The Extrinsic Motivation of Freedom at Work*

Teck Yong Tan†

This version: March 2017

Abstract

Why does a worker become more efficient when given freedom at work? Under incomplete contracts, a worker faces a ratchet effect of innovating when he is closely monitored: if the worker uncovers a more efficient production method, the firm, being aware of it, raises the future performance requirement; anticipating this, the worker never tries to innovate. When given freedom at work instead, the worker accrues private information about his innovation. The resulting information asymmetry generates information rent which feeds back as the worker's incentive to innovate and improve efficiency. This paper studies how a firm's strategic ignorance influences its incentive structure which has to simultaneously induce effort from the worker and endogenously generate asymmetric information *against* the firm. The resulting mechanism provides a novel rationale to why relationships are sometimes characterized by weak incentives and low-scale production at the early stages.

*I am grateful to Navin Kartik and W. Bentley MacLeod for their generous support on this paper. I have also benefited immensely from discussions with Yeon-Koo Che and Anh Nguyen. I also thank Charles Angelucci, Douglas Bernheim, Pierre-André Chiappori, Julien Combe, Marina Halac, Suehyun Kwon, Andrea Prat, Canice Prendergast, Daniel Rappoport, Mike Riordan, Bernard Salanié, Juuso Toikka, Joel Watson, Enrico Zanardo, and seminar participants at Columbia University, National University of Singapore and UC San Diego for very helpful comments and suggestions.

†teck.tan@columbia.edu. Department of Economics, Columbia University.

1 Introduction

> "If you give people freedom, they will amaze you. All you need to do is give them a little infrastructure and a lot of room." — Laszlo Bock (Google's SVP, People Operations).

Besides its many innovative products, Google is perhaps best known for giving its employees a lot of freedom at work and benefiting from it. But this phenomenon is not unique to Google. A report released by the LRN at the World Economic Forum in 2014 concluded that companies with more freedom in their employment relationships are associated with more innovations and better long-term successes.[1] There is also a plethora of other empirical evidence suggesting that autonomy provision to individuals often leads to improved performances.[2]

Why would firms benefit from not knowing what their workers are doing? This hypothesis does not square well with the prediction from the canonical principal-agent model where the principal generally benefits from more information about the agent's actions (Holmström, 1979). Relatedly, why does a worker become more efficient when given freedom at work? There is an extensive work in the psychology and management literature studying this question, and they generally attribute the positive effects from freedom provision to the worker's intrinsic motivation.[3] But if the worker's intrinsic motivation is the only explanation, why do we observe different types of firms having different kinds of monitoring environments? For example, why is freedom provision more often observed in startups than in factory assembly lines?

[1] The report is available online at "http://pages.lrn.com/the-freedom-report".

[2] For example, Andrews and Farris (1967) show that scientists are more likely to innovate under higher degree of freedom. Amabile et al. (1990) find that workers exhibit lower levels of creativity when being watched while working. Shalley et al. (2000) find that the autonomy to make decision increases employees' commitment to the organization and fosters innovation. Elloy (2005) finds that employees in self-managed teams exhibit higher levels of organizational commitment. Falk and Kosfeld (2006) find experimental evidence that agents react negatively to actions deemed controlling by reducing performances. More recently, Bloom et al. (2015) find that call center operators experience a 13% increase in overall performance when working from home.

[3] Intrinsic motivation are incentives to take an action that are derived through non-monetary rewards. In the psychology literature, Deci and Ryan (1987) suggest that autonomy is associated with greater interest and creativity (among other positive behaviors). In the management literature, intrinsic motivation from freedom takes the form of an empowering leadership style that encourages employees to develop self-control and initiative. Since autonomy leads to higher job satisfaction (e.g. Spreitzer, 2008), employees under less controlling work environments have a stronger sense of ownership in the success of the organization. For example, Amabile et al. (1996) identify freedom provision as an important managerial practice to foster creativity at work. Stern (2004) finds that scientists in pharmaceutical firms are willing to accept 15% less pay if they are given freedom on their project choice.

In this paper, I put aside a worker's intrinsic motivation and focus on understanding his extrinsic motivation (i.e. monetary incentives) to innovate and improve efficiency when given freedom at work.[4] The intuition behind the worker's extrinsic motivation is the following. When a worker is closely watched by his firm, any new knowledge gained from his innovation attempt is simultaneously learned by the firm. If the worker uncovers a more efficient work method, the firm, being aware of it, will respond by raising his future performance requirement; anticipating this, the worker never tries to innovate. If the worker is given freedom at work instead – for example, he is allowed to work from home – he accrues private information about any discovery made. This private information compels the firm to give information rent to the worker if the firm wants to tap on the worker's improved efficiency. The potential for information rent then feeds back as the worker's incentives to uncover better work methods in the first place. But giving a worker excessive freedom is costly to the firm due to the asymmetric information created. Hence, whether freedom provision is ultimately beneficial would depend on whether if the expected innovation benefits can outweigh the inefficiency due to the asymmetric information. This in turn provides a qualitative explanation to why, for example, startups and "tech firms" give their employees a lot of freedom at the workplace whereas a factory assembly line is often associated with tight monitoring.

While this notion that players can benefit from "strategic ignorance" has also been noted in the literature in other contexts,[5] the mechanism behind how this feature arises here is different, and the novelty of this paper lies in understanding how a firm's strategic ignorance influences its incentive structure which has to simultaneously induce effort from the worker and endogenously generate asymmetric information *against* the firm. The contract design under strategic ignorance involves an adverse selection problem with moral hazard, and there is an endogenously created mass point in an otherwise atom-less distribution over the agent's continuous private type space. This in turn leads to some interesting features in the optimal menu of contracts. The evolution of the optimal incentive structure also provides a novel rationale that explains why workers' incentives tend to be weak in the early stages of their career and why some economic relationships have to start off with low-scale production.

I consider a two-period principal-agent model with an embedded two-arm bandit problem.[6] In each period, the agent chooses between using an old technology (o) or a new

[4]For papers that study how extrinsic incentives affect intrinsic motivation, see for example Bénabou and Tirole (2003) and Sliwka (2007).

[5]See Crémer (1995) and other cited works in Section 8.

[6]I use the female pronoun for the principal and the male pronoun for the agent throughout.

technology (n). Output is binary in each period, and the probability of getting the high output with either technology is the agent's effort choice. The two technologies differ in their effort costs. The cost efficiency of o, denoted by θ^o, is commonly known – technology o represents the currently available work method inside the firm. On the other hand, the cost efficiency of n, denoted by θ^n, is ex-ante unknown to both players – technology n represents a previously untested work method. A higher cost efficiency implies a lower cost of effort,[7] and the common prior on θ^n is represented by a continuous distribution. Under the prior, o is ex-ante more cost-efficient than n in expectation (i.e. $\theta^o > E[\theta^n]$), but n is possibly more cost-efficient than o (i.e. $\Pr(\theta^n > \theta^o) > 0$). The agent can perfectly learn about θ^n by producing with n in period 1, which then creates an option value for period 2: if n turns out to be more cost-efficient than o (i.e. $\theta^n > \theta^o$), the agent can continue to use n in period 2; otherwise he can revert to using o. Hence, using n in period 1 is interpreted as an innovation attempt to uncover a more cost-efficient technology for period 2, but it comes at a cost of using the myopically less cost-efficient technology n in period 1. The contracting environment is highly incomplete – only the output is contractible and the principal can only make spot contracts. Hence the principal offers a bonus-pay contract to the agent at the start of each period.

Under a "*tight-monitoring*" environment (hereafter TM), the principal perfectly observes all of the agent's actions. This also implies that if n is used, the principal also learns about θ^n together with the agent. The agent thus faces a *ratchet effect* of innovating. If the agent produces with n in period 1 and uncovers that $\theta^n > \theta^o$, the principal will offer the agent a lower period-2 bonus since she knows the agent's cost of effort has decreased, thus extracting away all the innovation surplus. Anticipating no gain in period 2, the agent always uses o in period 1 since $\theta^o > E[\theta^n]$. Hence innovation is never possible under TM.

On the other hand, under a "*freedom-at-work*" environment (hereafter FW), the agent *privately* learns θ^n after using technology n in period 1. With asymmetric information, the principal solves a period-2 contracting problem that consists of both adverse selection and moral hazard – the principal has to elicit the agent's private information on θ^n with a menu of bonus-pay contracts (adverse selection), while each contract in the menu must provide incentives for the agent to make the appropriate technology and effort choices (moral hazard). The optimal period-2 menu has a bonus schedule that is increasing in the agent's innovation success and a fixed wage schedule that is decreasing in it. This is because an

[7] Formally, the cost of effort e on technology $\tau \in \{o, n\}$ is $\psi\left(\frac{e}{\theta^\tau}\right)$ where ψ is strictly increasing and convex. Hence a higher θ^τ implies a lower cost of effort and hence, a higher cost efficiency.

agent with a more cost-efficient technology incurs a lower cost to increase the probability of getting the bonus. Hence he is willing to accept a "riskier" contract with a lower fixed wage in exchange for a higher bonus. The increasing bonus schedule implies that the more successful innovations are utilized more intensively and efficiently.

An interesting feature of the optimal menu is the pooling of contracts for types with low cost efficiency, indicating that the principal optimally chooses not to distinguish an agent with a mildly successful innovation (i.e. θ^n that is slightly above θ^o) from an agent who failed in his innovation attempt (i.e. $\theta^n \leq \theta^o$). This feature arises due to the *mass point* of failed-innovator agents who revert to using o in period 2; these failed-innovator agents are essentially of the same type whose cost efficiency in period 2 is θ^o. If the principal wants to also distinguish the mildly successful innovators from these failed innovators (i.e. type θ^o), she has to further lower the bonus of the contract meant for θ^o. The loss in surplus from doing so is very costly to the principal because the atom-less distribution of any one type of successful innovator is incommensurate with the mass point at θ^o. Hence the possibility of innovation failure implies that innovations have to be sufficiently successful before they are "recognized" by the principal with increasingly powerful bonuses.

As in standard screening problems, the failed innovators get no information rent since they are the worst type. This implies that if the agent uses o in period 1, his expected payoff in period 2 is always zero. This is because when an agent enters period 2 without knowing the underlying value of θ^n, he will produce with o in period 2 since $\theta^o > E\left[\theta^n\right]$. This in turn makes him the worst-type agent in the screening problem who gets no information rent. Only an agent who has used n in period 1 and discovered that $\theta^n > \theta^o$ will get information rent in period 2. While the potential for rent in period 2 under FW creates incentives for the agent to use the ex-ante less cost-efficient n in period 1, the principal must also complement it with a low period-1 bonus. This is because under any period-1 bonus, the agent's optimal effort exerted on o is always higher than his optimal effort exerted on n because of $\theta^o > E\left[\theta^n\right]$. A high period-1 bonus would then encourage the agent to use o to maximize his likelihood of getting the period-1 bonus and forgo the potential period-2 rent that comes from experimenting on n.

Since the bonus represents the agent's opportunity cost of getting a low output, the requirement for a low period-1 bonus can be interpreted as the firm having to show tolerance to early failures by its worker to encourage him to innovate. This feature of low period-1 bonus in equilibrium also provides an alternative mechanism to explain why a worker's incentives tend to be weak in the early stages of his career, which differs from the explanation from the career concern literature (Holmström, 1999); and why some economic relationships have to start off with low stakes, which contrasts with the rationale in the reputation-building literature (e.g. Watson 1999, 2002; Halac 2012).

Finally, the contracts in equilibrium illustrate the evolution of the incentive structure under the two different monitoring environments. Under TM, the technology level remains at the cost efficiency of o in both periods and hence, the incentive structure stays relatively constant over time. On the other hand, the incentive structure under FW starts off with low incentive in period 1, followed by idiosyncratic changes in incentive in period 2 through the agent's choice from the menu of contracts, which is in turn dependent on the outcome of the agent's innovation attempt. This evolution can be interpreted as changes in job scopes and wage structures for the workers over time, which is akin to career progression or promotion opportunities inside the firm.

To make predictions on a firm's choice of monitoring environment, I consider the comparative statics of the equilibrium under TM and FW with respect to the underlying parameters. I show that when the distribution of θ^n becomes more favorable (in an appropriate sense), the principal's total expected payoff increases under FW, but it remains unchanged under TM. On the other hand, a higher θ^o increases the principal's total payoff under both FW and TM, but the increase is greater under TM. These results suggest that firms with important needs to improve on existing work methods – for example, young firms still learning about their operational needs or new-market firms (e.g. startups) which cannot mimic efficient procedures from others or from their history – are more likely to give their employees freedom at the workplace. On the other hand, when the gains from improvement are small – for example, mechanical jobs and relatively homogenous works (e.g. factory assembly lines) where there are already well-established experiences that can be adapted and used – firms are more likely to monitor their employees tightly.

The rest of the paper will proceed as follows. Section 2 first illustrates the intuition behind a worker's extrinsic motivation from freedom at work with a numerical example. Section 3 outlines the main model, and Sections 4 and 5 solve for the equilibrium under the TM and FW environments respectively. Section 6 does the comparative statics, and Section 7 discusses some extensions, including how more general types of monitoring environments can be modeled. The discussion on related literature is deferred to Section 8 after the results have been presented, and Section 9 concludes. All omitted proofs are found in Appendix A.

2 A Numerical Example

Consider a two-period principal-agent model with risk-neutral parties and no discounting. In each period $t \in \{1, 2\}$, the agent can produce an output of $Y = 6$ for the principal using one of two available technologies $\tau_t \in \{o, n\}$, or the agent can take his zero per-period outside option; he can use a different technology across periods. Producing with technology τ incurs cost c^τ which is constant across the two periods. $c^o = 5$ is commonly known. On the other hand, c^n is ex-ante unknown to both players, and they share a common prior of:

$$c^n = \begin{cases} 10 & \text{, with probability } 0.5 \\ 1 & \text{, with probability } 0.1 \\ 0 & \text{, with probability } 0.4 \end{cases}$$

$E[c^n] = 5.1 > c^o$ implies that n is in expectation more costly than o. If the agent uses n in period 1, the value of c^n is realized at the end of period 1; otherwise it remains unknown. The output is the only contractible term, and the principal cannot enforce any long-term contract. Hence a contract in each period t is a simple payment b_t from the principal to the agent in exchange for output $Y = 6$ in that period.

2.1 Tight Monitoring (TM)

Consider a tight-monitoring (TM) environment where the principal perfectly observes all of the agent's actions. If n is used, the principal also learns the value of c^n with the agent.

Under this arrangement, if c^n is still unknown in period 2, the principal will offer $b_2 = c^o$ for the agent to produce using o. If c^n is known, the principal will offer $b_2 = \min\{c^o, c^n\}$ for the agent to use the more cost-efficient technology. Under either case, the worker earns zero payoff in period 2. Anticipating this, under any offer $b_1 \geq c^o$, the agent will produce with o in period 1 since $c^o < E[c^n]$. The equilibrium is thus uniquely $b_1 = b_2 = c^o$. The principal's profit is $2 \times (Y - c^o) = 2$, and she can never induce the agent to try out n under TM.[8]

This result illustrates two key issues. The first is a *ratchet effect*: because the principal cannot commit not to lower future payment after a more cost-efficient technology is uncovered, the agent responds by not trying to uncover one. The second issue, which is first made by Aghion and Tirole (1994), is that the inability to contract on the innovation ex-ante can

[8] The "sell the firm to the agent" arrangement does not work here because the principal does not have the ability to pledge the period-2 surplus to the agent at start of period 1.

cause under-investment in the innovative activity. In the context here, the inability to induce the agent to choose $\tau_1 = n$ is also due to the uncontractibility of the c^n realization.

2.2 Freedom-at-Work (FW)

Consider a freedom-at-work (FW) environment where the principal does not observe any of the agent's actions. Importantly, if the agent uses n in period 1, the agent *privately* learns the value of c^n. The revenue Y remains publicly observed and contractible.

I will show that the agent chooses $\tau_1 = n$ in equilibrium now and the principal's total expected payoff is also increased. To see why, suppose that in period 2, the principal conjectures that the agent produced with $\tau_1 = n$ and privately knows c^n. The agent will produce whenever the offered bonus is such that $b_2 \geq \min\{c^o, c^n\}$. Table 1 below summarizes the principal's expected period-2 profit for the relevant b_2 offers, which implies that the optimal contract is $b_2 = 1$.

Table 1: Principal's period-2 payoffs

b_2 offer	Probability of the agent producing in period 2	Principal's expected payoff in period 2
5	$Pr\left[\min\{c^n, c^o\} \leq 5\right] = 1$	$1 \times [6 - 5] = 1$
1	$Pr[c^n \leq 1] = 0.5$	$0.5 \times [6 - 1] = 2.5$
0	$Pr[c^n = 0] = 0.4$	$0.4 \times [6 - 0] = 2.4$

Next, consider the agent's period-1 action when anticipating that $b_2 = 1$. The agent's expected period-2 payoff for producing with $\tau_1 = n$ is 0.4,[9] while his expected period-2 payoff for producing with $\tau_1 = o$ is 0 as he will be taking his outside option in period 2. Since $E[c^n] - c^o < 0.4$, the agent chooses $\tau_1 = n$ now. It is readily verified that the optimal period-1 offer is $b_1 = 4.7$, and the principal's total expected payoff is now 3.8 which is higher than that under TM.

Notice that the principal benefits from creating information asymmetry against herself in period 2. The information asymmetry compels the principal to give rent to the agent in the event of a very successful innovation (i.e. $c^n = 0$) which then channels back as the agent's incentives to experiment on n in period 1. Notice also that relative to TM, the principal's total expected payoff under FW is increased while the agent's payoff is unchanged. This implies that the creation of informational asymmetry is weakly Pareto-improving, which

[9]With probability 0.4, c^n is 0, in which case the agent earns a payoff of $b_2 - 0 = 1$.

stands in contrast to the well-understood result that information asymmetry is detrimental to efficient trade under complete contracts (Akerlof 1970; Myerson and Satterthwaite 1983).

2.3 Corporate Culture and the Persistence of Monitoring Environment

While not strictly needed to get the results, the model implicitly assumes that the monitoring environment is unchanged over time. In practice, the monitoring technology in a firm depends on features such as the workspace arrangement between the managers (monitors) and the workers, and on the availability and effectiveness of surveillance facilities. Many of these features are costly to change, which thus justifies the assumption regarding the persistence in the monitoring environment in the model.

More importantly, the essence of "freedom-at-work" (or the lack of it) here is about the *trust* that a worker has on his managers not secretly watching over him and trying to learn what he has uncovered. Such trust takes time to build, evolves from the experiences of both the worker himself and his co-workers, and it should thus be part of the identity and *culture* of the firm. As Hermalin (2013) writes:[10] "*[...] corporate culture is not a precisely defined concept. It can be seen as encompassing the norms and customs of a firm, its informal and unwritten rules of behavior. [...] Consequently, given normal personnel turnover, the culture will persist over time, evolving slowly if at all.*" Given the persistence of culture, the essence of the monitoring environment (TM or FW) should remain fixed at least in the short run, which justifies why the monitoring environment is fixed throughout the game in the model.

2.4 Other Applications

While being framed as an agency problem in organizations, the intuitions here can be applied more generally to other economic relationships that require the agent to make uncontractible experimentation. For example, in a long-term procurement relationship, from the result in TM, the incentives for the seller to make buyer-specific innovation attempts diminish when the innovation gains are expected to be observed and appropriated by the buyer. This might explain why buyers would choose suppliers who are located far away (analogous to FW) despite the transportation costs incurred.

[10] See page 458.

The notion can also be related to the delegation-versus-centralization problem.[11] Under delegation, the principal gives up decision rights to the agent; while under centralization, the principal keeps the decision rights. Shin and Strausz (2014) suggest that the party with the decision rights to the input should also have private information about its evolution; this notion is also implied in Riordan (1990). Hence delegation, analogous to FW, facilitates the agent in accruing private information; while centralization, analogous to TM, does not. The results here then suggest that innovations by the agent is more likely to take place under delegation.

3 The Main Model

The example above illustrates the intuition of how freedom provision motivates a worker to innovate. But since innovation attempt is modeled as producing with previously untested technologies, what the example lacks are details on the production environment. In this section, I introduce the main model which generalizes the example and introduces moral hazard into production – this would allow us to study how the monitoring environment influences the incentive structure in a firm.

As previously, consider a two-period principal-agent model with risk-neutral parties who do not discount the future. In each period, the agent chooses to exert effort on one of two available technologies $\tau \in \{o, n\}$ where the technologies have the same interpretations as previously; the agent also does not have to use the same technology across periods. The effort choice set is the continuum $[0, 1]$ for both periods. Both technologies generate a stochastic per-period revenue of $y \in \{0, Y\}$ for the principal with $Y > 0$, and the probability of getting Y with either technology is the agent's effort choice that period. Hence the two technologies have the same productivity. To save on time subscript, period-1 effort will be denoted by e and period-2 effort by its Greek letter counterpart ε. There will not be any time subscript in the main model; instead subscripts will exclusively denote partial derivative.

The cost of effort depends on the technology chosen. An effort level e (respectively ε) exerted on technology τ incurs a private cost of $\psi\left(\frac{e}{\theta^\tau}\right)$ (respectively $\psi\left(\frac{\varepsilon}{\theta^\tau}\right)$) to the agent, where the cost function $\psi(\cdot)$ satisfies Assumption 1 below. $\theta^\tau \in (0, 1]$ is the effort cost efficiency for technology τ which is constant across periods; a higher value of θ^τ indicates a more cost-efficient technology.

[11] See for example, Holmström (1977); Szalay (2005); Alonso and Matouschek (2008); Amador and Bagwell (2013).

Assumption 1. *For all* $x \in [0,1)$, $\psi(x)$ *is continuously differentiable, strictly increasing and strictly convex. Moreover,* $\psi(0) = \psi'(0) = 0$, $\lim_{x\to 1}\psi(x) = \infty$, *and* $0 \leq \psi'''(x) \leq \frac{2\left(\psi''(x)\right)^2}{\psi'(x)}$.[12]

The Inada condition for $\psi(\cdot)$ in Assumption 1 ensures that the agent's optimal effort choice is always interior. The bounds on the third derivative of $\psi(\cdot)$ is a technical condition that ensures that the marginal effect of incentives on the agent's optimal effort choice is increasing in θ^τ.

In terms of information, θ^o is commonly known. On the other hand, θ^n is ex-ante unknown to both players and their common prior on θ^n is represented by a distribution function $F(\cdot)$ on $\left[\underline{\theta}, \bar{\theta}\right]$, with $0 < \underline{\theta} < \bar{\theta} \leq 1$. If n is used in period 1 with $e > 0$, the value of θ^n will be realized at the end of period 1. Hence using n in period 1 is also an experimentation to uncover a possibly more cost-efficient technology and is thus interpreted as an innovation attempt.

Assumption 2. *F has an atom-less density f which is differentiable and strictly positive in its support. Moreover* $E[\theta^n] \leq \theta^o < \bar{\theta}$, *and* $\frac{d}{d\theta^n}\left(\frac{1-F(\theta^n)}{f(\theta^n)}\right) \leq 0$.

The assumption $E[\theta^n] \leq \theta^o$ implies that o is ex-ante more cost-efficient than the innovative activity n. The non-increasing inverse hazard rate property is a standard assumption in the screening literature that ensures that the screening problem is well-behaved, although as shown later, the screening problem in period 2 will still exhibit "bunching".

I assume that the agent has a zero limited liability constraint every period so that no positive transfer can be made from the agent to the principal. The agent's outside option in periods 1 and 2 are respectively 0 and $U^0 > 0$, where U^0 satisfies Assumption 3 below. On the other hand, the principal has a zero outside option for both periods. In addition, in period 1, the principal's only revenue source is the realization of y; while in period 2, on top of y, the principal also receives a fixed gain of trade of U^0 as long as the agent accepts her period-2 contract offer (to be defined). This fixed gain of trade ensures that the agent's higher period-2 outside option never prohibits the principal from entering the relationship in period 2.[13]

[12] Examples of functions that satisfy Assumption 1 include $\psi(x) = -\log(1-x) - x$, and $\psi(x) = \frac{x^{m_2}}{(1-x)^{m_1}}$ for any $m_1 \geq 1$ and $m_2 > m_1$.

[13] The agent's higher period-2 outside option can be motivated through the accumulation of a worker's general human capital over time which is left unmodeled here. His higher level of general human capital in period 2 then makes his mere presence worth U^0 to the firm, thus explaining the principal's period-2 fixed gain of trade. The worker's outside option of U^0 in period 2 is then consistent with a competitive market

Assumption 3. *U^0 satisfies* $U^0 \geq \max_{\varepsilon} \left\{ \varepsilon Y - \psi\left(\varepsilon/\bar{\theta}\right)\right\} > 0$.

The main purpose of assuming a high outside option U^0 for the agent in period 2 is to make it impossible for the agent to accrue any period-2 rent (net of outside option) through his limited liability constraint. Hence any period-2 rent that the agent accrues under a different monitoring environment (to be described) can be entirely attributed to the environment. The general results will be largely unchanged if U^0 is instead smaller or is zero, but the comparison of rents will then be shrouded by the rents that always exist due to the agent's limited liability.[14]

3.1 Contracts

The revenue y in each period is the only contractible term throughout; neither the effort choice, technology choice nor the experimentation outcome can be contracted upon. Effort is not contractible for the usual non-verifiability issues. τ is not contractible as n represents an innovative attempt which is in general difficult to identify and describe ex-ante. Innovative activities typically involve coming out with new ideas or exploring previously untested approaches, both of which often end up with no tangible outcomes to show for. It is thus difficult to distinguish between a worker who is carrying out an honest experimentation from a worker who is merely shirking. In the same vein, the experimentation outcome θ^n is also assumed to be non-contractible because of the difficulties in describing the innovation before it is realized and also in verifying the value of an innovation by a third party.[15]

The principal has no commitment power to enforce long-term contracts. Hence, contracting takes place at the beginning of each period, and a contract specifies a non-negative fixed transfer and a non-negative bonus for output Y. Like the notation for effort, period-1 contract is denoted by Roman alphabets and period-2 contract by their Greek letter coun-

for labor supply while ignoring the issues of competitive screening on the θ^n realization; for instance, the innovation θ^n can be firm-specific.

[14] Most of the results can be replicated with $U^0 = 0$ and the agent having unlimited liability. However, this leads to an unattractive feature in the optimal contracts whereby the agent makes a positive fixed transfer to the principal.

[15] Holmström (1989) provides a very good discussion on why innovation attempts are often difficult-to-measure activities, citing reasons that innovation projects are often multi-staged and unpredictable, and it is often not clear ex-ante what is the correct course of action that the agent should take. Aghion and Tirole (1994) also argue how innovations are always unpredictable and hence difficult to contract on before they are realized. Moreover, innovations are often complex such that even if parties can agree on the innovation ex-ante, courts can still lack the requisite technical knowledge to enforce the contract, or that the innovation can take a long time for its full value to be realized such that an agreement on the immediate value is impossible.

terparts. The contracts are thus (a, b) and (α, β) respectively for periods 1 and 2, where a and α are the fixed wages, and b and β are the bonuses. Henceforth, whenever unspecified, all payoffs refer to expected payoffs.

