EVOLUTIONARY ADVANTAGE IN THE LENS OF CONTRACTING STRATEGY

ABSTRACT

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We introduce the theoretical framework of evolutionary advantage through the lens of contracting strategy. It differs from static competitive advantage in that the evidence of evolutionary advantage transpires over time when the economic context has evolved. Using game theory, we examine across a mutually exclusive yet collectively exhaustive spectrum of economic contexts how firm and environmental characteristics entail different contracting strategies and, more important, different levels of performance. Our analyses lead to a theoretical framework that implies (a) Firms with evolutionary advantage can use simpler, less costly contracting alternatives in more diverse economic contexts. (b) Firms with evolutionary advantage can organize transactions more efficiently when forced into more complex contracting alternatives. (c) Firms with evolutionary advantage can deliver high levels of performance across diverse contexts, while firms without it perform just as well when the context is predictable but fails when it turns unpredictable. (d) Evolutionary advantage explains heterogeneity in performance of a single firm over time, when the economic context evolves, and among multiple firms, when the context is unpredictable. This paper establishes the mediating role between resources / dynamic capabilities and firm performance and addresses three perceived weaknesses in the extant literature of resource- and dynamic capability-based views.

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1 We use ‘economic environment’ and ‘economic context’ interchangeably.
INTRODUCTION

The industrial organization view of strategy (Bain, 1968; Porter, 1979, 1980, 1985) pioneered the study of external determinants of firm performance, primarily dynamics of the industry. The resource-based view (RBV; Rumelt, 1984; Wernerfelt, 1984; Barney, 1986, 1991; Peteraf, 1993), which was initially developed to complement the industrial organization view with the study of internal determinants of firm performance (Mahoney & Pandian, 1992; Barney, 2002; Peteraf & Barney, 2003), has grown to be the premier theory in explaining competitive advantage and firm performance. While originally static in its nature (Priem & Butler, 2001), the enrichment of dynamic capabilities (Teece & Pisano, 1994; Teece, Pisano, & Shuen, 1997; Helfat & Peteraf, 2003) has essentially rectified this shortcoming of RBV. Nevertheless, be it resource- or dynamic capabilities-based view, the key to firm performance is always sustained competitive advantage.

That competitive advantage is ‘sustained’, however, appears confusing. While RBV focuses on static environments, under which a certain set of resources or capabilities could be similarly valuable over time, the literature of dynamic capabilities has flourished under the realization that addressing environmental changes is essential for performance (Audia, Locke, & Smith, 2000). Indeed, the majority of researchers (Teece, 2000, 2007; Winter, 2003; Zahra, Sapienza, & Davidsson, 2006; Helfat et al., 2007) adopt the entrepreneurial perspective (Schumpeter, 1934) and believe dynamic capabilities to reflect manager’s skills or capabilities to adapt in times of change. Some scholars (Eisenhardt & Martin, 2000; Zollo & Winter, 2002) follow more closely the insight of evolutionary economics (Nelson & Winter, 1982) and believe dynamic capabilities capture routines or processes that achieve the same goal. Regardless of which tradition we look at, there is an undeniable neglect of external determinants as if dynamic
capabilities were universally powerful regardless of how the environment changes. In other words, it is dubious whether competitive advantage that can be sustained through one change of environment can also prove itself sustainable in successive following changes. Hence, it appears that the resource / dynamic capability view that was to complement the industrial organization view has largely separated from external analyses.

Further, dynamic capabilities are conceptualized to help firms adapt by adjusting their configurations of resources and capabilities (Eisenhardt & Martin, 2000; Helfat et al., 2007). Some scholars have further demarcated two ‘levels’ of capabilities: normal capabilities used to solve problems, and ‘dynamic’ capabilities that are only used to adjust normal capabilities (Winter, 2003; Zahra et al., 2006). This construction suffers from the same critique of infinite regress in RBV (Collis, 1994), for it suggests higher-order capabilities are more valuable than first-order capabilities – yet still higher-order capabilities may exist – indeed, regardless of whether dynamic capabilities are routines or skills, the capability to develop these routines or skills must be a higher-order capability. This logic continues ad infinitum. Exactly which level is ‘sustained’ becomes unobvious.

Lastly, recent studies have decoupled competitive advantage from firm performance (Newbert, 2008), as competitive advantage is conceptualized as a mediator between resources and performance – and the mediating relationship is subject to the influence of various moderators. Since the role of dynamic capabilities is to adjust the configuration of resources so as to gain competitive advantage, dynamic capabilities must also be decoupled from performance, a view echoed by Eisenhardt and Martin (2000). Consequently, sustained competitive advantage may not entail sustained performance.

In the light of these three weaknesses in our understanding of competitive advantage in
changing environments, we propose a new concept that differs from sustained competitive advantage – *evolutionary competitive advantage*, or evolutionary advantage. The understanding of evolutionary advantage builds on the mediating role of competitive advantage and incorporates internal as well as external analyses. It also circumvents the multi-level conceptualization and captures instead resources / capabilities that are valuable not only when changes occur but also when they do not, thereby reunifying the analyses of resources and dynamic capabilities regardless of whether changes occur in the environment. We show that the weaknesses we have laid out above are not inherent defects of resource- or dynamic capability-based views; they are the by-product of forcing *sustained* competitive advantage into changing environments.

To address this goal with rigour at the theoretical level entails several technical challenges: *(a)* The environment must be changing continually in a single dimension yet stable across other dimensions. This way, we can extract clean results that link evolutionary advantage to performance under changes along this dimension of the environment. *(b)* We must compare various environments which all satisfy the first requirement and yet differ in how predictable changes take place. This way, we can truly appreciate the unifying quality of the theory of evolutionary advantage. *(c)* Optimally, we should simultaneously decompose evolutionary advantage and retain firm as the level of analysis. In consideration of these challenges, we find the lens of contract to be an adequate option: It can be easily analysed through a game-theoretic model, which readily addresses the first two challenges. Further, it enables us to decompose the effects of evolutionary advantage on performance along the temporal dimension. Hence, while our analysis echoes the spirit of micro-foundations approach (Lippman & Rumelt, 2003; Felin & Foss, 2005; Gavetti, 2005; Teece, 2007), we do away with multiple levels of analyses.
Specifically, we adopt a simple model of long-term supply relationship. Inherent to its simplicity lies our intention to circumvent the common reasons for incomplete contracting (Tirole, 1999). Each day the daily demand becomes public information, and a firm tries to meet this demand. Its effort is limited by the supply of raw material it has received the day before. As Figure 1 illustrates, we examine a spectrum of abstractions of economic context and identify, in relation to firm characteristics, performance implications of a spectrum of contracting options for the transaction. Both spectra are mutually exclusive yet collectively exhaustive. This forms our technical analyses. They reveal how qualities of the firm as well as the environment together affect whether a contracting alternative is insufficient, optional, or necessary. Through the lens of contracting strategy, we establish the link between resources / dynamic capabilities and performance mediated by evolutionary advantage.

