LOTTERIES AS A MEAN OF FINANCING PUBLIC GOODS

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1. INTRODUTION

1.1. Foreword

For centuries lotteries of any kind have been used for replenishment and for financing public goods. Both public and private organisations and funds have been attracted to hold lotteries for that purpose.

Below several historic examples (United Kingdom and Soviet Union) of the effective contribution of lotteries to the solutions of economic problems are considered.

In 1559 following a lasting war and associated blockade England faced a serious economic crisis. To overcome the newly emerged difficulties Queen Elizabeth I held a lottery with an edition of forty thousand tickets. Gold, money and precious canvases were the prizes. The revenues from the lottery were enough not only to stabilise the financial situation in the country, but also to develop military and civil capabilities. That experience was used later when another lottery was held to finance the construction of the British Museum /10/.

Another example is the USSR, where all lotteries were controlled by the state and contributed to the national economy. In 1980 the Olympic Games in Moscow were to a great extent funded through the revenues from a lottery 'Sportloto', which was popular at the time. /11/. It should be noted that in the USSR all lotteries were socially oriented. The new phenomenon of conducting lotteries for personal gain is relatively recent for the post-soviet countries.

Currently, the majority of lotteries all over the world are fully or partially socially oriented. They are controlled and supervised by the World Lotteries Association (WLA), one of the main requirements of which is to redirect part of revenues from any lottery for public needs. /12/.

1.2. Thesis

The aims of this paper are:

- Analysis of the existing models of financing public goods by lotteries to detect the substantial factors that influence lottery participants' behaviour, which are not included in that models;

- Introduction of new models of public goods financing by lotteries free from the disadvantages of the existing models.

1.3. Lotteries as a means of non-budgetary financing of public goods

A simplified principle of the majority of the lotteries can be formulated as follows: participants make contributions (directly or by buying tickets), and their contributions go the 'pool'. Then comes the lottery draw and a winner (several winners) gets a prize (several prizes) provided by the accumulated funds. The prize may consist of cash or any other good. Each participant's chance to win the prize is equal to his share in the "pool" because the ticket prices are fixed and each ticket has the same chance of winning. Thus, in case there are lottery tickets (discrete case), the chance of a participant to win is equal to his share of tickets in the total ticket pool.

In a general (continuous) case, when a participant can spend any disposable amount on a lottery, the situation remains the same. One may assume that the price of a ticket is the greatest common divisor of contributions of participants and each participant has bought a certain number of tickets sum of face values of which equals to his/her contribution.

Let us assume that participants are rational. In other words they seek to maximise their well-being[1]and ceteris paribus prefer a bigger expected prize to a smaller one. Also lotteries with similar expected prizes are equally preferable for them. In this case such participants are seen as risk-neutral. Within the framework of this paper it means that participants base their decision about the amount of their contributions only on the expected prize. Values of a prize (prizes) or chance for winning alone are not important for them.

Assumption 1.

Within the framework of this model there is no difference whether there is one prize or several prizes in the lottery.

Proof.

Suppose we have $\boldsymbol{t}$ prizes $\{\boldsymbol{r_1}..\boldsymbol{r_t}\}$ and their total value is $\boldsymbol{R}$. Then, since the chances of winning each of them are equal to $\frac{x_i}{G}\boldsymbol{rd}$ for participant $(\boldsymbol{i})$ and prize $(\boldsymbol{d})$, the total expected prize of a participant $(\boldsymbol{i})$ is equal to $\frac{x_i}{G}\boldsymbol{r_1} + \cdots + \frac{x_i}{G}\boldsymbol{r_t} = \frac{x_i}{G}\boldsymbol{R}$, where $(\boldsymbol{G})$ is the total 'pool'.

Hereinafter the prize means the sum of all prizes $(\boldsymbol{R} = \boldsymbol{r_1} + \cdots + \boldsymbol{r_t})$.

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[1]*For more details see -part 2.1.*

1.4. Problems of financing public good through lotteries

Let us consider the society as a set of participants ($n > 0$), who maximise their welfare function by finding an optimal contribution to the lottery or by contributing voluntary.

Within the framework of this paper each participant ($0 < i \leq n$) has initial wealth ($w_i \geq 0$), part of which is contributed to financing public good. From that public good a participant (i) gets utility, which is a function h_i (such that $h_i(0) = 0$, $h_i' \geq 0$ and $h_i'' \leq 0$) from the sum ($G - R$) that was spent on the public good by the whole society.

We assume that each contributor believes that the money is used for the purposes that have been declared, while other living expense (flat rent, food spending, maintenance etc.) do not affect his/her welfare within the framework of the models analysed in this paper.

The main problem of public goods financing is a 'free rider problem', which can be formulated as follows. Any member of the society (including potential lottery participants) has a free access to a public good due to its non-excludability. /5/. Then, most probably he/she might rather prefer not to pay for the public good, without substantial decrease of the share of the public good that will remain available for him/her personally. For this reason direct voluntary financing of public good by citizens is not particularly efficient.

Because of this, we consider lotteries as one of the means of attracting private financing for indirect financing of public good without budgetary allocations for these purposes /

Let us assume that public good is financed exclusively by voluntary contributions. In this case each participant maximises his/her welfare function:

$$U_i = w_i - x_i + h_i(G),\ \text{г д е}\ \ G = \Sigma x_j$$

The paper is based on the following assumption: $\forall i, \forall G > 0: h_i(G) < w_i$. In other words within the framework of the model we believe that nobody will be ready to give <u>all</u> his/her money for public good, since no public good supplies all basic needs of an individual. Thus we always have $h_i \geq x_i$. Otherwise, having decreased x_i, an individual would have increased U_i. Thus, our conclusion is that $\forall i$: $x_i \leq h_i < w_i$, or in other words, that the optimal contribution of any member of the society is smaller than his/her initial wealth.

The first order condition is not only necessary but also sufficient. Sufficiency is determined by the concavity of the performance function and convexity of the admissible set.

We suppose that two types of society members can be defined according to their lottery participation.

