The Firm-Growth Imperative: A Theory of Production and Personnel Management*

Preliminary and Incomplete

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Abstract

We develop a model in which a firm makes a sequence of production decisions and has to motivate each of its employees to exert effort. The firm motivates its employees through incentive pay and promotion opportunities, which may differ across different cohorts of workers. We show that the firm benefits from reallocating promotion opportunities across cohorts, resulting in an optimal personnel policy that is seniority-based. Our main contribution is to highlight a novel time-inconsistent motive for firm growth: when the firm adopts an optimal personnel policy, it may pursue future growth precisely to create promotion opportunities for existing employees.

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1 Introduction

To meet the increased demand for explosives brought about by World War I, DuPont expanded its work force from 5,000 in 1914 to 85,000 in 1918. But executives were keenly aware that the war would not last forever, and they formed plans for post-war diversification in part to ensure their employees would continue to have jobs. In addition to entering into related chemical-based industries, they made investments in other industries to have "a place to locate some managerial personnel who might not be absorbed by the expansion into chemical-based industries." (Chandler, 1962, p. 90)

This example illustrates that good management requires planning ahead. It requires planning future production both to adapt to future business conditions, and to set up future career opportunities for current employees. Such opportunities are abundant in fast-growing firms and can be used to great effect in motivating workers. And in declining firms, they may be scarce or nonexistent, requiring the firm to rely on alternative ways to provide motivation. Production plans therefore affect the kinds of personnel policies the firm should adopt.

At the same time, the firm's personnel policies influence its future production plans, as many practitioners and management scholars have long highlighted. Jensen, for example, illustrates this point by arguing that using promotions to motivate employees "creates a strong organizational bias toward growth to supply the new positions that such promotion-based reward systems require." (1986, p. 2) Such growth is fundamentally backward-looking in nature and is often derided as wasteful, but it may serve an important purpose. Production plans and personnel policies therefore interact in meaningful ways and should be designed together.

This paper is an attempt to understand how a firm's past production decisions impact its future production plans when firms motivate their employees through the use of long-term, career-based incentives. Existing economic theories are not well-suited to explore these issues, since they either focus on the determinants of firm growth without accounting for long-term employee incentives (Lucas, 1978; Jovanovic, 1982; Hopenhayn, 1992; Ericson and Pakes, 1995) or they focus on long-term incentives for individual employees without exploring their implications for the size of the firm's workforce (see Rogerson (1985) and Spear and Srivastava (1987) for early contributions and Biais, Mariotti, and Rochet (2013) for a recent survey with a focus on financial contracts). We contribute to the existing literature by highlighting a novel time-inconsistent motivation for firm growth, and in doing so, we are able to assess when and why firms should pursue seemingly unprofitable growth strategies.

Model In our model, a single principal interacts repeatedly with a pool of employees. The interaction between the principal and each employee is a dynamic moral hazard problem with a limited-liability constraint. In each period, the principal assigns each employee to one of two jobs: a bottom job and a top job. In each job, the worker faces a moral hazard problem and must be provided incentives to exert effort. In the top job, the firm can motivate the worker by paying a bonus for good performance. In the bottom job, the firm can motivate the worker through a combination of bonuses for good performance and the prospect of being promoted to the top job.

Workers' promotion prospects depend both on how many new top positions there will be in the next period—which is determined by the firm's growth prospects—and on how many people are in line for these positions today—which is determined both by firm's production decisions today and by the personnel policies it has in place. The firm's problem is therefore to make its production plans and design its personnel policies jointly. In the model, the firm's profit-maximization problem can be decomposed into two steps. First, the firm chooses its pay and promotion policies subject to a production plan, a sequence of production decisions—the number of top and bottom positions in each period. Second, it chooses its production plan.

Results and Implications Our first set of results shows how a firm optimally designs its personnel policies given its production plan. In particular, we show that optimal promotion policies can be implemented through a modified first-in-first-out rule that favors workers with more seniority. If promotion policies were seniority-blind, then workers' promotion prospects in each period are completely determined by the firm's production plans, which can vary from period to period, leading to uneven promotion opportunities across cohorts. The firm can reallocate these opportunities by basing its promotion policies on seniority, and favoring senior workers has the advantage of motivating them in all previous periods they have been with the firm.

Our second, and main, set of results explores the implications of the firm's optimal incentive provision for its production decisions. In particular, we show that the firm may want to put in place a time-inconsistent production plan: it may want to pursue growth for "career opportunity's sake," growing faster than one's business opportunities. Doing so creates promotion opportunities that can attract employees at a lower wage. Early in its life-cycle, a firm may therefore wish to pursue a "slow-to-hire" policy, operating smaller than is statically optimal, whereas later in its life-cycle, it may wish to become larger than is statically optimal.

Extensions The tools we develop to analyze optimal personnel policies also allow us to explore how firms should manage employees' careers when business conditions require the firm to downsize.

An optimal personnel policy for a firm that has to make permanent cuts involves a first-in-last-out layoff policy and seniority-based severance payments: all laid-off workers are paid a severance payment upon dismissal, and less senior workers are dismissed first and receive a smaller severance payment. If the cuts the firm has to make are only temporary, then an optimal personnel policy entails seniority-based temporary layoffs: less-senior employees are laid off, but once the firm begins hiring again, it rehires them before it hires new employees. Finally, we extend our tools to analyze personnel policies in environments in which production plans are stochastic. Optimal personnel policies again resemble an internal labor market, and seniority-based promotion policies can be optimal.

Literature Review This paper contributes to the literature on internal labor markets (see Gibbons (1997), Gibbons and Waldman (1999b), Lazear (1999), Waldman (2012), and Lazear and Oyer (2013) for reviews of the theory on and evidence for internal labor markets). A particular feature of our model is that optimal personnel policies involve seniority-based promotion rules. The existing literature argues that seniority-based promotion policies can help motivate employees to invest in firm-specific human capital, (Carmichael, 1983) reduce rent-seeking behavior, (Milgrom and Roberts, 1988; Prendergast and Topel, 1996) and to better capture information rents related to its workers' abilities (Waldman, 1990). In our model, basing promotion decisions on seniority allows a firm that experiences uneven growth to better provide incentives by reallocating promotion opportunities across different cohorts of workers.

Our paper also contributes to the literature on the determinants of firm growth. Standard models of firm growth have not explored the effects of the size of a firm's workforce on its production plans (Jovanovic (1982), Hopenhayn (1992), Ericson and Pakes (1995), Albuquerque and Hopenhayn (2004), Clementi and Hopenhayn (2006)). One exception is Bennett and Levinthal (2017). In Bennett and Levinthal's (2017) model, workers exert unobservable effort to improve the production process. Two key assumptions are that the moral hazard problem is more severe in larger firms and that process improvement exhibits diminishing returns. As a result, smaller firms can motivate workers more cheaply and, at the same time, grow faster on average. Bennett and Levinthal (2017) does not consider dynamic incentive provision, and as a result, optimal production plans are time-consistent. In contrast, our focus on dynamic incentive provision leads to time-inconsistent production plans.

Finally, our paper contributes to the literature on dynamic moral hazard problems. In dynamic moral hazard settings, nontrivial dynamics can arise when there are contracting frictions.[1] The

[1] Such dynamics are highlighted by DeMarzo and Fishman (2007) and Clementi and Hopenhayn (2006) in a financial

closest papers are Board (2011) and Ke, Li, and Powell (2018). In Board's (2011) model, firms hire one supplier in each period, and the focus of its analysis is on which supplier to utilize. The focus of our model is on the number of workers to hire in each period, and this is a choice variable of the firm and can change from period to period. Ke, Li, and Powell (2018) examines how organizational constraints affect firms' personnel policies in a stationary environment in which the size of the firm is constant. In this case, there are no gains to reallocating promotion opportunities across cohorts, and optimal personnel policies are seniority-blind. In our model, uneven growth leads to seniority-based personnel policies, and the need to provide long-term incentives leads the firm to adopt time-inconsistent production plans.

2 The Model

A firm interacts with a pool of risk-neutral workers in periods $t = 1, \dots, T$, where T may be infinite, and all players share a common discount factor $\delta \in (0, 1)$. The firm's labor pool consists of a large mass of identical workers, and the firm chooses a personnel policy, which we will describe below, to maximize its discounted profits.

Production requires two types of activities to be performed, and each worker can perform a single activity in each period. A worker performing activity $i \in \{1, 2\}$ in period t chooses an effort level $e_t \in \{0, 1\}$ at cost $c_i e_t$. A worker who chooses $e_t = 0$ is said to *shirk*, and a worker who chooses $e_t = 1$ is said to *exert effort*. We refer to a worker who exerts effort as *productive*. A worker's effort is his private information, but it generates a publicly observable signal $y_{i,t} \in \{0, 1\}$ with $\Pr[y_{i,t} = 1 | e_t] = e_t + (1 - q_i)(1 - e_t)$, that is, shirking in activity i is contemporaneously detected with probability q_i. If the firm employs masses $N_{1,t}$ and $N_{2,t}$ of productive workers in the two activities, revenues are $F_t(N_{1,t}, N_{2,t})$.

In each period, the firm assigns each worker to an activity $i_t \in \mathcal{A} \equiv \{0, 1, 2\}$, where activity 0 is a non-productive activity. The worker accepts the assignment or rejects the assignment and exits the firm's labor pool, receiving an outside option that yields utility 0. If the worker accepts the assignment, he then exerts effort e_t, his signal $y_{i,t}$ is realized, and then the firm pays the worker an amount $W_t \geq 0$. That is, the worker is protected by a limited-liability constraint. At the end of each period, each worker exogenously exits the firm's labor pool with probability d and receives 0

contracting setting (see Biais, Mariotti, and Rochet (2013) for a survey), by Li and Matouschek (2013), Halac (2012), Zhu (2012), Fong and Li (2017), and Barron and Powell (2017) in an employment setting, by Boleslavsky and Kelly (2014) in a regulatory setting, by Padro i Miquel and Yared (2012) in a political economy setting, by Andrews and Barron (2016), and Urgun (2017) in a supplier allocation setting, and by Li, Matouschek, and Powell (2017), Lipnowski and Ramos (2017), and Guo and Horner (2017) in a delegation setting.

in all future periods, and a group of new workers enters the firm's labor pool.

Define a worker's *employment history* to be a sequence $h^t = (0, \ldots, 0, A_\tau, \ldots, A_t) \in \mathcal{H}^t$, where $A_s \in \mathcal{A}$ specifies the activity he was assigned to in period s, and τ is the time at which he first enters the firm's labor pool. By convention, we say that a worker is assigned to activity 0 in each period before he is in the firm's labor pool. We will say that a worker who is assigned to activity 1 or 2 for the first time in period t is a *new hire in period t* and that he is a *cohort-t worker*. Define $L(h^t)$ to be the mass of workers with employment history h^t.

Before we define a contract between the firm and a worker, we pause to make two observations that will help simplify notation. First, if a worker is assigned to activity 1 or 2 and is not asked to exert effort this period, then we can instead assign him to activity 0 this period. Second, if a worker is assigned to activity 1 or 2 and is asked to exert effort, it is without loss of generality to pay him 0 in this period and in all future periods if his signal is equal to 0. This follows because when a worker's signal is 0, the worker must have shirked, and this is the harshest punishment possible.

Given these two observations, we can now define a contract between the firm and a worker. A *contract* is a sequence of *assignment policies* $P_{i,t} : \mathcal{H}^{t-1} \to [0,1]$ specifying the probability the worker is assigned to activity i given employment history h^t, and a sequence of *wage policies* $W_t : \mathcal{H}^t \to [0, \infty)$ specifying the wage the worker receives at the end of period t given his history. A *personnel policy* is a set of contracts the firm has with each worker in its labor pool.

