The Limits to Partial Banking Unions: A Political Economy Approach

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Abstract

This paper studies the welfare effects of a 'partial banking union' in which cross-country financial transfers that could be used towards bailouts are decided at the supranational level, but policymakers in member countries hold decision power over the distribution of funds. This allows the policymakers, who are partially self-interested, to extract rents in the bailout process. In equilibrium, such a banking union lowers the welfare of citizens in the country receiving transfers. Supranational fiscal rules are ineffective at reversing this result, but a Pareto improvement may be achieved if fiscal rules are combined with domestic reforms that reduce political rents.

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1 Introduction

Increased cross-border financial flows in the lead up to the 2007-2008 financial crisis facilitated increased investment, output and growth in both developed and developing economies.[1] These flows decreased during the financial crisis,[2] and the trouble experienced by the banking sector exposed a largely overlooked aspect of financial integration – that public support for banks in a financial crisis depends on national governments, and that this support generates significant cross-country spillovers. Naturally, the presence of such spillovers suggests that a supranational agreement in the form of a banking union could deliver a Pareto improvement. It would centralize public bailouts and ensure that all spillovers are taken into account by decision-makers. Yet, such centralization requires governments to give up their decision power over national banking systems. If this requirement is infeasible due to sovereignty concerns, then any supranational agreement must allow national governments to retain some decision power, leading to a hybrid system, or a 'partial banking union.' Once this agreement is in place, domestic political economy constraints may interfere with the functioning of this institution: policymakers may divert resources towards spending that provides them with local political rents. This raises the question of whether such a banking union actually improves welfare and achieves an efficient government intervention in the banking sector.

The tension between domestic and supranational institutions is illustrated by the Eurozone's reaction following the 2007-8 financial crisis. After lengthy negotiations, the European Banking Union was created as a mechanism for common bank regulation. But its Common Resolution Mechanism stopped short of achieving a fully centralized response to a banking crisis.[3] In particular, it does not have a functional system for determining cross-country transfers towards public bailouts before a crisis is under way.[4] While the procedures in

[1] Evidence of these effects is presented in Alfaro, Kalemli-Ozcan and Volosovych (2014) and Kalemli-Ozcan, Sorensen and Volosovych (2014).

[2] For more details, see Bertaut and Pounder (2009) and ECB (2015).

[3] For details on the Common Resolution Mechanism, see Hadjiemmanuil (2015).

[4] The provisions for this are contained in the rules of the Direct Recapitalization Instrument (DRI). A country may apply for DRI funds only after it is considered unable to fund

place are aimed at avoiding another major crisis, if such a crisis does happen, public bailouts in one country will, to a large extent, have to go through that country's government. The role that domestic political distortions play in such a situation was illustrated by the Spanish bailout of the savings and loan sector (the '*cajas*') during the financial crisis.[5] Local policymakers decided to rescue failing *cajas* by merging them based on political and regional motives rather than economic efficiency, as each of these institutions had significant political connections to one of the major Spanish parties.[6] These inefficient mergers led to the creation of larger troubled entities, increasing the cost of public bailouts and the pressure on public finances in Spain and in the Eurozone.

This paper considers the above facts and builds a model of a supranational arrangement over bank bailouts in which domestic policymakers have decision power over the distribution of bailout funds. The model considers two countries, each with banks that hold deposits made by citizens from both countries. In each country, a crisis can wipe away value from bank investments made with the citizens' deposits. The crisis opens up the possibility of public bailouts, where each government provides funds to banks in its country, in order to salvage productive investments. Since deposits in banks are held by citizens from both countries, a bailout in one banking system produces cross-country spillovers. A supranational institution can set financial transfers between countries; however, the supranational institution has limited powers: it cannot directly give these transfers to banks. It can only propose cross-country transfers and the overall spending on public bailouts, but each government decides how the bailout funds are distributed to banks in its country. Each government is partially self-interested: it is concerned about the welfare of citizens within its own country, but it also derives political rents from engaging in public bailouts. It can use the budget for three purposes: to provide public bailouts to banks, to extract political rents, and to provide a public good – for non-bailout purposes.

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the bailout from its own budget; more details in Hadjiemmanuil (2015).

[5] Discussed in greater detail in Garicano (2012) and Cuñat and Garicano (2010).

[6] Discussion and more details in Garicano (2012).

The paper's main result is that creating a partial banking union under domestic political economy distortions can reduce citizen welfare in the country that receives transfers rather than in the country that provides transfers. The result relies on the interaction of two forces, one domestic and one supranational. At the domestic level, the ability of policymakers to extract rents leads to a mismatch between their incentives and those of their citizens. The policymakers have the incentive to extract rents whenever funds are allocated to banks, and this reduces the welfare of citizens. At the supranational level, policymakers must agree on how much each country contributes to the funding of bailouts. The citizens do not have a seat at the table when this decision is made. The spending on bailouts provides policymakers with the additional benefit of political rents, so they evaluate the relative costs and benefits of bailouts differently than citizens. Therefore, they may agree to a partial banking union in which their country faces high spending on bailouts and low spending on other public goods, even if this reduces the welfare of their country's citizens. The supranational division of bailout costs is determined using a set of country weights, such that a lower country weight translates into a higher contribution to the funding of bailouts. The model shows that citizen welfare in the country receiving transfers decreases when the country carries a sufficiently low supranational weight relative to the weight placed by its government on political rents.

Policy discussions point to the lack of fiscal integration as a major obstacle to the full supranational centralization of public bailouts. Limits to government borrowing have been proposed as a solution to this obstacle.[7] After establishing the main result described above, the model addresses this argument. It considers the effect of fiscal rules that restrict government borrowing. Such rules reduce the policymaker's ability to borrow in order to finance public bailouts and to take rents, which increases citizen welfare in autarky; however, fiscal rules decrease the overall welfare gains from a partial banking union.

[7] Restrictions on the governments' ability to borrow have been under discussion in early 2016. For more details, see Bloomberg Business, "EU Weighs Bank State-Debt Limits to Ease Germany's Risk Concerns," January 31, 2016.

The reason for this is that fiscal rules cannot restrict rent seeking without also restricting government spending in general. If a policymaker cannot provide sufficient funds to banks, then more transfers have to come in from the other country in order to fund bailouts. The need for more transfers makes it costlier to implement a partial banking union. Moreover, fiscal rules may even further reduce welfare in the country receiving transfers, as limited access to public debt together with the need to fund bailouts lead to larger cuts in public good provision.

Though by themselves fiscal rules decrease the overall welfare gains from a partial banking union, they may be crucial complements to political reforms in the design of more complex partial banking unions. Instead of a partial banking union set up when fiscal rules are already in place, the paper also considers the alternative institutional structure in which countries can set fiscal rules conditional on joining a partial banking union. This allows citizens to constrain the expansion of spending on bailouts and political rents under the supranational agreement. The conditional fiscal rules therefore reduce the losses in welfare to citizens due to the supranational division of bailout costs – the second force driving the model's main result. This effect means that, for any positive country weight at the supranational level, conditional fiscal rules expand the maximum size of the domestic rent seeking distortion under which a Pareto improvement can be achieved in a partial banking union. By reducing the supranational distortion, conditional fiscal rules allow for a larger domestic distortion to exist without it leading to inefficiency in the supranational agreement. From a policy perspective, this means that even limited political or regulatory reforms that reduce, but no not eliminate, political rents can be valuable in the design of a welfare-improving partial banking union. Such reforms, complemented by conditional fiscal rules, can be attached to a partial banking union in order to obtain a Pareto improvement.

Related literature. The interplay between financial integration and fiscal policy has been vastly studied in the literature. Yet, the main focus for most of the work in this area has been on optimal policy design with a benevolent government. This includes the study of optimal fiscal policy coordination (Kehoe, 1987; Chari and Kehoe, 1990; Beetsma and Lans Bovenberg, 1998; Halac and Yared, 2015), fiscal rules in currency unions (Von Hagen and Eichengreen, 1996; Ferrero, 2009) or the role of fiscal transfers in providing efficient insurance within a currency union (Farhi and Werning, 2013). These papers abstract from the effects of political economy distortions. By contrast, this paper considers the issue of financial integration taking into account the political economy issues that emerge when policymakers are partially self-interested. Therefore, this paper is most closely related to the political economy work that considers the effects of different political institutions in the context of fiscal or financial integration (Tabellini, 1990; Lohmann, 1993; Persson and Tabellini, 1996*a*,*b*). Whereas that literature focuses mainly on the effects of different electoral institutions and the aggregation of voter preferences, this paper considers the issue of political rent seeking and examines the distortion this brings to supranational policies.

The link between financial integration and domestic public debt in the presence of political economy constraints has also been studied by Tabellini (1990) and Azzimonti, de Francisco and Quadrini (2014), who show how fiscal or financial integration can lead to higher public debt due to political economy biases. This paper, however, highlights a different channel for the increase in public spending, and implicitly public debt. Debt does not increase due to lower costs of borrowing (as in Tabellini, 1990) or the aggregation of heterogeneous voter preferences (as in Azzimonti, de Francisco and Quadrini, 2014), but rather because cross-country transfers increase rent seeking. The increase in debt is directly linked to the existence of supranational agreements in the absence of political integration. In a set of papers also motivated by the European supranational institutions, Persson and Tabellini (1996*a*,*b*) study cross-country insurance and the effect of fiscal transfers on welfare under different political decision-making institutions, specifically direct voting versus bargaining. This paper provides a complement to their results. While their papers highlight the inefficiencies that emerge under various institutions of collective choice – voting versus bargaining–, this paper considers inefficiencies rooted in domestic institutions – rent seeking. Moreover, it presents another channel through which domestic institutions affect supranational agreements: that of rule implementation (the allocation of transfers by the local policymaker) rather than rule selection (the collective choice of transfers).

The desirability of supranational controls over domestic spending has also been examined in Dewatripont and Seabright (2006), but in the context of a politician whose type is unknown to voters, and who uses domestic spending to signal his type. By contrast, this paper considers the role of supranational controls in a model without private information, where the politician has a direct preference for rent seeking.

Finally, the design of fiscal rules and their effect on government spending is explored in Corsetti and Roubini (1997) and Milesi-Ferretti (2004) in the context of politically motivated public spending, and in Halac and Yared (2014, 2015) from the perspective of optimal policy design. This paper models fiscal rules in line with this literature, but it focuses on the interaction between fiscal rules and financial integration through a partial banking union.

Organization. The rest of the paper is organized as follows. Section 2 presents the setup of the model. Section 3 gives the benchmark case with benevolent policymakers. Section 4 presents the main result of the paper. Section 5 considers the role of fiscal rules. Section 6 concludes, and the Appendix contains proofs and extensions.

2 Environment

Consider a two-period economy, with periods 0 and 1.[8] The economy consists of two countries. One country will be the provider of cross-country transfers, and it will be referred to as Financing, or F. The other country will be receiving transfers, and it will be referred to as Debtor, or D. An indepen-

[8] An infinite-horizon version of the model is presented in the Appendix D. It shows that the main results of the model can be extended to a dynamic environment with repeated crises.

dent supranational authority, denoted by S, plays the role of a Principal who proposes the terms of a partial banking union between countries. Each of the two countries is made up of a continuum of mass 1 of identical households and a continuum of mass 1 of identical banks.

2.1 Households

In period 0, all households from both countries start with a perfectly diversified portfolio of risky projects, in the form of deposits in banks. Households in country D hold total deposits z^D, a fraction α^D of which is deposited in banks in country D, while the remaining fraction is deposited in banks in country F.[9] Similarly, households in country F hold total deposits z^F, a fraction α^F of them is deposited in banks in country F and the remaining fraction is deposited in banks in country D.

Households derive utility from private consumption equal to the return from their deposits, c^i. They also consume a public good g^i provided by the government of their country, $i = D, F$. Household preferences in country i are given by[10]

$$U^i = u(c^i) + w(g^i) + \beta \left(u(z^i) + w(g^i_1) \right),$$

where $u(\cdot)$ and $w(\cdot)$ are increasing, strictly concave, continuously differentiable, $0 < u'(0) < \infty$, $0 < w'(0) < \infty$, $\lim_{g \to \infty} w'(g) = 0$.

2.2 Banks

Banks in each country hold identical risky investment projects which pay off at the end of period 0. They do not have any equity and can fund projects exclusively using household deposits. Their objective is to maximize the re-

[9] The assumption that households hold deposits in banks outside their country's borders is a simplification meant to capture the loans made by banks in one country to banks outside that country. Specifically for the case of the Eurozone, direct deposits by households in foreign banks represent a negligeable fraction of cross-border banking compared to the substantial cross-border loans made between banks.

[10] For ease of notation, I omit the subscripts for the period 0 variables and keep only the subscripts for the period 1 policies.

turns to their depositors. The initial investment made by banks in country $i \in \{D, F\}$ is denoted by I^i and consists of the deposits from both D and F households:

$$\begin{aligned} I^D &= \alpha^D z^D + (1-\alpha^F) z^F, \\ I^F &= (1-\alpha^D) z^D + \alpha^F z^F. \end{aligned}$$

The project return is subject to uncertainty. Following investment, an aggregate shock $\theta \in \Theta$ is realized in both countries. After the shock, projects become distressed –a fraction θ of all investment projects is lost, while the remaining $(1-\theta)$ fraction of all projects is intact. The intact portion of the project has a rate of return R in the next period. The distressed portion of projects does not produce any returns, unless additional funds are reinvested. After observing θ and prior to project completion, the banks in country i can reinvest x^i new funds into their projects –through a process called recapitalization– such that the total size of the projects is at most equal to the initial investment: $x^i \leq \theta I^i$.[11] Since there is no private loan market for banks to access reinvestment funds, any funds x^i must be provided by the government of country i. Another key assumption is that these recapitalization funds cannot be targeted towards the deposits of a particular household, since projects are funded with deposits from all households and cannot be broken apart. This ensures that both the D and F households benefit from the reinvestment, in proportion to their contribution to the total investment. At the end of period 0, the project is completed and returns $R((1-\theta)I^i + x^i)$ consumption units, where $R > 1/(1-\theta)\,, \forall \theta \in \Theta$.

In the second period, banks hold safe projects with rate of return $R_1 = 1$ and receive deposits from D and F households, in the same proportions as in period 0. The assumption of a second period creates a role for public debt in smoothing public good provision over time, as further discussed below.

[11] The liquidity shock is modeled as a simplified version of the one in Holmström and Tirole (1998).

2.3 Policymakers

Policymakers in each country have access to a budget e^i, $i \in \{D, F\}$ each period. In period 0, they can provide public goods g^i, recapitalization funds x^i and also to take on rents r^i. They may also take on debt $b^i \in [\underline{b}^i, b^{i,MAX}]$ at rate $1/\beta$, equal to the discount rate, with a lower limit $\underline{b}^i = -e^i/\beta$ and upper limit $b^{i,MAX} = e^i$. Finally, the policymakers can join a partial banking union, with terms described below. The partial banking union involves a transfer τ from country F to country D. Therefore, policymaker i faces the following budget constraint in period 0 :

$$r^i + x^i + g^i \leq e^i + \beta b^i + \tau^i,$$

where $\tau^D = \tau = -\tau^F$. In period 1, each policymaker provides public goods g_1^i and repays debt b^i, leading to budget constraint

$$g_1^i \leq e^i - b^i.$$

The rent seeking process is modelled as follows. The policymaker can use public funds to intervene in the banking sector: he can provide reinvestment funds x^i for the distressed projects described above (with rate of return $R > 1$), but he can also provide funds towards investments that have a rate of return of 1, a return that goes to the policymaker alone. The reinvestment of x^i is socially efficient since $R > 1$. The investment in the projects that only benefit the policymaker and have a rate of return of 1 is socially inefficient, and represents rent seeking.[12] The value of political rents is determined as in Grossman and Helpman (1994): the politician weights both household utility and the benefits coming from political rents. Therefore, the total spending towards

[12] Another way to think about the two types of projects is the following: the policymaker can choose the degree of efficiency the reinvestment in projects. The socially efficient intervention provides reinvestment funds x^i for the distressed banks. The policymaker can choose less efficient interventions, which provide reinvestment funds x^i to banks but also expand the scope of the original project. Only the original project has the rate of return R. The expansion of the project has a rate of return of 1, in the form of political rents that only benefit the policymaker.

reinvestments will be equal to $x^i + r^i$, but only x^i are recapitalizations that provide returns to households. The allocation of funds between rents and recapitalizations cannot be verified by the other country or by the supranational authority.

Policymaker i's utility is then given by

$$V^i = (1 - \gamma^i) v(r^i) + \gamma^i U^i,$$

where $i \in \{D, F\}$, $\gamma^i \in (0, 1)$ represents the weight placed on household utility relative to rents by government i, and $v(r^i)$ is the utility derived by the policymaker from rents.[13] The function $v(r)$ is increasing, concave, continuously differentiable, $0 < v'(0) < \infty$, $\lim_{r\to\infty} v'(r) = 0$. Since rents are extracted as part of the recapitalization process, they can only be extracted in the first period.[14] The private consumption of households is given by

$$c^i(x^i, x^j) = R(1-\theta)\alpha^i z^i + R\sigma^i x^i + R(1-\theta)\left(1-\alpha^i\right) z^i + R\left(1-\sigma^j\right) x^j,$$

where $\sigma^i = \alpha^i z^i / \left(\alpha^i z^i + (1-\alpha^j) z^j\right)$ represents the share of deposits owned by country i households in country i banks, $i \in \{D, F\}$.

2.4 The partial banking union

The supranational authority can set the terms of a partial banking union between the two countries. Specifically, a partial banking union consists of a positive transfer τ from country F to country D and a minimum reinvestment spending $\underline{x}$ that country D must commit to. The spending $\underline{x}$ represents the conditionality imposed on the receiving country by the supranational authority, since it specifies how much of the government budget must go to the banks as opposed to public goods. The main feature that differentiates this supranational arrangement from a full banking union is that the reinvestment

[13] This form of the utility function can be interpreted as the reduced form of an electoral process in which an elected politician faces a reelection constraint.

[14] A dynamic extension of the model presented in appendix D considers the case in which rents are also extracted in future periods.

spending $\underline{x}$ is done by the policymaker country D. The supranational authority lacks enforcement power, in that it cannot choose and enforce a specific allocation of funds to recapitalizations of bank projects and to rents. This is what makes the supranational agreement vulnerable to domestic political economy incentives. Any required spending $\underline{x}$ on total reinvestments is satisfied as long as

$$x^D + r^D \geq \underline{x}.$$

The terms $(\tau, \underline{x})$ of the partial banking union are chosen by a supranational authority that maximizes a weighted sum of D and F household utilities, with relative weight $\eta \in (0,1)$ on the D households,

$$\max_{\tau,\underline{x}} \{\eta U^D(x^D, x^F, g^D, g_1^D) + (1-\eta) U^F(x^F, x^D, g^F, g_1^F)\}, \tag{1}$$

under the condition that any chosen pair $(\tau, \underline{x})$ must be preferred by each policymaker to the outside option of no partial banking union. This requirement is translated to two participation constraints that must be satisfied for a partial banking union to be implemented:

$$\begin{aligned}(1-\gamma^D)v(r^D) + \gamma^D U^D(x^D, x^F, g^D, g_1^D) \geq \\ (1-\gamma^D)v(r^{D0}) + \gamma^D U^D(x^{D0}, x^{F0}, g^{D0}, g_1^{D0}), \quad (2)\\ (1-\gamma^F)v(r^F) + \gamma^F U^F(x^F, x^D, g^F, g_1^F) \geq \\ (1-\gamma^F)v(r^{F0}) + \gamma^F U^F(x^{F0}, x^{D0}, g^{F0}, g_1^{F0}), \quad (3)\end{aligned}$$

where the superscript 0 indicates the policies chosen in the outside option, without a partial banking union.

To ensure focus on the non-trivial case in which there is scope for transfers between countries, and to ensure country D is the one receiving transfers, the following assumption is made about government endowments.

Assumption 1 *The endowments of the D and F governments satisfy*

$$\begin{aligned} e^D(1+\beta) &< \theta I^D + g^{D*}(1+\beta) + r^{D*}, \\ e^F(1+\beta) &\geq \theta I^F + \theta I^D + g^{F*}(1+\beta) + r^{F*} + r^{D*}, \end{aligned}$$

where g^{i*} is defined implicitly by $w'(g^{i*}) = \sigma^i R u'(c^i(\theta I^i, \theta I^j))$, and r^{i*} is defined implicitly by $(1-\gamma^i)\, v'(r^{i*}) = \gamma^i \sigma^i R u'(c^i(\theta I^i, \theta I^j)),\ i, j \in \{D, F\},\ i \neq j$.