3.2 Monitoring Environments

I consider the principal's problem under two types of monitoring environments – *tight-monitoring* (TM) or a *freedom-at-work* (FW) – and the monitoring environment is fixed across the two periods; see Section 2.3 for a discussion on this assumption. The principal's revenue y each period is publicly observed under both environments. Moreover, under TM, the principal observes all the actions of the agent which include his technology and effort choices. Importantly, if the agent uses n, both parties would observe θ^n. On the other hand, under FW, the principal does not observe any of this information. This implies that if the agent uses n in period 1, the θ^n realization will be *privately* observed by the agent which then results in asymmetric information in period 2.[16]

3.3 Exploitation vs Exploration

As Schumpeter (1934) points out, a central concern for sustainability and development in firms is the relation between exploiting its current capabilities and exploring for new possibilities. Following March (1991), this is termed as the tension between exploitation and exploration. This tension arises here via its two-armed bandit problem setup in which o is a known arm (exploitation) while n is an unknown arm (exploration). The assumptions made on the costs reflect the nature of conventional work methods versus innovative activities. Innovative activities, as represented by n, tend to be more costly to engage in initially. On the other hand, the gains from exploration is learning – engaging in innovative activities might lead to the uncovering of a more efficient technology (i.e. a $\theta^n > \theta^o$) for future use.

Throughout the paper, the equilibrium concept used is the perfect Bayesian Equilibrium (henceforth equilibrium). An equilibrium that features the agent choosing o in period 1 will be termed an *exploitation equilibrium*, and an equilibrium that features the agent choosing n in period 1 will be termed an *exploration equilibrium*.

[16]The observability of effort and/or technology choices under FW will be outcome-irrelevant because they are not contractible (see Remark 4). The only substantial difference between the two environments is the observability of θ^n which determines if asymmetry in period-2 payoff-relevant information exists.

4 Tight-Monitoring (TM)

This section considers the TM environment. It first illustrates the ratchet effect of innovating under TM and then shows that the only equilibrium is an exploitation equilibrium.

Under TM, information is always symmetric in period 2. Hence the principal optimally makes the agent the residual claimant of the output by setting bonus $\beta = Y$ and fixed wage α such that the agent's participation constraint binds: if θ^n is known in period 2, then $\alpha = U^0 - \max_{\varepsilon}\left\{\varepsilon Y - \psi\left(\frac{\varepsilon}{\theta}\right)\right\}$ where $\theta = \max\{\theta^n, \theta^o\}$; if θ^n is unknown, then $\alpha = U^0 - \max_{\varepsilon}\left\{\varepsilon Y - \psi\left(\frac{\varepsilon}{\theta^o}\right)\right\}$. Under Assumption 3, these period-2 fixed wages are non-negative.

To see why α is set in anticipation that the agent uses o when θ^n is unknown in period 2, consider any bonus β and let ε^n be the agent's optimal effort for n under this bonus.[17] Notice that:

$$\begin{aligned}
\varepsilon^n\beta - E\left[\psi\left(\frac{\varepsilon^n}{\theta^n}\right)\right] &< \varepsilon^n\beta - \psi\left(\frac{\varepsilon^n}{E\left[\theta^n\right]}\right) && (1)\\
&\leq \varepsilon^n\beta - \psi\left(\frac{\varepsilon^n}{\theta^o}\right) && (2)\\
&\leq \max_{\varepsilon}\left\{\varepsilon\beta - \psi\left(\frac{\varepsilon}{\theta^o}\right)\right\}, && (3)
\end{aligned}$$

where inequality (1) follows from the convexity of $\psi(\cdot)$ and Jensen's inequality. Hence the agent's period-2 payoff is higher by using o when θ^n is unknown.

Since the agent's period-2 payoff is always his outside option U^0, he myopically maximizes only his period-1 payoff at the start of period 1. Under any acceptable period-1 contract (a, b), by the same argument as in (1) to (3), the agent will use o in period 1. This illustrates the ratchet effect: when the employer cannot commit not to expropriate all the efficiency gains, the worker responds by not trying to improve efficiency at all.[18] This thus provides an economic rationale for the "conventional wisdom" that a more intrusive working environment stiffens the worker's creativity.

Proposition 1. *Let* $e^o(b) := \arg\max_{e}\left\{eb - \psi\left(\frac{e}{\theta^o}\right)\right\}$. *Under TM, the optimal contracts are*

[17] ε^n depends on β but the argument is dropped to save on notation.

[18] The model implicitly assumes that the agent is unable to take his innovation to another firm to reap the benefits of it. Section 7.3 discusses this assumption.

unique:

$$\begin{aligned} a^* &= 0 \\ b^* &= \arg\max_b \{e^o(b)[Y-b]\} \\ \alpha^* &= U^0 - \max_\varepsilon \left\{\varepsilon Y - \psi\left(\frac{\varepsilon}{\theta^o}\right)\right\} \\ \beta^* &= Y \end{aligned}$$

(Ratchet effect) The agent cannot be induced to experiment on technology n in period 1 under TM.[19,20]

Remark 1. If the effort level is also contractible, the result that the agent cannot be induced to experiment on n under TM (i.e. ratchet effect) will still hold but with different contracts; in particular, the principal will contract on effort rather than output now. To see this, suppose that the principal can enforce payments that are contingent on the agent's effort, but the technology that the agent chooses to exert the effort on remains uncontractible; this implies that the moral hazard problem on effort is eliminated. In period 2, since information is symmetric, the principal will enforce the agent to exert first-best (conditional on the available information in period 2) effort level and then exactly compensates the agent for his forgone period-2 outside option U^0 and his effort cost based on the agent's more cost-efficient technology. Hence the agent always earns zero rent (net of outside option) in period

[19]Readers who are familiar with the incomplete contracts literature might be aware of the Maskin-Tirole critique (Maskin and Tirole, 1999) – if the information is commonly observed by both parties ex-post, one can possibly design a subgame perfect implementation mechanism (Moore and Repullo, 1988) that gives incentives for both parties to reveal the information. The possibility of doing so is assumed away here for a number of reasons. First, it is difficult to reconcile with the requirement that the principal can commit to playing the revelation game given that she cannot commit to long-term contracts. Second, since players are risk-neutral, renegotiation must not be allowed in order for the Maskin-Tirole mechanism to work, which again carries some tension with the limited commitment assumption. Third, such revelation mechanisms are highly complex and appears unrealistic given the dearth of evidence that such mechanisms do operate in practice. Moreover, Aghion et al. (2015) and Fehr et al. (2016) have also provided experimental evidence that such subgame perfect implementation mechanisms fail to induce truth-telling due to various irrational beliefs held by the subjects. The recent work of Aghion et al. (2012) has also shown that the Maskin-Tirole mechanism is not robust to small perturbation from common knowledge of perfect information; this theoretical prediction is also confirmed empirically by Aghion et al. (2015).

[20]The result that the agent cannot be induced to experiment with n in period 1 can still hold even if the principal can make long-term contracts on output y, if the parties cannot commit not to renegotiate at the start of period 2. With commitment to make long-term contracts on y, this becomes a hold-up problem. Che and Hausch (1999) show that ex-ante contracting is useless if investment is "cooperative"; in the context here, this can happen when θ^n is firm-specific and the firm can reap the benefit of a worker's innovation through other workers as well. A similar result also holds if the investment is "selfish" but possibly "complex"; see Hart and Moore (1999) and Segal (1999).

2. As a result, under any effort obligation in period 1, the agent will always choose to fulfill the effort obligation using o since it is the ex-ante more cost-efficient technology. While innovation is not possible under TM, the elimination of the moral hazard on effort does, however, increase the principal's period-1 payoff as there was a distortion previously due to the interaction between the effort moral hazard and the agent's limited liability when the effort was not contractible. However, it is readily verified that all subsequent results in the paper are unaffected.

5 Freedom-at-Work (FW)

The previous section has illustrated that only the exploitation equilibrium is sustainable under TM. This section considers the FW environment, focusing on deriving the principal-optimal exploration equilibrium.[21] The analysis begins at period 2 (Section 5.1) after the agent has accrued private information about θ^n from period 1. The principal screens the agent with a menu of bonus contracts; such menus can be interpreted here as workers self-selecting themselves into the firm's various projects and career tracks. Note that the agent's period-1 effort is chosen before the realization of θ^n; hence the realization of y in period 1 does not provide any information about the agent's period-1 technology choice nor on the realization of θ^n, and deviation by the agent in period 1 is thus never detectable. As a result, the principal's choice of the period-2 menu will be independent of the period-1 realization of y. The optimal menu thus determines both the agent's on-path and off-path period-2 payoffs. The optimal period-1 contract is then derived in Section 5.2 while taking into account these period-2 payoffs. The discussions on the implications of the optimal contracts are provided in Section 5.3.

5.1 Period 2

The principal's period-2 problem is one of adverse selection with moral hazard – the menu as a whole must elicit information about θ^n from the agent (adverse selection), while each

[21] One can also view this as the principal's cost-minimization problem to induce the agent to explore with probability one. The Supplementary Appendix considers other types of equilibrium, including exploitation equilibrium, and mixed equilibrium in which the agent plays a mixed strategy for his period-1 technology choice. It is shown there that if an exploitation equilibrium exists under FW, the principal's total expected payoffs from such an equilibrium cannot be higher than her payoffs from the unique (exploitation) equilibrium under TM in Proposition 1. Hence, if the principal is better off under FW than TM, the equilibrium must involve the agent exploring with positive probability in the first period.

individual contract has to provide him with incentives to make the appropriate technology and effort choices (moral hazard).[22] The analysis begins by defining the appropriate type space of the agent which will have a mass point of failed innovators. The moral hazard aspect of the problem is then subsumed by working with the agent's indirect utility. The optimal menu of contracts, which features "bunching" because of the mass point of failed innovators, is characterized in Proposition 2.

5.1.1 Type-space, Distribution of Types and Innovation Success

In period 2, if θ^n is less than θ^o, its exact value is irrelevant since the agent will use o. Hence the agent's payoff-relevant private information (i.e. type) is:

$$\theta := \max\left\{\theta^n, \theta^o\right\}. \tag{4}$$

The support of θ is $\left[\theta^o, \bar{\theta}\right]$ with a probability distribution:

$$\Pr\left(\theta \leq x\right) = F\left(\theta^o\right) + \int_{\theta^o}^{x} f(t)dt.$$

Type-θ^o agent is a failed innovator who reverts to o in period 2 and this happens with probability $F(\theta^o) \in (0,1)$ on the equilibrium path. A successful innovator has a type θ greater than θ^o and an agent with a higher type has a more successful innovation.

5.1.2 Subsuming the Moral Hazard Problem

After accepting a period-2 contract (α, β), the agent's effort choice problem is:

$$\max_{\varepsilon} \left\{\alpha + \varepsilon\beta - \psi\left(\frac{\varepsilon}{\theta}\right)\right\}. \tag{5}$$

Let $\varepsilon\left(\beta, \theta\right)$ denote the solution to (5), which is implicitly characterized by the first-order condition:

$$\beta = \frac{1}{\theta}\psi'\left(\frac{\varepsilon\left(\beta,\theta\right)}{\theta}\right). \tag{6}$$

By the revelation principle, attention can be restricted to only direct mechanisms with an obedience constraint given by condition (6). This is equivalent to considering the agent's

[22]See chapter 7 of Laffont and Martimort (2001) for an introduction to such hybrid models.

indirect utility. Let:

$$u(\beta,\theta) := \max_{\varepsilon} \left\{ \varepsilon\beta - \psi\left(\frac{\varepsilon}{\theta}\right) \right\} \tag{7}$$
$$= \varepsilon(\beta,\theta)\beta - \psi\left(\frac{\varepsilon(\beta,\theta)}{\theta}\right).$$

The agent's indirect utility under a contract (α, β) is thus $\alpha + u(\beta,\theta)$. By viewing α as a "transfer" and β as an "allocation", the problem becomes a pure screening problem with the agent having quasi-linear preferences. The properties of the agent's indirect utility are stated in Lemma 1. Henceforth subscript i of any function denotes its partial derivative with the argument i.

Lemma 1. *Both $\varepsilon(\beta,\theta)$ and $u(\beta,\theta)$ are continuously differentiable and strictly increasing in both their arguments. $\varepsilon(\beta,\theta)$ is strictly concave in β and strictly convex in θ. $u(\beta,\theta)$ is strictly convex in both its arguments. The agent's indirect utility satisfies the Spence-Mirrlees single-crossing condition: $u_{\theta\beta}(\beta,\theta) > 0 \; \forall \beta > 0$.*

Both the induced effort $\varepsilon(\beta,\theta)$ and the agent's variable part of his indirect utility $u(\beta,\theta)$ are increasing in the bonus and his type. The principal never offers an infinite bonus since $\varepsilon(\beta,\theta)$ is concave in β. The single-crossing condition arises due to the fact that an increase in bonus β benefits an agent with higher θ more because his marginal cost of effort is lower and he is thus willing to exert more effort which then gives him the bonus with a higher probability.[23]

5.1.3 The Optimal Contracting Problem

Consider a menu of contracts denoted by $\{\alpha(\theta), \beta(\theta)\}_{\theta\in[\theta^o,\bar{\theta}]}$. Let $U\left(\tilde{\theta};\theta\right) := \alpha\left(\tilde{\theta}\right) + u\left(\beta\left(\tilde{\theta}\right),\theta\right)$ be the indirect utility of type θ when he chooses the contract of type $\tilde{\theta}$, and let $\bar{U}(\theta) := U(\theta;\theta)$ be his indirect utility under truth-telling. The menu is feasible and incentive

[23]The convexity of $u(\beta,\theta)$ in β is a non-standard feature of the screening problem here. In textbook treatments of screening problems (for example, chapter 7 of Fudenberg and Tirole (1991)), the utility function of the agent is assumed to be concave in the allocation (which is the β here) to ensure that the problem is well-behaved. The convexity here suggests that the principal might benefit from offering stochastic bonuses to the agent. However this turns out not to be the case under optimality (see footnote 45 in the Appendix). For ease of exposition, I restrict attention to only deterministic contracts but note that doing so is without loss of generality.

compatible if:

$$\alpha(\theta), \beta(\theta) \geq 0 \quad , \quad \forall\theta \in \left[\theta^o, \bar{\theta}\right] \tag{LL-2}$$

$$\bar{U}(\theta) \geq U^0 \quad , \quad \forall\theta \in \left[\theta^o, \bar{\theta}\right] \tag{IR-2}$$

$$\theta \in \arg\max_{\tilde{\theta}} U\left(\tilde{\theta}; \theta\right) \quad , \quad \forall\theta \in \left[\theta^o, \bar{\theta}\right] \tag{IC-2}$$

(LL-2) is the agent's limited liability constraint, (IR-2) is his participation constraint and (IC-2) is his truth-telling constraint.[24] The problem of the principal is program $\mathcal{P}$:

$$\max_{\alpha(\cdot),\beta(\cdot)} \int_{\theta^o}^{\bar{\theta}} \left(\varepsilon\left(\beta\left(\theta\right), \theta\right)\left[Y - \beta(\theta)\right] - \alpha\left(\theta\right) + U^0\right) dF(\theta) \tag{8}$$

subject to (LL-2), (IR-2) and (IC-2).

The term U^0 in the objective comes from the principal's fixed gain of trade. Using standard envelope theorem arguments, (IC-2) can be simplified to the following:

Lemma 2. *A menu of contracts* $\{\alpha(\theta), \beta(\theta)\}_{\theta\in\left[\theta^o,\bar{\theta}\right]}$ *satisfies constraint (IC-2) if and only if* $\forall\theta \in \left[\theta^o, \bar{\theta}\right]$*, the following two conditions are satisfied:*

1. *(IC-2A):* $\bar{U}(\theta) = \bar{U}\left(\theta^o\right) + \int_{\theta^o}^{\theta} u_\theta\left(\beta(t), t\right) dt.$

2. *(IC-2B):* $\beta(\theta)$ *is non-decreasing.*

To solve program $\mathcal{P}$, I first consider the relaxed program that ignores constraint (LL-2) and then check ex-post that the solution to the relaxed program satisfies (LL-2). From (IC-2A), since $u_\theta\left(\beta(\theta), \theta\right)$ is strictly positive, the (IR-2) constraint is most stringent for type θ^o. In the absence of a non-negative constraint on the fixed wage (i.e. ignoring (LL-2)), $\alpha(\theta)$ in the objective function in (8) can be replaced by $\alpha(\theta) = \bar{U}(\theta) - u\left(\beta(\theta), \theta\right)$ and (IC-2A) of Lemma 2, while setting $\bar{U}(\theta^o) = U^0$ so that constraint (IR-2) of type θ^o binds. After some rearrangement, the relaxed version of program $\mathcal{P}$ can be reformulated as program $\mathcal{P}'$:

$$\max_{\beta(\cdot)\in\mathcal{B}} \int_{\theta^o}^{\bar{\theta}} v\left(\beta(\cdot), \theta\right) dF(\theta), \tag{9}$$

[24] The "-2" in the labels of the constraints refers to period 2. This is added to distinguish these constraints from the associated constraints of period 1 later.

where $\mathcal{B}$ is the set of non-decreasing functions mapping from domain $\left[\theta^o, \bar{\theta}\right]$ to $[0, \infty)$, and:

$$v\left(\beta(\cdot), \theta\right) := \underbrace{\varepsilon\left(\beta(\theta), \theta\right) Y - \psi\left(\frac{\varepsilon\left(\beta(\theta), \theta\right)}{\theta}\right)}_{s(\beta(\theta),\theta)} - \underbrace{\int_{\theta^o}^{\theta} u_\theta\left(\beta(t), t\right) dt}_{r(\beta(\cdot),\theta)}. \tag{10}$$

$v\left(\beta(\cdot), \theta\right)$ is the principal's period-2 payoff generated from type θ under a bonus schedule $\beta(\cdot)$. It consists of the social surplus $s\left(\beta(\theta), \theta\right)$ less the agent's information rent $r\left(\beta(\cdot), \theta\right)$. Program $\mathcal{P}'$ is thus maximizing this subject to a monotonicity constraint on the bonus schedule $\beta(\cdot)$. Doing integration by parts, (9) becomes:

$$\max_{\beta(\cdot)\in\mathcal{B}} \int_{\theta^o}^{\bar{\theta}} \phi\left(\beta(\theta), \theta\right) f(\theta) d\theta + s\left(\beta(\theta^o), \theta^o\right) F\left(\theta^o\right), \tag{11}$$

where

$$\phi\left(\beta, \theta\right) \;=\; s\left(\beta, \theta\right) - u_\theta\left(\beta, \theta\right) H(\theta), \tag{12}$$

and $H(\theta) := \frac{1-F(\theta)}{f(\theta)}$ is the inverse hazard rate ratio for $\theta > \theta^o$. The term $\int_{\theta^o}^{\bar{\theta}} \phi\left(\beta(\theta), \theta\right) f(\theta) d\theta$ is the principal's expected payoff from agents with a successful innovation, while the term $s\left(\beta(\theta^o), \theta^o\right) F(\theta^o)$ is the expected payoff from the failed innovators. The non-standard feature of this screening problem is the mass point at type θ^o, and its effect on the objective is reflected in the term $s\left(\beta(\theta^o), \theta^o\right) F(\theta^o)$ in (11).[25] The following proposition characterizes the solution.

Proposition 2. *(Optimal period-2 incentive scheme.) The solution to program $\mathcal{P}'$ in (9) is*

[25] Due to the mass point at θ^o, the distribution of θ does not satisfy the non-increasing inverse hazard ratio property. This implies that the standard procedure of "optimizing point-wise" for program $\mathcal{P}'$ in (11) will not work here. When this happens, the literature typically uses some form of control theoretic techniques to solve the screening problem. However, the mass point at type θ^o precludes a direct application of optimal control theory. Hellwig (2010) provides a technique to handle screening problems with mass points in the support. The basic idea there is to develop a (as he terms it) "pseudo-type" $\tilde{\theta}$ of the original type θ where the distribution of $\tilde{\theta}$ has no mass point. Non-smooth analysis techniques in optimal control theory can then be applied on the "pseudo" problem, and Hellwig proves that the solution to the "pseudo" problem is also the solution to the original problem. The "bunching" of contracts for the mass point type θ^o with types slightly higher than θ^o, as shown in Proposition 2, is a general phenomenon which is proven in Theorem 2.3 in Hellwig (2010). However, because the problem here is more specific – in particular, the mass point exists only at the lowest type of the support here – than the one considered by Hellwig, the optimal bonus schedule can be solved here via a simpler method. This method is more in line with developing the intuition behind the optimal menu here, and it also circumvents the use of optimal control theory techniques completely, which implies that there is no need to make *a priori* assumption that the optimal bonus schedule is differentiable.

unique. The optimal bonus schedule $\beta^*(\cdot)$ is:

$$\beta^*(\theta) = \begin{cases} \hat{\beta}(\theta) & , \ \forall \theta \in \left[\ddot{\theta}, \bar{\theta}\right] \\ \hat{\beta}(\ddot{\theta}) & , \ \forall \theta \in \left[\theta^o, \ddot{\theta}\right] \end{cases},$$

where $\hat{\beta}(\cdot)$ is characterized by:

$$s_\beta\left(\hat{\beta}(\theta), \theta\right) = \varepsilon_\theta\left(\hat{\beta}(\theta), \theta\right) H(\theta) \ , \quad \forall \theta \in \left[\theta^o, \bar{\theta}\right], \tag{13}$$

and $\ddot{\theta}$ is characterized by:

$$\int_{\theta^o}^{\ddot{\theta}} \phi_\beta\left(\hat{\beta}(\ddot{\theta}), \theta\right) f(\theta) d\theta + s_\beta\left(\hat{\beta}(\ddot{\theta}), \theta^o\right) F\left(\theta^o\right) = 0. \tag{14}$$

The optimal fixed wage schedule $\alpha^*(\cdot)$ is:

$$\alpha^*(\theta) = U^0 + \int_{\theta^o}^{\theta} u_\theta\left(\beta^*(t), t\right) dt - u\left(\beta^*(\theta), \theta\right) > 0 \ , \quad \forall \theta \in \left[\theta^o, \bar{\theta}\right]. \tag{15}$$

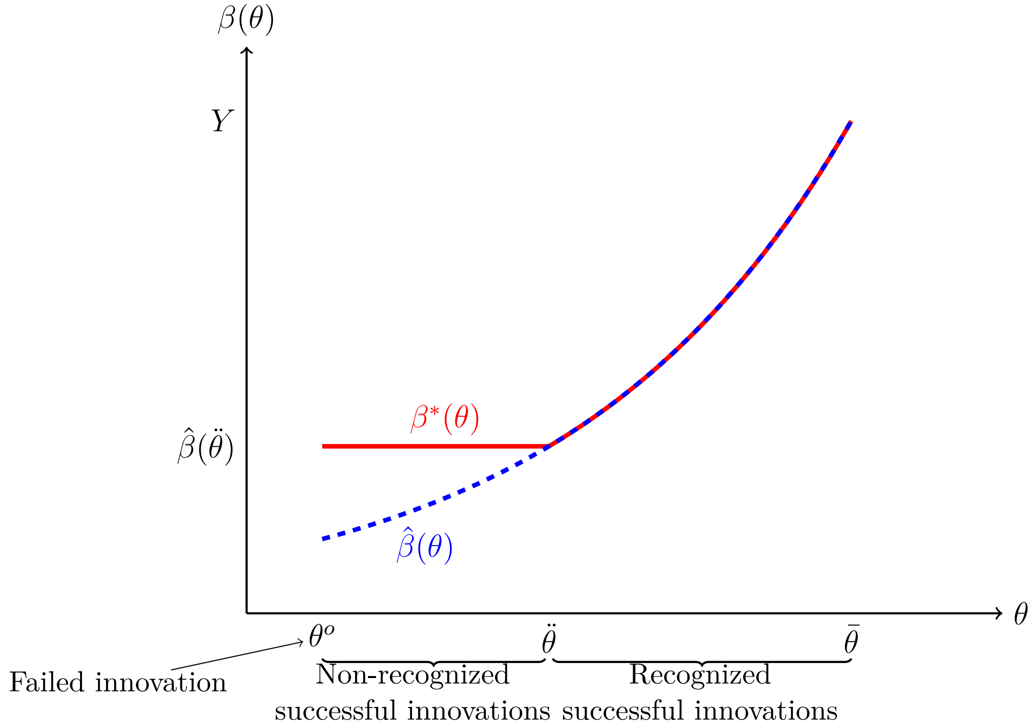
- *$\ddot{\theta}$ is strictly in the interior of $\left[\theta^o, \bar{\theta}\right]$.*
- *$\beta^*(\cdot)$ is continuous everywhere, differentiable everywhere except at $\ddot{\theta}$, and strictly increasing for $\theta > \ddot{\theta}$; and $\beta^*(\bar{\theta}) = Y$.*
- *$\alpha^*(\cdot)$ is continuous everywhere, differentiable everywhere except at $\ddot{\theta}$, and strictly decreasing for $\theta > \ddot{\theta}$.*

Under Assumption 3, the wage schedule $\alpha^*(\cdot)$ in (15) is strictly positive for all $\theta \in \left[\theta^o, \bar{\theta}\right]$ and hence satisfies (LL-2); Proposition 2 thus characterizes the optimal period-2 menu.

The form of the optimal bonus schedule $\beta^*(\cdot)$ is illustrated in Figure 1. To understand it, suppose for the moment that the principal is maximizing only the part of $\int_{\theta^o}^{\bar{\theta}} \phi\left(\beta(\theta), \theta\right) f(\theta) d\theta$ in the objective in (11), which is the expected payoff from the agents with a successful innovation. The optimal schedule for this problem is $\hat{\beta}(\cdot)$ in (13), which represents $\phi_\beta\left(\hat{\beta}(\theta), \theta\right) = 0$. The corresponding incentive compatible fixed wage schedule is $\hat{\alpha}(\theta) = U^0 - \int_{\theta^o}^{\theta} u_\theta\left(\hat{\beta}(t), t\right) dt - u\left(\hat{\beta}(\theta), \theta\right)$ (from (IC-2A) in Lemma 2.1). Under $\left\{\hat{\alpha}(\cdot), \hat{\beta}(\cdot)\right\}$, there is inefficiency for all types other than $\bar{\theta}$.[26] This is the consequence of the standard trade-off between efficiency and information rent provision in screening problems. The term $s_\beta\left(\hat{\beta}(\theta), \theta\right)$

[26] The first-best incentive is always $\beta = Y$ since it induces the first-best effort level for each θ.

