{k_r^j} \frac{(\ell_r^j + 1 + \tilde{S}_r^i)}{(k_r^j + 1)} \\ &= \frac{\left(k_r^j + 3 - (-1)^{\tilde{S}_r^i}\right)}{2(k_r^j + 3)} \end{aligned}$$

If i reveals $\tilde{S}_r^i = 0$, the posterior expectation for θ_r is expected to decrease (from i's perspective) —the opposite happens when $\tilde{S}_r^i = 1$. Note that the magnitude of the change in expectations depends on other senders revealing truthfully to decision-maker j: the more of them the lower i's expected influence. Recall that message strategies in terms of signal realizations are given by:

- Babbling: $\mathbf{m}_j^{i*} = \big\{(0,0);(0,1);(1,0);(1,1)\big\}$
- Revealing S_1^i only: $\mathbf{m}_j^{i*} = \big\{\{(0,0);(0,1)\};\{(1,0);(1,1)\}\big\}$
- Revealing S_2^i only: $\mathbf{m}_j^{i*} = \big\{\{(0,0);(1,0)\};\{(0,1);(1,1)\}\big\}$
- Revealing both signals: $\mathbf{m}_j^{i*} = \big\{\{(0,0)\};\{(1,0)\};\{(0,1)\};\{(1,1)\}\big\}$

Then, working out the expectations (conditional on signal realizations) I get the expressions for $\Delta(\delta_d)$ for each possible message strategy (Lemma 5 in Habermacher, 2018). In order to do that, I evaluate each message within a strategy against the relevant deviations in the equilibrium under play. If, for instance, i is expected to reveal S_1 in equilibrium and she observes signals $(0,0)$, the relevant deviations are either $(1,0)$ and $(1,1)$ [30] because any of these implies announcing $\tilde{S}_1^i = 1$ (information about S_2^i is not believed by the decision-maker in this case). It can be shown that the $\Delta(\delta_d)$ take the following forms:

- Revealing S_1^i only: $\Delta(\delta_d)_{S_1^i} = \left|\frac{w_{d1}}{(k_1^j+3)} - \frac{w_{d2}}{(k_2^j+3)}\right|$
- Revealing S_2^i only: $\Delta(\delta_d)_{S_2^i} = \left|\frac{w_{d2}}{(k_2^j+3)} - \frac{w_{d1}}{(k_1^j+3)}\right|$
- Revealing both: $\Delta(\delta_d) = \left|\frac{w_{d1}}{(k_1^j+3)} - \frac{w_{d2}}{(k_2^j+3)}\right|$

Plugging each of these into the IC constraint yields the corresponding condition in Lemma 1.

□

B.4 Proof of Proposition 1

Proof. To be updated. Having derived the necessary conditions for communication (Lemma 1), getting the Principal-optimal equilibrium consist of finding the system of beliefs consistent with each message strategy (if those exist). Let denote by $\mu_j^*\big(\mathbf{S}^i|m_j^{i*}\big)$ the beliefs that j has about i's information (her type) upon receiving message m_j^{i*}.

[30] Which of them, specifically, depends on the equilibrium beliefs that support that equilibrium.

Then, to sustain a Fully Revealing equilibrium for a sender whose preferences satisfy condition (1), the decision-maker would take i's message at face value. Formally, this implies equilibrium beliefs are:

$$\mu^*\left((0,0)|m^i=\{(0,0)\}\right)=1 \qquad \mu^*\left((1,0)|m^i=\{(1,0)\}\right)=1$$
$$\mu^*\left((0,1)|m^i=\{(0,1)\}\right)=1 \qquad \mu^*\left((1,1)|m^i=\{(1,1)\}\right)=1$$

This system of beliefs make condition (1) necessary and sufficient to sustain the Fully Revealing message strategy in equilibrium. Now, because the IC constraint for revealing one signal only is the same and the Principal always prefers full revelation, the former cannot arise as equilibrium message strategy.

□

B.5 Proof of Proposition 2

Proof. Informational gains arise when more agents reveal information to al least one decision-maker, as compared to those revealing to the Principal under centralization. Consider direct info gains first. For every agent that would reveal information to the Principal under centralization, there must exist an agent willing to reveal at least the same amount of information to the new decision-maker under delegation. For the gains to be strict, there must also exist at least one agent willing to reveal strictly more information to the new decision-maker.