1) Those for whom $\boldsymbol{h_i'(G_{-i}^*)} \leq \mathbf{1}$**,** will not participate in a lottery as the marginal utility of each dollar[2] spent is less than or equal to 1.

2) Those for whom $\boldsymbol{h_i'(G_{-i}^*)} > 1$ will spend positive sums for the public good until the marginal utility of a dollar spent equals 1.

Assumption 2.

In the Nash equilibrium (hereafter - equilibrium) financing public good exclusively through voluntary donations is not a socially optimal solution if there are multiple donors.

Proof.

Equilibrium conditions (if $\boldsymbol{G^*} > 0$):

1) $\forall \boldsymbol{i}\colon \boldsymbol{h_i'(G^*)} \leq \mathbf{1}$ - as otherwise at least for one participant intends to contribute more.

2) $\forall \boldsymbol{i}\colon \boldsymbol{x_i} > 0 \Rightarrow \boldsymbol{h_i'(G^*)} = \mathbf{1}$ (first order condition) - as otherwise ($\boldsymbol{h_i'(G^*)} < \mathbf{1}$) for at least one of those who had invested into the project it would have been more gainful to spend less.

Let us assume that at least two participants ($\boldsymbol{a}$ and $\boldsymbol{b}$) donated money. Then $\boldsymbol{h_{total}'(G^*)} = \sum \boldsymbol{h_i'(G^*)} \geq \boldsymbol{h_a'(G^*)} + \boldsymbol{h_b'(G^*)} = \mathbf{2} > \mathbf{1}$ **.** Thereby there is the underproduction of public good.

The only case where the socially optimal level of public good can be achieved is the following: $\exists \boldsymbol{a}\colon \boldsymbol{x_a} > 0$ и $\forall \boldsymbol{b} \neq \boldsymbol{a}\colon [\boldsymbol{x_b} = \mathbf{0}] \cap [\boldsymbol{h_b'(G^*)} = \mathbf{0}]$. However this case is a degenerate.

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Another problem of financing public goods by voluntary contributions is a lack of transparency in financing schemes. That makes potential donors suspicious about the proper use of funds /13/. However, this problem is not analysed in this paper

2. ANALYSIS OF THE EXISTING MODELS

Two of the most important existing models of financing public goods through lotteries, namely the Morgan model and the Franke-Leininger model, are

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[2] *We assume that the currency is US dollar.*

analysed below. Other models are not mentioned, as they seem less important and less relevant to the analysis.

2.1. Basic model

The analysis of financing public goods by holding lotteries is based mostly on the John Morgan's article/14/, where he considers the influence of lotteries upon the public goods provision level in case no external sources of finance are available.

As it is shown, assuming that financing of the public goods through lotteries is possible, the welfare function for a participant ($\boldsymbol{i}$) looks as follows:

in case of a voluntary donation for the public goods:

$$\boldsymbol{U_i = w_i - x_i + h_i(G)}, \text{where } \boldsymbol{G} = \sum \boldsymbol{x_j};$$

in case of a fixed prize lottery:

$$\boldsymbol{U_i = w_i - x_i + \frac{x_i}{G}R + h_i(G - R)}, \text{where } \boldsymbol{R} - \text{ is the value of the prize}$$

in case of a lottery with a prize being a share of the sum collected:

$$\boldsymbol{U_i = w_i - x_i + \frac{x_i}{G}\alpha G + h_i\big((1 - \alpha)G\big)}, \text{where } \boldsymbol{\alpha G} - \text{ is the prize value.}$$

Several important hypotheses (though not yet clearly formulated in the paper) can be found in the article. Firstly, any participant of a lottery, as well as a lottery organiser, knows the exact welfare function of any member of the society. Secondly, the society consists only of people who are in a position to take part in a lottery.

The welfare function of any member of society, who always seeks to maximise it, always contains the value of the aggregated fund $\boldsymbol{G} = \sum \boldsymbol{x_j}$. It is equal for everybody and somehow correct. Thus the contribution of each participant can be defined by the following set of equations:

$$\begin{cases} \dfrac{dU_1(x_1..x_n)}{dx_1} = \mathbf{0}; x_1 > 0 \\ \quad .. \\ \dfrac{dU_i(x_1..x_n)}{dx_i} = \mathbf{0}; x_i > 0 \\ \quad .. \\ \dfrac{dU_n(x_1..x_n)}{dx_n} = \mathbf{0}; x_n > 0 \end{cases}$$

Due to quasi-linearity and monotony of function $\boldsymbol{U_i}$ with fixed prize a system with $\boldsymbol{n}$ equations and with $\boldsymbol{n}$ variables has only one solution $\{\boldsymbol{x_1} \dots \boldsymbol{x_n}\}$ in non-negative values./*Morgan J. Financing Public Goods by Means of Lotteries, Review of Economic Studies, 2000. p. 768, Preposition 2*/

The main problem caused by the assumption about the quasi-linear welfare function is the under-production of the public goods as compared to the socially optimal level. In other words, if participants (who are assumed to be rational) maximise their welfare functions, the public welfare function $\sum \boldsymbol{U_j}$ does not reach its maximum (socially optimal level of public good).

The reason for the public good underproduction in case of its voluntary funding is as follows: defining his/her donation a participant does not consider the external effects, which have a positive external effect upon the other members of society, as he is not interested in increasing their wellbeing. As a result the participant provides insufficient financing of public good as compared to the optimal level.

Leaving aside the proof details that the article by Morgan provides, let us sum up the most important conclusions of the author:

1) Allocation of participants’ contributions that suits the society’s best is unachievable.

2) In equilibrium fixed prize lotteries provide a bigger amount of public good than voluntary donations.

3) Lotteries with fixed prize fully crowd out voluntary donations.

4) Lotteries with the prize as a share of total pool are less effective than lotteries with a fixed prize.

The major advantage of the Morgan's model is its relative simplicity as well as a quite small number of variables and functions that clearly determine a participants’ choice of the contribution amount. Such a simplified basic model is easy to analyse and it leads to conclusions that would be impossible to make in a more complex model.

