Modeling Legal Modularity

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Abstract

Law employs modular structures to manage the complexity among legal actors. Property, torts, contracts, intellectual property, and doctrines in other areas of the law reduce information costs in similar ways by chopping up the world of interactions between parties into manageable chunks—modules—that are semi-autonomous. These modules employ boundaries—whether "real" boundaries as in real property law or "abstract" boundaries as in intellectual property, torts, and contracts—to hide information so as to make law less context-dependent and, hence, more modular. Previous explications of modularity in law have been qualitative. Here, borrowing from numerical measures of modularity in network theory, we offer the beginnings of a quantitative model of legal modularity. We posit that our "network science" approach to jurisprudential issues can be adapted to quantify many other important aspects of legal systems.

1. Introduction

Law is a complex system. As such we should expect many of its properties to be nontrivially related to its parts. Put another way, the micro-foundations of law are easy to take for granted and not at all easy to relate to macro-behavior.

This micro-macro connection requires managing complexity itself. If any legal actor could relate to any other legal actor in any possible way in principle, specifying the constraints on that behavior and predicting behavior under constraints would quickly become intractable. Instead, when economists model two-party actions and then add up the effects across society, they are silently making strong assumptions about how behavior does *not* interact, and the legal rules being modeled likewise assume away large amounts of information as irrelevant. This problem is particularly central to property law. Many debate whether property is a "law of things" or is better characterized as being about bundles of rights. On the former view, ask where our notion of things comes from and why things should matter in property law. If, on the other hand, property is a bundle of rights, sometimes captured with the metaphor of the "bundle of sticks," why do some sticks go with other sticks and why are some packages more likely than others? Why doesn't the law disaggregate legal relations all the time?

If the system of actors and their interactions over valued resources form a complex system, and in turn the legal system that constrains and sometimes constitutes these interactions is a complex system, we can analyze this system in terms of modularity. Modules are parts of a system within which interaction is relatively intense and between which it is relatively sparse. Crucially modularity is a matter of degree. Without assuming that things or aggregates are

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important, we can measure the set of interactions captured by the law – through devices like nuisance, easement, covenants, zoning as well as the law of trespass, and ask to what extent they form modules.

Network theory has developed tools for making these notions more precise, and we employ these tools to show how to measure legal modularity and how to quantify theorizing about information in law. Network theory is increasingly being used in economics, and it offers a way to think about a wide range of complex systems made of interacting entities.

After setting out we the significance of modularity in law in Part II, we show in Part III how to model modularity using the tools of network theory and its algorithms for finding community structure. Part IV then presents some implications and extensions, and Part V concludes.

2. A Primer on Legal Modularity

Law as a complex system benefits from modularity. This modularity is easy to overlook. Even the basic ontology of persons, actions, and things, out of which legal rules are built, exhibits a modular structure. Each person, activity, or thing could be analyzed further, and it is worth asking why legal rules are stated at the level of abstractness that they are, rather than in a more (or less) fine-grained way.

One strand of legal analysis emphasizes the possibility of finer grain. Wesley Hohfeld (1913, 1917) devised a scheme of jural relations (rights and corresponding duties, privileges and no-rights, powers and liabilities, and immunities and disabilities) that offered the possibility of reducing coarse notions like "rights" into smaller and more accurate relations. The Legal Realists took this is a reductionist direction with the "bundle of rights" picture of property. Instead of being a law of things or centered on rights to exclude, "property" was just a label for socially important aggregates of more fundamental legal relations (Grey, 1980).

The problem of grain has entered property theory through the application of modularity. Developed in early work of Herbert Simon (1981 [1969]) and more recently in network theory, modularity has been applied to organizations (Baldwin & Clark, 2000; Langlois, 2002; Sanchez & Mahoney, 1996). In law, modularity has been applied to property (Smith, 2012) and other areas of private law (Smith, 2011; Lau ms.).

In the following, we will draw on both the modular and Hohfeldian approaches. From the former, we adopt the notion of modularity and from the latter we will borrow the vocabulary of constituent legal relations. Crucially, we will endogenize things, by build the familiar legal ontology from the ground up. Modularity will not be assumed but will emerge (or not) from the application of network theory to the cluster of basic Hohfeldian legal relations. In a sense, this will even allow us to provide a micro grounding for the bundle of sticks.

3. A Mathematical Model of Legal Modularity

As Section 2 described, modularity in the law reduces information costs by shielding the multitude of bilateral legal relations that exist among legal actors. Here, we begin by constructing a "Hohfeldian" network of individual legal actors connected to one another by various sets of legal relations. Then, adapting the work of Newman and Girvan (2004) and Newman (2006), we offer a means to naturally divide the network into relevant modules and to quantify the level of modularity present in the network.

3.1 Complex Hohfeldian Networks

In Section 2, we briefly noted that the bilateral, "atomic" relations among legal actors were Hohfeldian in nature. In other words, in the framework of the early 20th century legal theorist Wesley Hohfeld, legal relations can be selected from one of eight categories: right, privilege, duty, no-right, power, immunity, disability, liability (Hohfeld, 1913). Here, for simplicity, we focus on the first-order relations: right, privilege, duty, and no-right. In this framework, one legal actor's "right" implies a "duty" on the part of some other legal actor (to perform or abstain from performing some action), and the absence of a right (a "no-right") of one legal actor implies a "privilege" on the part of another legal actor (to perform or abstain from performing some action). In view of this Hohfeldian "correlativity" of rights with duties and no-rights with privileges, it is sufficient to specify whether a given actor has a duty or privilege vis-à-vis another actor with respect to a particular action.

