On the Origin of Money*

INCOMPLETE

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Abstract

The widely accepted account of the origin of money (see, e.g., Menger 1892) is that it grew out of the inefficiency of barter. This efficiency view of the origin of money, however, does not square well with historical and anthropological evidence that money often grew out of the destruction of some fairly sophisticated credit arrangements, and that its introduction was not necessarily associated with an increase in prosperity. In this paper, we develop a model of the origin of money that is consistent with the facts that trade can flourish without money and its use need not trigger prosperity. We show that the introduction of money can be explained by the fact that governments may be able to better tax agents if trade is conducted through money instead of credit even if credit trade is more efficient than the monetary trade. Our paper thus provide support for a fiscal theory of the origin of money.

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1 Introduction

The standard economic explanation for the origin of money is that it grew out of the inconveniences of barter, namely, lack of double coincidence of wants (Jevons 1875; Menger 1892), lack of divisibility of commodities (Smith 1776), and private information about the quality of goods (Brunner and Meltzer 1971; Alchian 1977). Modern monetary theory has been successful in showing formally why agents prefer a medium of exchange over barter (Kiyotaki and Wright 1989; Williamson and Wright 1994; Barnerjee and Maskin 1996). However appealing, this explanation for the origin of money does not square well both with existing evidence and introspection. History and anthropology alike emphasize that money usually replaced fairly sophisticated credit arrangements and that in the few documented instances of barter economies, barter grew out of the demise of money. Introspection suggests that an economy with credit may achieve better allocations than an economy with barter.

In this paper we propose a fiscal theory of the origin of money as a medium of exchange. The economy is populated by private agents who use credit if the technology used to keep track of past transactions is efficient enough, but prefer money when this technology fails to record transactions. We study two different credit arrangements common in history: bilateral (nominative) credit and multilateral credit (gift-giving). The economy is also populated by a ruler who may be interpreted either as a government or as clerics.[1] The ruler enjoys consumption through the taxation of private agents. It is endowed with two technologies that can be used for both trade and tax collection: it can monitor and record private transactions or it can issue money. Our main results are as follows.

First, if the government is benevolent and the monitoring technology is efficient enough or not too expensive, the credit technology sustains trade in equilibrium, and is always superior to the monetary equilibrium in terms of welfare. Intuitively, credit is more efficient because it decorrelates consumption and monetary holdings, i.e. some object in scarce supply. Conversely, credit is not an equilibrium and money is preferred by both society and the ruler if the monitoring is inefficient or too expensive. This notion captures the idea that credit system collapses if the size of the population is too large, if trades occurs amongst strangers as in Townsend (1989) or if

[1]The anthropological evidence is that people paid tax to the religiosity even in the absence of a state. More precisely they were paying duties to the person(s) in charge of the production and management of religious symbols.

transactions occur so infrequently that a record-keeping device is needed to keep track of them. This result echoes the familiar idea on the essentiality of money, that money achieves desirable allocations that could not be achieved otherwise (Hahn, 1973). It echoes the notion that money is an imperfect substitute for memory (Ostroy, 1973; Kocherlakota, 1998), and provides guidance to qualify the anthropological evidence emphasizing that money is used in primitive societies in very specific transactions .

Second, consistently with the historical origin of coinage, we introduce a self-interested ruler, and allow him to choose between implementing the monitoring technology and issuing money. We show that the ruler usually introduces money even though credit is feasible and is a superior equilibrium to money from the societal point of view. This new result is driven by the fact that money is a superior technology for tax collection purposes, with money the ruler has one more instrument to tax than under credit. In other words, under the credit arrangement, the government tax uniformly while under money it can condition it on money holdings. Intuitively, monitoring works through individual punishments which prevent a deviation from the equilibrium path. This allows to achieve a better allocation from the society point of view but it does not provides any extra benefit to the government. In contrast, the monetary arrangement creates extra benefits to agents who comply, as you get to trade with other agents and you get to pay your future taxes in specie if you produce for the government. Interestingly, these extra benefits are the more relevant the more inferior is the monetary arrangement from a societal point of view. This result guides our understanding of history, as discussed in Section 5.

We believe that our results fit well with the available anthropological and historical evidence. Our first result fits well with the anthropological evidence on primitive societies. There media of exchanges were not used in day-to-day transactions for which multilateral credit systems such as gift-giving were preferred (Polanyi, 1944). People used media of exchanges in a small subset of very specific and infrequent transactions such as bridewealth or compensation of bloodcrime, for which they have difficulties in keeping track of past actions (Quiggin, 1949; Graeber, 2011).

Our second set of results in which we consider a profit-maximizer ruler fits well with the history of the introduction of money in the antiquity. The evidence can be summarized as follows. During the Antiquity societies had sophisticated bilateral credit well before medium of exchange. Historians place a great role to the "government"—be it a warrior, a king or an emperor—in promoting the widespread use of money. Credit was widespread in societies of sedentary farmers and traders of the Near and Middle East of Asia (Einzig, 1948; Van Reden, 2010). Money was introduced late compared to other technologies. Agriculture was invented in the Middle East between 11,500 and 9,000 BCE and was instrumental to the settlement of hunters-gatherers (Braidwood, 1952; Janick et al., 1974). Writing, a Babylonian invention which was crucial to record debts, dated back to 3,200 BCE (Bazerman, 2009). Coins were first introduced hundreds of kilometers away from Babylon (today's Iraq) in the Greek colonies of contemporary Western Turkey in 630-580 BCE (Wallace, 1987). This is a millennium after the invention in Babylon of futures and options, and a legal framework for debt repayment which dated from the 1,750 BCE (Swan, 2000). Once invented in the Greek offshoots of western Turkey, coinage spread quickly to other Greek cities in Europe but it took a while before its non-Greek neighbors adopted it (Howgego, 1995). For example it took a century and a half before the neighboring Persian empire minted its first coins (Babelon, 1893) and two other centuries before its introduction in Egypt (Van Reden, 2007) or Babylon (Howgego, 1995). This suggests that those economies without coinage used at least as efficient transaction technology. For example, in Egypt a networks of bank operated a payment and credit system and used a common unit of account to operate transfers between payers and payees both for taxes and everyday purchases. There is a consensus that coinage was introduced when a ruler also reorganized its fiscal apparatus of tax collection, as for example in Persia (Babelon, 1893) or in Egypt (Le Rider, 2003).

Our paper contributes to both the economics and historical literature on the origin of money. Compared to most historical and anthropological accounts of the origin of money (Howgego, 1995; Graeber, 2011), we show that the relation between money and taxation does not follow directly from the seigniorage revenues but arises because money changes the nature of taxation. Our paper is also related to the few recent papers dealing with the reasons for the introduction of coinage by Croesus in Lydia during the 7th and 6th centuries BC (Velde, 2012; Melitz, 2015). Compared to those papers, and to other theory papers on the origin of money, money in our analysis is not explained by the difficulty of bartering.

The rest of the paper is organized as follows. Section 2 describes our physical environment and then describes the two exchange technologies that we consider: credit and money. Section 3 considers the pure credit economy and the pure monetary economy, and then compares them. Section 4 considers the co-existence of credit and money. Section 5 summarizes the historical evidence on the origins of money and relates it to our model. Section 6 concludes and the Appendix contains omitted proofs and details

2 Baseline Environment

Time is discrete and indexed by $t \geq 0$. The economy is populated by a continuum of mass one of nonatomic agents and by one large agent, the government. Both the agents and the government are infinitely lived, have the same discount factor $\beta \in (0, 1)$, and maximize the present discounted sum of their per-period expected payoffs.