At the same time the Morgan's' model is based on an assumption about the "absolute information". In other words, it is presumed that any participant as well as the lottery organiser knows the utility function of each member of the society.

Considering the potential of social research (polls etc.), one would argue that a lottery organiser may know the preferences of various members of the society and thus can approximate their welfare functions. However the idea that every participant knows the utility function of any other participant is impugnable and turns the model into an abstract and unreal concept.

In the third part of his work Morgan analyses the symmetrical reaction of participants, but he considers the society to be totally homogeneous, which is also a disputable assumption.

2.2. Model that takes into consideration the difference in nominal values of lottery tickets

The model of Franke and Leininger /2/ is based on the Morgan model. The authors analyse changes in results that occur if instead of being able to spend any available money on lottery contributions participants can only buy fixed-price lottery tickets. The price for the tickets would be individual for each member of the society and would be set by the organiser of the lottery depending on the public good utility function for each participant. Thus the achievable set of all possible contributions for each participant is discrete[3]. In this case, provided the preferences are heterogeneous, a socially optimal level of public good provisions is achievable.

As in the Morgan model, the weak point of the Franke-Leininger model is the assumption that participants know each other's utility functions. This assumption is unrealistic, especially if the lotteries (national or regional ones) have a large number of participants.

Even if a lottery organiser somehow knows the utility functions of participants, it is unlikely that there may be introduced different ticket prices for different people when selling lottery tickets; this assumption is quite theoretical. Firstly, if the price of a ticket were not fixed participants would not trust the organisers. As a result lottery tickets would not be sold out. Secondly, a real participant is not likely to buy a ticket for an individually set price when he/she knows that there are lower prices for some other participants. Moreover a rational

[3] *In the Morgan model it is continuous.*

participant might ask someone, who has different set prices, to buy tickets for him/her.

Another Franke-Leininger model problem (an operational problem) is associated with transactional costs and communication imperfections. In other words any ticket-selling point should have access to a database, which consists of information about all participants' welfare functions. If this condition is not met in most of them (and usually these are kiosks that have no permanent access to Internet) the procedure of buying/selling lottery tickets becomes much more complicated (or even impossible). Thus it impugns the success of a lottery.

In conclusion one might argue that though with the assumptions of the Franke-Leininger model the socially optimal level of public good provision is achievable, the unrealistic approach reduces its academic value.

3. INTRODUCTION OF NEW MODELS

Due to the shortcomings of the existing models, the new models of financing public good through lotteries are introduced below.

3.1. Model 1 "Assumption of relative expectations"

To eliminate the unrealistic assumption of the aforementioned models that all lottery participants know the welfare functions for each member of the society, the new models are based on the "assumption of relative expectations".

The idea of the assumption of relative expectations is as follows. If society is relatively homogeneous (the target group of lotteries is the middle class /15/), a participant choosing a sum that he/she is going to spend only knows the number of potential participants. Supposing that society is homogeneous, which might not be the real condition, each participant expects the others to spend as much as he does.

Taking the Morgan model as a basic one (for example, for lotteries with fixed prize), a participants' $(\boldsymbol{i})$ goal looks as follows:

$$\boldsymbol{max}(\boldsymbol{w_i} - \boldsymbol{x_i} + \frac{\boldsymbol{x_i}}{\boldsymbol{n x_i}}\boldsymbol{R} + \boldsymbol{h_i}(\boldsymbol{n x_i} - \boldsymbol{R}))$$
$$\textit{s.t. } 0 \leq \boldsymbol{x_i} < \boldsymbol{w_i}.$$

The participants' $(\boldsymbol{i})$ welfare function is:

$$\boldsymbol{U_i} = \boldsymbol{w_i} + \frac{\boldsymbol{R}}{\boldsymbol{n}} - \boldsymbol{x_i} + \boldsymbol{h_i}(\boldsymbol{n x_i} - \boldsymbol{R}).$$

The participants do not know (they can not know and will never know) either the amount of others' contributions or total value of the accumulated lottery fund. Thus they have no incentive to change the amount of their contributions up till the lottery drawing. That leads to the Nash equilibrium.

One of the disputable aspects of this concept is as follows: if after the draw a participant sees that the collected fund ($\boldsymbol{G^*}$) differs from what he/she expected ($\boldsymbol{G_{exp}}$), he/she will doubt the equality of all participants' contributions in future lotteries. Thus the initial "assumption of relative expectations" might be rejected. This situation may lead to the following reasoning.

According to the results of previous lotteries a participant defines the coefficient of ratio between his contribution and that of an average participant and concludes that in the following lotteries the fund collected will be $\boldsymbol{nz_i}$ times bigger than the contribution he had chosen, where $\boldsymbol{z_i} = \frac{\boldsymbol{G^*}}{\boldsymbol{G_{i\,exp}}}$ (of the previous lottery). Then:

$$\boldsymbol{U_i = w_i + \frac{R}{nz_i} - x_i + h_i(nz_i x_i - R).}$$

Since $\boldsymbol{z_i}$ is invariable for each particular lottery and has no impact upon posterior analysis (whether included or not) to simplify the model we assume that $\boldsymbol{z_i = 1}$.

Following the first order condition, if a participant's optimal contribution is more than zero, then:

$$\boldsymbol{1 = h_i'(nx_i - R)} \qquad (1)$$

Assumption 3.

Assuming relative expectations, in equilibrium there are no participants that might have been contributed to public good provided a certain amount of the total fund, but did not contribute to it. In other words:

$$\forall \boldsymbol{i}: [\boldsymbol{h_i'(0) > 1 \ \Rightarrow x_i > 0}].$$

Proof.

If $\boldsymbol{x_i = \frac{R}{n}}$ and accumulated fund equals $\boldsymbol{0}$, the value of public good utility function for a participant is equal to zero, and marginal utility reaches its maximum value.