![Figure 1. Two pillars of our analysis](image-url)

Our theoretical framework of evolutionary advantage anchors on two features in our analyses: (a) We study a spectrum of economic contexts that all share continually changing demand yet differ in how the demand changes over time. This helps us establish the concept of evolutionary advantage. (b) We dissect contracting strategy along the temporal dimension, which
allows for understanding evolutionary advantage by decomposing it in a spirit that echoes, but differs from, the micro-foundations approach in strategy (see review in Foss & Pedersen, forthcoming). The four main hypotheses in this framework focus on the link between evolutionary advantage and performance: 

(i) Firms with evolutionary advantage can use simpler, less costly contracting alternatives in more diverse economic contexts. 

(ii) Firms with evolutionary advantage can organize transactions more efficiently when forced into more complex contracting alternatives. 

(iii) Firms with evolutionary advantage can deliver high levels of performance across diverse environments, while firms without it perform just as well when the environment is predictable but fails when it turns unpredictable. 

(iv) Evolutionary advantage explains heterogeneity in performance of a single firm over time, when the economic context evolves, and among multiple firms, when the context is unpredictable.

Our main contribution is the establishment of mediating role of evolutionary advantage between resources / dynamic capabilities and performance. In doing so, we readily address the three points of weakness in extant literature. Therefore, this paper adds tangible value to the literature of resource-based view (RBV) and dynamic capabilities. First, our construction incorporates both internal and external analyses for firm performance and, in spirit, the resource-base and industrial organization views. It also helps explain why empirical support for the direct link between resources and performance has been modest (Armstrong & Shimizu, 2007; Newbert, 2007). Secondly, all environments we analyse are continually changing – we do not distinguish ‘dynamic’ capabilities as ‘higher-level’, for we believe ‘static’ environment to be an exception rather than the rule. We do concur, nevertheless, that some resources / capabilities may not deliver evolutionary advantage in general but in some specific environment may deliver static competitive advantage. Thirdly, by introducing the mediating role of evolutionary
advantage, we essentially point out that the literatures of RBV and dynamic capability indeed have focused on the direct link between resource / capability and performance while in fact they are most useful in explaining (evolutionary) competitive advantage.

The remainder of this paper proceeds as follows. We first lay out a simple model that forms the basis of our contracting strategy analysis and that opens the door to understanding evolutionary advantage. Then, we assess the performance implications of firm characteristics vis-à-vis various contracting options across three abstractions of economic context that are mutually exclusive yet collectively exhaustive. Based on our findings, we lay out a theoretical framework of evolutionary advantage through the lens of contracting strategy and present four main hypotheses aimed at inspiring future empirical research. Lastly, we discuss the value addition to the literature of competitive strategy, implications for empirical research, and limitations.

MODEL

Evolutionary advantage is only relevant to long-term transactions, and for it to transpire the economic context must have direct impact on the transaction. Therefore, we focus on a long-term supply relationship subject to exogenous demand between two firms, a seller and a buyer. The seller produces a good that is the key component for the buyer’s product; it is the only limiting factor. The buyer faces continually changing exogenous demand, which is not shared with any competitor; we can understand this as the result of differentiation. In order to avoid confusion, we use good for the trade between the two firms and final product for what the buyer sells on the market. We also refer to the process the seller makes the good as production and that the buyer makes the final product assembly.

As Figure 2 illustrates, the transaction begins after negotiation. The seller adjusts its
capacity at the beginning of Day 1 before producing the good that is traded at the end of the day. On Day 2, it repeats this process with possible adjustment in capacity according to the volume it trades with the buyer. Meanwhile, having received the good from the seller, the buyer assembles the final product on Day 2 and sells it on the market. This process repeats infinitely. Without loss of generality, we let one unit of good correspond with one unit of final product. We also normalize the production and assembly costs to zero. The cost of negotiation is non-negative. It is noteworthy that the strategy insights remain unchanged if the final product reaches the market any other following day. Hence, we let production and assembly take the same amount of time only for simplicity.

Several variables correspond with this story: (i) Exogenous demand, \(d_t\). (ii) Volume, \(v_t\). (iii) Unit price, \(p\). (iv) The seller’s capacity, \(k_t\). The seller adjusts its capacity to \(k_t\) before production, so \(v_t \leq k_t\). Without adjustment, \(k_t = k_{t-1}\). (v) Price of final product, \(\pi\). Subscript \(t\) denotes the date where applicable.

A number of other variables and parameters appear in our analysis: (vi) Excess, \(e_t\), is the volume of unsold final product. It becomes positive when the buyer assembles more than the market demands. (vii) Excess has to be stored. Unit cost of storage is \(\sigma\) per day. (viii) Residual demand, \(r_t\), is the demand left after absorbing excess from the previous day; \(r_t = \max\{0, d_t - e_{t-1}\}\). (ix)
The adjustment of capacity may be costly. Unit cost of new capacity for the seller is \( \gamma \). Keeping unused capacity may also be costly. Unit cost of unused capacity is \( \phi \) per day. Finally, the daily discount rate is \( \delta; \delta \in (0,1) \). In our analysis, we generally consider the costs of storage, \( \sigma \), new capacity, \( \gamma \), and unused capacity, \( \phi \), to be positive. They are the parameters that capture firm-specific characteristics.

In this context, contract is a set \( \{p,f,R\} \), where \( p \) is the unit price, \( f \) the volume schedule, and \( R \) a set of rules. Specifically, \( p \) can be either an arbitrary number or a function, and its value lies in \([0,\pi]\). The volume schedule, \( f \), captures the expectation of demand the firms agree on; it can be an arbitrary natural number, a function, or a transition probability distribution. The set of rules, \( R \), include terms that govern how firms use the parameters \( p \) and \( f \) to carry out the transfer of good and payment. The optimal forms of \( p \) and \( f \), as we show in our analysis, are determined by firm and environmental characteristics. The terms prescribed in \( R \) vary correspondingly.

In our analysis, we frequently refer to efficiency: A contract is more efficient if at least one firm is better off and neither is worse off. It is efficient if it maximizes the combined ex ante payoffs to both firms. Lastly, the feasible contract that maximizes the combined ex ante payoffs to both firms is optimal. An optimal contract may be inefficient.

For clarity and brevity, we restrict our attention in five ways. (a) Unmet demand does not carry to the next day. (b) The market price, \( \pi \), is fixed. (c) Reduction of capacity is free. (d) The seller may optimally maintain some unused capacity, i.e. the cost of unused capacity is lower than that of new capacity, \( \phi < \gamma \). (e) All variables and parameters are observable and verifiable.

We should also clarify that, while we restrict our attention to demand, we do not claim it is the sole possible factor in the environment that affects long-term transactions.