There are two types of non-storable consumption goods in the economy: a special indivisible good and a general divisible good. The agents can produce and consume both types of consumption good, while the government cannot produce either type of good and can consume only the general good. However, an agent cannot consume the special good that he produces. The cost of producing one unit of the special good is $c > 0$, while the cost of producing $x \geq 0$ units of the general good is x. The agents obtain utility $u > c$ from consuming one unit of the special good and the government and the agents obtain utility x from consuming $x \geq 0$ units of the general good. Thus, only the trade of the special good generates gains from trade.

Agents can trade the special good in a decentralized market. This activity is taxed by the government, though. The sequence of events in a period is as follows. First, the agents decide whether to stay in the market and trade or move to autarky. Autarky is absorbing, and its payoff to zero. The agents who stay in the market first meet with the government and pay taxes. Then they are randomly and anonymously matched in pairs. Only one agent in a match can produce the special good, i.e., be the producer, and production is voluntary. Moreover, the producer can produce only one unit of the special good. For ease of exposition, we say that the meetings with the government take place in the morning and the meetings in the market take place in the afternoon.

The assumption that the government derives utility from the consumption of general goods can be replaced with the assumption that the government and the agents derive utility from a public good which the government can produce using general goods, but that the government is self-interested and so does not care about the agents' payoffs. We discuss this alternative model at the end of the next section.

In the absence of a technology of exchange, agents have no incentive to produce in the market and trade does not take place. We consider two different technologies of exchange: credit and money. We first describe the economy with credit and then describe the economy with money.

Credit Economy

In the credit economy there exists a monitoring technology that keeps track of the agents' behavior in the market. This technology is costly to use, though. In each period, the agents who stay in the market pay $\phi(u-c)/2$ units of the general good to use this technology, where $\phi \in [0,1]$. The parameter ϕ measures the amount of resources that society needs to spend in order to keep the monitoring technology in place.

The monitoring technology assigns to each agent either a good or a bad label. All agents start with a good label and a bad label is absorbing, i.e., once an agent gets a bad label he stays with this label forever after. An agent with a good label can get a bad label only if he is a producer in his market meeting, his partner has a good label, and the agent fails to produce to his partner. In this case, the agent gets a bad label with probability $\rho \in [0,1]$. The parameter ρ measures the efficiency of the monitoring technology.

Trade takes place as follows. After two agents meet in the afternoon, they observe each other's label and a coin toss determines which of the two agents is the producer; the other agent is the consumer. We consider a "credit arrangement" in which trade takes place only if both agents in a match have a good label. Hence, an agent with a bad label is permanently excluded from trade, and so has no incentive to stay in the market. This arrangement provides agents with the greatest incentive to trade.[2] The expected gains from trade to an agent with a good label are $(u-c)/2$.

[2]Our analysis shows that the surplus that the government can extract from agents is increasing in the agents' incentive to trade. Thus, considering equilibria in which agents with a bad label are permanently excluded from trade is without loss of generality; no other credit arrangement which respects the agents' incentives to trade allows the government to extract more surplus from agents.

A policy for the government is a sequence $\pi = \{x_t\}_{t=1}^{\infty}$, where x_t is the quantity of the general good that an agent who stays in the market in period t produces to the government in the morning of period t. The government chooses the policy π that maximizes its payoff. We restrict attention to policies in which $x_t \equiv x$. We discuss this assumption in the next section.

The agents' participation constraints restrict the ability of the government to extract surplus from them. Indeed, agents must have an incentive to stay in the market, and high enough taxes will discourage them from doing so. As we are going to see in the next section, the agents' incentive compatibility constraints for trade—a producer must have an incentive to produce to his partner in the afternoon—also restrict the government's ability to extract surplus from agents.

Our model of credit is not a model of bilateral credit. Instead, it is a model of multilateral credit (or gift-giving) in which agents are "indebted" to society as a whole. We chose this model of credit for its simplicity. In the Appendix, we show that we obtain the same results in a model of bilateral credit. Moreover, as we discuss in Section 5, our model of multilateral credit is more consistent with credit arrangements observed in some primitive societies than a model of bilateral credit.

Monetary Economy

In the monetary economy there exists a durable indivisible good, money, which has no intrinsic value and can be produced only by the government, at zero cost. Each agent can hold at most one unit of money and money holdings are not observable by the government. Money is randomly distributed by the government to a fraction $m_0 \in [0, 1]$ of agents at the beginning of period 0.

Trade takes place as follows in the monetary economy. After two agents meet in the afternoon, they observe each other's money holdings. Production only takes place in a meeting in which one agent has one unit of money and the other agent has zero units of money, in which case the agent with no money produces to the other agent in exchange for the latter's money. We discuss our assumption of indivisible money and a unit upper bound on money holdings in the next section.

A policy for the government is now a list $\pi = \left(m_0, \{(x_t^0, q_t^0, x_t^1, q_t^1)\}_{t=0}^{\infty}\right)$, where $x_t^i \in \mathbb{R}$ is the quantity of the general good that an agent with $i \in \{0, 1\}$ units of money who stays in the market in period t produces to the government in the morning of period t and $q_t^i \in [0, 1]$ is the probability that this agent enters his meeting in the afternoon of period t with one unit of money. Thus, the government can set taxes in terms of goods and money. Notice that we allow the transfers x_t^i to be negative, i.e., the government can transfer general goods to the agents.[3] The government chooses the policy π to maximize its payoff. Let m_t be the fraction of agents with one unit of money in the morning of period t. We restrict attention to policies in which $m_t \equiv m$ and $(x_t^i, q_t^i) \equiv (x^i, q^i)$ for all $i \in \{0, 1\}$. We discuss this assumption in the next section.

As in the credit economy, the agents' participation and incentive compatibility constraints restrict the government's ability to extract surplus from them. Since the agents' money holdings are not observable by the government, the government policy must also be such that agents have an incentive to truthfully reveal their money holdings to the government.[4]

3 Credit versus Money

In this section we first analyze the credit economy and then analyze the monetary economy. After that, we compare both economies and determine under which conditions the government prefers the monetary economy over the credit economy. Our main result is that in some cases the government prefers the monetary economy even if the credit economy generates greater gains from trade. We conclude by discussing some of our assumptions.

3.1 Credit Economy

Since the government is restricted to choose policies in which the agents pay the same tax to trade in every period, we can describe any policy π by its associated per-period tax x. Let $V^c = V^c(x)$ be the present discounted lifetime payoff to an agent with a good label who chooses to stay in the market in a given period; this payoff does not depend on t. The agents have an incentive to stay in the market as long as $V^c \geq 0$. Moreover, an agent with a good label who stays in the market has

[3]Since the government cannot produce general goods, not all policies in the monetary economy are feasible. We discuss feasibility in the next section.