Due to the decreasing nature of $\boldsymbol{h}'_i$, increasing $\boldsymbol{x}_i$ causes decrease of $\boldsymbol{h}'_i(\boldsymbol{n}\boldsymbol{x}_i - \boldsymbol{R})$. Accordingly a rational participant increases his/her contribution until the $\boldsymbol{h}^i_i(\boldsymbol{n}\boldsymbol{x}_i - \boldsymbol{R})$ function value equals $\mathbf{1}$.

Thus the contribution of any member of society $(\boldsymbol{i})$ for whom $\boldsymbol{h}'_i(\mathbf{0}) > 1$ is greater than $\frac{\boldsymbol{R}}{\boldsymbol{n}}$. Obviously $\frac{\boldsymbol{R}}{\boldsymbol{n}} > \mathbf{0}$. Thereby there are no participants for whom the marginal utility of public good is greater than 1 (regardless of the financing amount) and who would choose to be a 'free rider' (would not contribute to public good).

This assumption does not lead directly to a socially optimal level of public good. However, it helps to get rid of one of the gravest problems of the public good financing theory, namely, classifying those who de-facto are not 'free riders' as 'free riders'. It simplifies the analyses of the models below and under certain assumptions makes it possible to achieve the optimal level of public good provision.

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Thus by introducing the assumption of relative expectations we get rid of an unrealistic hypothesis of absolute information, which appears in both Morgan and Franke-Leininger models.

In the models below an assumption of relative expectations is the basic one.

3.2. Model 2 "Moral society"

3.2.1. Moral society concept

Considering public goods like orphanages or humanitarian assistance to other countries a typical everyman, who is neither an orphan nor a starving one, should be indifferent to existence of such public goods. However people give alms to beggars, transfer money to support the poor, sponsor animal shelters etc. /16/. In that case not everyone is motivated by fear of homeless dogs, by an attempt to inculcate a taste for charity to beggars in case they get ruined themselves or by a possible reincarnation in the area of a humanitarian catastrophe.

In such cases for an individual it is not the sum collected by the society for a good deed that matters, but his/her own contribution and the self-satisfaction caused by the personal participation in a good deed. /17/ Within the framework of the Morgan model or the Franke-Leininger one such type of behaviour is irrational. However one cannot but notice such irrational behaviour and its importance /18, 19/. Let us name this irrationality "moral utility ".

Let us assume that moral utility ($\boldsymbol{m_i(x_i)}$) for a participant ($\boldsymbol{i}$) is an individual function that defines the participant's satisfaction by a particular donation ($\boldsymbol{x_i}$).

The personal (non-altruistic) component of welfare for each particular participant is logically predictable. It is also quite obvious that its function can take a numerical value. Let us try and prove it intuitively.

Suppose, a wealthy man walks along a pedestrian tunnel and sees a beggar. For some he decides to give the beggar a certain sum of money. But how much? Let us model the situation: what would he say if at the moment he was interviewed?

- *Would you give him a dollar (euro, rouble, pound etc. Dollars by default)?*
- *Yes, I would.*
- *Two dollars?*
- *I'd rather say, I would.*
- *What about three dollars?*
- *Well… Three is perhaps too much - no, I would not.*

It can be seen that the value of the potential donation exceeds its moral utility between 2 and 3 dollars ($\boldsymbol{m_i(1) > 1, m_i(2) > 2, m_i(3) < 3}$).

How does an individual make his choice about the alms to a homeless beggar amount within the framework of that simplified model? Suppose, the participant can donate not only acceptable discrete sum, which can be composed of face values of banknotes and coins he/she has at the moment, but any continuous[4] sum bordered by his total wealth. Then the welfare function at the moment is:

$$\boldsymbol{U_i = w_i - x_i + m_i(x_i)}.$$

The participant subconsciously maximises the function choosing $\boldsymbol{x_i}$, which leads to the following first order condition (if $\boldsymbol{m_i'(0)} > 1$):

$$\boldsymbol{m_i'(x_i) = 1}$$

What conclusions regarding the moral utility function can be made?

[4] *Suppose, the hypothetic beggar 'accepts' alms by withdrawing money from plastic cards without transactional expenses.*

Firstly, with zero contribution the moral utility function of the contribution equals to zero:

$$\boldsymbol{m_i(0) = 0.}$$

Secondly, the moral utility function is non-decreasing: the more money a participant donates for the good course the higher is the participant's satisfaction.

$$\boldsymbol{m_i' \geq 0}$$

Thirdly, as the contribution further increases the related rise of moral utility of the contribution increases less intensively (as it can be seen from the case with a beggar). Thus moral utility is a decreasing function.

$$\boldsymbol{m_i''} < 0$$

An important assumption of the moral utility concept is the fact that when maximizing the function each participant acts intuitively and cannot define its exact value. Furthermore, neither social research nor other methods that a lottery organiser might apply to determine participants' public good utility functions may define their moral utility. On the contrary, the public good utility function can be digitally defined for each member of society. One can define the amount of money the participant is ready to exchange for the existence of a public good such as services of a hospital under construction (vs. his/her expenses for similar services in already existing hospitals), new pedestrian tunnel (vs. opportunity cost of time wasted for going around to the closest cross-road) etc. This amount (unless force majeure events occur) remains constant for each member of society. Instead, one's moral utility function depends on a number of changeable factors (such as cash in the pocket, mood, beggar's appearance etc.).

3.2.2. Applying moral society concept and assumption of relative expectations to the basic model

Applying moral satisfaction from the contribution to public good to a participant's welfare in case of a fixed-prize lottery, one gets the following formula:

$$\boldsymbol{U_i = w_i + \frac{R}{n} - x_i + m_i(x_i) + h_i(nx_i - R)}$$

A participant maximises his/her welfare function. Application of the first order (if $\boldsymbol{m_i'(0) + h_i'(0)} > 1$) condition leads to:

$$\mathbf{1} = \boldsymbol{m}_i'(\boldsymbol{x}_i) + \boldsymbol{h}_i'(\boldsymbol{n}\boldsymbol{x}_i - \boldsymbol{R})$$

Suppose[5] in most cases participation in lotteries and other voluntary forms of financing public good gives a participant moral satisfaction greater than personal utility of the public good ($\boldsymbol{m}_i(\boldsymbol{x}_i) \gg \boldsymbol{h}_i(\boldsymbol{n}\boldsymbol{x}_i - \boldsymbol{R})$).