Suppose the Principal delegates y_1 to agent j and retain decision power on y_2; recall that θ_1 is more important for the former and θ_2 for the latter. Now, for every $h \in N$ such that β_1^h satisfies (3); then there must exist a $i \in N$ such that b_1^i satisfy (1); otherwise, k_1^j is lower than k_1^{C}.[31] In addition, there exists an agent i such that:

$$|b_1^i w_{11} + b_2^i w_{21}| > \frac{1}{2}\left[\frac{(w_{11})^2+(w_{21})^2}{(k_1^{\mathrm{C}}+3)} - \frac{w_{11}w_{12}+w_{21}w_{22}}{(k_2^{\mathrm{C}}+3)}\right]$$
$$|b_1^i - b_1^j| \leq \frac{1}{2}\left[\frac{w_{11}}{(k_1^j+3)} - \frac{w_{12}}{(k_2^j+3)}\right]$$

Multiplying the second inequality by w_{1r} I get that its right-hand side is strictly lower than the corresponding term of the expression above —when $k_1^{\mathrm{C}} = k_1^j$. Thus I get to the condition that $|b_1^i - b_1^j| < |b_1^i + b_2^i \frac{w_{21}}{w_{11}}|$.

Indirect informational gains mean that in equilibrium $k_2^{\mathrm{P}2} > k_2^{\mathrm{C}}$, such that the above equations must hold for y_2. The previous argument then leads to: for every $h \in N$ such that β_2^h satisfies (3), there must exist a $i \in N$ such that b_2^i satisfies (1) and, in addition, there exist exists an agent i such that:

$$|b_1^i w_{12} + b_2^i w_{22}| > \frac{1}{2}\left[\frac{(w_{12})^2+(w_{22})^2}{(k_2^{\mathrm{C}}+3)} - \frac{w_{11}w_{12}+w_{21}w_{22}}{(k_1^{\mathrm{C}}+3)}\right]$$
$$|b_2^i| \leq \frac{1}{2}\left[\frac{w_{22}}{(k_2^j+3)} - \frac{w_{21}}{(k_1^j+3)}\right]$$

Again, multiplying the last inequality be w_{22} evidences that its RHS is larger than that of the first one. This reduces the necessary conditions for IIG to $w_{22}|b_2^i| < |b_1^i w_{12} + b_2^i w_{22}|$. It follows that the previous is only possible when $|b_1^i|$ is sufficiently large. □

[31] Indeed, any of the agents revealing under delegation are revealing both signals.

B.6 Proof of Proposition 3

Proof. Suppose the equilibrium organizational structure involve some form od delegation, and let $j', j'' \in \{P, 1, 2, ...\}$ be decision makers for y_1 and y_2, respectively. Then $U^{\mathrm{B}} - U^{\mathrm{C}} \geq 0$, which yields:

$$\frac{(w_{11})^2}{6}\left[\frac{1}{(k_1^{\mathrm{C}}+2)} - \frac{1}{(k_1'+2)}\right] + \frac{(w_{12})^2}{6}\left[\frac{1}{(k_2^{\mathrm{C}}+2)} - \frac{1}{(k_2'+2)}\right] +$$
$$\frac{(w_{21})^2}{6}\left[\frac{1}{(k_1^{\mathrm{C}}+2)} - \frac{1}{(k_1''+2)}\right] + \frac{(w_{22})^2}{6}\left[\frac{1}{(k_2^{\mathrm{C}}+2)} - \frac{1}{(k_2''+2)}\right] \geq (b_1')^2 + (b_2'')^2$$

Now let $w \equiv w_{11} = w_{22}$ and assume $k_1^{\mathrm{C}} = k_2^{\mathrm{C}} = 0$. Then, taking the square root the above inequality, along with the previous assumptions yields:

$$\left[\frac{w^2}{6}\left[1 - \frac{1}{(k_1'+2)} - \frac{1}{(k_2''+2)}\right] + \frac{(1-w)^2}{6}\left[1 - \frac{1}{(k_2'+2)} - \frac{1}{(k_1''+2)}\right]\right]^{\frac{1}{2}} \geq \left[(b_1')^2 + (b_2'')^2\right]^{\frac{1}{2}}$$

Then the expression in Proposition 3 follows, noticing that the RHS of the above represents the module of a vector with components b_1' and b_2'' and that the maximal loss imply it holds with equality. □

B.7 Proof of Lemma 7

Proof. Let $w \equiv w_{11} = w_{22}$, let κ be an non-negative integer, and let $\varepsilon \in \Re_+$ with associated integer n_ε.

Let $i \in N_\varepsilon$ be an agent whose preferences satisfy equation (3) with respect to θ_r for a given κ; that is:

$$||\mathbf{b}^i - \mathrm{Proj}_{\lambda_1}(\mathbf{b}^i)|| \leq \frac{\left[(w)^2 + (1-w)^2\right]^{\frac{1}{2}}}{2\,(\kappa+3)}$$

Now consider an agent j with the following preferences: $b_1^j = b_2^i$ and $b_2^j = b_1^i$. I need to show 1) $j \in N_\varepsilon$, and 2) $\mathbf{b}^j$ satisfies equation (3) for θ_2. Proving the first claim is straightforward, since j's preference vector is just i's with its components swapped. This says that both i and j agents have exactly the same conflict of interest with the Principal.