In our analysis, we use the Nash bargaining solution (Nash, 1950) as the solution concept:
(a) It enjoys a set of axiomatic properties, including symmetry and Pareto-optimality, that suit perfectly our goal of identifying optimal contracting strategy. (b) It is independent of the specific format of contract negotiation and thereby broadens the applicability of our results. (c) It is easily tractable even in complex, evolving stochastic environment.

DETERMINISTIC ENVIRONMENT

From the researcher’s perspective, an abstraction of environment captures some fundamental features in the model. From an objective point of view, an abstraction reflects some predominant characteristics an economic environment possesses. In three consecutive sections, we assess how different abstractions of environment entail different optimal contracting strategies and, more important, different levels of performance, in relation to characteristics of the firms. To this end, whereas traditionally one paper focuses on a single abstraction, we analyse a mutually exclusive yet collectively exhaustive spectrum of them: in ascending order of complexity, we examine deterministic, stationary stochastic, and evolving stochastic environments. Specifically, we answer how qualities of the firm as well as the environment together affect whether a contracting alternative is insufficient, optional, or necessary. Through the lens of contracting strategy, we establish the link between resources / dynamic capabilities and performance mediated by evolutionary advantage when the economic context of a transaction evolves.

Deterministic demand sits at the heart of the simplest abstraction of economic environment. It entails that demand is known beforehand through a deterministic process. Ostensibly, market alone suffices given the simplicity of the environment and verifiability of all variables. However this holds with certainty only when a transaction is not repeated.

When the transaction repeats itself infinitely, market suffices only if the unit cost of
capacity is lower than the price of the good, i.e. $\gamma \leq p$. Otherwise, inefficient distribution of the exposure to changing demand becomes evident each time the seller faces the decision to adjust capacity. For example, with a price $p \in (0, \pi)$, the seller would not invest in new capacity to meet an additional unit of demand that does not recur and lasts no longer than $n$ days if $np < \pi$. However, as long as there exists an $m \in \{1, 2, \ldots\}$ such that $m\pi > \gamma$, it is more efficient for the transaction and more beneficial to the buyer if the seller made this investment. Here, $\pi$ is the market price of the final product and $\gamma$ is the unit cost of capacity.

Since such failure of constant pricing arises from inefficient distribution of the exposure to exogenous demand, a natural response is to redistribute the exposure by introducing flexible pricing. An efficient choice is prescribed by the Nash bargaining solution (Nash, 1950) conditional on whether additional capacity is needed: $p_{\text{flex}} = \frac{\pi + xy}{2}$ with $x = 1$ if additional capacity is needed for this additional unit and 0 otherwise. The flexible price, $p_{\text{flex}}$, simply combines the two state-dependent Nash bargaining solutions, $\frac{\pi}{2}$ and $\frac{\pi + \gamma}{2}$. It ensures the firms divide evenly the surplus from the trade of a unit of good. With $p_{\text{flex}}$, if $\pi \geq \gamma$, all demands are met. If $\pi < \gamma$, the buyer would only purchase the good if subsequent demands the additional capacity satisfies make up for the upfront loss. Anticipating this, the seller would only invest in capacity that the buyer would split the cost of. This is also the condition for the long-term transaction to be efficient. We summarize the preceding analysis in the following proposition.

**Proposition 1.** If demand is known beforehand, the following hold:

i) In a market with price $p$, firms trade efficiently without contract if the seller’s cost of new capacity, $\gamma$, is no higher than $p$; that is, if $\gamma \leq p$.

ii) In a market with price $p$, the seller under invests when firms trade without contract if the seller’s cost of new capacity, $\gamma$, is higher than $p$; that is if $\gamma > p$. 

11
iii) In a market with flexible pricing, \( p_{\text{flex}} \) firms trade efficiently without contract.

The intuition here is straightforward. In particular, \( p_{\text{flex}} \) leads the transaction in the same way an efficient contract would. It reflects some real-world orders under capacity constraint, where additional units above the original capacity are priced higher (e.g. Oren, Smith, & Wilson, 1985). Flexible prices other than \( p_{\text{flex}} \) may implement efficiency but is as restrictive as constant pricing while retaining all the complications of flexible pricing. With a constant price, if \( \gamma > p \) efficiency is unobtainable because the condition under which the seller invests differs from the one for efficient trading, which requires cost-sharing infeasible without flexible pricing. Specifically, the seller invests only if it can recuperate its investment.

Without a contract, spot transaction is not renegotiation-proof unless the price is exogenous. While both firms are generally better off if the transaction is more efficient, many negative sum outcomes offer one side higher payoffs. Hence, in Proposition 1 we consider the prices (\( p \) and \( p_{\text{flex}} \)) to be exogenous, perhaps the result of convention or market interaction. Nevertheless, we reckon that flexible pricing is more precarious, for frequent changes in unit price are more likely to result in miscommunications and/or provoke disagreements.

As Figure 3 illustrates, the only firm-specific characteristic that stands out is the unit cost of new capacity, \( \gamma \). It captures the seller’s ability to adjust its production: If it can obtain new capacity easily (cheaply), perhaps trading with a constant price without contract already suffices. Otherwise, even when market demand is perfectly predictable, firms can benefit from flexible pricing. Ceteris paribus, with a lower \( \gamma \) the seller has better chance of trading under efficiency, for flexible pricing may be difficult to arrange. In fact, a lower \( \gamma \) benefits both firms. Hence, the seller with a lower \( \gamma \) is more attractive to the buyer. Other firm-specific characteristics, including the costs of storage, \( \sigma \), and unused capacity, \( \phi \), are indeed also part of the consideration. They
influence (ii) in Proposition 1. When both are higher, the seller under-invests more severely and the efficiency of constant pricing drops further, and vice versa.

![Figure 3. Firm characteristic, contracting options, and performance](image)

**STATIONARY STOCHASTIC ENVIRONMENT**

A step up in complexity is stationary stochastic demand. It entails that changes in the economic environment is no longer perfectly predictable, yet that demand can nevertheless be forecasted probabilistically. Technically, we model demand as a time-homogenous Markov chain; that is, the transition probability distribution (transition matrix) is stationary over time, and the conditional probability distribution of future demands depends only on current demand (Markov property). Stationary stochastic demand renders spot transaction inefficient for many firms, though a single adaptive contract would suffice regardless of whether the true transition matrix is known.

We say spot transaction is *usually* inefficient because *sometimes* it is — it remains so even in the most complex environment. In our specific model, the condition is $\gamma \leq \pi$; that is, an
additional unit of capacity costs no more than the market price of the final product. Obviously, with \( p_{\text{flex}} \) the trade each single day is efficient under spot transaction regardless of the type of environment, for it removes all constraints of production that underlie the need for a contract. In all following analysis, we give our attention to the case where this condition is not satisfied.