[4]The restriction to policies in which agents truthfully reveal their money holdings to the government is without loss of generality. Indeed, let π be a policy such that in some period t agents with one unit of money have an incentive to claim that they have zero units of money; as we discuss later, we only need to check the incentives of agents with one unit of money. Now consider the policy $\widehat{\pi}$ that coincides with π in every period other than t and is such that $\widehat{x}_t^i \equiv x_t^0$, $\widehat{q}_t^0 = q_t^0$, and $\widehat{q}_t^1 = 1$. The government's payoff from $\widehat{\pi}$ is that same as its payoff from π. Moreover, under $\widehat{\pi}$, agents with one unit of money in period t are indifferent between truthfully revealing their money holdings or not.

an incentive to trade in the afternoon if, and only if,

$$-c + \beta \max\{0, V^c\} \geq \beta(1-\rho) \max\{0, V^c\}; \quad (1)$$

as agents have the option of moving to autarky, the continuation payoff to an agent with a good label is $\max\{0, V^c\}$.[5] A necessary condition for (1) is that $V^c > 0$, in which case (1) becomes

$$V^c \geq \frac{c}{\beta\rho}. \quad (2)$$

Hence, agents have an incentive to stay in the market as long as there are incentives to trade.

Without any loss, we can restrict attention to policies for which (2) holds, otherwise there are no gains from trade and agents have no incentive to stay in the market, in which case the government's payoff is zero. Under (2), it is immediate to see that

$$V^c = \frac{1}{1-\beta}\left[-x + (1-\phi)\frac{(u-c)}{2}\right],$$

in which case we can rewrite (2) as

$$x \leq x^c(\phi, \rho, \beta) = (1-\phi)\frac{(u-c)}{2} - \frac{(1-\beta)c}{\beta\rho}. \quad (3)$$

Clearly, the government wants to set the tax x as high as (3) allows, subject to the constraint that $x \geq 0$. We then have the following result.

Proposition 1. *Credit trade is feasible if, and only if,* $x^c = x^c(\phi, \rho, \beta) \geq 0$*, in which case the government's flow payoff is* $G^c = x^c$*.*

It is easy to see that x^c is decreasing in ϕ and increasing in ρ and β. Indeed, by reducing the agents' payoff from trading in the market, an increase in the cost of operating the monitoring technology reduces the ability of the government to tax the agents. To understand why an increase in the efficiency of the monitoring technology makes the government better off, notice from (2) that an increase in ρ reduces the payoff that agents need to derive from trade in order to be willing to produce in the market. This, in turn, allows the government to extract more rents from the agents

[5] Recall that an agent with a bad label exits the market, and so has a continuation payoff of zero.

and still respect their incentives to trade. Likewise, an increase in β also relaxes (2), as it makes it more costly for an agent to be excluded from trade, and so allows the government to extract more rents from the agents without destroying their incentives to trade.

3.2 Monetary Economy

The assumption that the government must tax agents in the same way in every period and the stock of money in the economy remains constant over time implies that a policy in the monetary economy is described by a list (m, x^0, q^0, x^1, q^1), where $(x_t^i, q_t^i) \equiv (x^i, q^i)$ for all $i \in \{0, 1\}$ and $m_t \equiv m$. Since $m_{t+1} = m_t q^1 + (1 - m_t)q^0$ for all $t \geq 0$, the restriction that $m_t \equiv m$ implies that m must be such that $m = mq^1 + (1-m)q^0$. Notice if that if $q^1 < 1$, then $q^0 > 0$, otherwise $m = 0$ and monetary trade does not take place. Likewise, if $q^1 = 1$, then $q^0 = 0$, otherwise $m = 1$ and monetary trade also does not take place. Both of these facts are useful in what follows.

Since the government cannot produce general goods, not all policies are feasible: transfers of general goods to some group of agents must be funded by the general goods collected from the other agents in the economy. The total amount of general goods that the government collects in a given period is $mx^1 + (1 - m)x^0$, which is also the government's flow payoff in any period. Thus, a policy is feasible if, and only if, $mx^1 + (1 - m)x^0 \geq 0$. Since the government has the option of setting $x^1 = x^0 = 0$ and obtain a payoff of zero, the optimal policy for the government is necessarily feasible. Thus, we can ignore feasibility constraints in what follows.

As in the credit economy, we can restrict attention to policies in which the agents have an incentive to stay in the market and trade. Let W^i and V^i be the present discounted payoff to an agent with $i \in \{0, 1\}$ units of money in the morning and afternoon, respectively. Then:

$$W^0 = -x^0 + q^0 V^1 + (1 - q^0)V^0; \tag{4}$$

$$W^1 = -x^1 + q^1 V^1 + (1 - q^1)V^0; \tag{5}$$

$$V^0 = m[-c + \beta W^1] + (1 - m)\beta W^0; \tag{6}$$

$$V^1 = (1 - m)[u + \beta W^0] + m\beta W^1. \tag{7}$$

The interpretation of equations (4) to (7) is straightforward. For instance, an agent with zero units of money in the morning produces x^0 units of the general good to the government and receives one unit of money from the government with probability q^0. Likewise, an agent with zero units of money in the afternoon meets someone with money with probability m, in which case he produces in exchange for one unit of money, and meets someone without money with probability $1-m$, in which case he remains with zero units of money.

It follows from (4) and (5) that

$$V^1 - V^0 = (1-m)u + mc. \tag{8}$$

Hence, (6) and (7) imply that

$$W^1 - W^0 = x^0 - x^1 + (q^1 - q^0)\left[(1-m)u + mc\right]. \tag{9}$$

We then have the following result.

Lemma 1. *The payoffs W^0 and V^0 satisfy:*

$$W^0 = -x^0 + q^0\left[(1-m)u + mc\right] + V^0;$$
$$V^0 = -\frac{mc}{1-\beta} - \frac{\beta}{1-\beta}\left[mx^1 + (1-m)x^0\right] + \frac{\beta}{1-\beta}m\left[(1-m)u + mc\right].$$

A corollary of Lemma 1 is that changes in q^0 and q^1 that keep m constant do not affect V^0 and affect W^0 only through the change in q^0. Likewise, changes in x^0 and x^1 that keep the government's flow payoff $mx^1 + (1-m)x^0$ constant also do not affect V^0 and affect W^0 only through the change in x^0. We make use of these two facts later on.

Agents have an incentive to stay in the market if $W^i \geq 0$ for all $i \in \{0,1\}$. Moreover, because of the unit upper bound on money holdings, an agent with one unit of money has an incentive to reveal his money holdings truthfully to the government if

$$W^1 \geq -x^0 + V^1; \tag{10}$$

agents with zero units of money cannot hide their money holdings from the government since the government can always ask to see an agent's money holdings if he claims to have one unit of money. It follows from (5) and (7) to (9) that (10) is equivalent to

$$x^0 - x^1 \geq (1 - q^1)\left[(1-m)u + mc\right].$$

Finally, an agent with no money in the afternoon has an incentive to trade if

$$-c + \beta W^1 \geq \beta W^0, \tag{11}$$

while an agent with money in the afternoon has an incentive to trade if

$$u + \beta W^0 \geq \beta W^1. \tag{12}$$

Since the government can tax agents differently depending on their money holdings, we need to ensure that both incentive compatibility constraints for trade are satisfied.[6] It follows from (9) that we can rewrite (11) and (12) as

$$u \geq \beta\left\{x^0 - x^1 + (q^1 - q^0)\left[(1-m)u + mc\right]\right\} \geq c.$$

Given that $V^1 > V^0$ by (8), we have that

$$-x^0 + V^1 \geq -x^0 + q^0 V^1 + (1 - q^0)V^0 = W^0.$$

Hence, $W^0 \geq 0$ and (10) imply that $W^1 \geq 0$, and so we can ignore the participation constraint of the agents with one unit of money. Thus, the government's problem is

$$\begin{array}{rll} \max_{(m,x^0,q^0,x^1,q^1)} & mx^1 + (1-m)x^0 & \\ \text{s.t.} & m = mq^1 + (1-m)q^0 & \\ & -x^0 + q^0\left[(1-m)u + mc\right] + V^0 \geq 0 & (IR^0) \\ & x^0 - x^1 \geq (1-q^1)\left[(1-m)u + mc\right] & (NHC) \\ & \beta\left\{x^0 - x^1 + (q^1 - q^0)\left[(1-m)u + mc\right]\right\} \geq c & (IC^0) \\ & u \geq \beta\left\{x^0 - x^1 + (q^1 - q^0)\left[(1-m)u + mc\right]\right\} & (IC^1) \end{array}.$$

[6]In typical models of monetary trade, agents with money are always willing to part with their money holdings in exchange for consumption goods.