The firm's period-t profits are

$$F_t(N_{1,t}, N_{2,t}) - \sum_{h^t \in H^t} W_t(h^t) L(h^t),$$

and each worker's period t utility is $W_t(h^t) - c_i e_t$. The firm's problem is to choose $\{W_t, P_{i,t}\}$ to maximize its expected discounted profits, and given the contract he faces, each worker chooses his acceptance decisions and effort decisions to maximize his expected discounted utility. Throughout most of the analysis, we will be focusing on contracts for which if a worker is ever assigned to activity 0 after he has been assigned to activity 1 or 2, he is assigned to activity 0 in all future periods. We will refer to such contracts as *full-effort contracts* because they motivate the worker to exert effort in every period they have been employed by the firm. In Section 6.2, we discuss situations in which the firm would optimally choose contracts that permit workers to shirk in some periods.

Finally, we define a *production path* to be a sequence $N = \{N_{1,t}, N_{2,t}\}_{t=1}^T$ that specifies the mass of productive workers in each activity in each period. We will say that a production path is *steady* if $N_{2,t+1} < (1-d)(N_{1,t} + N_{2,t})$ and $N_{i,t+1} \geq N_{i,t}(1-d)$ for $i = 1, 2$. The first condition for a steady production path says that the number of top positions in the firm does not grow too fast—it ensures that, in each period, there are enough incumbent workers to fill all the activity 2 positions. The second condition says that the firm does not shrink too fast.

3 Preliminaries

Our analysis decomposes the firm's overall problem into two steps. First, given any production path N, we derive properties of optimal personnel policies that induce a mass $N_{i,t}$ of workers assigned to activity i to exert effort in period t. This section takes the production path as given, sets up the firm's cost-minimization program, and develops several intermediate results to simplify the analysis. In particular, we show that the firm's cost-minimization problem is equivalent to minimizing the rents that are paid to new hires.

The second step of the firm's problem is to choose an optimal production path N^* given the external environment it faces. Section 5 analyzes the second step of the firm's problem.

3.1 Cost-Minimization Problem

Recall that a personnel policy is a set of contracts the firm has with each worker in its labor pool, where each contract describes the assignment policy and the wage policy the worker is subject to. Given a production path N, we will say that a personnel policy *implements* N if, given the personnel policy, a mass $N_{1,t}$ and $N_{2,t}$ of workers exerts effort in activities 1 and 2 in period t. Denote a worker's *initial-hire history* by $n^t = (0, \ldots, 0, n_t)$, where $n_t \in \{1, 2\}$. The first lemma shows that the problem of characterizing cost-minimizing personnel policies can be simplified by focusing on a smaller class of personnel policies. All the proofs are in the appendix.

Lemma 1. *Given N if there is an optimal personnel policy, there is an optimal personnel policy in which workers with the same employment history face the same wage and assignment policies.*

In order to specify the firm's program, define $c(h^t) = c_{A_t}$ if $A_t \in \{1, 2\}$ and 0 otherwise, and $q(h^t) = q_{A_t}$ if $A_t \in \{1, 2\}$. Denote by $w(h^t)$ the wage the worker receives if $y_{A_t,t} = 1$ and by $p_i(h^t)$ the probability the worker is assigned to activity i in the next period, conditional on remaining in the labor pool. Denote by $h^t A_{t+1} = (A_1, \ldots, A_{t+1})$ the concatenation of h^t with A_{t+1}. For all workers in the labor pool, we have

$$L(h^t i) = (1-d)\, p_i(h^t)\, L(h^t).$$

Given a production path N, the firm's problem is to minimize its wage bill

$$\min_{w,p_i} \sum_{t=1}^{T} \sum_{h^t \in \mathcal{H}^t} \delta^{t-1} L\left(h^t\right) w\left(h^t\right)$$

subject to the following constraints.

Promise-Keeping Constraints. If we denote by $v\left(h^t\right)$ the worker's expected discounted payoffs at time t given employment history h^t, then workers' payoffs have to be equal to the sum of their current payoffs and their continuation payoffs:

$$v\left(h^t\right) = w\left(h^t\right) - c\left(h^t\right) + \delta\left(1-d\right) \sum_{i \in \{1,2\}} p_i\left(h^t\right) v\left(h^t i\right). \tag{1}$$

Incentive-Compatibility Constraints. Productive workers prefer to exert effort in activity i if they cannot gain by shirking:

$$v\left(h^t\right) \geq \left(1 - q\left(h_t\right)\right) \left(w\left(h^t\right) + \delta\left(1-d\right) \sum_{i \in \{1,2\}} p_i\left(h^t\right) v\left(h^t i\right) \right)$$

or equivalently

$$v\left(h^t\right) \geq \frac{1 - q_{A_t}}{q_{A_t}} c_{A_t} \equiv R_{A_t}, \tag{2}$$

where we refer to the quantity R_{A_t} as the *incentive rent for activity* A_t. Note that these incentive-compatibility constraints imply that workers receive positive surplus in equilibrium, and they therefore imply that workers' participation constraints are also satisfied. We therefore do not include workers' participation constraints in the firm's problem.

Flow Constraints. In each period, the firm employs a mass $N_{i,t}$ workers in activity i:

$$\sum_{h^t | A_t = i} L\left(h^t\right) = N_{i,t}, \text{ for } i \in \{1,2\}. \tag{3}$$

Given these constraints, the firm maximizes its profits. For a given production path, the firm's discounted profits are equal to the total discounted surplus net of the rents it pays to workers. Given a production path, therefore, the firm's problem is to minimize these rents. Recall that a worker with employment history n^t is a worker who is first employed by the firm in period t.

Lemma 2. *Cost-minimizing personnel policies minimize the rents paid to new hires:*

$$\min \sum_{t=1}^{T} \sum_{h^t \in \mathcal{H}^t} \delta^t L\left(n^t\right) v\left(n^t\right)$$

subject to (1), (2), *and* (3).

Lemma 2 shows that for a given production path, the firm's profit-maximization problem is equivalent to minimizing the rents that are paid to new hires. Notice that Lemma 2 captures the idea that since different workers potentially enter the firm at different times, the rents they receive are calculated at the time they enter the firm.

4 Optimal Personnel Policies

The firm's cost-minimization problem involves choosing an assignment policy and a wage policy for each worker at every history and is not amenable to standard Lagrangian techniques. For ease of exposition, we will first focus our analysis on steady production paths, returning to "unsteady" production paths in Section 6.1. Recall that a production path N is steady if $N_{2,t+1} < (1-d)(N_{1,t}+N_{2,t})$ and $N_{i,t+1} \geq N_{i,t}(1-d)$ for $i = 1, 2$. In the lemma below, we show that we can focus on a narrower class of personnel policies without loss of generality.

Lemma 3. *Given a steady production path N, if there is an optimal personnel policy, there is an optimal personnel policy with the following three properties:*

(*i.*) $v\left(h^t\right) \leq R_2$ *if* $A_t = 1$ *and* $v\left(h^t\right) = R_2$ *if* $A_t = 2$.

(*ii.*) *All new workers are assigned to activity* 1.

(*iii.*) $p_2\left(h^t\right) = 1$ *if* $A_t = 2$ *and* $p_1\left(h^t\right) + p_2\left(h^t\right) = 1$ *if* $A_t = 1$.

The first part of Lemma 3 shows that in a cost-minimizing personnel policy, the firm does not gain by rewarding workers with rents exceeding R_2. Part 2 of the lemma shows that new hires are always assigned to activity 1 unless the firm has grown sufficiently rapidly that it must hire new workers directly into activity 2. The final part of the lemma shows that as long as the firm never shrinks abruptly, workers performing activity 2 will continue to do so, and workers performing activity 1 will either continue to perform activity 1 in the next period or will be "promoted" to activity 2.

Lemma 3 highlights several features that are consistent with Doeringer and Piore's description of internal labor markets: (1) there is a port of entry, (2) there is a well-defined career path, and (3) wages increase upon promotion. We will say that a personnel policy satisfying these three properties is an *internal labor market*. Lemma 3 immediately implies the following proposition.

Proposition 1. *If N is a steady production path, an internal labor market is an optimal personnel policy.*

Proposition 1 shows that in a relatively stable environment, a cost-minimizing personnel policy can be implemented as an internal labor market. This result generalizes Proposition 3 in Ke, Li, and Powell (2018). For the rest of this section, we will characterize further properties of cost-minimizing internal labor markets when N is a steady production path. We defer discussion of cost-minimizing personnel policies for other production paths to Section 6.1.

An internal labor market fully pins down each worker's payoffs, wages, and assignment probabilities once a worker has been promoted. It remains to describe workers' period-t wages and period-t promotion probabilities for cohort-τ workers who have not been promoted prior to period t. Abusing notation slightly, denote by $w_{1,t}^{\tau}$ the wage paid to a cohort-τ worker who is assigned to activity 1 in period t. Similarly, denote by p_t^{τ} such a worker's promotion probability in period t and $v_{1,\tau}^{\tau}$ the value he places on the job in period t. We will say that the worker is a *candidate for promotion* in period t if $p_t^{\tau} > 0$. The following proposition describes how wages and promotion prospects evolve in an optimal internal labor market.

Proposition 2. *If N is a steady production path, an internal labor market with the following three properties is an optimal personnel policy: (i.) $w_{1,t+1}^{\tau} \geq w_{1,t}^{\tau}$, (ii.) if $\tau < \tau'$, then $w_{1,t}^{\tau} \geq w_{1,t}^{\tau'}$, $p_t^{\tau} \geq p_t^{\tau'}$, and $v_{1,t+1}^{\tau} \geq v_{1,t+1}^{\tau'}$, and (iii.) if $p_t^{\tau}, p_t^{\tau'} \in (0,1)$, then $v_{1,t+1}^{\tau} = v_{1,t+1}^{\tau'}$.*

The first part of Proposition 2 describes wage dynamics for a single cohort within the firm and shows that wages in activity 1 exhibit returns to tenure. This feature that wages are backloaded in a worker's career is familiar from models of optimal long-term contracts (Becker and Stigler, 1974; Lazear, 1979; Ray, 2002).

The second and third parts of the proposition compare wage and promotion dynamics across cohorts. In particular, the second part shows that for cohorts that enter the firm earlier, their wages, promotion probabilities, and the value they place on the job are higher than for cohorts that enter later. These results reflect the idea that the firm might benefit from redistributing rents across cohorts in order to transfer slack from one cohort's incentive constraint to another's, a point which we expand upon below.

One implication of part 2 is that if workers in an earlier cohort are not candidates for promotion in period t, then neither are workers from a later cohort. The third part shows further that a particular pattern of promotions is optimal. In particular, it implies that we can divide the set of workers assigned to activity 1 in period t into two "buckets": workers who are candidates for promotion and workers who are not candidates for promotion. When positions in activity 2 open up, those workers who entered the firm earliest will become candidates for promotion. Any worker who is a candidate for promotion in period t will also be a candidate for promotion in period $t+1$ if he is not promoted in period t, and moreover, he will be promoted with a weakly higher probability in period $t+1$ than any other worker.

This promotion pattern bears similarities to familiar first-in-first-out (FIFO) promotion patterns, with one crucial difference. In particular, it is not true that a worker will become eligible for promotion only once every worker from earlier cohorts is promoted. Note that if a FIFO promotion policy is used, then the time-to-promotion for each cohort is completely determined by N. A FIFO policy is suboptimal because it may force the firm to pay higher wages to cohorts whose promotion opportunities are limited. In contrast, our policy allows the firm to choose the time to candidacy for each cohort, enabling the firm to smooth promotion opportunities across cohorts and thus reduce the wage bill.

4.1 Examples of Optimal Personnel Policies

We now describe two examples of the model in order to highlight the implications of Proposition 2. The first example examines optimal personnel policies for constant-growth production paths and shows that a seniority-blind internal labor market can be optimal in the sense that the firm cannot do better than to tie promotion prospects and wages directly to a worker's current activity assignment, ignoring information about how long he has been assigned to that activity. The second example illustrates why this argument breaks down when growth is not constant. Here, we illustrate why it may be strictly optimal to treat different cohorts differently within a given period.