The second part of the proof requires work out $||\mathbf{b}^j - \mathrm{Proj}_{\lambda_2}(\mathbf{b}^j)||$, which yield:

$$\begin{aligned} ||\mathbf{b}^j - \mathrm{Proj}_{\lambda_2}(\mathbf{b}^j)|| &= |b_1^j\,(1-w) + b_2^j\,w|\ \left[(w)^2 + (1-w)^2\right]^{-\frac{1}{2}} \\ &= |b_2^i\,(1-w) + b_1^i\,w|\ \left[(w)^2 + (1-w)^2\right]^{-\frac{1}{2}} \\ &= ||\mathbf{b}^i - \mathrm{Proj}_{\lambda_1}(\mathbf{b}^i)|| \end{aligned}$$

□

Appendix C Org. Structure, Information Acquisition and Communication

C.1 Proof of Lemma 4

Proof. The proof proceed by contradiction and I analyse the case of centralization, since that for decentralization follows the same logic with change in index for decision-makers. Let $(\{\mathbf{y}^*\}, \{\mathbf{m}^*, \mathfrak{s}^*\})$ be the equilibrium strategy profiles for the receiver and all agents, respectively. The equilibrium can be characterized by k_1^* and k_2^*.

Acquisition of S_1. Suppose that agent i's equilibrium info acquisition strategy is such that: $S_1 \in \mathfrak{s}^{i*}$ but condition (3) does not hold for S_1. Agent i thus cannot credibly reveal information about θ_1 despite having a signal about it — i.e. his equilibrium message strategy involves not revealing $S^i_{\theta_1}$. In addition, other agents base their message strategies on conjectures about k^*_1, but in equilibrium i is not part of that count.

At the information acquisition stage i's expected payoff of $\mathfrak{s}^{i*}$ is given by:

$$E\left[U^i\left(\mathbf{y}^*(\mathbf{m}^*(\mathfrak{s}^*)),\delta,b^i\right)\right] = -E\left[\left(\mathbf{y}_1\left(m^{i*}(\mathfrak{s}^{i*}),\mathbf{m}^*_{\text{-}i}\right)-\delta_1-b^i_1\right)^2+\left(\mathbf{y}_2\left(m^{i*}(\mathfrak{s}^{i*}),\mathbf{m}^*_{\text{-}i}\right)-\delta_2-b^i_2\right)^2\right]-C(\mathfrak{s}^{i*})$$

Now, consider the following deviation: $\hat{\mathfrak{s}}^i = \mathfrak{s}^{i*}\backslash\{S_1\}$. Note that this deviation does not affect neither k^*_1 nor k^*_2, thus i's overall influence in the receiver's decision will not be altered; that is:

$$y_d\left(m^i(\hat{\mathfrak{s}}^i),\mathbf{m}^*_{\text{-}i}\right) = y_d\left(m^i(\mathfrak{s}^{i*}),\mathbf{m}^*_{\text{-}i}\right)$$

for $d=\{1,2\}$

Note also that $C(\mathfrak{s}^{i*}) > C(\hat{\mathfrak{s}}^i)$, given $\#\mathfrak{s}^{i*} > \#\hat{\mathfrak{s}}^i$. Consequently,

$$E\left[U^i\left(\mathbf{y}^*(\mathbf{m}^*(\mathfrak{s}^*)),\delta,b^i\right)\right]-E\left[U^i\left(\mathbf{y}\left(m^i(\hat{\mathfrak{s}}^i),\mathbf{m}^*_{\text{-}i}\right),\delta,b^i\right)\right] = -C(\mathfrak{s}^{i*})+C(\hat{\mathfrak{s}}^i) < 0$$

So, $\hat{\mathfrak{s}}^i$ is a profitable deviation from $\mathfrak{s}^{i*}$. $\Rightarrow\Leftarrow$

Acquisition of S_2. The proof proceed in the same way as the previous, with i's equilibrium info acquisition strategy such that: $S_2 \in \mathfrak{s}^{i*}$ but condition (3) not holding for S_2, and the profitable deviation being $\hat{\mathfrak{s}}^i = \mathfrak{s}^{i*}\backslash\{S_2\}$.

Acquisition of both signals. The proof is similar to the previous, with i's proposed equilibrium strategy being $\mathfrak{s}^{i*} = \{S_1, S_2\}$. In this case, acquiring each individual signal is ex-ante incentive compatible since i is willing to reveal truthfully each of them (individually), but Full Revelation is not.