We claim that (IR^0) binds. Suppose not. Since (IC^0), (IC^1), and (NHC) depend only on $x^0 - x^1$ and m is independent of x^0 and x^1, the government can increase both x^0 and x^1 by the same positive amount $\varepsilon > 0$ without affecting these constraints. Given that (IR^0) is slack, this change in policy is feasible if ε is small enough. Thus, the government is behaving suboptimally. We have thus established the following result.

Lemma 2. (IR^0) *binds at an optimal solution to the government's problem.*

We can now establish our main result about monetary trade.

Proposition 2. *Monetary trade is feasible if, and only if,* $\beta u > c$*. When monetary trade is feasible, the government sets*

$$m = m^* = \frac{\beta u - c}{2\beta(u - c)} \tag{13}$$

and obtains flow payoff

$$G^m = m^*(1 - m^*)(u - c) - \frac{(1 - \beta)m^* c}{\beta}.$$

There are multiple optimal choices of (x^0, q^0, x^1, q^1) *when monetary trade is feasible. Any optimal policy is such that* $x^0 > 0$*,* $q^0 > 0$*,* $x^1 < 0$*, and* $q^1 < 1$.

It follows from Proposition 2 that any optimal policy for the government involves redistribution of general goods and money: the government transfers general goods from the agents without money to the agents with money and transfers money from the agents with money to the agents without money. If one interprets a tax on money holdings, i.e., $q^1 < 1$, as inflation, then optimal government policy is inflationary.

To understand why inflation is beneficial for the government, recall that $q^0 = 0$ if $q^1 = 1$. So, without inflation, the constraint that agents with one unit of money in the morning must be willing to reveal their money holdings to the government implies that $W^1 > W^0$. Thus, while the government is able to extract all surplus from the agents without money in the morning, it leaves some rents for the agents with money in the morning.

Now observe that while inflation can destroy the incentive of agents without money in the afternoon to produce in exchange for money, it increases the likelihood that an agent without money in the morning enters the market in the afternoon with money. As it turns out, (IC^0) is slack at the optimal government policy without inflation. Indeed, if (IC^0) binds, then $V^0 = \beta W^0 = 0$, and so $x^0 = 0$ by Lemma 1. Given that (NHC) implies that $x^0 \geq x^1$ when $q^1 = 1$ and $q^0 = 0$, the government's behavior is suboptimal. Thus, starting from no inflation, some inflation, i.e., a decrease in $q^1 - q^0$, is beneficial for an agent without money in the morning: it does not destroy his incentives for trade in the afternoon and makes it more likely that such an agent has money to trade in the afternoon. This, in turn, allows the government to extract more rents from agents without money in the morning.

The reason why the optimal government policy also involves $x^0 > x^1$ is that otherwise the agents with one unit of money in the morning would have an incentive to lie to the government when there is inflation.[7]

3.3 Comparing Credit and Money

To understand the forces at play which impact the government's payoff under credit and money, recall that the government's flow payoff in the credit economy is

$$G^c = (1-\phi)\frac{u-c}{2} - \frac{(1-\beta)c}{\beta\rho},$$

and the government's flow payoff in the monetary economy is

$$G^m = m^*(1-m^*)(u-c) - \frac{(1-\beta)m^*c}{\beta}.$$

If the monitoring technology is too costly or too inefficient, taxes under money are higher than taxes under credit. For instance, if ρ becomes too small, say because it becomes harder to keep track of behavior when the society is large or complex, taxes under credit will fall below zero. In this case, the government is better off under the monetary arrangement, but so is society, i.e., the credit arrangement cannot be sustained as an equilibrium even if there is no taxation. This echoes Neil Wallace's idea that "we use money with strangers, and we don't with people we know" (2013, page 06), which underpins the standard view on the essentiality of money, i.e., the idea that money

[7] In the proof of Proposition 2 we show that (IC^1) is also slack at the optimal government policy when there is no inflation. Thus, the government is able to increase $x^0 - x^1$ to ensure that (NHC) still holds after it decreases $q^1 - q^0$.

is essential because it allows to achieve desirable allocations that could not be achieved otherwise.

More interestingly, there are instances where, in the presence of taxes, the government is better off under money but, absent taxes, society is better off under credit. This is transparent when $\rho = 1$, so the technology is quite efficient; and $\phi < \frac{1}{2}$, so the surplus under credit, given by $(1-\phi)(u-c)/2$, is strictly larger than the surplus under money, given by $m^*(1-m^*)(u-c)$. In this case, if there is not taxation, society is clearly better off under credit. However, the government is better of under money if and only if

$$\frac{1}{2} - \phi < \frac{1}{2}\left[\left(1 + \frac{1-\beta}{\beta}\frac{c}{u-c}\right)^2 - 1\right].$$

For every $\beta < 1$, there exists $\underline{\phi}(\beta) < \frac{1}{2}$ such that the above condition holds for all $\phi \in \left(\underline{\phi}(\beta), \frac{1}{2}\right)$. The intuition for this result runs as follows. Both under credit and under money, the government appropriates the surplus from market exchange but she needs to compensate the agent in the current period for her effort in the previous period. Interestingly, even when $\rho = 1$ and monitoring is perfect, the amount of surplus the government must give up in order to preserve market exchange is always higher under credit than under money. This is so because, while agents are uniformly taxed under credit, under money, the government can actively use her policy instruments to condition taxes on money holdings. In particular, she only needs to give up surplus to agents who hold money in the current period.

Summarizing, the only reason credit may dominate money from the government's perspective is because it creates more surplus. If the difference between the surplus produced under each technology reduces, say because the monitoring technology worsens or because money becomes a more efficient medium of exchange, the government may prefer to tax under the monetary arrangement even though the credit arrangement is better from the society's point of view. This suggests a different view on the essentiality of money, one that is not driven by welfare considerations but by money's efficiency in transferring surplus from the society to the government.

3.4 Discussion

To be written.

4 Co-Existence

So far in our analysis we have not allowed for the co-existence of credit and money. In order to allow for this possibility we now assume that the monitoring technology is limited and only keeps track of the behavior of a fraction $\alpha(\phi) > \phi$ of the market meetings. For ease of exposition, we let $\rho = 1$ in what follows.