Constant Growth Production Path Suppose $T = \infty$ and that for $i = 1, 2$, the production path N satisfies $N_{i,t+1} = (1 + g) N_{i,t}$. We refer to such production paths as *constant-growth production paths.* If N is a constant-growth production path, the optimal internal labor market takes a simple form. If an internal labor market satisfies $(i.)$ $w\left(h^t\right) = w_{A_t}$, $(ii.)$ $p_2\left(h^{t-1}1\right) = p_2$, $(iii.)$ $v\left(h^t\right) = v_{A_t}$, we will say that it is *seniority-blind.*

Corollary 1. *If N is a constant-growth production path with $g \geq -d$, then a seniority-blind internal labor market is optimal.*

Corollary 1 illustrates that when the firm grows at a constant rate, it is optimal to condition wages, future assignment probabilities, and worker values only on workers' current activity assignments. Under a constant-growth production path, if every cohort is treated symmetrically, then each cohort has the same promotion opportunities and therefore, the value of relaxing each cohort's incentive constraints are equalized. Favoring one cohort over another relaxes one's incentive constraint and tightens the other's, and there are no gains from doing so.

Under constant-growth production paths, wages and promotion probabilities are particularly easy to calculate. Corollary 2 provides expressions for these objects and shows how they depend on the growth rate g.

Corollary 2. *Suppose N is a constant-growth production path with growth rate $g \geq -d$ and $N_{1,t}/N_{2,t} = s$. Then the promotion rate for workers performing activity 1 is*

$$p^* = \frac{1}{s}\frac{g+d}{1-d},$$

wages for workers assigned to activity 2 are $w_2^* = c_2 + (1-\delta(1-d)) R_2$, and there is a threshold growth rate $\hat{g}$ such that wages for workers assigned to activity 1 are

$$w_1^* = \begin{cases} 0 & \text{if } g \geq \hat{g} \\ c_1 + (1-\delta(1-d)) R_2 - \delta \frac{g+d}{s} (R_2 - R_1) & \text{if } g \leq \hat{g}. \end{cases}$$

Average wages paid in each period, $(sw_1^* + w_2^*)/(1+s)$ are decreasing in the firm's growth rate.

Corollary 2 shows that wages paid to workers assigned to activity 2 are independent of the firm's growth rate, while wages paid in activity 1 are decreasing in the growth rate. Faster growth rates allow the firm to provide better promotion opportunities, reducing the expected time to promotion for workers currently assigned to activity 1, which in turn allows the firm to reduce their wages. Once the growth rate exceeds $\hat{g}$, however, workers' promotion rates are sufficiently high that they are motivated to exert effort in activity 1 solely by their promotion opportunities, even if their limited-liability constraint binds. Further increases in the growth rate therefore increase workers' continuation payoffs, and since the firm can not extract this surplus by reducing wages further, this leads to an increase in the ex ante rents received by workers.

Variable Growth-Rate Production Path The previous example shows that optimal internal labor markets take a simple form when the firm's growth rate is constant because the firm treats all cohorts the same conditional on their current activity assignment. We now illustrate why it may be optimal to treat different cohorts differently conditional on activity assignment when the firm's growth rate is not constant. To make this point as clearly as possible, we make a number of strong assumptions in the example, but the forces we illustrate hold more broadly.

Suppose that $T = 3$ and that the firm's organizational span, $N_{1,t}/N_{2,t}$, is constant and equal to s. Let $g_t = (N_{i,t+1} - N_{i,t})/N_{i,t}$ denote the firm's growth rate from period t to period $t+1$. Let $d = 0, \delta = 1$, $s = 1$, and $q_1 = q_2 = 1/2$, which results in $R_i = c_i$.

Suppose the firm treats cohorts 1 and 2 identically in period 2. That is, the firm chooses $w_{1,2}^1 = w_{1,2}^2$ and $p_2^1 = p_2^2$. Then it must be the case that $p_2^1 = p_2^2 = g_2$. We now make the following assumptions.

Assumption 1. $g_2 > \frac{c_1}{c_2 - c_1}$.

Assumption 2. $c_1 > \frac{(1-g_1)}{(1-g_1)(1+g_2)+1} c_2$

Assumption 1 ensures that if the firm chooses $p_2^1 = p_2^2$, then even if cohort-2 workers earn a wage of zero in period 2, they will earn rents that strictly exceed R_1. The second assumption ensures that if the firm chooses $p_2^1 = p_2^2$ and $w_{1,2}^1 = w_{1,2}^2 = 0$, then in order to satisfy cohort 1 workers' incentive constraints in period 1, the firm must pay them strictly positive wages in period 1.

Proposition 3. *Under Assumptions* 1 *and* 2, *any seniority-blind internal labor market is strictly suboptimal.*

To see why a seniority-blind internal labor market is strictly suboptimal, begin with an optimal seniority-blind internal labor market. Assumption 1 ensures that such an internal labor market will pay workers $w_2^1 = w_2^2 = 0$, and workers' incentive constraints will be slack in the second period. Assumption 2 ensures that $w_1^1 > 0$, and cohort-1 workers' incentive constraints will bind in the first period. The firm can do better, however, by reducing the probability that cohort-2 workers are promoted, increasing the probability that cohort-1 workers are promoted, and reducing cohort-1 workers' first period wages. The proof of Proposition 3 constructs such a perturbation in a way that maintains the firm's flow constraint and ensures all incentive constraints are satisfied.

By favoring cohort-1 workers in period 2, the firm can transfer slack from cohort-2 workers' first-period incentive constraints to cohort-1 workers' first-period incentive constraints so that, instead of paying cohort-1 workers with wages, the firm can use promotion opportunities that would have gone to cohort-2 workers if the two cohorts were treated the same in period 2, and this allows the firm to reduce its overall wage bill. This perturbation is always feasible unless either $w_1^1 = 0$ or cohort-2's promotion probability in period 2 falls to the point where their second-period incentive constraint holds with equality, and $w_2^2 = 0$, so the optimal internal labor market favors cohort-1 over cohort-2 as much as is feasible in period 2.

5 Personnel Policies and Production Paths

In the previous section, we explored how differences in the firm's production path shape the characteristics of the internal labor market it puts in place. We now turn to the firm's problem of choosing an optimal production path and implementing it with a cost-minimizing personnel policy in order to understand how the firm's internal labor market shapes how the firm's production path will respond to changes in the external environment.

The firm's problem is to choose a production path and a personnel policy to maximize its discounted profits. In general, the problem of choosing a production path is complicated because the firm may prefer to choose a production path that is not steady. Our goal is to explore the differences between the firm's optimal production path and the production path it would choose in a more static environment, in a way that we will make precise.

Using Lemma 3, part (*i.*), we may assume that all workers assigned to activity 2 in period t receive the same wage, independent of when they started working at the firm, that is, $w_{2,t}^{\tau} = w_{2,t}$ for all τ. In addition, there are no demotions, so that the assignment policy can be summarized by the probability that workers are promoted from activity 1 to activity 2 in the next period. Denote by p_t^{τ} the promotion probability at the end of period t for cohort-τ workers who perform activity 1 in period t and by $w_{1,t}^{\tau}$ the wage a cohort-τ worker receives if he is assigned to activity 1 in period t. Define H_t to be the number of new workers the firm hires in period t and assigns to activity 1. Denote by z_t^s the fraction of cohort-s workers who were assigned to activity 1 in their first period of employment and remain in activity 1 in period t.

Assuming the firm puts in place an optimal internal labor market given their choice of production path, the firm's optimal production path solves

$$\max_{\{N_{1,t},N_{2,t},H_t\}_{t=1}^{T}} \sum_{t=1}^{T} \left[F_t\left(N_{1,t}, N_{2,t}\right) - w_{2,t}N_{2,t} - \sum_{s=1}^{t} H_s w_{1,t}^s z_t^s \right]$$

subject to the flow constraint for $N_{1,t}$

$$H_t + \sum_{s=1}^{t-1} H_s z_t^s = N_{1,t},$$

and the flow constraint for $N_{2,t}$

$$N_{2,t-1}\left(1-d\right) + \sum_{s=1}^{t-1} H_s z_{t-1}^s p_{t-1}^s = N_{2,t}.$$

The Lagrangian for this constrained maximization problem is

$$\begin{aligned} \mathcal{L} \;=\; & \sum_{t=1}^{T} \delta^t \left[F_t\left(N_{1,t}, N_{2,t}\right) - w_{2,t}N_{2,t} - \sum_{s=1}^{t} H_s w_{1,t}^s z_t^s \right] \\ & + \sum_{t=1}^{T} \delta^t \mu_t \left[H_t + \sum_{s=1}^{t-1} H_s z_t^s - N_{1,t} \right] \\ & + \sum_{t=1}^{T} \delta^t \eta_t \left[N_{2,t} - N_{2,t-1}\left(1-d\right) - \sum_{s=1}^{t-1} H_s z_{t-1}^s p_{t-1}^s \right], \end{aligned}$$

where $\delta^t \mu_t$ is the Lagrange multiplier for the flow constraint for $N_{1,t}$ and $\delta^t \eta_t$ is the Lagrange multiplier for the flow constraint for $N_{2,t}$. The optimality conditions for the problem are

$$\frac{\partial F_t}{\partial N_{1,t}} = w_{1,t}^t + \sum_{\tau=t+1}^{T} \delta^{\tau-1} z_\tau^t \left(w_{1,\tau}^t - \mu_\tau + p_\tau^t \eta_\tau \right) \tag{1}$$

and

$$\frac{\partial F_t}{\partial N_{2,t}} = w_{2,t}^t - \eta_t + \delta \left(1-d\right) \eta_{t+1}, \tag{2}$$

where we show in the appendix that $\mu_t \geq 0$ and $\eta_t \geq 0$ for all t.

These two conditions illustrate how the firm's production decisions, and hence, their hiring decisions differ from the statically optimal decisions. Take activity 1 in period t, for example, the statically optimal decision solves $\partial F_t / \partial N_{1,t} = w_{1,t}^t$. Here, production decisions also need to take into account dynamic effects on worker incentives. In particular, if the firm increases $N_{1,t}$, it will hire a new worker into activity 1 in period t, and that worker will receive $w_{1,\tau}^t$ in all future periods he is assigned to activity 1. Further, in each period $\tau > t$ that cohort-t worker is assigned to activity 1, the firm will hire one fewer worker into activity 1, saving μ_τ.

If the firm hires an extra worker into activity 1 in period t, then it will have an effect for each future period τ. This effect can be decomposed into two channels. First, because one extra worker is hired, it reduces the promotion prospects for other workers in the firm. This effect is captured by the $p_\tau^t \eta_\tau$ term if the worker is promoted in period τ. Second, because this worker is hired in period t instead of period τ, his future wage in period τ, $w_{1,\tau}^t$, is in general different from the wage a cohort-τ worker would receive. This effect is captured by the $w_{1,\tau}^t - \mu_\tau$ term.

Recall from Proposition 2 that $w_{1,\tau}^t \geq w_{1,\tau}^{t+1}$, suggesting it is more costly to hire a given worker in period t relative to any future period, and it can be shown that if $T < \infty$, then $w_{1,\tau}^t \geq \mu_\tau$ in each period $\tau > t$. Both of these dynamic considerations imply that, relative to the statically optimal production decision, the firm distorts the number of activity-1 positions downward in each period. Moreover, this distortion is more severe in early periods of production.

In terms of activity 2 in period t, the statically optimal decision solves $\partial F_t / \partial N_{2,t} = w_{2,t}^t$. Here, there are two additional considerations. First, an increase in $N_{2,t}$ allows the firm to promote more workers that were hired prior to period t. This is reflected in the η_t term. Second, an increase in $N_{2,t}$, holding all future $N_{2,\tau}$'s constant, implies that there will be fewer promotion opportunities in period $t+1$. This is reflected in the $-\delta \left(1-d\right) \eta_{t+1}$ term. In general, it is difficult to determine which of these two effects dominate in any given period for the optimal production path.

To make more progress on how the number of top positions will be affected by internal labor markets, we consider a two-period example with a fixed-factor production function in which revenues in period t depends on a demand parameter θ_t and on $N_t \equiv \min\{N_{1,t}/s, N_{2,t}\}$. This specification fixes the firm's optimal organizational span $s = N_{1,t}/N_{2,t}$. We can therefore write the firm's revenues in period t as only a function of the demand parameter and N_t. We assume the revenue function takes the form $\theta_t \ln N_t$.