3.1.1 "Fully" Modular Networks

A landowner's rights against trespass and privileges of use vis-à-vis third parties are a quintessential example of Hohfeldian relations at work. For instance, suppose we have a large tract of land L, comprising three sub-regions, I, J, K, as well as commons between the sub-regions. See Fig. 1.

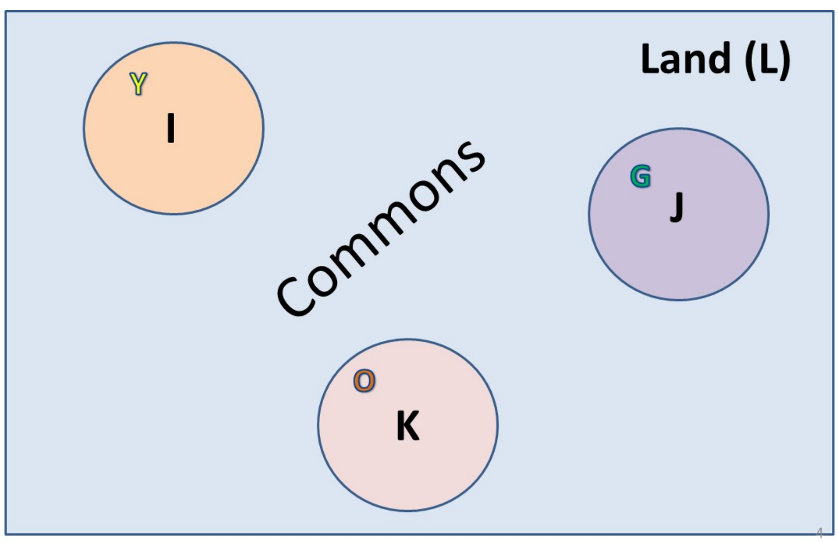


Figure 1. Three Subplots and a Commons on a Parcel of Land.

Assume that each of the sub-regions, I, J, K, are ideal for various economic activities (e.g., grazing cattle, growing fruit, etc.), and that in the absence of some kind of legal restrictions, overuse would occur. One of several options to achieve efficient use is to "privatize" the tracts so that an individual owns each plot, which allows for more optimal decisionmaking regarding usage (Demsetz, 1967; Ellickson, 1993). Here, we assume that Yellow (Y) owns I, Orange (O) owns K, and Green (G) owns J.

Ownership implies rights in the owner and concomitant duties in third parties along with privileges in the owner and concomitant no-rights in third parties. In a pure "exclusionary" regime, each "owner" of each plot has (1) an absolute (Hohfeldian) privilege to do what she pleases on her land (assume there are no negative externalities); (2) an absolute (Hohfeldian) right that others not enter the owner's land or interfere with uses on the land; and (3) an absolute (Hohfeldian) privilege to undertake action in the commons.

If we represent the legal actors as nodes in a network and legal relations as edges between nodes, we can create a traditional network or graph representation of the exclusionary property regime (removing plot K for simplicity). See Fig. 2.

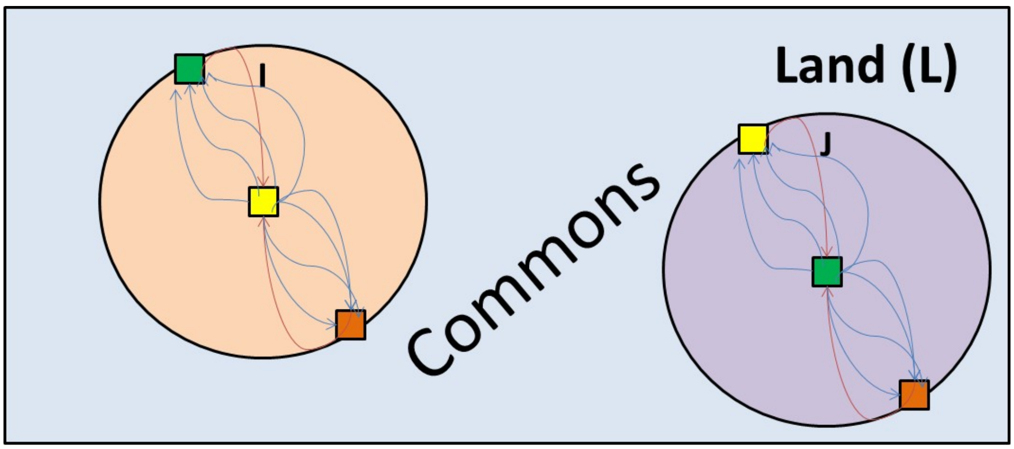


Figure 2. Two Subplots, a Commons, and Legal Actors
Connected by Modular Hohfeldian Legal Relations on a Parcel of Land.

In order to properly represent the "location" of the legal relations via the edges, we place the actors at the location where an action of interest is performed. For simplicity, we place the owner of each plot at the center of the plot, so that owner Y is placed in the center of plot I and owner G is placed at the center of plot J. Because we are concerned here solely with trespass, we place the non-owners at the boundary of each plot. So for plot I, the legal actors G and Y are placed at the boundary; for plot J, the legal actors Y and O are placed at the boundary.