Thus, taking into consideration similar nature of the functions (signs of the first and the second derivatives) one can assume that in case of maximization of welfare function the marginal moral utility for a participant does not sufficiently differ from the sum of marginal moral and personal utilities. In other words, marginal personal utility from public good is quite small:

$$\boldsymbol{h}_i'(\boldsymbol{n}\boldsymbol{x}_i - \boldsymbol{R}) \leq \frac{\mathbf{1}}{\boldsymbol{n}}$$

It means that if relative expectations are rational enough (in other words if $\boldsymbol{E}(\boldsymbol{h}_j'(\boldsymbol{G} - \boldsymbol{R}) - \boldsymbol{h}_j'(\boldsymbol{n}\boldsymbol{x}_j - \boldsymbol{R})) \leq \boldsymbol{E}(\boldsymbol{m}_j'(\boldsymbol{x}_i)))$, the derivative of sum of utility functions (the sum of marginal utilities) from public good for each participant should not exceed 1 ($\sum \boldsymbol{h}_i'(\boldsymbol{G} - \boldsymbol{R}) \leq \mathbf{1}$).

Let us emphasize that the lottery organiser and/or the participants can not define the moral utility function (see the last paragraph of section 3.2.1 'Moral society concept'). In view of that and also according to the assumption that moral utility is a purely personal function let us assume that total social welfare function is a sum of welfare functions of its participants excluding moral utility.

Assumption 4.

If an average share of marginal moral utility in moral utility function is relatively high, then it is possible to reach the optimal social level of public good. In other words:

$$[\boldsymbol{E}(\boldsymbol{m}_i'(\boldsymbol{x}_i)) \gg \boldsymbol{E}(\boldsymbol{h}_i'(\boldsymbol{n}\boldsymbol{x}_i - \boldsymbol{R}))] \Rightarrow [\exists \boldsymbol{R}: \sum \boldsymbol{h}_i' = \mathbf{1}]$$

Proof.

$$\boldsymbol{U}_{total} = \sum \left(\boldsymbol{w}_i + \frac{\boldsymbol{R}}{\boldsymbol{n}} - \boldsymbol{x}_i + \boldsymbol{h}_i(\boldsymbol{G} - \boldsymbol{R})\right) = \boldsymbol{W} + \boldsymbol{R} - \boldsymbol{G} + \sum \boldsymbol{h}_i(\boldsymbol{G} - \boldsymbol{R})$$

To achieve the socially optimal level apply the first order condition to the social welfare as to a function, which domain is the total fund value:

$$\sum \boldsymbol{h}_i'(\boldsymbol{G} - \boldsymbol{R}) = \mathbf{1}$$

[5] *See section '3.2.3 –Survey'*

As it is proved above:

$$\sum \boldsymbol{h}_i'(\boldsymbol{G}-\boldsymbol{R}) \leq \mathbf{1}$$

Consequently there is no overproduction of public good.

Suppose with a certain value of prize $\boldsymbol{R}=\boldsymbol{R}^*$ there occurs the overproduction of public good: $\sum \boldsymbol{h}_i'(\boldsymbol{G}-\boldsymbol{R}^*) < \mathbf{1}$.

Let us notice that if $\boldsymbol{R}=\boldsymbol{G},\ \sum \boldsymbol{h}_i'(\boldsymbol{G}-\boldsymbol{R}) = \sum \boldsymbol{h}_i'(\mathbf{0}) > \mathbf{1}$.

That is only true for a non-degenerate case. Unrealistic cases such as "society does not need public good" ($\sum \boldsymbol{h}_i'(\mathbf{0}) < 1$) are not analysed.

As function $\sum \boldsymbol{h}_i'$ is continuous (as a sum of continuous functions) on the ray $[\mathbf{0};\ +\infty)$, and particularly on the on interval $[\mathbf{0};\ \boldsymbol{G}-\boldsymbol{R}^*]$, then by the intermediate value (Bolzano's) theorem there is such $\boldsymbol{R}$ from segment $[\boldsymbol{R}^*;\ \boldsymbol{G}]$ that makes $\sum \boldsymbol{h}_i'(\boldsymbol{G}-\boldsymbol{R}) = \mathbf{1}.$

Thus, in a non-degenerate case a socially optimal level of public good is achievable.

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3.2.3. <u>Survey</u>

Having impugned Morgan's thesis that lotteries fully crowd out voluntary donations for public goods, there was a survey conducted in a social network "VK". The following text was given:

"*If you do not doubt the target use of the collected funds, which form of a donation would be preferable for you – direct donation for the construction of a hospital of any sum you are ready to spend or contribution of the same amount of money to a lottery where all the funds contributed minus fixed prize of the winners will be allocated to the construction of the same hospital?*"

445 participated in the polls. The results were as follows:

Direct donation – 239 (53,7%).
Fixed prize lottery – 123 (27,6%).
Both options are equally preferable (including "I will not contribute at all.") – 83 (18,7%).

Though most respondents were of more or less the same age and belonged to the same social group, following assumptions can be made based on the above-mentioned survey.

Firstly, the moral satisfaction from financing public goods by lottery is at average lower than the moral satisfaction from direct donation of the same amount.

Secondly, while taking a decision about the amount of their contribution, many participants consider the moral factor quite seriously (applicable to the given social group and to the declared target use of funds).

Therefore one may conclude that lotteries do not crowd out direct donations because along with those attracted by a chance of winning a prize and their own personal utility of public good there will be others, who are motivated by moral. A combination of these two approaches allows more efficient fundraising than each of the two taken separately.

3.3. Model 3 "Trust in luck"

In all the above-mentioned models people participate in lotteries only because the lottery fund is used to finance public goods.