The firm's problem is to choose both a production path and a personnel policy to maximize its discounted profits. Given our analysis from the previous section, the firm solves:

$$\max_{N_1, N_2, \{w_{i,1}, w_{i,2}\}} \theta_1 \ln N_1 - N_1 (s w_{1,1} + w_{2,1}) + \delta (\theta_2 \ln N_2 - N_2 (s w_{1,2} + w_{2,2}))$$

subject to the workers' incentive-compatibility constraints and to the firm's flow constraint.

Given the analysis in Section 4, workers assigned to activity 2 receive rents R_2 in each period, which pins down $w^*_{2,1}$ and $w^*_{2,2}$. Moreover, it is without loss of generality to set $w^*_{1,2}$ such that workers assigned to activity 1 in period 2 receive rents R_1. The key part of the analysis, then, is to choose N_1, N_2, and $w_{1,1}$ subject to the relevant incentive constraint and flow constraint. For now, we assume that θ_1 and θ_2 will be such that the firm's optimal production path will be a steady production path. For a more detailed analysis of this problem, see the appendix.

To describe the firm's constraints, denote the firm's growth rate from period 1 to period 2 by $g = (N_2 - N_1)/N_1$, and denote the promotion rate in the first period by p. The incentive constraint for workers assigned to activity 1 in the first period is given by

$$w_{1,1} - c_1 + \delta (1 - d) (pR_2 + (1 - p) R_1) \geq R_1,$$

and the flow constraint is given by

$$N_{2,2} = (1 - d) (N_{2,1} + pN_{1,1}).$$

Solving this constrained-maximization problem is standard and is carried out in the appendix. The following proposition highlights how production paths depend on future, current, and past demand parameters.

Proposition 4. *The solution to the program above satisfies the following: (i.) $dN^*_1/d\theta_2 \geq 0$, (ii.) $0 \leq d \log N^*_t / d \log \theta_t \leq 1$, and (iii.) $dN^*_2/d\theta_1 \geq 0$, and these inequalities are strict whenever $\theta_2/\theta_1 \in (\underline{\ell}, \bar{\ell})$ for some $\underline{\ell} < \bar{\ell}$ which are independent of θ_1 and θ_2.*

The first part of Proposition 4 shows that firms expand production in anticipation of future demand conditions. When the firm expects better conditions in the next period, it adjusts by hiring more workers in this period, holding fixed this period's demand conditions. This result holds because higher growth in the future creates promotion opportunities for workers today, which allows the firm to reduce the wages of workers currently assigned to activity 1, reducing the cost of hiring more workers today.

The second part of the proposition shows that firms facing better contemporaneous demand conditions will be larger. However, production levels in a given period will be stickier than they would be in a static model. Note that in a static model, a firm chooses N_t to maximize $\theta_t \ln N_t - N_t (sw_{1,t} + w_{2,t})$ and therefore, the elasticity of firm size with respect to the demand parameter is equal to one. In contrast, when the demand parameter increases by one percent, the firm size increases by less than one percent. This stickiness reflects the fact that changes in current firm size affect the cost of motivating workers both in the past and in the future.

The third part of the proposition shows that there are lingering effects of past demand conditions. In particular, better demand conditions in the first period lead the firm to increase its size in the first period. The firm can do so at a lower cost if it expands workers' promotion opportunities by also increasing its size in the second period. One implication is that two firms facing the same demand conditions in the second period may operate at different sizes because they had different demand conditions in the past. The firm that had better demand conditions in the past will be larger precisely in order to provide promotion opportunities for the workers it hired in the past. This implication formalizes the idea, present in the early works of Baker (1986), Jensen (1986), and Baker, Jensen, and Murphy (1988), that organizations may be biased towards growth in order to provide more opportunities for career advancement.

More generally, increasing the size of the firm in period t affects not only contemporaneous profits but also the profits in past and future periods. In particular, an increase in the size of the firm in period t decreases the firm's labor costs in periods prior to t, and it increases the firm's labor costs in periods after t. If we were to take as given the wages the firm faces in period t, the statically optimal firm size would equate the marginal revenue product of labor to the marginal cost of labor, or $\theta_t / N_t = sw_{1,t} + w_{2,t}$. Taking dynamic considerations into account, however, the firm will optimally set $\theta_t / N_t = sw_{1,t} + w_{2,t} + \lambda_t$ for some λ_t, which reflects the dynamic benefits and costs of increasing firm size in period t. In the two-period example in this section, we have $\lambda_1 \geq 0$ and $\lambda_2 \leq 0$, suggesting that the firm will be smaller than statically optimal in the first period and larger than statically optimal in the second. More generally, earlier growth leads to internal labor market congestion in later periods and later growth reduces congestion in previous periods.

6 Discussion and Extensions

In this section, we first characterize properties of optimal personnel policies in environments in which the production plan is unsteady. We next explore whether and why a firm might want to adopt a partial-effort contract: a contract in which some workers are not expected to exert effort in some periods. Finally, we show how our main model can be extended to analyze environments in which production paths are stochastic.

6.1 Unsteady Environments

The analysis in Section 4 presumed that the firm's production path N was a steady production path. That is, we assumed that for each t, $N_{2,t+1} \leq (1-d)(N_{1,t}+N_{2,t})$ and, for each i, $N_{i,t+1} \geq (1-d)N_{i,t}$. In this section, we explore characteristics of optimal personnel policies when N is not a steady production path. We will say that N experiences *breakneck growth at* $t+1$ if $N_{2,t+1} > (1-d)(N_{1,t}+N_{2,t})$, and we will say that N involves *deep downsizing in* i *at* $t+1$ if $N_{i,t+1} < (1-d)N_{i,t}$, and *deep downsizing for the firm overall* at $t+1$ if $N_{1,t+1}+N_{2,t+1} < (1-d)(N_{1,t}+N_{2,t})$.

For our analysis of optimal personnel policies under deep downsizing, we expand the number of activities an employee can be assigned to in each period to include a null activity in which the employee's effort has no impact on the firm's production. We therefore an employee's time-t activity assignment by $A_s \in \{0,1,2\}$, where $A_s = 0$ denotes the null activity, and we denote by $p_{0,t}^{\tau}$ the probability that a cohort-τ worker will be assigned to the null activity in $t+1$ conditional on remaining with the firm and having good performance in each period of employment.

6.1.1 Breakneck Growth

Suppose N experiences breakneck growth for the first time at $t+1$, that is, even if the firm promotes all workers assigned to activity 1 in period t, it must place some new hires at $t+1$ into activity 2. This implies that all workers hired prior to period $t+1$ must earn a continuation payoff of R_2 at the beginning of period $t+1$. We can then break the optimal personnel policy problem up into two problems.

We first solve for the optimal personnel policies in periods $1,\ldots,t$, treating t as effectively the last period of production but with the requirement that all incumbent workers at period t receive R_2 in continuation payoffs. For the second problem, we solve for optimal personnel policies in periods after $t+1$, and we take as given that all workers in cohorts prior to $t+1$ will initially be assigned to activity 2 and will therefore receive rents equal to R_2. In other words, the analysis can be carried out chunk-by-chunk, where each chunk starts with a period in which breakneck growth occurs and ends with the next period in which breakneck growth occurs. Within each chunk, the optimal personnel policy minimizes the rents that are paid to new hires assigned to activity 1, and the same type of analysis as in Section 4 can be applied, so the main results continue to hold.

6.1.2 Deep Downsizing

When firms go through periods of deep downsizing, managing personnel can be more complicated. In this section, we explore some features of personnel policies that might arise. If deep downsizing is permanent, in the sense that once there is deep downsizing in one period, there is deep downsizing in all future periods, the firm will never hire new workers, and it will shrink faster than by attrition alone. When this is the case, in order to motivate workers in their last period of employment, the firm has to pay severance pay to workers that it will not employ in the future. Proposition 5 describes optimal personnel policies in this case.

Proposition 5. *Suppose N satisfies $N_{1,t+1} < (1-d)\,N_{1,t}$, $N_{2,t+1} > (1-d)\,N_{2,t+1}$, and $N_{1,t+1} + N_{2,t+1} < (1-d)\,(N_{1,t} + N_{2,t})$ for all t. There is an optimal personnel policy in which (i.) laid-off workers receive severance pay, (ii.) if $\tau < \tau'$, then $p_{0,t}^{\tau} \leq p_{0,t}^{\tau'}$, and (iii.) conditional on being laid off, workers with more seniority receive greater severance pay.*

This proposition describes an optimal personnel policy for a firm that must downsize in every future period. The first part shows that when employees are laid off, they are paid severance pay in their last period of employment. Severance pay is necessary to maintain employees' incentives to exert effort in their last period of employment. The second part of the proposition shows that an optimal personnel policy exhibits a last-in-first-out pattern for layoffs: employees with more seniority are less likely to be laid off in each period. The final part shows that if employees of different cohorts are laid off in the same period, their severance payments are higher the longer they have been employed by the firm.

We now discuss some features of optimal personnel policies for a firm experiencing temporary deep downsizing, that is, the firm must downsize in one period, and there is a future period at which it will need to hire again. We will say that a worker is **permanently laid off** in period t if he is assigned to activity 0 in all future periods with probability 1. We say that a worker is **temporarily laid off** in period t if he is assigned to activity 0 in period $t+1$ and is assigned to activity 1 or 2 in a future period with positive probability. The next proposition partially characterizes optimal personnel policies when a firm experiences temporary deep downsizing.

Proposition 6. Suppose there is a t_1 at which $N_{1,t_1+1} < (1-d)\,N_{1,t_1}$ and $N_{1,t_1+1} + N_{2,t+1} < (1-d)\,(N_{1,t_1} + N_{2,t_1})$ and there exists a $t_2 > t_1$ at which $N_{1,t_2+1} + N_{2,t_2+1} > (1-d)\,(N_{1,t_2} + N_{2,t_2})$.

Then (a) no workers are permanently laid off in period t_1, and (b) $v^{\tau}_{1,t_2+k} \geq v^{t_2+k}_{1,t_2+k}$ for all $\tau < t_2$ and for all $k \geq 1$.

The conditions for Proposition 6 imply that the firm must downsize at t_1, and at time t_2+1, it recovers and must hire workers into one of the two positions. This proposition shows that whenever this is the case, the firm favors rehiring laid-off workers. If, instead, the firm hired new workers, it would have to pay them rents in their first period of employment. By rehiring laid-off workers, the firm can allocate these rents to these workers and reduce the overall rents it has to pay. The second part of the proposition shows that temporarily laid off workers will be rehired before the firm hires a worker who has never worked for the firm in the past, and moreover, these workers receive higher continuation payoffs than new hires. The rationale for this result is similar to the logic underlying why seniority-based promotions can be optimal.

6.2 Partial Effort Contracts and Sabbaticals

Throughout our analysis, we have focused on full-effort contracts. In this section, we will construct an example in which we show how, and why, it may be optimal to put workers on "sabbaticals"—that is, an optimal contract may ask incumbent workers to exert no effort in a period. When promotion prospects in the future look more promising than they do today, one way to relax a worker's incentive constraint today is to not ask them to exert effort in the next period. Doing so, of course, comes at the cost that the firm will need to hire other workers in the next period who will exert effort. If future promotion prospects are sufficient to guarantee those workers can be motivated at zero cost, then such a sabbatical policy will reduce the firm's overall wage bill.

To be concrete, consider again the three-period example from Section 4.1. In particular, suppose that the firm's organizational span, $N_{1,t}/N_{2,t}$, is constant and equal to s. Let $g_t = \left(N_{i,t+1} - N_{i,t}\right)/N_{i,t}$ denote the firm's growth rate from period t to period $t+1$. Let $d = 0, \delta = 1$, $s = 1$, and $q_1 = q_2 = 1/2$, which results in $R_i = c_i$. Moreover, assume that $g_1 = 0$. This condition, together with the assumption that $d = 0$, implies that the firm will not want to hire any new workers at $t = 2$ in a full-effort contract. We will now provide conditions under which a full-effort contract is dominated by a sabbatical contract in which the firm asks cohort-1 workers to shirk in period 2, and at the same time, it will hire new workers in period 2 and assign them to activity 1.