The legal relations between each actor with respect to a given action on a given plot are indicated by directed edges. Red arrows represent Hohfeldian duties on the party of the third parties to the owner—here, not to enter a particular plot. Blue arrows represent Hohfeldian privileges of the owner to perform various actions on the plot, such as grazing cattle, as well as the owner's privilege to enter the plot.[1] (For simplicity, we ignore the various privileges of parties with respect to each other that relate to actions on the commons.)

In the event that all of the legal relations that are connected to one another in the "Hohfeldian" network are within a given boundary, then in graph theory terms, the network is

[1] We assume a level of abstractness for actions, which themselves could be analyzed for modular structure.

"modular" (or, "decomposable") in the sense that it can be partitioned without any loss of information. For instance, a social network—say among "friends" on a social networking service such as Facebook—is fully modular between two communities when the first community of individuals (here, a group of friends) have no links to a second community of individuals (here, another group of friends). For instance, Figure 3 shows two dense networks of friends ("communities") that are not in any manner linked to one another.

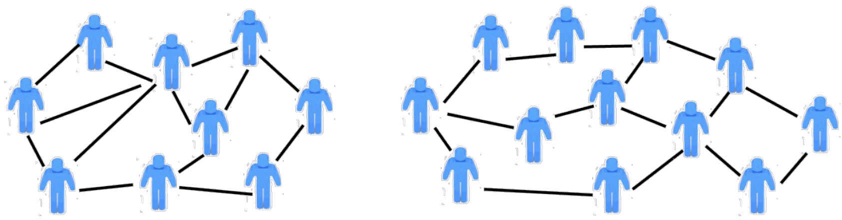


Figure 3. Two Independent ("Fully Modular") Communities of Friends.

Complete modularity is present in the real property schematic of Figure 1, because the entire plot L can be subdivided into two separate plots, I and J, and the legal analysis of each subplot can proceed without paying any attention to the other subplot.[2] In other words, all of the relevant legal relations occur at or inside the boundary of a subplot, which follows from the fact that there are no relations connecting one subplot to the other.

3.1.2 Partially Modular Networks

However, as we noted in Section 2, legal relations governing property (or any area of law for that matter) are typically not completely modular. For instance, easements, covenants, nuisance law, and regulation all concern privilege or duty relations that in essence "cross" the boundary of a piece of land. Partial modularity (or "near" decomposability, Simon, 1981) occurs when there are multiple, stable communities formed by dense networks that are connected to one other loosely enough that the communities retain enough form so as to function as effectively independent modules.

For instance, in Figure 4, the previously unconnected friend communities are now loosely connected via a single connection (indicated by the thick red edge) between two individuals.

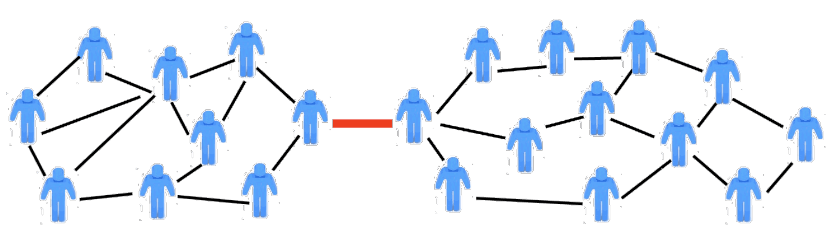


Figure 4. Two Communities of Friends Connected Solely by One Relationship.

[2] Although the subplots I and J are modular with respect to one another, because there is structure within each subplot that is not wholly modular, the *entire system* is not, as Section 3.3 describes, fully modular.

The single connection destroys full modularity between the two communities, but essentially leaves intact the independence of each community. As we show in Section 3.2, there are standard techniques to quantify the amount of system-wide modularity destroyed by connections between communities.

In the context of our real property hypothetical, deviation from full legal modularity occurs when the following three types of first-order relations arise: (1) duties of an owner inside the boundary of the owned plot; (2) privileges of a third-party across or inside the boundary of the owner's plot; (3) duties of a third-party outside the boundary that affect the owner's plot. In the parlance of Smith (2002), such relations shift the relevant legal regime from one of pure exclusion to a mixed regime of exclusion and governance. An example of how these "governance" relations appear in the Hohfeldian network is shown in Figure 5.

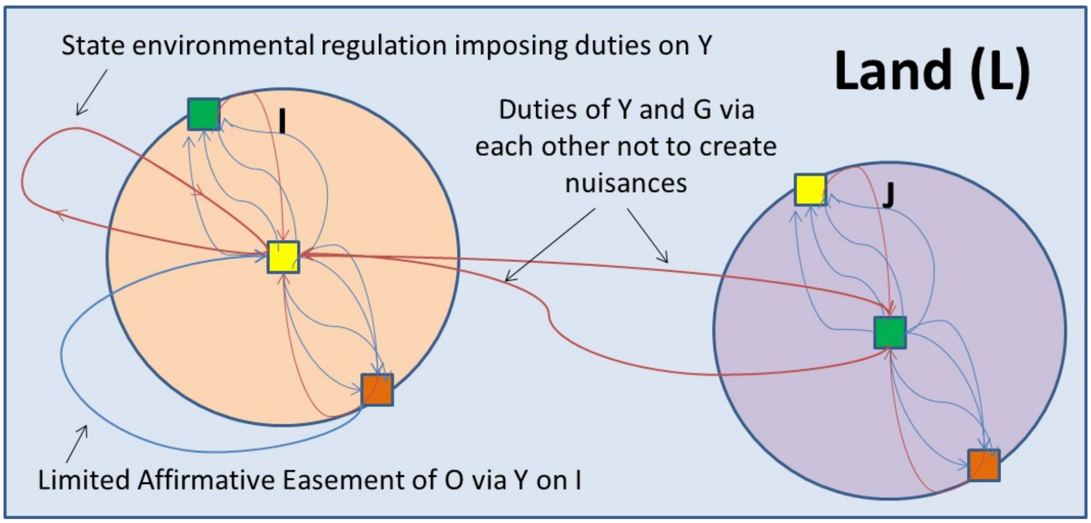


Figure 5. Two Subplots, a Commons, and Legal Actors
Connected by Partially Modular Hohfeldian Legal Relations on a Parcel of Land.