Figure 1: Optimal bonus schedule $\beta^*(\cdot)$



The blue dotted graph represents $\hat{\beta}(\cdot)$. The red bold graph represents the optimal bonus schedule $\beta^*(\cdot)$; the schedule is flat for $\theta \in \left[\theta^o, \ddot{\theta}\right]$ and strictly increasing for $\theta > \ddot{\theta}$.

in (13) represents the marginal gain in surplus from type θ from increasing his bonus; the term $\varepsilon_\theta \left(\hat{\beta}(\theta), \theta\right) H(\theta)$ represents the marginal increase in information rent provision from doing so.

To elaborate on the trade-off, holding fixed a bonus level, the agent's optimal effort choice, which determines his probability of getting his bonus, is positively correlated to the level of his innovation success θ. Hence an agent with a low θ gets the bonus with a lower probability and thus has to be given a higher fixed wage to satisfy his participation constraint. This creates incentives for the agent to under-report his type. The principal counters this by lowering the bonus of the agents with low θ; doing so makes these contracts less attractive to the agents with higher types and thus reduces the rent given to them. As a result, $\hat{\beta}(\cdot)$ is upward-sloping.

Next, suppose now that the principal is instead maximizing only the part of $s\left(\beta(\theta^o), \theta^o\right) F\left(\theta^o\right)$ in the objective function in (11), which is the expected payoff from the failed innovations. The solution to this problem is to set the bonus at the first-best level $\beta^o = Y$, and the fixed transfer at $\alpha^o = U^0 - s(Y, \theta^o)$. This is equivalent to making the failed-innovator agents the residual claimant while ensuring that their participation constraint binds.

In the presence of both failed and successful innovations – that is, maximizing the full objective in (11) – the principal cannot offer the contract schedule $\left\{\hat{\alpha}(\cdot), \hat{\beta}(\cdot)\right\}$ together with the contract (α^o, β^o). This is because such a set of contracts violates (IC-2B) of Lemma 2.2: any agent with a successful innovation will take contract (α^o, β^o) instead of the contract meant for him.[27]

Consider giving the failed-innovator type θ^o a bonus of $\ddot{\beta} > \hat{\beta}(\theta^o)$ instead. Let $\ddot{\theta} > \theta^o$ be the type in which $\hat{\beta}\left(\ddot{\theta}\right) = \ddot{\beta}$. From the discussion above, the bonus for any type $\theta \in (\theta^o, \ddot{\theta})$ must also be raised to be at least $\ddot{\beta}$ to satisfy (IC-2B). This implies that when maximizing the expected payoff from the successful innovations, there is now the additional constraint that the bonus schedule must be everywhere above $\ddot{\beta}$. A $\ddot{\beta}$ that is higher than $\hat{\beta}(\theta^o)$ creates an additional loss to the principal due to her inability to lower the information rent as necessary according to $\hat{\beta}(\cdot)$. On the other hand, whenever $\ddot{\beta}$ is below Y, the principal faces a loss from not being able to maximize the expected payoff from the agents who have a failed innovation.

Hence the principal trades off between maximizing the expected payoff from the successful innovations with that from the failed innovations by optimally choosing $\ddot{\beta}$; this trade-off is reflected in (14). The first term $\int_{\theta^o}^{\ddot{\theta}} \phi_\beta\left(\hat{\beta}(\ddot{\theta}), \theta\right) f(\theta) d\theta$ represents the marginal gain in the principal's expected payoff from distorting the bonus of types in $\left(\theta^o, \ddot{\theta}\right]$ to below the level of $\hat{\beta}(\ddot{\theta})$; the marginal cost of this distortion on the payoff from the failed innovations is the second term in (14), $s_\beta\left(\hat{\beta}(\ddot{\theta}), \theta^o\right) F(\theta^o)$. Because $F(\theta^o)$, the probability of having a failed innovation, is strictly positive, this marginal cost is strictly positive as well, which then implies that the optimal $\ddot{\beta}$ will always be higher than $\hat{\beta}(\theta^o)$. This thus creates a "bunching" region where all types in $\left[\theta^o, \ddot{\theta}\right]$ receive the same incentive contract. The implication is that only the more successful innovations $\theta \in \left(\ddot{\theta}, \bar{\theta}\right]$ are "recognized" and distinguished with increasingly powerful incentive contracts; the mildly successful innovations $\theta \in \left(\theta^o, \ddot{\theta}\right]$ are "not recognized" and given the same incentive contracts as if their innovation was a failure.

Remark 2. The principal's period-2 payoff in the exploration equilibrium under FW is always higher than her period-2 payoff in the exploitation equilibrium under TM. This is because her period-2 payoff under TM is $s(Y, \theta^o)$ (see Proposition 1), while program $\mathcal{P}$ has a feasible

[27]If an agent of type $\theta > \theta^o$ chooses his contract $\left(\hat{\alpha}(\theta), \hat{\beta}(\theta)\right)$, he earns a rent of $\int_{\theta^o}^{\theta} u_\theta\left(\hat{\beta}(t), t\right) dt$. If he takes contract (α^o, β^o) instead, he earns a rent of $\int_{\theta^o}^{\theta} u_\theta(Y, t)\, dt$. Since $\hat{\beta}(t) < Y\ \forall t < \bar{\theta}$, this is higher than $\int_{\theta^o}^{\theta} u_\theta\left(\hat{\beta}(t), t\right) dt$.

but not optimal solution $\beta(\theta) = Y$ and $\alpha(\theta) = U^0 - s(Y, \theta^o)$ $\forall\theta$ which achieves the payoff $s(Y, \theta^o)$.

5.1.4 On-path and Off-path Payoffs of the Agent.

Let R be the agent's expected period-2 information rent (net of outside option) for choosing n in period 1. R is given by:

$$R = E\left[\bar{U}(\theta)\right] - U^0 = \int_{\theta^o}^{\bar{\theta}} \left(\int_{\theta^o}^{\theta} u_\theta\left(\beta^*(t), t\right) dt \right) dF(\theta)$$
$$= \int_{\theta^o}^{\bar{\theta}} u_\theta\left(\beta^*(\theta), \theta\right) \left[1 - F(\theta)\right] d\theta \quad > 0. \tag{16}$$

If the agent deviates to o in period 1, he will not know θ^n. Since $\theta^o \geq E\left[\theta^n\right]$, by the same argument in (1) to (3), the deviating agent will use o in period 2. This implies that the deviating agent is of type θ^o, and $\bar{U}(\theta^o) = U^0$.

Corollary 1. *If the agent uses technology n in period 1, his expected period-2 payoff is $R + U^0$. If he deviates to using technology o in period 1, his expected period-2 payoff is U^0.*

Remark 3. Because the agent reverts to o whenever the experimentation outcome is such that $\theta^n \leq \theta^o$, the distribution of θ^n below θ^o does not have any substantial effect on the analysis here. Hence all results follow through even if the regularity conditions and the decreasing inverse hazard rate ratio property on the distribution of θ^n, as stated in Assumption 2, hold only for $\theta^n \in \left(\theta^o, \bar{\theta}\right]$.

Remark 4. Corollary 1 will continue to hold if the agent's choice of effort and/or technology is also observed by the principal but is not contractible. The only difference is that when the agent deviates to choosing $\tau = o$ in period 1, the principal will know about it; instead of offering the menu of contracts in Proposition 2 in period 2 in this case, the principal will offer the contract $\beta = Y$ and $\alpha = U^0 - s(Y, \theta^o)$ which still gives the agent an expected period-2 payoff of U^0.

5.2 Period 1

This subsection completes the equilibrium characterization by deriving the optimal period-1 contract. Under a period-1 contract (a, b), the agent's total expected payoffs for using n

with period-1 effort e is $a + W^n(e,b)$, where:

$$W^n(e,b) := eb - E\left[\psi\left(\frac{e}{\theta^n}\right)\right] + R + U^0. \tag{17}$$

His total expected payoffs for deviating to o is $a + W^o(e,b)$ where:

$$W^o(e,b) := eb - \psi\left(\frac{e}{\theta^o}\right) + U^0. \tag{18}$$

For $\tau \in \{o, n\}$, let:

$$\bar{W}^\tau(b) \quad := \quad \max_e \; W^\tau(e,b) \tag{19}$$

$$e^\tau(b) \quad := \quad \arg\max_e \; W^\tau(e,b) \tag{20}$$

Lemma 3. *$e^\tau(b)$ is strictly increasing for both $\tau \in \{o, n\}$, and $e^o(b) > e^n(b)$ $\forall b > 0$. There exists a bonus threshold $\bar{b} > 0$ such that $\bar{W}^n(b) \geq \bar{W}^o(b)$ if and only if $b \leq \bar{b}$. The threshold $\bar{b}$ is non-decreasing in R.*

The first property of $e^\tau(\cdot)$ is intuitive: higher bonus induces higher effort. At any bonus b, the agent chooses higher effort under o than under n since o is ex-ante more cost-efficient in expectation. To understand the property of $\bar{W}^\tau(\cdot)$, observe that in deciding which technology to use in period 1, the agent trades off between the payoffs across the two periods. $R > 0$ implies that n maximizes his period-2 payoff while o, being more cost-efficient ex-ante, maximizes his period-1 payoff. Hence, if the principal makes the period-1 incentives too powerful, the agent's total payoffs from using o in period 1 will be higher than that from choosing n .

The following provides the definition for an exploration equilibrium:

Definition 1. A period-1 contract (a, b), together with the agent using technology n with effort e^* in period 1, forms an equilibrium if the following conditions are satisfied:

$$a + \bar{W}^n(b) \geq U^0 \tag{IR-1}$$

$$\bar{W}^n(b) \geq \bar{W}^o(b) \tag{IC-τ-1}$$

$$e^n(b) = e^* \tag{IC-e-1}$$

$$a, b \geq 0 \tag{LL-1}$$

(IR-1) is the agent's participation constraint where the right hand side is the sum of the outside options across the two periods. (IC-τ-1) is the agent's technology choice incentive compatibility constraint, (IC-e-1) is his effort choice incentive compatibility constraint, and (LL-1) is his limited liability constraint.

Proposition 3. *Under FW, the optimal period-1 contract that induces the agent to choose technology n in period 1 is unique:*

$$\begin{aligned} a^* &= 0 \\ b^* &= \min\left\{\hat{b}, \bar{b}\right\} \end{aligned}$$

where $\hat{b} := \arg\max_{b} \{e^n(b)\,[Y - b]\}$.

If the agent always uses n in period 1, the principal's maximized period-1 payoff subject to constraints (IR-1) and (IC-e-1) is achieved by providing bonus level $\hat{b}$ in Proposition 3. Under Lemma 3, (IC-τ-1) is satisfied if and only if $\hat{b} \leq \bar{b}$. If $\hat{b} > \bar{b}$, then the optimal bonus becomes $\bar{b}$ because the principal's period-1 objective function $e^n(b)\,[Y - b]$ is strictly concave in b.

5.3 Discussion on the Optimal Contracts

5.3.1 Low Incentives in Period 1

Proposition 3 illustrates that low early incentives are required to motivate exploration and this feature has a number of interesting interpretations. First, the bonus in the model reflects the agent's opportunity cost of getting a low output. The requirement for low early incentives thus implies that firms must show tolerance to early failures by their workers in order to encourage them to innovate. This complements Manso's (2011) point that *long-term* contracts that encourages exploration must both tolerate early failures and reward long-term successes.

Second, it illustrates a mechanism, which is in terms of innovation incentives, that explains why workers' incentives tend to be weak in the early stages of their career. The prevalent explanation for this phenomenon is the career concern model (Holmström 1999; Gibbons and Murphy 1992) where low incentives would be sufficient to induce workers to exert effort in their early career due to the reputation concern regarding their ability. In contrast, low early incentives are necessary here to induce the workers to take on risky experimentation instead. This feature where low incentives are sometimes needed to increase the overall surplus has also been noted in other contexts. For example, in the multi-tasking literature (Holmström and Milgrom, 1991), the principal might lower the incentives of one task to create incentives for the agent to work on other more important tasks. The parallel here is not drawn directly on the multi-tasking aspect of the model since the principal cannot give incentives for a different choice of τ; rather, the point is one can view the two periods here as different "tasks", and low incentives in period 1 induce the agent to put emphasis on the period-2 rent, which would then encourage him to experiment on n in period 1.

Finally, the requirement for low period-1 bonus implies that the expected surplus from the principal-agent relationship is also small in period 1. That relationship sometimes has to "start small" is reminiscent of the reputation building literature (e.g. Sobel 1985; Ghosh and Ray 1996; Watson 1999, 2002; Halac 2012).[28] In this literature, there is ex-ante asymmetric information about the players' type and hence, the early stages of relationships are characterized by low stakes while the asymmetric information among the players is gradually resolved. In contrast, information is ex-ante symmetric here, and the players "start small" here to create asymmetric information rather than resolving it.

5.3.2 Cost of Innovation and Share of Innovation Surplus in Period 2

There are three types of costs incurred to innovate. First it involves the use of the ex-ante inefficient technology n; second there is a limit to the period-1 surplus due to the upper bound $\bar{b}$ on the period-1 bonus; third there is a cost due to the asymmetric information in period 2. The first type of cost is an inherent cost to innovate. The second type is a friction which has been discussed above. I discuss the third type of cost now.

In the absence of experimentation, the period-2 social surplus (net of outside options) is $\max_{\varepsilon}\left\{\varepsilon Y-\psi\left(\frac{\varepsilon}{\theta^o}\right)\right\}=s\left(Y,\theta^o\right)$ (see Proposition 1); $s\left(Y,\theta^o\right)$ thus acts as the benchmark surplus. With experimentation, under the optimal menu of contracts, the social surplus (net of outside options) generated from innovation θ is $s\left(\beta^*(\theta),\theta\right)$. Hence the *social innovation surplus of* θ is defined as:

$$Innov(\theta):=s\left(\beta^*(\theta),\theta\right)-s\left(Y,\theta^o\right),$$

while the ex-ante *expected social innovation surplus* is defined as:

$$Innov \;:=\; E\left[Innov(\theta)\right] \;=\; \int_{\theta^o}^{\bar{\theta}} s\left(\beta^*(\theta),\theta\right)dF(\theta) \;-\; s\left(Y,\theta^o\right).$$

[28]The term "start small" is taken from Watson (1999, 2002).

The *agent's innovation surplus of* θ is defined as his period-2 rent (net of outside option) under the optimal menu of contracts, which is $\bar{U}(\theta) - U^0 = \int_{\theta^o}^{\theta} u_\theta\left(\beta^*(t), t\right) dt$.

Proposition 4. *$Innov(\theta^o) < 0$ and $Innov > 0$. The agent's innovation surplus is strictly positive for all $\theta > \theta^o$, and it is strictly increasing and strictly convex in θ.*

Although innovation is socially beneficial in expectation ex-ante (i.e. $Innov > 0$), innovation failure is always costly (i.e. $Innov(\theta^o) < 0$), and the agency costs involved implies that a successful innovation is socially beneficial ex-post only if it turns out to be sufficiently successful. This is seen by noticing that $Innov(\theta)$ is negative when θ is only slightly above θ^o.

On the other hand, the agent's innovation surplus is never negative; it is strictly positive whenever he has a successful innovation (i.e. $\theta^n > \theta^o$), and it is zero when he fails to innovate (i.e. $\theta^n \leq \theta^o$). This implies that the principal subsidizes the agent when the innovations are only mildly successful (i.e. θ^n is slightly above θ^o); in return the principal is able to extract a larger share of the social innovation surplus when the innovation turns out to be highly successful.

It is intuitive that the agent's innovation surplus is increasing in his innovation success. What is less obvious is that this gain is convex. In practice, it is difficult to distinguish the degree of innovation success from its monetary payoff to the innovator, since the latter is often used as proxy measure for the former. However there are suggestive evidences for the presence of this convexity feature in gains related to innovation (see Trajtenberg, 1990; Hall et al., 2005).[29] In this model, the convexity feature arises from the principal's solution to the screening problem. This suggests that the high concentration of value at the upper-end of the innovation spectrum is attributed to users trying to discern very good innovations from the rest, instead of reflecting the true values of the innovations to the users.

6 Tight-Monitoring Versus Freedom-at-Work

This section studies how the performance of the two monitoring environments are affected by the innovation potential (i.e. distribution of θ^n) and the efficiency of the existing technology

[29] Trajtenberg (1990) uses patent citation counts as the measure of innovation success and find that the marginal value of patent citation is increasing. Hall et al. (2005) examine data on patent citations and market value, and they find that the market-value premium is disproportionately concentrated at highly-cited patents. For a more anecdotal form of support, the *Wall Street Journal* also reports that almost all of the profits from venture-backed start-ups come from only 25% of all the venture-backed companies, although all the companies almost always have created something new (see "The Venture Capital Secret", September 20 2012, The Wall Street Journal).

(i.e. the θ^o value). The results here speak to why and when would different firms adopt different monitoring environments in practice.[30]

Throughout this section, whenever unspecified, all payoffs will refer to *ex-ante* expected payoffs. In addition, the contracts and the payoffs under FW will refer to the contracts and the resulting payoffs of the principal-optimal exploration equilibrium under FW as characterized in Propositions 2 and 3. Likewise, the contracts and the payoffs under TM will refer to the contracts and the resulting payoffs of the unique exploitation equilibrium under TM in Proposition 1. The results of this section are summarized in Table 2 below.

6.1 Innovation Potential: The Distribution of θ^n

Under TM, the contracts and payoffs of both players are independent of the distribution of θ^n. Hence, I only consider the effects of the distribution of θ^n under FW. To do so, I first parametrize the set of distributions of θ^n by a parameter k. Let $F(\cdot;k)$ be a distribution on $\left[\underline{\theta}, \bar{\theta}(k)\right]$, and let $\mathcal{F} := \{F(\cdot;k)\}$ denote a family of such distributions.

Assumption 4. *$\mathcal{F}$ satisfies the following assumptions:*

1. *For all $F(\cdot;k) \in \mathcal{F}$, the distribution $F(\cdot;k)$ has a density function $f(\cdot;k)$ that satisfies Assumption 2.*
2. *$k'' > k'$ implies that $\bar{\theta}(k'') \geq \bar{\theta}(k')$ and $H(\theta^n;k'') > H(\theta^n;k')$ $\forall \theta^n \in \left[\underline{\theta}, \bar{\theta}(k')\right]$.*
3. *For any $\theta^n \in \left[\underline{\theta}, \bar{\theta}(k)\right)$, $f(\theta^n;k)$ is absolutely continuous in k.*

Assumption 4.1 implies that all the results from Section 5 apply. Assumption 4.2 implies that the parameter k orders the distributions in terms of both the upper bound of the support and the inverse hazard rate ratio point-wise, while the lower bound is fixed within the family of distributions. Assumption 4.3 is a regularity condition on the family of distributions. An example of $\mathcal{F}$ is the *family of uniform distributions* in which θ^n is uniformly distributed over $[\underline{\theta}, k]$:

$$f(\theta^n;k) = \begin{cases} \frac{1}{k-\underline{\theta}} & , \ \theta^n \in [\underline{\theta}, k] \\ 0 & , \ \theta^n \notin [\underline{\theta}, k] \end{cases} \tag{21}$$

[30] Another comparative static that can be readily carried out is on the players' discount factor. Recall that the principal's period-1 payoff is higher under TM than FW since o is ex-ante more cost-efficient than n, while her period-2 payoff is higher under FW than TM (Remark 2). Moreover, when the agent's period-2 rent increases under FW, the upper bound $\bar{b}$ on the period-1 bonus also increases, which then implies that the principal is free to offer better period-1 incentives if necessary. Hence, when the players' weight on the period-2 payoffs increases, FW becomes more likely to be favored over TM , and vice-versa.

It is well-known that inverse hazard rate dominance implies first-order stochastic dominance under common support; this relationship remains when the dominant distribution has a higher upper bound:

Remark 5. Under Assumption 4, $k'' > k'$ implies that $F(\theta^n; k'') < F(\theta^n; k')\ \forall \theta^n \in \left[\underline{\theta}, \bar{\theta}(k'')\right)$.[31]

Under a family of distributions $\mathcal{F}$ that satisfies Assumption 4, a higher k implies that the distribution (i) has a higher upper bound, (ii) is more concentrated at more successful innovations (first-order stochastic dominance from Remark 5), and (iii) has less risk (second-order stochastic dominance). k thus provides a measure of the ex-ante innovation potential of the innovative activity n, and a higher k implies that the innovative activity has more potential ex-ante.

I develop some comparative statics on the optimal contracts and payoffs with respect to k next. The following notations will be useful in explaining the comparative statics. Let:

$$V\Big(\beta(\cdot); k\Big) := \int_{\theta^o}^{\bar{\theta}(k)} v\Big(\beta(\cdot), \theta\Big) dF(\theta; k); \tag{22}$$

let the optimal bonus schedule associated with distribution $F(\cdot; k) \in \mathcal{F}$ be:

$$\beta^*(\cdot; k) \quad := \quad \underset{\beta(\cdot)\in\mathcal{B}(k)}{\arg\max}\ V\Big(\beta(\cdot); k\Big), \tag{23}$$

where $\mathcal{B}(k)$ is the set of non-decreasing functions mapping from domain $\left[\theta^o, \bar{\theta}(k)\right]$ to $[0, \infty)$. The principal's maximized period-2 payoff is thus:

$$\mathcal{V}(k) := V\Big(\beta^*(\cdot; k); k\Big), \tag{24}$$

and the agent's expected period-2 rent is, from (16):

$$R(k) := \int_{\theta^o}^{\bar{\theta}(k)} u_\theta\left(\beta^*(\theta; k), \theta\right)\left[1 - F(\theta; k)\right] d\theta.$$

In addition, let $\hat{b}(k)$ and $\bar{b}(k)$ respectively be the bonus $\hat{b}$ and threshold $\bar{b}$, characterized in the optimal period-1 contract in Proposition 3, that are associated with distribution $F(\cdot; k)$.

Proposition 5. *Suppose that $k'' > k'$. Under FW:*

1. *(Period-2 Bonus Schedule.)* $\beta^*(\theta; k'') < \beta^*(\theta; k')\ \forall \theta \in \left[\theta^o, \bar{\theta}(k')\right]$.[32]

[31] A proof of Remark 5 is provided in the Supplementary Appendix.

[32] $\beta^*(\cdot; k')$ is not defined for $\theta \in \left(\bar{\theta}(k'), \bar{\theta}(k'')\right]$.

2. *(Period-2 Payoff.)* $\mathcal{V}(k'') > \mathcal{V}(k')$.

3. *(Period-1 Payoff.) If* $\hat{b}(k'') \leq \bar{b}(k'')$*, or if* $R(k'') \geq R(k')$*, then the principal's expected period-1 payoff is strictly higher from an innovative activity with potential* k'' *than from one with potential* k'*.*

Proposition 5.1 states that when n has more potential ex-ante, the period-2 bonus offered will be lower. To understand it, recall from (IC-2A) in Lemma 2 that the information rent (net of outside option) for a type-θ agent is the integral of $u_\theta\Big(\beta(t), t\Big)$ over all types t below θ. When the principal further distorts the bonus schedule downward for all types below θ, the rents given to all agents with types above θ are reduced. The downside of doing so is that it lowers the surplus generated by the lower types. However, when k is higher, the density of θ is more concentrated at the higher types. Hence the savings from rent-provision to the higher types outweighs the loss in surplus generated from the lower types, thus making such a downward distortion of the bonus schedule profitable for the principal.

From (IC-2A) in Lemma 2, $\frac{d}{d\theta}\bar{U}(\theta) = u_\theta\Big(\beta^*(\theta; k), \theta\Big)$ whenever the derivative exists, and this is increasing in the bonus. Proposition 5.1 thus also implies that when there is a higher likelihood of more successful innovations, the agent's marginal gain from a better innovation decreases:

Corollary 2. *If the innovative activity has more potential ex-ante, the agent's marginal gain from having a better innovation decreases.*

With regards to the principal's period-2 payoff (Proposition 5.2), the effect from the change in the bonus schedule due to a change in k (as described in Proposition 5.1) is of only a second-order magnitude at the optimum. Instead, the first-order effect comes from the change in distribution of θ. It is verified in the proof for Proposition 5.2 that the surplus generated by a higher θ increases faster than the rent provided (i.e. $v\left(\beta(\cdot), \theta\right)$ is increasing in θ). Hence a higher k, which implies higher density at more successful innovations, results in a higher period-2 payoff for the principal.

As for period 1 (Proposition 5.3), a higher k also implies that the ex-ante cost efficiency of n, which is the technology used in period 1 under FW, is higher. Under any positive period-1 bonus, the agent optimally exerts more effort on a n with a higher k; this in turn leads to a higher period-1 payoff for the principal. In the absence of any restriction on the period-1 bonus that motivates the agent to use n in period 1, her period-1 payoff is always increasing in the ex-ante efficiency of n. This explains why when $\hat{b}(k) \leq \bar{b}(k)$, the principal's period-1 payoff is increasing in k

However, if the bonus threshold is binding (i.e. $\hat{b}(k) > \bar{b}(k)$), the principal's period-1 payoff can depend on how k affects the bonus threshold $\bar{b}(k)$. In general, the bonus threshold is increasing in both the ex-ante efficiency of n and the agent's expected rent (see Lemma 3). Hence, if $R(k)$ is also increasing in k, the bonus threshold will also be increasing in k, which in turn implies that a n that is more efficient ex-ante also has a less restrictive bonus threshold. In this case, the principal's period-1 payoff will be increasing in k, which then explains the second part of Proposition 5.3.[33] Corollary 3 provides a particular parametrization of the model in which $R(k)$ can be shown to be always increasing in k; its proof is found in the Supplementary Appendix.

Corollary 3. *Suppose that $\mathcal{F}$ is the family of uniform distributions in (21) and $\psi(x) = \xi x^2$ where $\xi > 0$ is sufficiently large such that $\arg\max_e \left\{eY - \psi\left(e/\bar{\theta}(k)\right)\right\} < 1$ $\forall k$.*[34] *Then $R(k)$ is strictly increasing, and the principal's total expected payoffs under FW is strictly increasing in the ex-ante innovation potential of the innovative activity.*

From Proposition 5.2-5.3, if $R(k)$ is increasing – such as in the case of quadratic cost function with uniform distribution in Corollary 3 – the principal's total payoffs under FW are strictly increasing in k. Since her payoffs under TM are independent of k, the results thus provide a prediction for the choice of monitoring environment as a function of k, as summarized in Corollary 4:

Corollary 4. *Suppose that $R(k)$ is increasing. Then there exists a threshold potential k^* for the innovative activity n such that the principal prefers FW to TM if and only if $k > k^*$.*

6.2 Efficiency of Existing Technology: The Value of θ^o

Next, I consider how the contracts and payoffs change with θ^o while holding the distribution of θ^n fixed. For an efficiency level of θ^o, let $\beta^*(\cdot|\theta^o)$ be the associated optimal period-2 bonus schedule $\beta^*(\cdot)$ as characterized in Proposition 2, let $\hat{\beta}(\cdot|\theta^o)$ be the associated bonus schedule $\hat{\beta}(\cdot)$ as characterized in (13), and let $\ddot{\theta}(\theta^o)$ be the associated type $\ddot{\theta}$ as characterized in (14).