Let the set of possible demand be \( D = \{0, 1, \ldots, D - 1\} \). The corresponding \( D \times D \) transition matrix is \( F \). Let \( \omega_t \) be the observed state at \( t \); it is a vector in which the \((d_t + 1)\)th element is 1 and all others are 0. On future date \( s \), the expected state is \( \omega_s|t = \omega_t F^{s-t} \). We order the complete set of possible demands each day as a column vector \((0, 1, \ldots, D-1)\) and use \( S \) to represent it; \( S \) has \( D \) members. The expected demand on future date \( s \) given observed state at \( t \) is \( \mathbb{E}d_s|t = \omega_s|t S = \omega_t F^{s-t} S \).

We primarily focus on the case where the transition matrix is known. Apparently, current demand, \( d_t \), must be a contingent variable in the rules of the contract \((R)\) in order to make predictions of future demands. Current excess, \( e_t \), is no less important. Essentially, the firms trade each day if the expected residual demand, \( \mathbb{E}r_{t+1} = \mathbb{E}d_{t+1} - e_t = \omega_t F S - e_t \), is positive. As \( p_{\text{flex}} \) ensures alignment of interest that paves the way for efficient trading under stochastic demand, it must also be adopted.

Even if the true transition matrix is unknown, firms can optimally approach efficient trading through Bayesian updating. Essentially, firms agree to trade as if the true transition matrix were known, while they also agree to update its parameters each day following the Bayes rule. As the transaction continues and the number of observations increases, the posterior approaches the true value and the efficiency of the contract improves. We omit further technical details on the likelihood function and the updating process, for despite considerable complexity they bring no additional strategy insight. Interested readers can refer to Strelioff, Crutchfield, and
Hubler (2007). Proposition 2 summarizes the above observations and prescribes the set of rules, $R$, of the adaptive contract.

**Proposition 2.** In stationary stochastic environment, the following hold:

i) If the seller’s unit cost of new capacity, $\gamma$, is lower than the market price of the final product, $\pi$, firms can trade efficiently with flexible price $p_{\text{flex}}$.

ii) Otherwise, for efficiency firms need an adaptive contract that satisfies

- a. Volume depends on excess, expected demand, and the seller’s capacity.
- b. The seller reduces capacity if keeping it unused is more costly than reinvesting.
- c. It increases a unit of demanded capacity if its return outweighs its cost.
- d. For the efficient contract, (a) to (c) prescribe the set of rules ($R$), while the true transition matrix, $F$, is the volume schedule ($f$). Unit price is $p_{\text{flex}}$.
- e. If $F$ is unknown, Bayesian updating of this contract approaches efficiency.

We give the numeric details as well as the proof of (ii) in Proposition 2 in the appendix. The intuitions of (i) and (iii) directly follow our preceding analysis. In the real world, many transactions take place without a contract, even though real-world environment cannot always be abstracted as “deterministic”. This is attributable to two considerations. First comes contracting cost: Spot transaction is favourable unless the increase of efficiency by having a contract offsets this cost. The more predictable the environment is, the less a contract increases the efficiency of the transaction. The second is constraints in production: As (i) in Proposition 2 highlights, without these constraints each period can be independently efficient sans contract (in the spirit of Williamson, 1979).

The contracting decisions prescribed by Proposition 2 are dependent on a set of firm-
specific characteristics. Ideally, firms would have the costs of new capacity, $\gamma$, unused capacity, $\sigma$, and storage, $\phi$, as low as possible. Figure 4 captures the qualitative results: Similar to the argument in the previous subsection, if $\gamma$ is sufficiently low a contract is unnecessary. In the efficient contract, the expectation of excess is always 0, eliminating $\sigma$ from \textit{ex ante} considerations. \textit{Ceteris paribus}, with the adaptive contract the seller invests more responsively to changes in demand if $\gamma$ is lower and/or if $\phi$ is higher. Conversely, it divests more responsively to changes if $\gamma$ is higher and/or if $\phi$ is lower. Therefore, with lower $\gamma$ and lower $\phi$ the firms have higher chance of meeting all the demand.

![Figure 4. Firm characteristic, contracting options, and performance](image)

The adaptive contract prescribed by (ii) in Proposition 2 maximizes payoffs to both firms and is therefore renegotiation-proof. It is also efficient when the transition matrix is known. Meanwhile, even though we say firms can slowly approach efficiency through Bayesian updating when the transition matrix is unknown, we must admit that the calculations involved are complicated and cannot be realistically adopted by two firms without causing issues in
coordination and perhaps dispute. This accentuates the particularly strong position bestowed on firms that can avoid contracting thanks to low cost of new capacity, \( \gamma \), for efficiency is guaranteed for them. Meanwhile, it does not contradict the advantages enjoyed by firms with low costs of unused capacity, \( \sigma \), and storage, \( \phi \), for they also enjoy more efficient transactions compared to firms that have to bear these costs at higher levels.

**EVOLVING STOCHASTIC ENVIRONMENT**

For transactions that last years or even decades, the trend of demand may evolve over time. Evolving stochastic environment entails that not only changes in demand are not perfectly predictable but also the stochastic process that governs the demand evolves over time and can only be probabilistically forecasted. In this context, an adaptive contract alone cannot be efficient, for its underlying assumption of stationary transition matrix is invalid. It is only temporarily efficient at the time it is written; it is only temporarily renegotiation-proof but not so inter-temporally. Updates to the contract are therefore crucial for maintaining efficiency in the long run.

Before we continue, it is important to define formally “evolving stochastic environment”. A subordinated stochastic process (common in finance literature; Clark, 1973) with Markov property provides perhaps the simplest abstraction: A second stochastic process with Markov property, \( \Theta \), is the subordinator that guides the evolution of demand: Say the demand follow stochastic process \( d_{t+1} \sim X_{\theta_t}(d_t) \), whose parameters follow a second stochastic process \( \theta_{t+1} \sim \Theta(\theta_t) \). Then, \( X \) is the subordinated stochastic process, and its subordinator, \( \Theta \), influences \( X \) through the transition matrix, \( F \), which captures \( \theta \). Essentially, the trend of demand as reflected in \( F \) evolves. The difference between stationary and evolving stochastic environments is therefore salient: Whereas in the former the state in the future is dependent on the current state,
in the latter the way changes in states take place in the future is dependent on the current way changes take place. Consequently, while in stationary stochastic environment the Bayesian posterior converges, in evolving stochastic environment there are more unknown parameters than observable relations and therefore the Bayesian posterior cannot converge (Nyarko, 1991).

In this context, a single contract without adaptation cannot be efficient: Even if firms know exactly how the second stochastic process governs the evolution of demand beforehand, they can only write a temporarily efficient contract that becomes inefficient over time. — Firms can only contract on their expectation, which is a single point in a broad spectrum of possible future states. Hence, firms must update their contract in order to maintain efficiency. It is noteworthy that with evolution “over time” we are still referring to the ex ante efficiency (before the value of demand is realized) of the contract.