In Figure 5, additional legal relations appear that cross the boundaries of the subplots. First, governmental environmental regulation that imposes "governance" obligations on owner Y sets up a red duty relation between the owner and the State. (For ease of visualization, we indicate this relation as a line that extends from owner Y outside the boundary and back to owner Y.)[3] Second, there is a license indicated by a blue privilege relation that extends from legal actor O that crosses the boundary into owner Y. (Alternatively we could model an easement with a duty relation.) Third, there are two nuisance obligations indicated by red edges that extend from owner Y to owner G and vice-versa. These additional legal relations destroy full modularity between the subplots, instead resulting in partial modularity. Like social and other types of

[3] However, this loop is not intended to be "self-referential" in the usual sense of network theory. In any event, we ignore legal relations vis-à-vis the State in the treatment below, but we briefly address the issue of public law in Section 4.

networks, in order to determine how much modularity has been erased, it is necessary to devise a quantitative measure of modularity, which we turn to next.

3.2 Quantifying the Modularity of the Network

3.2.1 General Methods for Identifying Communities of Interest

To quantify the level of modularity in a network, one must have a means to determine how a network can be decomposed into communities that are roughly independent.[4] In our property example in Section 3.1, the artificial "legal" boundary of each subplot implicitly represented the "community" of interest. However, in many legal situations, there is no external boundary that can be simply identified and associated with a given community. For instance, the various legal relationships in tort law between individuals do not typically create an artificial boundary that allows one to quickly identify "communities" of legal relations. Indeed, even in real property law, ideally the aim is to ignore the external, physical boundary in order to determine the effective boundary and locus of legal relations. For instance, the effective boundary that results from how real property relations are enforced and adjudicated may include a buffer zone that is an effective expansion of the nominal, de jure boundary. (Or, conversely, the effective boundary may be an effective contraction of the nominal boundary.) Thus, in the analysis that follows, we abstract away from the nominal, physical boundary and examine only the Hohfeldian network of legal relations itself. This allows us to more accurately depict communities and modularity not only for real property examples, but also to extend our analysis to areas of law where there are no such boundaries (such as torts, contracts, intellectual property, and so forth).[5]

In the citation network literature, hierarchical clustering techniques may be used to "discover[] natural divisions of (social) networks" and identify groups (Newman and Girvan, 2004).[6] In the "agglomerative" technique, communities are determined by building up the network from the ground up, that is, from the lowest level of relations between nodes in the network. However, as Newman and Girvan (2004) explain, agglomerative techniques have not been shown to determine community structure well in a variety of networks. Rather, using a novel "divisive" technique—which starts with the existing network and splits it up from the top down—Newman and Girvan (2004) provide a highly accurate, as well as computationally simple and relative fast, method to determine community structure.[7] As such, we adapt their method to modeling Hohfeldian networks of legal relations.

[4] This assumes that the network exhibits some community structure. In rare cases, a network may simply be composed of individuals so haphazardly connected to one another, there is essentially no community structure.
[5] Although we believe that the concept of modularity plays a role in public law, given that the role of modularity has been analyzed primarily in the domain of private law (Smith, 2002, 2004, 2007, 2011), we generally confine our discussion accordingly.
[6] In contrast to hierarchical clustering, "graph partitioning" may be used to determine sub-network properties but only is feasible when community structure is essentially known ex ante (Newman and Girvan, 2004).
[7] Although the community identification approach of Newman and Girvan (2004) has been improved (e.g., Girvan (2006)), the basic approach of the improvements is the same as the original method. Because the original method is simpler to convey, we use that approach here. Of course, our method could be improved in basically the same manner.