[33] It is not clear how $R(k)$ changes with k in general. If a higher k were to lower $R(k)$ significantly, then $\bar{b}(k)$ can be decreasing in k. In turn, if this decrease is also significant, the more restrictive $\bar{b}(k)$ can then cause the principal's period-1 payoff to be decreasing in k as well. Numerical examples however suggest that such a phenomenon would not happen; although $\bar{b}(k)$ tends to bind for high values of k (i.e. $\bar{b}(k) < \hat{b}(k)$), $R(k)$ appears to be always increasing in k, which then implies that $\bar{b}(k)$ is always non-decreasing in k.

[34] This cost function violates the Inada condition that $\lim_{x\to 1} \psi(x) = \infty$, but since the principal will never offer a bonus greater than Y, the agent's effort choice is still always interior, which in turn ensures that all the results in Section 5 hold.

Proposition 6. *When the efficiency level of technology o increases from $\theta^{o'}$ to $\theta^{o''}$:*

1. *Under TM, the principal's expected payoffs in both periods increase.*[35]

2. *Under FW:*

   (a) $\hat{\beta}(\theta|\theta^{o'}) = \hat{\beta}(\theta|\theta^{o''})\ \forall\theta \in \left[\theta^{o''}, \bar{\theta}\right]$,[36] *while* $\ddot{\theta}\left(\theta^{o''}\right) > \ddot{\theta}\left(\theta^{o'}\right)$. *Hence* $\beta^*\left(\theta^{o''}|\theta^{o''}\right) > \beta^*\left(\theta^{o'}|\theta^{o'}\right)$.

   (b) *The principal's expected period-2 payoff increases.*

Proposition 6.1 is straightforward: since o is used in both periods under TM, when o becomes more efficient, both the offered bonus and the principal's payoffs in both periods become higher.

Proposition 6.2a states that when θ^o is higher, the optimal bonus schedule "bunches" at a higher level, but the bonuses at the "non-bunching" region are unchanged. The latter property is readily seen from condition (13) which illustrates that the schedule $\hat{\beta}(\cdot)$ is independent of the value of θ^o. Intuitively, this is because $\hat{\beta}(\cdot)$ is determined by a *local* trade-off between surplus maximization and information rent minimization. As for the former property, when θ^o is higher, the probability of having a failed innovator (i.e. an agent who draws $\theta^n < \theta^o$) increases. The principal will then optimally increase the "bunching" bonus level because the principal now puts more emphasis on increasing the surplus generated by the failed innovators. Since the failed-innovator type has a mass point, such a change in bonus schedule significantly increases the principal's period-2 payoff, as stated in Proposition 6.2b

Although a higher θ^o increases the principal's period-2 payoff under both types of monitoring environments, the magnitude of increase is always higher under TM (Proposition 7 below). Let $D^{TM}\left(\theta^{o''}, \theta^{o'}\right)$ be the difference in the principal's period-2 payoff under TM when the efficiency of o changes from $\theta^{o'}$ to $\theta^{o''}$. Let $D^{FW}\left(\theta^{o''}, \theta^{o'}\right)$ be the analogous difference in the principal's period-2 payoff under FW.

Proposition 7. *For any* $\theta^{o''} > \theta^{o'}$, $D^{TM}\left(\theta^{o''}, \theta^{o'}\right) > D^{FW}\left(\theta^{o''}, \theta^{o'}\right) > 0$.

Proposition 7 rests on two main observations. First, as just mentioned, the main effect of θ^o on the principal's period-2 payoff under FW is on the surplus from the failed-innovator type agent which occurs with a probability of less than 1. On the other hand, under TM, a

[35] The results are unchanged even if effort is contractible under TM (see Remark 1).

[36] $\hat{\beta}(\theta|\theta^{o''})$ is undefined for $\theta \in \left[\theta^{o'}, \theta^{o''}\right)$.

more efficient o always has an effect since o is used with probability 1 in period 2. Second, under TM, the principal provides first-best incentives for the agent to use o, whereas under FW, when o is used by the agent in the event of an innovation failure, the incentive provided is distorted downward.

The effect of θ^o on period 1 under FW is however ambiguous. If the period-1 bonus threshold does not bind (i.e. $\hat{b} \geq \bar{b}$), the period-1 bonus and the principal's period-1 payoff under FW will be independent of the value of θ^o since n is used instead. However, if the period-1 bonus threshold binds, then the effect on the principal's period-1 payoff depends on how θ^o affects the bonus threshold $\bar{b}$. The ambiguity then arises because it is in general unclear how a higher θ^o affects the agent's expected rent which will in turn affects the threshold: a higher bonus for the lower types implies that the rent of all agents with a successful innovation is higher, but the higher probability of innovation failure implies that the agent is also getting a rent with lower probability.

Table 2: Effect of n with more potential, and a more efficient o

	When n has more potential ($k \uparrow$)			When o becomes more efficient ($\theta^o \uparrow$)		
	Period-2 bonus	Principal's payoff		Period-2 bonus	Principal's payoff	
		Period 2	Period 1		Period 2	Period 1
TM	No change	No change	No change	Higher	Higher	Higher
FW	Lower everywhere	Higher	Higher[a]	Higher for lower types; no change for the higher types	Higher, but by a smaller magnitude than under TM	Ambiguous

[a]Assuming that the conditions in Proposition 5.3 holds; see Corollary 3.

Nevertheless, the principal's period-1 payoff under FW is bounded above by $e^n\left(\hat{b}\right)\left[Y-\hat{b}\right]$ (see Proposition 3) which is strictly lower than her period-1 payoff under TM whenever $E\left[\theta^n\right] \leq \theta^o$. On the other hand, while her period-2 payoff under FW is always higher than that under TM (Remark 2), it is readily verified that as θ^o approaches $\bar{\theta}$, the principal's period-2 payoffs under the two different monitoring environments converge. Hence, when θ^o is sufficiently high, the loss in the principal's period-2 payoff under TM relative to under FW will be offset by the gain in period-1 payoff:

Corollary 5. *There exists $\delta > 0$ such that the principal always prefers TM to FW if $\theta^o >$*

$\bar{\theta} - \delta$.

6.3 Implications on the Choice of Monitoring Environment

The comparative static results in this section suggest that one is more likely to observe a FW type of environment when the firm has a constant need to adapt and improve upon existing work methods. Within industries, young firms (e.g. startup firms) should be more likely than old firms to adopt a FW type of environment as they are still learning about their own operational needs and improving upon the current operating procedures. Across industries, firms whose business models are based on creating new products (e.g. "tech firms") are unlikely to be able to mimic efficient procedures from others or from their own history, and they thus have high gains from their employees constantly trying to find better operating strategies. Similarly, firms in volatile market conditions face a constant need to adapt their procedures to the changing market conditions; firms in such industries should then be expected to adopt a FW type of environment which promotes innovations from their employees. On the other hand, when the gains from improvement from the status quo method are small, the jobs are expected to be carried out under a TM type of environment. These could include mechanical jobs and relatively homogenous businesses where the experience of others can be readily adapted and used.

7 Discussion

7.1 General Monitoring Environments

The key difference between TM and FW lies in the information available to the principal in period 2. In contrast to most principal-agent models, a more informative signal generating process, in the sense of Blackwell (1951), does not necessarily improve the social surplus nor the ex-ante expected payoffs of the principal. This is readily observed by noting that the perfectly informative signal generating process corresponds to TM while the perfectly uninformative one corresponds to FW. This phenomenon arises because of the incompleteness in the contracting environment.

One can consider more general types of monitoring environments by allowing for a more general structure of information flow to the principal post-experimentation. To formalize this idea, suppose now that the principal never observes the agent's effort and technology choices in both periods, and she also does not observe the θ^n realization directly if it is realized. Instead, at the start of period 2, the principal observes a signal $\sigma \in \Sigma$, where Σ is a finite set. If the agent uses n in period 1, the conditional probability distribution of σ given the θ^n realization is $\Pi\left(\sigma \middle| \theta^n\right)$; and if the agent uses o in period 1, the conditional probability distribution of σ is $\Pi\left(\sigma \middle| \theta^o\right)$. Signal σ is also observed by the agent but is non-verifiable by a third party and hence not contractible. A $\{\Sigma, \Pi\}$-pair is thus a general form of monitoring environment.

On the equilibrium path of an exploration equilibrium, upon observing signal σ in period 2, the principal forms a posterior belief about θ^n. Using Bayes theorem, the density of the posterior is:

$$f^\sigma\left(\theta^n\right) = \frac{\Pi\left(\sigma \middle| \theta^n\right) f\left(\theta^n\right)}{\int_{\underline{\theta}}^{\bar{\theta}} \Pi\left(\sigma \middle| t\right) f\left(t\right) dt}, \qquad \forall \theta^n.$$

The principal's screening problem after observing σ will be program $\mathcal{P}$ in (8) but with the distribution replaced by her posterior now.[37] The parties' ex-ante expected payoffs in period 2 is then obtained by taking expectation over the probability distribution of σ.[38]

The choice of $\{\Sigma, \Pi\}$ is a choice of the distribution over the principal's posterior, where each posterior is associated to a set of period-2 payoffs for the two parties. This is an information control problem and is related to the literature on Bayesian persuasion (Kamenica and Gentzkow, 2011). The difference is that in the persuasion literature, the principal chooses an information structure to "persuade" the agent to take an ex-post action in her favor; in contrast, here the principal chooses an information structure that commits herself to take an ex-post action (the bonus offers in period 2) which then influences the agent's actions both ex-ante (period 1) and ex-post (period 2).

This is also reminiscent of the seminal work of Aghion and Tirole (1997) which provides a theory of authority using asymmetric information as the main building block. Aghion and Tirole define "formal authority" as the formal right to make decisions, while "real authority" is having the information to effectively make decisions. Hence the principal trades off between giving up control (ceding formal authority) or committing to have less information (ceding

[37] That is, the distribution of $\theta \in \left[\theta^o, \bar{\theta}\right]$ is $\Pr\left[\theta \leq x\right] = F^\sigma\left(\theta^o\right) + \int_{\theta^o}^{x} f^\sigma(t) dt$, where F^σ is the distribution representing the posterior.

[38] Some of the assumptions made here are without loss of generality. First, the assumption that the agent also observes σ is not important. Even if the agent does not observe σ, the menu offered by the principal, which depends on her now privately known posterior distribution of θ^n, must still satisfy the same set of agent's truth-telling constraints for all realizations of θ^n. Next, when the focus is on exploration equilibrium, what the principal observes when the agent uses $\tau = o$ in period 1 does not matter, because an agent who uses o always earns zero rent regardless of the posterior held by the principal.

real authority), with incentivizing the agent to gather information that improves the quality of decision-making. In the same spirit, this paper uses the information observed by the principal as a cornerstone that determines the size of the surplus generated in the relationship through innovation attempts by the agent, and how it is eventually shared between the principal and the agent.

7.2 Innovative Effort

The degree of innovation success in the baseline model is completely dictated by luck since it is just a draw of θ^n. In reality, a worker should be able to affect the probability of getting an innovation and its degree of success. In the Supplementary Appendix, the baseline model is extended to allow for the agent to privately choose another dimension of effort in period 1 when he uses the innovative technology n. This additional dimension of effort is termed as *innovative effort* i, which is distinct from *productive effort* e and ε. Exerting a higher innovative effort level i will increase both the agent's probability of getting an innovation and its degree of success, but it comes at a cost of decreasing marginal productivity of productive effort in period 1.

The distinction between effort for production and effort for better innovation is motivated by the view that the process of generating output and that of generating innovation are different, and better innovation is often created at the expense of lowering productivity during the innovating stage. For example, when trying out a new machine, a worker might devote time to understanding its mechanics. This takes away time from current production but improves his proficiency in using the new machine in future. The choice of innovative effort i thus studies to what extent is a worker willing to neglect his current main job, which his current pay is dependent on, to focus on coming out with better innovations, to which he might benefit from in the future?

The main result in the extension establishes that the agent can only be induced to exert either zero or maximum innovative effort level. This is because the agent's gain from a better innovation is his expected period-2 rent which is inherently convex in his innovation success level (this is analogous to Proposition 4). The convexity then makes it difficult to strike a balance between incentive provision for productive and innovative efforts. This result suggests an implicit need for specialization in organizations at each stage of their development in which the focus at any one time can only be on either innovation or production, but not both:

> "Both exploration and exploitation are essential for organizations, but they com-

pete for scarce resources. As a result, organizations make explicit and implicit choice between the two" - March (1991), pg. 71.

7.3 Outside Option for Innovation

By having the agent's period-2 outside option being independent of θ^n, the model is implicitly assuming that the worker is unable to take his innovation to another firm and reap the benefits there. For example, the innovation could be firm-specific, or it could be difficult for other firms to verify the innovation. More generally, this assumption is also in line with a firm's need to also protect its own innovations from being stolen by its workers, which implies that the firm has in place a default arrangement that prevents its workers from taking *any* innovation to another firm. In practice, this is often carried out through "assignment of inventions" and "non-compete" clauses in employment contracts.

An "assignment of inventions" clause cedes intellectual property rights of any invention created by the worker during his tenure in the firm to his employer. Cherensky (1993) observes that "*virtually all technical employees agree, as a condition of employment, to assign to the employer all rights to inventions conceived by the employee while at work, or in subject matters related to work, or while using any resources of the employer*". Howell (2012) writes about how such agreements "*have become important and commonplace facets of employment agreements*". On the other hand, a "non-compete" clause forbids employees from leaving the firm to work for a rival or setting up a competing business. The *Economist* reports that about 90% of managerial and technical employees in the United States have signed such "non-compete" agreements, and the classifications of "rival" or "competing" businesses are defined very broadly.[39] These agreements typically have a lifespan of up to two but can go as many as five years.

8 Related Literature

This paper is related to the ratchet effect literature. Freixas et al. (1985) and Laffont and Tirole (1987, 1988) are early theoretical contributions on the ratchet effect. In the labor literature, the ratchet effect arises as the workers rationally restricting output as they expect their employers to respond to higher performances with more demanding requirements later (Lazear, 1986; Gibbons, 1987). Kanemoto and MacLeod (1992) and Carmichael and

[39]See "*Ties that bind*", Dec 14th 2013, The *Economist*.

MacLeod (2000) identify market conditions that mitigate the ratchet effect in labor markets. In this paper, instead of engaging in output restriction, the ratchet effect concerns the worker (not) choosing the principal's desired technology. More substantially, the ratchet effect literature assumes that the agent has ex-ante superior information over the principal and studies the principal's problem due to short-term contracting; in contrast, information is ex-ante symmetric in this paper. Two notable exceptions are Bhaskar (2014) and Bhaskar and Mailath (2015) who also identify a ratchet effect under ex-ante symmetric uncertainty. However, the ratchet effect in these two papers arises from the agent shirking and gaining rent from the principal holding wrong (in particular, more pessimistic) beliefs about the underlying state, which is different from here. Moreover, while the principal suffers from the agent's ability to create differential beliefs in Bhaskar (2014) and Bhaskar and Mailath (2015), the principal can benefit here from the agent having superior information over the underlying state as illustrated under FW.

The theme of strategic ignorance where the principal can benefit from less information on the agent's actions, which is at the core of this paper, has been previously noted in the literature in various contexts (e.g. Riordan, 1990; Crémer, 1995; Aghion and Tirole, 1997; Fingleton and Raith, 2005; Prat, 2005; Taylor and Yildirim, 2011; Shin and Strausz, 2014).[40] However, this paper differs from the above mentioned paper in two aspects. First, the mechanism is different – reduced monitoring here serves as a commitment device for the principal to share the innovation surplus with the agent. Second, the notion of strategic ignorance is stronger here in the sense that the principal also has to appropriately design

[40] Riordan (1990) considers vertical relationships and illustrates how incentives for the supplier to make buyer-specific investments diminish when the two are integrated. Crémer (1995) highlights the benefit of an "arm's length relationship" where the principal does not observe the agent's intrinsic ability and effort choice; in doing so, the principal's threat of firing the agent after a low performance becomes credible which then deters the agent from shirking. Aghion and Tirole (1997) show that a principal can provide better incentives for information acquisition for effective decision-making by committing not to observe the information collected by an agent with mis-aligned preference on project choice. Fingleton and Raith (2005) considers how observing the actions of the agent who bargains on behalf of the principal can lead to overly aggressive bargaining behavior by the agent in order to signal his bargaining ability. Prat (2005) shows that when the principal observes the agent's action directly, the agent who has career concerns might disregard his useful information and acts in a conformist way to mimic a good agent to gain reputation, thus hurting the principal. Taylor and Yildirim (2011) show that the principal can provide better incentives for the agent to improve the quality of the project if she can commit not to observe the agent's ability (i.e. "blind review" on the project quality). Shin and Strausz (2014) consider a setting in which the agent has ex-ante private information and the principal is unable to elicit the agent's information in period 1 due to her inability to enforce long-term contracts; they then show that by allowing the agent to accrue private information on another dimension, the inability to elicit information in period 1 can be alleviated, and this can benefit the principal if early information revelation is important.

the incentive structure for the agent to generate information asymmetry against herself.[41,42]

This paper is also related to the strategic experimentation literature. Bergemann and Hege (1998, 2005), Hörner and Samuelson (2013), and Halac et al. (2016) (among others) study optimal contracting on the experimentation outcome to solve the moral hazard problem. In this literature, the uncertainty and learning are typically modeled on the productivity; whereas here, they are modeled on the cost of effort instead. This is a deliberate choice and it highlights the difference in focus. A model here that is more in line with this literature would be to suppose that the probability of getting Y in period 1 with n is $p^n e$, where p^n is an ex-ante unknown productivity parameter to be learned via the experimentation. Under such a model specification, the principal's posterior on p^n under FW will depend on the realization of y in period 1. Two substantial issues then arise. First the realization of y in period 1 now carries information about the experimentation outcome since it is affected by the underlying value of p^n. Outcome y being contractible then implies that the experimentation outcome is at least partially contractible. However, the focus in this paper is on motivating experimentation when the experimentation outcome is totally uncontractible. The second issue is a technical one resulting from the agent's ability to affect y in period 1 through his effort e. The types of private beliefs, and hence the off-equilibrium payoffs, that can be generated would complicate the analysis significantly and shroud the main points.[43]

The way in which the tension between experimentation and forgoing current productivity is incorporated in a two-armed bandit problem here is similar to Manso (2011) (see also Ederer, 2013; Ferreira et al., 2014). Besides the difference in perspective on the contractibility of the experimentation outcome just mentioned, this paper also differs from Manso (2011) in terms of how the agent derives incentives to innovate. In Manso (2011), the agent's incentives for innovation are hard; they come from the principal's ability to write long-term contracts or from the rent derived due to the agent's limited liability. In this paper, the incentives are soft; it is a potential information rent derived from the asymmetric information that the

[41]Information asymmetry is absent in models of career concerns which thus further stand in contrast to this paper. The model of Ortner and Chassang (2015) also features the principal benefiting from generating asymmetric information, but the asymmetric information is generated between the agents to make collusion between them harder; in contrast, the principal benefits from generating asymmetric information against herself here.

[42]The strategic ignorance theme is also reminiscent of the highly influential work by Bernheim and Whinston (1998). They show that when some aspect of performance is not contractible, it can be optimal to also not contract on some verifiable aspects of the performance. Their focus is on optimal incompleteness in contracts – a phenomenon they call "strategic ambiguity" – whereas this paper focuses on optimal (un)observability of the agent's actions.

[43]A simple version of this issue arises in an extension with innovative effort which is modeled in the Supplementary Appendix.

principal generates against herself.

More broadly, the screening problem in period 2 in this paper also concerns the effects of the information structure on the optimal contracts in the principal-agent framework. Early works by Lewis and Sappington (1991, 1993) and Sobel (1993) study the principal's preference over the amount of information possessed by the agent. Subsequent contributions by Crémer and Khalil (1992), Lewis and Sappington (1997), Crémer et al. (1998a,b), Szalay (2005, 2009) and Iossa and Martimort (2015) consider the agent's incentives to acquire information. This literature takes the agent's cost of information acquisition as given and considers writing a menu of contracts that affects the agent's incentives to gather information. In contrast, the menu of contracts in this paper *cannot* be used to affect the agent's information acquisition incentives (i.e. incentives to experiment on n) since the menu is offered only after the information acquisition process in period 1; instead the principal influences the agent's information acquisition incentives via the monitoring environment, which affects the agent's expected gain from information acquisition, and via the period-1 contract, which determines his opportunity cost of information acquisition.

9 Conclusion

This paper studies motivating innovation when the innovation is not contractible ex-ante and the firm is unable to commit to future rewards. I illustrate how the monitoring environment affects a worker's incentives to innovate and improve the efficiency inside the firm, and I characterize the incentive structure required to complement the monitoring environment. By allowing its workers to accrue private information about their innovation, the firm is compelled to share any innovation surplus with its workers via an information rent. This helps to explain why workers perform better when given more freedom at work, and how workplaces are sometimes organized to indirectly distribute rents to the workers.

References

Aghion, P., E. Fehr, R. Holden, and T. Wilkening (2015). The role of bounded rationality and imperfect information in subgame perfect implementation: An empirical investigation.

Aghion, P., D. Fudenberg, R. Holden, T. Kunimoto, and O. Tercieux (2012). Subgame-perfect implementation under information perturbations*. *The Quarterly Journal of Economics 127*(4), 1843–1881.

Aghion, P. and J. Tirole (1994). The management of innovation. *The Quarterly Journal of Economics*, 1185–1209.

Aghion, P. and J. Tirole (1997). Formal and real authority in organizations. *The Journal of Political Economy 105*(1), 1–29.

Akerlof, G. A. (1970). The market for "lemons": Quality uncertainty and the market mechanism. *The Quarterly Journal of Economics 84*(3), 488–500.

Alonso, R. and N. Matouschek (2008). Optimal delegation. *The Review of Economic Studies 75*(1), 259–293.

Amabile, T. M., R. Conti, H. Coon, J. Lazenby, and M. Herron (1996). Assessing the work environment for creativity. *Academy of Management Journal 39*(5), 1154–1184.

Amabile, T. M., P. Goldfarb, and S. C. Brackfleld (1990). Social influences on creativity: Evaluation, coaction, and surveillance. *Creativity Research Journal 3*(1), 6–21.

Amador, M. and K. Bagwell (2013). The theory of optimal delegation with an application to tariff caps. *Econometrica 81*(4), 1541–1599.

Andrews, F. M. and G. F. Farris (1967). Supervisory practices and innovation in scientific teams. *Personnel Psychology 20*(4), 497–515.

Bénabou, R. and J. Tirole (2003). Intrinsic and extrinsic motivation. *Review of Economic Studies 70*(3), 489–520.

Bergemann, D. and U. Hege (1998). Venture capital financing, moral hazard, and learning. *Journal of Banking & Finance 22*(6-8), 703–735.

Bergemann, D. and U. Hege (2005). The financing of innovation: Learning and stopping. *RAND Journal of Economics 36*, pp–719.

Bernheim, B. D. and M. D. Whinston (1998). Incomplete contracts and strategic ambiguity. *American Economic Review*, 902–932.

Bhaskar, V. (2014). The ratchet effect re-examined: A learning perspective.

Bhaskar, V. and G. J. Mailath (2015). The curse of long horizons.

Blackwell, D. (1951). Comparison of experiments. In *Proceedings of the Second Berkeley Symposium on Mathematical Statistics and Probability*, Volume 1, pp. 93–102. University of California Press.

Bloom, N., J. Liang, J. Roberts, and Z. J. Ying (2015). Does working from home work? evidence from a chinese experiment. *The Quarterly Journal of Economics 130*(1), 165–218.

Carmichael, H. L. and W. B. MacLeod (2000). Worker cooperation and the ratchet effect. *Journal of Labor Economics 18*(1), 1–19.

Che, Y.-K. and D. B. Hausch (1999). Cooperative investments and the value of contracting. *American Economic Review*, 125–147.

Cherensky, S. (1993). A penny for their thoughts: Employee-inventors, preinvention assignment agreements, property, and personhood. *California Law Review*, 595–669.

Crémer, J. (1995). Arm's length relationships. *The Quarterly Journal of Economics 110*(2), 275–295.

Crémer, J. and F. Khalil (1992). Gathering information before signing a contract. *The American Economic Review*, 566–578.

Crémer, J., F. Khalil, and J.-C. Rochet (1998a). Contracts and productive information gathering. *Games and Economic Behavior 2*(25), 174–193.

Crémer, J., F. Khalil, and J.-C. Rochet (1998b). Strategic information gathering before a contract is offered. *Journal of Economic Theory 1*(81), 163–200.

Deci, E. L. and R. M. Ryan (1987). The support of autonomy and the control of behavior. *Journal of Personality and Social Psychology 53*(6), 1024.

Ederer, F. (2013). Incentives for parallel innovation.

Elloy, D. F. (2005). The influence of superleader behaviors on organization commitment, job satisfaction and organization self-esteem in a self-managed work team. *Leadership & Organization Development Journal 26*(2), 120–127.

Falk, A. and M. Kosfeld (2006). The hidden costs of control. *American Economic Review 96*(5), 1611–1630.

Fehr, E., M. Powell, and T. Wilkening (2016). Handing out guns at a knife fight: Behavioral limitations of subgame-perfect implementation.

Ferreira, D., G. Manso, and A. Silva (2014). Incentives to innovate and the decision to go public or private. *Review of Financial Studies 27*(1), 256–300.

Fingleton, J. and M. Raith (2005). Career concerns of bargainers. *Journal of Law, Economics, and organization 21*(1), 179–204.

Freixas, X., R. Guesnerie, and J. Tirole (1985). Planning under incomplete information and the ratchet effect. *Review of Economic Studies 52*(2), 173–191.

Fudenberg, D. and J. Tirole (1991). *Game Theory*. The MIT Press, Cambridge, MA.

Ghosh, P. and D. Ray (1996). Cooperation in community interaction without information flows. *Review of Economic Studies 63*(3), 491–519.

Gibbons, R. (1987). Piece-rate incentive schemes. *Journal of Labor Economics*, 413–429.

Gibbons, R. and K. Murphy (1992). Optimal incentive contracts in the presence of career concerns: Theory and evidence. *Journal of Political Economy 100*(3), 468–505.

Halac, M. (2012). Relational contracts and the value of relationships. *The American Economic Review 102*(2), 750–779.

Halac, M., N. Kartik, and Q. Liu (2016). Optimal contracts for experimentation. *The Review of Economic Studies 83*(3), 1040–1091.

Hall, B., A. Jaffe, and M. Trajtenberg (2005). Market value and patent citations. *RAND Journal of Economics 36*(1), 16–38.

Hart, O. and J. Moore (1999). Foundations of incomplete contracts. *The Review of Economic Studies 66*(1), 115–138.

Hellwig, M. F. (2010). Incentive problems with unidimensional hidden characteristics: A unified approach. *Econometrica 78*(4), 1201–1237.