According to Proposition 1 in Habermacher (2018), i will reveal the signal with the greatest marginal influence in equilibrium. The profitable deviation (on-path) thus is to acquire only that signal which, following the previous arguments, leaves the agent better-off. □

C.2 Proof of Lemma 5

Proof. To be completed. The intuitions are similar to the case in which the agent observes both signals, but incentives for communication in this case depend only on other senders revealing the same information. To see this recall that the credibility loss when the agent observes both signals arises when their realizations update the corresponding beliefs in opposite directions. In an equilibrium in which he is believed to report one signal only, he has incentives to lie when this signals goes against his bias and the other towards it. This is because he *believes* the induced movement in decision(s) should be smaller than if he reveals his signal truthfully, and he prefers to reveal the most favourable signal. But the strength of these incentives to lie depend on how many other sender reveal this signal in equilibrium; that is, how much smaller should he believes the induced movement should be depends on how important is expected his signal to be in equilibrium.

When the agent acquires only one signal any information transmitted moves both decision in the same direction (according to the correlation). Communication of S^i_1 thus does not depend on information the decision-maker has about S^i_2 (and vice-versa) because the agent specialized in θ_1 does not know about the other state. The IC constraints then look similar to (1) and (3) without the minus term associated to the other state on the RHS. □

C.3 Equilibrium Information Acquisition and Cost-effectiveness condition

Before the proof of Lemma 6 I derive the information-acquisition IC constraint. The ex-ante expected utility for agent i is given by:

Observation 2. *Let* $k_r^* \equiv k_r\left(\mathbf{m}^i(\mathfrak{s}^{i*}, \mathfrak{s}^{-i}), \mathbf{m}^{-i}((\mathfrak{s}^{i*}, \mathfrak{s}^{-i}))\right)$ *and* $\hat{k}_r \equiv k_r\left(\mathbf{m}^i(\hat{\mathfrak{s}}^i, \mathfrak{s}^{-i}), \mathbf{m}^{-i}((\mathfrak{s}^{i*}, \mathfrak{s}^{-i}))\right)$ *for* $\theta_r = \{\theta_1, \theta_2\}$. *Let* $\mathfrak{s}^{i*}$ *denote i's information acquisition strategy in an equilibrium characterized by* $(\mathbf{y}^*, \mathbf{m}^*, \mathfrak{s}^*)$. *Then, i's ex-ante expected utility from* $\mathfrak{s}^{i*}$ *is given by:*

$$E\left[U^i\left(\mathbf{m}^*, \mathfrak{s}^{i*}, \mathfrak{s}^{-i*}, \boldsymbol{\delta}, \mathbf{b}^i\right)\right] = -\left[(b_1^i)^2 + (b_2^i)^2\right] - \frac{\left[(w_{11})^2 + (w_{21})^2\right]}{6\left(k_1^* + 2\right)} - \frac{\left[(w_{12})^2 + (w_{22})^2\right]}{6\left(k_2^* + 2\right)}$$

Observation 3. *Let* $(\mathbf{y}^*, \mathbf{m}^*, \mathfrak{s}^*)$ *be equilibrium strategy profiles. Then,* $\mathfrak{s}^{i*}$ *is incentive compatible for agent i if and only if, for every alternative* $\hat{\mathfrak{s}}^i$*:*

$$\frac{[(w_{11})^2 + (w_{21})^2]}{6}\left[\frac{1}{\left(\hat{k}_1 + 2\right)} - \frac{1}{\left(k_1^* + 2\right)}\right] + \frac{[(w_{12})^2 + (w_{22})^2]}{6}\left[\frac{1}{\left(\hat{k}_2 + 2\right)} - \frac{1}{\left(k_2^* + 2\right)}\right] \geq \left[C(\mathfrak{s}^{i*}) - C(\hat{\mathfrak{s}}^i)\right] \tag{20}$$

Proof of Lemma 6

Proof. I first derive the cost-effectiveness condition (11) and then the maximum number of agents for which acquiring a given piece of information is cost-effective –condition (14). In order to derive cost-effectiveness, I consider each possible info acquisition strategy in equilibrium.

Recall that k_r^* is the number of agents revealing truthfully their signals in equilibrium, which includes i's message strategy in the equilibrium under consideration. In other words, in equilibria in which i does not acquire S_1^i, k_1^* does not count him; but in any deviation in which he does acquire it, $\hat{k}_1 = k_1^* + 1$. Here I need to make two clarifications. The first relates to the effect of i's deviation on other agents' message strategies: if there were agents that acquired S_1 but were not willing to reveal it when $k_1 = k_1^* + 1$, then this will make only one of them to change his message strategy —i.e. when one of these agents stop revealing, then $k_1 = k_1^*$ again. As a consequence, $\hat{k}_1 = \{k_1^*, k_1^* + 1\}$. I take the most conservative of these approaches by making $\hat{k}_1 = k_1^* + 1$ whenever i acquires S_1 off-path.