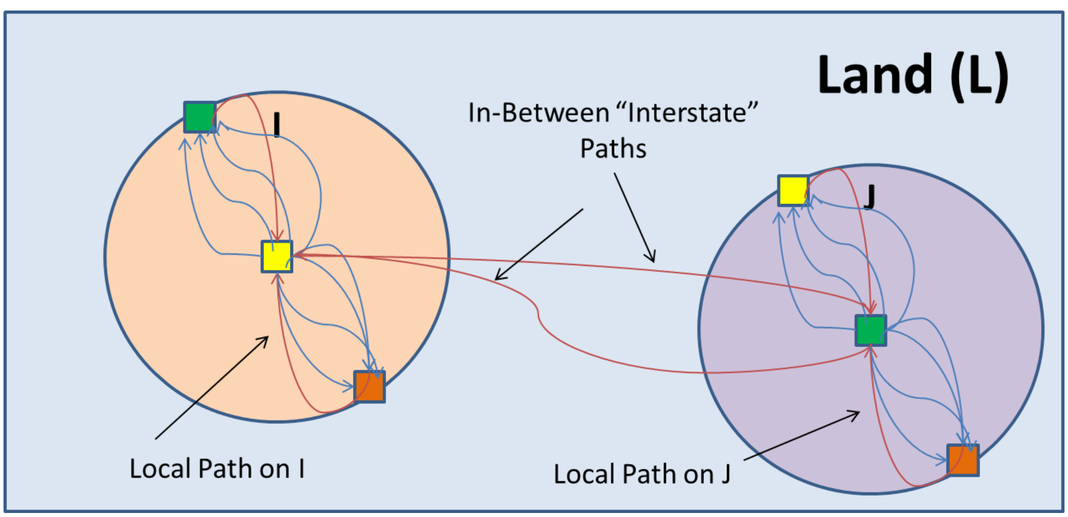
3.2.2 Subdividing Hohfeldian Networks

Before calculating the modularity of a system, it is necessary to identify the communities of interest in the system. The approach of Newman and Girvan (2004) to community identification is to remove edges from the network with the highest measure of "betweenness"— namely, those edges that primarily lie "between" rather than "within" communities. As edges are removed, communities are exposed, and the process may repeated to identify sub-communities.

Newman and Girvan (2004) offer two approaches to determining the betweenness of edges—"shortest path" and "random walk." In the shortest path approach, one examines the shortest path (or paths) between all pairs of vertices in the network, examining how often a particular edge occurs in each of the shortest paths. Edges that occur more frequently have a higher "betweenness" measure than edges that do not. For instance, in a road network connecting various parts of a city, a freeway that connects many sub-regions within a city would have a higher betweenness score than a cul-de-sac. Another approach is to use a "random walk," in which betweenness is measured by the expected number of times a particular edge will be traversed in a random walk between to vertices. Because Newman and Girvan (2004) show that the random walk method essentially provides results similar to the shortest path method, yet is more computationally demanding, we apply the shortest path method here.[8]

In examining the shortest paths, for simplicity, we ignore path direction (e.g., privileges vs. duties), and discard easements, covenants, and legal relations vis-a-vis the State. We also do not adjust for the size of each edge—in actuality, some relations may be treated as more important than others. As discussed by Newman and Girvan (2004), the shortest path approach can be adapted to such extensions; thus, our treatment is without loss of such generality.

Figure 6 is adapted from Figure 5, and represents solely the relations of interest for the treatment to follow.



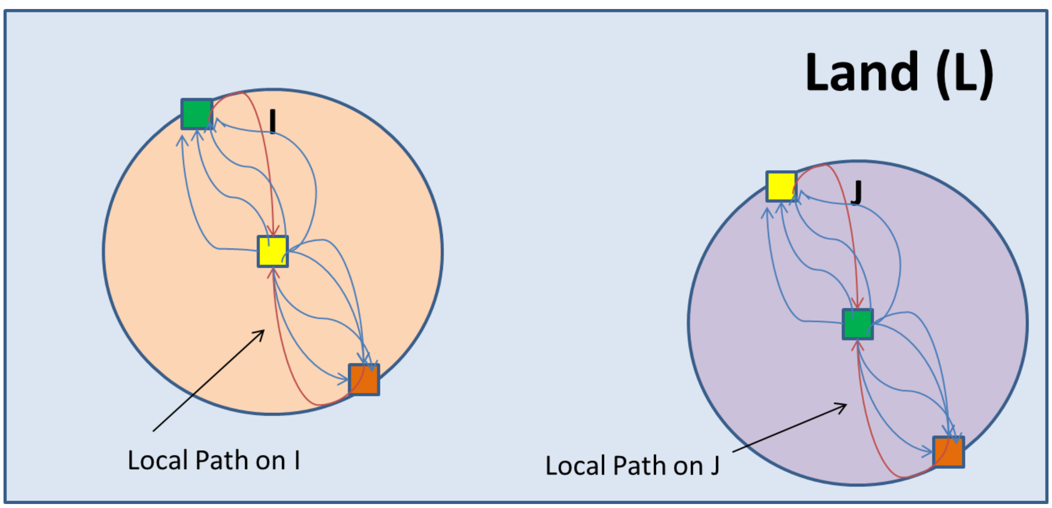
Using the network on Figure 6, we can engage in the first step of identifying relevant communities by calculating a betweenness score for every edge. For example, suppose our full path of interest is from O (on I) to O (on J) (i.e., orange node to orange node). In this case, there

[8] In particular, the shortest path method computes the betweenness for a system with n vertices and m edges in time $O(mn)$.

are five paths from O (on I) to Y (on I) (i.e., orange node to yellow node on I); two paths from Y (on I) to G (on J) (i.e., yellow node on I to green node on J); and five paths from G (on J) to O (on J) (i.e., green node on J to orange node on J). This makes for a total of 50 possible paths from O (on I) to O (on J): Orange-Yellow on I (5 paths) * Yellow to Green (I-J) (2 paths) * Green to Orange on J (5 paths) = 50 paths.

Next, one can calculate the frequency of each edge occurring in the 50 paths. For the five paths from O to Y (on I), each edge appears in 10 of the 50 paths (since there is one of five paths that can be taken on each route), or 20% of the time; for the two paths from Y (on I) to G (on J), each one appears 50% of the time (since one or the other is necessary on each route); and for the five paths from G (on J) to O (on J), each edge appears 20% of the time. Thus, the edges with the highest betweenness scores for this route are the two connecting I and J.