Hermalin, B. E. (2013). Leadership and corporate culture. In R. Gibbons and J. Roberts (Eds.), *The Handbook of Organizational Economics*, pp. 432 – 478. Princeton University Press.

Holmström, B. (1977). *On Incentives and Control in Organizations.* Ph. D. thesis, Stanford University.

Holmström, B. (1979). Moral hazard and observability. *Bell Journal of Economics 10*(1), 74–91.

Holmström, B. (1989). Agency costs and innovation. *Journal of Economic Behavior & Organization 12*(3), 305–327.

Holmström, B. (1999). Managerial incentive problems: A dynamic perspective. *Review of Economic Studies*, 169–182.

Holmström, B. and P. Milgrom (1991). Multitask principal-agent analyses: Incentive contracts, asset ownership, and job design. *Journal of Law, Economics, & Organization*, 24–52.

Hörner, J. and L. Samuelson (2013). Incentives for experimenting agents. *The RAND Journal of Economics 44*(4), 632–663.

Howell, P. A. (2012). Whose invention is it anyway-employee invention-assignment agreements and their limits. *Wash. JL Tech. & Arts 8*, 79.

Iossa, E. and D. Martimort (2015). Pessimistic information gathering. *Games and Economic Behavior 91*, 75–96.

Kamenica, E. and M. Gentzkow (2011). Bayesian persuasion. *American Economic Review 101*, 2590–2615.

Kanemoto, Y. and W. B. MacLeod (1992). The ratchet effect and the market for secondhand workers. *Journal of Labor Economics*, 85–98.

Laffont, J.-J. and D. Martimort (2001). *The Theory of Incentives: The Principal-Agent Model.* Princeton University Press.

Laffont, J.-J. and J. Tirole (1987). Comparative statics of the optimal dynamic incentive contract. *European Economic Review 31*(4), 901–926.

Laffont, J.-J. and J. Tirole (1988). The dynamics of incentive contracts. *Econometrica 56*(5), 1153–1175.

Lazear, E. P. (1986). Salaries and piece rates. *Journal of Business*, 405–431.

Lewis, T. R. and D. E. Sappington (1991). All-or-nothing information control. *Economics Letters 37*(2), 111–113.

Lewis, T. R. and D. E. Sappington (1993). Ignorance in agency problems. *Journal of Economic Theory 61*(1), 169–183.

Lewis, T. R. and D. E. Sappington (1997). Information management in incentive problems. *Journal of Political Economy 105*(4), 796–821.

Manso, G. (2011). Motivating innovation. *The Journal of Finance 66*(5), 1823–1860.

March, J. G. (1991). Exploration and exploitation in organizational learning. *Organization Science*, 71–87.

Maskin, E. and J. Tirole (1999). Unforeseen contingencies and incomplete contracts. *Review of Economic Studies*, 83–114.

Milgrom, P. and I. Segal (2002). Envelope theorems for arbitrary choice sets. *Econometrica 70*(2), 583–601.

Moore, J. and R. Repullo (1988). Subgame perfect implementation. *Econometrica 56*(5), 1191–1220.

Myerson, R. B. and M. A. Satterthwaite (1983). Efficient mechanisms for bilateral trading. *Journal of Economic Theory 29*(2), 265–281.

Ortner, J. and S. Chassang (2015). Making collusion harder: Asymmetric information, collusion, and crime.

Prat, A. (2005). The wrong kind of transparency. *American Economic Review 95*(3), 862–877.

Riordan, M. (1990). What is vertical integration? In O. E. W. Masahiko Aoki, Bo Gustafsson (Ed.), *The Firm as a Nexus of Treaties*, pp. 94–111. London: Sage.

Salanié, B. (2005). *The Economics of Contracts: A Primer*. MIT press.

Schumpeter, J. A. (1934). *The Theory of Economic Development: An Inquiry into Profits, Capital, Credit, Interest, and the Business Cycle*, Volume 55. Transaction publishers.

Segal, I. (1999). Complexity and renegotiation: A foundation for incomplete contracts. *The Review of Economic Studies 66*(1), 57–82.

Shalley, C. E., L. L. Gilson, and T. C. Blum (2000). Matching creativity requirements and the work environment: Effects on satisfaction and intentions to leave. *Academy of Management Journal 43*(2), 215–223.

Shin, D. and R. Strausz (2014). Delegation and dynamic incentives. *The RAND Journal of Economics 45*(3), 495–520.

Sliwka, D. (2007). Trust as a signal of a social norm and the hidden costs of incentive schemes. *American Economic Review 97*(3), 999–1012.

Sobel, J. (1985). A theory of credibility. *Review of Economic Studies 52*(4), 557–573.

Sobel, J. (1993). Information control in the principal-agent problem. *International Economic Review 34*(2), 259–69.

Spreitzer, G. M. (2008). Taking stock: A review of more than twenty years of research on empowerment at work. *Handbook of Organizational Behavior*, 54–72.

Stern, S. (2004). Do scientists pay to be scientists? *Management Science 50*(6), 835–853.

Szalay, D. (2005). The economics of clear advice and extreme options. *Review of Economic Studies 72*(4), 1173–1198.

Szalay, D. (2009). Contracts with endogenous information. *Games and Economic Behavior 65*(2), 586–625.

Taylor, C. R. and H. Yildirim (2011). Subjective performance and the value of blind evaluation. *Review of Economic Studies 78*(2), 762–794.

Trajtenberg, M. (1990). A penny for your quotes: Patent citations and the value of. *The Rand Journal of Economics 21*(1), 172.

Watson, J. (1999). Starting small and renegotiation. *Journal of Economic Theory 85*(1), 52–90.

Watson, J. (2002). Starting small and commitment. *Games and Economic Behavior 38*(1), 176–199.

A Appendix: Proofs

Proof of Proposition 1.

Proof. The main text establishes the period-2 contract (α^*, β^*). Given the agent's limited liability and zero outside option in period 1, $a^* = 0$. b^* will be unique if $e^o(b)\,[Y-b]$ is continuous and strictly concave. Verifying this is similar to the proof of Proposition 3 and is thus omitted. □

Proof of Lemma 1.

Proof. $\varepsilon\,(\cdot)$ is differentiable using the implicit function theorem on (6). Differentiability of $u\,(\cdot)$ then follows. Omitting the arguments, the partial derivatives of $\varepsilon\,(\cdot)$ and $u\,(\cdot)$ are: $\varepsilon_\beta = \frac{\theta^2}{\psi''} > 0$, $\varepsilon_\theta = \frac{\psi'}{\psi''} + \frac{\varepsilon}{\theta} > 0$, $\varepsilon_{\beta\beta} = -\frac{\theta^3\psi'''}{(\psi'')^3} < 0$, $\varepsilon_{\theta\theta} = \frac{\psi'[2(\psi'')^2 - \psi'.\psi''']}{\theta(\psi'')^3} > 0$, $\varepsilon_{\beta\theta} = \frac{\theta[2(\psi'')^2-\psi'.\psi''']}{(\psi'')^3} > 0$; and $u_\beta = \varepsilon > 0$, $u_\theta = \psi' \cdot \frac{\varepsilon}{\theta^2} > 0$, $u_{\beta\beta} = \varepsilon_\beta > 0$, $u_{\theta\theta} = \frac{(\psi')^2}{\theta^2(\psi'')^2} > 0$, $u_{\beta\theta} = \varepsilon_\theta > 0$. □

Proof of Lemma 2.

Proof. Follows from standard arguments. See for example Salanié (2005) Chapter 2.3.1. □

Proof of Proposition 2.

The proof proceeds in two steps. In Step 1, I consider the auxiliary problem $\tilde{\mathcal{P}}'$:

$$\Phi\left(\ddot{\beta}\right) := \max_{\beta(\cdot)\in\mathcal{B}(\ddot{\beta})} \int_{\theta^o}^{\bar{\theta}} \phi\left(\beta(\theta), \theta\right) f(\theta) d\theta, \qquad (\tilde{\mathcal{P}}')$$

where $\mathcal{B}(\ddot{\beta})$ is the set of all non-decreasing functions mapping from domain $\left[\theta^o, \bar{\theta}\right]$ to $[\ddot{\beta}, \infty)$.[44] $\Phi\left(\ddot{\beta}\right)$ is thus the principal's maximum expected payoff from the successful innovators when she has to give at least bonus $\ddot{\beta}$ to the failed innovator type. It will be shown that for any $\ddot{\beta}$, the solution to $\tilde{\mathcal{P}}'$ is unique. In Step 2, the optimal $\beta^*(\theta^o)$ is pinned down by solving:

$$\max_{\ddot{\beta}} \; \Phi\left(\ddot{\beta}\right) + s\left(\ddot{\beta}, \theta^o\right) F\left(\theta^o\right). \qquad (A.1)$$

[44]There is an abuse of notation of $\mathcal{B}(\cdot)$ from (23) in the main text.

Since the solution to $\tilde{\mathcal{P}'}$ is unique, the solution to (A.1) together with its corresponding solution for $\tilde{\mathcal{P}'}$ is the solution to program $\mathcal{P}'$ in (11).

(Step 1) Solving the Problem $\tilde{\mathcal{P}'}$:

The following properties of $\phi(\cdot)$ and its maximal will be useful to characterize the solution of $\tilde{\mathcal{P}'}$:

Lemma A.1. *$\phi(\beta,\theta)$ is continuously differentiable in both its arguments. For any $\theta \in \left[\theta^o, \bar{\theta}\right]$, $\arg\max_{\beta\geq 0}\phi(\beta,\theta)$ has a unique solution. Define $\hat{\beta}(\theta) := \arg\max_{\beta\geq 0}\phi(\beta,\theta)$. For any $\theta \in \left[\theta^o, \bar{\theta}\right]$:*

1. $\phi(\beta,\theta)$ is strictly concave in β for $\beta \in [0, Y]$.
2. *$\hat{\beta}(\theta)$ is characterized by $\phi_\beta\Big(\hat{\beta}(\theta),\theta\Big) = 0$.*
3. *$\hat{\beta}(\bar{\theta}) = Y$.*
4. *$\hat{\beta}(\theta)$ is differentiable and $\frac{d}{d\theta}\hat{\beta}(\theta)$ is continuous.*
5. *$\hat{\beta}(\theta)$ is strictly increasing in θ.*
6. *$\phi^*(\theta) := \phi\Big(\hat{\beta}(\theta),\theta\Big)$ is continuous in θ.*

Proof. Differentiability of $\phi(\cdot)$ follows from differentiability of $\varepsilon(\cdot)$ and $\psi(\cdot)$.

$$\begin{aligned}\phi_\beta(\beta,\theta) &= \varepsilon_\beta(\beta,\theta)Y - \frac{1}{\theta}\psi'\left(\frac{\varepsilon(\beta,\theta)}{\theta}\right) - u_{\beta\theta}(\beta,\theta)H(\theta)\\ &= \varepsilon_\beta(\beta,\theta)\left[Y-\beta\right] - \varepsilon_\theta(\beta,\theta)H(\theta),\end{aligned} \tag{A.2}$$

where the last line uses (6) and $u_{\beta\theta}(\beta,\theta) = \varepsilon_\theta(\beta,\theta)$ (see proof of Lemma 1). Next:

$$\phi_{\beta\beta}(\beta,\theta) = \varepsilon_{\beta\beta}(\beta,\theta)\left[Y-\beta\right] - \varepsilon_\beta(\beta,\theta) - \varepsilon_{\beta\theta}\Big(\beta,\theta\Big)H(\theta). \tag{A.3}$$

Hence $\phi(\cdot)$ is strictly concave in β for $\beta \in [0, Y]$ (point 1).[45] From (A.2), $\phi_\beta(\beta,\theta) < 0$ whenever $\beta > Y$. Hence the optimal β must be less than Y. Together with point 1, $\hat{\beta}(\theta)$ is unique and is characterized by the FOC; this thus proves point 2. Point 3 then follows from $H(\bar{\theta}) = 0$. Next:

$$\phi_{\theta\beta}(\beta,\theta) = \varepsilon_{\theta\beta}(\beta,\theta)\left[Y-\beta\right] - \varepsilon_{\theta\theta}(\beta,\theta)H(\theta) - \varepsilon_\theta(\beta,\theta)H_\theta(\theta). \tag{A.4}$$

[45] From here, it is readily noted that the principal's objective is strictly concave in β. Hence offering stochastic contracts will never be optimal for the principal.

Using point 2 and FOC (A.2), $Y-\hat{\beta}(\theta)=\frac{\varepsilon_\theta\left(\hat{\beta}(\theta),\theta\right)H(\theta)}{\varepsilon_\beta\left(\hat{\beta}(\theta),\theta\right)}$. Substitute this into (A.4) and with some algebra (and dropping the arguments without confusion):

$$\begin{aligned}\phi_{\theta\beta}(\beta,\theta)\Big|_{\beta=\hat{\beta}(\theta)} &= \varepsilon_{\beta\theta}\frac{\varepsilon_\theta H(\theta)}{\varepsilon_\beta}-\varepsilon_{\theta\theta}H(\theta)-\varepsilon_\theta H_\theta(\theta)\\ &= \frac{[2(\psi'')^2-\psi'\psi''']}{\theta^2(\psi'')^2}\varepsilon H(\theta)-\varepsilon_\theta H_\theta(\theta)>0\end{aligned}$$

where the inequality follows Assumptions 1 and 2. By the implicit function theorem, $\frac{d}{d\theta}\hat{\beta}(\theta)=-\frac{\phi_{\theta\beta}\left(\hat{\beta}(\theta),\theta\right)}{\phi_{\beta\beta}\left(\hat{\beta}(\theta),\theta\right)}>0$ which is also continuous. This establishes points 4 and 5. Since $\phi(\cdot)$ is continuous in $\theta\in\left[\theta^o,\bar{\theta}\right]$, by Berges Maximum theorem , ϕ^* is continuous which establishes point 6. □

Lemma A.1.2 is condition (13) in the proposition. The next lemma completes Step 1:

Lemma A.2. *Fix a $\ddot{\beta}>0$ and let $\tilde{\beta}^*(\cdot)$ be the solution to program $\tilde{\mathcal{P}}'$. $\tilde{\beta}^*(\cdot)$ is uniquely characterized as the following:*

1. *If $\ddot{\beta}\geq Y$, then $\tilde{\beta}^*(\theta)=\ddot{\beta}\ \forall\theta\in\left[\theta^o,\bar{\theta}\right]$.*
2. *If $\ddot{\beta}\leq\hat{\beta}(\theta^o)$, then $\tilde{\beta}^*(\theta)=\hat{\beta}(\theta)\ \forall\theta\in\left[\theta^o,\bar{\theta}\right]$.*
3. *If $\ddot{\beta}\in\left(\hat{\beta}(\theta^o),Y\right)$, then*

$$\tilde{\beta}^*(\theta)=\begin{cases}\hat{\beta}(\theta) & ,\forall\theta>\ddot{\theta}\\ \ddot{\beta} & ,\forall\theta\leq\ddot{\theta}\end{cases}\tag{A.5}$$

where $\ddot{\theta}:=\hat{\beta}^{-1}\left(\ddot{\beta}\right)$.

Proof. The case of $\ddot{\beta}\geq Y$ is trivially true since $\phi(\beta,\theta)<0\ \forall\theta$ for $\beta>Y$. The case for $\ddot{\beta}\leq\hat{\beta}(\theta^o)$ follows from $\hat{\beta}(\cdot)$ being point-wise optimal for all $\theta\in\left[\theta^o,\bar{\theta}\right]$ (Lemma A.1). For the case of $\ddot{\beta}\in\left(\hat{\beta}(\theta^o),Y\right)$, first note that $\ddot{\theta}$ is well-defined since $\hat{\beta}(\cdot)$ is strictly increasing (Lemma A.1.5). Consider some $\tilde{\beta}(\cdot)$ that differs from $\tilde{\beta}^*(\cdot)$ in (A.5) for some non-zero measure subset of $\left[\theta^o,\bar{\theta}\right]$. Suppose first that $\tilde{\beta}(\cdot)$ differs from $\tilde{\beta}^*(\cdot)$ for some non-zero measure subset of $\left[\ddot{\theta},\bar{\theta}\right]$. This implies that:

$$\int_{\ddot{\theta}}^{\bar{\theta}}\phi\left(\tilde{\beta}^*(\theta),\theta\right)f(\theta)d\theta=\int_{\ddot{\theta}}^{\bar{\theta}}\phi\left(\hat{\beta}(\theta),\theta\right)f(\theta)d\theta>\int_{\ddot{\theta}}^{\bar{\theta}}\phi\left(\tilde{\beta}(\theta),\theta\right)f(\theta)d\theta$$

since $\hat{\beta}(\cdot)$ is the unique point-wise optimal for $\phi(\cdot)$. Next suppose that $\tilde{\beta}(\cdot)$ differs from $\tilde{\beta}^*(\cdot)$ for some non-zero measure subset of $\left[\theta^o, \ddot{\theta}\right]$ instead. Note that $\tilde{\beta}(\theta) \geq \ddot{\beta}\ \forall\theta \in \left[\theta^o, \bar{\theta}\right]$ since $\tilde{\beta}(\cdot) \in \mathcal{B}(\ddot{\beta})$. Moreover, $\phi_{\beta\beta}(\beta, \theta) < 0$ (Lemma A.1.1) implies that $\forall\theta \in \left[\theta^o, \bar{\theta}\right]$, $\phi(\beta, \theta) < \phi(\beta', \theta)$ when $\beta > \beta' \geq \hat{\beta}(\theta)$. Since $\ddot{\beta} \geq \hat{\beta}(\theta)\ \forall\theta \in \left[\theta^o, \ddot{\theta}\right]$ and $\tilde{\beta}(\theta) \geq \ddot{\beta}$, this implies that:

$$\int_{\theta^o}^{\ddot{\theta}} \phi\left(\tilde{\beta}^*(\theta), \theta\right) f(\theta) d\theta = \int_{\theta^o}^{\ddot{\theta}} \phi\left(\ddot{\beta}, \theta\right) f(\theta) d\theta > \int_{\theta^o}^{\ddot{\theta}} \phi\left(\tilde{\beta}(\theta), \theta\right) f(\theta) d\theta.$$

Hence any $\tilde{\beta}(\cdot)$ that differs from $\tilde{\beta}^*(\cdot)$ in (A.5) is strictly worse than $\tilde{\beta}^*(\cdot)$. □

(Step 2) Solve for $\ddot{\beta}$:

Lemma A.3. *$\Phi\left(\ddot{\beta}\right)$ is differentiable, strictly decreasing and strictly concave for $\ddot{\beta} \leq Y$.*

Proof. $\Phi\left(\ddot{\beta}\right)$ can be written as:

$$\Phi\left(\ddot{\beta}\right) = \int_{\theta^o}^{\hat{\beta}^{-1}\left(\ddot{\beta}\right)} \phi\left(\ddot{\beta}, \theta\right) f(\theta) d\theta + \int_{\hat{\beta}^{-1}\left(\ddot{\beta}\right)}^{\bar{\theta}} \phi\left(\hat{\beta}(\theta), \theta\right) f(\theta) d\theta.$$

Since $\hat{\beta}(\cdot)$ is strictly increasing and differentiable (Lemma A.1), $\hat{\beta}^{-1}(\beta)$ is also strictly increasing and differentiable with derivative $\frac{1}{\hat{\beta}'\left(\hat{\beta}^{-1}(\beta)\right)}$ which is continuous since $\hat{\beta}'(\cdot)$ is continuous (Lemma A.1.4). Hence Φ is differentiable by Leibniz's rule:

$$\frac{d}{d\ddot{\beta}}\Phi\left(\ddot{\beta}\right) = \underbrace{\left(\phi\left(\ddot{\beta}, \ddot{\theta}\right) f(\ddot{\theta}) - \phi\left(\hat{\beta}(\ddot{\theta}), \ddot{\theta}\right) f(\ddot{\theta})\right)}_{=0} \left(\frac{1}{\hat{\beta}'\left(\hat{\beta}^{-1}(\ddot{\beta})\right)}\right) + \int_{\theta^o}^{\hat{\beta}^{-1}\left(\ddot{\beta}\right)} \underbrace{\phi_\beta\left(\ddot{\beta}, \theta\right)}_{<0} f(\theta) d\theta < 0.$$

$\phi_\beta\left(\ddot{\beta}, \theta\right) < 0$ for all $\theta \in \left[\theta^o, \ddot{\theta}\right)$ since $\phi_{\beta\beta} < 0$ (Lemma A.1.1) and $\ddot{\beta} > \hat{\beta}(\theta)$. Differentiating again:

$$\frac{d^2}{d\ddot{\beta}^2}\Phi\left(\ddot{\beta}\right) = \underbrace{\phi_\beta\left(\hat{\beta}(\ddot{\theta}), \ddot{\theta}\right)}_{=0} f(\ddot{\theta}) \left(\frac{1}{\hat{\beta}'\left(\hat{\beta}^{-1}(\ddot{\beta})\right)}\right) + \int_{\theta^o}^{\hat{\beta}^{-1}\left(\ddot{\beta}\right)} \underbrace{\phi_{\beta\beta}\left(\ddot{\beta}, \theta\right)}_{<0} f(\theta) d\theta < 0.$$

□

Lemma A.4. *$s\left(\ddot{\beta}, \theta^o\right) F(\theta^o)$ is differentiable, strictly increasing and strictly concave for $\ddot{\beta} < Y$.*

Proof. Differentiability of $s(\cdot)$ follows from differentiability of $\varepsilon(\cdot)$ and $\psi(\cdot)$.

$$\begin{aligned} \frac{d}{d\ddot{\beta}} s\left(\ddot{\beta}, \theta^o\right) &= \varepsilon_\beta\left(\ddot{\beta}, \theta^o\right) Y - \frac{\varepsilon_\beta\left(\ddot{\beta}, \theta^o\right)}{\theta^o} \psi'\left(\frac{\varepsilon\left(\ddot{\beta}, \theta^o\right)}{\theta^o}\right) \\ &= \varepsilon_\beta\left(\ddot{\beta}, \theta^o\right)\left[Y - \ddot{\beta}\right] \end{aligned}$$

which is positive whenever $\ddot{\beta} < Y$, and $\frac{d^2}{d\ddot{\beta}^2} s\left(\ddot{\beta}, \theta^o\right) = \varepsilon_{\beta\beta}\left(\ddot{\beta}, \theta^o\right)\left[Y - \ddot{\beta}\right] - \varepsilon_\beta\left(\ddot{\beta}, \theta^o\right) < 0$. □

Lemmas A.3 and A.4 jointly imply that $\Phi\left(\ddot{\beta}\right) + s\left(\ddot{\beta}, \theta\right) F\left(\theta^o\right)$ is strictly concave in $\ddot{\beta} \in \left[\hat{\beta}(\theta^o), Y\right]$. The solution for (A.1) must be in the interior of $\left[\hat{\beta}(\theta^o), Y\right]$. To see why, notice that $\frac{d}{d\ddot{\beta}}\Phi\left(\ddot{\beta}\right)\Big|_{\ddot{\beta}=\hat{\beta}(\theta^o)} = 0$ while $\frac{d}{d\ddot{\beta}} s\left(\ddot{\beta}, \theta^o\right)\Big|_{\ddot{\beta}=\hat{\beta}(\theta^o)} > 0$. Hence $\ddot{\beta} > \hat{\beta}(\theta^o)$. Similarly, $\frac{d}{d\ddot{\beta}}\Phi\left(\ddot{\beta}\right)\Big|_{\ddot{\beta}=Y} < 0$ while $\frac{d}{d\ddot{\beta}} s\left(\ddot{\beta}, \theta^o\right)\Big|_{\ddot{\beta}=Y} = 0$. Hence $\ddot{\beta} < Y$. Thus, the solution to (A.1) is characterized by its first-order condition which is condition (14) in the proposition. The strict concavity also implies that the solution is unique and thus, the solution to $\mathcal{P}'$ is unique. Notice that $\tilde{\beta}^*(\cdot)$ in ($\tilde{\mathcal{P}}'$) is continuous everywhere, differentiable everywhere except (possibly) at $\ddot{\theta}$, and strictly increasing for $\theta \geq \ddot{\theta}$. Since these properties hold for any $\ddot{\beta}$, they carry over to $\beta^*(\cdot)$. Using (IC-2A) in Lemma 2:

$$\alpha^*(\theta) = \bar{U}(\theta) - u\left(\beta^*(\theta), \theta\right) = U^0 + \int_{\theta^o}^{\theta} u_\theta\left(\beta^*(t), t\right) dt - u\left(\beta^*(\theta), \theta\right).$$

$\beta^*(\theta) \leq Y \implies u\left(\beta^*(\theta), \theta\right) \leq u\left(Y, \theta\right) \leq u\left(Y, \bar{\theta}\right) \leq U^0$; hence $\alpha^*(\theta) > 0$ is differentiable almost everywhere, with $\frac{d}{d\theta}\alpha^*(\theta) = -u_\beta\left(\beta^*(\theta), \theta\right)\beta^*_\theta(\theta) \leq 0$ whenever the derivative exists.

Proof of Lemma 3.

Proof. Concavity of $W^\tau(e, b)$ in e implies that $e^\tau(b)$ is characterized by the respective first-order condition:

$$b = \frac{1}{\theta^o}\psi'\left(\frac{e^o(b)}{\theta^o}\right) \tag{A.6}$$

$$b = E\left[\frac{1}{\theta^n}\psi'\left(\frac{e^n(b)}{\theta^n}\right)\right] \tag{A.7}$$

Since $W^\tau(e,b)$ is super-modular, using the implicit function, $e^\tau(b)$ is differentiable and $\frac{d}{db}\left(e^\tau(b)\right) > 0$:

$$\frac{d}{db}\left(e^o(b)\right) = \frac{1}{\left(\frac{1}{\theta^o}\right)^2 \psi''\left(\frac{e^o(b)}{\theta^o}\right)} \quad , \quad \frac{d}{db}\left(e^n(b)\right) = \frac{1}{E\left[\left(\frac{1}{\theta^n}\right)^2 \psi''\left(\frac{e^n(b)}{\theta^n}\right)\right]}. \tag{A.8}$$

Next (A.6) and (A.7) imply that for any $b > 0$:

$$\frac{1}{\theta^o}\psi'\left(\frac{e^o(b)}{\theta^o}\right) = E\left[\frac{1}{\theta^n}\psi'\left(\frac{e^n(b)}{\theta^n}\right)\right] > \frac{1}{E\left[\theta^n\right]}\psi'\left(\frac{e^n(b)}{E\left[\theta^n\right]}\right) \geq \frac{1}{\theta^o}\psi'\left(\frac{e^n(b)}{\theta^o}\right).$$

Since $\psi'(\cdot)$ is strictly increasing, $e^o(b) > e^n(b)$.