Assumption 1 is necessary in order to establish the need for cross-country transfers. The first inequality restricts e^D to be sufficiently small such that the D government does not fully recapitalize its banks if no partial banking union is in place, i.e., $x^{D0} < \theta I^D$. The second inequality ensures that the F government's endowment e^F is sufficiently large so that full recapitalizations are provided to the F banks even if transfers are made to country D $\left(\text{so } x^F = \theta I^F\right)$. Since the F country recapitalizes its banks without outside transfers, this establishes that, if transfers are feasible, then the F country will be the country providing transfers, while the D country will be the one receiving transfers.

3 Benchmark with benevolent policymakers

We begin the analysis with the benchmark in which the two policymakers are benevolent. This provides a baseline case from which we can analyze the effects of political economy distortions. Each policymaker i maximizes the same utility as that of households in its country, U^i. The supranational authority proposes a transfer τ and a minimum reinvestment expenditure $\underline{x}$ in order to maximize the weighted sum of household utilities,

$$\max_{\tau, \underline{x}} \eta U^D + (1-\eta) U^F, \tag{4}$$

subject to the participation of each policymaker:

$$U^D(x^D, x^F, g^D, g_1^D) \geq U^D(x^{D0}, x^{F0}, g^{D0}, g_1^{D0}), \quad (5)$$

$$U^F(x^F, x^D, g^F, g_1^F) \geq U^F(x^{F0}, x^{D0}, g^{F0}, g_1^{F0}), \quad (6)$$

where, as described above, the superscript 0 indicates policies chosen without the banking union in place.

Without rent seeking, there is no mismatch between policymakers and households. This baseline problem leads to the immediate benchmark result on the welfare effect of a partial banking union.

Proposition 1 *With benevolent governments, a partial banking union always achieves a Pareto improvement.*

Proof. In Appendix B, section B.1.1. ■

The result emerges because the economic benefits of a partial banking union are the same for policymakers and households. The incentives of policymakers and households are aligned, so any agreement that is accepted by policymakers necessarily benefits households. Analytically, the participation constraint of policymaker i requires that $U^i \geq U^{i0}$. If a positive transfer ($\tau > 0$) is optimal, and a partial banking union is implemented, then it must be the case that at least one country's welfare is improved over the outside option of no partial banking union. The participation constraints ensure that the other country's utility does not decrease below its value under no partial banking union. Therefore, whenever a partial banking union is implemented, it leads to a Pareto improvement.

Starting from this result, we proceed to introduce rent seeking in the decision problem of the policymakers. The following set of results show how the mismatch in incentives between policymakers and households interacts with the supranational weighting of household utilities to overturn this benchmark result.

4 Full model of partial banking union

We return to the model with domestic rent seeking, in which there is a mismatch between the utility of policymakers and that of households. The utility of policymakers now balances the value of political rents and the value of household utility. In proposing a partial banking union, the supranational authority chooses a pair $(\tau, \underline{x})$ that maximizes the weighted sum of household utilities, as described in (1), subject to the participation constraints (2) and (3). The veto power than policymakers have over the supranational policies ensures that they do not implement policies which lower their utilities. Yet, since the policymakers are no longer perfectly aligned with households in their policy preferences, this does not guarantee that household utility does not decrease. This is captured in the main result of the model:

Proposition 2 *A partial banking union does not achieve a Pareto improvement in household welfare if* $\eta \leq \eta^*$ *where* $\eta^* \in (0, 1)$: *it increases household welfare in the country providing transfers, but it lowers household welfare in the country receiving transfers.*

Proof. In Appendix B, section B.2.1. ■

The result comes out of the interaction of two forces: one domestic and one supranational. At the domestic level, rent seeking creates a mismatch between the incentives policymakers and those of households. The policymaker in country D derives benefits from rents, so transfers from country F increase rent seeking. But transfers also increase other government spending, not only rents, so the domestic distortion does not by itself reduce household welfare in a partial banking union. At the supranational level, there is another mismatch between policymakers and households. Any supranational agreement must be accepted by policymakers. They must agree on how much each country contributes to the funding of bailouts – the total reinvestment $\underline{x}$, which includes both rents and recapitalizations. Households do not have a seat at the table when this decision is made. The spending on reinvestments provides policymakers with the additional benefit of political rents, so their evaluation of the benefits relative to the costs of the supranational agreement is different than that of households. It is biased in favor of the agreement. The supranational division of bailout costs is determined according to the relative weights η for country D and $(1-\eta)$ for country F. A lower weight η means that country D must cover a higher share of the funding of bailouts. When η is sufficiently low ($\eta < \eta^*$) the cost for D households of the supranational agreement is higher than the benefit they receive from transfers. Yet, since rents make the relative benefit higher for policymaker D, the agreement is still accepted.

To see that both forces are necessary in order to obtain the result, consider what happens when only one of these forces is present. Without rent seeking, Proposition 1 shows that a Pareto improvement is always achieved. Without the supranational restriction that only policymakers have a seat at the table, the households can oppose any supranational agreement that is reducing their welfare. The loss in welfare emerges when policymakers have rent seeking incentives and households do not have a seat at the table at the supranational level, where the division of bailout costs is decided. The domestic mismatch between the incentives of policymakers and those of households is mirrored at the supranational level, where agreements can be reached with terms unfavorable to households.

The mismatch in incentives only leads to a reduction in welfare for households in the country receiving transfers. For the country providing transfers, country F, the policymaker and the households receive the same consumption from more spending on recapitalizations in country D. The cost of providing transfer τ is less public good provision in country F, but also fewer rents for policymaker F. Therefore, the cost of making transfer τ is relatively higher for policymaker F than it is for F households, since households do not value the political rents. In this case, policymaker F is biased against accepting the agreement, and any supranational policy that is acceptable to policymaker F increases the welfare of F households.

Analytically, the reasoning for Proposition 2 is as follows. Given $(\tau, \underline{x})$, policymaker D chooses $\zeta^D \equiv \left\{r^D, x^D, g^D, g_1^D, b^D\right\}$, to solve

$$\max_{\zeta^D} \left(1-\gamma^D\right) v(r^D) + \gamma^D \left[u(c^D(x^D, x^F)) + w(g^D) + \beta w(g_1^D)\right] \tag{7}$$

subject to

$$\begin{aligned} r^D + x^D + g^D &\leq e^D + \beta b^D + \tau, & \text{(8a)} \\ r^D + x^D &\geq \underline{x}, & \text{(8b)} \\ g_1^D &\leq e^D - b^D, & \text{(8c)} \\ x^D &\leq \theta I^D. & \text{(8d)} \end{aligned}$$

Constraints (8a) and (8c) are the budget constraints of the D government in periods 0 and 1, respectively. Constraint (8b) specifies the minimum required spending on reinvestments under the partial banking union, and constraint (8d) gives the maximum level of recapitalizations given the loss to projects.

The first relationship coming out of problem (7) is that an increase in recapitalizations x^D cannot be achieved without an increase in rents r^D. With an interior solution, the first-order conditions to the politician's problem lead to

$$\left(1-\gamma^D\right) v'(r^D) = \gamma^D \sigma^D R u'(c^D(x^D, x^F)).$$

The politician's utility is concave in both rents and recapitalizations, so any incentive to increase recapitalizations will also give the politician the incentive to increase rents. A policy $\underline{x}$ that increases recapitalizations implies $r^D > r^{D0}$ and $x^D > x^{D0}$.

The second relationship leading to the result is that the value of U^D is positively linked to η. This comes out of the supranational authority's problem (1). The weight η determines the share of the costs borne by country D when more reinvestment spending is decided by the supranational authority. As η increases, country D's bears a relatively smaller share of the costs. Specifically, the first-order conditions to problem (1) lead to

$$\begin{aligned}(1-\eta)\left(1-\sigma^D\right)Ru'(c^F)+\eta\sigma^D Ru'(c^D)\frac{\partial x^D}{\partial \underline{x}} &= \eta w'(g^D),\\ (1-\eta)w'(g^F)\left(-\frac{\partial g^F}{\partial \tau}\right) &= \eta w'(g^D).\end{aligned}$$

Applying the Envelope Theorem in this maximization problem then leads to the result that an increase in η increases equilibrium transfers τ and decreases equilibrium reinvestment $\underline{x}$. Given problem (7), this implies an increase in D household utility.

To see the role of η more clearly, consider two extreme cases. If $\eta = 0$, then the supranational authority does not take into account the utility of D households. In this case, it assigns country D the maximum costs that policymaker D can take on given the participation constraint (2). When constraint (2) binds, the increased political rents imply $v(r^D) > v(r^{D0})$ and $U^D(x^{D0}, x^{F0}, g^{D0}, g_1^{D0}) > U^D(x^D, x^F, g^D, g_1^D)$. If $\eta = 1$, then the supranational authority does not take into account the utility of F households. The utility of D households is maximized, transfers are maximized subject to the participation constraint of policymaker F, and so $U^D(x^D, x^F, g^D, g_1^D) > U^D(x^{D0}, x^{F0}, g^{D0}, g_1^{D0})$. Then, $\eta^* \in (0,1)$ denotes the value at which

$$U^D(x^D(\eta^*), x^F(\eta^*), g^D(\eta^*), g_1^D(\eta^*)) = U^D(x^{D0}, x^{F0}, g^{D0}, g_1^{D0}). \tag{9}$$

To shed more light on the condition that emerges in Proposition 2, a lower bound for η^* can be established as a function of the "baseline" rent seeking in country D without the banking union, r^{D0}.

Corollary 1 *The threshold η^* satisfies*

$$\eta^* \geq \frac{1}{1+\Phi(\theta)}, \tag{10}$$

where

$$\Phi(\theta) = \frac{w'\left(e^D - r^{D0}(1+\beta)^{-1} - \theta I^D\right)}{w'\left(e^F - \theta I^F(1+\beta)^{-1}\right)}.$$

Proof. In Appendix B, section B.2.2. ■

The bound described in (10) is derived at the extreme case in which D banks receive the maximum recapitalization. It helps highlight the relationship between the domestic and supranational forces at the root of the main result: high domestic rent seeking outside of a banking union means that fewer funds are used by politicians for socially efficient spending. This implies a high marginal benefit to households from additional recapitalizations, and therefore a high benefit from joining a partial banking union. This higher benefit means that improvements to household welfare can be achieved even if the weight η on country D is small – even if country D must provide a large part of the additional bailout spending from its own resources rather than from outside transfers.

The following results further explore the determinants of η^*, by performing some comparative statics.[15]

Corollary 2 *The cutoff value η^* decreases as e^F increases; it also decreases as γ^F increases.*

Proof. In Appendix B section B.2.3. ■

Corollary 2 shows that a higher difference in government budgets decreases the minimum supranational weight that country D must carry at the supranational level in order to overcome the distortion due to rent seeking. A higher budget for country F means that more of the bailout costs can be covered through transfers, thus easing the costs to country D households of the agreement policymakers accept at the supranational level. Similarly, a higher value of γ^F means that policymaker F places more weight on household welfare, and implicitly recapitalizations. Then, the policymaker is more willing to take on a higher share of the cost of bailouts.

Corollary 3 *A decrease in α^F leads to an increase in the cutoff value η^* if $\alpha^D + \alpha^F \leq 1$ and e^D is sufficiently small relative to e^F.*

[15] Appendix A presents a numerical analysis of the model under different scenarios regarding the size of rents relative to recapitalizations. As shown in these results, the implied cutoff η^* can be substantial.

Proof. In Appendix B section B.2.4. ■

Corollary 3 explores the effect of more financial integration. A decrease in α^F means that more of the assets of country F households are held by country D's banks – so there is more financial integration. The result shows that the threshold η^* below which a partial banking union is Pareto inefficient increases in environments with high financial integration and high differences in country incomes. These two factors combine to create high spillovers from bailouts. If financial integration is high ($\alpha^D + \alpha^F \leq 1$), then domestic bailouts bring relatively little benefit to domestic households, as many of their assets are abroad. Yet, bailouts bring a high benefit to households abroad. A supranational agreement then entails a large increase in recapitalizations. But this also means a large increase in rent seeking. As long as country D contributes any funds to these bailouts – which is the case whenever $\eta \leq \eta^*$–, more financial integration increases the supranational distortion described in Proposition 2: higher bailout spending is accepted by policymakers, who benefit from more rents.

The results in both corollaries show that the reduction in D household welfare comes from the interplay of domestic rent seeking and supranational allocation of bailout spending. Having established this main result, we next move to explore a potential solution to the inefficiency caused by the partial banking union. The next section considers the role of fiscal rules that limit public debt in reducing rent seeking in a partial banking union.

5 A Partial Banking Union with Fiscal Rules

Fiscal integration has been argued to be a necessary complement to financial integration. This section shows that this argument might not hold when fiscal integration is achieved through fiscal rules and there is only partial financial integration.

I model fiscal rules as the policy of each country setting an upper limit on debt at the beginning of period 0, contingent on the loss θ.[16] As in Halac and

[16] In Appendix C, I present a version of the model in which fiscal rules cannot be made

Yared (2015), I consider and contrast two types of fiscal rules: domestic and supranational. Domestic fiscal rules are set in a decentralized fashion, with each country choosing its fiscal rule independently. Supranational fiscal rules are set by the supranational authority which maximizes the weighted sum of household welfare.

5.1 Domestic Fiscal Rules

First, consider the case in which fiscal rules are set at the domestic level, by each country. A debt limit $\bar{b}^D(\theta)$, contingent on the loss θ, is set in country D so as to maximize D household utility, without anticipating the partial banking union.[17] The key assumption is that the fiscal rule is set by households through a constitutional process that allows them to set this rule before the policymaker makes any policy decisions.[18] The debt limit $\bar{b}^D(\theta)$ is chosen as the solution to the following problem:

$$\max_{\left\{\bar{b}^D, x^{D0}, g^{D0}, b^{D0}, r^{D0}\right\}} u(c^D(x^{D0}, x^{F0})) + w(g^{D0}) + \beta w(e^D - b^{D0}) \tag{11}$$

subject to

$$\gamma^D R\sigma^D u'(c^D(x^{D0}, x^{F0})) = (1-\gamma^D)v'(r^{D0}), \tag{12a}$$

$$R\sigma^D u'(c^D(x^{D0}, x^{F0})) = w'(g^{D0}), \tag{12b}$$

$$r^{D0} + x^{D0} + g^{D0} \leq e^D + \beta b^{D0}, \tag{12c}$$

$$b^{D0} \leq \bar{b}^D(\theta). \tag{12d}$$

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contingent on the realization of shock θ. The main results are qualitatively similar. In compiling a new dataset of fiscal rules in use around the world, Budina et al. (2012) find a recent evolution of fiscal rules towards rules that offer more flexibility in response to shocks. Therefore, the modelling choice of shock-contingent fiscal rules seems more relevant for current policy debates.

[17] In Appendix C, I show that the main results (except for Corollary 5) hold in the case in which the fiscal rules are set anticipating the partial banking union.

[18] The problem can be interpreted as a one-period reduced-form representation of a dynamic model in which past policymakers (with preferences aligned with those of households) have chosen fiscal rules that are binding for current and future policymakers.

Constraints (12a)-(12b) are the equilibrium conditions derived from the D policymaker's maximization problem with debt limit $\overline{b}^D(\theta)$. Constraint (12c) is the budget constraint of the D government, and constraint (12d) represents the limit on public debt imposed by the fiscal rule.

The problem for the F country is analogous. From the above setup, the following benchmark result is established.

Proposition 3 *Without a partial banking union, domestic fiscal rules (weakly) increase household welfare in each country.*

Proof. In Appendix B, section B.3.1. ■

Since rent seeking is only possible in period 0, restricting the government's access to funds in that period reduces rents. The limit on debt prevents the government from borrowing too much in order to engage in spending on the financial sector.

Having seen the effects of fiscal rules on household welfare, consider now the creation of a partial banking union. Denote by $\overline{U^i}(\theta, \overline{b}^i, \tau, \underline{x})$ the indirect household utility in country $i \in \{D, F\}$ given θ, when the debt limit is $\overline{b}^i$, and the terms of the partial banking union are $(\tau, \underline{x})$. The supranational authority must propose the transfer τ and reinvestment requirement $\underline{x}$ taking into account the debt limits $\overline{b}^D$ and $\overline{b}^F$ in each country. The problem it faces is

$$\max_{\tau, \underline{x}} \eta \overline{U^D}(\theta, \overline{b}^D, \tau, \underline{x}) + (1-\eta)\overline{U^F}(\theta, \overline{b}^F, \tau, \underline{x}) \tag{13}$$

subject to

$$\begin{aligned}(1-\gamma^D)v(r^D) + \gamma^D \overline{U^D}(\theta, \overline{b}^D, \tau, \underline{x}) \geq\ & (1-\gamma^D)v(r^{D0}) \\ & + \gamma^D \overline{U^D}(\theta, \overline{b}^D, 0, 0), \end{aligned} \tag{14}$$

$$\begin{aligned}(1-\gamma^F)v(r^F) + \gamma^F \overline{U^F}(\theta, \overline{b}^F, \tau, \underline{x}) \geq\ & (1-\gamma^F)v(r^{F0}) \\ & + \gamma^F \overline{U^F}(\theta, \overline{b}^F, 0, 0). \end{aligned} \tag{15}$$

Constraints (14) and (15) represent the participation constraints for the D and F governments, respectively. The participation constraints make it clear that the fiscal rules are set outside of the partial banking union, and therefore they remain in place even if the partial banking union is not accepted.

In order to compare the equilibria with and without fiscal rules, we first derive the condition under which policymaker F provides full recapitalizations ($x^F = \theta I^F$) when the domestic fiscal rule $\bar{b}^F(\theta)$ is set by households.

Lemma 1 *There exists $\overline{\gamma^F}$ such that $\forall \gamma^F \geq \overline{\gamma^F}$, policymaker F provides full recapitalizations ($x^F = \theta I^F$) when domestic fiscal rules are in place.*

Proof. In Appendix B section B.3.2. ■

Lemma 1 shows that full recapitalizations are still done in equilibrium as long as policymaker F values the welfare of households sufficiently relative to rent seeking. In this case, households in country F do not place highly restrictive limits on public debt. This case is relevant because full recapitalizations in country F ensure that transfers to country D will be feasible in a partial banking union.

Proposition 4 *For $\gamma^F \geq \overline{\gamma^F}$, there exists threshold $\eta^{**} > 0$ such that a partial banking union with domestic fiscal rules does not achieve a Pareto improvement over no banking union whenever $\eta < \eta^{**}$.*

Proof. In Appendix B, section B.3.3. ■

The result is driven by the same domestic and supranational forces as those described in Proposition 2. The main effect of the fiscal rules is to change the supranational division of bailout spending between the two countries. Fiscal rules limit policymaker D's ability to borrow in order to smooth out the cost of bailouts over time. This makes it more costly for the policymaker to comply with the reinvestment requirement $\underline{x}$. Since the supranational authority balances the distribution of costs across countries, this means that a higher cross-country transfer will be proposed. While fiscal rules increase the cross-country transfers, their effect on public goods in country D is ambiguous: the higher transfers have a positive effect on public good provision, while the reinvestment requirement $\underline{x}$ reduces public good provision, since it can no longer be financed with as much debt. This ambiguity does not allow for an immediate comparison between η^* and η^{**}. The relationship between the two cutoffs is a function of the elasticity of the utility from private consumption, $u(c)$, and the elasticity of the utility from the public good, $w(g)$.

The following results explore the welfare changes generated by a partial banking union in the presence of domestic fiscal rules.

Corollary 4 *The welfare of households in country F is lower under the partial banking union with domestic fiscal rules in country D, compared to the case without fiscal rules in country D.*

Proof. In Appendix B, section B.3.4. ■

The intuition is that fiscal rules in the D country limit the ability of policymaker D to fund recapitalizations using public debt. As described above, the effect of the fiscal rule is to determine an increase in the cross-country transfer. This reduces household welfare in the country providing these transfers.

Finally, the following result compares the loss in welfare from joining a partial banking union when fiscal rules are in place to the loss in welfare from joining a partial banking union when no fiscal rules are in place.