We consider a "mixed arrangement" where agents truthfully announce their money holdings in their meeting with the government and, in non-monitored meetings between an agent with money and an agent without money, the latter produces to the former in exchange for money. In turn, in monitored meetings, they observe each other's label and a coin toss determines the producer. As in the previous section, we restrict attention to stationary policies (m, x^0, q^0, x^1, q^1).

Consider first the scenario where the government wants to maximize the agent's ex-ante welfare subject to his incentive compatibility, participation, and truth-telling constraints. The expected payoff of an agent with a good label and with one unit of money at the beginning of a period is

$$V_1^\omega = -\phi \frac{u-c}{2} + \alpha(\phi) \left(\frac{u-c}{2} + \beta V_1^\omega \right) + [1 - \alpha(\phi)] \left[m\beta V_1^\omega + (1-m)(u + \beta V_0^\omega) \right],$$

while the expected payoff of an agent with a good label and no money is

$$V_0^\omega = -\phi \frac{u-c}{2} + \alpha(\phi) \left(\frac{u-c}{2} + \beta V_0^\omega \right) + [1 - \alpha(\phi)] \left[m(-c + \beta V_1^\omega) + (1-m)\beta V_0^\omega \right].$$

If an agent refuses to produce in a monitored meeting, he does not need to contribute to the monitoring technology but he is permanently excluded from consumption in all future monitored meetings. Thus, an agent produces in monitored meeting meetings if and only if $-c + \beta V_i^\omega \geq \beta V_i^{\overline{\omega}}$, where $V_i^{\overline{\omega}}$ is the expected payoff of holding $i \in \{0, 1\}$ units of money and only participating in exchange in non-monitored meetings. We can rewrite this condition as

$$\beta \left[\alpha(\phi) - \phi \right] (u - c) \geq 2(1 - \beta)c. \tag{14}$$

We assume that agents are patient enough and (14) hold. In turn, an agent without money produces in a non-monitored meeting if and only if $-c + \beta V_1^\omega \geq \beta V_0^\omega$, which can be rewritten as

$$\alpha(\phi) \equiv \alpha^\omega(\phi) < \frac{\beta\left[(1-m)u + mc\right] - c}{\beta(1-m)\left(u-c\right)}, \tag{15}$$

and monitoring must be sufficiently limited. Indeed, if monitoring is abundant, agents have no incentive to produce in exchange for money, since they will rarely participate in a non-monitored meeting. This reasoning implicitly assumes that an agent does not use money in monitored meetings as a way to induce a partner without money to undertake the production opportunity. Precisely, it requires $\left(u-c\right)/2 + \beta V_0^\omega \geq -c + \beta V_1^\omega$, i.e., an agent without money does not accept money in exchange for his production in a monitored meeting. This condition can be rewritten as

$$\beta\left\{c + \left[1-\alpha(\phi)\right]\left[(1-m)\left(u-c\right)\right]\right\} \leq \left(u+c\right)/2. \tag{16}$$

The ex-ante welfare is

$$(1-\beta)U^\omega = \left[\alpha(\phi) - \phi\right]\frac{u-c}{2} + \left[1-\alpha(\phi)\right]m(1-m)(u-c),$$

and the optimal choice of money supply is equal to $1/2$, as in the environment under the pure monetary arrangement. Note that (16) holds when $m = 1/2$.

Consider now the scenario where the government wants to maximize her own payoff. Let W_i^g be the present discounted payoff of an agent with $i \in \{0,1\}$ units of money at the beginning of the morning, and V_i^g be the present discounted payoff of an agent with $i \in \{0,1\}$ units of money at the beginning of the afternoon. We have

$$W_i^g = -x^i - \phi\frac{u-c}{2} + q^i V_1^g + (1-q^i)V_0^g,$$

and

$$\begin{aligned} V_1^g &= \alpha(\phi)\left(\frac{u-c}{2} + \beta W_1^g\right) + \left[1-\alpha(\phi)\right]\left[m\beta W_1^g + (1-m)(u + \beta W_0^g)\right] \\ V_0^g &= \alpha(\phi)\left(\frac{u-c}{2} + \beta W_0^g\right) + \left[1-\alpha(\phi)\right]\left[m(-c + \beta W_1^g) + (1-m)\beta W_0^g\right]. \end{aligned}$$

The same arguments used in the proof of Lemma 1 imply that, in the determination of the optimal policy, it suffices to find (x^0, q^0, x^1, q^1) which solve for

$$W_0^g = 0, \tag{17}$$

$$-c + \beta W_1^g = \beta W_0^g, \tag{18}$$

and

$$W_1^g = -x^0 - \phi \frac{u-c}{2} + V_1^g, \tag{19}$$

together with the stationarity condition (**??**). The first constraint corresponds to the incentive of an agent without money to participate in the market, the second constraint corresponds to the incentive of an agent without money to produce in a non-monitored meeting, and the latter constraint captures the incentive of an agent with money to truthfully announce his money holdings in the meeting with the government.[8] After some computation, we obtain that, in the optimal policy, we must have

$$x^1 = [\alpha(\phi) - \phi] \frac{u-c}{2} + q^1 \{c + [1 - \alpha(\phi)] (1-m) (u-c)\} - \frac{c}{\beta},$$

and

$$x^0 = [\alpha(\phi) - \phi] \frac{u-c}{2} + q^0 \{c + [1 - \alpha(\phi)] (1-m) (u-c)\} .$$

Intuitively, the government faces a trade-off between taxation of general goods and taxation of money holdings. Expected taxes are given by $x(\phi, m) = mx^1 + (1-m)x^0$, which can be rewritten as

$$x(\phi, m) = [\alpha(\phi) - \phi] \frac{u-c}{2} + [1 - \alpha(\phi)] m(1-m)(u-c) - \frac{(1-\beta)mc}{\beta}.$$

As expected, $x(\phi, m)$ combines elements of the pure credit and the pure monetary arrangement. The government chooses m in order to maximize $x(\phi, m)$. The optimal money supply is

$$m(\phi) = \frac{\beta [1 - \alpha(\phi)] u - [1 - \beta\alpha(\phi)] c}{2\beta [1 - \alpha(\phi)] (u-c)}.$$

[8] As in the pure monetary arrangement, the participation constraint and the incentive compatibility constraint of an agent with money are implied by (17) and (18). However, in the presence of credit, we also need to make sure that an agent wants to produce in a monitored meeting. This condition is satisfied whenever (14) holds. Finally, we need to make sure that money is not used in monitored meetings. This requires $(u-c)/2 + \beta V_0^g \geq -c + \beta V_1^g$. Since $V_1^g = \phi(u+c)/2 + (1-\phi)[(1-m)u + mc]$, and $V_0^g = \phi(u-c)/2$, this condition is always satisfied, irrespective of the value of m.

A positive money supply requires

$$\alpha(\phi) \equiv \alpha^{g}(\phi) < \frac{\beta u - c}{\beta\,(u-c)}, \tag{20}$$

i.e., there must be a relatively large number of non-monitored meetings. Indeed, if most meetings are monitored, the government has no incentive to introduce money in the economy and simply taxes as in a pure credit arrangement. Note that $m(\phi) < \widehat{m}$ is strictly decreasing in ϕ and it converges to $\widehat{m}$ when ϕ goes to zero. Thus, the presence of credit further increases the inefficiency of monetary trades as compared to the environment under the pure monetary arrangement. Intuitively, m cannot be too far away from $1/2$ in the economy without credit because trades can only be conducted with money. As a result, the impact of the money supply on the extensive margin of trade is critical. This is not the case anymore when trade can also be conducted with credit.