Suppose the rest of the fund after payments to the happy owner of a lucky ticket is not used for public good and thus do not provide any additional welfare to the participants. Then for a fixed prize lottery the welfare function of a participant looks as follow:

$$\boldsymbol{U_i = w_i + \frac{R}{n} - x_i}$$

Obviously, in that case any participant would not spend money on lottery tickets at all. Let us notice that a lottery is held only if the fund accumulated is higher than $\boldsymbol{R}$. If $\boldsymbol{x_i < \frac{R}{n}}$ an individual would assume that the sum collected is equal to $\boldsymbol{nx_i < R}$. In other words the money is returned and each participant preserves his/her initial wealth $\boldsymbol{w_i}$. Accordingly cases where $\boldsymbol{x_i < \frac{R}{n}}$ are not analysed. Thereby lotteries where funds are not spent on social needs after the obligatory payments to winners should not exist. However there are such lotteries and they succeed. It is notoriously that in a significant amount of lotteries the remaining share is the organisers' revenue and has no socially important goal /13/. Why then the participants take part in such lotteries?

A possible answer is that people trust in luck. A participant of a lottery is sure that he\she is luckier than the others. Thus though the real expected prize remains unchanged (and equals to $\boldsymbol{R}\frac{x_i}{G}$), a participant, initially defining the amount of his/her contribution, imaginatively increases the expected gain supposing that the others are less lucky.

If $\boldsymbol{f_i}$ is the ratio of the alleged (imaginary) over-estimation of the potential gain of a participant $(\boldsymbol{i})$, then

$$\boldsymbol{U_i = w_i + f_i \frac{R}{n} - x_i.}$$

What does $\boldsymbol{f}$ depend on?

In some lotteries (Russian "Sportloto" or "Gosloto" as an example) nearly every participant believes that he/she will win the main prize; consequently, there is mass participation /21/. At the same time other lotteries face a shortage of participants (such as Russian "Art lottery", "DOSAAF lottery" etc.) /22/. A possible explanation of the phenomenon is the effect of advertisement.

Billboards with pictures of lucky winners surrounded by cash and slogans “You are the next one!” or “The luck is on your side!” make potential participants believe that they will definitely win: they only need to buy a ticket and the Fortune will take care of all the rest.

The more advertising is used (assuming that the advertisement is made by professionals and reaches its aim), the higher is the overestimation ratio. However the amount of advertisement is proportional to the amount of money spent on it.

Since advertising has different impact on each person the overestimation ratio appears to be each participant’s individual function, which domain is the amount of money spent on advertising the lottery ($\boldsymbol{\varepsilon}$). In other words, the more money is spent on advertising, the higher is the overestimation ratio $\boldsymbol{f_i' \geq 0}$.

However it is known that the more is a product advertised, the less efficient is it’s sales growth on a later stage, in other words $\boldsymbol{f_i'' \leq 0}$ /20/. One can argue that in the absence of advertisement, which distorts participants’ perceptions of reality, participants have a better understanding of the true state of affairs $(\boldsymbol{f_i(0) = 1})$.

In a case where lottery contributes to public good, the trust in luck effect does not disappear. A participant maximises the following function:

$$\boldsymbol{U_i = w_i - x_i + f_i(\varepsilon)\frac{R}{n} + h_i(nx_i - R - \varepsilon.}$$

As a result the common welfare function of the society is:

$$\boldsymbol{U_{total}} = \sum(\boldsymbol{w_i} - \boldsymbol{x_i} + \boldsymbol{f_i}(\boldsymbol{\varepsilon})\frac{\boldsymbol{R}}{\boldsymbol{n}} + \boldsymbol{h_i}(\boldsymbol{n}\boldsymbol{x_i} - \boldsymbol{R} - \boldsymbol{\varepsilon})).$$

Though in a non-degenerate case (in other words if $\boldsymbol{R}$ is sufficiently smaller than $\sum \boldsymbol{x_j}$) the socially optimal level still remains unachievable, qualified advertising may increase lotteries' contribution to public good funding.

Having determined the function of a participant's contribution from the value of the prize and the amount of advertising as $\boldsymbol{x_i}(\boldsymbol{R}, \boldsymbol{\varepsilon})$, we assume that in a non-degenerate case with $\boldsymbol{\varepsilon}^*$ being small $\boldsymbol{x_i}(\boldsymbol{R}, \boldsymbol{\varepsilon}^*) > \boldsymbol{x_i}(\boldsymbol{R}, \boldsymbol{0})$, which is also supported by the following.

If $\boldsymbol{\varepsilon} = \boldsymbol{0}$, $\boldsymbol{h}'_{\boldsymbol{i}}(\boldsymbol{n}\boldsymbol{x_i} - \boldsymbol{R}) = \boldsymbol{1}$, but if $\boldsymbol{\varepsilon} > \boldsymbol{0}$, $\boldsymbol{f}'_{\boldsymbol{i}}(\boldsymbol{\varepsilon})\frac{\boldsymbol{R}}{\boldsymbol{n}} + \boldsymbol{h}'_{\boldsymbol{i}}(\boldsymbol{n}\boldsymbol{x_i} - \boldsymbol{R} - \boldsymbol{\varepsilon}) = \boldsymbol{1}$, consequently, $\boldsymbol{h}'_{\boldsymbol{i}}(\boldsymbol{n}\boldsymbol{x_i} - \boldsymbol{R} - \boldsymbol{\varepsilon}) < \boldsymbol{1}$. In other words, due to $\boldsymbol{h}''_{\boldsymbol{i}} \leq \boldsymbol{0}$, the contribution is larger than in case of no advertising at all.

At the same time a negative effect of advertising is also possible. A situation where advertising grows beyond a certain level at which it starts to have negative effects on sales[6]; However that case is not analysed in our paper.

3.4. <u>Model 4 "Additional issue of lottery tickets with buyback"</u>

Let us analyse such a case, which de-facto is a fraud, which allows an organiser to increase the amount of public good funding through lotteries.