Next let $\Delta(b) = \bar{W}^o(b) - \bar{W}^n(b)$. Since $\lim_{b\to 0} e^\tau(b) = 0\ \forall \tau$, $\lim_{b\to 0}\Delta(b) = -R < 0$. Hence there exists small enough b such that $\Delta(b) < 0$. By the envelope theorem, $\Delta'(b) = e^o(b) - e^n(b) > 0\ \forall b > 0$; this implies that $\Delta(b)$ is strictly increasing and thus can cross 0 at most once. If $\Delta(\cdot)$ crosses 0, then $\bar{b}$ is defined by $\bar{b} = \Delta^{-1}(0)$. If $\Delta(\cdot)$ is bounded above by 0, then $\bar{b} = \infty$. That $\bar{b}$ is non-decreasing in R is immediate from the observation that $\Delta(b)$ is point-wise decreasing in R. □

Proof of Proposition 3.

Proof. Given the explanation in the main text, I only have to prove that $e^n(b)\left[Y - b\right]$ is strictly concave in b. $e^n(\cdot)$ is continuously differentiable from (A.8). Differentiating (A.7) w.r.t b twice:

$$-E\left[\left(\frac{1}{\theta^n}\right)^3 \psi'''\left(\frac{e^n(b)}{\theta^n}\right)\right]\left(\frac{d}{db}\left(e^n(b)\right)\right)^2 - E\left[\left(\frac{1}{\theta^n}\right)^2 \psi''\left(\frac{e^n(b)}{\theta^n}\right)\right]\frac{d^2}{db^2}\left(e^n(b)\right) = 0$$

Since $\psi'''(\cdot) \geq 0$, it must be true that $\frac{d^2}{db^2}\left(e^n(b)\right) \leq 0$. Next differentiating $e^n(b)\left[Y-b\right]$ twice gives $\frac{d^2}{db^2}\left(e^n(b)\left[Y-b\right]\right) = \left(\frac{d^2}{db^2}\left(e^n(b)\right)\right)\left[Y-b\right] - 2\left(\frac{d}{db}\left(e^n(b)\right)\right) < 0$. □

Proof of Proposition 4.

Proof. $Innov(\theta^o) < 0$ is immediate. For $Innov > 0$, using the formulation in (9) for program $\mathcal{P}$, $\max_{\beta(\cdot)\in\mathcal{B}} \int_{\theta^o}^{\bar{\theta}} v\left(\beta(\cdot),\theta\right) dF(\theta) = \int_{\theta^o}^{\bar{\theta}} s\left(\beta^*(\theta),\theta\right) dF(\theta) - R \geq s\left(Y,\theta^o\right)$ (see Remark 2). Since $R > 0$, $Innov$ must be strictly positive. That the agent's innovation surplus is strictly increasing and strictly convex in θ follows from straightforward differentiation on (IC-2A) in

Lemma 2. □

The following result will be useful for the proof of the next three propositions.

Lemma A.5. *$s_\theta(\beta,\theta) = \varepsilon_\theta(\beta,\theta)[Y-\beta] + u_\theta(\beta,\theta)$ and $s_{\theta\beta}(\beta,\theta) = \varepsilon_{\theta\beta}(\beta,\theta)[Y-\beta]$. Hence $\forall \beta \leq Y$, $s_\theta(\beta,\theta) > 0$ and $s_{\theta\beta}(\beta,\theta) > 0$.*

Proof.

$$\begin{aligned} s_\theta(\beta,\theta) &= \varepsilon_\theta(\beta,\theta)Y - \psi'\left(\frac{\varepsilon(\beta,\theta)}{\theta}\right)\left[\frac{\theta\varepsilon_\theta(\beta,\theta)-\varepsilon(\beta,\theta)}{\theta^2}\right] \\ &= \varepsilon_\theta(\beta,\theta)\left[Y - \frac{1}{\theta}\psi'\left(\frac{\varepsilon(\beta,\theta)}{\theta}\right)\right] + \psi'\left(\frac{\varepsilon(\beta,\theta)}{\theta}\right)\frac{\varepsilon(\beta,\theta)}{\theta^2} \\ &= \varepsilon_\theta(\beta,\theta)[Y-\beta] + u_\theta(\beta,\theta) \end{aligned}$$

By noting that $\varepsilon_\theta(\beta,\theta) = u_{\theta\beta}(\beta,\theta)$, it is immediate that $s_{\theta\beta}(\beta,\theta) = \varepsilon_{\theta\beta}(\beta,\theta)[Y-\beta]$. □

Proof of Proposition 5.

Throughout this proof, let $\hat{\beta}(\cdot;k)$ be the bonus schedule characterized in (13) for distribution $F(\cdot;k)$, let $\ddot{\theta}(k)$ be the corresponding maximum bunching type characterized in (14),[46] and let $\ddot{\beta}(k)$ denote the bunching bonus level $\hat{\beta}\left(\ddot{\theta}(k);k\right)$.

Proof of Proposition 5.1.

Proof. Since $\varepsilon_{\beta\beta} < 0$ and $\varepsilon_{\beta\theta} > 0$, inspection of (13) reveals that $H(\theta;k'') > H(\theta;k') \implies \hat{\beta}(\theta;k'') < \hat{\beta}(\theta;k')$. The result is then established by showing $\ddot{\beta}(k'') < \ddot{\beta}(k')$. Rewrite (14) as:

$$\int_{\theta^o}^{\hat{\beta}^{-1}\left(\ddot{\beta}(k);k\right)} \phi_\beta\left(\ddot{\beta}(k),\theta\right) f(\theta;k)d\theta + s_\beta\left(\ddot{\beta}(k),\theta^o\right) F(\theta^o;k) = 0 \qquad \text{(A.9)}$$

[46] Note that there is an abuse of notation of $\ddot{\theta}(\cdot)$ relatively to how it is used in Section 6.2.

where $\hat{\beta}^{-1}(\cdot;k)$ is the inverse function of $\hat{\beta}(\cdot;k)$. By the implicit function theorem, $\frac{d}{dk}\ddot{\beta}(k) = -\frac{\frac{\partial(eq\ (A.9))}{\partial k}}{\frac{\partial(eq\ (A.9))}{\partial \ddot{\beta}}}$. From Lemma A.3 and A.4, $\frac{\partial(eq\ A.9)}{\partial \ddot{\beta}} < 0$. I show that $\frac{\partial(eq\ A.9)}{\partial k} < 0$ next.

$$\begin{aligned}
\frac{\partial(eq\ A.9)}{\partial k} &= \underbrace{\phi_\beta\left(\ddot{\beta}(k), \ddot{\theta}(k)\right)}_{=0} f\left(\ddot{\theta}(k);k\right)\left(\frac{\partial}{\partial k}\hat{\beta}^{-1}\left(\ddot{\beta}(k);k\right)\right) \\
&+ \int_{\theta^o}^{\hat{\beta}^{-1}\left(\ddot{\beta}(k);k\right)}\left[\underbrace{\varepsilon_\beta\left(\ddot{\beta}(k),\theta\right)}_{>0}\underbrace{\left(Y-\ddot{\beta}(k)\right)}_{>0} f_k(\theta;k) + \underbrace{\varepsilon_\theta\left(\ddot{\beta}(k),\theta\right)}_{>0}\underbrace{F_k(\theta;k)}_{<0}\right] d\theta \\
&+ \underbrace{s_\beta\left(\ddot{\beta}(k),\theta^o\right)}_{>0}\underbrace{F_k(\theta^o;k)}_{<0}.
\end{aligned}$$

To sign the second line,[47] notice that if $f_k(\theta;k) \leq 0$, then $\frac{\partial(eq\ A.9)}{\partial k} < 0$. Suppose that $f_k(\theta;k) > 0$ instead. Since $\theta \leq \hat{\beta}^{-1}\left(\ddot{\beta}(k);k\right)$ in this region, $\ddot{\beta}(k) \geq \hat{\beta}(\theta;k)$. From (A.2) and (A.3):

$$\begin{aligned}
& \varepsilon_\beta\left(\ddot{\beta}(k),\theta\right)\left[Y-\ddot{\beta}(k)\right] - \varepsilon_\theta\left(\ddot{\beta}(k),\theta\right)H(\theta;k) \leq 0 \\
\iff \quad & Y-\ddot{\beta}(k) \leq \frac{\varepsilon_\theta\left(\ddot{\beta}(k),\theta\right)H(\theta;k)}{\varepsilon_\beta\left(\ddot{\beta}(k),\theta\right)}.
\end{aligned}$$

Substitute this into the above:

$$\begin{aligned}
& \varepsilon_\beta\left(\ddot{\beta}(k),\theta\right)\left(Y-\ddot{\beta}(k)\right) f_k(\theta;k) + \varepsilon_\theta\left(\ddot{\beta}(k),\theta\right)F_k(\theta;k) \\
\leq\ & \varepsilon_\beta\left(\ddot{\beta}(k),\theta\right)\left(\frac{\varepsilon_\theta\left(\ddot{\beta}(k),\theta\right)H(\theta;k)}{\varepsilon_\beta\left(\ddot{\beta}(k),\theta\right)}\right) f_k(\theta;k) + \varepsilon_\theta\left(\ddot{\beta}(k),\theta\right)F_k(\theta;k) \\
=\ & \varepsilon_\theta\left(\ddot{\beta}(k),\theta\right)\left[H(\theta;k)f_k(\theta;k) + F_k(\theta;k)\right]
\end{aligned}$$

which is negative since $\frac{d}{dk}H(\theta;k) > 0$ and $\frac{d}{dk}H(\theta;k) = -\frac{1}{f(\theta;k)}\left[H(\theta;k)f_k(\theta;k) + F_k(\theta;k)\right]$. $\square$

[47] To get the second line, it might be helpful to note that $\phi_\beta\left(\ddot{\beta}(k),\theta\right)f(\theta;k) = \varepsilon_\beta\left(\ddot{\beta}(k),\theta\right)\left(Y-\ddot{\beta}(k)\right)f(\theta;k) - \varepsilon_\theta\left(\ddot{\beta}(k),\theta\right)\left[1-F(\theta;k)\right]$.

Proof of Proposition 5.2.[48]

Proof. $\int_{\theta^o}^{\bar{\theta}} v(\beta(\cdot),\theta) f(\theta)d\theta = v\left(\beta(\cdot),\bar{\theta}\right) - s(\beta(\theta^o),\theta^o) F(\theta^o) - \int_{\theta^o}^{\bar{\theta}} \frac{d}{d\theta}[v(\beta^*(\cdot),\theta)] F(\theta)d\theta$ from an integration by part. Program $\mathcal{P}'$ in (9) can thus be written as:

$$\int_{\theta^o}^{\bar{\theta}} v(\beta(\cdot),\theta) dF(\theta) = v\left(\beta(\cdot),\bar{\theta}\right) - \int_{\theta^o}^{\bar{\theta}} \frac{d}{d\theta}[v(\beta^*(\cdot),\theta)] F(\theta)d\theta.$$

Suppose $k'' > k'$. It is without loss to define $\beta^*(\theta;k') = Y$ for $\theta \in \left[\bar{\theta}(k'),\bar{\theta}(k'')\right]$; $\beta^*(\cdot;k')$ is non-decreasing in $\left[\underline{\theta},\bar{\theta}(k'')\right]$ and is thus feasible under distribution $F(\cdot;k'')$. This implies:

$$\begin{aligned}
\mathcal{V}(k'') &\geq v\left(\beta^*(\cdot;k'),\bar{\theta}(k'')\right) - \int_{\theta^o}^{\bar{\theta}(k'')} \frac{d}{d\theta}[v(\beta^*(\cdot;k'),\theta)] F(\theta;k'')d\theta, \\
\mathcal{V}(k') &= v\left(\beta^*(\cdot;k'),\bar{\theta}(k')\right) - \int_{\theta^o}^{\bar{\theta}(k')} \frac{d}{d\theta}[v(\beta^*(\cdot;k'),\theta)] F(\theta;k')d\theta.
\end{aligned}$$

Hence

$$\begin{aligned}
\mathcal{V}(k'') - \mathcal{V}(k') \geq\ & v\left(\beta^*(\cdot;k'),\bar{\theta}(k'')\right) - v\left(\beta^*(\cdot;k'),\bar{\theta}(k')\right) && \text{(A.10)} \\
& - \int_{\bar{\theta}(k')}^{\bar{\theta}(k'')} \frac{d}{d\theta}[v(\beta^*(\cdot;k'),\theta)] F(\theta;k'')d\theta && \text{(A.11)} \\
& - \int_{\theta^o}^{\bar{\theta}(k')} \frac{d}{d\theta}[v(\beta^*(\cdot;k'),\theta)] (F(\theta;k'') - F(\theta;k'))\, d\theta. && \text{(A.12)}
\end{aligned}$$

For (A.10), using the definition of v in (10):

$$\begin{aligned}
v\left(\beta^*(\cdot;k'),\bar{\theta}(k'')\right) - v\left(\beta^*(\cdot;k'),\bar{\theta}(k')\right) &= s\left(Y,\bar{\theta}(k'')\right) - s\left(Y,\bar{\theta}(k')\right) - \int_{\bar{\theta}(k')}^{\bar{\theta}(k'')} u_\theta(\beta^*(\theta;k'),\theta)\, d\theta \\
&= \int_{\bar{\theta}(k')}^{\bar{\theta}(k'')} s_\theta(Y,\theta)\, d\theta - \int_{\bar{\theta}(k')}^{\bar{\theta}(k'')} u_\theta(Y,\theta)\, d\theta, \qquad \text{(A.13)}
\end{aligned}$$

[48]Standard envelope theorem results (eg. Milgrom and Segal (2002)) cannot be directly applied here because the feasible set $\mathcal{B}(k)$ is dependent on k. The proposition also does not require differentiability of $\mathcal{V}(k)$.

and, for (A.11):

$$
\begin{aligned}
& \int_{\bar{\theta}(k')}^{\bar{\theta}(k'')} \frac{d}{d\theta}\left[v\left(\beta^*(\cdot;k'),\theta\right)\right] F(\theta;k'')dt \\
= & \int_{\bar{\theta}(k')}^{\bar{\theta}(k'')} \frac{d}{d\theta}\left[s\Big(\underbrace{\beta^*(\theta;k')}_{=Y},\theta\Big) - \int_{\theta^o}^{\theta} u_\theta\left(\beta(t;k'),t\right)dt\right] F(\theta;k'')d\theta \\
= & \int_{\bar{\theta}(k')}^{\bar{\theta}(k'')} \left[s_\theta\left(Y,\theta\right) - u_\theta\left(Y,\theta\right)\right] F(\theta;k'')d\theta. \qquad \text{(A.14)}
\end{aligned}
$$

From Lemma A.5, $s_\theta\left(Y,\theta\right) = u_\theta\left(Y,\theta\right)$; hence the terms in (A.13) and (A.14) are both 0. Hence:

$$
\mathcal{V}(k'') - \mathcal{V}(k') \;\; \geq \;\; -\int_{\theta^o}^{\bar{\theta}(k')} \frac{d}{d\theta}\left[v\left(\beta^*(\cdot;k'),\theta\right)\right] \underbrace{\left[F(\theta;k'') - F(\theta;k')\right]}_{<0} d\theta.
$$

which is strictly positive since, using Lemma A.5:

$$
\begin{aligned}
\frac{d}{d\theta}v\left(\beta^*(\cdot;k'),\theta\right) &= s_\beta\left(\beta^*(\theta;k'),\theta\right)\beta^*_\theta(\theta;k) + s_\theta\left(\beta^*(\theta;k'),\theta\right) - u_\theta\left(\beta^*(\theta;k'),\theta\right) \\
&= s_\beta\left(\beta^*(\theta;k'),\theta\right)\beta^*_\theta(\theta;k') + \varepsilon_\theta\left(\beta^*(\theta;k'),\theta\right)\left[Y - \beta^*(\theta;k')\right] \qquad \text{(A.15)} \\
&> 0. \qquad \text{(A.16)}
\end{aligned}
$$

□

Proof of Proposition 5.3.

Proof. For a period-1 bonus b, let the agent's optimal period-1 effort for n with potential k be:

$$
\begin{aligned}
e^n(b;k) &:= \arg\max_e \left\{ eb - \int_{\underline{\theta}}^{\bar{\theta}(k)} \psi\left(\frac{e}{\theta^n}\right) f(\theta^n;k)d\theta^n \right\} \\
&= \arg\max_e \left\{ eb - \psi\left(\frac{e}{\bar{\theta}(k)}\right) - \int_{\underline{\theta}}^{\bar{\theta}(k)} \frac{e}{(\theta^n)^2}\psi'\left(\frac{e}{\theta^n}\right) F(\theta^n;k)d\theta^n \right\},
\end{aligned}
$$

$e^n(b;k'') > e^n(b;k')$ since the objective is strictly super-modular in e and k. First suppose that $\hat{b}(k'') \leq \bar{b}(k'')$. The principal's period-1 payoff under k'' is $e^n\left(\hat{b}(k'');k''\right)\left[Y - \hat{b}(k'')\right]$, and:

$$\begin{aligned} e^n\left(\hat{b}(k''); k''\right)\left[Y-\hat{b}(k'')\right] &\geq e^n\left(\hat{b}(k'); k''\right)\left[Y-\hat{b}(k')\right] \\ &> e^n\left(\hat{b}(k'); k'\right)\left[Y-\hat{b}(k')\right] \\ &\geq \max_{b\leq \bar{b}(k')}\left\{e^n\left(b; k'\right)\left[Y-b\right]\right\}. \end{aligned}$$

Suppose that $R(k'') \geq R(k')$ instead. This implies $\bar{b}(k'') \geq \bar{b}(k') \geq b^*(k')$ where $b^*(k') := \min\left\{\hat{b}(k'), \bar{b}(k')\right\}$; $b^*(k')$ is a feasible bonus under k''. Hence

$$\max_{b\leq \bar{b}(k'')}\left\{e^n(b; k'')\left[Y-b\right]\right\} \geq e^n\left(b^*(k'); k''\right)\left[Y-b^*(k')\right] > e^n\left(b^*(k'); k'\right)\left[Y-b^*(k')\right].$$

□

Proof of Proposition 6.

Proof. Proposition 6.1 is straightforward and thus omitted. For Proposition 6.2a, note that θ^o affects only the bunching bonus $\hat{\beta}\left(\ddot{\theta}\right)$. From the proof of Proposition 2 , $\frac{\partial(\text{LHS of (14)})}{\partial\hat{\beta}(\ddot{\theta})} < 0$. Moreover:

$$\frac{\partial}{\partial\theta^o}(\text{LHS of (14)}) = -\phi_\beta\left(\hat{\beta}(\ddot{\theta}), \theta^o\right) f(\theta^o) + s_\beta\left(\hat{\beta}(\ddot{\theta}), \theta^o\right) f(\theta^o) + s_{\beta\theta}\left(\hat{\beta}(\ddot{\theta}), \theta^o\right) F(\theta^o).$$

Using the results in the proof of Proposition 2, $\hat{\beta}(\ddot{\theta}) > \hat{\beta}(\theta^o) \implies \phi_\beta\left(\hat{\beta}(\ddot{\theta}), \theta^o\right) < 0$; $\hat{\beta}\left(\ddot{\theta}\right) < Y \implies s_\beta\left(\hat{\beta}(\ddot{\theta}), \theta^o\right) > 0$. From Lemma A.5, $s_{\beta\theta}\left(\hat{\beta}(\ddot{\theta}), \theta^o\right) = \varepsilon_{\beta\theta}\left(\hat{\beta}(\ddot{\theta}), \theta^o\right)\left[Y-\hat{\beta}(\ddot{\theta})\right] > 0$. Hence $\frac{\partial}{\partial\theta^o}(\text{LHS of (14)}) > 0$. By the implicit function theorem, $\hat{\beta}(\ddot{\theta})$ is increasing in θ^o.

I prove Proposition 6.2b next. Let $\theta^{o''} > \theta^{o'}$. $\beta^*(\cdot|\theta^{o'})$ is feasible under $\theta^{o''}$ imply that:

$$\begin{aligned} \int_{\theta^{o''}}^{\bar{\theta}} v\left(\beta^*(\cdot|\theta^{o''}), \theta\right) dF(\theta) &\geq \int_{\theta^{o''}}^{\bar{\theta}} v\left(\beta^*(\cdot|\theta^{o'}), \theta\right) dF(\theta) \\ &= \int_{\theta^{o''}}^{\bar{\theta}} \phi\left(\beta^*(\theta|\theta^{o'}), \theta\right) f(\theta)d\theta + s\left(\beta^*(\theta^{o''}|\theta^{o'}), \theta^{o''}\right) F(\theta^{o''}) \end{aligned} \tag{A.17}$$

Hence:

$$\begin{aligned}
&\int_{\theta^{o''}}^{\bar\theta} v\left(\beta^*(\cdot|\theta^{o''}),\theta\right)dF(\theta) - \int_{\theta^{o'}}^{\bar\theta} v\left(\beta^*(\cdot|\theta^{o'}),\theta\right)dF(\theta) \\
\geq\ & s\left(\beta^*(\theta^{o''}|\theta^{o'}),\theta^{o''}\right)F(\theta^{o''}) - s\Big(\underbrace{\beta^*(\theta^{o'}|\theta^{o'})}_{\leq\beta^*(\theta^{o''}|\theta^{o'})},\theta^{o'}\Big)F(\theta^{o'}) - \int_{\theta^{o'}}^{\theta^{o''}}\phi\left(\beta^*(\theta|\theta^{o'}),\theta\right)f(\theta)d\theta \quad (A.18)\\
\geq\ & \underbrace{s\left(\beta^*(\theta^{o''}|\theta^{o'}),\theta^{o''}\right)F(\theta^{o''}) - s\Big(\beta^*(\theta^{o''}|\theta^{o'}),\theta^{o'}\Big)F(\theta^{o'})}_{=\int_{\theta^{o'}}^{\theta^{o''}}\left[s_\theta\left(\beta^*(\theta^{o''}|\theta^{o'}),\theta\right)F(\theta)+s\left(\beta^*(\theta^{o''}|\theta^{o'}),\theta\right)f(\theta)\right]d\theta} - \int_{\theta^{o'}}^{\theta^{o''}}\phi\left(\beta^*(\theta|\theta^{o'}),\theta\right)f(\theta)d\theta \quad (A.19)\\
\geq\ & \int_{\theta^{o'}}^{\theta^{o''}} s_\theta\left(\beta^*(\theta^{o''}|\theta^{o'}),\theta\right)F(\theta)d\theta + \int_{\theta^{o'}}^{\theta^{o''}} u_\theta\left(\beta^*(\theta|\theta^{o'}),\theta\right)H(\theta)f(\theta)d\theta. \quad (A.20)
\end{aligned}$$

where inequality (A.18) follows from (A.17), and (A.20) follows from $\beta^*(\theta^{o''}|\theta^{o'}) \geq \beta^*(\theta^o|\theta^{o'})$ $\forall\theta \in \left[\theta^{o'},\theta^{o''}\right]$. Proposition 6.2b thus follows from $u_\theta(\beta,\theta) > 0$ and $s_\theta(\beta,\theta) > 0$ (Lemma A.5). □

Proof of Proposition 7.

Proof. I follow the notation for $\beta^*(\cdot|\theta^o)$ used in the proof of Proposition 6. Let $\theta^{o''} > \theta^{o'}$. Note that $D^{TM}\left(\theta^{o''},\theta^{o'}\right) = s\left(Y,\theta^{o''}\right) - s\left(Y,\theta^{o'}\right)$. $s_{\theta\beta}(\beta,\theta) > 0$ (Lemma A.5) implies:

$$D^{TM}\left(\theta^{o''},\theta^{o'}\right) > s\left(\beta^*(\theta^{o''}|\theta^{o''}),\theta^{o''}\right) - s\left(\beta^*(\theta^{o''}|\theta^{o''}),\theta^{o'}\right) = \int_{\theta^{o'}}^{\theta^{o''}} s_\theta\left(\beta^*(\theta^{o''}|\theta^{o''}),\theta\right)d\theta. \quad (A.21)$$

Next, without loss, let $\beta^*(\theta|\theta^{o''}) = \beta^*(\theta^{o''}|\theta^{o''})$ $\forall\theta \in \left[\theta^{o'},\theta^{o''}\right)$; this implies that $\beta^*(\cdot|\theta^{o''})$ is feasible under $\theta^{o'}$. Hence $\int_{\theta^{o'}}^{\bar\theta} v\left(\beta^*(\cdot|\theta^{o'}),\theta\right)dF(\theta) \geq \int_{\theta^{o'}}^{\bar\theta} v\left(\beta^*(\cdot|\theta^{o''}),\theta\right)dF(\theta)$, and:

$$\begin{aligned}
\int_{\theta^{o'}}^{\bar{\theta}} v\left(\beta^*(\cdot|\theta^{o''}),\theta\right)dF(\theta) &= \int_{\theta^{o'}}^{\bar{\theta}} \phi\left(\beta^*(\theta|\theta^{o''}),\theta\right)f(\theta)d\theta + s\Big(\underbrace{\beta^*(\theta^{o'}|\theta^{o''})}_{=\beta^*(\theta^{o''}|\theta^{o''})},\theta^{o'}\Big)F(\theta^{o'}) \\
&= \int_{\theta^{o''}}^{\bar{\theta}} \phi\left(\beta^*(\theta|\theta^{o''}),\theta\right)f(\theta)d\theta + s\left(\beta^*(\theta^{o''}|\theta^{o''}),\theta^{o'}\right)F(\theta^{o'}) \\
&\quad + \int_{\theta^{o'}}^{\theta^{o''}} \Big[s\Big(\underbrace{\beta^*(\theta|\theta^{o''})}_{=\beta^*(\theta^{o''}|\theta^{o''})},\theta\Big)f(\theta) - u_\theta\Big(\underbrace{\beta^*(\theta|\theta^{o''})}_{=\beta^*(\theta^{o''}|\theta^{o''})},\theta\Big)[1-F(\theta)]\Big]d\theta \\
&= \underbrace{\int_{\theta^{o''}}^{\bar{\theta}} \phi\left(\beta^*(\theta|\theta^{o''}),\theta\right)f(\theta)d\theta + s\left(\beta^*(\theta^{o''}|\theta^{o''}),\theta^{o''}\right)F(\theta^{o''})}_{=\int_{\theta^{o''}}^{\bar{\theta}} v\left(\beta^*(\cdot|\theta^{o''}),\theta\right)dF(\theta)} \\
&\quad - \int_{\theta^{o'}}^{\theta^{o''}} \Big[s_\theta\left(\beta^*(\theta^{o''}|\theta^{o''}),\theta\right)F(\theta) + u_\theta\Big(\beta^*(\theta^{o''}|\theta^{o''}),\theta\Big)[1-F(\theta)]\Big]d\theta
\end{aligned}$$

where the last equality follows from integration by parts on $\int_{\theta^{o'}}^{\theta^{o''}} s\Big(\beta^*(\theta^{o''}|\theta^{o''}),\theta\Big)f(\theta)d\theta$. Hence:

$$\begin{aligned}
D^{FW}\left(\theta^{o''},\theta^{o'}\right) &= \int_{\theta^{o''}}^{\bar{\theta}} v\left(\beta^*(\cdot|\theta^{o''}),\theta\right)dF(\theta) - \int_{\theta^{o'}}^{\bar{\theta}} v\left(\beta^*(\cdot|\theta^{o'}),\theta\right)dF(\theta) \\
&\leq \int_{\theta^{o'}}^{\theta^{o''}} \Big[s_\theta\left(\beta^*(\theta^{o''}|\theta^{o''}),\theta\right)F(\theta) + u_\theta\Big(\beta^*(\theta^{o''}|\theta^{o''}),\theta\Big)[1-F(\theta)]\Big]d\theta \qquad \text{(A.22)}
\end{aligned}$$

(A.21) and (A.22) jointly imply that

$$\begin{aligned}
& D^{TM}\left(\theta^{o''},\theta^{o'}\right) - D^{FW}\left(\theta^{o''},\theta^{o'}\right) \\
> & \int_{\theta^{o'}}^{\theta^{o''}} \left[s_\theta\left(\beta^*(\theta^{o''}|\theta^{o''}),\theta\right) - u_\theta\left(\beta^*(\theta^{o''}|\theta^{o''}),\theta\right)\right][1-F(\theta)]\,d\theta \\
= & \int_{\theta^{o'}}^{\theta^{o''}} \varepsilon_\theta\left(\beta^*(\theta^{o''}|\theta^{o''}),\theta\right)\left[Y-\beta^*(\theta^{o''}|\theta^{o''})\right][1-F(\theta)]\,d\theta \\
> & \ 0
\end{aligned}$$

where the last equality follows from Lemma A.5. □

B Supplementary Appendix (For Online Publication Only)

B.1 Other Equilibria under FW

B.1.1 Exploitation Equilibrium under FW

This section considers the possibility of an exploitation equilibrium under FW in which the agent chooses o in period 1. In this case, θ^n is unknown in period 2 on the equilibrium path, and following the argument in (1) to (3) the agent can only be induced to choose o in period 2. The principal then optimally sets $\beta^* = Y$ and $\alpha^* = U^0 - \max_\varepsilon \left\{\varepsilon Y - \psi\left(\frac{\varepsilon}{\theta^o}\right)\right\} = U^0 - s\left(Y, \theta^o\right)$ where $s\left(\cdot\right)$ is as defined in (10).