The second clarification relates to what happens when i acquires a signal and does not reveal it. Since other agents' message strategies will depend on conjectures about k_r, i not revealing the signal acquired off-path will not affect their equilibrium behaviour at the communication stage. In other words, Lemma 4 holds: i gains nothing from acquiring a signal he will not reveal.

Let first consider the acquisition of both signals in equilibrium; that is $\mathfrak{s}^{i*} = \{S_1^i, S_2^i\}$. The IC constraint (20) for each possible alternative strategy become:

1) $\tilde{\mathfrak{s}}^i = \{\emptyset\}$

$$\sum_{\theta_r} \frac{(w_{1r})^2 + (w_{2r})^2}{6(k_r^* + 2)(k_r^* + 3)} \geq C(S_1^i, S_2^i)$$

2) $\tilde{\mathfrak{s}}^i = \{S_r^i\}$

$$\frac{(w_{1\tilde{r}})^2 + (w_{2\tilde{r}})^2}{6(k_{\tilde{r}}^* + 2)(k_{\tilde{r}}^* + 3)} \geq C(S_{\tilde{r}}^i)$$

Now, when the equilibrium strategy consists of one signal only, $\mathfrak{s}^{i*} = \{\tilde{S}_r^i\}$, the IC constraints become:

3) $\tilde{\mathfrak{s}}^i = \{\emptyset\}$

$$\frac{(w_{1r})^2 + (w_{2r})^2}{6(k_r^* + 2)(k_r^* + 3)} \geq C(S_r^i)$$

4) $\tilde{\mathfrak{s}}^i = \{S^i_{\tilde{r}}\}$

$$\frac{(w_{1r})^2 + (w_{2r})^2}{6(k^*_r + 2)(k^*_r + 3)} \geq \frac{(w_{1\tilde{r}})^2 + (w_{2\tilde{r}})^2}{6(k^*_{\tilde{r}} + 2)(k^*_{\tilde{r}} + 3)}$$

5) $\tilde{\mathfrak{s}}^i = \{S^i_1, S^i_2\}$

$$\frac{(w_{1\tilde{r}})^2 + (w_{2\tilde{r}})^2}{6(k^*_{\tilde{r}} + 2)(k^*_{\tilde{r}} + 3)} < C(S^i_{\tilde{r}})$$

It is straightforward to note that case 3) represents the necessary condition to acquire any individual signal S^i_r, since it implies case 1) (in which it holds for both signals) and case 4) (in which the agent acquires the signal that would have the highest influence). This case corresponds to equation (11).

Now I work out the expression for the maximum number of agents to acquire a given signal. According to the Cost-effectiveness condition (11), as k_r increases the ex-ante expected utility of acquiring S_r decreases. So, the maximum number of agents who will acquire that signal is given by the largest k^* for which the cost-effectiveness condition hold. Re-arranging this condition I get the following polynomial:

$$-(k^*_r)^2 - 5k^*_r - \left[6 - \frac{(w_{1r})^2 + (w_{2r})^2}{6\,C(S^i_r)}\right] \geq 0$$

Then, solving for the highest positive root I get K^{C}_r in (14). □

Proof of equation (12). For any dimensional non separable strategy in which i fully reveals some realizations and play babbling on the others, the expected payoff in equation (20) becomes.

$$\frac{[(w_{11})^2 + (w_{21})^2]}{6}\left[\frac{1}{(\hat{k}_1 + 2)} - \frac{1}{2(k^*_1 + 2)} - \frac{1}{2(k^*_1 + 3)}\right] + \frac{[(w_{12})^2 + (w_{22})^2]}{6}\left[\frac{1}{(\hat{k}_2 + 2)} - \frac{1}{2(k^*_2 + 2)} - \frac{1}{2(k^*_2 + 3)}\right] \geq \left[C(\mathfrak{s}^{i*}) -\right.$$

Because i fully reveals his signals half of the time (in expectation) –where k^*_r indicates the equilibrium number of agents revealing information about θ_r apart from i. Then, solving for deviations as in the previous result, I get that acquiring both signals to play a DNS message strategy is cost effective if:

$$\frac{(w_{11})^2 + (w_{21})^2}{6(k^*_1 + 2)(k^*_1 + 3)} + \frac{(w_{12})^2 + (w_{22})^2}{6(k^*_2 + 2)(k^*_2 + 3)} \geq 2C(S^i_1, S^i_2)$$