In addition to Assumption 1 from Section 4.1, we make the following additional assumptions.

Assumption 3. $c_1 > 2c_2$.

Assumption 4. $g_2 > 2$.

Assumption 3 ensures that if a cohort-1 worker exerts effort in both periods, then even if he promoted with probability 1 at the end of the second period, the firm would have to pay him a positive wage in at least one of the first two periods in order to motivate him. Assumption 4 is a sufficient condition that ensures that there are enough promotion opportunities even if additional workers are hired in period 2.

Proposition 7. *Under Assumptions* $1-4$, *any full-effort contract is strictly suboptimal.*

To see why Proposition 7 holds, note that in a full-effort contract, the hardest incentive constraint to satisfy is the incentive constraint for cohort-1 workers in the first period. To relax this constraint, the firm needs to increase cohort-1 workers' continuation payoffs. One way to do so is to increase their promotion prospects between periods 2 and 3, which we highlighted in Section 4.1. If, however, their incentive constraint still binds even if they are going to be promoted with probability 1, then another instrument available to the firm to relax their first-period incentive constraint is to put them on sabbatical in the second period, saving on their effort costs. If the firm does so, however, it will have to hire new workers to exert effort in the second period. Assumption 4 ensures that there are sufficient promotion opportunities at the end of period 2 to motivate any cohort-2 workers while still paying them zero wages.

In contrast to the existing work on hiring and sourcing decisions (Board, 2011; Ke, Li, and Powell, 2018), which highlight the benefits of biasing such decisions towards insiders, the optimal partial-effort contract here suggests that the firm's personnel policies can exhibit an "outsider bias." To see why this outsider bias arises, note that, in the example above, an important requirement is that there are abundant promotion opportunities in period 3. When promotion opportunities are abundant in period 3, the cost of hiring new workers into period 2 is small, so the firm may be willing to do so in order to reduce the wages it has to pay to cohort-1 workers. Our results, therefore, show that future production plans impact current hiring and sourcing decisions.

6.3 Stochastic Production Paths

In this section, we show how our analysis can be extended to allow for stochastic production paths. Suppose N is a stochastic process, with $(N_{1,t}, N_{2,t}) \in \mathcal{M} = \{m^1, \ldots, m^K\}$, where K is possibly infinite. Denote the vector $(N_{1,t}, N_{2,t})$ by $N(m^k)$. Let $f_{k'k} = \Pr\left[m_t = m^{k'} \middle| m_{t-1} = m^k\right]$. Let $m^t = (M_1, \ldots, M_t) \in \mathcal{M}^t$, $h^t = (A_1, \ldots, A_t)$, and $\hat{h}^t = (m_1, A_1, \ldots, m_t, A_t)$. The firm's objective function is to choose a personnel policy to

$$\max \sum_{t=1}^{T} \delta^{t-1} \sum_{m^t \in \mathcal{M}^t} \Pr\left[m^t\right] \left[F_t\left(N\left(M_t\right)\right) - \sum_{\hat{h}^t} \Pr\left[\hat{h}^t \middle| m^t\right] \hat{W}\left(\hat{h}^t\right) \hat{L}\left(\hat{h}^t \middle| m^t\right)\right]$$

where

$$\hat{L}\left(\hat{h}^t M_{t+1} A_{t+1}\right) = (1-d)\, p_{A_{t+1}}\left(\hat{h}^t M_{t+1}\right) f_{M_{t+1} M_t} \hat{L}\left(\hat{h}^t\right),$$

and

$$\hat{L}\left(\hat{h}^t \middle| m^t\right) = \hat{L}\left(\hat{h}^t\right) / \Pr\left[m^t\right].$$

The firm faces promise-keeping constraints

$$v_{i,t,k} = w_{i,t,k} - c_i + \delta(1-d)\sum_{k'=1}^{K} f_{k'k} \sum_{j=0}^{2} p_j\left(\hat{h}^t m_{t+1}\right) v_{j,t+1,k'}$$

incentive-compatibility constraints

$$v_{i,t,k} \geq qc_i + (1-q)\, v_{i,t,k}$$

or $v_{i,t,k} \geq R_i$. Finally, the flow constraints are

$$\sum_{\hat{h}^t} \hat{L}\left(\hat{h}^t M_t i\right) = N_i\left(M_t\right).$$

We will say that N is **steady** if for all k, k' such that $f_{k'k} > 0$, $N_i\left(m^{k'}\right) \geq (1-d)\, N_i\left(m^k\right)$ for $i = 1, 2$, and $N_2\left(m^{k'}\right) \leq (1-d)\left(N_1\left(m^k\right) + N_2\left(m^k\right)\right)$.

The analysis of personnel policies under deterministic production paths carries over to the case in which production paths are stochastic, as long as they are steady. The key reason is that each worker is risk-neutral, and their continuation payoffs depend on their expected promotion probability. In particular, notice that the right-hand side of these incentive constraints are the same as they are when production paths are deterministic. Optimal personnel policies again resemble an internal labor market, and seniority-based promotions serve to motivate workers in an optimal personnel policy.

7 Conclusion and Discussion

In this paper, we develop a model of production and personnel management. We first show that optimal personnel policies resemble internal labor markets in which seniority plays an important role in promotion and wage decisions. Our main result sheds light on Baker, Jensen, and Murphy's (1988) observation that an "important problem with promotion-based reward systems is that they require organizational growth to feed the reward system." (p. 600) Indeed, in order to make use of promotion-based incentives, the firm has to grow faster than would be productively efficient. Yet, our model shows that doing so can be optimal ex ante, if not ex post. This time-inconsistency in the firm's production plans results from the optimal provision of long-term incentives to its employees.

Our model is a first step in understanding the interaction between production plans and personnel policies, and it leaves out many factors. For instance, we assume employees are risk-neutral and make binary effort choices, and the firm has full commitment power. In addition, employees are homogeneous, and there is no human capital acquisition or uncertainty about their productivity.[2] Future work that incorporates these factors can improve our understanding of personnel policies that firms adopt in richer environments and how they interact with firm growth.

The firm-growth imperative we highlight abstracts from, but has implications for, the important strategic choices firms have to make when they decide to expand. Interwar DuPont, for example, pursued growth through diversification, expanding into other lines of business rather than expanding its existing business. One important issue they had to address was whether to expand organically or through acquisition. Our model suggests that organic growth may create additional career opportunities for existing employees that growth through acquisition might not. Future work examining the personnel implications of different ways of expanding can help improve our understanding of the dynamics of corporate strategy.

[2] Many papers examine how these different features affect personnel and supplier dynamics and hence firm-level productivity dynamics but do not speak directly to the dynamics of firm size. For papers emphasizing the role of supplier heterogeneity, see Board (2011), DeVaro and Waldman (2012), DeVaro and Morita (2013), Andrews and Barron (2016), Board, Meyer-ter-Vehn, and Sadzik (2017); for papers emphasizing human capital acquisition, see Gibbons and Waldman (1999, 2006); for papers emphasizing risk aversion and continuous effort, see Harris and Holmstrom (1982) and Holmstrom and Ricart i Costa (1986), Chiappori, Salanie, and Valentin (1999); for papers emphasizing lack of commitment, see Malcomson (1984), MacLeod and Malcomson (1988)

Appendix

Lemma 1. *Given N if there is an optimal personnel policy, there is an optimal personnel policy in which workers with the same employment history face the same wage and assignment policies.*

Proof of Lemma 1. If there is an optimal personnel policy in which two workers with the same employment history receive different wage and/or assignment policies, then we can consider an alternative assignment and wage policy that is a public randomization between these policies, and if both players are subject to this same alternative policy, their incentive constraints and the firm's flow constraints remain satisfied.■

Lemma 2. *Cost-minimizing personnel policies minimize the rents paid to new hires:*

$$\min \sum_{t=1}^{T} \sum_{h^t \in \mathcal{H}^t} \delta^t L\left(n^t\right) v\left(n^t\right)$$

subject to (1), (2), *and* (3).

Proof of Lemma 2. The PDV of the firm's wage bill, times δ is

$$\sum_{t=1}^{T} \sum_{h^t \in \mathcal{H}^t} \delta^t L\left(h^t\right) w\left(h^t\right).$$

For all workers who currently work in the firm in period t, that is for those for which $A_t \neq 0$, the flow constraint gives us

$$\begin{aligned} L\left(h^t 1\right) &= (1-d) p_1\left(h^t\right) L\left(h^t\right) \\ L\left(h^t 2\right) &= (1-d) p_2\left(h^t\right) L\left(h^t\right). \end{aligned}$$

In addition, for $i \in \{1,2\}$, we can write $N_{i,t} = \sum_{h^t | A_t = i} L\left(h^t\right)$. We can write the period-t wages paid to workers with employment history h^t as $L\left(h^t\right) w\left(h^t\right)$, which equals

$$\begin{aligned} & L\left(h^t\right) v\left(h^t\right) + L\left(h^t\right) c\left(h^t\right) - \delta(1-d) L\left(h^t\right)\left(p_1\left(h^t\right) v\left(h^t 1\right) + p_2\left(h^t\right) v\left(h^t 2\right)\right) \\ = \; & L\left(h^t\right) v\left(h^t\right) + L\left(h^t\right) c\left(h^t\right) - \delta L\left(h^t 1\right) v\left(h^t 1\right) - \delta L\left(h^t 2\right) v\left(h^t, 2\right), \end{aligned}$$

where the first equality plugs in the promise-keeping constraint for workers with employment history h^t, and the second equality plugs in the flow constraint.

The total wage bill is the sum of these expressions over time and over employment histories and is therefore

$$
\begin{aligned}
& \sum_{t=1,h^t}^{T} \delta^t L\left(h^t\right) w\left(h^t\right) \\
= & \sum_{t=1}^{T} \sum_{h^t \in \mathcal{H}^t} \delta^t \left(L\left(h^t\right) v\left(h^t\right) + L\left(h^t\right) c\left(h^t\right) - \delta L\left(h^t 1\right) v\left(h^t 1\right) - \delta L\left(h^t 2\right) v\left(h^t 2\right)\right) \\
= & \sum_{t=1}^{T} \sum_{h^t \in \mathcal{H}^t} \delta^t L\left(h^t\right) c\left(h^t\right) + \sum_{t=1}^{T} \sum_{h^t \in \mathcal{H}^t} \delta^t \left(L\left(h^t\right) v\left(h^t\right) - \delta L\left(h^t 1\right) v\left(h^t 1\right) - \delta L\left(h^t 2\right) v\left(h^t 2\right)\right) \\
= & \sum_{t=1}^{T} \delta^t \left(N_{1,t} c_1 + N_{2,t} c_2\right) + \sum_{t=1}^{T} \delta^t L\left(n^t\right) v\left(n^t\right),
\end{aligned}
$$

where recall that $L\left(n^t\right)$ are the new workers hired into the firm in period t. It follows that the firm's objective is simply to minimize

$$
\sum_{t=1}^{\infty} \delta^t L\left(n^t\right) v\left(n^t\right),
$$

which establishes the lemma.

Lemma 3. *Given a steady production path N, if there is an optimal personnel policy, there is an optimal personnel policy with the following three properties:*
(i.) $v\left(h^t\right) \leq R_2$ if $A_t = 1$ and $v\left(h^t\right) = R_2$ if $A_t = 2$.
(ii.) All new workers are assigned to activity 1.
(iii.) $p_2\left(h^t\right) = 1$ if $A_t = 2$ and $p_1\left(h^t\right) + p_2\left(h^t\right) = 1$ if $A_t = 1$.

Proof of Lemma 3. To establish part (*i.*), we will first show that for all h^t, we do not need to have both $w\left(h^t\right) > 0$ and $v\left(h^t\right) > R\left(A_t\right)$. To establish this intermediate result, there are two cases to consider. First, suppose the worker is a new hire in period t. In this case, if both $w\left(h^t\right) > 0$ and $v\left(h^t\right) > R\left(A_t\right)$, the firm can reduce the wage bill by reducing $w\left(h^t\right)$ without violating the incentive constraint. Second, if the worker was a new hire prior to period t, the firm can reduce $w\left(h^t\right)$ and increase $w\left(h^{t-1}\right)$ to maintain $v\left(h^{t-1}\right)$. This establishes the intermediate result and shows that it is without loss of generality to focus on personnel policies in which in each period, either the minimum wage constraint is binding or the IC constraint is binding. We will use this result to establish part (*i.*), but we do not make use of it in our other results.