Corollary 5 *Consider a partial banking union that achieves full recapitalizations ($x^D = \theta I^D$). Then, there exists $\widetilde{\eta} \in (0, \eta^{**})$ such that $\forall \eta < \widetilde{\eta}$, having domestic fiscal rules in country D increases the welfare losses to households from joining a partial banking union.*

Proof. In Appendix B, section B.3.5. ■

Fiscal rules may increase household welfare compared to having no fiscal rules, both with and without a banking union. What Corollary 5 shows is that the drop in welfare going into a partial banking union is higher when fiscal rules are in place. The result emerges because a low value of η means that the supranational authority allocates a high share of the bailout costs to country D. With a limited ability to borrow due to the fiscal rule, the D government must finance the spending on the banking sector by significantly reducing public good provision in period 0. This lowers the utility of D households, and the welfare loss is higher than in the alternative scenario in which there are no fiscal rules.[19]

This case highlights the pitfall of domestic fiscal rules: if the country carries a low weight at the supranational level, domestic constraints on spending increase the cost of implementing the agreement. The benefit of fiscal rules in terms of reducing rents is offset by the supranational transfers, which allow rents to increase. This creates a situation in which policymaker D still derives a higher relative benefit from the supranational agreement due to rent seeking, while the households face higher relative costs.

The above corollaries show that domestic fiscal rules and a partial banking union provide opposing incentives to the policymakers in terms of rent seeking. If the fiscal rules are effective in reducing rent seeking and increasing D household utility, then a partial banking union undermines these benefits to households by providing policymakers with a means to increase rents.

5.2 Supranational Fiscal Rules

We contrast the domestic fiscal rules with centralized, supranational fiscal rules. When the same decision-maker can select both the terms of the partial banking union and the fiscal rules, it internalizes the opposing incentives to the policymakers in terms of rent seeking. The supranational authority must then choose between implementing fiscal rules and creating a partial banking union. If the financial spillovers are high, then the partial banking union is implemented without any fiscal rules.

To outline the problem for the supranational authority, denote the supranational fiscal rules by $\overline{B}^i$ for country i, and consider an environment in which there are no domestic fiscal rules. Instead, the supranational authority proposes fiscal rules along with the transfer and minimum reinvestment requirement $(\tau, \underline{x})$. If the partial banking union is not implemented, the outside option

[19] Corollary 5 discusses a comparison between a partial banking union and no banking union, with domestic fiscal rules in place in both cases. It can still be the case that household welfare in country D is higher in a banking union with fiscal rules compared to a banking union without fiscal rules.

for policymakers in each country is to choose policies under no banking union and no fiscal rules. The problem for the supranational authority is therefore given by

$$\max_{\overline{B}^D, \overline{B}^F, \tau, \underline{x}} \eta \overline{U^D}(\theta, \overline{B}^D, \tau, \underline{x}) + (1-\eta)\overline{U^F}(\theta, \overline{B}^F, \tau, \underline{x})$$

subject to

$$\begin{aligned} (1-\gamma^D)v(r^D) + \gamma^D \overline{U^D}(\theta, \overline{B}^D, \tau, \underline{x}) &\geq (1-\gamma^D)v(r^{D0}) \\ &\quad + \gamma^D \overline{U^D}(\theta, b^{D0}, 0, 0), \qquad (16) \\ (1-\gamma^F)v(r^F) + \gamma^F \overline{U^F}(\theta, \overline{B}^F, \tau, \underline{x}) &\geq (1-\gamma^F)v(r^{F0}) \\ &\quad + \gamma^F \overline{U^F}(\theta, b^{F0}, 0, 0). \qquad (17) \end{aligned}$$

Constraints (16) and (17) represent the participation constraints for the D and F governments, respectively.

The analysis of the above problem leads us to the following result.

Proposition 5 *Supranational fiscal rules do not change the result of Proposition 2: a partial banking union does not achieve a Pareto improvement if $\eta \leq \eta^*$; it increases household welfare in country F, but it lowers household welfare in country D.*

Proof. In Appendix B, section B.3.6. ■

The supranational authority internalizes the fact that fiscal rules and the partial banking union have opposing effects on rent seeking in country D. Therefore, it faces a trade-off between fiscal rules and the partial banking union. It can impose fiscal rules in order to reduce rent seeking, but this increases the cost of bailouts under the partial banking union. The reason why the cost of bailouts increases is that public debt cannot be used to smooth out this cost over time. The supranational authority anticipates that a partial banking union requires increasing spending on bailouts. If the benefits of a partial banking union are valued more highly by the supranational authority than the benefits of fiscal rules, then imposing fiscal rules would be counter to the supranational authority's objective. When $\eta \leq \eta^*$, we already know from

Proposition 2 that a partial banking union is created even though welfare decreases for country D households. This means that the overall benefits of the partial banking union are higher than the losses in household welfare due to rent seeking. Therefore, when $\eta \leq \eta^*$ the supranational authority prefers to form the partial banking union and not impose fiscal rules.

The above results show that fiscal rules could be a useful instrument in reducing rent seeking; however, they impose higher costs to creating a partial banking union. The next section considers an alternative institutional setting in which fiscal rules may help improve the welfare outcomes of a partial banking union.

5.3 Conditional fiscal rules and political reforms

Consider the following alternative institutional structure. Instead of creating a partial banking union once domestic fiscal rules are in place, each country can set domestic fiscal rules conditional on the partial banking union being formed – that is, only conditional on a partial banking union being adopted and not conditional on the terms τ and $\underline{x}$. This means that there is one fiscal rule in place if the country enters a partial banking union and another fiscal rule in place (or no fiscal rule at all) if the country does not enter the partial banking union. By doing this, each country's citizens essentially place constraints on what the policymakers can negotiate at the supranational level. These constraints allow households to improve on the outcome presented in Proposition 2.

Proposition 6 *A partial banking union with conditional fiscal rules achieves a Pareto improvement in household welfare* $\forall \eta > \widetilde{\eta}^*$, *where* $\widetilde{\eta}^* < \eta^*$.

Proof. In Appendix B, section B.3.7. ■

The result emerges because the fiscal rules reduce the size of the supranational distortion discussed in the description of the result from Proposition 2. By reducing the supranational distortion, the conditional fiscal rules expand the set of country weights under which a Pareto improvement can be obtained.

Policymaker D might benefit from a supranational agreement which specifies large bailouts, with a high share of their costs borne by country D; however, the limit of public debt imposed by the conditional fiscal rules does not allow for such an agreement to be implemented. The share of costs assigned to country D must be decreased in order to comply with the fiscal rules.

The above result requires conditional fiscal rules rather than fiscal rules chosen independently of the partial banking union. With unconditional fiscal rules, setting tighter fiscal rules may not actually reduce the supranational distortion. This happens because domestic, unconditional fiscal rules constrain the policymaker *outside* of the partial banking union as well. They increase the incentive for the policymaker to accept a supranational agreement, by making the outside option worse for the policymaker. This additional effect acts towards increasing the supranational distortion, because policymaker D's bias towards accepting the agreement increases.

Another way to view the role of conditional fiscal rules is that they allow for a Pareto improvement for larger domestic rent seeking distortions. For any supranational weight η on country D, we can compute a maximum domestic distortion under which a Pareto improvement can be achieved. One measure of the domestic distortion is the relative weight γ^D placed by the policymaker on household utility versus rents. Let $\underline{\gamma^D}(\eta) \leq 1$ denote the smallest value of γ^D at which a Pareto improvement is obtained in a partial banking union. Then, another way to state Proposition 6 is that conditional fiscal rules decrease $\underline{\gamma^D}(\eta)$. Changes in γ^D mean changes in the value of political rents. If these changes come through political or regulatory reforms, then a smaller reform is sufficient for a Pareto improvement, if conditional fiscal rules can be attached to the partial banking union.

The above result shows that conditional fiscal rules and political reforms have complementary effects, and they can be used together in order to achieve a Pareto improvement. To derive the conditions under which a Pareto improvement can always be achieved, we establish the following Lemma.

Lemma 2 *There exist functions $w(g)$, $u(c)$ and $v(r)$ such that the participation constraint for policymaker D – constraint (68)– is not satisfied under if*

$\bar{b}^D = -\frac{e^D}{\beta}$.

Proof. In Section B.3.8. ■

Under the conditions of the above Lemma, the fiscal rules can be sufficiently constraining for the policymaker such that we obtain for following result.

Corollary 6 *Consider a partial banking union in which the participation of each country can be conditioned on an increase in γ^D and on domestic fiscal rules. A Pareto improvement is achieved for $\gamma^D < 1$ and a conditional fiscal rule $\bar{b}^D$.*

Proof. In Appendix B, section B.3.9. ■

Conditional fiscal rules and a higher γ^D both reduce rent seeking, one by restricting the budget available to the policymaker when rents are extracted (through the limit on debt-taking), and the other by reducing the value of rents in the policymaker's utility function. Even though policymaker D has the incentive to increase rents in a partial banking union due to the cross-country transfer, these two policies act together to bring rents back down. A higher value of γ^D means that more of the bailout spending is used for recapitalizations rather than rents. This also increases the welfare for households in country F, and it offsets the negative welfare effect that fiscal rules set by country D have on F households.

A key feature of this policy solution is that both the fiscal rules and the increase in γ^D are conditions for joining the partial banking union. Country D must set the fiscal rule so as to ensure that household utility does not decrease in the resulting partial banking union. If the partial banking union is conditioned on an increase in γ^D, then it also ensures a welfare gain for the F households.

6 Conclusions

This paper presented a model of a partial banking union with domestic rent seeking. It showed that implementing such a supranational agreement can reduce the welfare of citizens in the country receiving transfers. The result is driven by the an interaction of domestic and supranational forces: the domestic mismatch between the benefits of bailouts to policymakers versus citizens and the supranational allocation of bailout costs between the two countries. Fiscal rules meant to reduce rent seeking may work well without a partial banking union, but they may lower the welfare gains from a partial banking union.

Despite its simplicity, the model captures several main features of government intervention in the banking sector. First, it captures the diffused costs of bank bailouts. When public funds are used to recapitalize a distressed bank, the costs are spread over the entire taxpayer base, through a reduction in other public goods or an increase in public debt. Second, it captures the cross-border spillovers from government intervention. It also captures the tension that banks generate, as institutions with cross-border operations, but backed by national authorities. Moreover, it captures the inability of governments to target public funds just to domestic stakeholders, because investments are made with funds from both domestic and foreign sources. Finally, the distressed investment projects and the aggregate loss to the banking sector provide a clear motivation for government intervention and supranational transfers. Of course, other sectors of the economy share some of these features. For example, environmental policies generate significant spillovers and carry a high benefit of supranational agreements; however, it is not clear that they create the same incentives for higher public debt, increased public spending and the development of assets that make it impossible for policymakers to target policies to particular constituencies. The model is therefore aimed at capturing key elements of the politics of supranational agreements over banking sector interventions.

Two main policy implications come out of this model. First, effective supranational fiscal rules may not be implemented together with a partial banking union, since the two institutions have offsetting effects on domestic incentives for rent seeking. This calls into question the policy proposals for a gradual implementation of a fiscal union together with a banking union. Without a full banking union in place, the viability of a fiscal union with supranational rules may be threatened. Second, the welfare losses stemming from the inability to fully centralize bank bailouts can be mitigated through a combination of conditional fiscal rules and political reforms. These two policies act as complements. This result suggests that even small reforms towards reducing the politicization of bank bailouts could have significant welfare effects when enacted together with conditional fiscal rules.

Finally, the model opens up several avenues for further research. The portfolio choices of households have so far been taken as exogenous. Allowing for an endogenous allocation of assets in response to the supranational agreement could shed light on the dynamics of investment and public good provision under a partial banking union. Also, the model has taken as given the structure of the supranational institution that proposes the partial banking union. Understanding how the supranational weighting of countries is developed could help illuminate how supranational institutions evolve in the absence of political integration.

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A Appendix A – Numerical Example (For Online Publication)

This section explores the implications of the model using a simple numerical simulation. It focuses on the upper limit η^* to the set of country weights under which a partial banking union leads to a welfare loss for the country receiving transfers.

Consider the following example which assumes logarithmic utility functions and symmetry between the two countries:

$$\begin{aligned} u(c^i) &= \zeta \log(1 + c^i(x^i, x^j)), \\ w(g^i) &= \vartheta \log(1 + g^i), \\ v(r^i) &= \rho \log(1 + r^i), \end{aligned}$$

where $i, j \in \{D, F\}$, $i \neq j$.

The parameter values are chosen so that the resulting equilibrium policies $(x^i + r^i)$ and g^i represent plausible shares of the government budget. It is useful to notice the variation in the values these shares take in the data. Below is data on relevant Euro area countries provided by Eurostat and the European Central Bank for the 2008-2013 period. It provides a summary of the share of spending on bailouts out of the total government expenditure.

Country	Total general	Financial needs for	Bailouts
	government	government	
	expenditure	bailouts	
			(% of total
		(2008-2013)	government
	(2013, % of GDP)	(% of 2013 GDP)	expenditure)
Spain	49.6	4.9	10.86
Ireland	39.7	37.3	93.95
Portugal	49.9	10.4	20.84
Greece	60.8	24.8	40.79
Cyprus	41.4	10.5	25.36
Germany	44.5	8.8	19.78
France	57	0	0
Italy	51	0.2	0.39
Netherlands	46.4	6.1	13.15
Belgium	55.6	3.9	7.01
Austria	50.9	3.1	6.09
Euro Area	49.6	5.1	10.28
Source: Eurostat Database and Maurer and Grussenmeyer (2015), Table 2.			

The parameters used in this example are $R = 1.1$, $\theta = 0.15$, $\alpha^D = \alpha^F = 0.6$, $\gamma^D = \gamma^F = 0.8$. The ratio of financial assets to government budget is set to approximate that of Spanish bank deposits to total government expenditure in 2010,[20] which was approximately 5, so $z^D/e^D = 15/3$, and $z^F = z^D$ in the baseline model, for symmetry. The following parameters are used for the utility functions: $\rho = 1.1$, $\vartheta = 1$ and two different values for ζ, in order to capture two different ratios of total spending on bailouts $\left(x^D + r^D\right)$ to public goods $\left(g^D\right) : \zeta_1 = 6.4$ leads to a ratio of approximately 0.11, $\zeta_2 = 6.7$ leads to a ratio of approximately 0.2 . The starting ratio of r^D to $x^D + r^D$ (without a partial banking union) is 0.17 when $\zeta = \zeta_1$, and 0.033 when $\zeta = \zeta_2$.

[20] A summary of Spanish private sector bank deposits in Spanish banks is available in the Reuters database "Deposits held in Spanish, Italian, Greek and Irish banks", March 17, 2013

Using the above functional forms and parameters, the following graphs show the resulting cutoff η^* obtained through the following steps

- given any feasible pair $(\tau, \underline{x})$ and η, the optimal policies $\{x^D, r^D, g^D, g_1^D\}$ for policymaker D are derived from problem (7); the equivalent problem is solved for F.
- at each η the equilibrium pair $(\tau, \underline{x})$ solves the problem for the supranational authority, given in (1).[21]
- η^* is the minimum value of η at which $U^D\left(x^D, g^D, g_1^D\right) > U^D(x^{D0}, g^{D0}, g_1^{D0})$.

Figure 1 graphs the value of η^* as against the ratio e^F/e^D, where the change in e^F/e^D is due to increases in e^F.

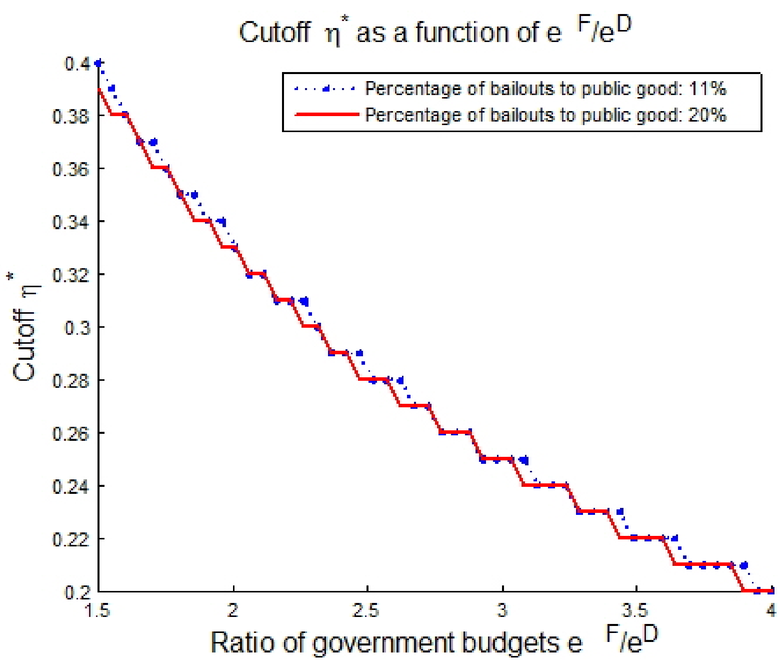

Figure 1:Cutoff η^* as a function of the ratio of government budgets

[21] A vector of 100 possible values is assumed for η.

The graph shows that an increase in e^F lowers the cutoff η^* below which a partial banking union is Pareto inefficient. As described in Corollary 2, a higher government budget in country F leads to country F being assigned a higher share of the bailout costs decided at the supranational level. The costs of joining the partial banking union are then smaller for country Country D. In terms of the two distortions discussed in Proposition 2, the supranational distortion is decreased. Also, notice that when bailout spending is higher, η^* is weakly decreasing for any level of e^F/e^D. The reason is again that the higher costs of bailouts are divided at the supranational level between the two countries, and these costs reduce household utility.

Figure 2 graphs the value of η^* as a function of the ratio of private assets in country F to private assets in country D, z^F/z^D. The change in z^F/z^D comes from changes in z^F while keeping z^D constant.

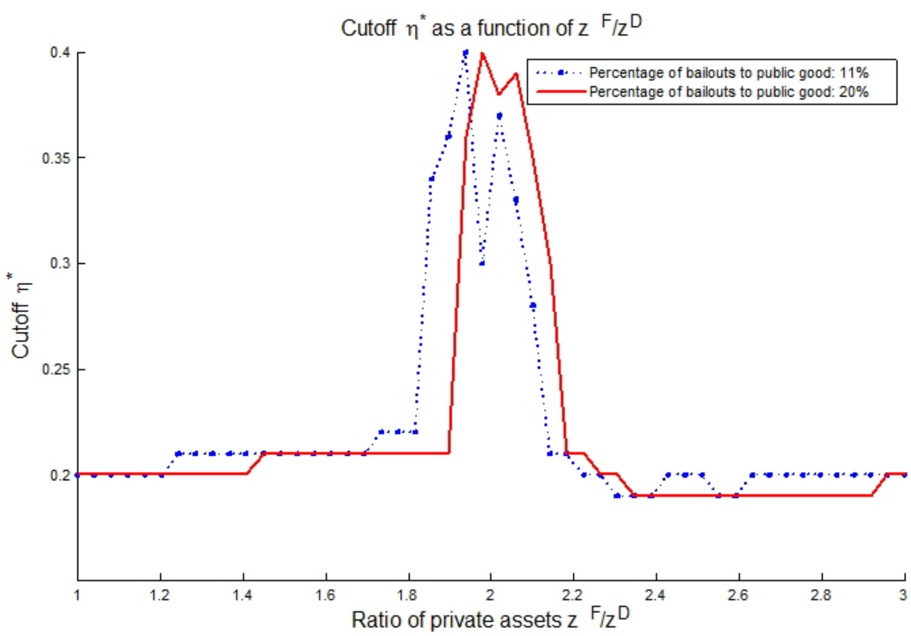

Figure 2: Cutoff η^* as a function of private asset ratio

The plots in Figure 2 show that η^* is not a monotone function of z^F.

Whether η^* increases or decreases depends on the change in the marginal benefit of rents relative to the marginal benefit of recapitalizations and relative to the marginal benefit of public goods. This happens because an increase in z^F has two opposing effects on the welfare of D households. First, an increase in z^F increases the assets held by country $D's$ banks. This makes bailouts costlier in country D, as more assets must be rescued in a bailout. Second, an increase in z^F increases the spillovers from bailouts, increasing the benefit for country F from providing transfers to country D. This means that policymaker F accepts a higher share of the bailout costs decided at the supranational level. Therefore, combing these two effects, the overall effect of an increase in z^F on D households is ambiguous.