Let us assume that each participant of a lottery (for whom $\boldsymbol{h}'_{\boldsymbol{i}}(\boldsymbol{0}) > 1$) has chosen an optimal (for him/her) amount of contribution ($\boldsymbol{x}^*_{\boldsymbol{i}}$). Thus:

$$\boldsymbol{1} = \boldsymbol{h}'_{\boldsymbol{i}}(\boldsymbol{n}\boldsymbol{x_i} - \boldsymbol{R}).$$

Then $\boldsymbol{G}^* = \sum \boldsymbol{x}^*_{\boldsymbol{i}}$;

$$\boldsymbol{U_{total}} = \boldsymbol{W} + \boldsymbol{R} - \boldsymbol{G}^* + \sum \boldsymbol{h_i}(\boldsymbol{G}^* - \boldsymbol{R}).$$

Let us assume that the society is heterogeneous in its preferences. Thus, public good is under-produced.

[6] *More known as Claus Moser effect /20/.*

$$\sum \boldsymbol{h}_i'(\boldsymbol{G}^* - \boldsymbol{R}) > 1$$

Suppose $\sum \boldsymbol{h_i}(\boldsymbol{G}^*) < \mathbf{1}$. In other words, in the absence of a prize the money collected would guarantee over-production of public good.

Assumption 5.

In a heterogeneous society additional issue of tickets allows organiser of a lottery to bring the public good financing to the socially optimal level.

Proof.

Let us assume that the organiser of a lottery, whose aim is the benefit of society and the reaching the socially optimal level of public good issues an additional number of lottery tickets for the sum $\boldsymbol{K}$ and re-purchases them using the money that was collected from the participants. Thus he/she keeps the fund value constant. Moreover, the organiser does not inform participants about those actions. Also the organiser decides that if the tickets he/she acquires that way win, the gain will be spent for public good.

This additional (and illegal) issue of lottery tickets produces a negative externality by decreasing the expected gain of each participant $(\boldsymbol{i})$ because $\frac{\boldsymbol{x_i}}{\boldsymbol{G}^*} < \frac{\boldsymbol{x_i}}{\boldsymbol{G}^*+\boldsymbol{K}}$. However taking into consideration that the organiser of a lottery does not appropriate prize if he/she wins, the expected real expenses on public good are $\boldsymbol{G}^* - \boldsymbol{R}\frac{\boldsymbol{G}^*}{\boldsymbol{G}^*+\boldsymbol{K}} > \boldsymbol{G}^* - \boldsymbol{R}$**.**

Due to continuity of the function of sum of derivatives of utilities of public good in segment $[\boldsymbol{G}^* - \boldsymbol{R}; \boldsymbol{G}^*]$, by the intermediate value (Bolzano's) theorem there will be such $\boldsymbol{K}^*$, that $\sum \boldsymbol{h}_i'\left(\boldsymbol{G}^* - \boldsymbol{R}\frac{\boldsymbol{G}^*}{\boldsymbol{G}^*+\boldsymbol{K}^*}\right) = \mathbf{1}$

One has to emphasise that this model, which is obviously a fraud, is absolutely conditional, non-applicable within legal framework and impermissible in real life, as the size of the issue of lottery tickets must be determined, declared in advance and known by all participants of the lottery.

Another option may be to increase the nominal issue up to an unachievable number. In that case participants of the lottery from the very beginning are aware that the issue will not be fully covered and the case is close to the aforementioned model.

Following the unverified information from mass media one may argue that de-facto organisers use a scheme implying that they keep some lottery tickets out of declared issue unsold. That produces a negative external effect similar to the abovementioned one. One cannot exclude the possibility that such actions are aimed not at the increase the public good financing, but at a personal gain of dishonest lottery organisers/13/.

3.5. <u>Model 5 "Maximising public good"</u>

In the abovementioned, which is based on the Morgan model, there is an assumption that the lottery organiser controls variables such as size of a prize, expenses on advertising etc. in order to make the level of public good as close as possible to the socially optimal one. That assumption is disputable. Not only potential participants of a lottery use public goods. Let us prove it with the following example.

Suppose the lottery was used to finance a pedestrian tunnel construction. It is obvious that the tunnel will be used not only by participants of the lottery, but also by children, tourists, beggars etc., who did not participate. Later the tunnel will remain useful though the number of those who had once contributed to the construction will decrease and the number of those who did not participate in the lottery will increase.

Thus in a long term perspective the maximization of total utility of public good for potential participants of the lottery is not the most rational approach to the problem. Suppose the task of lottery organisers is more practical, namely, to provide maximum financing to public good by means of a lottery.

Assumption 6.
Within the framework of this model the optimal level of financing of public good is achievable.

Proof.

Regardless of the lottery format (fixed prize or a share of the total return), participants ceteris paribus choose the amount of their contributions based on the prize value as the only exogenous variable[7]. Each participant's function of dependence of his/her contribution from the prize can be defined as $\boldsymbol{x_i(R)}$(or $\boldsymbol{x_i(\alpha)}$). The function may or may not be either monotonous or continuous anyway its values are nonnegative for any R.

Therefore the amount of money spent on public good is equal to $\boldsymbol{S(R) = \sum x_j(R) - R}$. The prize value as well as the total fund is limited by zero as the minimum value and by the initial wealth of the society ($\sum \boldsymbol{w_j}$) as the maximum value. As $\boldsymbol{S(R)}$ is the function defined on the set of non-negative rational numbers, which domain and range are bounded, there should exist its' maximum.

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[7] *In case of Model 3 "Trust in Luck" advertisement spending also matters*.

Thus there is always a prize $\boldsymbol{R}^*$ that brings the public good financing $\boldsymbol{S}(\boldsymbol{R}^*)$ to its maximum. In the case of advertising there is a function of the two variables $\boldsymbol{S}(\boldsymbol{R};\ \boldsymbol{\varepsilon})$. That does not change the inference.

■

3.6. Model 6 "Generalised model"

Bringing together all the aforementioned models excluding Model 4 one faces the following number of assumptions.

The organiser of a lottery maximises the volume of public good influencing participants by setting the prize amount and by advertising.

Each participant assumes that other participants spend as much as he does.

Participants not only want to win a prize but also face the utility of public good and moral satisfaction from a good deed.