Contractual adaptation is one solution to incorporate such updates. It differs from both contract renegotiation and incomplete contract, despite some apparent resemblance. Complete contracts must have already exhausted all opportunities of mutually beneficial improvements (Dewatripont & Maskin, 1990). The possibility of renegotiation after defection reduces the prospect of cooperative outcome, for costly sanction against defection loses credibility (Farrell & Maskin, 1989). Similarly, after each voluntary renegotiation the updated contract is temporarily renegotiation-proof — opportunities of mutually beneficial improvement arise only over time, for, as our analysis shows, an inter-temporally renegotiation-proof contract is infeasible. Contractual adaptation is not an extension of incomplete contract, either. A contract may be incomplete due to indescribability of contingencies (Hart & Moore, 1999), renegotiation (Wernerfelt, 2007), or spatial complexity (Segal, 1999), etc. In contrast, contractual adaptation responds to temporal complexity. While other types of incompleteness expose firms to the
infamous hold-up problem (Williamson, 1975, 1985; Grossman & Hart, 1986; Hart & Moore, 1990), which stands to reduce the contract’s efficiency (Hart & Moore, 1988), temporal complexity brings no new hold-up hazard.

Intuitively, when actual demand deviates significantly from expectation, over time the original terms become increasingly inefficient. Whenever inefficiency surpasses a threshold, update becomes preferable. Notably, this voluntary update of existing contract is Pareto-improving, which draws a sharp contrast to the often zero-sum renegotiation driven by opportunism (e.g. Fudenberg & Tirole, 1990; Edlin & Hermalin, 2000; Schmitz, 2002; Plambeck & Taylor, 2007).

The literature has indeed explored some theoretical solutions that purport to manage the evolution of environment. A complete contingent contract (e.g. Svensson, 1981) is the most straightforward: if firms can anticipate all permutations of the future, updates are redundant. However, the evolutionary problem we face differs from the core problem contingent contracts address, which is how an ex ante contract can manage interim contingency revelation to achieve ex post efficiency (Rothschild & Stiglitz, 1976). In contrast, evolution is continual, and correspondingly the number of contingencies grows exponentially over time; it is impossible to write a contingent contract for infinite horizon.

Another popular type of contract adopts the insight of folk theorem (Fudenberg & Maskin, 1986; Fudenberg, Levine, & Maskin, 1994): firms can set up some rules of cooperation and enforce them by peer retaliation (Rubinstein, 1979). However, the focus of this literature is sustaining cooperation among firms with conflicting interests. It is questionable whether a renegotiation-proof equilibrium solution (Farrell & Maskin, 1989) exists when the environment is evolving continually. Even if such solution does exist, the detailed interaction between the
firms over time should share some insight with the updating of contract we examine here. For these reasons, we do not have to go further into the details of the evolving stochastic environment — we are only concerned with the performance implications when the needs to update the contract arise, not with how they arise.

Indeed, contractual adaptation is only one of the possible reactions to this predicament. A common subject in the literature also helps firms update the contract – multiple short-term contracts (Crawford, 1988). Each short-term contract is a finite-duration variant of the adaptive contract with duration, $T$. They approach the update differently by substituting the adaptation of the old contract with the drafting of a new one. More important, a new contract is written regardless of the efficiency of the old one. Hence, while more flexible than a single long-term contract, multiple short-term contracts fare worse in precision compared to contractual adaptation.

![Figure 5. Multiple short-term contracts and contractual adaptation](image)

Specifically, a new contract is written independent of inefficiency: Either $T$ is too short,
which incurs unnecessary drafting cost, or it is too long, which entails loss from avoidable inefficiency. Hence, multiple short-term contracts cannot match the efficiency of contractual adaptation. Potentially, if each short-term contract is renewable as is without cost, unnecessary drafting cost can be avoided: when the threshold of inefficiency is not breached, firms renew the original contract for free. This special variant becomes contractual adaptation if $T=1$. Therefore, contractual adaptation and multiple short-term contracts indeed share the same spirit: *efficiency through flexibility*. Figure 5 captures the differences between multiple short-term contracts and contractual adaptation. Proposition 3 summarizes the above observations.

**Proposition 3.** Under evolving stochastic demand, the following hold:

i) Contractual adaptation ensures efficiency regardless of firm characteristics.

ii) Multiple short-term contracts are less efficient than contractual adaptation due to lower precision but more efficient than a single contract thanks to higher flexibility.

iii) With the option of free renewal, multiple short-term contracts turns into contractual adaptation as the duration of each short-term contract becomes $1$.

Because each short-term contract is simply a finite-duration variant of the adaptive contract, the discussion of firm-specific characteristics after Proposition 2 is echoed here. One additional point worth noting is that the costs of new capacity, $\gamma$, unused capacity, $\sigma$, and storage, $\phi$, have *evolutionary implications* on the transaction. *Ceteris paribus*, the lower they are, the less frequent the needs to update the contract arise. In case contractual adaptation cannot be arranged and firms have to rely on multiple short-term contracts, having lower $\gamma$, $\sigma$, and $\phi$ allows the firms to retain relatively higher efficiency. Further, *ceteris paribus*, when firms adopt multiple short-term contracts, they can set a longer duration, $T$, if their costs are lower. Therefore, these firm-
specific characteristics are not only carriers of static competitive advantage but also the core of evolutionary advantage.

As regards contractual adaptation, we have also proven that these firm-specific characteristics directly affect three critical aspects: (a) They collectively determine how responsive firms are in adapting to the evolving economic context. (b) When firms go through contractual adaptation, they influence how costly the process may be. (c) Above all, they affect how efficient the firms can trade in the long run. This part of the analysis is particularly involved yet provides merely a confirmation to the above observation – some firm characteristics are carriers of evolutionary advantage – hence, we omit it from the text here.

**EVOLUTIONARY ADVANTAGE IN THE LENS OF CONTRACT**

In the above analyses, we have focused on contracting strategy: we addressed primarily the question of optimal contracting alternatives as well as their performance implications given firm and environmental characteristics. Table 1 summarizes these findings. In comparing the results from across the spectrum of economic contexts, that some firm characteristics consistently affect performance becomes evident. These characteristics capture some resources and dynamic capabilities firms possess; hence, ostensibly our work merely suggests that resources and dynamic capabilities contribute to competitive advantage. Newbert (2008) has shown that competitive advantage mediates resources and performance. In the same vain, Teece (2007) argues that dynamic capabilities foster business practices that underlie competitive advantage, which ultimately leads to higher performance. Therefore, on the surface we seem to add no new insight to the literature – but that is without considering the continually changing economic context as well as our dissection of contracting strategy along the temporal dimension. These two unique features allow us to delineate evolutionary competitive advantage, or
evolutionary advantage, that differs from static competitive advantage.