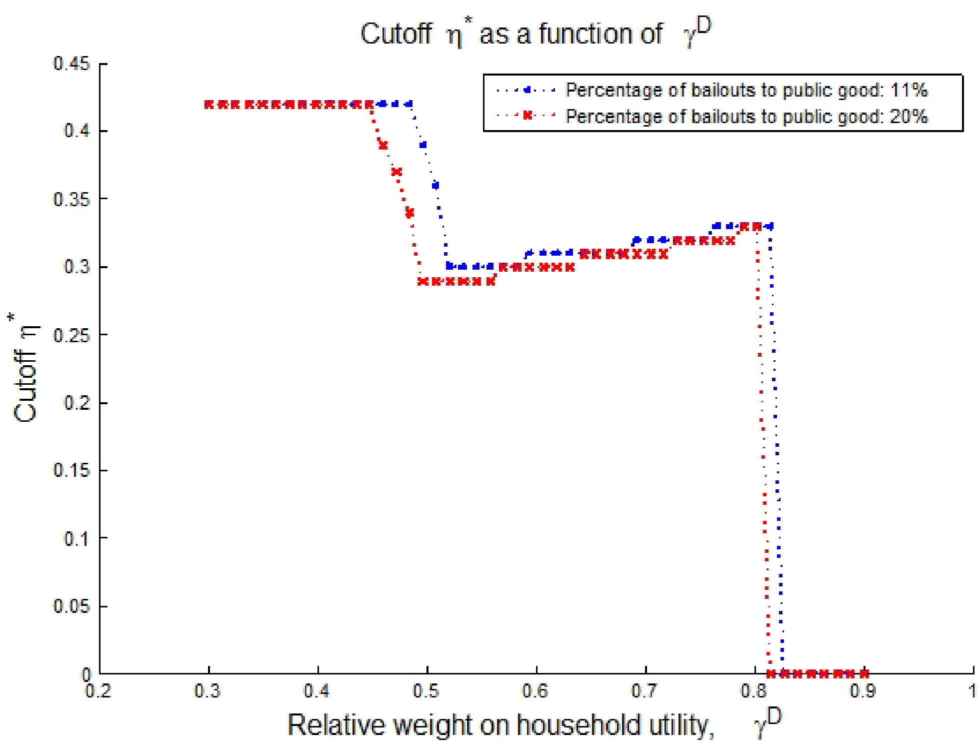

Figure 3: Cutoff η^* as function of the value of political rents

Figure 3 graphs the value of η^* for different values of γ^D, the relative weight placed by policymaker D on household utility.

A higher value of γ^D implies a lower domestic rent seeking distortion. The policymaker values household utility relatively more, so rent seeking is less of a concern. Yet, the value of η^* is not a monotone function of γ^D. This happens because here too we have two forces affecting the utility of D households. An increase in γ^D lowers rents, both with and without a partial banking union. In the partial banking union, however, a higher γ^D has two additional effects. First, the additional increase in spending on reinvestments decided at the supranational level is smaller. This happens because country D uses more funds for recapitalizations even without the partial banking union, so the need for additional spending is lower. This effect pushes η^* down, by reducing the supranational distortion. Second, the supranational authority has the incentive to allocate a higher share of the costs of reinvestments to country D. This is a redistributive effect: the utility of households in country D is higher due to γ^D being higher and they therefore face a smaller marginal cost of reinvestment spending. This redistributive pushes η^* up. These two additional effects exist only in the partial banking union and they push η^* in opposite directions. The end result is the non-monotonicity shown in Figure 3.

B Appendix B – Proofs (For Online Publication)

B.1 Proofs from Section 3

B.1.1 Proof of Proposition 1

Consider the problem for the policymaker in country i with a partial banking union $(\tau, \underline{x})$. Let $\zeta^i \equiv \{x^i, g^i, g_1^i, b_1^i\}$ be the policies chosen by policymaker i, with $i \in \{D, F\}$. Policymaker D solves

$$\max_{\zeta^D} u(c^D(x^D, x^F)) + w(g^D) + \beta w(g_1^D), \tag{18}$$

subject to

$$\begin{aligned} x^D + g^D &\leq e^D + \beta b_1^D + \tau, && (19a)\\ x^D &\geq \underline{x} && (19b)\\ g_1^D &\leq e^D - b_1^D, && (19c)\\ b_1^D &\in \left[-e^D/\beta, e^D\right], && (19d)\\ x^D &\leq \theta I^D. && (19e) \end{aligned}$$

According to Assumption 1, constraint (19e) does not bind.

Policymaker F solves

$$\max_{\zeta^F} u(c^F(x^F, x^D)) + w(g^F) + \beta w(g_1^F), \tag{20}$$

subject to

$$\begin{aligned} x^F + g^F &\leq e^F + \beta b - \tau, && (21a)\\ g_1^F &\leq e^F - b_1^F, && (21b)\\ b_1^F &\in \left[-e^F/\beta, e^F\right], && (21c)\\ x^F &\leq \theta I^F. && (21d) \end{aligned}$$

According to Assumption 1, constraint (21d) binds for policymaker F.

By the Envelope Theorem, problem (18) is strictly concave in e^D and τ, and problem (20) is strictly concave in e^F and τ.

In choosing $(\tau, \underline{x})$, the supranational authority faces the following maximization problem:

$$\max_{\tau,\underline{x}}\{\eta\left[u(c^D(x^D,x^F))+w(g^D)+\beta w(g_1^D)\right]$$
$$+(1-\eta)\left[u(c^F(x^F,x^D))+w(g^F)+\beta w(g_1^F)\right]\}$$

subject to

$$U^i(x^i,x^j,g^i,g_1^i)\geq U^i(x^{i0},x^{j0},g^{j0},g_1^{i0}), \tag{22}$$

where $i,j\in\{D,F\}$, $i\neq j$, and $\{x^{i0},g^{i0},g_1^{i0}\}$ denote the solution to policymaker $i's$ maximization problem when $\tau=0$, $\underline{x}=0$.

The supranational authority's objective function is a sum of utilities maximized in (18) and (20), so it is a strictly concave function of τ. Then, any solution to the supranational authority's problem that involves $\tau>0$ implies a strict increase in the utility of the supranational authority. Given the participation constraints of the two governments, (22), it follows that the utility of households in at least one country must increase.

B.2 Proofs from Section 4

B.2.1 Proof of Proposition 2

Assumption 1 guarantees that $\tau\leq e^F-e^{F*}$, where

$$e^{F*}=\frac{\theta I^F}{1+\beta}+g^{F*}+\frac{r^{F*}}{1+\beta},$$

with g^{F*} and r^{F*} defined in Assumption 1. This means that full recapitalizations are provided in country F ($x^F=\theta I^F$) even if transfers are made to country D.

Step 1. *The policymakers' problem*

Consider a partial banking union with terms τ and $\underline{x}$. Let be the λ^D, ϑ^D, and $\beta\mu^D$ be the Lagrange multipliers on constraints (8a), (8b), and (8c), respectively. The first-order conditions to problem (7) when constraint (8b) binds and there is an interior solution lead to

$$(1-\gamma^D)v'(r^D) = \gamma^D\sigma^D R u'(c^D), \tag{23a}$$

$$r^D + x^D = \underline{x}, \tag{23b}$$

$$g^D = g_1^D = e^D + \frac{\tau - \underline{x}}{1+\beta}. \tag{23c}$$

The maximization problem for policymaker F given $\{\tau, \underline{x}\}$ is to choose $\zeta^F = \{r^F, x^F, g^F, g_1^F, b^F\}$ to solve

$$\max_{\zeta^F} \left(1-\gamma^F\right) v(r^F) + \gamma^F \left[u(c^F(x^F, x^D)) + w(g^F) + \beta w(g_1^F)\right] \tag{24}$$

subject to

$$r^F + x^F + g^F \leq e^F + \beta b^F - \tau, \tag{25a}$$

$$g_1^F \leq e^F - b^F, \tag{25b}$$

$$b^F \in [\underline{b}^F, e^F], \tag{25c}$$

$$x^F \leq \theta I^F. \tag{25d}$$

The first-order conditions for an interior solution for r^F and g^F imply

$$(1-\gamma^F)v'(r^F) = \gamma^F w'(g^F), \tag{26a}$$

$$g_1^F = g^F, \tag{26b}$$

$$x^F = \theta I^F. \tag{26c}$$

Step 2. *The supranational authority's problem*

The supranational authority sets $\tau \geq 0$ and $\underline{x} \geq 0$ in order to maximize (1) given (2) and (3). The minimum reinvestment requirement is $r^D + x^D \geq \underline{x}$. Setting $\underline{x}$ at least equal to the policymaker's unconstrained choices is a weakly dominant strategy, so constraint (8b) holds with equality for policymaker D.

The policymaker's utility from rents r^D and recapitalizations x^D is concave and additive, so a binding $\underline{x}$ implies $r^D \geq r^{D0}$ and $x^D \geq x^{D0}$. Then $v(r^D) \geq v(r^{D0})$, and (2) is satisfied as long as

$$U^D(x^{D0}, x^{F0}, g^{D0}, g_1^{D0}) - U^D(x^D, x^F, g^D, g_1^D) \leq \frac{(1-\gamma^D)}{\gamma^D}\left[v(r^D) - v(r^{D0})\right]. \tag{27}$$

Step 3. *We show that if constraint (2) does not bind for some $\eta^C \in (0,1)$, then it does not bind $\forall \eta \geq \eta^C$.*

Assume there exists a value $\eta^C \in (0,1)$ at which (2) does not bind.

Case A. Corner solution for x^D.

If $\underline{x} = \underline{x}^* = \theta I^D + r^{D*}$, with r^{D*} defined implicitly by $\left(1-\gamma^D\right) v'(r^{D*}) = \gamma^D \sigma^D R u'(\theta I^D, \theta I^F)$. In this case, the maximum recapitalization is achieved at $\eta^C : x^D = \theta I^D$.

Let ι denote the Lagrange multiplier on constraint (3). Then, the first order-condition that determines τ is

$$\begin{aligned} w'(g^D)\frac{\partial g^D}{\partial \tau} = \frac{(1-\eta)}{\eta} w'(g^F)\left(-\frac{\partial g^F}{\partial \tau}\right)(1+\gamma^F \iota) \\ +\iota(1-\gamma^F)v'(r^F)\left(-\frac{\partial r^F}{\partial \tau}\right). \end{aligned} \tag{28}$$

Given this condition, an increase in η would increase τ, which is equivalent to increasing e^D, so

$$\frac{\partial U^D(x^D, x^F, g^D, g_1^D)}{\partial \eta} > 0, \tag{29}$$

$$r^D = r^{D*}, \tag{30}$$

and the policymaker's utility is also increasing; hence, (2) does not bind $\forall \eta \geq \eta^C$.

Case B: Interior solution for $\underline{x}$.

The first-order conditions to the supranational authority's maximization problem, in case of an internal solution $(\tau, \underline{x})$, are:

$$\begin{aligned}&\left[\frac{(1-\eta)}{\eta}\left(1-\sigma^D\right) Ru'(c^F)(1+\gamma^F\iota)\right.\\&\left.+\,\sigma^D Ru'(c^D)\right]\frac{\partial x^D}{\partial \underline{x}} = w'(g^D)\left(-\frac{\partial g^D}{\partial \underline{x}}\right)(1+\beta),\end{aligned} \tag{31}$$

$$\begin{aligned}w'(g^D)\frac{\partial g^D}{\partial \tau}(1+\beta) = \frac{(1-\eta)}{\eta}w'(g^F)\left(-\frac{\partial g^F}{\partial \tau}\right)(1+\beta)(1+\gamma^F\iota)\\+\iota(1-\gamma^F)v'(r^F)\left(-\frac{\partial r^F}{\partial \tau}\right).\end{aligned} \tag{32}$$

From (23a)-(23c), $\frac{\partial x^D}{\partial \underline{x}} > 0$, $\frac{\partial g^D}{\partial \underline{x}}(1+\beta) = -1$, $\frac{\partial g^D}{\partial \tau} = \frac{1}{(1+\beta)}$. From (26a), $0 < -\frac{\partial g^F}{\partial \tau} < 1$ and $0 < -\frac{\partial r^F}{\partial \tau} < 1$. Then, from (31) and (32) an increase in η implies $\frac{\partial \underline{x}}{\partial \eta} < 0$ and $\frac{\partial \tau}{\partial \eta} > 0$. So

$$\frac{\partial U^D(x^D, x^F, g^D, g_1^D)}{\partial \eta} > 0. \tag{33}$$

From (8b)

$$\frac{\partial V^D(x^D, x^F, g^D, g_1^D)}{\partial \underline{x}} \leq 0,$$

and

$$\frac{\partial U^D(x^D, x^F, g^D, g_1^D)}{\partial \underline{x}} \leq 0. \tag{34}$$

From (8a),

$$\frac{\partial V^D(x^D, x^F, g^D, g_1^D)}{\partial \tau} > 0,$$

and

$$\frac{\partial U^D(x^D, x^F, g^D, g_1^D)}{\partial \tau} > 0. \tag{35}$$

Then, constraint (2) does not bind for $\eta > \eta^C$.

Step 4. *We show that if for some $\eta^B \in (0,1)$ constraint (2) binds, then it binds $\forall \eta \leq \eta^B$.*

Since $\underline{x}$ is at least as high as policymaker $D's$ policy choices, $r^D \geq r^{D0}$. If

(2) binds, then τ is inferred implicitly from this constraint as

$$\gamma^D(1+\beta)w\left(e^D+\frac{\tau-\underline{x}}{1+\beta}\right)=(1-\gamma^D)v(r^{D0})+U^D(x^{D0},x^{F0},g^{D0},g^{D0})$$
$$-(1-\gamma^D)v(r^D(\underline{x}))-\gamma^D u\left(c^D(x^D(\underline{x}),\theta I^F)\right).$$

Case A. There is a corner solution for $\underline{x}=\underline{x}^*$, with $\underline{x}^*$ defined in Step 3.

A decrease in η would not change the value of $\underline{x}$ nor the value of τ. Hence, (2) binds $\forall\eta<\eta^B$.

Case B. If $\underline{x}<\underline{x}^*$.

Constraint (2) binding implies that the first-order conditions (31) and (32) become

$$\left[\frac{(1-\eta)}{\eta}\left(1-\sigma^D\right)Ru'(c^F)(1+\gamma^F\iota)+\sigma^D Ru'(c^D)\right]\frac{\partial x^D}{\partial\underline{x}}$$
$$-w'(g^D)\left(-\frac{\partial g^D}{\partial\underline{x}}\right)(1+\beta)>0, \tag{36}$$
$$w'(g^D)\frac{\partial g^D}{\partial\tau}+\frac{(1-\eta)}{\eta}w'(g^F)\frac{\partial g^F}{\partial\tau}<0. \tag{37}$$

Then, a decrease in η keeps the constraint (2) binding.

Step 5 *We show that there exists $\eta^{B*}\in(0,1)$ such that constraint (2) binds for $\eta<\eta^{B*}$ and it does not bind for $\eta>\eta^{B*}$.*

If $\eta=0$, the supranational authority maximizes the utility of the F households only, so τ is minimized and $\underline{x}$ is maximized given constraint (2). At $\eta=0$, the first-order conditions to the supranational authority's problem are given by (36) and (37). The left-hand side of (36) is strictly decreasing in η, and the left-hand side of condition (37) is strictly increasing in η. By the continuity of the utility functions it then follows that $\exists\eta^{B*}>0$ such that (36) and (37) hold with equality.

Step 6. *We show there exists $\eta^*\in(0,1)$ such that $U^D(x^{D0},x^{F0},g^{D0},g_1^{D0})=U^D(x^D,x^F,g^D,g_1^D)$.*

From Step 5, there exists $\eta^{B*}\in(0,1)$ such that constraint (2) binds. Since $r^D\geq r^{D0}$, it follows that $U^D(x^{D0},x^{F0},g^{D0},g_1^{D0})\geq U^D(x^D,x^F,g^D,g_1^D)$.

If $\eta = 1$, the supranational authority maximizes the utility of the D households, so the transfer τ will be at the maximum level at which the participation constraint for the F government holds. It then follows that $U^D(x^D, x^F, g^D, g_1^D) > U^D(x^{D0}, x^{F0}, g^{D0}, g_1^{D0})$ and $v(r^D) > v(r^{D0})$.

Given (29) in case A and (33) in case B, and the continuity of $U^D(x^D, x^F, g^D, g_1^D)$ it follows that there exists $\eta^* \in (0,1)$ such that.

$$U^D(x^{D0}, x^{F0}, g^{D0}, g_1^{D0}) - U^D(x^D(\eta^*), x^F(\eta^*), g^D(\eta^*), g_1^D(\eta^*)) = 0. \tag{38}$$

B.2.2 Proof of Corollary 1

The value of η^* satisfies

$$\begin{aligned} u(c^D(x^D(\eta^*), x^F)) + (1+\beta)w(g^D(\eta^*)) &= u(c^D(x^{D0}, x^{F0})) \\ &\quad + (1+\beta)w(g^{D0}). \end{aligned}$$

From the supranational authority's first-order condition (32), an internal solution for $(\tau, \underline{x})$ implies that

$$\eta^* = \frac{1}{1 + \frac{w'(g^D(\eta^*))}{w'(g^F(\eta^*))\left(-\frac{\partial g^F}{\partial \tau}\right)}}.$$

Define $\Delta x \equiv x^D - x^{D0}$ and let Δg^D be implicitly given by

$$\begin{aligned} u(c^D(x^{D0} + \Delta x, x^F)) + (1+\beta)w(g^{D0} - \Delta g^D) &= u(c^D(x^{D0}, x^F)) \\ &\quad + (1+\beta)w(g^{D0}). \end{aligned} \tag{39}$$

Then, $w'(g^D(\eta^*)) = w'(g^{D0} - \Delta g^D)$.

When $x^D = \theta I^D$, $\Delta x^{D,MAX} \equiv \theta I^D - x^{D0}$, and $\Delta g^{D,MAX}$ is given implicitly by

$$\begin{aligned} u(c^D(x^{D,MAX}, x^F)) + (1+\beta)w(g^{D0} - \Delta g^{D,MAX}) &= u(c^D(x^{D0}, x^{F0})) \\ &\quad + (1+\beta)w(g^{D0}) \end{aligned} \tag{40}$$

If $\tau(\eta^*) > 0$, then $g^F(\eta^*) < g^{F0}$. So

$$\begin{aligned}
\frac{w'(g^D(\eta^*))}{w'(g^F(\eta^*))\left(-\frac{\partial g^F}{\partial \tau}\right)} &\leq \frac{w'(g^{D0} - \Delta g^{D,MAX})}{w'(g^F(\eta^*))\left(-\frac{\partial g^F}{\partial \tau}\right)} \\
&\leq \frac{w'\left(g^{D0} - \frac{\Delta x^{D,MAX}}{1+\beta}\right)}{w'(g^{F0})}, \qquad (41)
\end{aligned}$$

where $\frac{\Delta x^{D,MAX}}{1+\beta} > \Delta g^{D,MAX}$ given the concavity of $u(\cdot)$ and $w(\cdot)$. Then,

$$\begin{aligned}
\frac{w'\left(g^{D0} - \frac{\Delta x^{D,MAX}}{1+\beta}\right)}{w'(g^{F0})} &\leq \frac{w'\left(g^{D0} + \frac{x^{D0}}{1+\beta} - \frac{\theta I^D}{1+\beta}\right)}{w'(e^F - \frac{\theta I^F}{1+\beta})} \\
&= \frac{w'\left(e^D - \frac{r^{D0}}{1+\beta} - \frac{\theta I^D}{1+\beta}\right)}{w'(e^F - \frac{\theta I^F}{1+\beta})}.
\end{aligned}$$

Then, from (41),

$$\frac{w'(g^D(\eta^*))}{w'(g^F(\eta^*))} \leq \frac{w'\left(e^D - \frac{r^{D0}}{1+\beta} - \frac{\theta I^D}{1+\beta}\right)}{w'(e^F - \frac{\theta I^F}{1+\beta})} = \Phi(\theta),$$

and so

$$\eta^* \geq \frac{1}{1+\Phi(\theta)}.$$

B.2.3 Proof of Corollary 2

The value η^* is defined as the value at which

$$\begin{aligned}
u(c^D(x^D(\eta^*), x^F)) + (1+\beta)w(g^D(\eta^*)) &= u(c^D(x^{D0}, x^{F0})) \\
&\quad +(1+\beta)w(g^{D0}), \qquad (42)
\end{aligned}$$

where $x^F = \theta I^F$, given Assumption 1.

The effect of increasing e^F

Case A: Corner solution with respect to x^D $\left(x^D = \theta I^D\right)$.