A participant's welfare function in the generalised model is:

in case of a voluntary contribution to the public good:

$$\boldsymbol{U_i = w_i - x_i + m_i(x_i) + h_i(nx_i)};$$

in case of a lottery with a fixed prize:

$$\boldsymbol{U_i = w_i - x_i + m_i(x_i) + f_i(\varepsilon)\frac{R}{n} + h_i(nx_i - R - \varepsilon)};$$

in case of a lottery with a prize calculated as a share of the total return:

$$\boldsymbol{U_i = w_i - x_i + m_i(x_i) + f_i(\varepsilon)\alpha x_i + h_i\big((1-\alpha)nx_i - \varepsilon\big)}.$$

4. MODEL OF THE TAXATION OF LOTTERIES

Let us analyse the impact of taxes on the financing of public good by means of lotteries. For that purpose the generalized model given above is used. Suppose there are three nominal types of taxes: a tax on a participant's contribution $(\boldsymbol{t_x})$, a tax on the prize (income) of a participant $(\boldsymbol{t_R})$, and a tax on the total fund collected $(\boldsymbol{t_G})$.

Suppose these taxes are charged proportionally to the taxable sum on standard tariff. In this case the function of the lottery participants' welfare is described as:

$$\boldsymbol{U_i = w_i - x_i + \frac{(1-t_x)x_i}{(1-t_x)G}(1-t_R)R + h_i((1-t_G)G(1-t_x) - R)}$$

or

$$\boldsymbol{U_i = w_i - x_i + \frac{x_i}{G}(1-t_R)R + h_i((1-t_G)G(1-t_x) - R)}$$

Let us consider how the tax amount influences contributions of participants.

Introduction of a tax on a contribution or the fund collected produces two oppositely directed effects. On the one hand it is taken into consideration that the marginal utility of public good is a decreasing function. Thus, assuming that each participant maximises his/her total utility, the decrease of the public good value will either increase the contribution of a participant or leave it unchanged.

On the other hand if a participant spends one dollar to buy a ticket he contributes only a share of that sum to the fund. In other words, if the maximum utility of public good used to be a function of the contribution ($\boldsymbol{x_i}$), then it is a function of a share of a contribution left after taxation ($\boldsymbol{(1-t_x)(1-t_G)x_i}$). Consequently, assuming the relative expectations model, due to the chain rule the first order condition looks as follows:

$$\boldsymbol{1 = (1-t_x)(1-t_G)h_i'.}$$

In other words, due to the decreasing marginal utility, a new equilibrium level of public good will be lower than the previous one.

So which of the two effects is stronger?

Assumption 8.

Assuming relative expectations, the increase in tax of a contribution or of the fund collected will decrease the value of a participant's contribution in case of an internal solution.

Proof.

Based on the assumption that the effect of both taxes ($\boldsymbol{t_x}$и$\boldsymbol{t_G}$) is similar, suppose that there is only one tax ($\boldsymbol{t}$).

$\boldsymbol{F = (1-t)h_i'((1-t)nx_i - R) - 1 = 0}$, then:

$$\frac{dx_i}{dt} = -\frac{\frac{dF}{dt}}{\frac{dF}{dx_i}} = -\frac{-t(-nx_i)h_i'' + (-1)h_i'}{(1-t)(1-t)nh_i''} = \frac{h_i' - tnx_ih_i''}{(1-t)^2nh_i''}$$

$$= \frac{h_i'}{(1-t)^2nh_i''} - \frac{tnx_i}{(1-t)^2n} < 0$$

■

Thus neither the introduction of the taxes analysed above nor the increase of the amount of tax will effectively maximise the public good.

Those who were ready to participate in the lottery without a tax on the contribution (internal solution) due to the introduction of a tax (or the increase of the tax rate) may reduce the amount of their contributions or even refuse to participate. Potential participants of a lottery, who had not yet decided whether to participate before the taxes were introduced (corner solution), are even less likely to become participants of a lottery in case of tax increase.

Accordingly, following the model of maximization of public good in the context of the assumption of relative expectations one can argue that the introduction of a new tax is inefficient aiming to fund public good through lotteries.

At the same time one can assume that taxes may have a stimulating role. For example legislative decrease of the tax basis of a participant of a lottery by the size of his/her contribution may attract more participants. In that case budget revenues from taxes on contribution of participants decrease but the fund of the lottery increases. After the obligatory payments will be made, the remaining part of the fixed share of fund will be spent on a public good. It may cause a positive balance and increased financing of a public good. The result in each case is determined by specific conditions of lottery organisation.

Other ways of stimulation public good funding by tax regulation are possible as well (direct partial payback of a lottery contribution to a participant, introduction of tax incentives for lottery participants etc.).

However the stimulating role of tax benefits in financing public good is beyond the framework of the research and is not analysed in this paper.

5. CONCLUSION

In this paper the existing models of financing public good by means of lotteries are analysed. Several other models are proposed so as to compensate certain shortcomings of the above-mentioned models. Those models are based on the assumption that lottery organiser knows the welfare function of each participant of a lottery except the moral component.

The analysis of the classical and newly introduced models allows one to make the following conclusions.

1. It is proved that the socially optimal level of providing public good is achievable (with the assumptions of Model 5 'Maximising public good'– 3.5).

2. The newly introduced notion of 'relative expectations' along with taking into consideration the influence of advertising and the moral utility factor for the participants of a lottery provides a new approach to the 'free-rider' problem.

3. The conclusion of the Morgan model that fixed prize lotteries fully crowd out voluntary contributions is refuted.

4. For each lottery there may be defined the size of a prize that maximises financing of public good.

5. To become a large-scale event a lottery requires advertising, which reflects the potential gain of participants and the moral component of their participation.

6. Lotteries as a mean of financing public goods, if organized properly and with adequate control of the funds collected, are definitely beneficial for all members of society regardless of their level of involvement.

The newly introduced models are theoretical hypotheses. Each of them admits further development and exploration of possible increasing efficiency of financing public good by means of lotteries

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