Table 1. Contracting alternatives across economic contexts

<table>
<thead>
<tr>
<th>Type</th>
<th>Variation</th>
<th>Environment</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Deterministic</td>
</tr>
<tr>
<td>No contract</td>
<td>Flexible</td>
<td>efficient if $\gamma \leq \pi$</td>
</tr>
<tr>
<td></td>
<td>Constant $p_{\text{flex}}$</td>
<td>efficient if $\gamma \leq p$</td>
</tr>
<tr>
<td></td>
<td>Flexible $p_{\text{flex}}$</td>
<td>efficient if $\gamma \leq p$</td>
</tr>
<tr>
<td>Adaptive Long-Term</td>
<td>unnecessary</td>
<td>efficient when $F$ known</td>
</tr>
<tr>
<td></td>
<td>Bayesian</td>
<td>approaches efficiency</td>
</tr>
<tr>
<td>Multiple Short-Term</td>
<td>Costly renewal</td>
<td>unnecessary</td>
</tr>
<tr>
<td></td>
<td>Free renewal</td>
<td>unnecessary</td>
</tr>
<tr>
<td>Adaptation</td>
<td>unnecessary</td>
<td>unnecessary</td>
</tr>
</tbody>
</table>

As we study a spectrum of economic contexts that all share continually changing exogenous demand yet differ in how the demand changes over time, we are able to link evolutionary advantage to characteristics of the environment. Environments can be predictable, which is reflected in our deterministic abstraction, and they can be unpredictable, which in the extreme is captured by our evolving stochastic abstraction. We find that the same firm characteristics indeed affect performance through different mechanisms, even though the directions of their effects remain the same. Therefore, while static competitive advantages may share the same channel in affecting firm performance, there is a spectrum of channels for evolutionary advantage dependent on characteristics of the environment: (a) Evolutionary advantage may entail that a firm can organize the transaction using simpler, less costly alternatives and thereby achieve higher performance. (b) In unpredictable environments, firms
with higher evolutionary advantage can consistently organize their transaction more efficiently.

(c) In highly predictable environments, evolutionary advantage coincides with static competitive advantage. Figure 6 presents a vivid illustration of these mechanisms in a single graph.

Figure 6. The effect of evolutionary advantage on firm performance in various environments

A Snapshot

Resource / Dynamic capability → Static Competitive Advantage → Performance

General Situation

Environment

Resource / Dynamic capability → Evolutionary Advantage → Performance

Figure 7. Static vs evolutionary competitive advantage
For this last reason, we postulate that competitive advantage should be considered from the evolutionary perspective, and that static competitive advantage is only a snapshot of a more general situation where the moderating role of the environment between evolutionary advantage and performance is negligible. Figure 7 illustrates this view.

We build our analysis through the lens of contract. Hence, we decompose evolutionary advantage by the myriad transaction-level mechanisms through which it affects firm performance along the temporal dimension. In this sense, our effort emulates in spirit the micro-foundations approach in strategy first called for by Lippman and Rumelt (2003) and echoed by Felin and Foss (2005), Gavetti (2005), and Teece (2007). Researches in the micro-foundations approach that touch on performance (Eisenhardt, Furr, & Bingham, 2010) or resources and dynamic capabilities (Foss, 2011; Hodgkinson & Healey, 2011; Argot & Ren, 2012; Helfat & Peteraf, 2015) go down the levels of analysis and focusing on how interactions among lower-level forces affect macro-level constructs. In contrast, we retain a focus on firms and instead dissect competitive advantage along the temporal dimension. Hence, we are able to uncover evolutionary insights.

The first and most straightforward revelation relates to the contracting options available to firms. Our analyses show that the external context of a transaction does not by itself determine the set of options firms can efficiently organize the transaction. Broadly speaking, some firms by virtual of their internal characteristics can enjoy simpler, less costly options unavailable to other firms. There is some resemblance between this revelation and the strategy insights captured by transaction cost economics (Williamson, 1975, 1985) and the hold-up problem (Grossman & Hart, 1986; Hart & Moore, 1990). Nevertheless, our argument is based on characteristics that firms have control of (resources and dynamic capabilities) across various economic contexts.
instead of on rigid features of a type of transaction in a determined institutional setting. Our finding also differs from the theory of optimal contract in economics (e.g. Fudenberg, Holmstrom, & Milgrom, 1990), whose focus is the analysis of the context of a transaction rather than the role of firm characteristics that underlie evolutionary advantage. This is the first mechanism through which evolutionary advantage affects firm performance. It applies across the entire spectrum of economic contexts. Hypothesis 1 provides a synopsis of this mechanism, and Figure 8 illustrates its impact on firm performance across various economic contexts.

**Hypothesis 1a.** Firms with evolutionary advantage can use simpler, less costly contracting alternatives given the economic context.

**Hypothesis 1b.** Firms with evolutionary advantage can use simpler, less costly contracting alternatives across diverse economic contexts.

While the insight of Hypothesis 1 goes beyond determined institutional settings and thereby beyond the prescriptions of transaction cost economics, we concede that oftentimes the
choice of contracting options is out of the hands of firms. Legislations, regulations, and conventions may all play a part in determining or confining this choice. Nevertheless, our analyses show that the effects of evolutionary advantage remains even when firms are forced into complex contracting alternatives. Various extant theories shed light on performance implications of a contract; a prime example may be agency theory / relational contract (Baker, Gibbons, & Murphy, 2002, 2011; Prendergast, 1999; Gibbons 2005). The novelty in our framework of evolutionary advantage is that this quality that could have allowed for higher performance through simpler contracting option nevertheless could allow for higher performance when simpler options are unavailable. That is, the benefit of evolutionary advantage is multifaceted. This sums up the second hypothesis. It again applies across the entire spectrum of economic contexts.

**Hypothesis 2.** *Firms with evolutionary advantage can organize transactions more efficiently when forced into more complex contracting alternatives.*

The third interesting result adds a colourful spin to the relationship between static and evolutionary advantage illustrated in Figure 7. As we have argued throughout this paper, firms with evolutionary advantage are poised to deliver high performance across diverse environments thanks to the myriad mechanisms encompassed in Hypotheses 1 and 2. One point that is also obvious here but not thoroughly emphasized in the extant literature concerns firms that lack evolutionary advantage: Our analyses ascertain that these firms would nevertheless deliver high performance when the economic context is predictable. If we believe that when the environment is highly predictable evolutionary advantage coincides with static competitive advantage, this practically suggests that firms without competitive advantage can flourish under the right circumstances. This echoes the notion of luck in extant literature (e.g. Barney, 1986; Porter,
Our insight, however, goes beyond subscribing to ‘luck’ as we clearly delineate (a) when luck is unimportant (with evolutionary advantage) and (b) what constitutes luck (economic context). It is captured by the following dual of hypotheses.

**Hypothesis 3a.** Firms with evolutionary advantage can deliver high levels of performance across diverse economic contexts.

**Hypothesis 3b.** Firms without evolutionary advantage perform just as well when the economic context is predictable but fails when the context turns unpredictable.