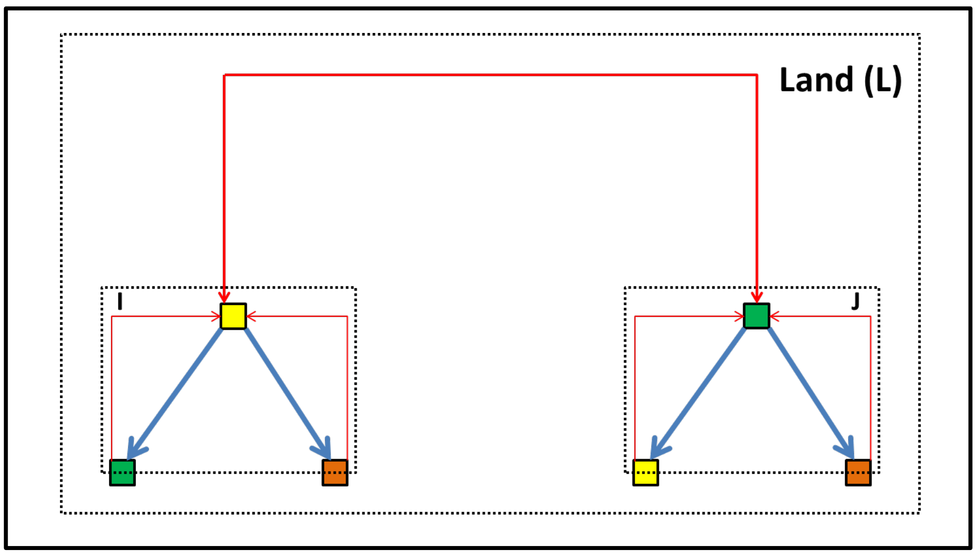
In the method of Newman and Girvan (2004), it is necessary to calculate betweenness scores for all possible paths—in other words, how often a given edge appears in every possible shortest path between each pair of nodes. We perform these calculations in the Appendix and show (expectedly) that the two edges connecting plots I and J (the "interstate" like paths) occur with the greatest frequency. Specifically, the Appendix finds that there are a total of __ shortest paths between all pairs of nodes, in which the edges between plot I and J occur __ times (__%), and the edges inside I and J each occur __ times (__%). Thus, the first step in the process of Neman and Girvan (2004) is to remove the two edges between plots I and J, resulting in Figure 7.



In Figure 7, the two nuisance relations imposing duties on the owners of subplots I and J have been removed, leaving two unconnected sets of legal relations of privileges and duties solely relating to I or to J. In this sense, the method of Newman and Girvan (2004) method identifies two separate communities of legal relations or "modules" in the sense of Smith (2012). Importantly, these communities are identified without any reference to the physical, external boundaries of I and J, but rather merely to the legal relations and legal actors regarding these plots.

The next step of the Newman and Girvan (2004) algorithm is to recalculate betweenness for each community in order to determine sub-communities. In Figure 7, however, each edge is completely symmetric to every other edge. Thus, no edge has a higher betweennness score than any other. Thus, removing any edge requires removing all edges, which simply leaves the owners of each plot, Y and G, which are single nodes and are not typically considered communities. In more complex examples involving asymmetric legal relations within communities, division of those communities into subcommunities would be feasible.

A common abstraction to display the division of a system into communities is the dendogram. Figure 8 displays a dendogram for the set of legal relations depicted in Figure 6, where the thickness of the edges represents the number of different underlying legal relations. The outer dotted line represents the division of the land L from the outside world, and the inner dotted lines represent the division of L into the two communities of legal relations and actors separately centered on the subplots I and J. Note that the inner dotted lines intersect the non-owners, as they reside right at the edge of the plot. (Again, the relations regarding the commons between the subplots are ignored for simplicity.)



3.1.1 Calculating Modularity Scores

As explored in Newman and Girvan (2004) and Newman (2006), the ability to remove edges of high betweenness so as to identify communities and sub-communities forms the basis for a quantitative measure of modularity for a given system. Using the approach of Newman (2006), we begin with a legal system of relations among legal actors, then divide the system into communities using the method of Newman and Girvan (2004). Next, we calculate the modularity, Q, of the system as-a-whole by determining "the number of edges falling within [communities] minus the expected number in an equivalent network with edges placed at random" normalized to 1 (Newman, 2006).[9]

In other words, when Q is equal to 1, the system is absolutely (i.e, 100%) modular. In the real property context, this would entail Blackstone's (1768) idealization of "property as . . . that sole and despotic dominion which one man claims and exercises over the external things of the world, in total exclusion of the right of any other individual in the universe." In this case—and as we