Under this period-2 contract, if the agent had deviated to $\tau = n$ in period 1 and privately knows θ^n, his period-2 payoff under innovation outcome $\theta = \max\left\{\theta^o, \theta^n\right\}$ is

$$\begin{aligned} & \alpha^* + \max_\varepsilon \left\{\varepsilon Y - \psi\left(\frac{\varepsilon}{\theta}\right)\right\} \\ = \; & U^0 + s\left(Y, \theta\right) - s\left(Y, \theta^o\right). \end{aligned}$$

His ex-ante expected period-2 payoff from deviation is then $\int_{\theta^o}^{\bar{\theta}} s\left(Y, \theta\right) dF(\theta) - s\left(Y, \theta^o\right) + U^0 > U^0$.

Consider period 1 next. Analogous to (17) and (18), let the agent's total expected payoffs for choosing technology τ in period 1 together with effort e and under a contract (a, b) be $a + \tilde{W}^\tau(e, b)$, where:

$$\begin{aligned} \tilde{W}^n(e,b) &:= eb - E\left[\psi\left(\frac{e}{\theta^n}\right)\right] + \int_{\theta^o}^{\bar{\theta}} s\left(Y, \theta\right) dF(\theta) - s\left(Y, \theta^o\right) + U^0, \\ \tilde{W}^o(e,b) &:= eb - \psi\left(\frac{e}{\theta^o}\right) + U^0. \end{aligned}$$

In period 1, on the equilibrium path, the principal always offers a zero period-1 fixed wage: $a^* = 0$. Let $\tilde{e}^\tau(b) := \arg\max_e \tilde{W}^\tau(e, b)$,[49] and $\bar{\tilde{W}}^\tau(b) := \max_e \tilde{W}^\tau(e, b)$. Following the proof of Lemma 3, it is readily verified that $\bar{\tilde{W}}^o(b) - \bar{\tilde{W}}^n(b)$ is strictly increasing in b, and there exists $\underline{b} > 0$ such that $\bar{\tilde{W}}^o(b) \leq \bar{\tilde{W}}^n(b)$ if and only if $b \leq \underline{b}$. Analogous to the argument for Proposition 3, the following result is immediate:

Proposition B.1. *(Exploitation equilibrium under FW.) Under FW, an exploitation equi-*

[49] $\tilde{e}^\tau(\cdot)$ is equivalent to $e^\tau(\cdot)$ in (20).

librium (i.e. the agent chooses technology o in period 1) exists if and only if $\underline{b}$ is finite. When $\underline{b}$ is finite, the optimal period-1 contract that induces the agent to choose technology o in period 1 is unique:

$$\begin{aligned} a^* &= 0 \\ b^* &= \max\left\{\tilde{b}, \underline{b}\right\} \end{aligned}$$

where $\tilde{b} := \arg\max_b \{\tilde{e}^o(b)\,[Y-b]\}$.

Notice that under this equilibrium, the principal's period-2 payoff is $s\,(Y, \theta^o)$ which is exactly what she gets in period 2 from the optimal contract under TM as characterized in Proposition 1. On the other hand, her expected period-1 payoff must be weakly lower given the additional constraint of $b^* \geq \underline{b}$. Hence:

Corollary B.1. *If an exploitation equilibrium exists under FW, the principal's total expected payoffs from this equilibrium is weakly lower than her total expected payoffs from the unique equilibrium under TM as characterized in Proposition 1.*

B.1.2 Mixed Exploration-Exploitation under FW

This section considers equilibria in which the agent chooses mixed strategy in his technology choice in period 1 under FW. Consider an equilibrium in which the agent plays technology n in period 1 with probability ρ. In period 2, if the principal knows that the agent had chosen o in period 1, she will offer bonus $\beta = Y$ and fixed fee $\alpha = U^0 - s\,(Y, \theta^o)$. If the principal knows that the agent had chosen n in period 1, she will offer the agent the menu of contracts characterized in Proposition 2. Notice that an agent with a successful innovation always prefers the first contract to his contract in the menu and will thus always report that he has chosen o in period 1. Hence the principal can never elicit information about the agent's period-1 technology choice *before* offering the agent a menu of contracts.

As in Section 5, the agent's payoff-relevant private type is still $\left[\theta^o, \bar{\theta}\right]$, where the agent's type when he had chosen o in period 1 is θ^o. The difference here from Section 5 is the principal's belief about the type distribution. Now, for any $\theta \in \left[\theta^o, \bar{\theta}\right]$:

$$Pr\left(\theta^n \leq \theta\right) = 1 - \rho + \rho\left[F(\theta^o) + \int_{\theta^o}^{\theta} f(t)dt\right] \tag{B.1}$$

where f and F are, as previously, the pdf and cdf of θ^n respectively.

Consider the following density function:

$$\tilde{f}(\theta^n;\rho) = \begin{cases} \left[\frac{1-\rho}{F(\theta^o)}+\rho\right] f(\theta^n) & , \ \theta^n \le \theta^o \\ \rho f(\theta^n) & , \ \theta^n > \theta^o \end{cases},$$

and let $\tilde{F}(\cdot;\rho)$ denote its corresponding cdf; it is readily verified that $\tilde{f}(\cdot;\rho)$ is a valid density for any $\rho \in [0,1]$. Notice that under $\tilde{f}(\cdot;\rho)$, for any $\theta \in \left[\theta^o, \bar{\theta}\right]$,

$$\begin{aligned} Pr(\theta^n \le \theta) &= \int_{\underline{\theta}}^{\theta^o} \left[\frac{1-\rho}{F(\theta^o)}+\rho\right] f(t)\,dt + \int_{\theta^o}^{\theta} \rho f(t)dt \\ &= 1-\rho+\rho\left[F(\theta^o)+\int_{\theta^o}^{\theta} f(t)dt\right] \end{aligned}$$

as in (B.1). In other words, if the principal conjectures that the agent plays technology n in period 1 with probability ρ, it is equivalent to her designing a period-2 menu of contracts based on the belief that θ^n follows a distribution of $\tilde{f}(\cdot;\rho)$. Notice that $\tilde{f}(\cdot;1) = f(\cdot)$. Moreover Assumption 2 is satisfied for $f(\theta^n;\rho)$ for all $\theta^n \in \left(\theta^o, \bar{\theta}\right]$ and $\rho \in (0,1]$. Hence all results from Section 5 hold for any $\rho \in (0,1]$ (see Remark 3).

Let $\tilde{H}(\cdot;\rho) := \frac{1-\tilde{F}(\cdot;\rho)}{\tilde{f}(\cdot;\rho)}$ be the inverse hazard rate associated with $\tilde{f}(\cdot;\rho)$. For all $\theta^n > \theta^o$:

$$\begin{aligned} \tilde{H}(\theta^n;\rho) &= \frac{\int_{\theta^n}^{\bar{\theta}} \rho f(t)dt}{\rho f(\theta^n)} \\ &= H(\theta^n). \end{aligned}$$

For $\theta^n \le \theta^o$:

$$\begin{aligned} \tilde{H}(\theta^n;\rho) &= \frac{\int_{\theta^n}^{\theta^o}\left[\frac{1-\rho}{F(\theta^o)}+\rho\right] f(t)dt + \int_{\theta^o}^{\bar{\theta}} \rho f(t)dt}{\left[\frac{1-\rho}{F(\theta^o)}+\rho\right] f(\theta^n)} \\ &= H(\theta^n) - \left[\frac{1-\rho}{1-\rho+\rho F(\theta^o)}\right]\frac{1-F(\theta^o)}{f(\theta^n)}. \end{aligned}$$

$F(\theta^o) < 1$ implies that $\tilde{H}(\theta^n;\rho)$ is strictly increasing in ρ for all $\theta^n \in \left(\theta^o, \bar{\theta}\right]$.[50] Hence $\rho \in (0,1]$ parametrizes the distribution as in Assumption 4 (with the upper bound of the support fixed for all ρ) and it is readily verified that all results in Section 6 apply with weak

[50] $\frac{d}{d\rho}\left[\frac{1-\rho}{1-\rho+\rho F(\theta^o)}\right] = \frac{-(1-\rho+\rho F(\theta^o)-(1-\rho)(-1+F(\theta^o)}{[1-\rho+\rho F(\theta^o)]^2} = -\frac{F(\theta^o)}{[1-\rho+\rho F(\theta^o)]^2} < 0$. Hence $\frac{d}{d\rho}\tilde{H}(\theta^n;\rho) > 0$.

inverse hazard rate dominance.[51] Hence:

Proposition B.2. *The principal's expected period-2 payoff under FW is strictly increasing in ρ, the principal's conjecture probability of the agent choosing technology n in period 1.*

Proof. Follows from Proposition 5.2. □

Let $R(\rho)$ be the agent's expected period-2 rent (net of outside option) given the principal's conjecture ρ. For the agent to mix between o and n in period 1, the bonus b in period 1 must satisfy:

$$\max_e \left\{ eb - E\left[\psi\left(\frac{e}{\theta^n}\right)\right] + R(\rho)\right\} = \max_e \left\{ eb - \psi\left(\frac{e}{\theta^o}\right)\right\}. \tag{B.2}$$

Let $\bar{b}(\rho)$ denote the bonus threshold characterized in Lemma 3 for $R = R(\rho)$. If $\bar{b}(\rho)$ is finite, then it is the unique solution bonus to (B.2); if $\bar{b}(\rho)$ is infinite, then there is no bonus level satisfying (B.2).

Proposition B.3. *Under FW, a mixed exploration-exploitation equilibrium in which the agent chooses n with probability ρ exists if and only if $\bar{b}(\rho)$ is finite.*

Suppose a mixed exploration-exploitation equilibrium for ρ exists. The principal's period-1 payoff is:

$$\left[\rho e^n\left(\bar{b}(\rho)\right) + (1-\rho)e^o\left(\bar{b}(\rho)\right)\right]\left(Y - \bar{b}(\rho)\right) \tag{B.3}$$

where $e^\tau(\cdot)$ is as defined in (20). The following provides a sufficient condition for when a mixed exploration-exploitation equilibrium is dominated by a pure exploration equilibrium for the principal:

Proposition B.4. *Suppose* $\arg\max_b \{e^n(b)[Y-b]\} < \bar{b}(1)$*. Then there exists* $\delta > 0$ *such that* $\forall \rho \in (1-\delta, 1)$*, the principal's total expected payoffs from the principal-optimal exploration equilibrium, as characterized in Section 5, is higher than her payoffs from a mixed exploration-exploitation equilibrium in which the agent chooses n with probability ρ, whenever such a mixed exploration-exploitation equilibrium exists.*

Proof. Taking the limit $\rho \to 1$ for (B.3) gives $e^n\left(\bar{b}(1)\right)\left(Y - \bar{b}(1)\right) \leq \max_b \{e^n(b)[Y-b]\}$. Under the assumption that $\arg\max_b \{e^n(b)[Y-b]\} < \bar{b}(1)$, the principal's period-1 payoff in the optimal pure exploration equilibrium is $\max_b \{e^n(b)[Y-b]\}$ (see Proposition 3). This implies that there exists $\delta > 0$ such that $\forall \rho \in (1-\delta, 1)$, the principal's period-1 payoff from

[51] That is, if Assumption 4.2 had been "$k'' > k'$ implies that $H(\theta^n; k'') \geq H(\theta^n; k')\ \forall \theta^n$ and $H(\theta^n; k'') > H(\theta^n; k')\ \forall \theta^n$ in some non-zero measure subset of $[\underline{\theta}, \bar{\theta}]$."

the principal-optimal pure exploration equilibrium is higher than her period-1 payoff from a mixed exploration-exploitation equilibrium in which the agent chooses n with probability ρ. The result of the proposition then follows given Proposition B.2. □

B.2 Additional Details for Section 6

B.2.1 Proof of Remark 5.

Lemma. *Under Assumption 4, $k'' > k'$ implies that $F(\theta^n; k'') < F(\theta^n; k')$ $\forall \theta^n \in \left[\underline{\theta}, \bar{\theta}(k'')\right)$.*

Proof. Since $F(\theta; k') = 1$ for $\theta \in \left[\bar{\theta}(k'), \bar{\theta}(k'')\right]$ and $f(\cdot; k'')$ is strictly positive over its support $\left[\underline{\theta}, \bar{\theta}(k'')\right]$, the lemma is trivially true in $\left[\bar{\theta}(k'), \bar{\theta}(k'')\right]$. Consider $\theta \in \left[\underline{\theta}, \bar{\theta}(k')\right]$ next. Notice that:

$$\begin{aligned} \int_{\underline{\theta}}^{\theta} \frac{1}{H(t;k)} dt &= \int_{\underline{\theta}}^{\theta} \frac{f(\theta;k)}{1-F(\theta;k)} dt \\ &= -\log\left[1-F(\theta;k)\right] \end{aligned}$$

For any $\theta \in \left[\underline{\theta}, \bar{\theta}(k')\right]$:

$$\begin{aligned} & \textstyle\int_{\underline{\theta}}^{\theta} \frac{1}{H(t;k')} dt > \int_{\underline{\theta}}^{\theta} \frac{1}{H(t;k'')} dt \\ \Longrightarrow \quad & -\log\left[1-F(\theta;k')\right] > -\log\left[1-F(\theta;k'')\right] \\ \Longrightarrow \quad & F(\theta;k') > F(\theta;k'') \end{aligned}$$

□

B.2.2 Proof of Corollary 3 and the Effect of k on the Agent's Period-2 Rent

I follow the notations $\hat{\beta}(\cdot; k)$, $\ddot{\theta}(k)$ and $\ddot{\beta}(k)$ as in the proof of Proposition 5.

Lemma B.1. *If $\hat{\beta}_{\theta k}(\theta; k) \leq 0$ $\forall \theta, k$, then $R(k)$ is strictly increasing in k.*[52]

Proof. From (16), $R(k) = \int_{\theta^o}^{\bar{\theta}(k)} u_\theta(\beta^*(\theta;k), \theta)\left[1-F(\theta;k)\right] d\theta$. Note that $u_\theta(\beta^*(\theta;k), \theta)$ is differentiable in θ everywhere except at $\ddot{\theta}(k)$. Define $Q(\theta;k) := \int_{\underline{\theta}}^{\theta}\left[1-F(t;k)\right] dt > 0$. Doing

[52] Since $f(\theta; k)$ is absolutely continuous in k, $\hat{\beta}_\theta(\theta; k)$ is differentiable in k almost everywhere.

integration by parts:

$$
\begin{aligned}
R(k) &= u_\theta\left(\beta^*(\theta;k),\theta\right)Q(\theta;k)\Big|_{\theta^o}^{\bar\theta(k)} - \int_{\theta^o}^{\bar\theta(k)}\left[\frac{d}{d\theta}u_\theta\left(\beta^*(\theta;k),\theta\right)\right]Q(\theta;k)d\theta \\
&= u_\theta\left(Y,\bar\theta(k)\right)Q(\bar\theta(k);k) - u_\theta\left(\ddot\beta(k),\theta^o\right)Q(\theta^o;k) \\
&\quad - \int_{\ddot\theta(k)}^{\bar\theta(k)}\left[\frac{d}{d\theta}u_\theta\left(\hat\beta(\theta;k),\theta\right)\right]Q(\theta;k)d\theta - \int_{\theta^o}^{\ddot\theta(k)}\left[\frac{d}{d\theta}u_\theta\left(\ddot\beta(k),\theta\right)\right]Q(\theta;k)d\theta.
\end{aligned}
$$

Differentiating with respect to k now:[53]

$$
\begin{aligned}
&\frac{d}{dk}R(k) \\
= &\underbrace{\left[u_{\theta\theta}\left(Y,\bar\theta(k)\right)Q(\bar\theta(k);k) + u_\theta\left(Y,\bar\theta(k)\right)\underbrace{Q_\theta(\bar\theta(k);k)}_{=1-F(\bar\theta(k);k)=0} - \underbrace{\left[\frac{d}{d\theta}u_\theta\left(\hat\beta(\bar\theta(k);k),\bar\theta(k)\right)\right]}_{=u_{\theta\theta}\left(Y,\bar\theta(k)\right)}Q(\bar\theta(k);k)\right]\bar\theta'(k)}_{=0} \\
&+\underbrace{u_\theta\left(Y,\bar\theta(k)\right)Q_k(\bar\theta(k);k) - u_\theta\left(\ddot\beta(k),\theta^o\right)Q_k(\theta^o;k)}_{=\text{Term 1}} - \underbrace{\left[\frac{d}{dk}u_\theta\left(\ddot\beta(k),\theta^o\right)\right]Q(\theta^o;k)}_{<0} \\
&\underbrace{+\left[\frac{d}{d\theta}u_\theta\left(\hat\beta\left(\ddot\theta(k);k\right),\ddot\theta(k)\right)\right]Q\left(\ddot\theta(k);k\right)\cdot\ddot\theta'(k) - \left[\frac{d}{d\theta}u_\theta\left(\ddot\beta(k),\ddot\theta(k)\right)\right]Q\left(\ddot\theta(k);k\right)\cdot\ddot\theta'(k)}_{=0} \\
&-\underbrace{\int_{\ddot\theta(k)}^{\bar\theta(k)}\frac{d}{dk}\left(\left[\frac{d}{d\theta}u_\theta\left(\hat\beta(\theta;k),\theta\right)\right]Q(\theta;k)\right)d\theta}_{=\text{Term 2}} - \underbrace{\int_{\theta^o}^{\ddot\theta(k)}\frac{d}{dk}\left(\left[\frac{d}{d\theta}u_\theta\left(\ddot\beta(k),\theta\right)\right]Q(\theta;k)\right)d\theta}_{=\text{Term 3}}
\end{aligned}
$$

Hence:

$$
\frac{d}{dk}R(k) > (\text{Term 1}) - (\text{Term 2}) - (\text{Term 3}).
$$

[53]The derivative exists almost everywhere.

Next:

$$\begin{aligned}
\text{Term 1} &= u_\theta\Big(\underbrace{\hat\beta(\bar\theta(k);k)}_{=Y},\bar\theta(k)\Big)Q_k(\bar\theta(k);k) - u_\theta\Big(\underbrace{\hat\beta\left(\ddot\theta(k);k\right)}_{=\ddot\beta(k)},\ddot\theta(k)\Big)Q_k\left(\ddot\theta(k);k\right) \\
&\quad + u_\theta\left(\ddot\beta(k),\ddot\theta(k)\right)Q_k\left(\ddot\theta(k);k\right) - u_\theta\left(\ddot\beta(k),\theta^o\right)Q_k(\theta^o;k) \\
&= \int_{\ddot\theta(k)}^{\bar\theta(k)}\frac{d}{d\theta}\left(u_\theta\left(\hat\beta(\theta;k),\theta\right)Q_k(\theta;k)\right)d\theta + \int_{\theta^o}^{\ddot\theta(k)}\frac{d}{d\theta}\left(u_\theta\left(\ddot\beta(k),\theta\right)Q_k(\theta;k)\right)d\theta \\
&= \int_{\ddot\theta(k)}^{\bar\theta(k)}\left(\left[\frac{d}{d\theta}u_\theta\left(\hat\beta(\theta;k),\theta\right)\right]Q_k(\theta;k) \;+\; u_\theta\left(\hat\beta(\theta;k),\theta\right)Q_{k\theta}(\theta;k)\right)d\theta \\
&\quad + \int_{\theta^o}^{\ddot\theta(k)}\left(\left[\frac{d}{d\theta}u_\theta\left(\ddot\beta(k),\theta\right)\right]Q_k(\theta;k) \;+\; u_\theta\left(\ddot\beta(k),\theta\right)Q_{k\theta}(\theta;k)\right)d\theta
\end{aligned}$$

and

$$\begin{aligned}
\text{Term 2} &= \int_{\ddot\theta(k)}^{\bar\theta(k)}\left[\frac{d}{d\theta}u_\theta\left(\hat\beta(\theta;k),\theta\right)\right]Q_k(\theta;k)d\theta + \int_{\ddot\theta(k)}^{\bar\theta(k)}\left[\frac{d^2}{d\theta dk}u_\theta\left(\hat\beta(\theta;k),\theta\right)\right]Q(\theta;k)d\theta \\
\text{Term 3} &= \int_{\theta^o}^{\ddot\theta(k)}\left[\frac{d}{d\theta}u_\theta\left(\ddot\beta(k),\theta\right)\right]Q_k(\theta;k)d\theta + \int_{\theta^o}^{\ddot\theta(k)}\left[\frac{d^2}{d\theta dk}u_\theta\left(\ddot\beta(k),\theta\right)\right]Q(\theta;k)d\theta
\end{aligned}$$

Hence

$$\begin{aligned}
&(\text{Term 1}) - (\text{Term 2}) - (\text{Term 3}) \\
= &\int_{\ddot\theta(k)}^{\bar\theta(k)} u_\theta\left(\hat\beta(\theta;k),\theta\right)Q_{k\theta}(\theta;k)d\theta \;+\; \int_{\theta^o}^{\ddot\theta(k)} u_\theta\left(\ddot\beta(k),\theta\right)Q_{k\theta}(\theta;k)d\theta \\
&-\int_{\ddot\theta(k)}^{\bar\theta(k)}\underbrace{\left[u_{\theta\beta\beta}\left(\hat\beta(\theta;k),\theta\right)\hat\beta_\theta(\theta;k)\hat\beta_k(\theta;k) + u_{\theta\beta}\left(\hat\beta(\theta;k),\theta\right)\hat\beta_{\theta k}(\theta;k) + u_{\theta\theta\beta}\left(\hat\beta(\theta;k),\theta\right)\hat\beta_k(\theta;k)\right]}_{=\frac{d^2}{d\theta dk}u_\theta\left(\hat\beta(\theta;k),\theta\right)}Q(\theta;k)d\theta \\
&-\int_{\theta^o}^{\ddot\theta(k)}\underbrace{\left[u_{\theta\theta\beta}\left(\ddot\beta(k),\theta\right)\ddot\beta_k(k)\right]}_{=\frac{d^2}{d\theta dk}u_\theta\left(\ddot\beta(k),\theta\right)}Q(\theta;k)d\theta
\end{aligned}$$

Since $Q_{k\theta}(\theta;k) = -F_k(\theta;k) > 0$, the first line is positive. From the proof of Lemma 1, $u_{\theta\beta} > 0$ and it is readily verified that $u_{\theta\theta\beta}, u_{\theta\beta\beta} > 0$; from the proof of Proposition 5.1, $\ddot\beta_k(k) < 0$. Hence the last line is positive. If $\hat\beta_{\theta k}(\theta;k) \le 0$, then the second line is also positive. This in turn implies that $\frac{d}{dk}R(k) > 0$. □

Corollary 3 is then established by checking that the condition $\hat\beta_{\theta k}(\theta;k) \le 0$ holds under the given parametrization.

Proof of Corollary 3

Proof. When $\psi(x) = \xi x^2$, it is readily verified that $\varepsilon(\beta, \theta) = \frac{1}{2\xi}\beta\theta^2$. From (13), $\hat{\beta}(\theta) = \frac{\theta Y}{2H(\theta)+\theta}$. When $\mathcal{F}$ is the family of uniform distributions in (21), $H(\theta; k) = k - \theta$. Hence $\hat{\beta}(\theta; k) = \frac{\theta}{2k-\theta}Y$ and it is readily verified that $\hat{\beta}_{\theta k}(\theta; k) < 0$. The result then follows from Lemma B.1. □

B.3 Innovative Effort

The section considers the extension discussed in Section 7.2. In particular, I extend the baseline model to incorporate innovative effort in a stylized way and develop some insights on the nature of innovative effort.

Suppose now that in period 1, when the agent chooses technology n, on top of the choice of e, the agent also gets to privately choose another dimension of effort denoted by $i \in [0, \bar{i}]$ with $\bar{i} < \frac{1}{2}$. One should interpret i as an *innovative effort* which is exerted only during the innovative stage (period 1), as opposed to e and ε which are *productive efforts* that are exerted in both periods. Let p be a commonly known marginal productivity parameter for technology n with the assumption that $p \in (\bar{i}, 1 - \bar{i})$.[54] When the agent exerts innovative effort i and productive effort e on technology n in period 1, the probability of getting outcome Y in that period is $(p - i)e$. On the other hand, come period 2, the probability of getting outcome Y under a period-2 productive effort level ε becomes $(p + i)\varepsilon$. Hence the choice of innovative effort i is a deterministic transfer of marginal productivity of productive effort for technology n across periods.

The rest of the model remains unchanged from Section 3. The reason to why this formulation can be an appropriate representation for the issue at hand will be made clear below after Lemma B.2. Since technology o is unaltered, it is immediate that Proposition 1 continues to hold here. The focus thus remains on inducing the agent to explore in period 1 under FW and studying the effect of having the additional dimension of innovative effort i.