In this case, we have a corner solution with respect to x^D, so $\frac{\partial \underline{x}}{\partial \eta^*} = \frac{\partial \underline{x}}{\partial e^F} = 0$. Applying the Envelope Theorem in (42), we obtain

$$\frac{\partial \eta^*}{\partial e^F} = -\frac{\partial \tau}{\partial e^F} \left(\frac{\partial \tau}{\partial \eta^*} \right)^{-1}.$$

From (32), applying the Envelope Theorem, $\frac{\partial \tau}{\partial e^F} > 0$ and $\frac{\partial \tau}{\partial \eta^*} > 0$, so

$$\frac{\partial \eta^*}{\partial e^F} < 0.$$

Case B: Internal solution with respect to x^D ($x^D < \theta I^D$)
Applying the Envelope Theorem in (42), we obtain

$$\frac{\partial \eta^*}{\partial e^F} = -\frac{\Gamma}{\sigma^D R u'(c^D) \frac{\partial x^D}{\partial \underline{x}} \frac{\partial \underline{x}}{\partial \eta^*} + w'(g^D) \left(\frac{\partial \tau}{\partial \eta^*} - \frac{\partial \underline{x}}{\partial \eta^*} \right)},$$

where

$$\Gamma \equiv \sigma^D R u'(c^D) \frac{\partial x^D}{\partial \underline{x}} \frac{\partial \underline{x}}{\partial e^F} + w'(g^D) \left(\frac{\partial \tau}{\partial e^F} - \frac{\partial \underline{x}}{\partial e^F} \right).$$

From (31), applying the Envelope Theorem, $\frac{\partial \underline{x}}{\partial e^F} > 0$.
From (32), applying the Envelope Theorem, $\frac{\partial \tau}{\partial e^F} > 0$ and $\frac{\partial g^D}{\partial e^F} > 0$, so

$$\frac{\partial \tau}{\partial e^F} - \frac{\partial \underline{x}}{\partial e^F} > 0.$$

Also, from (42), at η^*,

$$\sigma^D R u'(c^D) \frac{\partial x^D}{\partial \underline{x}} = w'(g^D). \tag{43}$$

This implies

$$\sigma^D R u'(c^D) \frac{\partial x^D}{\partial \underline{x}} \frac{\partial \underline{x}}{\partial \eta^*} + w'(g^D) \left(\frac{\partial \tau}{\partial \eta^*} - \frac{\partial \underline{x}}{\partial \eta^*} \right) = w'(g^D) \frac{\partial \tau}{\partial \eta^*},$$

and applying the Envelope Theorem in (31) and (32), $\frac{\partial \tau}{\partial \eta^*} > 0$. So

$$w'(g^D)\frac{\partial \tau}{\partial \eta^*} > 0. \tag{44}$$

Also, (43) implies

$$\Gamma = w'(g^D)\frac{\partial \tau}{\partial e^F} > 0. \tag{45}$$

Then, (44) and (45) imply

$$\frac{\partial \eta^*}{\partial e^F} < 0.$$

The effect of increasing γ^F

Case A: Corner solution with respect to x^D $\left(x^D = \theta I^D\right)$.

In this case, the corner solution implies $\frac{\partial \underline{x}}{\partial \eta^*} = \frac{\partial \underline{x}}{\partial \gamma^F} = 0$. Applying the Envelope Theorem in (42), we obtain

$$\frac{\partial \eta^*}{\partial \gamma^F} = -\frac{\partial \tau}{\partial \gamma^F}\left(\frac{\partial \tau}{\partial \eta^*}\right)^{-1}.$$

From (32), applying the Envelope Theorem, $\frac{\partial \tau}{\partial \gamma^F} > 0$ and $\frac{\partial \tau}{\partial \eta^*} > 0$, so

$$\frac{\partial \eta^*}{\partial \gamma^F} < 0.$$

Case B: Internal solution with respect to x^D $(x^D < \theta I^D)$

As above, applying the Envelope Theorem in (42), we obtain

$$\frac{\partial \eta^*}{\partial \gamma^F} = -\frac{\Xi}{\sigma^D R u'(c^D)\frac{\partial x^D}{\partial \underline{x}}\frac{\partial \underline{x}}{\partial \eta^*} + w'(g^D)\left(\frac{\partial \tau}{\partial \eta^*} - \frac{\partial \underline{x}}{\partial \eta^*}\right)}, \tag{46}$$

where

$$\Xi \equiv \sigma^D R u'(c^D)\frac{\partial x^D}{\partial \underline{x}}\frac{\partial \underline{x}}{\partial \gamma^F} + w'(g^D)\left(\frac{\partial \tau}{\partial \gamma^F} - \frac{\partial \underline{x}}{\partial \gamma^F}\right).$$

From (43), the above (46) can be simplified to

$$\frac{\partial \eta^*}{\partial \gamma^F} = -\frac{\partial \tau}{\partial \gamma^F}\left(\frac{\partial \tau}{\partial \eta^*}\right)^{-1}.$$

From (31) and (32),

$$\frac{\partial \tau}{\partial \gamma^F} > 0,$$

so

$$\frac{\partial \eta^*}{\partial \gamma^F} < 0.$$

B.2.4 Proof of Corollary 3

The value η^* is defined in (42) above. The effect of a change in $\left(-\alpha^F\right)$ on σ^D and σ^F is given by

$$\begin{aligned}\frac{\partial \sigma^D}{\partial\left(-\alpha^F\right)} &= -\frac{\alpha^D z^D z^F}{\left[\alpha^D z^D + (1-\alpha^F) z^F\right]^2} = -\frac{\sigma^D z^F}{I^D}, \\ \frac{\partial \sigma^F}{\partial\left(-\alpha^F\right)} &= \frac{-z^F (1-\alpha^D) z^D}{\left[\alpha^F z^F + (1-\alpha^D) z^D\right]^2} = -\frac{(1-\sigma^F) z^F}{I^F}.\end{aligned}$$

Applying the Envelope Theorem in (42), and using the above expressions, we obtain

$$\frac{\partial \eta^*}{\partial\left(-\alpha^F\right)} = \frac{\Psi}{\sigma^D R u'(c^D)\frac{\partial x^D}{\partial \underline{x}}\frac{\partial \underline{x}}{\partial \eta^*} + w'(g^D)\left(\frac{\partial \tau}{\partial \eta^*} - \frac{\partial \underline{x}}{\partial \eta^*}\right)}, \tag{47}$$

where

$$\begin{aligned}
\Psi \equiv\ & \sigma^D R u'(c^{D0}) \left(-\frac{z^F}{I^D} x^{D0} + \frac{\partial x^{D0}}{\partial(-\alpha^F)} + \frac{(1-\sigma^F)}{\sigma^D}\theta z^F \right) \\
& -\sigma^D R u'(c^D) \left(-\frac{z^F}{I^D} x^D + \frac{\partial x^D}{\partial(-\alpha^F)} + \frac{(1-\sigma^F)}{\sigma^D}\theta z^F \right) \\
& +(1+\beta) w'(g^{D0}) \frac{\partial g^{D0}}{\partial(-\alpha^F)} \\
& -w'(g^D) \left(\frac{\partial \tau}{\partial(-\alpha^F)} - \frac{\partial \underline{x}}{\partial(-\alpha^F)} \right) \\
& -\sigma^D R u'(c^D) \frac{\partial x^D}{\partial \underline{x}} \frac{\partial \underline{x}}{\partial(-\alpha^F)}.
\end{aligned}$$

Without a partial banking union, the first-order conditions to the policymaker's problem (7) without a banking union lead to

$$(1-\gamma^D) v'(r^{D0}) = \sigma^D R u'(c^{D0}), \tag{48}$$

$$(1-\gamma^D) v'(r^{D0}) = \gamma^D w'(g^{D0}). \tag{49}$$

Applying the Envelope Theorem in (48) and (49),

$$\begin{aligned}
(1-\gamma^D) v''(r^{D0}) \frac{\partial r^{D0}}{\partial(-\alpha^F)} &= \gamma^D \left(\sigma^D R\right)^2 u''(c^{D0}) \left(-\frac{z^F}{I^D} x^{D0} + \frac{\partial x^{D0}}{\partial(-\alpha^F)} \right. \\
& \quad \left. + \frac{(1-\sigma^F)\theta z^F}{\sigma^D} \right), \\
(1-\gamma^D) v''(r^{D0}) \frac{\partial r^{D0}}{\partial(-\alpha^F)} &= \gamma^D w''(g^{D0}) \frac{\partial g^{D0}}{\partial(-\alpha^F)}.
\end{aligned}$$

From the budget constraint in country D :

$$\frac{\partial r^{D0}}{\partial(-\alpha^F)} + (1+\beta) \frac{\partial g^{D0}}{\partial(-\alpha^F)} + \frac{\partial x^{D0}}{\partial(-\alpha^F)} = 0.$$

From the above conditions, we derive

$$\frac{\partial r^{D0}}{\partial(-\alpha^F)} = \frac{z^F}{\Lambda^0} \left(\frac{1-\sigma^F}{\sigma^D}\theta - \frac{x^{D0}}{I^D} \right), \tag{50}$$

where

$$\Lambda^0 \equiv 1 + (1+\beta)\frac{(1-\gamma^D)v''(r^{D0})}{\gamma^D w''(g^{D0})} + \frac{(1-\gamma^D)v''(r^{D0})}{\gamma^D\left(\sigma^D R\right)^2 u''(c^{D0})}.$$

With a partial banking union:

Case A: Corner solution with respect to x^D ($x^D = \theta I^D$)

When the supranational authority's problem gives the corner solution $x^D = \theta I^D$, $\frac{dx^D}{d(-\alpha^F)} = 0$.

If $\alpha^D + \alpha^F \leq 1$, then $(1-\sigma^F)/\sigma^D > 1$, and so

$$\frac{(1-\sigma^F)}{\sigma^D} > 1.$$

Since $\frac{\partial x^D}{\partial(-\alpha^F)} < 0$, $\frac{\partial \underline{x}}{\partial(-\alpha^F)} > 0$. Applying the Envelope Theorem to the supranational authority's problem then leads to

$$\frac{\partial \tau}{\partial\left(-\alpha^F\right)} > 0. \tag{51}$$

Using (48), (49) and (50), Ψ can be simplified to

$$\begin{aligned}\Psi \quad = \quad & \sigma^D R u'(c^{D0})\left(\frac{1-\sigma^F}{\sigma^D}\theta - \frac{x^{D0}}{I^D}\right)\left(1-\frac{1}{\Lambda^0}\right) \\ & -\sigma^D R u'(c^D)\theta z^F\left(\frac{(1-\sigma^F)}{\sigma^D}-1\right) \\ & -w'(g^D)\left(\frac{\partial \tau}{\partial\left(-\alpha^F\right)} - \frac{\partial \underline{x}}{\partial\left(-\alpha^F\right)}\right).\end{aligned}$$

The upper bound on $\frac{\partial \tau}{\partial(-\alpha^F)} - \frac{\partial \underline{x}}{\partial(-\alpha^F)}$ is 0, so a sufficient condition for $\Psi \geq 0$ is

$$\frac{u'(c^D)\left(\frac{(1-\sigma^F)}{\sigma^D}-1\right)}{u'(c^{D0})\left(\frac{1-\sigma^F}{\sigma^D}\theta - \frac{x^{D0}}{I^D}\right)} \leq 1 - \frac{1}{\Lambda^0}. \tag{52}$$

Notice that $u'(c^{D0})\left(\frac{1-\sigma^F}{\sigma^D} - \frac{x^{D0}}{\theta I^D}\right)$ is a decreasing function of e^D. Therefore, condition (56) is satisfied if e^F is sufficiently large relative to e^D, such that

$u'(c^{D0})\left(\frac{1-\sigma^F}{\sigma^D}-\frac{x^{D0}}{\theta I^D}\right)$ is sufficiently small and $x^D=\theta I^D$.

Case B: Internal solution with respect to x^D ($x^D<\theta I^D$)

From (23a)

$$\begin{aligned}(1-\gamma^D)v''(r^D)\frac{\partial r^D}{\partial\left(-\alpha^F\right)} &= \gamma^D\left(\sigma^D R\right)^2 u''(c^D)\left(-\frac{z^F}{I^D}x^D+\frac{\partial x^D}{\partial\left(-\alpha^F\right)}\right.\\ &\quad \left.+\frac{(1-\sigma^F)\theta z^F}{\sigma^D}\right),\end{aligned}$$

and from condition (23b),

$$\frac{\partial r^D}{\partial\left(-\alpha^F\right)}+\frac{\partial x^D}{\partial\left(-\alpha^F\right)}=0,$$

so

$$\frac{\partial x^D}{\partial\left(-\alpha^F\right)}=\frac{1}{\Lambda}\left(\frac{z^F}{I^D}x^D-\frac{(1-\sigma^F)\theta z^F}{\sigma^D}\right), \tag{53}$$

where

$$\Lambda=1+\frac{(1-\gamma^D)v''(r^D)}{\gamma^D\left(\sigma^D R\right)^2 u''(c^D)}.$$

Finally, from (23a) and (23b),

$$\frac{\partial x^D}{\partial \underline{x}}=\frac{1}{\frac{\gamma^D(\sigma^D R)^2 u''(c^D)}{(1-\gamma^D)v''(r^D)}+1}=1-\frac{1}{\Lambda}.$$

If $\alpha^D+\alpha^F\leq 1$, then $(1-\sigma^F)/\sigma^D>1$, and so

$$\frac{(1-\sigma^F)}{\sigma^D}\theta-\frac{x^D}{I^D}>0,\ \forall x^D<\theta I^D.$$

In this case, in (53), $\frac{\partial x^D}{\partial(-\alpha^F)}<0$; applying the Envelope Theorem to the supranational authority's problem then leads to

$$\frac{\partial\tau}{\partial\left(-\alpha^F\right)}>0. \tag{54}$$

Notice that the upper bound on $\frac{\partial\tau}{\partial(-\alpha^F)}$ is given by the case in which the supranational authority sets $\frac{\partial\tau}{\partial(-\alpha^F)} = \frac{\partial\underline{x}}{\partial(-\alpha^F)}$. In this case, the first-order condition to the supranational authority's problem is

$$\begin{aligned}&\left[(1-\eta)\left(1-\sigma^D\right)Ru'(c^F)\right.\\&\left.+\eta\sigma^D Ru'(c^D)\right]\frac{\partial x^D}{\partial\underline{x}} = (1-\eta)w'(g^F)\left(-\frac{\partial g^F}{\partial\tau}\right)(1+\beta).\end{aligned}$$

The total effect of the change in $\left(-\alpha^F\right)$ on x^D is negative, so

$$\frac{dx^D}{d\left(-\alpha^F\right)} = \frac{\partial x^D}{\partial\left(-\alpha^F\right)} + \frac{\partial x^D}{\partial\underline{x}}\frac{\partial\underline{x}}{\partial\left(-\alpha^F\right)} < 0.$$

Then, the upper bound on $\frac{\partial\tau}{\partial(-\alpha^F)}$ is

$$\frac{\partial\tau}{\partial\left(-\alpha^F\right)} < -\frac{\partial x^D}{\partial\left(-\alpha^F\right)}\left(\frac{\partial x^D}{\partial\underline{x}}\right)^{-1}. \tag{55}$$

From (43), (48), (54), and (55),

$$\begin{aligned}\Psi \;\geq\; & \sigma^D Ru'(c^{D0})z^F\left(\frac{1-\sigma^F}{\sigma^D}\theta - \frac{x^{D0}}{I^D}\right)\left(1-\frac{1}{\Lambda^0}\right)\\ & -\sigma^D Ru'(c^D)z^F\left(\frac{1-\sigma^F}{\sigma^D}\theta - \frac{x^D}{I^D}\right)\end{aligned}$$

A sufficient condition for $\Psi \geq 0$ is then that

$$\frac{u'(c^D)\left(\frac{1-\sigma^F}{\sigma^D} - \frac{x^D}{\theta I^D}\right)}{u'(c^{D0})\left(\frac{1-\sigma^F}{\sigma^D} - \frac{x^{D0}}{\theta I^D}\right)} \leq 1 - \frac{1}{\Lambda^0}, \tag{56}$$

which is satisfied as long as $u'(c^D)\left(\frac{1-\sigma^F}{\sigma^D} - \frac{x^D}{\theta I^D}\right)$ is sufficiently lower than $u'(c^{D0})\left(\frac{1-\sigma^F}{\sigma^D} - \frac{x^{D0}}{\theta I^D}\right)$.

Notice that $u'(c^{D0})\left(\frac{1-\sigma^F}{\sigma^D} - \frac{x^{D0}}{\theta I^D}\right)$ is a decreasing function of e^D. Also, the expression $u'(c^D)\left(\frac{1-\sigma^F}{\sigma^D} - \frac{x^D}{\theta I^D}\right)$ is a decreasing function of $\underline{x}$. From (31) and (32), $\underline{x}$ is an increasing function of e^F. Therefore, condition (56) is satisfied if

e^F is sufficiently large relative to e^D.

B.3 Proofs from Section 5

B.3.1 Proof of Proposition 3

Step 1. If $\bar{b}^{D*}(\theta)$ is a solution to the maximization problem (11), then $\bar{b}^{D*}(\theta) < e^D$.

Denote by $\overline{U^D}(\bar{b}^D)$ the value of the D household utility given the solution to (11) subject to (12a)-(12d). If constraint (12d) is not binding, then

$$\frac{\partial \overline{U^D}(\bar{b}^D)}{\partial \bar{b}^D} \leq 0.$$

Consider the case when constraint (12d) binds. Then, $\overline{U^D}(\bar{b}^D)$ is a continuous and differentiable function of $\bar{b}^D$, since $u(c^D)$ and $w(g^D)$ are continuously differentiable functions of $\bar{b}^D$. Also, policymaker D's indirect utility function is given by

$$\overline{V^D}(\bar{b}^D) = (1-\gamma^D)v(r^{D0}) + \gamma^D \overline{U^D}(\bar{b}^D),$$

and

$$\begin{aligned} \overline{V^D}(\bar{b}^D) &= \max_{\{r^D, x^D, g^D\}} (1-\gamma^D)v(r^{D0}) + \gamma^D u(c^D(x^{D0}, x^{F0})) \\ &\quad + \gamma^D w(g^{D0}) + \gamma^D \beta w(e^D - \bar{b}^D) \end{aligned} \tag{57}$$

subject to

$$r^{D0} + x^{D0} + g^{D0} \leq e^D + \beta \bar{b}^D$$

Then, the change in household utility due to the change in the binding debt limit $\bar{b}$ is given by

$$\begin{aligned} \frac{\partial \overline{U^D}(\bar{b}^D)}{\partial \bar{b}^D} &= \sigma^D R u'(c^D(x^{D0}, x^{F0})) \frac{\partial x^{D0}}{\partial \bar{b}^D} \\ &\quad + w'(g^{D0}) \frac{\partial g^{D0}}{\partial \bar{b}^D} - \beta w'(e^D - \bar{b}^D). \end{aligned} \tag{58}$$

The first order-conditions to problem (57) give

$$\begin{aligned} \gamma^D \sigma^D R u'(c^D(x^{D0}, x^{F0})) &= (1-\gamma^D) v'(r^{D0}), \\ \gamma^D w'(g^{D0}) &= (1-\gamma^D) v'(r^{D0}). \end{aligned}$$

Therefore,

$$\frac{\partial r^{D0}}{\partial \bar{b}^D} = \beta \Psi^{-1}, \tag{59}$$

where

$$\Psi \equiv 1 + \frac{1-\gamma^D}{\gamma^D} \frac{v''(r^{D0})}{w''(g^{D0})} + \frac{1-\gamma^D}{\gamma^D} \frac{v''(r^{D0})}{(\sigma^D R)^2 u''(c^{D0})}.$$

From (59), $\frac{\partial r^D}{\partial \bar{b}} > 0$. Re-writing (58) as

$$\frac{\partial \overline{U^D}(\bar{b}^D)}{\partial \bar{b}^D} = w'(g^{D0}) \left(-\frac{\partial r^D}{\partial \bar{b}} \right) - \beta w'(e^D - \bar{b}^D),$$

it follows that

$$\left. \frac{\partial \overline{U^D}(\bar{b}^D)}{\partial \bar{b}^D} \right|_{\bar{b}^D = e^D - g^{D0}} < 0.$$

Therefore, at the optimal level, $\bar{b}^* \leq b^{D0} = e^D - g^{D0}$, so the debt constraint is binding.

Step 2. If $\bar{b}^{F*}(\theta)$ is a solution to the maximization problem for the F country, then $\bar{b}^{F*}(\theta) \leq e^F$. The problem for country F is analogous to the problem for country D. Hence the function $\overline{U^F}(\bar{b}^F)$ is maximized at $\bar{b}^{F*} \leq e^F - g^{F0}$.