More interestingly, by comparing (15) and (20), we obtain that, for all $m > 0$, there exists a region of parameters $\alpha(\phi) \in (\alpha^{\omega}(\phi), \alpha^{g}(\phi))$ where money cannot be part of the equilibrium if the objective of the government is to maximize the welfare, but money is part of the equilibrium if the objective of the government is to maximize her own payoff. Intuitively, even if monitoring is relatively abundant, the agent has an incentive to produce in exchange for money in a non-monitored meeting so he can reduce the transfer of general goods he needs to make to the government in the following period.

5 Historical Evidence

The search for the origin of money is a concern of economists since Smith (1776). Following Smith (1776), Jevons (1875) and Menger (1892) explained that money emerged out of the inconvenience of bartering goods. In their analysis, the willingness to save on the costs created by the difficulty of finding a double coincidence of wants explains why private agents have (learned to) used medium of exchange.[9] They took as evidence of the origin of money in the inconveniences of trades the use of very different goods as medium of exchanges in many different places, characterized by very

[9] A long tradition in economics argues that money saved on the difficulty associated with bartering, from the issue of the absence of divisibility of most goods discussed by Smith (1776) to the cost associated with recognizing the value of goods in barter economies emphasized by Alchian (1977), see Alvarez and Bignon (2013) for details.

different polities. Among the goods that they mentioned (and that are cited in many textbooks), one finds cattle, salt, tobacco, species of shells, etc

Anthropology and history have examined the empirical validity of early economists claims on the origin of money. Anthropology provides analysis mostly on primitive societies, where primitive means a community of people not governed by a state. History and numismatic have provided arguments explaining the reason for the introduction of money in ancient societies governed by a centralized polity, i.e. by a ruler able to tax other agents. Here money is identified with coinage. Early economists writing about money often used the two sets of evidence interchangeably, ignoring the role of the organization of the polity in the circulation of money.

In this section we review the primary evidence that history and anthropology gathered to examine the claim that the counterfactual situation to monetary circulation was the difficulties of bartering, and on the type of payment system that they replaced. We also examine the evidence regarding the motives for initiating the circulation of money. The discussion of the coexistence of money and credit is divided in three parts, to account for the differences in the meaning and use of credit and money in primitive societies (that we call thereafter communities) versus in societies with a centralized polity, and to account for the two forms of credit used in societies that had a circulating medium of exchanges

5.1 At the origin (1): Primitive media

Available anthropological evidence suggest the widespread use of gift-giving system of exchanges alongside the use of some media of exchanges in very specific transactions.[10]

In primitive or tribal communities, everyday trades were mediated by a system of reciprocal exchanges or gift-giving (Mauss, 1925; Polanyi, 1944), where the reciprocity in the exchanges takes the form of informally enforced agreements to give goods, services, information in exchange for future compensation in kind (**?**Kranton, 1996). There are also some trades that are mediated by a medium of exchanges, a list of which may be found in Einzig (1948) or Quiggin (1949). It is a noted fact that goods used media of exchanges were also usually a religious symbol (Laum, 1924)

[10]This leads some anthropologists to argue that "money" of primitive societies does not have the same understanding as the money today, see notably the debate between the anthropologists working in **?**'s tradition and Melitz (1970) on whether money was and is a general medium of exchanges in primitive versus contemporary societies.

and in fixed supply.[11] People may have accumulate them as a symbol of prestige or power. When used in payment, they accounted for a transaction vis-a-vis a specific group or clan in specific instances such as blood crime such as the acknowledgement of the bond triggered by a marriage or in mortuary payment, see Graeber (2011) for a recent survey.[12] Primitive media (e.g. cowries) were assigned to the specific purchases, such as acknowledging the gift of the bride's nose or her ears (Dalton, 1965), which made each denomination a non-universal medium of marriage.

The view that in those societies, money was preceded by barter had been challenged by Einzig (1948) and Quiggin (1949) who provided evidence that money was preceded by credit, even in remote places. Einzig's conclusion from the available evidence is that avoiding the inconvenience of barter was never the main reason for their use. Quiggin (1949, p. 5) noticed that the inconveniences [of barter] are avoided in simpler societies by elaborate customs of credit, deferred payments or payment by services. Both authors concludes that some of the previous evidence was mistaken in that some of the goods were not used as medium of exchanges but only as a store of value or as a symbol of wealth (e.g. Quiggin, 1949, p. 3).

The results of our model without a self-interested ruler suggests that primitive societies use multilateral credit when the was efficient enough and that when the record-keeping technology was too inefficient, people prefer using some medium of exchange to account for their trades. The media of exchanges can be understand within this context. Indeed marriage or mortuary payment were unusual or infrequent instances, and the use of some object could have helped to record the transactions.

5.2 At the origin (2): Coinage in societies of the Mediterranean and Near– and Middle-East Antiquity

Almost all currency used throughout the history of have been issued by a public authority or a sovereign ruler. This leads Knapp (1905) to claim that money is a creature of the law. But here

[11] Explorers and early anthropologists such as Temple (1899) assigned the label of currency or money to any good or objects that they thought were used as medium of exchanges in those communities. Thilenius (1921) distinguished between useful objects used in exchange and objects of conventional form, practically useless, mere tokens of value. Goldberg (2005) have shown that all of the objects so far that had been labeled as fiat money by those early anthropologists had in fact some intrinsic value.

[12] See Melitz (1970) and Pryor (1977) for critical discussions of the differences made by Polanyi (1944) and (Dalton, 1965) between primitive versus contemporary moneys.

money means a very specific object, coins (to the point that in the 19th century money was defined as being coins). Indeed there is no doubt that ancient societies used some kind of unit of account, even before the invention of coinage. The issue is whether people made payments with some form of physical medium of exchanges–e.g. coins–or by bartering goods against other goods or whether payments were made using some transfer from one account to another in some form of credit registry (record keeping device). In this section we summarize the available evidence on the following issues. What was the alternative payment system used by traders, when coins were invented? Was coinage invented to provide the means of payment necessary to help traders escape the inconveniences of barter? Was elaborated credit system available to traders? Could traders pay using bank wired transfer? What was the motives of sovereign when striking (new) coins?

We first summarize the available evidence in terms of the counterfactual situation to the use of coinage: What types of payment technology and record keeping devices did the ancient societies use before Croesus introduced coinage? Recent histories of money indicates that irregular pieces of silver ingots could have been used as a means of payments together with barley, wheat and other types of commodities (Le Rider, 2001; Van Reden, 2010). Some historians dispute the fact that those economies were monetary economy, denying those (very) imperfect pieces of silver any medium of exchange role, see notably Seweet (1958); Renger (1995) and (Seaford, 2004) on Babylonian Mesopotamia. One must noted that it took about two centuries after coinage was invented for the emperor of Persia Darius I to introduce coinage in his empire. Yet some other historians disputed that Babylon was an economy without a medium of exchanges on the ground that some trades were pay using silver (Powell, 1999; Goddeeris, 2002). Still everyone agrees that the high value of those cut pieces of silver was too high to have been used in payment of day-to-day transactions, see Powell (1996) on Babylonian Mesopotamia and Howgego (1995) on much later Imperial Rome.