Now, i would prefer to acquire both signals and play DNS strategy to acquire only S_r and reveal it for sure if:

$$\frac{(w_{1\tilde{r}})^2 + (w_{2\tilde{r}})^2}{6(k^*_{\tilde{r}} + 2)(k^*_{\tilde{r}} + 3)} - \frac{(w_{1r})^2 + (w_{2r})^2}{6(k^*_r + 2)(k^*_r + 3)} \geq 2C(S^i_{\tilde{r}})$$

Which is easily shown that never holds when $w_{11} = w_{22}$ and $w_{d1} + w_{d_2} = 1$. □

C.4 Cost-effectiveness under delegation

Observation 4 (Observation 3 bis)**.** *Let* $(\mathbf{y}^*, \mathbf{m}^*, \mathfrak{s}^*)$ *be equilibrium strategy profiles. Let index the decision-maker for* y_d *as* $j(d)$*, and* $k^{j(d)}_r$ *be the number of agents revealing* S_r *to* $j(d)$ *on the equilibrium path. Then,* $\mathfrak{s}^{i*}$ *is incentive compatible for agent i if and only if, for every alternative information acquisition strategy* $\hat{\mathfrak{s}}^i$*:*

$$\sum_{y_d=\{y_1,y_2\}} \sum_{\theta_r=\{\theta_1,\theta_2\}} \frac{(w_{dr})^2}{6}\left[\frac{1}{(\hat{k}^{j(d)}_r + 2)} - \frac{1}{(k^{j(d)*}_r + 2)}\right] \geq \left[C(\mathfrak{s}^{i*}) - C(\hat{\mathfrak{s}}^i)\right] \quad (21)$$

It follows from the ex-ante expected utility (bias-variance decomposition, see Appendix B.2), given each decision is taken by a different decision-maker, such that k_rs are indexed by who decides on each issue.

Proof. As before, agent i will not invest in signals he is not willing to reveal on-path (Lemma 4). But there are two decision-makers under delegations and, thus, communication IC could refer to any of them (or both). From(21) it is easy to see that the necessary condition to acquire S_r^i requires that i is willing to reveal it to at least one decision-maker, say:

$$C(S_r^i) \leq \frac{(w_{dr})^2}{6(k_r^j+2)(k_r^j+3)}$$

For at least one y_d.

Now, investing in both signals will be cost-effective in two generic cases. When i is willing to reveal at least one signal to a different decision-maker, then the previous condition should also hold such that each signal matches a different decision-maker —i.e. each r is paired with a different d. When i is willing to reveal both signals to a single j the RHS of the above expression becomes larger. As a consequence the above expression –equation (13)– is a necessary condition for investing in any individual signal.

In order to get the expression for the maximum number of agents to invest in S_r I need to analyse also two cases. The minimal incentives to reveal will be given by the case in which all agents are willing to reveal S_r to decide on the dimension it is less important. This will define the minimum upper-bound, since the CE condition becomes

$$C(S_r^i) \leq \min_{w_{dr}} \left\{ \frac{(w_{dr})^2}{6(k_r^j+2)(k_r^j+3)} \right\}$$

Now consider the case in which all agents are willing to reveal *both signals* to *both decision-makers*. This is the maximum upper-bound. In such a case, the CE condition will be just like the centralization case; that is:

$$C(S_r^i) \leq \frac{(w_{1r})^2+(w_{2r})^2}{6(k_r^j+2)(k_r^j+3)}$$

Then, following the same steps as the proof of Lemma 6 (Appendix C.3) I have the first and the second expressions in square bracket in equation (15), respectively. □

C.5 Proof of Proposition 4

Proof. By Lemma 4 and Proposition 3 I established that on the equilibrium path the info acquisition strategy must be CE and revealing the acquired information must be IC at the communication stage. In addition, by overt information acquisition, the Principal observes agents' choices of information, this allows her to know the relevant message space. The previous imply that equilibrium communication is characterized by Proposition 1 in Habermacher (2018) –conditional on i having acquired the corresponding information.

But cost-effectiveness can impose restrictions on equilibrium communication: when i cannot 'afford' to acquire all the information he is willing to reveal on-path. Consider the case in which i would reveal both signals but only one of them is cost-effective, which means the expression in case 1) of the previous proof does not hold. But if i is willing to reveal both signals he must be willing to reveal each of them individually. So despite he cannot afford both, he will acquire one signal if cost-effective, and equilibrium communication will involve revealing truthfully the signal he acquired. Which signal i's will actually acquire in this case will depend on his ex-ante expected utility –see case 4) in the previous proof.

Similar argument applies in any equilibrium in which i is willing to reveal S_r^i only. If acquiring it is CE, then he reveals that information in equilibrium; if not, he does not reveal any information –indeed, he has no message to send and the receiver observes that.

Finally, no information is transmitted when i's preferences are such that he is not willing to reveal any information, or when acquiring any signal is not cost-effective.