B.3.2 Proof of Lemma 1

Consider the rents r^{F*} and public good g^{F*} defined implicitly by:

$$\begin{aligned} w'(g^{F*}) &= \sigma^F R u'(c^F(\theta I^F, \theta I^D)), & (60) \\ (1-\gamma^F) v'(r^{F*}) &= \gamma^F \sigma^F R u'(c^F(\theta I^F, \theta I^D)). & (61) \end{aligned}$$

Let $\bar{b}^{F*}(\gamma^F) = \beta^{-1}\left(-e^F + \left(\theta I^F + r^{F*}(\gamma^F) + g^{F*}\right)\right)$. Then, by construction, policymaker F's maximization problem yields solutions $\left\{r^{F*}, g^{F*}, \theta I^F\right\}$. The rule $\bar{b}^{F*}(\gamma^F)$ gives the minimum budget in period 0 needed to obtain $x^F = \theta I^F$ when $x^D = \theta I^H$. A fiscal limit $\bar{b}^F > \bar{b}^{F*}$ is preferred by the F households if

$$w'(g^{F*})\frac{\partial g^{F*}}{\partial \bar{b}^{F*}} - \beta w'(g_1^{F*}) \geq 0, \tag{62}$$

where

$$g_1^{F*} \equiv e^F - \bar{b}^{F*}(\gamma^F).$$

From (60) and (61),

$$\begin{aligned} \gamma^F w'(g^{F*}) &= (1-\gamma^F)v'(r^{F*}), \\ \frac{\partial g^{F*}}{\partial \gamma^F} &= 0. \end{aligned}$$

and applying the Envelope Theorem in policymaker F's problem, we obtain

$$\frac{\partial r^{F*}}{\partial \gamma^F} = \frac{w'(g^{F*}) + v'(r^{F*})}{(1-\gamma^F)v''(r^{F*})} < 0.$$

The effect of increasing γ^F in (62) is given by

$$w''(g_1^{F*})\frac{\partial r^{F*}}{\partial \gamma^F} > 0. \tag{63}$$

For $\gamma^F \to 1$, $r^{F*} \to 0$, so (62) holds since $w'(g^{F*}) > w'(g_1^{F*})$ for any binding fiscal rule. Then, $\exists \overline{\gamma^F} < 1$, such that (62) is satisfied $\forall \gamma^F > \overline{\gamma^F}$.

B.3.3 Proof of Proposition 4

The existence of a binding fiscal rule only changes the equilibrium policies $\left\{r^D, x^D, g^D, g_1^D\right\}$ coming out of policymaker D's constrained maximization problem. It does not change the problem for the supranational authority.

Step 1. *The policymakers' problem*

With a binding debt limit $\bar{b}^D(\theta)$ in country D, the policymaker's problem is

$$\max_{\{x^D,g^D,r^D,b^D\}} \left(1-\gamma^H\right) v(r^D) + \gamma^H \left[u(c^D(x^D,x^F)) \right.$$
$$\left. + w(g^D) + \beta w(e^D - b^D)\right]$$

subject to

$$\begin{aligned} r^D + x^D + g^D &\leq e^D + \beta b^D + \tau, \\ r^D + x^D &\geq \underline{x} \\ b^D &\leq \bar{b}^D(\theta). \end{aligned}$$

The first-order conditions with a binding rule $\underline{x}$ lead to

$$\begin{aligned} \gamma^D R \sigma^D u'(c^D) &= (1-\gamma^D) v'(r^D), \\ r^D + x^D &= \underline{x}, \\ g^D &= e^D + \beta b^D - \underline{x}. \end{aligned}$$

The maximization problem for policymaker F facing debt limit $\bar{b}^F$ is

$$\max_{\{x^F,g^F,r^F,b^F\}} \left(1-\gamma^F\right) v(r^F) + \gamma^F \left[u(c^F(x^F,x^D)) + w(g^F) + \beta w(e^D - b^F)\right]$$

subject to

$$\begin{aligned} r^F + x^F + g^F &\leq e^F + \beta b^F - \tau, \\ g_1^F &\leq e^F - b^F, \\ b^F &\leq \bar{b}^F, \\ x^F &\leq \theta I^F. \end{aligned}$$

Therefore, the above conditions (together with Assumption 1) imply

$$\begin{aligned} (1-\gamma^F) v'(r^F) &= \gamma^F w'(g^F), \\ x^F &= \theta I^F. \end{aligned}$$

Steps 2-4 are analogous to the proof of Proposition 2**.**

It then follows that there exists $\eta^{**} \in (0,1)$ such that.

$$U^D(\theta, \bar{b}^D, 0, 0) - U^D(\theta, \bar{b}^D, \tau, \underline{x}) = 0.$$

B.3.4 Proof of Corollary 4

Let $(\tau, \underline{x})$ denote the equilibrium policy chosen by the supranational authority without fiscal rules, and by $(\tau^{FR}, \underline{x}^{FR})$ the equilibrium policy chosen by the supranational authority with fiscal rule $\bar{b}^D$ in country D and fiscal rule $\bar{b}^F$ in country F.

Consider a fiscal rule in country D that sets a binding debt limit $\bar{b}^D$. From the first-order conditions to the D government's problem, it follows that

$$\frac{\partial g^D}{\partial \bar{b}^D} > 0.$$

Then, a decrease in debt from the non-binding value b^D to $\bar{b}^D$ in first-order condition (32) implies an increase in τ to some $\tau^{FR} > \tau$.

From condition (31) it follows that $\underline{x}^{FR} < \underline{x}$. Given the D government's first-order conditions, then

$$x^D(\underline{x}^{FR}) < x^D(\underline{x})$$

and

$$g^F(\tau^{FR}, \underline{x}^{FR}) < g^F(\tau, \underline{x}).$$

Therefore, the utility of the Financing households is given by

$$\begin{aligned} U^F &= u(c^F(x^F, x^D(\underline{x}^{FR}))) + w(g^F(\tau^{FR}, \underline{x}^{FR})) \\ &\quad + \beta w(g_1^F(\tau^{FR}, \underline{x}^{FR})). \end{aligned}$$

From policymaker F's problem, $u^F(c^F)$ is an increasing function of x^D, $g^F = g_1^F$, and $w(g^F)$ is an increasing function of g^F. Then, U^F decreases if debt in country D is limited to $\bar{b}^D$.

B.3.5 Proof of Corollary 5

Consider a fiscal rule that sets a binding debt limit of $\bar{b}^D$, lower than b^{D0}, the debt chosen by the D government in the equilibrium without fiscal rules. Denote by $\left(r^D, x^D, g^D, g_1^D\right)$ the policies chosen by the D government given $(\tau, \underline{x})$ and no fiscal rules, by $\left(r^{D0}, x^{D0}, g^{D0}, g_1^{D0}\right)$ are the policies chosen by the D government without a banking union and without fiscal rules, by $\left(\overline{r^D}, \overline{x^D}, \overline{g^D}, \overline{g_1^D}\right)$ the policies chosen by the D government given policies $\left(\tau^{FR}, \underline{x}^{FR}\right)$ and fiscal rules $\left(\bar{b}^D, \bar{b}^F\right)$, and by $\left(\overline{r^{D0}}, \overline{x^{D0}}, \overline{g^{D0}}, \overline{g_1^{D0}}\right)$ the policies chosen by the D government without a banking union, but under fiscal rule $\bar{b}^D$ in country D.

From the proof to Proposition 2, Step 5, without fiscal rules, $\exists \eta^{B*} < \eta^*$ such that the participation constraint for policymaker D binds $\forall \eta < \eta^{B*}$. Given proof to Proposition 4, which is analogous to that of Proposition 2, $\exists \overline{\eta^{B*}} < \eta^{**}$, such that the participation constraint for policymaker D binds $\forall \eta < \overline{\eta^{B*}}$. Let $\widetilde{\eta^B} = \min\{\eta^{B*}, \overline{\eta^{B*}}\}$. Then, $\forall\ \eta < \widetilde{\eta^B}$,

$$(1-\gamma^D)v(r^D) + \gamma^D U^D(x^D, x^F, g^D, g_1^D) = (1-\gamma^D)v(r^{D0}) + \gamma^D U^D(x^{D0}, x^{F0}, g^{D0}, g_1^{D0}) \quad (65)$$

and

$$(1-\gamma^D)v(\overline{r^D}) + \gamma^D U^D(\overline{x^D}, \overline{x^F}, \overline{g^D}, \overline{g_1^D}) = (1-\gamma^D)v(\overline{r^{D0}}) + \gamma^D U^D(\overline{x^{D0}}, \overline{x^{F0}}, \overline{g^{D0}}, \overline{g_1^{D0}}) \quad (66)$$

where by Assumption 1 and the condition that $\gamma^F \geq \overline{\gamma^F}$ (described in Lemma 2), $x^F = x^{F0} = \overline{x^F} = \overline{x^{F0}} = \theta I^F$.

A binding fiscal rule decreases the outside option for the D government, since for $\bar{b}^D < b^{D0}$,

$$(1-\gamma^D)v(r^{D0}) + \gamma^D U^D(x^{D0}, x^{F0}, g^{D0}, g_1^{D0}) > (1-\gamma^D)v(\overline{r^{D0}}) + \gamma^D U^D(\overline{x^{D0}}, \overline{x^{F0}}, \overline{g^{D0}}, \overline{g_1^{D0}}).$$

Moreover, since the fiscal rules maximize D household utility,

$$U^D(\overline{x^{D0}}, \overline{x^{F0}}, \overline{g^{D0}}, \overline{g_1^{D0}}) > U^D(x^{D0}, x^{F0}, g^{D0}, g_1^{D0}).$$

Consider the case in which $\underline{x}^{FR} = \underline{x}^* = \theta I^D + r^{D*}$, with r^{D*} defined implicitly by $\left(1 - \gamma^D\right) v'(r^{D*}) = \gamma^D \sigma^D R u'(\theta I^D, \theta I^F)$. In this case, the maximum recapitalization is achieved, so $x^D = \overline{x^D} = \theta I^D$. Conditions (65) and (66), together with $v(r^D) = v(\overline{r^D})$ lead to

$$\begin{aligned} U^D(\overline{x^{D0}}, \overline{x^{F0}}, \overline{g^{D0}}, \overline{g_1^{D0}}) - U^D(\overline{x^D}, \overline{x^F}, \overline{g^D}, \overline{g_1^D}) \quad > \quad & U^D(x^{D0}, x^{F0}, g^{D0}, g_1^{D0}) \\ & - U^D(x^D, x^F, g^D, g_1^D). \end{aligned}$$

B.3.6 Proof of Proposition 5

Consider the case in which $\eta \leq \eta^*$, with η^* defined in Proposition 2.

Denote by $(\tau, \underline{x})$ the equilibrium supranational policy without fiscal rules. Consider introducing a fiscal rule $\overline{B}^D > b^D$ (i.e., that binds at $(\tau, \underline{x},)$). Denote by $(\tau^{FR}, \underline{x}^{FR})$ the optimal policy for the supranational authority when the fiscal rule is $\overline{B}^D$. Also, denote by (r^D, x^D, g^D, g_1^D) policymaker $D's$ utility maximizing policy choices under $(\tau, \underline{x})$ and no fiscal rules, and by g^F the public good provision in country F. Denote by $(\overline{r^D}, \overline{x^D}, \overline{g^D}, \overline{g_1^D})$ the policy choices in country D under $(\tau^{FR}, \underline{x}^{FR})$ and fiscal rule $\overline{B}^D$, and by $\overline{g^F}$ the public good provision in country F.

Without the first rule, the first-order condition (32) implies that with an internal solution for $(\tau, \underline{x})$,

$$\eta w'(g_1^D) = (1 - \eta)\, w'(g^F) \frac{\partial g^F}{\partial \tau}.$$

With a binding fiscal rule $\overline{B}^D$,

$$\eta w'(\overline{g^D}) = (1 - \eta)\, w'(\overline{g^F}) \frac{\partial \overline{g^F}}{\partial \tau},$$

but $w'(\overline{g_1^D}) < w'(g_1^D)$, so condition (32) becomes

$$\eta w'(\overline{g_1^D}) < (1-\eta)\, w'(\overline{g^F})\frac{\partial \overline{g^F}}{\partial \tau}.$$

Then, reducing $\overline{B}^D$ while keeping $(\tau, \underline{x},)$ constant increases the utility of the supranational authority. Since this holds true for all $\overline{B}^D > b^D$ and all $(\tau, \underline{x},)$, it implies that the supranational authority is maximized at $\overline{B}^D = b^D$.

B.3.7 Proof of Proposition 6

Let $b^D(\eta^*)$ denote the equilibrium debt at η^* under the partial banking union. Consider a fiscal rule that marginally decreases the debt $b^D : \bar{b}^D = b^D - \varepsilon$, where $\varepsilon \to 0$.

At $\bar{b}^D$

$$\sigma^D R u'(c^D(x^{D0}, x^{F0}))\frac{\partial x^{D0}}{\partial \bar{b}^D} + w'(g^{D0})\frac{\partial g^{D0}}{\partial \bar{b}^D} - \beta w'(g_1^{D0}) = 0.$$

By the Envelope Theorem,

$$\frac{\partial \eta^*}{\partial \bar{b}^D} = -\frac{\sigma^D R u'(c^D)\frac{\partial x}{\partial \underline{x}}\frac{\partial \underline{x}}{\partial \bar{b}^D} + w'(g^D)\left(\beta + \frac{\partial \tau}{\partial \bar{b}^D} - \frac{\partial \underline{x}}{\partial \bar{b}^D}\right) - \beta w'(g_1^D)}{\sigma^D R u'(c^D)\frac{\partial x}{\partial \underline{x}}\frac{\partial \underline{x}}{\partial \eta^*} + w'(g^D)\left(\frac{\partial \tau}{\partial \eta^*} - \frac{\partial \underline{x}}{\partial \eta^*}\right)}.$$

From the first-order conditions to the supranational authority's problem, (31) and (32), $\frac{\partial \underline{x}}{\partial \eta^*} < 0$, $\frac{\partial \tau}{\partial \eta^*} > 0$, $\frac{\partial \tau}{\partial \bar{b}^D} < 0$ and $\frac{\partial \underline{x}}{\partial \bar{b}^D} > 0$.

As shown in the proof to Corollary 2, at $\eta^*, \sigma^D R u'(c^D)\frac{\partial x}{\partial \underline{x}} = w'(g^D)$.

So the above expression simplifies to

$$\frac{\partial \eta^*}{\partial \bar{b}^D} = -\frac{\frac{\partial \tau}{\partial \bar{b}^D} + \beta\left(1 - \frac{w'(g_1^D)}{w'(g^D)}\right)}{\frac{\partial \tau}{\partial \eta^*}}.$$

Since $\frac{w'(g_1^D)}{w'(g^D)} \to 1$ as $\varepsilon \to 0$, it follows that

$$\frac{\partial \eta^*}{\partial \bar{b}^D} > 0.$$

Therefore, there exists a fiscal rule $\bar{b}^D$ that marginal decreases debt and leads to a decrease η^*.

With $\bar{b}^D$, the problem for the supranational authority is the same as under domestic fiscal rules,the only difference being the outside option of policymaker D :

$$V^{D0} = \left(1-\gamma^D\right) v(r^{D0}) + \gamma^D u(c^D(x^{D0}, \theta I^F)) + (1+\beta)\,\gamma^D w(g^{D0}).$$

Then, the same steps as in the proof of Proposition 2 show that $\forall \eta \geq \widetilde{\eta}^*$,

$$u(c^D(x^D(\eta), x^F)) + w(g^D(\eta)) + \beta w(g_1^D(\eta)) \geq u(c^D(x^{D0}, x^{F0})) + w(g^{D0}) + \beta w(g^{D0}).$$

B.3.8 Proof of Lemma 2

Consider the case in which $\bar{b}^D = -\frac{e^D}{\beta}$. In this case, a partial banking union with positive transfers is the solution to the following system of equations, with unknowns $\{x^{D,MIN}, r^{D,MIN}, r^F, g^F, \tau\}$:

- the first order-condition to the supranational authority's maximization problem (1):

$$[\eta\sigma^D R u'(c^D(x^{D,MIN}, \theta I^F)) + (1-\eta)\left(1-\sigma^D\right) R u'(c^D(x^{D,MIN}, \theta I^F))]\frac{\partial x^{D,MIN}}{\partial \tau} = (1-\eta) w'(g^F)\frac{\partial g^F}{\partial \tau}$$

- the first order-conditions to policymaker D's problem and policymaker

F's problem:

$$\begin{aligned} \left(1-\gamma^{D}\right) v'(r^{D,MIN}) &= \gamma^{D}\sigma^{D}Ru'(c^{D}(x^{D,MIN},\theta I^{F})) \\ \left(1-\gamma^{F}\right) v'(r^{F}) &= \gamma^{F}w'(g^{F}) \end{aligned}$$

- the result of applying the Envelope Theorem in the first-order conditions to the policymakers' problem:

$$\frac{\partial x^{D,MIN}}{\partial \tau} = \frac{\left(1-\gamma^{D}\right) v''(r^{D,MIN})}{\gamma^{D}\left(\sigma^{D}R\right)^{2} u''(c^{D}(x^{D,MIN},\theta I^{F})) + \left(1-\gamma^{D}\right) v''(r^{D,MIN})}$$

$$\frac{\partial g^{F}}{\partial \tau} = \frac{\left(1-\gamma^{F}\right) v''(r^{F})}{\gamma^{F}w''(g^{F}) + \left(1-\gamma^{F}\right) v''(r^{F})}$$

- the government budget constraints in period 0 when $\overline{b}^{D} = -\frac{e^{D}}{\beta}$:

$$\begin{aligned} r^{D,MIN} + x^{D,MIN} &= \tau \\ (1+\beta)g^{F} + r^{F} &= (1+\beta)e^{F} - \theta I^{F} - \tau \end{aligned}$$

Consider the case in which the above system has a solution with $\tau > 0$. Then, policymaker D's participation constraint does not bind if:

$$\begin{aligned} &\left(1-\gamma^{D}\right) v(r^{D,MIN}) + \gamma^{D}u(c^{D}(x^{D,MIN},\theta I^{F})) + \beta\gamma^{D}w(0) \\ &+\beta\gamma^{D}w(2e^{D}) < \left(1-\gamma^{D0}\right) v(r^{D0}) + \gamma^{D0}u(c^{D}(x^{D0},\theta I^{F})) \\ &\quad + (1+\beta)\gamma^{D0}w(g^{D0}). \end{aligned} \tag{67}$$

Condition (67) is satisfied if $w(0) < M$, where

$$\begin{aligned} M &= \left(1-\gamma^{D0}\right) v(r^{D0}) + \gamma^{D0}u(c^{D}(x^{D0},\theta I^{F})) \\ &\quad + (1+\beta)\gamma^{D0}w(g^{D0}) - \left(1-\gamma^{D}\right) v(r^{D,MIN}) \\ &\quad -\gamma^{D}u(c^{D}(x^{D,MIN},\theta I^{F})). \end{aligned}$$

B.3.9 Proof of Corollary 6

Denote the political reform as an increase of γ^D, up to a maximum value $\gamma^{D*} < 1$. If the political reform is a condition to the partial banking union, then the supranational authority selects a value for $\gamma^D \leq \gamma^{D*}$.

Denote the initial value of γ^D as γ^{D0}. Let $\left(r^{D0}, x^{D0}, g^{D0}, g_1^{D0}\right)$ be the policies chosen by the D government without a banking union at the initial value of $\gamma^D = \gamma^{D0}$.

Country D households set $\overline{b}^D$, which applies if a partial banking union is formed. The supranational authority sets $\underline{x}$, τ and $\gamma^D \in [\gamma^{D0}, \gamma^{D*}]$ given $\overline{b}^D$. The problem for the supranational authority is

$$\max_{\{\tau, \underline{x}, \gamma^D\}} \eta \overline{U^D}(\theta, \overline{b}^D, \tau, \underline{x}) + (1-\eta) U^F(\theta, b^F, \tau, \underline{x}),$$

subject to

$$\begin{aligned} (1-\gamma^D)v(r^D) + \gamma^D \overline{U^D}(\theta, \overline{b}^D, \tau, \underline{x}) \quad &\geq \quad (1-\gamma^{D0})v(r^{D0}) \\ &\quad\quad + \gamma^{D0} U^D(\theta, b^{D0}, 0, 0), \end{aligned} \tag{68}$$

$$\begin{aligned} (1-\gamma^F)v(r^F) + \gamma^F U^F(\theta, b^F, \tau, \underline{x}) \quad &\geq \quad (1-\gamma^F)v(r^{F0}) \\ &\quad\quad + \gamma^F U^F(\theta, b^{F0}, 0, 0). \end{aligned} \tag{69}$$

Denote the solution to this problem by $\tau(b^D)$, $\underline{x}(b^D)$, and γ^{Ds}.