For part (*i.*), there are two cases to consider. First, suppose $w\left(h^t\right) > 0$. Then by the previous result, we have $v\left(h^t\right) = R\left(h^t\right) \leq R_2$. Next, suppose $w\left(h^t\right) = 0$. We can then consider all histories that follow h^t. With probability 1, the workers must eventually receive a strictly positive wage, or else there must be some employment history following h^t at which his incentive constraint is violated. If $w\left(h^t\right) = 0$, we can write $v\left(h^t\right)$ as

$$
v\left(h^t\right) = \sum_{h^\tau} \Pr\left[h^\tau \middle| h^t\right] \left(\sum_{s=0}^{\tau-t-1} \delta^s \left(-c_1\right) + \delta^{\tau-t} v\left(h^\tau\right)\right),
$$

where h^τ is the first history following h^t such that $w\left(h^\tau\right) > 0$. We can again use the previous result to get

$$
\begin{aligned}
v\left(h^{t}\right) &= \sum_{h^{\tau}} \Pr\left[h^{\tau} \mid h^{t}\right]\left(\sum_{s=0}^{\tau-t-1} \delta^{s}\left(-c_{1}\right)+\delta^{\tau-t} R\left(h^{\tau}\right)\right) \\
&< \sum_{h^{\tau}} \Pr\left[h^{\tau} \mid h^{t}\right] R\left(h^{\tau}\right) \leq R_{2},
\end{aligned}
$$

which establishes part $(i.)$.

For part $(ii.)$, note that part $(i.)$ implies that $v\left(h^{t}\right) \leq R_{2}$ if $A_{t}=1$. As a result, if a new worker is assigned to activity 2, it is better instead to assign them to activity 1 and promote an existing worker assigned to activity 1 to instead be assigned to activity 2. This would relax the existing workers' incentive constraints and reduce the wage bill.

Finally, for part $(iii.)$, suppose $p_{1}\left(h^{t}\right)+p_{2}\left(h^{t}\right)<1$. Because $N_{i,t+1} \geq(1-d) N_{i,t}$ for $i=1,2$, we must have that $L\left(n^{t+1}\right)>0$, so there must be positive hiring into either position 1 or position 2. We will construct a perturbation to the personnel policy in which any rents that would be paid out to new hires are paid out, instead, to currently employed workers. This perturbation will introduce slack into some current employees' incentive constraints, and it will not increase the total wage bill. If a positive mass of new workers is hired and assigned to activity 1, $L\left(\vec{0}1\right)$, let $\tilde{p}_{1}\left(h^{t}1\right)=p_{1}\left(h^{t}1\right)+\varepsilon$, and let $\tilde{L}\left(\vec{0}1\right)=L\left(\vec{0}1\right)-\varepsilon(1-d) L\left(h^{t}\right)$. This perturbation preserves the flow constraint, and it relaxes workers' incentive constraints in periods $s \leq t$ for those workers with history h^{t}. This perturbation therefore weakly decreases the firm's overall wage bill. A similar perturbation can be constructed if $L\left(\vec{0}2\right)>0$. Result (i) of this lemma implies that $v\left(h^{t}\right)=R_{2}$ if $A_{t}=2$, which implies that $p_{2}\left(h^{t}\right)=1$ if $A_{t}=2$.■

Proposition 1. *If N is a steady production path, an internal labor market is an optimal personnel policy.*

Proof of Proposition 1. Follows directly from Lemma 3 and the definition of an internal labor market.

Proposition 2. *If N is a steady production path, an internal labor market with the following three properties is an optimal personnel policy: $(i.)$ $w_{1,t+1}^{\tau} \geq w_{1,t}^{\tau}$, $(ii.)$ if $\tau<\tau'$, then $w_{1,t}^{\tau} \geq w_{1,t}^{\tau'}$, $p_{t}^{\tau} \geq p_{t}^{\tau'}$, and $v_{1,t+1}^{\tau} \geq v_{1,t+1}^{\tau'}$, and $(iii.)$ if $p_{t}^{\tau}, p_{t}^{\tau'} \in(0,1)$, then $v_{1,t+1}^{\tau}=v_{1,t+1}^{\tau'}$.*

Proof of Proposition 2. First, note that for any t, in any optimal personnel policy, it must be the case that $v\left(h^{t}\right) \geq v\left(n^{t}\right)$, that is, new hires receive lower rents than incumbent workers. Suppose to the contrary that $v\left(h^{t}\right)<v\left(n^{t}\right)$. We can then "switch" the future history of a worker with employment history h^{t} with a new worker. This switch preserves the total wage bill and relaxes the incentive constraints of workers whose employment histories are consistent with h^{t}.

Now, suppose $\tau_{1}<\tau_{2}$. We can write the rents workers from each cohort receive in period t if they are assigned to activity 1 as follows:

$$
\begin{aligned}
v_{1,t}^{\tau_{1}} &= w_{1,t}^{\tau_{1}}-c_{1}+\delta(1-d)\left(p_{t}^{\tau_{1}} R_{2}+\left(1-p_{t}^{\tau_{1}}\right) v_{1,t+1}^{\tau_{1}}\right) \\
v_{1,t}^{\tau_{2}} &= w_{1,t}^{\tau_{2}}-c_{1}+\delta(1-d)\left(p_{t}^{\tau_{2}} R_{2}+\left(1-p_{t}^{\tau_{2}}\right) v_{1,t+1}^{\tau_{2}}\right).
\end{aligned}
$$

Take a rent $v_{1,t}^{\tau 1}$. We can always reduce $w_{1,t}^{\tau 1}$ by ε and increase $v_{1,t+1}^{\tau 1}$ by $\varepsilon / [\delta (1-d)(1-p_t^{\tau 1})]$, maintaining rents $v_{1,t}^{\tau 1}$, unless either $w_{1,t}^{\tau 1} = 0$ or $v_{1,t+1}^{\tau 1} = R_2$. We can do this similar for the τ_2 cohort. Let $\bar{w}_{1,t}^1$ and $\bar{w}_{1,t}^2$ denote the resulting activity-1 wages at which this procedure terminates and $\bar{v}_{1,t+1}^{\tau 1}$ and $\bar{v}_{1,t+1}^{\tau 2}$ the resulting continuation payoffs. There are four cases to consider: (*i.*) $\bar{w}_{1,t}^1 = 0$, $\bar{w}_{2,t}^1 = 0$; (*ii.*) $\bar{w}_{1,t}^1 > 0$, $\bar{w}_{1,t}^2 > 0$; (*iii.*) $\bar{w}_{1,t}^1 = 0$, $\bar{w}_{1,t}^2 > 0$; and (*iv.*) $\bar{w}_{1,t}^1 > 0$, $\bar{w}_{1,t}^2 = 0$.

The first observation is that case (*iv.*) is impossible, because it would imply that $v_{1,t}^{\tau 1} > v_{1,t}^{\tau 2}$. If $\bar{w}_{1,t}^1 > 0$, this implies that $\bar{v}_{1,t+1}^{\tau 1} = R_2$, and as a result, it must be the case that cohort-τ_1's continuation payoff weakly exceeds cohort-τ_2's continuation payoff, and so do the wages.

Next, in case (*ii.*), $\bar{v}_{1,t+1}^{\tau 1} = \bar{v}_{1,t+1}^{\tau 2} = R_2$, so both cohorts have the same continuation payoffs. Moreover, if $v_{1,t}^{\tau 1} \leq v_{1,t}^{\tau 2}$, it must be the case that $\bar{w}_{1,t}^{\tau 1} \leq \bar{w}_{1,t}^{\tau 2}$. Define $\tilde{p}_t = \left(L_{1,t} p_t^{\tau 1} + L_{2,t} p_t^{\tau 2}\right) / \left(L_{1,t} + L_{2,t}\right)$, where $L_{i,t}$ is the mass of cohort-i workers assigned to activity 1 in period t. Promoting both cohorts at rate $\tilde{p}_t$ maintains the flow constraints, and it does not affect $v_{1,t}^{\tau 1}$ or $v_{1,t}^{\tau 2}$, so such a personnel policy is optimal if the original personnel policy is optimal, and it satisfies property (*ii.*) of the proposition. It also satisfies property (*i.*), which means that after period t, both cohorts earn wages $\hat{w} = c_1 + (1 - \delta (1-d)) R_2$, which must weakly exceed $\bar{w}_{1,t}^{\tau 1}$ and $\bar{w}_{1,t}^{\tau 2}$, or else $v_{1,t}^{\tau 1}$ or $v_{1,t}^{\tau 2}$ would exceed R_2. Moreover, property (*iii.*) is satisfied because $\bar{v}_{1,t+1}^{\tau 1} = \bar{v}_{1,t+1}^{\tau 2} = R_2$.

In case 3, $\bar{v}_{1,t+1}^{\tau 2} = R_2$, which necessarily exceeds $\bar{v}_{1,t+1}^{\tau 1}$. If $p_t^{\tau 1} \leq p_t^{\tau 2}$, then properties (*i.*) and (*ii.*) are automatically satisfied. We can then decrease $p_t^{\tau 1}$ by ε, increase $p_t^{\tau 2}$ by $L_{1,t}\varepsilon / L_{2,t}$. This perturbation does not affect $v_{1,t}^{\tau 2}$, since $\bar{v}_{1,t+1}^{\tau 2} = R_2$, and in order to maintain $v_{1,t}^{\tau 1}$, we increase $\tilde{v}_{1,t+1}^{\tau 1}$. We can keep doing this until either $\tilde{p}_t^{\tau 1} = 0$ or $\tilde{p}_t^{\tau 2} = 0$. Now, suppose $p_t^{\tau 1} > p_t^{\tau 2}$. Then choose $\tilde{p}_t^{\tau 1} = \tilde{p}_t^{\tau 2} = \left(L_{1,t} p_t^{\tau 1} + L_{2,t} p_t^{\tau 2}\right) / \left(L_{1,t} + L_{2,t}\right)$. This construction maintains cohort-τ_2's continuation payoff. Increase $\bar{v}_{1,t+1}^{\tau 1}$ to $\tilde{v}_{1,t+1}^{\tau 1}$ which maintains the same continuation payoff for cohort-τ_1. This construction satisfies properties (*i.*) and (*ii.*). Further, we can alter this construction just as we did in the proof of case 2 in order to construct an optimal personnel policy that satisfies property (*iii.*).

Finally, consider case 1. Set $\tilde{p}_t^{\tau 1} = \tilde{p}_t^{\tau 2} = \left(L_{1,t} p_t^{\tau 1} + L_{2,t} p_t^{\tau 2}\right) / \left(L_{1,t} + L_{2,t}\right)$, and choose $\tilde{v}_{1,t+1}^{\tau 1}$ and $\tilde{v}_{1,t+1}^{\tau 2}$ to maintain the same continuation payoffs for both cohorts. Since $v_{1,t}^{\tau 1} \leq v_{1,t}^{\tau 2}$, it must be the case that $\tilde{v}_{1,t+1}^{\tau 1} \leq \tilde{v}_{1,t+1}^{\tau 2}$. This establishes properties (*i.*) and (*ii.*), and we can use a similar argument as above to construct an optimal personnel policy that satisfies property (*iii.*).■

Corollary 1. *If N is a constant-growth production path with $g \geq -d$, then a seniority-blind internal labor market is optimal.*

Proof of Corollary 1. First, by Proposition 1, an optimal personnel policy involves no demotions or forced turnover. This implies that, given a constant growth rate g, there is a maximal rate of promotion for workers assigned to activity 1 in period t, and this rate of promotion is equal to

$$p_t = \frac{N_{2,t+1} - (1-d) N_{2,t}}{(1-d) N_{1,t}} = \frac{1}{s} \frac{g+d}{1-d} \equiv p(g),$$

where $N_{1,t}/N_{2,t} = s$ and $N_{i,t+1}/N_{i,t} = 1+g$.