Last but not least, the theory of evolutionary advantage explains heterogeneity in performance across dimensions: When the economic context evolves over time, firms with heterogeneous evolutionary advantage diverge in levels of performance – a prediction studying static competitive advantage cannot arrive at. In unpredictable environments, evolutionary advantage also explains performance heterogeneity across firms – again a prediction studying static competitive advantage when the environment is predictable cannot produce. Hypothesis 4 summarizes these insights.

**Hypothesis 4a.** Evolutionary advantage explains heterogeneity in performance of a firm when the economic context evolves.

**Hypothesis 4b.** Evolutionary advantage explains heterogeneity in performance among multiple firms, when the economic context is unpredictable.

Here, we should reemphasize that in our framework firms are both takers in reacting to changes in the environment and managers in changing themselves to garner evolutionary advantage. In this, our perspective differs from that of evolutionary economics (Hannan & Freeman, 1977; Nelson & Winter, 1982, 2002): We establish the link between resources / dynamic capabilities and performance mediated by evolutionary advantage when the economic
context of a transaction evolves. In contrast, evolutionary economics understands firm survival when the socioeconomic context of an organization evolves.

**DISCUSSION**

In this section, we discuss a few questions that may pop into one’s mind when reading the theoretical framework of evolutionary advantage. Our goal is twofold: to help readers understand the value addition of our framework to the vast literature in competitive strategy, and to provide a primer that will hopefully inspire future empirical work on evolutionary advantage. Through this discussion, our theoretical contribution becomes clear.

**Value addition**

Our main contribution is the establishment of mediating role of evolutionary advantage between resources / dynamic capabilities and performance. In doing so, we readily address the three points of weakness in extant literature, which are outlined in the introduction, and beyond. Therefore, this paper adds tangible value to the literature of resource-based view (RBV) and dynamic capabilities.

First, our construction succinctly illustrated by Figure 7 incorporates both internal and external analyses for firm performance. Similar to existing literature, we concur that evolutionary advantage reflects some innate qualities of a firm, be it skills / capabilities (Teece, 2000, 2007; Winter, 2003; Zahra, Sapienza, & Davidsson, 2006; Helfat et al., 2007), routines / processes (Eisenhardt & Martin, 2000; Zollo & Winter, 2002), or resources. However, we fully decouple evolutionary advantage from performance. While firms with evolutionary advantage still deliver higher performance, external determinants nevertheless impact these firms (Figure 6, Figure 8). This should help explain why empirical support for the direct link between resources and performance has been modest (Armstrong & Shimizu, 2007; Newbert, 2007). In this sense, our
construction inherently combines the resource-base and industrial organization views.

Secondly, we include resources and ‘normal’ capabilities as determinants of evolutionary advantage, a point confirmed by our game-theoretic model. Indeed, we have uncovered a myriad of mechanisms through which the same firm characteristics could impact performance; exactly which one is dominant depends on the economic context. More important, all environments we analyse are continually changing – we do not distinguish ‘dynamic’ capabilities as ‘higher-level’ (Winter, 2003; Zahra et al., 2006), for we believe ‘static’ environment to be an exception rather than the rule. We do concur, nevertheless, that some resources / capabilities may not deliver evolutionary advantage in general but in some specific environment may deliver static competitive advantage. Related to our inclusion of resources is that we do not believe the determinants of evolutionary advantage to be exclusively built rather than bought.

Thirdly, by introducing the mediating role of evolutionary advantage, we essentially point out that the literatures of RBV and dynamic capability indeed have focused on the direct link between resource / capability and performance. Eisenhardt and Martin (2000) reckon that dynamic capabilities are important when markets are dynamic, while Zollo and Winter (2002) as well as Zahra et al. (2006) suggest that dynamic capabilities are useful even when the environment is not very dynamic. Further, Eisenhardt and Martin (2000) argue that dynamic capabilities are necessary but insufficient for ‘competitive advantage’. Reflections on our construction show that these apparent disagreements indeed arise from substituting advantage for performance. Indeed, resources and capabilities determine evolutionary advantage that is necessary but insufficient for performance – the environment has its role, too. The effects of evolutionary advantage transpire differently in different types of changing environments – in some it is more important than in others.
Throughout the paper, we frequently use *firm characteristics*, which reflect unique combinations of resources and capabilities, instead of naming individual resources and capabilities. While given the specific firms and transaction involved one may pinpoint the exact resources and/or dynamic capabilities, we seek to develop an evolutionary perspective generally applicable across industry lines. Consequently, we only scrutinize qualities that are *results* of unique combinations of resources and capabilities. In our analysis, for example, lower cost of new capacity is an important firm-specific characteristic. It captures different sets of resources between a firm in the food industry and one in the automotive industry, for capacity constraint may transpire in the supply chain for the former while in capital investment for the latter. This choice echoes the dominant view that in changing environments the important capabilities are firm-specific (Teece *et al.*, 1997; Makadok, 2001). It also supports the view that, while the factor market determines the price of resources, its value is not fully reflected by that price (Penrose, 1959/1995).

Beyond resources and capabilities, our paper also contributes to the evolutionary perspective of contracting strategy. In view of firm and environmental characteristics, we present a performance-based explanation when contractual adaptation is insufficient, when it is optional, and when it is necessary. This answers the call of Bell, den Ouden, and Ziggers (2006) to investigate the condition as well as consequences of contractual adaptation. More important, we point out that a firm’s evolutionary advantage determines the set of contracting options available in each type of economic context. In particular, firms with particularly high evolutionary advantage can benefit from spot transaction even in changing environments. Hence, we provide an argument for market that goes beyond the prescriptions of transaction cost economics (Williamson, 1975, 1985).
A primer for empirical works

Two avenues of empirical research naturally follow the ideas developed in this paper. The first seeks to verify and explore the effects of evolutionary advantage vis-à-vis other known factors such as industry- or country-effects. This would strengthen our understanding of how big are the impacts of evolutionary competitive advantage compared to static competitive advantage, for currently a significant proportion of variance in performance is not accounted for (Hawawini, Subramanian, & Verdin, 2003). The challenge resides in measuring evolutionary advantage. It requires researchers to determine first when the environment is highly predictable and when it is not so, and second when the environment is evolving, perhaps due to changes in the socioeconomic context. The knowledge of these temporal qualities will point researchers to identify evolutionary advantages and pinpoint their sources.

The second compelling avenue of research may seek to expand the idea of evolutionary advantage beyond transaction in the spirit of Figure 7. Indeed, while we aim to deliver the broadest, most generally applicable analyses, it is impractical to cover in a game-theoretic model every aspect of a high-level construct like evolutionary / competitive advantage in every setting. Nevertheless, our intuition suggests some connection between evolutionary and static competitive advantage. In contrast to the first avenue, this stream preferably would focus on a single industry in a single country in order to exclude spurious effects. The key is to observe both when the economic context has been highly predictable and when it has been highly unpredictable.