We solve the problem starting from the supranational authority's problem, and then determine the choice of $\overline{b}^D$ made by the country D households.

Step 1. Supranational authority's problem

Notice that with $\gamma^{D*} > \gamma^{D0}$, setting $\gamma^{Ds} = \gamma^{D*}$ is increasing the utility of households in both countries and the utility of the supranational authority. Moreover, if $U^{D0} \geq v(r^{D0})$, then policymaker D's participation constraint will continue to hold $\forall \gamma^{D*} \geq \gamma^{D0}$. Therefore $\gamma^{Ds} = \gamma^{D*}$ in the partial banking union.

Claim 1 *The supranational authority either sets $\gamma^{Ds} = \gamma^{D*}$, or $\gamma^{Ds} = \gamma^{D**} < \gamma^{D*}$ and the participation constraint for policymaker D binds at γ^{D**}.*

Proof. Assume $\gamma^D < \gamma^{D*}$ and the participation constraint (68) does not bind at $\bar{b}^D = b^D$ (when the debt limit is non-binding). Then marginally increasing γ^D increases the utility of the supranational authority, while the participation constraints still holds. So $\gamma^{Ds} < \gamma^{D*}$ cannot be optimal. ■

Step 2. Country D's equilibrium choice of $\bar{b}^D$

Consider the following strategy for D households:

- If $U^D(\theta, b^D, \tau, \underline{x}|\gamma^{Ds}) \geq U^{D0}$ (i.e., household utility in a partial banking union without conditional fiscal rules is higher than autarky), then $\bar{b}^D$ is chosen to maximize utility in the partial banking union, hence

$$U^D(\theta, \bar{b}^D, \tau, \underline{x}|\gamma^{Ds}) \geq U^D(\theta, b^D, \tau, \underline{x}|\gamma^{Ds}) \geq U^{D0}. \tag{70}$$

- If $U^D\theta, b^D, \tau, \underline{x}|\gamma^D) < U^{D0}$ (i.e., household utility in a partial banking union without conditional fiscal rules is higher than autarky), the debt limit $\bar{b}^D$ is chosen to maximize utility in the partial banking union, so there are two possible outcomes:
  - if $U^D(\theta, \bar{b}^D, \tau, \underline{x}|\gamma^{Ds}) \geq U^{D0}$, then set $\bar{b}^D$ as the solution to

$$\max_{\bar{b}^D} U^D(\theta, b^D, \tau, \underline{x}), \tag{71}$$

    subject to $\tau(\bar{b}^D)$, $\underline{x}(\bar{b}^D)$, $\gamma^D = \gamma^{Ds}$.
  - if $U^D(\theta, \bar{b}^D, \tau, \underline{x}|\gamma^{Ds}) < U^{D0}$, then
    * if there exists $\bar{b}^{D*} \geq -\frac{e^D}{\beta}$ such that $\tau(\bar{b}^{D*}) > 0$, $\underline{x}(\bar{b}^{D*}) > 0$, constraint (68) binds and

$$v(r^D(\bar{b}^{D*})) \leq \frac{(1-\gamma^{D0})}{(1-\gamma^{Ds})} v(r^{D0}) - \frac{(\gamma^{Ds}-\gamma^{D0})}{(1-\gamma^{Ds})} U^{D0},$$

      then set the debt limit as the lowest value $\bar{b}^{D*} \geq -\frac{e^D}{\beta}$ that satisfies the above conditions.[22]

[22] In this case, setting debt limit $\bar{b}^{D*}$ leads to $U^D(\theta, \bar{b}^{D*}, \tau, \underline{x}|\gamma^{Ds}) \geq U^{D0}$.

∗ if $\bar{b}^{D*}$ does not exist, then

· if the conditions outlined in Lemma 2 are satisfied, set $\bar{b}^{D} = -\frac{e^D}{\beta}$

· if the conditions outlined in Lemma 2 are not satisfied, then set $\bar{b}^{D}$ at the solution to (71).

If a partial banking union is implemented, then the participation constraint for policymaker F is satisfied, so

$$\begin{aligned}&(1-\gamma^F)v(r^F) + \gamma^F u(c^F(x^F, x^D)) \\ &+\gamma^F w(g^F) + \beta\gamma^F w(g_1^F) \geq (1-\gamma^F)v(r^{F0}) \\ &+\gamma^F u(c^F(x^{F0}, x^{D0})) + \gamma^F w(g^{F0}) + \beta\gamma^F w(g_1^{F0}).\end{aligned} \tag{72}$$

Since $\tau > 0$, rents in country F satisfy $r^F < r^{F0}$, and (72) implies

$$\begin{aligned}&u(c^F(x^F, x^D)) + w(g^F) + \beta w(g_1^F) \geq \\ &u(c^F(x^{F0}, x^{D0})) + w(g^{F0}) + \beta w(g_1^{F0}).\end{aligned} \tag{73}$$

The above strategies for the supranational authority and the D households lead to a unique subgame perfect equilibrium in which a partial banking union achieves a Pareto improvement $\forall \eta > \widehat{\eta^*}$, where $\widehat{\eta^*} < \widetilde{\eta^*}$. This follows from

$$\frac{\partial \widetilde{\eta^*}}{\partial \gamma^{Ds}} = -\frac{\Gamma}{\sigma^D R u'(c^D) \frac{\partial x^D}{\partial \underline{x}} \frac{\partial \underline{x}}{\partial \eta^*} + w'(g^D)\left(\frac{\partial \tau}{\partial \eta^*} - \frac{\partial \underline{x}}{\partial \eta^*}\right)},$$

where

$$\Gamma \equiv \sigma^D R u'(c^D) \frac{\partial x^D}{\partial \underline{x}} \frac{\partial \underline{x}}{\partial \gamma^{Ds}} + w'(g^D)\left(\frac{\partial \tau}{\partial \gamma^{Ds}} - \frac{\partial \underline{x}}{\partial \gamma^{Ds}}\right).$$

From (43),

$$\frac{\partial \widetilde{\eta^*}}{\partial \gamma^{Ds}} = -\frac{\partial \tau}{\partial \gamma^{Ds}} \left(\frac{\partial \tau}{\partial \eta^*}\right)^{-1}.$$

From (32), $\frac{\partial \tau}{\partial \gamma^{Ds}} > 0$, so

$$\frac{\partial \widetilde{\eta^*}}{\partial \gamma^{Ds}} < 0.$$

If the conditions of Lemma 2 are satisfied, then $\widehat{\eta}^* = 0$.

C Appendix C – Alternative Fiscal Rules (For Online Publication)

C.1 Domestic Fiscal Rules That Anticipate the Partial Banking Union

Given a debt limit $\overline{b}^D(\theta)$ in the D country and a debt limit $\overline{b}^F(\theta)$ in country F, the supranational authority determines policies $(\tau, \underline{x})$ to solve

$$\max_{\tau,\underline{x}}\{\eta\overline{U^D}(\theta,\overline{b}^D,\tau,\underline{x})+(1-\eta)\overline{U^F}(\theta,\overline{b}^F,\tau,\underline{x})\}$$

subject to

$$\begin{aligned}(1-\gamma^D)v(r^D)+\gamma^D\overline{U^D}(\theta,\overline{b}^D,\tau,\underline{x}) &\geq (1-\gamma^D)v(r^{D0}) \\ &\quad +\gamma^D\overline{U^D}(\theta,\overline{b}^D,0,0), \qquad (74)\\ (1-\gamma^F)v(r^F)+\gamma^F\overline{U^F}(\theta,\overline{b}^F,\tau,\underline{x}) &\geq (1-\gamma^F)v(r^{F0}) \\ &\quad +\gamma^F\overline{U^F}(\theta,\overline{b}^F,0,0). \qquad (75)\end{aligned}$$

where (74) and (75) are the participation constraints for policymakers D and F, respectively.

Denote the equilibrium policies chosen by the supranational authority by $(\tau, \underline{x})$. The debt limit $\overline{b}^D(\theta)$ for country D is set so as to maximize D household utility, anticipating policies τ, $\underline{x}$:

$$\max_{\left\{\bar{b}^D,x^D,g^D,b^D,r^D\right\}} u(c^D(x^D,x^F))+w(g^D)+\beta w(e^D-b^D) \qquad (76)$$

subject to

$$\gamma^D R\sigma^D u'(c^D(x^D, x^F)) = (1-\gamma^D)v'(r^D), \tag{77a}$$
$$R\sigma^D u'(c^D(x^D, x^F)) = w'(g^D), \tag{77b}$$
$$r^D + x^D + g^D \leq e^D + \beta b^D + \tau(\bar{b}^D), \tag{77c}$$
$$r^D + x^D \geq \underline{x}(\bar{b}^D), \tag{77d}$$
$$b^D \leq \bar{b}^D(\theta). \tag{77e}$$

Constraints (77a)-(77b) are the equilibrium conditions derived from the D policymaker's maximization problem with debt limit $\bar{b}^D(\theta)$. Constraint (77c) is the budget constraint of the D government, constraint (77d) is the reinvestment constraint, and constraint (77e) represents the limit on public debt imposed by the fiscal rule.

The problem for the F country is analogous, without the reinvestment constraint. From the above setup, we obtain the corresponding result to Proposition 4:

Proposition 7 *There exists threshold $\widehat{\eta} > 0$ such that a partial banking union with domestic fiscal rules does not achieve a Pareto improvement compared to no banking union whenever $\eta < \widehat{\eta}$*

Proof. In Section C.3.1. ■

The equivalent of Corollary 4 follows.

Corollary 7 *Domestically set fiscal rules in country D decrease the welfare of F households in the partial banking union, compared to the case without fiscal rules.*

Proof. The proof is analogous to that of Corollary 4. ■

Notice that the only result that does not emerge when domestic fiscal rules are set anticipating a partial banking union is Corollary 5. If full recapitalizations are performed, $x^D = \theta I^D$ and the participation constraint for policymaker D binds, then, using the notation from the proof to Corollary 5,

$$\begin{aligned}(1-\gamma^D)v(r^D)+\gamma^D U^D(x^D,x^F,g^D,g_1^D) &= (1-\gamma^D)v(r^{D0}) \\ &\quad +\gamma^D U^D(x^{D0},x^{F0},g^{D0},g_1^{D0}).\end{aligned}$$

The full recapitalization $x^D=\theta I^D$ implies $r^D=\overline{r^D}=r^{D*}$, where $(1-\gamma^D)v'(r^D)=\gamma^D u(c^D(\theta I^D,\theta I^F))$. The binding participation constraint for policymaker D and then implies

$$U^D(\overline{x^D},\overline{x^F},\overline{g^D},\overline{g_1^D})=U^D(x^D,x^F,g^D,g_1^D).$$

Therefore, under the conditions of Corollary 5, domestic fiscal rules cannot lead to an increase in the losses to household utility from a partial banking union, since these losses are at the maximum possible level to begin with.

C.2 Domestic Fiscal Rules Non-contingent on θ

Consider the case in which $\Theta=[\underline{\theta},\overline{\theta}]$ and domestic fiscal rules cannot be made contingent on the value of θ, and they are set without the anticipation of the partial banking union. The debt limit $\bar{b}^D$ for country D is set so as to maximize expected household utility:

$$\max_{\bar{b}^D}\mathbb{E}_\theta[u(c^D(x^D(\theta),x^F(\theta),\theta))+w(g^D(\theta))+\beta w(e^D-b^D(\theta))] \tag{78}$$

subject to

$$\begin{aligned}\gamma^D\sigma^D Ru'(c^D(x^D,x^F,\theta)) &= (1-\gamma^D)v'(r^D(\theta)), &\text{(79a)}\\ \sigma^D Ru'(c^D(x^D,x^F,\theta)) &= w'(g^D(\theta)), &\text{(79b)}\\ w'(g^D(\theta)) &= \mathbf{1}_{\{b^D<\bar{b}^D\}}w'(e^D-b^D(\theta)), &\text{(79c)}\\ r^D(\theta)+x^D(\theta)+g^D(\theta) &\le e^D+\beta b^D(\theta), &\text{(79d)}\\ b^D(\theta) &\le \bar{b}^D. &\text{(79e)}\end{aligned}$$

Problem (78) can be simplified by noticing that if $\bar{b}^D$ binds for some $\widetilde{\theta} \in \Theta$, then it binds for all $\theta > \widetilde{\theta}$. For $\Theta = [\underline{\theta}, \overline{\theta}] \subset \mathbb{R}$, problem (78) can then be expressed as:

$$\max_{\bar{b}^D} \mathbb{E}_{\theta \geq \widetilde{\theta}(\bar{b}^D)}[u(c^D(x^D(\theta), x^F(\theta), \theta)) + w(g^D(\theta)) + \beta w(e^D - \bar{b}^D)] +$$
$$\mathbb{E}_{\theta < \widetilde{\theta}(\bar{b}^D)}[u(c^D(x^D(\theta), x^F(\theta), \theta)) + w(g^D(\theta)) + \beta w(e^D - b^D(\theta))], \quad (80)$$

subject to (79a)-(79e).

In order to ensure that the objective in program (80) is concave in $\bar{b}^D$, we make the following assumption about the government's utility from rent seeking:

Assumption 2 *For any set of feasible policies $\{x^D, g^D, r^D\}$ and $\theta \in \Theta$ that satisfy*

$$\begin{aligned} \gamma^D \sigma^D R u'(c^D(x^D, x^F, \theta)) &= (1 - \gamma^D)\, v'(r^D), \\ \gamma^D w'(g^D) &= (1 - \gamma^D)\, v'(r^D), \\ x^D + g^D + r^D &\leq e^D(1 + \beta), \end{aligned}$$

the following conditions are also satisfied:

$$\frac{u'''(c^D)}{(\sigma^D R)\, u''(c^D)^2} \geq \frac{\gamma^D}{(1 - \gamma^D)} \frac{v'''(r^D)}{v''(r^D)^2}, \quad (81)$$

$$\frac{w'''(g^D)}{w''(g^D)^2} \geq \frac{\gamma^D}{(1 - \gamma^D)} \frac{v'''(r^D)}{v''(r^D)^2}, \quad (82)$$

where $u'''(c^D)$, $w'''(g^D)$, and $v'''(r^D)$ denote the third derivatives of the utility functions.

We proceed to analyze the problem by establishing the following lemmas.

Lemma 3 *The objective function (80) is strictly concave in $\bar{b}^D$ and the maximization problem has a unique solution $\bar{b}^{D*} \in [-e^D/\beta, e^D]$.*

Proof. In Section C.3.2. ∎

Lemma 4 *There exists $\theta^G \in \Theta$, $\theta^G < \overline{\theta}$ such that the debt limit imposed by the domestic fiscal rule is binding for policymaker D if $\theta \geq \theta^G$.*

Proof. In Section C.3.3. ■

The above lemmas establish that the solution to problem (80) is unique, and that the fiscal rule is binding for a subset of the possible realizations of θ.

This setup captures the main trade-off of non-contingent fiscal rules: on the one hand, they limit the government's ability to engage in excessive spending in the first period; since part of first-period spending goes towards rents, the debt limit is beneficial to households because it reduces rents; on the other hand, fiscal rules limit government's ability to borrow in order to recapitalize banks in period 0.

For country F, we assume the analogous decision problem to (80), such that debt limit $\overline{b}^F \leq e^F$ is set. The following Lemma ensures that $\forall\theta$, full recapitalization of F banks are performed even under the fiscal rule ($x^F(\theta) = \theta I^F$).

Lemma 5 *There exists $\overline{\gamma^F}^*$ such that $\forall \gamma^F \geq \overline{\gamma^F}^*$, policymaker F provides full recapitalizations ($x^F = \theta I^F$) when domestic fiscal rules are in place.*

Proof. In Section C.3.4. ■

Lemma 5 gives the equivalent result to that of Lemma 1.

Consider the supranational authority's problem with debt limits as described above. Analyzing the equivalent problem to problem (13), we obtain the following results.

Proposition 8 *For $\gamma^F \geq \overline{\gamma^F}^*$, there exists a threshold η^{***} such that a partial banking union under domestic fiscal rules achieves a Pareto improvement compared to no banking union whenever $\eta > \eta^{***}$.*

Proof. Analogous to the proof to Proposition 4. ■

The fiscal rules change the cost of funding recapitalizations, but they do not change the trade-off faced by the supranational authority between increasing recapitalizations and reducing public good provision. Therefore, the intuition from the case without fiscal rules carries over to the case with fiscal rules.

We can also derive the equivalents of Corollaries 4 and 5.

Corollary 8 *Domestic fiscal rules in country D decrease the welfare of F households in the partial banking union, compared to the case without fiscal rules in country D.*

Proof. Same as the proof to Corollary 4. ■

Finally, the equivalent of Corollary 5 holds even if the fiscal rule is not made contingent on θ. The proof is analogous to the proof to Corollary 5.

C.3 Proofs for Sections C.1 and C.2

C.3.1 Proof of Proposition 7

From the proof to Proposition 2, if the debt limit $\bar{b}^D$ is not binding, then $\widehat{\eta} = \eta^*$.

If the debt limit is binding both with and without the partial banking union, then the problem is analogous to the problem considered in Proposition 4.

If the debt limit binds only in the partial banking union and it does not bind without the partial banking union, then $\bar{b}^D \in (b^{D0}, b^D)$. In this case the outside option for policymaker D is

$$V^{D0} = (1-\gamma^D)v(r^{D0}) + \gamma^D U^D(x^{D0}, x^{F0}, g^{D0}, g_1^{D0}),$$

and the proof is the same as the proof for Proposition 4.

C.3.2 Proof of Lemma 3

Denote by $U^{D0}(\theta)$ the value of the D household utility given the solution $\{r^{D0}(\theta), x^{D0}(\theta), g^{D0}(\theta), g_1^{D0}(\theta), b^{D0}(\theta)\}$ to policymaker D's maximization problem without the partial banking union and without the fiscal rule. Also, denote by $\overline{U^{D0}}(\theta, \bar{b}^D)$ the value of D household utility given the solution to policymaker D's maximization problem without a partial banking union, but with a fiscal rule $\bar{b}^D$. Finally, let $\widetilde{\theta}(\bar{b}^D)$ denote the value of θ at which $b^{D0} = \bar{b}^D$ (so $g_1^{D0}(\widetilde{\theta}) = e^D - \bar{b}^D$). Given $f(\theta)$ the p.d.f. for θ over $\Theta = [\underline{\theta}, \overline{\theta}]$, the function maximized by problem (80) is

$$EU(\bar{b}^D) = \int_{\underline{\theta}}^{\widetilde{\theta}(\bar{b}^D)} U^{D0}(\theta) f(\theta) d\theta + \int_{\widetilde{\theta}(\bar{b}^D)}^{\overline{\theta}} \overline{U^{D0}}(\theta, \bar{b}^D) f(\theta) d\theta.$$

The function $\overline{U^{D0}}(\theta, \bar{b}^D)$ is a continuous and differentiable function of $\bar{b}^D$, since $u(c)$, $w(g)$ and $v(r)$ are continuously differentiable. Also, $\widetilde{\theta}(\bar{b}^D)$ is differentiable since it is a continuous function of $u(\cdot)$, $w(\cdot)$ and $v(\cdot)$, derived from the solution $\bar{b}^D$ to policymaker D's problem. Taking the first-derivative with respect to $\bar{b}^D$, we obtain

$$\begin{aligned} \frac{\partial EU(\bar{b}^D)}{\partial \bar{b}^D} &= U^{D0}(\widetilde{\theta}(\bar{b}^D)) f(\widetilde{\theta}) \frac{\partial \widetilde{\theta}(\bar{b}^D)}{\partial \bar{b}^D} + \int_{\widetilde{\theta}(\bar{b}^D)}^{\overline{\theta}} \frac{\partial \overline{U^{D0}}(\theta, \bar{b}^D) f(\theta)}{\partial \bar{b}^D} d\theta \\ &\quad - \overline{U^{D0}}(\widetilde{\theta}(\bar{b}^D), \bar{b}^D) f(\widetilde{\theta}) \frac{\partial \widetilde{\theta}(\bar{b}^D)}{\partial \bar{b}^D}. \end{aligned}$$

Notice that for $\theta = \widetilde{\theta}$, we have $U^{D0}(\widetilde{\theta}) = \overline{U^{D0}}(\widetilde{\theta}, \bar{b}^D)$, so

$$\frac{\partial EU(\bar{b}^D)}{\partial \bar{b}^D} = \int_{\widetilde{\theta}(\bar{b}^D)}^{\overline{\theta}} \frac{\partial \overline{U^{D0}}(\theta, \bar{b}^D) f(\theta)}{\partial \bar{b}^D} d\theta.$$