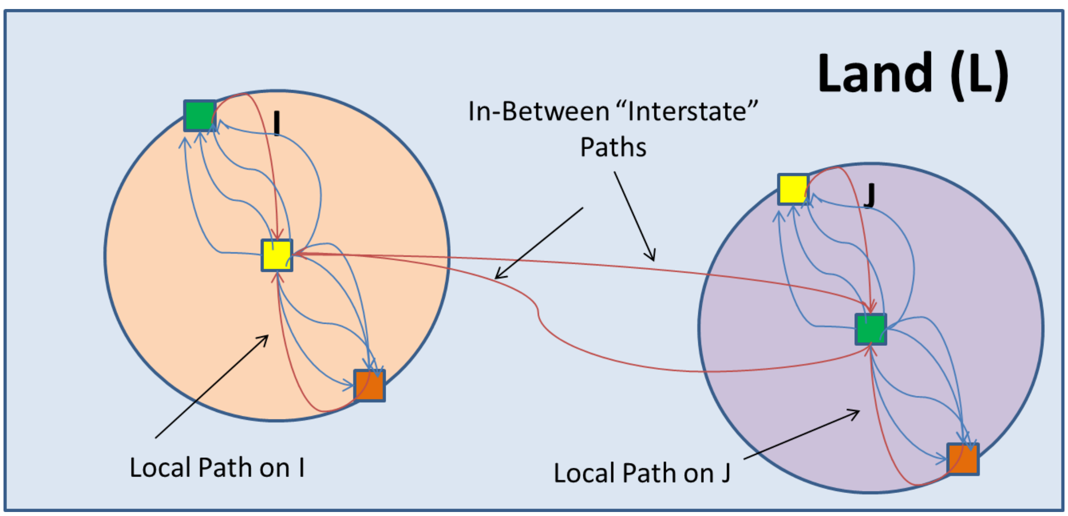
[9] Newman and Girvan (2004) provide a similar approach to Newman (2006) to determine modularity, but Newman (2006) is a substantial improvement and more straightforward, so we adopt it here.

show, even at much lower modularity scores—all legal relations are contained within the boundary of each owner's plot, with no interactions between the owners of various plots or other third parties beyond the boundary of an owner's plot.

When Q is equal to 0, the network is exactly the expected one—namely, a network that is most likely to occur given an exogenous set of vertices arranged in space. In this instance, the modularity of the system is roughly half that of a fully modular one. For instance, in the real property context, one would expect roughly an equal mix of legal relations inside and outside the boundaries of subplots when Q is 0. In the language of Smith (2002), such a system is roughly equal in "exclusion" and "governance" strategies.

Finally, when Q is –1 the system is absolutely indivisible—no communities emerge and each node stands on its own. When no legal relation is part of a module, the entire system is one of pure "governance" (Smith, 2002).

For simplicity, following Newman (2006), we describe how to calculate modularity when the system of interest contains two communities.[10] First, we calculate the actual number of edges falling between two vertices, i and j. Let A_{ij} be an element of a matrix **A** (the "adjacency" matrix), which represents the number of the actual edges between i and j. If we assume there are n total vertices, then **A** will be an $n \times n$ matrix.



Recall from Figure 6 (reproduced above) that there are six nodes in our two-community Hohfeldian network. If we label G on I as node 1, Y on I as 2, O on I as 3, Y on J as 4, G on J as 5, and O on J as 6, the A_{ij} are as follows:

- $A_{12}, A_{21}, A_{23}, A_{32}, A_{45}, A_{54}, A_{56}, A_{65} = 5$
- $A_{25}, A_{52} = 2$
- All other $A_{ij} = 0$.

As such, matrix **A** can be represented as follows:

[10] Newman (2006) describes methods for three or more communities, which can be readily adapted to the approach described here.

$$\begin{bmatrix} 0 & 5 & 0 & 0 & 0 & 0 \\ 5 & 0 & 5 & 0 & 2 & 0 \\ 0 & 5 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 5 & 0 \\ 0 & 2 & 0 & 5 & 0 & 5 \\ 0 & 0 & 0 & 0 & 5 & 0 \end{bmatrix}$$

Now that the total paths between each pair of vertices has been calculated, the next step is to determine the expected number of edges between each pair of vertices if edges are placed at random between the two vertices.

To determine this for a given graph, one must first calculate the total number of edges, m, in the network. This can be calculated by summing the total number of edges, k_i, emanating from each node i, and dividing the total by two:

$$m = (\tfrac{1}{2}) \sum_i k_i$$

For any given graph, following Newman (2006), the expected number of edges between two nodes, i and j, is:

- $k_i k_j / 2m \equiv k_{ij}/2m \equiv \bar{k}_{ij}$

In our hypothetical, we have the following:[11]

- $m = 22$ (total edges)
- $k_{12,\ 21,\ 23,\ 32,\ 45,\ 54,\ 56,\ 65} = 60$; $k_{13,\ 31,\ 46,\ 64} = 25$
- Thus, $\bar{k}_{12,21,\ 23,\ 32,\ 45,\ 54,\ 56,\ 65} = (60/44)$; $\bar{k}_{13,31,\ 46,\ 64} = (25/44)$

The next step is to determine $A_{ij} - k_i k_j/2m$, but only for those pairs of nodes, i and j, that appear in the same community. This limitation is effective because the measure of the expected number of edges effectively incorporates the structure of the entire network. In other words, modularity compares the number of actual linkages within the nodes of a given community with the expected linkages within the nodes in the same community, effectively taking into account the structure of the entire network.

Thus, for any given pair of vertices, one then calculates the following:

- $(A_{ij} - k_i k_j/2m)\,(s_i s_j + 1)$, where $s_i = 1$ vertex i belongs to community 1 and $s_i = -1$ if vertex i belongs to community 2 (and similarly for s_j)

Then, to calculate the total system modularity, Q, one sums across all pairs of vertices in the network and normalizes to 1 with the weighting favor ($1/4m$) as follows:

- $Q = \left(\frac{1}{4m}\right) \sum_{ij} (A_{ij} - k_i k_j/2m)\,(s_i s_j + 1) = \left(\frac{1}{4m}\right) \sum_{ij} (A_{ij} - \bar{k}_{ij})\,(s_i s_j + 1)$