The absence of money left room for other trading technology used in payment of consumption and taxes. Historians concurred that before the introduction of coinage, ancient societies relied mostly either on some form of gift-giving, like in archaic Greece (Van Reden, 2010), or in societies with active markets used to trade commodities on some form of elaborate credit system, like in Mesopotamia (Renger, 1995) or in Egypt (Van Reden, 2007). In Mesopotamia, clay tablets were used to record the debt generated by everyday transactions (Goddeeris, 2002) while Egypt used papyrus. Those economies used common units of account, for example silver or barley in Mesopotamia and grains in Egypt but credit instead of money as a means of payment. It is worth noticing that agents in those societies could easily be located, and a legal system exist to enforce debt repayment (e.g. the Hammurabi code in Babylon dated 1683 BC). The monitoring technology could also easily be implemented in villages or small societies. It could have been easily implemented in societies in which citizens were embedded in various social bonds with neighbors and other citizens that not only helped to identify traders but also increase the punishment in case of default. Finally although credit systems may have been more costly to operate than the monetary technology with large population, the Babylonian or Egyptian history show that credit could have been sustained even in societies with pretty large population and a taste for organizing trades.

We now turn to discuss the reasons that have been put forward to explain the introduction of coinage in ancient societies. The invention of money is usually credited with the introduction of the first form of coinage by the Greek king Croesus who reigned over Lydia (contemporary southwest of Turkey) during the fifth century BC. The invention consisted in standardizing the weight of pieces of metal and hammering a symbol on them-either a figure or an attribute of the ruler. The pieces of metal appear as small irregular disks made of electrum, an alloy of silver and gold that was naturally found in the nearby river of Croesus' kingdom (Le Rider, 2001). Before the introduction of coinage, irregular (and of very different weight) of pieces of silver ingots were found in hoards.

A prediction of our model is that an economy will shift from a payment system with credit to a system using money when the fiscal needs of the ruler increases. To date, the three oft-mentioned explanations of the reasons why coinage was introduced in Lydia are as follows: (i) the payment of war-related expenses such as the wages of mercenaries (Cook, 1958; Graeber, 2011), (ii) its usefulness as a means of receiving taxes (Kraay, 1976) and (iii) as a show of political power (Le Rider, 2001). Kraay (1964) noted that the explanation for the introduction of coinage is unlikely to be rooted in the need to pay retail trade, as there was a small supply of small denominations in all but a few states.[13] There is also an ample historical discussion on the relation between the issuance

[13] Howgego (1990) noticed that this argument may have been overstated. Yet Velde (2012) has shown that the first

of new coins and the pressing fiscal needs caused by wars or games in Rome (Crawford, 1970; Howgego, 1990). There is also an argument that proposed that the monetization of the Roman economy economy followed from the requirement to pay tax with coins (Hopkins, 1980). These are not our argument.

Following (Cook, 1958; Graeber, 2011), there is an fierce historical debate on whether there is any correlation between between the coinage activity (the volume of coins minted) and the occurrence of wars. Our model does not require this type of evidence. We argue that monetization took place because it allows the ruler to tax more than when people pay using credit and not that the profit derived by the ruler from the coinage activity was the source of greater fiscal resources.

An especially interesting case of transition from credit to money is Egypt after its invasion by Alexander the Great. Before the invasion, pre-Hellenistic Egypt did not pay with coins but with credit: Payments were made through the network of public banks and ordering a payment did not required the presence of the account holder (Van Reden, 2010). Historians agree that it took the invasion of Egypt by Alexander the great in 332 for Egypt to initiate a substantial volume of minting. Newell (1923) documented the increased in the number of types of coins minted, see Van Reden (2007) for the survey of recent additional evidence. Le Rider (2003) surveys the consensual historical view that this new coinage activity was related to the much higher fiscal needs of the state, linked notably to the foundation of the new city of Alexandria.[14] Yet even during the Hellenistic period, archeological evidence from Egyptian bank account registers show that there were still taxes and other day-to-day purchases paid using bank transfer even after the conquest of Egypt by the Greeks, see Clarysse et al. (2011).

5.3 Money and spheres of credit across spaces [to be completed]

Even after the invention of coinage, it is conventional wisdom that for almost all economies maybe except the most developed one that (i) money and credit always coexisted to some extent and (ii) money was not of common use in the payments made in some regions. For example, during the

series of coins were divided in various denominations (coins of different weights) and provided evidence that those coins were most probably used as medium of exchanges.

[14]Cleomen, the governor in charge of organizing the construction, levied substantial resources, notably gold and silver, on the clergy and decided to make the government the unique intermediary on the wheat market so as to increase prices, see chapter 6 in Le Rider (2003).

Roman empire from 1st BC to 5th AD, for which some historians denied any significant role to banks and credit (Finley, 1973), recent historical evidence have shown the importance and wide-range of financial services offered by banks and credit in the making of payments and in the financing of the economy (Andreau, 1999; Geva, 2011). Money and credit have also coexisted for centuries in Middle-Ages Europe between the fifth and the 15th centuries (De Roover, 1948; Spufford, 1988), in Modern Europe during the period from the 16th to the 19th century (Van der Wee, 1977), in Japan (Kuroda, 2013) and China (Yang, 1952).

6 Conclusion

To be written.

7 Appendix

Proof of Lemma 1

The fact about W^0 follows immediately from (4) and (8). Hence, (6) and (9) imply that

$$\begin{aligned} V^0 &= -mc + \beta m(W^1 - W^0) + \beta W^0 \\ &= -mc + \beta m \left\{ x^0 - x^1 + (q^1 - q^0)\left[(1-m)u + mc\right] \right. \\ &\qquad \left. + \beta \left\{ -x^0 + q^0\left[(1-m)u + mc\right] + V^0 \right\} \right\}. \end{aligned}$$

The expression for V^0 follows from straightforward algebra.

Proof of Proposition 2

Necessity. We begin by showing that monetary trade is not feasible if $\beta u \leq c$. First notice that monetary trade is feasible only if $m \in (0, 1)$. Now observe, by Lemma 1, that if $m \in (0, 1)$, then

$$V^0 \leq \frac{m}{1-\beta}\left\{\beta m\left[(1-m)u + mc\right] - c\right\} < \frac{m}{1-\beta}(\beta u - c).$$

On the other hand, (IC^0) requires that $V^0 \geq \beta W^0$ and (IR^0) requires that $W^0 \geq 0$. This establishes necessity.

Sufficiency. Suppose that $\beta u > c$. We divide the proof of sufficiency in three parts. We first show that $q^1 = 1$ is suboptimal for the government. We then show that (IC^0) must bind if $q^1 < 1$. To conclude, we use the fact that (IR^0) and (IC^0) bind at an optimal solution to the government's problem to compute the optimal government policy.

Step 1. Suppose that $q^1 = 1$ (and so $q^0 = 0$). We first show that it is optimal for the government to set $x^1 = x^0$ in this case. Notice that (NHC) becomes $x^0 \geq x^1$ when there is no inflation. Let then $x^0 > x^1$. If (IC^0) binds, then $V^0 = \beta W^0$, and so Lemma 2 implies that $x^0 = V^0 = 0$. This, in turn, implies that $mx^1 + (1-m)x^0 < 0$, which is suboptimal for the government. Suppose now that (IC^0) is slack and consider a deviation for the government in which it increases x^1 and decreases x^0 in such a way that $mx^1 + (1-m)x^0$, and thus V^0, remains constant. This deviation relaxes (IC^1) and (IR^0) and keeps the government's payoff the same. Moreover, since (IC^0) is slack, this deviation is feasible as long as the change in x^1 and x^0 is small enough. But then, by Lemma 2, the government is behaving suboptimally. Thus, it is optimal for the government to set $x^1 = x^0$ in the absence of inflation.