For the case of delegation the same argument applies, with Lemmas 1 and 4, and Corollary 2. □

C.6 Proof of Corollary 3

Proof. As previously, when the cost does not impose actual restrictions on information acquisition, communication in the most informative equilibrium will be dictated by IC constraints in Lemma 2. In the case of two agents and linear costs, CE does not restrict agents' communication strategies if the cost of any signal is lower than the ex-ante expected utility in the equilibrium under consideration ($k_r^* = 1$); that is:

$$C(S_r^i) \leq \frac{(w_{1r})^2 + (w_{2r})^2}{6(k_r^* + 2)(k_r^* + 3)}$$
$$c \leq \frac{(w)^2 + (1-w)^2}{72}$$

In such a case, the most informative equilibrium has A_1 acquiring and revealing S_1^1 only if his preferences satisfy the corresponding IC constraint (3) for $k_1^* = 1$. In parallel, he should not be willing to reveal both signals.[32] The equivalent argument applies for A_2 with respect to S_2^2.

When $\frac{(w)^2+(1-w)^2}{72} < c \leq \frac{(w)^2+(1-w)^2}{36}$ it is not CE to acquire both signals, so agents will acquire the signal each of them is willing to reveal. In other words, the necessary and sufficient condition for specialization in this case is that (3) holds for different signals. □

C.7 Proof of Proposition 6

Proof. Let $\kappa < n$ be the maximum number of agents willing to reveal S_r to both decision-makers, suppose $\kappa > 0$. For any of these agents, cost-effectiveness under delegation –condition (13)– is given by:

$$C(S_r^i) \leq \frac{(w_{1r})^2 + (w_{2r})^2}{6(k_r^j + 2)(k_r^j + 3)}$$

But for any other agent, the most favourable CE condition is:

$$C(S_r^i) \leq \frac{(\hat{w}_{dr})^2}{6(k_r^j + 2)(k_r^j + 3)}$$

Then, $C(S_r^i) \leq \frac{(w_{1r})^2+(w_{2r})^2}{6(\kappa+2)(\kappa+3)}$ implies that CE holds for at most κ agents under delegation if they reveal S_r to both decision makers. But the CE will not hold for other agents because any of these will at most reveal one signal and conditions on the cost make it not CE —i.e. $C(S_r^i) > \frac{(\hat{w}_{dr})^2}{6(\kappa+2)(\kappa+3)}$.

On the other hand, equation (14) determines the maximum number of agents for which acquiring S_r is CE under centralization. But since $C(S_r^i) \leq \frac{(w_{1r})^2+(w_{2r})^2}{6(\kappa+3)(\kappa+4)}$, it should be greater or equal to $\kappa + 1$. Then, $K_r^{\text{D}} = \kappa < K_r^{\text{C}}$ □

C.8 Proof of Corollary 4

Proof. $\kappa = 1$ implies that only informed decision-makers acquire both signals under delegation. Noting that $6(\kappa+2)(\kappa+3) = 72$, Proposition 6 implies that $K_r^{\text{C}} > K_r^{\text{D}} = 1$. In addition, from Proposition 3 we know the the maximum bias the Principal will tolerate when $k_r^j = 1$; that is:

$$||\hat{\mathbf{b}}^{\text{D}}|| = \left[\frac{w^2}{6}\left(1 - \frac{2}{3}\right) + \frac{(1-w)^2}{6}\left(1 - \frac{2}{3}\right)\right] = 1 - 2w(1-w)$$

If there are no agents j' and j'' such that $||\mathbf{b}^{\text{D}}|| \leq ||\hat{\mathbf{b}}^{\text{D}}||$, then the informational gains under delegation do not compensate the distributional loss and centralization yields higher ex-ante expected utility to the

[32] Note that if condition (3) holds for S_2^1 (but still not those for full revelation) he will still find optimal to acquire S_1^1 only, *given A_2 is acquiring the other signal in equilibrium.*

Principal. If, on the other hand, there were such agents j' and j'', the Principal will prefer delegation only when there is no agent i for whom full revelation is IC —i.e. informational gains are non-zero. $\square$

References

Aghion, P. and J. Tirole (1997). Formal and real authority in organizations. *Journal of Political Economy 105*(1), 1–29.

Alonso, R., W. Dessein, and N. Matouschek (2008). When does coordination require centralization? *American Economic Review 98*(1), 145–79.

Alonso, R., W. Dessein, and N. Matouschek (2015). Organizing to adapt and compete. *American Economic Journal: Microeconomics 7*(2), 158–87.

Ambrus, A. and S. Takahashi (2008). Multi-sender cheap talk with restricted state spaces. *Theoretical Economics 3*(1), 1–27.

Andersson, U. and M. Forsgren (2000). In search of centre of excellence: Network embeddedness and subsidiary roles in multinational corporations. *Management International Review 40*(4), 389–350.