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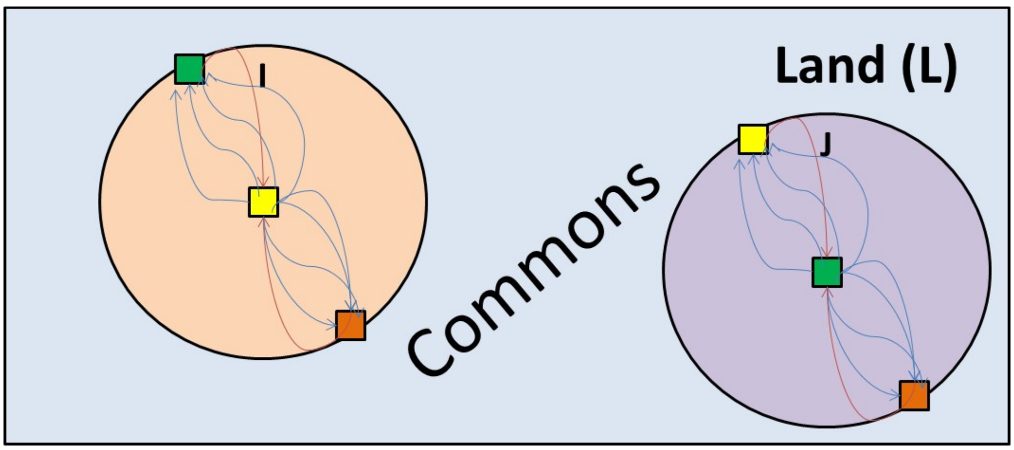
[11] Here, we ignore the expected edges between pairs of nodes in different communities, since—as the next step will show—these quantities are not used in calculating the modularity measure, Q.

For our hypothetical, one can now calculate the total modularity, Q:

- Q = 1/88 * (8 * (5–(60/44)) + 4 * (–25/44)) = 0.30

The first term represents (1/4*m*) where *m* is the total number of edges (here, 22). The term 8 * (5–(60/44)) concerns the two sets of four pairs of nodes within each community (here, pairs 12, 21, 23, 32, 45, 54, 56, 65) connected by five edges (for which the expected number of edges is (60/44). The term 4 * (–25/44) concerns the two sets of two pairs of nodes within each community (here, pairs 13, 31, 46, 64) connected by no edges (for which expected number of edges is (25/44).

Again, the pairs 25 and 52 are not counted, because vertex 2 and vertex 5 are in different communities. Recall these pairs represent nuisance obligations of one plot owner to another. However, suppose we return to the initial graph in which there were no such duties, which is reproduced below (see Figure 2).



In this case, there are 20 total edges and the A_{ij} and $\bar{k}_{ij}$ are as follows:

- $A_{12}, A_{21}, A_{23}, A_{32}, A_{45}, A_{54}, A_{56}, A_{65} = 5$
- $A_{25}, A_{52} = 0$
- All other $A_{ij} = 0$.
- $k_{12, 21, 23, 32, 45, 54, 56, 65} = 50$; $k_{13, 31, 46, 64} = 25$
- Thus, $\bar{k}_{12, 21, 23, 32, 45, 54, 56, 65} = (5/4)$; $\bar{k}_{13, 31, 46, 64} = (5/8)$

In this event, Q is as follows:

- $Q = \left(\frac{1}{4m}\right) \sum_{ij} (A_{ij} - \bar{k}_{ij}) (s_i s_j + 1) = 1/80 * (8 * (5–(5/4)) + 4 * (–5/8))$

which is equal to 0.34. From a legal standpoint, we considered the network of Figure 2 fully modular from a legal standpoint, because there were no legal interactions between the two subplots. However, in the Newman (2006) methodology, this system is still only partially modular, because the structure of the network is examined within each community, and this structure is to some degree similar (though not fully similar) to what one would expect in a random sub-network within a given community. From a legal standpoint, of course, the bundles within each subplot are generally provided as a package. Thus, to the extent modularity is of interest because of concerns about high information costs of "unbundled" relations—a modularity score of 0.30 (as in the example when nuisance obligations are present) is, from this perspective, fairly close to what can be considered "fully" modular in the legal sense, namely, a score of 0.34.

If we divide the modularity score of the system with edges between communities between a similar system without any such edges, we can define the relative modularity as follows:

- $Qr = Qs/Qc$, where Qs is the modularity of the entire system (i.e., with all edges) and Qc is the modularity of decomposed system (i.e., with any edges between communities removed)

Thus, for our hypothetical, the relative modularity would be (0.30/0.34) = 0.88. In this sense, modularity of the system with a few nuisance relations is quite close in quantitative modularity to the one with all the legal relations located within each subplot. Thus, consistent with the analysis of Smith (2012), a legal system with large numbers of relations inside the relevant boundaries and a small number traversing the relevant boundaries is—from a legal perspective—"nearly" decomposable.

4. Discussion & Extensions

4.1 Theoretical and Practical Implications

[Boundaries are automatically generated …]

[Abstract modularity (speed limits, etc.)]

[Application to "social relations" theories of property (the bundle is "very tight" when modularity score is high)]

[Modularity score as measure of information costs.]

4.2 Qualifications and Extensions

[Examples with different #s of use rights (e.g., grazing land, trespass to chattels, water rights, etc.).]

[Duties to the state; easements/covenants; etc.]

[Using directed networks to determine those actors with substantial rights and those with substantial duties; use of higher-order relations to determine "power" centers within the node; broker nodes that carry information between nodes; etc.—See Newman PPT.]

5. Conclusion

[Insert.]

References

Appendix A – Calculation of Betweenness Scores