Limitation

In this paper, we present a theoretical framework of evolutionary advantage through the lens of contracting strategy. We do this by analysing a simple model of long-term supply
relationship subject to continually changing exogenous demand. While conceptually evolutionary advantage may extend beyond contracting and transactions between/among firms, such breadth is not showcased by our model. The breadth of evolutionary advantage is highlighted by Figure 7, in which we postulate that static competitive advantage is a snapshot of a more general situation. It has, nonetheless, been alluded to in the framework we have laid out – particularly through Hypotheses 3 and 4.

Critics may also point to the perceived rigidity of a game theoretic model and argue that it goes against the purpose of a theoretical framework, which is supposed to be open-ended for future works to build on. Conceding this limitation, we should nevertheless point out that our model serves as an illustrative example, a window from which we peak at a broader framework. Its ‘rigidity’ goes hand in hand with its rigour, which has helped us delineate qualities of evolutionary advantage in the lens of contracting strategy. Flexibility in adopting this perspective in future works is unhindered by our choice of model.

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**DETAILS AND PROOF OF (ii) IN PROPOSITION 2**

**Details:**

We use $\Phi_{s,k}$ to denote the cumulative probability (equivalent of CDF) from $d=0$ up to $d=k$ at time $s$; $\Phi_{s,k} = \sum_{i=0}^{k} (\omega_t F^{s-t})_{i+1}$.

When the seller reduces capacity, $k_t$ is the largest natural number for which

$$\frac{\gamma}{2} \sum_{m=1}^{\infty} (\delta^m (1 - \Phi_{t+m,k-1}) \prod_{n=0}^{m-1} \Phi_{t+n,k-1}) \geq \phi \sum_{m=1}^{\infty} (\delta^m \prod_{n=1}^{m} \Phi_{t+m,k-1}),$$

where the left term is the present discounted cost of reinvesting in this unit of capacity in the future while the right term is the present discounted cost of keeping this unit of unused capacity. When it increases capacity, $k_t$ is the largest natural number for which

$$\pi \sum_{m=1}^{\infty} (\delta^m \prod_{n=1}^{m} (1 - \Phi_{t+n,k-1})) \geq \gamma,$$

where the left term is the present discounted return from this unit of capacity.

If $e_t > 0$ and $\mathbb{E}r_{t+1} \leq 0$, $v_t=0$; that is, without expected residual demand no trade takes place. If $e_t > 0$ and $\mathbb{E}r_{t+1} > 0$, $v_t = \mathbb{E}r_{t+1}$; that is, trade volume would be the same as the expected residual demand when current excess is positive. If $e_t=0$ and $k_t \geq d_t$, $v_t=d_t$; that is, trade volume
would be the same as the current demand when there is no excess or capacity constraint. If \( e_t = 0 \) and \( k_{t-1} < d_t \), \( v_t = k_t \); that is, trade volume would be determined by the seller’s choice of capacity when there is no excess but there is capacity constraint.

Proof:

Let \( \omega_t \) be the initial state. \( \omega_{s \mid t} = \omega t F^{s-t} \) is the expected state at \( s \).

b) If \( k_{t-1} \leq \omega_t F - e_t \), the seller would not divest.

If \( k_{t-1} > \omega_t F - e_t \), the seller would choose a \( k_t, k_t \leq k_{t-1} \), such that it is the largest natural number for which the present discounted costs of reinvesting in another unit is no smaller than all present discounted costs of one additional unit of unused capacity. The present discounted costs of this unit of unused capacity is \( \phi(\delta \Phi_{t+1,k-1} + \delta^2 \Phi_{t+1,k-1} \Phi_{t+2,k-1} + \ldots) \), which equals \( \phi \sum_{m=1}^{\infty} (\delta^m \prod_{n=1}^{m} \Phi_{t+m,k-1}) \). The present discounted costs of reinvesting is

\[
\frac{\gamma}{2} \left( (1 - \Phi_{t+1,k-1}) \delta + (1 - \Phi_{t+2,k-1}) \Phi_{t+1,k-1} \delta^2 + \ldots \right),
\]

which equals \( \frac{\gamma}{2} \sum_{m=1}^{\infty} (\delta^m (1 - \Phi_{t+m,k-1}) \prod_{n=0}^{m-1} \Phi_{t+n,k-1}) \). Therefore, the seller chooses the largest natural number for \( k_t \) such that

\[
\frac{\gamma}{2} \sum_{m=1}^{\infty} (\delta^m (1 - \Phi_{t+m,k-1}) \prod_{n=0}^{m-1} \Phi_{t+n,k-1}) \geq \phi \sum_{m=1}^{\infty} (\delta^m \prod_{n=1}^{m} \Phi_{t+m,k-1}) .
\]

c) If \( e_{t-1} > 0 \), it would not invest, because it already has over-capacity.

If \( e_{t-1} = 0 \) and \( k_{t-1} \geq d_t \), it would not invest, either, for it is unnecessary.

If \( e_{t-1} = 0 \) and \( k_{t-1} < d_t \), it would choose a \( k_t, k_t = k_{t-1} \), such that it is the largest natural number for which all present discounted returns for one unit of capacity is larger than the immediate cost of its investment. The present discounted returns for this unit of capacity is \( \frac{\pi}{2} ((1 - \Phi_{t+1,k-1}) \delta + (1 - \Phi_{t+1,k-1}) (1 - \Phi_{t+2,k-1}) \delta^2 + \ldots) \), which equals \( \frac{\pi}{2} \sum_{m=1}^{\infty} (\delta^m \prod_{n=1}^{m} (1 - \Phi_{t+n,k-1})) \).
Therefore, the condition is \( \frac{\pi}{2} \sum_{m=1}^{\infty} (\delta^m \prod_{n=1}^{m} (1 - \Phi_{t+n,k-1})) \geq \frac{\gamma}{2} \), which equals
\[
\pi \sum_{m=1}^{\infty} (\delta^m \prod_{n=1}^{m} (1 - \Phi_{t+n,k-1})) \geq \gamma.
\]

\( d \) (b) and (c) are the unique solutions to maximize both firms’ payoffs given known transition matrix, \( F \), and unit price, \( p_{flex} \). We only need to prove the efficiency of the actions under (a).

If \( e_t > 0 \) and \( \mathbb{E}r_{t+1} \leq 0 \), apparently \( v_t = 0 \).

If \( e_t > 0 \) and \( \mathbb{E}r_{t+1} > 0 \), \( v_t \leq \mathbb{E}r_{t+1} \) units. Meanwhile, \( v_t \geq \mathbb{E}r_{t+1} \) because \( k_{t-1} > \mathbb{E}r_{t+1} \):

Each day the seller tries to eliminate excess. Positive \( e_t \) means \( d \) has been lower than expected. Because no further investment is needed, \( v_t = \mathbb{E}r_{t+1} \) is optimal.

If \( e_t = 0 \) and \( k_{t-1} \geq d_t \), by the same token, \( v_t = d_t \).

Similarly, if \( e_t = 0 \) and \( k_{t-1} < d_t \), \( v_t = k_t \).

QED