Argenziano, R., S. Severinov, and F. Squintani (2016). Strategic information acquisition and transmission. *American Economic Journal: Microeconomics 8*(3), 119–55.

Austen-Smith, D. (1990). Information transmission in debate. *American Journal of Political Science 34*(1), 124.

Austen-Smith, D. (1993). Information acquisition and the orthogonal argument. In W. A. Barnett, M. J. Hinich, and N. J. Schofield (Eds.), *Political Economy: Institutions, Competition, and Representation*, pp. 407–436. Cambridge University Press.

Austen-Smith, D. and W. H. Riker (1987). Asymmetric information and the coherence of legislation. *The American Political Science Review 81*(3), 897–918.

Battaglini, M. (2002). Multiple referrals and multidimensional cheap talk. *Econometrica 70*(4), 1379–1401.

Battaglini, M. (2004). Policy advice with imperfectly informed experts. *Advances in Theoretical Economics 4*(1), 1–32.

Baumgartner, F. R. and B. D. Jones (2009). *Agendas and instability in American politics* (2nd ed.). The University of Chicago Press.

Boutellier, R., O. Gassmann, and M. von Zedtwitz (2008). *Managing Global Innovation* (3rd ed.). Springer.

Brett, J. M. (2007). *Negotiating Globally* (2nd ed.). John Wiley and Sons.

Chakraborty, A. and R. Harbaugh (2007). Comparative cheap talk. *Journal of Economic Theory 132*(1), 70 – 94.

Crawford, V. P. and J. Sobel (1982). Strategic information transmission. *Econometrica 50*(6), 1431–1451.

Dessein, W. (2002). Authority and communication in organizations. *The Review of Economic Studies 69*(4), 811–838.

Dessein, W., A. Galeotti, and T. Santos (2016, June). Rational inattention and organizational focus. *American Economic Review 106*(6), 1522–36.

Dessein, W. and T. Santos (2006). Adaptive organizations. *Journal of Political Economy 114*(5), 956–995.

Dewan, T., A. Galeotti, C. Ghiglino, and F. Squintani (2015). Information aggregation and optimal structure of the executive. *American Journal of Political Science 59*(2), 475–494.

Dewan, T. and F. Squintani (2015). In defense of factions. *American Journal of Political Science*.

Di Pei, H. (2015). Communication with endogenous information acquisition. *Journal of Economic Theory 160*, 132 – 149.

Ecker, B., S. van Triest, and C. Williams (2013). Management control and the decentralization of r&d. *Journal of Management 39*(4), 906–927.

Farrell, J. (1993). Meaning and credibility in cheap-talk games. *Games and Economic Behavior 5*(4), 514–531.

Galeotti, A., C. Ghiglino, and F. Squintani (2013). Strategic information transmission networks. *Journal of Economic Theory 148*(5), 1751 – 1769.

Gassmann, O. and M. von Zedtwitz (1999). New concepts and trends in international r&d organization. *Research Policy 28*(2-3), 231–250.

Gilligan, T. and K. Krehbiel (1987). Collective decisionmaking and standing committees: An informational rationale for restrictive amendment procedures. *Journal of Law, Economics, and Organization 3*(2), 287–335.

Gilligan, T. W. and K. Krehbiel (1989). Asymmetric information and legislative rules with a heterogeneous committee. *American Journal of Political Science 33*(2), 459–490.

Habermacher, D. F. (2018). Information aggregation in multidimensional cheap talk.

Krishna, V. and J. Morgan (2001a). Asymmetric information and legislative rules: Some amendments. *The American Political Science Review 95*(2), 435–452.

Krishna, V. and J. Morgan (2001b). A model of expertise. *The Quarterly Journal of Economics 116*(2), 747–775.

Levy, G. and R. Razin (2007). On the limits of communication in multidimensional cheap talk: A comment. *Econometrica 75*(3), 885–893.

Lindsey, D. (2017). Diplomacy through agents. *International Studies Quarterly 61*(3), 544–556.

Liu, S. and D. Migrow (2018). Designing organizations in volatile markets.

Migrow, D. (2017). Designing communication hierarchies in organizations.

Morgan, T. C. (1990). Issue linkages in international crisis bargaining. *American Journal of Political Science 34*(2), 311–333.

Rantakari, H. (2008). Governing adaptation. *The Review of Economic Studies 75*(4), 1257–1285.

Trager, R. F. (2010). Diplomatic calculus in anarchy: How communication matters. *American Political Science Review 104*(2), 347368.

Trager, R. F. (2011). Multidimensional diplomacy. *International Organization 65*(3), 469–506.

Trager, R. F. (2017). *Diplomacy: Communication and the Origins of International Order*. Cambridge University Press.