Now observe that $x^1 = x^0$ implies that (IC^1) is slack. We know from the main text that (IC^0) is also slack when the government is behaving optimally in the absence of inflation. Let then $\pi = (m, x, 0, x, 1)$ be an optimal non-inflationary policy for the government and consider the following alternative policy: $\widetilde{\pi} = (m, x^0, q^0, x^1, q^1)$, where $x^0 = x + \varepsilon(\lambda)$, $q^0 = \lambda$, $x^1 = x - \varepsilon(\lambda)(1-m)/m$, and $q^1 = 1 - \lambda(1-m)/m$, with $\lambda > 0$ and $\varepsilon(\lambda) = (1-m)\lambda\left[(1-m)u + m\right]$. Clearly, (IC^0) and (IC^1) hold when λ is small enough. Given that $mx^1 + (1-m)x^0 = x$ and $mq^1 + (1-m)q^0 = m$, we have that W^0 changes to

$$-\varepsilon(\lambda) + \lambda\left[(1-m)u + mc\right] + W^0 = m\lambda\left[(1-m)u + mc\right] + W^0 > W^0$$

by Lemma 1. Moreover,

$$x^0 - x^1 = \frac{\varepsilon(\lambda)}{m} = \frac{(1-m)\lambda}{m}\left[(1-m)u + m\right] = (1-q^1)\left[(1-m)u + m\right].$$

Thus, (IR^0), (NHC), (IC^0), and (IC^1) hold under $\widetilde{\pi}$ if λ is small enough. Since (IR^0) is slack, $\widetilde{\pi}$ is suboptimal by Lemma 2. However, the government's payoff under π and $\widetilde{\pi}$ is the same, and so π is suboptimal as well. This concludes the first step.

Step 2. We now show that (IC^0) binds if $q^1 < 1$. We start with the following lemma.

Lemma 3. *Either (IC^0) or (NHC) bind at an optimal solution to the government's problem.*

Proof. Suppose not and consider a reduction in x^0 and an increase in x^1 so that $mx^1 + (1-m)x^0$ is constant. This change is feasible since (NHC) slack implies that $x^0 > x^1$. Now observe from Lemma 1 that such a change keeps V^0 constant, and thus relaxes (IR^0). Since such a change also keeps the government's payoff constant, its bevahior is suboptimal by Lemma 2. ∎

Suppose that $q^1 < 1$. Notice that $q^0 > 0$, otherwise $m = 0$ by the steady-state condition for the stock of money, in which case monetary trade does not take place. We claim that $q^1 = 0$ is not feasible. Indeed, if $q^1 = 0$, then the steady-state condition for the stock of money implies that $m = q^0/(1+q^0) \leq 1/2$. Now observe that (NHC) and $q^1 = 0$ imply that

$$x^0 \geq x^1 + (1-m)u + mc \geq (1-m)u + mc.$$

Hence,

$$mx^1 + (1-m)x^0 \geq (1-m)\left[(1-m)u + mc\right],$$

and so

$$V^0 < \frac{\beta}{1-\beta}\left[(1-m)u + mc\right](2m-1) < 0$$

by Lemma 1. However, $V^0 \geq \beta W^0 = 0$ by (IC^0) and Lemma 2, a contradiction. Thus, $q^1 > 0$.

Suppose now that (IC^0) is slack. Then (NHC) is binding by Lemma 3, and so a necessary condition for (IC^0) is that $q^0 < 1$. Now observe that (NHC) binding and Lemma 1 imply that

$$V^0 = -\frac{mc}{1-\beta} - \frac{\beta}{1-\beta}x^0 + \frac{\beta}{1-\beta}(1-q^1)m[(1-m)u+mc] + \frac{\beta}{1-\beta}m[(1-m)u+mc].$$

Since $V^0 = x^0 - q^0\left[(1-m)u + mc\right]$ by Lemma 2 and $(1-m)q^0 = m(1-q^1)$ by the steady-state condition for the stock of money, we then have that

$$\begin{aligned} x^0 &= -mc + [(1-m)u + mc]\left\{(1-\beta)q^0 + \beta(1-q^1)m + \beta m\right\} \\ &= -mc + [(1-m)u + mc]\left\{q^0(1-\beta m) + \beta m\right\}. \end{aligned}$$

Hence, using the fact that (NHC) binds one more time, we have that

$$\begin{aligned} mx^1 + (1-m)x^0 &= x^0 - (1-q^1)m\left[(1-m)u + mc\right] \\ &= -mc + \left[(1-m)u + mc\right]\left\{\beta m + q^0 m(1-\beta)\right\}. \end{aligned}$$

Therefore, by increasing q^0 and reducing q^1 in such a way that m remains constant, the government increases its payoff; this change is feasible since $q^0 < 1$, $q^1 > 0$, and (IC^0) is slack. Thus, the government is not maximizing its payoff if (IC^0) is slack.

Step 3. We now use the fact that (IR^0) and (IC^0) bind at a solution to the government's problem to compute the optimal government policy. Consider a policy (m, x^0, q^0, x^1, q^1) such that (IR^0) and (IC^0) bind and $q^1 = 1 - (1-m)q^0/m$, so that the steady-state condition for the stock of money holds. Since (IR^0) and (IC^0) binding imply that $V^0 = 0$, we then have that

$$x^0 = q^0\left[(1-m)u + mc\right]. \tag{21}$$

Now observe that (IC^0) binding also implies that

$$\begin{aligned} x^1 &= x^0 + (q^1 - q^0)\left[(1-m)u + mc\right] - \frac{c}{\beta} \\ &= q^1\left[(1-m)u + mc\right] - \frac{c}{\beta}, \end{aligned} \tag{22}$$

and so

$$\begin{aligned} mx^1 + (1-m)x^0 &= \left[mq^1 + (1-m)q^0\right]\left[(1-m)u + mc\right] - \frac{mc}{\beta} \\ &= m\left[(1-m)u + mc\right] - \frac{mc}{\beta}. \end{aligned}$$

Ignoring (NHC), it is immediate to see that the government maximizes its payoff by setting $m = m^*$ given by (13) to obtain a flow payoff of

$$G^m = m^*\left[(1-m^*)u + m^*c\right] - \frac{m^*c}{\beta} = m^*(1-m^*)(u-c) - \frac{(1-\beta)m^*c}{\beta}.$$

To finish sufficiency, observe that if $m = m^*$, then (21) and (22) imply that (NHC) becomes

$$\frac{c}{\beta} \geq (1 - q^0)\frac{\beta u + c}{2\beta}. \tag{23}$$

Clearly, there are multiple choices of (x^0, q^0, x^1, q^1) consistent with optimality. By (23), we have that $q^0 \geq (\beta u - c)/(\beta u + c) > 0$, and so $q^1 \leq 2\beta c/(\beta u + c) < 1$. Thus, $x^0 > 0$ and $x^1 < 0$ by (21) and (22). This concludes the proof.

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