Denote the marginal productivity of productive effort for technology n in period 2 by $\pi(i) := p + i$. Previously an innovation is considered successful if the θ^n realization is higher that θ^o. This is no longer the case here because of the difference in the marginal productivity of productive effort between the two technologies in period 2, which is dependent on the

[54] p was 1 in the baseline model.

choice of i in period 1.

Lemma B.2. *For any $\beta > 0$, $\max_{\varepsilon}\left\{\pi(i)\varepsilon\beta - \psi\left(\frac{\varepsilon}{\theta^n}\right)\right\} \geq \max_{\varepsilon}\left\{\varepsilon\beta - \psi\left(\frac{\varepsilon}{\theta^o}\right)\right\}$ if and only if $\pi(i)\theta^n \geq \theta^o$.*

Proof. Let $p^n = \pi(i)$ and $p^o = 1$. For $\tau \in \{o, n\}$, let $\varepsilon^\tau = \arg\max_\varepsilon \left\{p^\tau \varepsilon\beta - \psi\left(\frac{\varepsilon}{\theta^\tau}\right)\right\}$ which is uniquely characterized by

$$p^\tau \beta = \frac{1}{\theta^\tau}\psi'\left(\frac{\varepsilon^\tau}{\theta^\tau}\right). \tag{B.4}$$

Hence

$$\max_{\varepsilon}\left\{p^\tau\varepsilon\beta - \psi\left(\frac{\varepsilon^\tau}{\theta^\tau}\right)\right\} = \frac{\varepsilon^\tau}{\theta^\tau}\psi'\left(\frac{\varepsilon^\tau}{\theta^\tau}\right) - \psi\left(\frac{\varepsilon^\tau}{\theta^\tau}\right).$$

Convexity of ψ implies that $x\psi'(x) - \psi(x)$ is increasing in x for any positive x. Hence for any $\beta > 0$:

$$\begin{aligned}
& \max_\varepsilon \left\{(\pi(i)\varepsilon)\beta - \psi\left(\tfrac{\varepsilon}{\theta^n}\right)\right\} \geq \max_\varepsilon \left\{\varepsilon\beta - \psi\left(\tfrac{\varepsilon}{\theta^o}\right)\right\} \\
\iff \quad & \tfrac{\varepsilon^n}{\theta^n}\psi'\left(\tfrac{\varepsilon^n}{\theta^n}\right) - \psi\left(\tfrac{\varepsilon^n}{\theta^n}\right) \geq \tfrac{\varepsilon^o}{\theta^o}\psi'\left(\tfrac{\varepsilon^o}{\theta^o}\right) - \psi\left(\tfrac{\varepsilon^o}{\theta^o}\right) \\
\iff \quad & \tfrac{\varepsilon^n}{\theta^n} \geq \tfrac{\varepsilon^o}{\theta^o} \\
\iff \quad & p^n\theta^n\beta \geq p^n\theta^n\beta \\
\iff \quad & \pi(i)\theta^n \geq \theta^o
\end{aligned}$$

where the second last line follows from (B.4). This thus establishes the result. □

Lemma B.2 describes the agent's period-2 technology choice after choosing $\tau = n$ in period 1 with innovative effort level i. It is also clear that $\max_\varepsilon \left\{\pi(i)\varepsilon\beta - \psi\left(\frac{\varepsilon}{\theta^n}\right)\right\}$ is increasing in $\pi(i)$. From this, the efficiency of technology n in period 2 and hence, the degree of innovation success, can be represented by $\pi(i)\theta^n$. Since $\pi(i)$ is increasing in i, exerting i increases both the probability and the degree of innovation success, and is thus an appropriate representation of innovative effort. Although the inter-temporal transfer of marginal productivity due to innovative effort i is deterministic, the marginal effect of innovative effort on the degree of innovation success is θ^n, which is stochastic.

B.3.1 Period 2 with Innovative Effort

To fix idea, let us first suppose that the agent's choice of innovative effort i in period 1 is also observed by the principal. $\pi(i)$ is thus common knowledge in period 2. The agent's private type is now (with an abuse of notation on θ) $\theta := \max\{\theta^o, \pi(i)\theta^n\} \in \left[\theta^o, \bar{\theta}(i)\right]$, where $\bar{\theta}(i) := \pi(i)\bar{\theta}$. Let $f(\cdot|i)$ and $F(\cdot|i)$ denote the pdf and cdf of $\pi(i)\theta^n$ respectively.[55] The distribution of θ is then $Pr(\theta \leq x|i) = F(\theta^o|i) + \int_{\theta^o}^{x} f(t|i)\, dt$.

As in expression (7), it will be convenient to work with the agent's indirect utility function. The analog of $u(\beta, \theta)$ in (7) is defined here as:

$$u(\beta, \theta|i) := \max_{\varepsilon} \left\{ \pi(i)\varepsilon\beta - \psi\left(\frac{\varepsilon}{\theta}\right) \right\}.$$

It is readily verified that the properties in Lemma 1 continue to hold for any i. The optimal menu of period-2 contracts is thus characterized as in Proposition 2 with the appropriate modifications to the distribution and indirect utility functions as described.

When the principal does not observe the agent's choice of innovative effort i in period 1, she designs the menu of contracts in period 2 based on her conjecture on i. Note that the outcome of y in period 1 does not affect her conjecture. Consistency in beliefs in equilibrium requires that the principal's conjecture is correct, and this issue will be handled later.

Complications arise regarding which contract an agent who has deviated in period 1 to an innovative effort $\tilde{i} \neq i$ will optimally choose in period 2, and what is his resulting (off-equilibrium path) payoffs. In particular, the agent's private type space is now two-dimensional ($\tilde{i}$ and θ^n), and multidimensional screening problems are in general very difficult to handle.

The advantage of the formulation here is that under Lemma B.2, the agent's payoff-relevant private information can still be summarized by the one-dimensional variable (abusing notation of θ again) $\theta := \max\left\{\pi(\tilde{i})\theta^n, \theta^o\right\}$ where $\tilde{i}$ is his privately known innovative effort exerted in period 1. Notice that if θ is in $\left[\theta^o, \bar{\theta}(i)\right]$, the type-space according to the principal's conjecture, the agent will still truthfully report his type since each contract in the menu satisfies the truth-telling constraint. The only concern is what an agent with type $\theta > \bar{\theta}(i)$ will choose; such an agent can exist if $\tilde{i} > i$. The following lemma establishes that such an agent will optimally claim to be $\bar{\theta}(i)$, the highest type that the principal expects to exist:

Lemma B.3. *Suppose the principal conjectures that the agent has exerted innovative effort i in period 1 and thus offers him a menu of contracts $\beta^* : \left[\theta^o, \bar{\theta}(i)\right] \to [0, Y]$ and $\alpha^* : \left[\theta^o, \bar{\theta}(i)\right] \to \mathbb{R}$ according to Proposition 2. Facing this menu of contracts, an agent with type $\theta \in \left[\theta^o, \bar{\theta}(i)\right]$ will report his type truthfully, and an agent with type $\theta > \bar{\theta}(i)$ will report his type to be $\bar{\theta}(i)$.*

[55] $f(z|i) = \frac{1}{\pi(i)} f\left(\frac{z}{\pi(i)}\right)$ and $F(z|i) = F\left(\frac{z}{\pi(i)}\right)$.

Proof. Incentive compatibility for type $\bar{\theta}(i)$ implies that for all $\tilde{\theta} < \bar{\theta}(i)$:

$$\left(\beta\left(\bar{\theta}(i)\right), \bar{\theta}(i)\right) - u\left(\beta\left(\tilde{\theta}\right), \bar{\theta}(i)\right) \geq \alpha\left(\tilde{\theta}\right) - \alpha\left(\bar{\theta}(i)\right).$$

The Spence-Mirrlees condition (see Lemma 1) implies that for any $\theta > \bar{\theta}(i)$,

$$\begin{aligned} u\left(\beta\left(\bar{\theta}(i)\right), \theta\right) - u\left(\beta\left(\tilde{\theta}\right), \theta\right) &> u\left(\beta\left(\bar{\theta}(i)\right), \bar{\theta}(i)\right) - u\left(\beta\left(\tilde{\theta}\right), \bar{\theta}(i)\right) \\ &\geq \alpha\left(\tilde{\theta}\right) - \alpha\left(\bar{\theta}(i)\right). \end{aligned}$$

The last inequality then establishes that type $\theta > \bar{\theta}(i)$ prefers the contract of $\bar{\theta}(i)$ to that of any $\tilde{\theta} < \bar{\theta}(i)$. □

With this, the agent's expected period-2 information rent both on and off equilibrium path are readily computed. Let $R\left(\tilde{i}\middle|i\right)$ be the agent's expected period-2 information rent (net of outside option U^0) under innovative effort level $\tilde{i}$ and facing a menu of contracts designed for an agent with innovative effort i (i.e. the principal conjectures i).

Lemma B.4. *The agent's expected information rent, $R\left(\cdot\middle|i\right)$, is strictly increasing and strictly convex for any principal's conjecture i.*

Proof. Suppose that the principal conjectures the innovative effort to be i and offers the optimal menu of contracts $\{\alpha^*(\cdot|i), \beta^*(\cdot|i)\}$ as characterized in Proposition 2. Let $rent\left(\theta^n, \tilde{i}\middle|i\right)$ be the information rent (net of outside option U^0) of an agent who has exerted innovative effort $\tilde{i}$ in period 1, has a θ^n realization and faces a menu of contracts designed by a principal with conjecture i. From the result in Lemma B.3:

$$rent\left(\theta^n, \tilde{i}\middle|i\right) = \begin{cases} 0 & \text{if } \pi(\tilde{i})\theta^n < \theta^o, \\ \int_{\theta^o}^{\pi(\tilde{i})\theta^n} u_\theta\left(\beta^*(t|i), t\right) dt & \text{if } \theta^o \leq \pi(\tilde{i})\theta^n \leq \bar{\theta}(i), \\ \int_{\theta^o}^{\bar{\theta}(i)} u_\theta\left(\beta^*(t|i), t\right) dt + \int_{\bar{\theta}(i)}^{\pi(\tilde{i})\theta^n} u_\theta\left(Y, t\right) dt & \text{if } \bar{\theta}(i) < \pi(\tilde{i})\theta^n. \end{cases}$$

$rent\left(\theta^n, \cdot|i\right)$ is continuous and piece-wise differentiable for all θ^n and i:

$$\frac{d}{d\tilde{i}} rent\left(\theta^n, \tilde{i}\middle|i\right) = \begin{cases} 0 & \text{if } \pi(\tilde{i})\theta^n < \theta^o, \\ u_\theta\left(\beta^*\left(\pi(\tilde{i})\theta^n\middle|i\right), \pi(\tilde{i})\theta^n\right) \cdot \theta^n & \text{if } \theta^o \leq \pi(\tilde{i})\theta^n \leq \bar{\theta}(i), \\ u_\theta\left(Y, \pi(\tilde{i})\theta^n\right) \cdot \theta^n & \text{if } \bar{\theta}(i) < \pi(\tilde{i})\theta^n. \end{cases}$$

By noting that u_θ is positive and strictly increasing in both its argument, and that $\beta^*\left(\pi(\tilde{i})\theta^n \middle| i\right) \leq Y$ for $\pi(\tilde{i})\theta^n \leq \bar{\theta}(i)$, the derivatives are strictly positive and increasing everywhere for $\pi(\tilde{i})\theta^n \geq \theta^o$. Hence $rent\left(\theta^n, \cdot|i\right)$ is increasing and convex. This implies that $E\left[rent\left(\theta^n, \tilde{i}|i\right)\right]$ is strictly increasing in $\tilde{i}$. To see that it is also convex, note that expectation is a linear operator which preserves convexity. □

B.3.2 Period 1 with Innovative Effort

With the agent's period-2 expected rent both on and off equilibrium paths accounted for, I consider the innovative effort level that can be induced in equilibrium next. Under a contract (a, b) that prescribes the agent to choose technology n in period 1 together with an innovative effort i (where i then forms the principal's conjecture in period 2), the agent's total expected payoff for choosing n together with productive effort e and an innovative effort $\tilde{i}$ is $a + W^n(\tilde{i}, e, b|i)$ where:

$$W^n(\tilde{i}, e, b|i) := (p - \tilde{i})eb - E\left[\psi\left(\frac{e}{\theta^n}\right)\right] + R\left(\tilde{i}\middle|i\right) + U^0.$$

His total expected payoff for deviating to choose technology o instead is $a + W^o(e, b)$ as in (18). With the additional choice of innovative effort now, the equilibrium definition is slightly different from that in Definition 1. The differences are incorporated in the following definition:

Definition B.1. Under the setting with innovative effort, a contract (a, b), together with the agent choosing technology n, productive effort e^* and innovative effort i^*, forms an equilibrium if the following conditions are satisfied:

$$\begin{aligned}
\text{(IR-1}i\text{)}: &\quad a + W^n(i^*, e^*, b|i^*) \geq U^0 \\
\text{(IC-}\tau\text{-1}i\text{)}: &\quad W^n(i^*, e^*, b|i^*) \geq \max_e W^o(e, b) \\
\text{(IC-}e,i\text{-1}i\text{)}: &\quad (e^*, i^*) \in \arg\max_{e,\tilde{i}} W^n(\tilde{i}, e, b|i^*) \\
\text{(LL-1}i\text{)}: &\quad a, b \geq 0
\end{aligned}$$

The next proposition, illustrated in Figure 2, characterizes the set of possible equilibria associated with each contract (a, b).

Proposition B.5. *Under FW in the setting with innovative effort, for any principal's conjecture* $i \in [0, \bar{i}]$, *there exist thresholds* $\bar{b}^l(i)$ *and* $\bar{b}^h(i)$, *with* $0 < \bar{b}^l(i) \leq \bar{b}^h(i)$, *such that:*

1. *For all* $b \leq \bar{b}^l(i)$*:*

$$\max_e W^o(e,b) \leq \max_{e,\tilde{i}} W^n(\tilde{i},e,b|i) = \max_e W^n\left(\bar{i},e,b|i\right).$$

In words, the agent's optimal actions are to choose technology n *in period 1 and maximum innovative effort* $i = \bar{i}$ *(together with the appropriate productive effort* e *on* n*).*

2. *If* $\bar{b}^l(i) < \bar{b}^h(i)$*, then for all* $b \in \left[\bar{b}^l(i), \bar{b}^h(i)\right]$*:*

$$\max_e W^o(e,b) \leq \max_{e,\tilde{i}} W^n(\tilde{i},e,b|i) = \max_e W^n\left(0,e,b|i\right).$$

In words, the agent's optimal actions are to choose technology n *in period 1 and minimum innovative effort* $i = 0$ *(together with the appropriate productive effort* e *on* n*).*

3. *For all* $b \geq \bar{b}^h(i)$*:*

$$\max_e W^o(e,b) \geq \max_{e,\tilde{i}} W^n(\tilde{i},e,b|i).$$

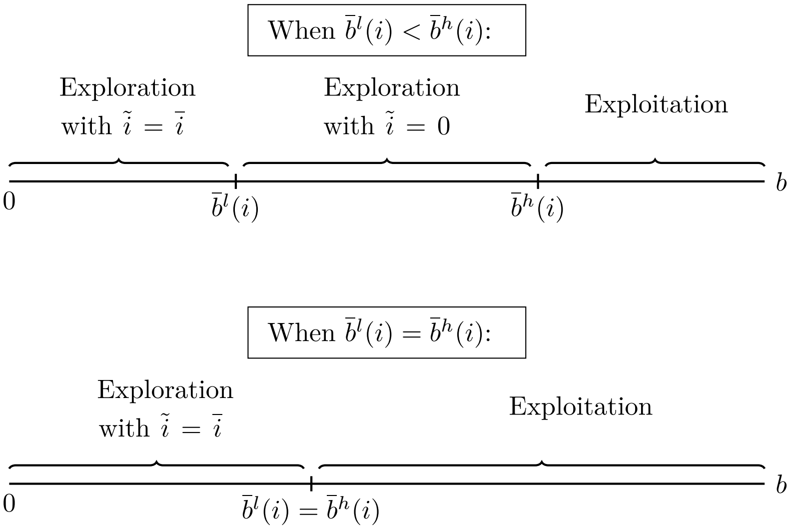
In words, the agent's optimal action is to choose technology o *(together with the appropriate productive effort* e *on* o*).*

The following result is then immediate:

Corollary B.2. *Under FW in the setting with innovative effort, in an exploration equilibrium in which the agent uses technology* n *in period 1, the agent is either choosing zero or maximum innovative effort.*

The intuition behind the result is similar to that for Lemma 3. When deciding whether to engage in the innovative activity or not, the agent trades off between future expected payoffs of innovation and current payoffs from production. In addition, here, there is the added consideration of "how intense should one be exploring the new technology". As in the baseline model, low-powered incentives in period 1 decreases the opportunity cost of forgoing early production. This encourages innovative initiative and moreover, it increases the intensity of the innovative initiative. Furthermore, because the gain from better innovation is convex (Lemma B.4), whenever the agent finds it profitable to exert additional intensity for exploration, he will exert maximum intensity which thus explains Corollary B.2. The proof of Proposition B.5 is as follows:

Figure 2: The agent's best response given a bonus b under principal's conjecture i.



Proof. First note that given any contract (a, b) with prescribed innovative effort i, the optimal $\tilde{i}$ and e choices for the agent exist since the choice set is compact and the objective is continuous. I first show that at the optimal, $\tilde{i}$ must be either 0 or $\bar{i}$ which proves Corollary B.2. Suppose for a contradiction that under some $b > 0$, the optimal choices are e and $\tilde{i} \in (0, \bar{i})$. Let $\tilde{i}' = \max\left\{0, 2\tilde{i} - \bar{i}\right\}$. $\tilde{i}$ is optimal implies that:

$$\begin{aligned}
& W^n(\tilde{i}, e, b|i) \geq W^n(\tilde{i}', e, b|i) \\
\iff \quad & -\tilde{i}eb + R\left(\tilde{i}\middle|i\right) \geq -\tilde{i}'eb + R\left(\tilde{i}'\middle|i\right) \\
\iff \quad & \frac{R\left(\tilde{i}\middle|i\right) - R\left(\tilde{i}'\middle|i\right)}{\tilde{i} - \tilde{i}'} \geq eb
\end{aligned}$$

Recall that $R\left(\cdot\middle|i\right)$ is strictly increasing and strictly convex (Lemma B.4). Since $\bar{i} - \tilde{i} \geq \tilde{i} - \tilde{i}'$, this implies that $\frac{R\left(\bar{i}\middle|i\right) - R\left(\tilde{i}\middle|i\right)}{\bar{i} - \tilde{i}} > \frac{R\left(\tilde{i}\middle|i\right) - R\left(\tilde{i}'\middle|i\right)}{\tilde{i} - \tilde{i}'}$. Hence:

$$\begin{aligned}
& \frac{R\left(\bar{i}\middle|i\right) - R\left(\tilde{i}\middle|i\right)}{\bar{i} - \tilde{i}} > eb \\
\implies \quad & -\bar{i}eb + R\left(\bar{i}\middle|i\right) > -\tilde{i}eb + R\left(\tilde{i}\middle|i\right) \\
\implies \quad & W^n(\bar{i}, e, b|i) \geq W^n(\tilde{i}, e, b|i)
\end{aligned}$$

which contradicts $\tilde{i}$ being optimal. This then implies that $\tilde{i} = 0$ or $\tilde{i} = \bar{i}$ in equilibrium.

Hence, given the contract, the agent is essentially only choosing from 3 options in period 1: (i) $\tau = n$ and $\tilde{i} = \bar{i}$, (ii) $\tau = n$ and $\tilde{i} = 0$, or (iii) $\tau = o$, each together with the corresponding optimal productive effort level. Using the same steps as in the proof of Lemma 3 and noting that $R(\cdot|i)$ is increasing, the following three claims can be established:

Claim 1. Suppose the agent is considering only between options (i) and (ii). Then there exists a threshold $\bar{b}^{(1)}(i) > 0$ such that $\max_e W^n(\bar{i}, e, b|i) \geq \max_e W^n(0, e, b|i)$ if and only if $b \leq \bar{b}^{(1)}(i)$.

Claim 2. Suppose the agent is considering only between options (i) and (iii). Then there exists a threshold $\bar{b}^{(2)}(i) > 0$ such that $\max_e W^n(\bar{i}, e, b|i) \geq \max_e W^o(e, b)$ if and only if $b \leq \bar{b}^{(2)}(i)$.

Claim 3. Suppose the agent is considering only between options (ii) and (iii). Then there exists a threshold $\bar{b}^{(3)}(i) > 0$ such that $\max_e W^n(0, e, b|i) \geq \max_e W^o(e, b)$ if and only if $b \leq \bar{b}^{(3)}(i)$.

Set $\bar{b}^l(i) = \min\{\bar{b}^{(1)}(i), \bar{b}^{(2)}(i)\}$ and $\bar{b}^h(i) = \max\{\bar{b}^{(2)}(i), \bar{b}^{(3)}(i)\}$, and the three statements in the proposition will follow. To see why, first notice that the statements 1 and 3 of the proposition are readily verified to be true. The non-obvious part is the second statement about $b \in \left[\bar{b}^l(i), \bar{b}^h(i)\right]$. Consider all 6 possible permutations on the order of $\bar{b}^{(1)}(i), \bar{b}^{(2)}(i)$ and $\bar{b}^{(3)}(i)$ now. Note that whenever $\bar{b}^l(i) = \bar{b}^h(i)$, the statement is trivially true. To ease notation, I drop the argument of i from now on. Also, recall that $\bar{W}^o(b) = \max_e W^o(e, b)$, and I denote $\bar{W}^n(\tilde{i}, b|i) := \max_e W^n(\tilde{i}, e, b|i)$.

Case 1: $\bar{b}^{(1)} \leq \bar{b}^{(2)} \leq \bar{b}^{(3)}$**:** Hence $\bar{b}^l = \bar{b}^{(1)}$ and $\bar{b}^h = \bar{b}^{(3)}$. For any $b \in \left[b^l, b^h\right]$, $\bar{W}^n(0, b|i) \geq \bar{W}^n(\bar{i}, b|i)$ since $b \geq \bar{b}^{(1)}$, and $\bar{W}^n(0, b|i) \geq \bar{W}^o(b)$ since $b \leq \bar{b}^{(3)}$.

Case 2: $\bar{b}^{(1)} \leq \bar{b}^{(3)} \leq \bar{b}^{(2)}$**:** Hence $\bar{b}^l = \bar{b}^{(1)}$ and $\bar{b}^h = \bar{b}^{(2)}$. If $\bar{b}^{(3)} = \bar{b}^{(2)}$, then the argument of Case 1 applies. Moreover, it cannot be the case that $\bar{b}^{(3)} < \bar{b}^{(2)}$. If not, consider a $b \in \left(\bar{b}^{(3)}, \bar{b}^{(2)}\right)$. $b > \bar{b}^{(3)}$ implies that $\bar{W}^n(0, b|i) < \bar{W}^o(b)$, $b < \bar{b}^{(2)}$ implies that $\bar{W}^o(b) < \bar{W}^n(\bar{i}, b|i)$, and $b > \bar{b}^{(1)}$ implies that $\bar{W}^n(\bar{i}, b|i) < \bar{W}^n(0, b|i)$, which then implies $\bar{W}^n(0, b|i) < \bar{W}^n(0, b|i)$ (contradiction).

Case 3: $\bar{b}^{(2)} \leq \bar{b}^{(1)} \leq \bar{b}^{(3)}$**:** Hence $\bar{b}^l = \bar{b}^{(2)}$ and $\bar{b}^h = \bar{b}^{(3)}$. If $\bar{b}^{(2)} = \bar{b}^{(1)}$, then the argument of Case 1 applies. Moreover, it cannot be the case where $\bar{b}^{(2)} < \bar{b}^{(1)}$. If not, consider a $b \in \left(\bar{b}^{(2)}, \bar{b}^{(1)}\right)$. $b < \bar{b}^{(1)}$ implies that $\bar{W}^n(0, b|i) < \bar{W}^n(\bar{i}, b|i)$, $b > \bar{b}^{(2)}$ implies that $\bar{W}^n(\bar{i}, b|i) <$

$\bar{W}^o(b)$, and $b < \bar{b}^{(3)}$ implies that $\bar{W}^o(b) < \bar{W}^n(0,b|i)$, which then implies $\bar{W}^n(0,b|i) < \bar{W}^n(0,b|i)$ (contradiction).

Case 4: $\bar{b}^{(2)} \leq \bar{b}^{(3)} \leq \bar{b}^{(1)}$**:** Hence $\bar{b}^l = \bar{b}^{(2)}$ and $\bar{b}^h = \bar{b}^{(3)}$. If $\bar{b}^{(2)} = \bar{b}^{(3)}$, then $\bar{b}^l = \bar{b}^h$ and the statement is trivially true. Moreover, it cannot be the case where $\bar{b}^{(2)} < \bar{b}^{(3)}$. If not, consider a $b \in \left(\bar{b}^{(2)}, \bar{b}^{(3)}\right)$. $b_1 > \bar{b}^{(2)}$ implies that $\bar{W}^n(\bar{i},b|i) < \bar{W}^o(b)$, $b < \bar{b}^{(3)}$ implies that $\bar{W}^o(b) < \bar{W}^n(0,b|i)$, and $b_1 < \bar{b}^{(1)}$ implies that $\bar{W}^n(0,b|i) < \bar{W}^n(\bar{i},b|i)$, which then implies $\bar{W}^n(\bar{i},b|i) < \bar{W}^n(\bar{i},b|i)$ (contradiction).

Case 5: $\bar{b}^{(3)} \leq \bar{b}^{(1)} \leq \bar{b}^{(2)}$**:** Hence $\bar{b}^l = \bar{b}^{(1)}$ and $\bar{b}^h = \bar{b}^{(2)}$. If $\bar{b}^{(1)} = \bar{b}^{(2)}$, then $\bar{b}^l = \bar{b}^h$ and the statement is trivially true. Moreover, it cannot be the case where $\bar{b}^{(1)} < \bar{b}^{(2)}$. If not, consider a $b \in \left(\bar{b}^{(1)}, \bar{b}^{(2)}\right)$. $b > \bar{b}^{(3)}$ implies that $\bar{W}^n(0,b|i) < \bar{W}^o(b)$, $b < \bar{b}^{(2)}$ implies that $\bar{W}^o(b) < \bar{W}^n(\bar{i},b|i)$, and $b > \bar{b}^{(1)}$ implies that $\bar{W}^n(\bar{i},b|i) < \bar{W}^n(0,b|i)$, which then implies $\bar{W}^n(0,b|i) < \bar{W}^n(0,b|i)$ (contradiction).

Case 6: $\bar{b}^{(3)} \leq \bar{b}^{(2)} \leq \bar{b}^{(1)}$**:** Hence $\bar{b}^l = \bar{b}^h = \bar{b}^{(2)}$ and the statement is trivially true. $\square$