Next, suppose all workers assigned to activity 1 are promoted at rate $p(g)$, and define $w_1(g)$ to be the wage that guarantees a worker assigned to activity 1 who is paid $w_1(g)$ and promoted to activity 2 at rate $p(g)$, at which point he will receive rents R_2, receives R_1.

$$R_1 = w_1(g) - c_1 + \delta (1-d) \left((1 - p(g)) R_1 + p(g) R_2 \right).$$

Define $\hat{g}$ to satisfy $w_1(\hat{g}) = 0$. Note that if $g < \hat{g}$, then in order to satisfy incentive compatibility for workers assigned to activity 1, a firm must pay them strictly positive wages, and if $g > \hat{g}$, then workers incentive constraints are slack even if they receive zero wage.

We now consider two cases. First, suppose $g \geq \hat{g}$. In this case, let $p_1^*(h^t) = p(g)$, $w_1^* = 0$, and $w_2^* = c_2 + (1 - \delta(1-d)) R_2$, which is the lowest wage the firm can pay workers assigned to activity 2 in each period while maintaining their incentives. This personnel policy satisfies $(IC), (PK)$, and (FL) and is therefore a feasible personnel policy. Moreover, it minimizes the firm's wage bill, because it is impossible to reduce wages for either activity without violating workers' incentive constraints. For $g \geq \hat{g}$, therefore, a seniority-blind internal labor market is optimal.

Next, suppose $g < \hat{g}$. Set $p_1^*(h^t)$ and w_2^* as above, and set

$$w_1^* = c_1 + (1 - \delta(1-d)) R_1 - \delta \frac{g+d}{s} (R_2 - R_1).$$

This personnel policy satisfies $(IC), (PK)$, and (FL) and is therefore a feasible personnel policy. Moreover, under this personnel policy, each worker receives ex ante rents R_1 at the time they are hired, except for those workers who are hired in the first period directly into activity 2. Such workers receive ex ante rents R_2. By Lemma 2, we have

$$\sum_{t=1}^{T} \sum_{h^t \in \mathcal{H}^t} \delta^t L(n^t) v(n^t) \geq \sum_{t=1}^{T} \delta^t L(n^{t-1} 1) R_1 + L(2) R_2,$$

and the constructed personnel policy achieves this lower bound and is feasible. It is therefore optimal. This completes the proof.■

Corollary 2. *Suppose N is a constant-growth production path with growth rate $g \geq -d$ and $N_{1,t}/N_{2,t} = s$. Then the promotion rate for workers performing activity 1 is*

$$p^* = \frac{1}{s} \frac{g+d}{1-d},$$

wages for workers assigned to activity 2 are $w_2^* = c_2 + (1 - \delta(1-d)) R_2$, and there is a threshold growth rate $\hat{g}$ such that wages for workers assigned to activity 1 are

$$w_1^* = \begin{cases} 0 & \text{if } g \geq \hat{g} \\ c_1 + (1 - \delta(1-d)) R_2 - \delta \frac{g+d}{s} (R_2 - R_1) & \text{if } g \leq \hat{g}. \end{cases}$$

Average wages paid in each period, $(s w_1^* + w_2^*)/(1+s)$ are decreasing in the firm's growth rate.

Proof of Corollary 2. The expressions for p^*, w_1^*, and w_2^* are derived in the proof of Corollary 1. The result that the firm's average per-period wages are decreasing in the firm's growth rate follows because w_2^* is independent of g, and w_1^* is decreasing in g.■

Proposition 3. *Under Assumptions 1 and 2, any seniority-blind internal labor market is strictly suboptimal.*

Proof of Proposition 3. Recall that $T = 3$ and that the firm's organizational span, $N_{2,t}/N_{1,t}$, is constant and equal to s. Let $g_t = (N_{i,t+1} - N_{i,t})/N_{i,t}$ denote the firm's growth rate from period t to period $t+1$. Let $d = 0, \delta = 1$, $s = 1$, and $q_1 = q_2 = 1/2$, which results in $R_i = c_i$.

Suppose the firm puts in place a seniority-blind internal labor market, that is, the firm chooses $w_{1,2}^1 = w_{1,2}^2$ and $p_2^1 = p_2^2$. Then a straightforward calculation gives us that $p_2^1 = p_2^2 = g_2$. Now, suppose that the firm chooses $p_2^2 = g_2$ and $w_{1,2}^2 = 0$. The rents that cohort 2 workers receive in their first period of employment are therefore $-c_1 + g_2 R_2 + (1 - g_2) R_1$, which by Assumption 1 strictly exceed R_1.

Next, notice that if cohort-1 workers are paid $w_{1,1}^1 = w_{1,2}^1 = 0$ and are promoted with probability $p_2^1 = g_2$ in the second period, then their first-period rents are given by

$$-c_1 + g_1 R_2 + (1 - g_1) [-c_1 + g_2 R_2 + (1 - g_2) R_1].$$

Assumption 2 ensures that such rents are smaller than R_1, so that in order to satisfy cohort-1 workers' incentive constraints in period 1, the firm must pay them strictly positive wages in either period 1 or 2.

Take an optimal seniority-blind internal labor market. Under such a policy, the second-period promotion probability is $p_2^1 = p_2^2 = g_2$, and the incentive constraint for cohort 2 workers is slack. Now consider the following perturbation. Increase cohort-1 workers' promotion probability to $\tilde{p}_2^1 = p_2^1 + \varepsilon$ and reduce the period-2 promotion probability for cohort-2 workers to $\tilde{p}_2^2 = p_2^2 - \varepsilon N_{1,2}^1 / N_{1,2}^2$, where $N_{i,t}^\tau$ is the number of cohort-τ workers assigned to activity i in period t. Under this perturbation, cohort-2 workers' incentive constraints in period 2 remain satisfied for ε sufficiently small. For cohort-1 workers, their incentive constraint in periods 1 and 2 are both slack. We can therefore reduce either $w_{1,1}^1$ or $w_{1,2}^1$, at least one of which must be positive under Assumption 1. This strictly increases the firm's profits, implying that the optimal seniority-blind internal labor market is strictly suboptimal.■

Proposition 4. *The solution to the program above satisfies the following: (i.) $dN_1^*/d\theta_2 \geq 0$, (ii.) $0 \leq d \log N_i^* / d \log \theta_i \leq 1$, and (iii.) $dN_2^*/d\theta_1 \geq 0$, and these inequalities are strict whenever $\theta_2/\theta_1 \in (\underline{\ell}, \bar{\ell})$ for some $\underline{\ell} < \bar{\ell}$ which are independent of θ_1 and θ_2.*

Proof of Proposition 4. Let $T = 2$, and denote by $w_1^S = c_1 + (1 - \delta(1 - d)) R_1$ and $w_2^S = c_2 + (1 - \delta(1 - d)) R_2$ the wages required to motivate a worker assigned to each activity in the first period if they are going to stay in that activity in the second period.

Production is $f_t(N_1, N_2) = f_t(\min\{N_1, sN_2\}) = \theta_t \ln N_2$. The firm's problem is to

$$\max_{N_{2,1}, N_{2,2}, w_{1,1}, w_{1,2}, w_{2,1}, w_{2,2}} \theta_1 \ln N_{2,1} - N_{2,1}(s w_{1,1} + w_{2,1}) + \delta(\theta_2 \ln N_{2,2} - N_{2,2}(s w_{1,2} + w_{2,2}))$$

subject to incentive-compatibility, flow, and promise-keeping. Since the second period is the final period, to motivate workers assigned to activity i, their wages must be at least $w_{i,2}^* = c_i + R_i$, and optimally, they will be exactly this amount. In the first period, write

$$v_1 = w_{1,1} - c_1 + \delta(1 - d)(pR_2 + (1 - p) R_1),$$

where

$$p = \frac{N_{2,2} - (1 - d) N_{2,1}}{(1 - d) N_{1,1}} = \frac{N_{2,1}}{N_{1,1}} \frac{1 + g - (1 - d)}{(1 - d)} = \frac{1}{s} \frac{g + d}{1 - d}$$

and note that $p \in [0, 1]$ as long as $g \geq -d$ and $g \geq -d + s(1 - d)$ (which obviously implies the other condition).

Define $\hat{w}_{1,1}$ by

$$\begin{aligned} R_1 &= \hat{w}_{1,1} - c_1 + \delta(1-d)\left(R_1 + \frac{1}{s}\frac{g+d}{1-d}(R_2 - R_1)\right) \\ &= \hat{w}_{1,1} - c_1 + \delta(1-d)R_1 + \delta\frac{g+d}{s}(R_2 - R_1) \end{aligned}$$

to be the wage which, if the worker received $\hat{w}_{1,1}$ in the first period and was promoted at rate p, his incentive constraint would hold with equality. Rearranging, this value is

$$\hat{w}_{1,1} = c_1 + (1 - \delta(1-d))R_1 - \frac{g+d}{s}(R_2 - R_1).$$

It is nonnegative if and only if $g \leq \hat{g} = \frac{1}{R_2 - R_1}\frac{sw_1^S}{\delta} - d$. If this is the case, the worker's incentive constraint will optimally bind in the first period. If $g > \hat{g}$, then the worker's incentive constraint will not bind in the first period, but the limited liability constraint will. The optimal first-period wage for activity 1 is therefore

$$w_{1,1}^* = \begin{cases} 0 & \text{if } g \geq \hat{g} \\ w_1^s - \delta\frac{g+d}{s}(R_2 - R_1) & \text{if } g \leq \hat{g}. \end{cases}$$

The optimal first-period wage for activity 2 is $w_{2,1}^* = w_2^S$, and the first-period wage bill is

$$N_{2,1}\left(sw_{1,1}^* + w_{2,1}^*\right) = \begin{cases} N_2 w_2^S & \text{if } g \geq \hat{g} \\ N_2\left(sw_1^S - \delta(g+d)(R_2 - R_1)\right) + N_2 w_2^S & \text{if } g \leq \hat{g}. \end{cases}$$

To solve for the optimal growth rate, there are three cases to consider. First, suppose the optimal growth rate satisfies $g \geq \hat{g}$. Then the firm's problem is to

$$\max_{N_{2,1}, N_{2,2}} \theta_1 \ln N_{2,1} - N_{2,1} w_2^S + \delta\left(\theta_2 \ln N_{2,2} - N_{2,2}\left(sw_{1,2}^* + w_{2,2}^*\right)\right),$$

and the first-order conditions are

$$\frac{\theta_1}{N_{2,1}^*} = w_2^S$$

and

$$\frac{\theta_2}{N_{2,2}^*} = sw_{1,2}^* + w_{2,2}^*.$$

These conditions imply that

$$1 + g^* = \frac{N_{2,2}^*}{N_{2,1}^*} = \frac{\theta_2}{\theta_1}\frac{w_2^S}{sw_{1,2}^* + w_{2,2}^*}.$$

This is in fact the solution to the problem if this g^* satisfies $g^* \geq \hat{g}$.

For the second case, suppose $g \leq \hat{g}$. Then the firm's problem is

$$\max_{N_{2,1},N_{2,2}} \theta_1 \ln N_{2,1} - sN_{2,1}w_1^S - N_{2,1}w_2^S + \delta \left(N_{2,2} - (1-d) N_{2,1}\right)(R_2 - R_1)$$
$$+\delta \left[\theta_2 \ln N_{2,2} - N_{2,2}\left(sw_{1,2}^* + w_{2,2}^*\right)\right].$$

The first-order conditions are

$$\frac{\theta_1}{N_{2,1}^*} = sw_1^S + w_2^S + \delta (1-d)(R_2 - R_1)$$

and

$$\frac{\theta_2}{N_{2,2}^*} = sw_{1,2}^* + w_{2,2}^* - (R_2 - R_1),$$

which implies an optimal growth rate of

$$1 + g^* = \frac{N_{2,2}^*}{N_{2,1}^*} = \frac{\theta_2}{\theta_1}\frac{sw_1^S + w_2^S + \delta (1-d)(R_2 - R_1)}{sw_{1,2}^* + w_{2,2}^* - (R_2 - R_1)}$$

This is in fact a solution to the problem if the resulting g^* satisfies $g^* \leq \hat{g}$. The third case is that $g = \hat{g}$, and this is a boundary condition.

There are therefore three regions of the parameter space for θ_2 to consider: $\theta_2 \in [0, \underline{\theta}_2] \cup \left[\underline{\theta}_2, \bar{\theta}_2\right] \cup [\bar{\theta}_2, \infty)$, where

$$\underline{\theta}_2 = \theta_1 \frac{sw_{1,2}^* + w_{2,2}^* - (R_2 - R_1)}{sw_1^S + w_2^S + \delta (1-d)(R_2 - R_1)} \frac{\delta (1-d)(R_2 - R_1) + sw_1^S}{\delta (R_2 - R_1)},$$

and

$$\bar{\theta}_2 = \theta_1 \frac{sw_{1,2}^* + w_{2,2}^*}{w_2^S} \frac{\delta (1-d)(R_2 - R_1) + sw_1^S}{\delta (R_2 - R_1)}$$

In Region 1, where $\theta_2 \in [0, \underline{\theta}_2]$, which is the low-growth region, we therefore have

$$\begin{aligned}
N_1^* &= \frac{\theta_1}{sw_1^S + w_2^S + \delta (1-d)(R_2 - R_1)} \\
N_2^* &= \frac{\theta_2}{sw_{1,2}^* + w_{2,2}^* - (R_2 - R_1)} \\
w_{i,2}^* &= c_i + R_i \\
w_{2,1}^* &= w_2^S \\
w_{1,1}^* &= w_1^S - \delta \frac{g+d}{s}(R_2 - R_1).
\end{aligned}$$

In Region 2, $\theta_2 \in \left[\underline{\theta}_2, \bar{\theta}_2\right]$, which is the region in which $g^* = \hat{g}$, we have

$$\begin{aligned}
N_1^* &= \frac{\theta_1 + \delta\theta_2}{w_2^S + \delta\left(1+\hat{g}\right)\left(sw_{1,2}^* + w_{2,2}^*\right)} \\
N_2^* &= \frac{\left(\theta_1 + \delta\theta_2\right)\left(1+\hat{g}\right)}{w_2^S + \delta\left(1+\hat{g}\right)\left(sw_{1,2}^* + w_{2,2}^*\right)} \\
w_{i,2}^* &= c_i + R_i \\
w_{2,1}^* &= w_2^S \\
w_{1,1}^* &= 0.
\end{aligned}$$

In Region 3, $\theta_2 \in [\bar{\theta}_2, \infty)$, which is the high-growth region, we have

$$\begin{aligned}
N_1^* &= \frac{\theta_1}{w_2^S} \\
N_2^* &= \frac{\theta_2}{sw_{1,2}^* + w_{2,2}^*} \\
w_{i,2}^* &= c_i + R_i \\
w_{2,1}^* &= w_2^S \\
w_{1,1}^* &= 0.
\end{aligned}$$

All of the associated comparative statics follow from these expressions.■

Proposition 5. *Suppose N satisfies $N_{1,t+1} < (1-d)\,N_{1,t}$, $N_{2,t+1} > (1-d)\,N_{2,t+1}$, and $N_{1,t+1} + N_{2,t+1} < (1-d)\left(N_{1,t} + N_{2,t}\right)$ for all t. There is an optimal personnel policy in which (i.) laid-off workers receive severance pay, (ii.) if $\tau < \tau'$, then $p_{0,t}^{\tau} \le p_{0,t}^{\tau'}$, and (iii.) conditional on being laid off, workers with more seniority receive greater severance pay.*

Proof of Proposition 5. Using a similar argument as in the proof of Proposition 2, we may assume that $v_{1,t}^{\tau}$ is decreasing in τ. That is, later-cohort workers value being assigned to activity 1 in period t more than newer-cohort workers. Given an optimal personnel policy, we now construct an optimal personnel policy with the desired properties by specifying $w_{1,t}^{\tau}, p_{0,t}^{\tau}, p_{2,t}^{\tau}, v_{1,t+1}^{\tau}$, and $v_{0,t+1}^{\tau}$. To do this, we proceed in three steps.

First, we will assign promotion opportunities in each period to workers so that workers with positive promotion probabilities all receive the same continuation payoff if they are not promoted, and the average rate of promotion for workers satisfies the flow constraint for activity 2, that is, $p_{2,t} = \left[N_{2,t+1} - (1-d)\,N_{2,t}\right] / \left[(1-d)\,N_{1,t}\right]$. In particular, it can be shown that there exists a k such that for all $\tau > k$, we have $p_{2,t}^{\tau} = 0$, and for all $\tau, \tau' \le k$, promotion probabilities will satisfy the following two sets of equations. First, for all $\tau, \tau' \le k$

$$\frac{1 - p_{2,t}^{\tau}}{1 - p_{2,t}^{\tau'}} = \frac{v_{1,t}^{\tau} + c_1 - \delta\,(1-d)\,R_2}{v_{1,t}^{\tau'} + c_1 - \delta\,(1-d)\,R_2},$$

which ensures that, if they receive a wage of 0 this period, workers promoted with positive probability receive the same continuation conditional on not being promoted. Second, the flow constraint for activity 2 is satisfied $\sum_{\tau=1}^{k} N_{1,t}^{\tau} p_{2,t}^{\tau} = p_{2,t} N_{1,t}$. These two sets of equations pin down k and $p_{2,t}^{\tau}$ for all τ. Given the associated promotion probabilities, we can write, for each τ,

$$v_{1,t}^{\tau} = -c_1 + \delta (1-d) \left(p_{2,t}^{\tau} R_2 + \left(1 - p_{2,t}^{\tau}\right) \tilde{v}_{t+1}^{\tau} \right).$$

Notice that our construction ensures that $\tilde{v}_{t+1}^{\tau} = \tilde{v}_{t+1}^{\tau'}$ for all $\tau, \tau' \leq k$, and $\tilde{v}_{t+1}^{\tau} \geq \tilde{v}_{t+1}^{\tau'}$ for all $k \leq \tau \leq \tau'$.

In the second step, we construct wages $w_{1,t}^{\tau}$ and continuation payoffs $\hat{v}_{t+1}^{\tau}$ for each cohort to guarantee that each cohort is promoted with the same probability as in the previous step, they receive the same payoffs $v_{1,t}^{\tau}$, and $\hat{v}_{t+1}^{\tau} \leq R_2$ for all τ. That is,

$$w_{1,t}^{\tau} = \max \left\{ v_{1,t}^{\tau} + c_1 - (1 - \delta (1-d)) R_2, 0 \right\}.$$

This implies that we can write

$$v_{1,t}^{\tau} = w_{1,t}^{\tau} - c_1 + \delta (1-d) \left(p_{2,t}^{\tau} R_2 + \left(1 - p_{2,t}^{\tau}\right) \hat{v}_{t+1}^{\tau} \right).$$

Notice that this construction implies there is some k' such that $w_{1,t}^{\tau} = 0$ for all $\tau \geq k'$.

Finally, we construct severance probabilities $p_{0,t}^{\tau}$ so that workers with the least seniority are laid off first, and we construct severance values $v_{0,t+1}^{\tau}$ so that the incentive constraints for laid-off workers remain satisfied. To this end, let $v_{0,t+1}^{\tau} = \hat{v}_{t+1}^{\tau} = v_{1,t+1}^{\tau}$, and write $p_{0,t}^{\tau} = \left(1 - p_{2,t}^{\tau}\right) \rho_t^{\tau}$, where ρ_t^{τ} is the probability of being laid off conditional on not being promoted. The flow constraint for activity 1 requires that the number of workers who are laid off is equal to the number of workers the firm has to get rid of, or

$$\sum_{\tau=1}^{t} p_{0,t}^{\tau} N_{1,t}^{\tau} = (1-d) (N_{1,t} + N_{2,t}) - (N_{1,t+1} + N_{2,t+1}).$$

This constraint implies there exists a k'' such that $\rho_t^{\tau} = 1$ for all $\tau > k''$, and $\rho_t^{\tau} = 0$ for all $\tau < k''$. This constructed policy satisfies all the conditions in the statement of the proposition.■

Proposition 6. Suppose there is a t_1 at which $N_{1,t_1+1} < (1-d) N_{1,t_1}$ and $N_{1,t_1+1} + N_{2,t+1} < (1-d)(N_{1,t_1} + N_{2,t_1})$ and there exists a $t_2 > t_1$ at which $N_{1,t_2+1} + N_{2,t_2+1} > (1-d)(N_{1,t_2} + N_{2,t_2})$. Then (a) no workers are permanently laid off in period t_1, and (b) $v_{1,t_2+k}^{\tau} \geq v_{1,t_2+k}^{t_2+k}$ for all $\tau < t_2$ and for all $k \geq 1$.

Proof of Proposition 6. Suppose $L\left(n^{t_2+1}\right) > 0$ and there is a positive mass of workers who worked for the firm by t_2 but are assigned to activity 0 in period t_2 and receive payoff v_{0,t_2+1}^{τ} for some $\tau < t_2$. Suppose such workers are permanently laid off. There are then two cases: either $v_{0,t_2+1}^{\tau} \geq v_{i,t_2+1}^{t_2+1}$ or $v_{0,t_2+1}^{\tau} < v_{i,t_2+1}^{t_2+1}$.

In the first case, consider an alternative personnel policy in which the firm does not hire the new worker and instead rehires the old worker and treats him the way the firm would have treated the new worker but pays him an additional $v_{0,t_2+1}^{\tau} - v_{i,t_2+1}^{t_2+1}$ in period t_2+1. This new personnel policy still satisfies the flow constraint for activity i, and it satisfies the promise-keeping constraint and the incentive constraints for the re-hired worker, and it pays out less in rents to new hires, so it reduces the overall wage bill.

In the second case in which $v_{0,t_2+1}^{\tau} < v_{i,t_2+1}^{t_2+1}$, similarly consider an alternative personnel policy in which the firm does not hire the new worker and instead rehires the old worker and treats him exactly the same ways as the firm would have treated the new worker. This new personnel policy is again feasible and reduces the overall wage bill because it pays out less in rents to new hires. This establishes property (a).

For part (b), if it is ever the case that $v^{\tau}_{1,t_2+k} < v^{t_2+k}_{1,t_2+k}$, then we can instead give the new worker initial rents of v^{τ}_{1,t_2+k} and give the cohort-τ worker period $t_2 + k$ rents of $v^{t_2+k}_{1,t_2+k}$. The associated personnel policy relaxes the cohort-τ worker's incentive constraint for all periods $t \leq t_2 + k$, and it reduces the initial rents of the cohort-$t_2 + k$ worker while maintaining their incentive constraint. It therefore reduces the firm's overall wage bill.■

Proposition 7. *Under Assumptions 1-4, any full-effort contract is strictly suboptimal.*

Proof of Proposition 7. In this setting, the wages in period 3 are fixed and equal to $w^*_i = 2c_i$ for $i = 1, 2$. Suppose the firm does not hire new workers in period 2. Under a full-effort contract, it is impossible to motivate cohort-1 workers assigned to activity 1 in both periods if the firm sets wages for activity 1 equal to zero, since for such workers, $v^1_{1,1} \leq -c_1 - c_1 + (2c_2 - c_2)$ with equality if the worker is promoted with probability 1 in period 3. By Assumption 3, $v^1_{1,1} < 0$.

Next, we show that the firm can set the wages for activity 1 to be zero in both periods 1 and 2 if it hires new workers in period 2 and does not ask for effort from cohort-1 workers. Specifically, notice that the second-period growth rate of the firm is sufficiently high to ensure that even if the firm hires all new workers in period 2, it will be able to promote all cohort-1 and cohort-2 workers. In this case, cohort-1 workers receive $v^1_{1,1} = -c_1 + (2c_2 - c_2) > 0$, and similarly, cohort-2 workers receive $v^2_{1,2} = -c_1 + (2c_2 - c_2) > 0$. This shows that full-effort contracts are strictly suboptimal.■

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