Then,

$$\frac{\partial^2 EU(\bar{b}^D)}{\partial \bar{b}^{D^2}} = \int_{\widetilde{\theta}(\bar{b}^D)}^{\overline{\theta}} \frac{\partial^2 \overline{U^{D0}}(\theta, \bar{b}^D) f(\theta)}{\partial \bar{b}^{D^2}} d\theta - \frac{\partial \overline{U^{D0}}(\widetilde{\theta}, \bar{b}^D) f(\widetilde{\theta})}{\partial \bar{b}^D} \frac{\partial \widetilde{\theta}(\bar{b}^D)}{\partial \bar{b}^D}.$$

But $\frac{\partial \overline{U^{D0}}(\widetilde{\theta}, \bar{b}^D) f(\widetilde{\theta})}{\partial \bar{b}^D} = 0$ since any increase in $\bar{b}^D$ would make the debt constraint ($b^D(\widetilde{\theta}) \leq \bar{b}^D$) slack. Therefore,

$$\frac{\partial^2 EU(\bar{b}^D)}{\partial \bar{b}^{D^2}} = \int_{\widetilde{\theta}(\bar{b}^D)}^{\bar{\theta}} \frac{\partial^2 \overline{U^{D0}}(\theta, \bar{b}^D) f(\theta)}{\partial \bar{b}^{D^2}} d\theta.$$

Then

$$\frac{\partial^2 \overline{U^{D0}}(\theta, \bar{b}^D) f(\theta)}{\partial \bar{b}^{D^2}} < 0 \Leftrightarrow \frac{\partial^2 EU(\bar{b}^D)}{\partial \bar{b}^{D^2}} < 0. \tag{83}$$

The change in household utility due to the change in the binding debt limit $\bar{b}^D$ is given by

$$\frac{\partial \overline{U^{D0}}(\theta, \bar{b}^D)}{\partial \bar{b}^D} = \sigma^D R u'(c^D) \frac{\partial x^D}{\partial \bar{b}^D} + w'(g^D) \frac{\partial g^D}{\partial \bar{b}^D} - \beta w'(e^D - \bar{b}^D).$$

Then,

$$\begin{aligned} \frac{\partial^2 \overline{U^{D0}}(\theta, \bar{b}^D)}{\partial \bar{b}^{D^2}} &= \Bigg[\left(\sigma^D R\right)^2 u''(c^D(x^D, x^F, \theta)) \left(\frac{\partial x^D}{\partial \bar{b}}\right)^2 \\ &\quad + w''(g^D) \left(\frac{\partial g^D}{\partial \bar{b}^D}\right)^2 + \beta w''(e^D - \bar{b}^D) \Bigg] \\ &\quad + w'(g^D) \left(-\frac{\partial^2 r^D}{\partial \bar{b}^{D^2}}\right) \end{aligned}$$

The first-order conditions to the Home government's problem give

$$\begin{aligned} \gamma^D \sigma^D R u'(c^D(x^D, x^F, \theta)) &= (1 - \gamma^D) v'(r^D), \\ \gamma^D w'(g^D) &= (1 - \gamma^D) v'(r^D). \end{aligned}$$

Then,

$$\begin{aligned} \gamma^D \left(\sigma^D R\right)^2 u''(c^D(x^D, x^F, \theta)) \frac{\partial x^D}{\partial \bar{b}^D} &= (1 - \gamma^D) v''(r^D) \frac{\partial r^D}{\partial \bar{b}^D}, \\ \gamma^D w''(g^D) \frac{\partial g^D}{\partial \bar{b}^D} &= (1 - \gamma^D) v''(r^D) \frac{\partial r^D}{\partial \bar{b}^D}, \end{aligned}$$

and

$$\frac{\partial x^D}{\partial \bar{b}^D} + \frac{\partial r^D}{\partial \bar{b}^D} + \frac{\partial g^D}{\partial \bar{b}^D} = \beta.$$

Combining the above conditions,

$$\frac{\partial r^D}{\partial \bar{b}^D} = \beta \left[1 + \frac{\left(1-\gamma^D\right) v''(r^D)}{\gamma^D \left(\sigma^D R\right)^2 u''(c^D)} + \frac{\left(1-\gamma^D\right) v''(r^D)}{\gamma^D w''(g^D)} \right]^{-1}.$$

So

$$\begin{aligned}\frac{\partial^2 r^D}{\partial \bar{b}^{D^2}} &= -\left(\frac{\partial r^D}{\partial \bar{b}^D}\right)^3 \frac{1}{\beta} \left(\frac{1-\gamma^D}{\gamma^D} v''(r^D)\right)^2 \cdot \\ &\quad \cdot \left(\frac{\gamma^D}{1-\gamma^D} \frac{v'''(r^D)}{v''(r^D)^2} \left(\frac{1}{\left(\sigma^D R\right)^2 u''(c^D)} + \frac{1}{w''(g^D)} \right) \right. \\ &\quad \left. - \frac{u'''(c^D)}{\left(\sigma^D R\right) u''(c^D)^3} - \frac{w'''(g^D)}{w''(g^D)^3} \right).\end{aligned}$$

By Assumption 2,

$$\frac{\partial^2 r^D}{\partial \bar{b}^{D^2}} \geq 0.$$

This, together with the concave increasing functions $u(c^D)$, $w(g^D)$ implies

$$\frac{\partial^2 \overline{U^{D0}}(\theta, \bar{b}^D)}{\partial \bar{b}^{D^2}} < 0$$

and

$$\frac{\partial^2 EU(\bar{b}^D)}{\partial \bar{b}^{D^2}} < 0.$$

Given the strict concavity of the objective function, it follows that the maximization problem has a unique solution $\bar{b}^{D*} \in [-e^D/\beta, e^D]$.

C.3.3 Proof of Lemma 4

From the proof to Lemma 3, the first-order condition for the household expected utility maximization problem is given by

$$\int_{\widetilde{\theta}(\overline{b}^D)}^{\overline{\theta}} \frac{\partial \overline{U^{D0}}(\theta, \overline{b}^D) f(\theta)}{\partial \overline{b}^D} d\theta = 0. \tag{84}$$

Claim 2 *Consider the case in which $\overline{b}^D < b^{D*}(\overline{\theta})$, where $b^{D*}(\overline{\theta})$ is the level of debt at which the utility of D households is maximized when $\theta = \overline{\theta} \equiv \max_\theta \Theta$.*
Proof. *Let $\overline{b}^D < b^{D*}(\overline{\theta})$. Then, $\forall \theta < \overline{\theta}$, $\frac{\partial \overline{U^{D0}}(\theta, \overline{b}^D)}{\partial \overline{b}^D} < 0$, due to the concavity of $\overline{U^{D0}}(\theta, \overline{b}^D)$. Since it is set to maximize Home household utility, $\overline{b}^D$ is lower than the level of debt that maximizes policymaker D's utility when $\theta = \overline{\theta}$.,It follows that $\widetilde{\theta}(\overline{b}^D) < \overline{\theta}$. So, for all nondegenerate probability distribution functions $f(\theta)$, we have*

$$\int_{\widetilde{\theta}(\overline{b}^D)}^{\overline{\theta}} \frac{\partial \overline{U^{D0}}(\theta, \overline{b}^D)}{\partial \overline{b}} f(\theta) d\theta < 0.$$

Then, $\overline{b}^D = b^{D*}(\overline{\theta})$ cannot be the solution to (84). ■

Since $\overline{b}^* < b^{D*}(\overline{\theta})$, it follows that $\exists \theta^G \in \Theta$ such that $\forall \theta \geq \theta^G$, $\overline{V^{D0}}(\theta, \overline{b}^D) < V^{D0}(\theta)$.

C.3.4 Proof of Lemma 5

From Lemma 1, $\forall \theta \in \Theta$, there exists $\overline{\gamma^F}(\theta)$ such that $x^F = \theta I^F \; \forall \gamma^F \geq \overline{\gamma^F}(\theta)$. Then, it follows that $\overline{\gamma^F}^* = \max_\theta \{\overline{\gamma^F}(\theta)\}$.

D Appendix D – Dynamic Model (For Online Publication)

The following extension develops the model in a dynamic setting, in which future recapitalizations are possible. Debt accumulation has different effects on the continuation utilities of D and F households, which implies that the supranational authority's preferences over debt are different from those of the policymakers. The dynamic model shows that the results to the two-period model carry over to the infinite-horizon environment.

Consider the setup described for period 0 in the two-period model. This setup is repeated every period, leading to the following timing each period $t \geq 0$:

1. The households receive their respective endowments ω^D and ω^F, and the governments receive endowments e^D and e^F; banks make investments in projects;
2. Shock $\theta_t \in \Theta$ is realized and observed by all agents;
3. The supranational authority offers an agreement $(\tau_t, \underline{x}_t)$ first to country F, then to country D, and each policymaker decides whether to accept or reject it;
4. Policymaker i decides policies $\alpha_t^i = \{x_t^i, g_t^i, r_t^i, b_t^i\}$, $i \in \{D, F\}$;

Policymaker F can decide each period whether to accept or reject the supranational agreement offered that period, and this decision is denoted by $\varrho_t^F \in \{0, 1\}$, with $\varrho_t^F = 1$ for acceptance. The utility for policymaker F is given by

$$V_0^F = \mathbb{E} \sum_{t=0}^{\infty} \beta^t \left[(1-\gamma^F) v(r_t^F) + \gamma^F u(c^F(x_t^F, x_t^D, \theta_t)) + \gamma^F w(g_t^F) \right].$$

Policymaker D also decides participation in the agreement each period, denoted by $\varrho_t^D \in \{0,1\}$, and derives expected utility:

$$V_0^D = \mathbb{E}\sum_{t=0}^{\infty}\beta^t\left[(1-\gamma^D)v(r_t^D) + \gamma^D u(c^D(x_t^D, x_t^F, \theta_t)) + \gamma^D w(g_t^D)\right].$$

Lastly, the supranational authority offers $(\tau_t, \underline{x}_t)$ each period, and derives the following expected utility:

$$\begin{aligned} S_0 \quad = \quad & \mathbb{E}\sum_{t=0}^{\infty}\beta^t\left[\eta\left(u(c^D(x_t^D, x_t^F, \theta_t)) + w(g_t^D)\right)\right. \\ & \left. + (1-\eta)\left(u(c^F(x_t^F, x_t^D, \theta_t)) + w(g_t^F)\right)\right]. \end{aligned}$$

As in the two-period model, we assume that the income of country F is sufficiently high such that full recapitalizations are provided in country F.

Assumption 3 *The endowment F governments satisfies*

$$e^F \geq \theta I^F + \theta I^D + g^{F*} + r^{F*} + r^{D*},$$

$\forall\theta \in \Theta$, where g^{F*} is defined implicitly by $w'(g^{F*}) = \sigma^F R u'(c^F(\theta I^F, \theta I^D))$, and r^{i*} is defined implicitly by $(1-\gamma^i)\,v'(r^{i*}) = \gamma^i\sigma^i R u'(c^i(\theta I^i, \theta I^j))$, $i, j \in \{D, F\}$, $i \neq j$.

Assumption 3 ensures that full recapitalizations can be performed in country F even if transfers are made to country D.

D.1 Equilibrium Concept

I consider the pure strategy Markov Perfect Equilibria of this game, in which strategies only depend on the current state of the world and not on the entire history of the game. The current state of the world in period t consists of the outstanding debt $b_{t-1} = (b_{t-1}^D, b_{t-1}^F)$ and the liquidity shock in the current period, θ_t. A Markov Perfect Equilibrium (MPE) is defined as a set of strategies $\left\{\{\tau_t, \underline{x}_t\}, \alpha_t^D, \alpha_t^F, \varrho_t^D, \varrho_t^F\right\}$ such that these strategies depend only on the current payoff-relevant state of the economy $\left\{b_{t-1}^D, b_{t-1}^F, \theta_t\right\}$ and on the prior actions within the same period as described in the timing of events. Therefore, an MPE is given by a set of strategies $\{\tau_t(b_{t-1}, \theta_t),\ \underline{x}_t(b_{t-1}, \theta_t),\ \varrho_t^F(b_{t-1}, \theta_t),\ \varrho_t^D(b_{t-1}, \theta_t), \alpha_t^D, \alpha_t^F\}$.[23]

The above framework with separable utility functions, discount factor $\beta < 1$ and bounded instantaneous utilities (given the finite government resources) satisfies the conditions for the existence of a Markov Perfect Equilibrium to this game.[24]

D.2 Analysis

Policymaker D decides policy in country D, given the partial banking union terms offered by the supranational authority. Each period, the state of the economy can be summarized by the outstanding debt in both countries $b_{t-1} = (b_{t-1}^D, b_{t-1}^F)$ and the shock θ_t, all of which are observed before policy is decided. Let $V(b, \theta, \tau, \underline{x})$ denote the maximum expected utility for the politician at the beginning of a period in which the state is given by $(b, \theta, \tau, \underline{x})$. Policymaker D chooses a policy vector $\alpha^D = \{r^D, x^D, g^D, b^{D\prime}\}$ with $x^D \geq 0,\ g^D \geq 0,\ r^D \geq 0,\ b^{D\prime} \geq \underline{b}^D$, and a decision to participate in the partial banking union $\varrho^D \in \{0, 1\}$ to solve:

$$\begin{aligned} V^D(b, \theta, \tau, \underline{x}) = \max_{\alpha^D, \varrho^D} \{ &(1-\gamma^D)v(r^D) + \gamma^D u(c^D(x^D, x^F, \theta)) + \gamma^D w(g^D) \\ &+ \beta \mathbb{E}_{\theta'} \left[V^D(b', \theta', \tau'(b', \theta'), \underline{x}'(b', \theta')) \right] \} \end{aligned} \tag{85}$$

subject to

$$r^D + x^D + g^D \leq e^D + \varrho^D \tau + \beta b^{D\prime} - b^D, \tag{86a}$$

$$r^D + x^D \geq \varrho^D \underline{x}, \tag{86b}$$

$$b^{D\prime} \in \left[\underline{b}^D, b^{D,MAX} \right], \tag{86c}$$

$$x^D \leq \theta I^D. \tag{86d}$$

[23] Formally, the set of stretegies is written as $\{\tau_t(b_{t-1}, \theta_t),\ \underline{x}_t(b_{t-1}, \theta_t),\ \varrho_t^F(\tau_t, \underline{x}_t | b_{t-1}, \theta_t), \varrho_t^D(\tau_t, \underline{x}_t, \varrho_t^F | b_{t-1}, \theta_t),\ x_t^i(\tau_t, \underline{x}_t, \varrho_t^F, \varrho_t^D | b_{t-1}, \theta_t),\ g_t^i(\tau_t, \underline{x}_t, \varrho_t^F | b_{t-1}, \theta_t),\ r_t^i(\tau_t, \underline{x}_t, \varrho_t^F, \varrho_t^D | b_{t-1}, \theta_t), b_t^i(\tau_t, \underline{x}_t, \varrho_t^F, \varrho_t^D | b_{t-1}, \theta_t)\}$

[24] By Theorem 13.2 in Fudenberg and Tirole (1991).

Constraint (86a) is the resource constraint of the economy. Constraint (86b) is the required reinvestment $\underline{x}$ as part of the partial banking union. Finally, conditions (86c) and (86d) give the limits on debt and recapitalizations, respectively. The feasible upper value for debt is given by $b^{i,MAX} \equiv e^i/(1-\beta)$, where $i \in \{D, F\}$ and the lower bound is exogenously given, $\underline{b}^i > -\infty$.

Policymaker D's problem can reduced to the case where $\varrho^D = 1$ in all periods, given the equilibrium strategy of the supranational authority. The supranational authority is expected to follow the equilibrium policy functions in all future periods, while the current period's agreement $(\tau, \underline{x})$ can be a deviation from that. Therefore, the expected utility of policymaker D from participating in the partial banking union is

$$\begin{aligned} V^D(b,\theta,\tau,\underline{x}) = \max_{\alpha^D}\{&(1-\gamma^D)v(r^D) + \gamma^D u(c^D(x^D,x^F,\theta)) + \gamma^D w(g^D) \\ &+ \beta \mathbb{E}_{\theta'}\left[V^D(b',\theta',\tau'(b',\theta'),\underline{x}'(b',\theta'))\right]\} \end{aligned} \tag{87}$$

subject to

$$r^D + x^D + g^D \leq e^D + \tau + \beta b^{D\prime} - b^D, \tag{88a}$$

$$r^D + x^D \geq \underline{x}, \tag{88b}$$

$$b^{D\prime} \in [\underline{b}^D, b^{D,MAX}], \tag{88c}$$

$$x^D \leq \theta I^D. \tag{88d}$$

The constraints (88a)-(88d) are the equivalents of constraints (86a)-(86d) with $\varrho^D = 1$.

Similarly, policymaker F's problem is to choose $\alpha^F = \{r^F, x^F, g^F, b^{F\prime}\}$ to solve the analogous planning problem:

$$\begin{aligned} V^F(b,\theta,\tau,\underline{x}) = \max_{\alpha^F}\{&(1-\gamma^F)v(r^F) + \gamma^F u(c^F(x^F,x^D,\theta)) + \gamma^F w(g^F) \\ &+ \beta \mathbb{E}_{\theta'}\left[V^F(b',\theta',\tau'(b',\theta'),\underline{x}'(b',\theta'))\right]\} \end{aligned} \tag{89}$$

subject to

$$r^F + x^F + g^F \leq e^F - \tau + \beta b^{F\prime} - b^F, \tag{90a}$$

$$b^{F\prime} \in [\underline{b}^F, b^{F,MAX}], \tag{90b}$$

$$x^F \leq \theta I^F. \tag{90c}$$

Constraint (90c) holds with equality in all periods given Assumption 3.

If policymaker $i \in \{D, F\}$ does not participate in the agreement in the current period, let $\alpha^{i,out} = \{x^{i,out}, r^{i,out}, g^{i,out}, b^{i\prime,out}\}$ denote the vector of policies chosen by the policymaker in the current period. Also, let $b^{\prime,out} = \{b^{D\prime,out}, b^{F\prime,out}\}$ denote the outstanding debt in both countries in the next period. The outside option for the home government is derived from

$$\begin{aligned} V^{D0}(b,\theta) \;=\; & \max_{\alpha^{D,out}} \{(1-\gamma^D)v(r^{D,out}) + \gamma^D u(c^D(x^{D,out}, x^{F,out}, \theta)) + \gamma^D w(g^{D,out}) \\ & + \beta \mathbb{E}_{\theta'} \left[V^D(b^{\prime,out}, \theta', \tau'(b^{\prime,out}, \theta'), \underline{x}'(b^{\prime,out}, \theta')) \right] \end{aligned} \tag{91}$$

subject to

$$r^{D,out} + x^{D,out} + g^{D,out} \leq e^D + \beta b^{D\prime out} - b^D, \tag{92a}$$

$$b^{D\prime,out} \in \left[\underline{b}^D, b^{D,MAX}\right], \tag{92b}$$

$$x^{D,out} \leq \theta I^D. \tag{92c}$$

Since the policymaker stays out of the banking union in the current period, the transfer is not received and there is no bound on current intervention.

Similarly, the utility of the F country in case of no agreement this period is given by

$$\begin{aligned} V^{F0}(b,\theta) \;=\; & \gamma^F u(c^F(x^{F,out}, x^{D,out}, \theta)) + \gamma^F w(g^{F,out}) + (1-\gamma^F)v(r^{F,out}) \\ & + \beta \mathbb{E}_{\theta'} \left[V^F(b^{\prime,out}, \theta', \tau'(b^{\prime,out}, \theta'), \underline{x}'(b^{\prime,out}, \theta')) \right], \end{aligned} \tag{93}$$

subject to

$$
\begin{aligned}
r^{F,out}+x^{F,out}+g^{F,out} &\leq e^F+\beta b^{F\prime out}-b^F, && (94a)\\
b^{F\prime,out} &\in \left[\underline{b}^F, b^{F,MAX}\right], && (94b)\\
x^{F,out} &\leq \theta I^D. && (94c)
\end{aligned}
$$

Lastly, the supranational authority seeks to maximize a weighted sum of household utilities. The supranational authority chooses $(\tau, \underline{x})$, $\underline{x} \geq 0$, $\tau \geq 0$, given $b = (b^D, b^F)$, θ, and knowing the equilibrium policies that will be chosen by the policymakers, and the outside options described by $V^{D0}(b,\theta)$ and $V^{F0}(b,\theta)$. The problem for the supranational authority is given by:

$$
\begin{aligned}
S(b,\theta) = \max_{\tau,\underline{x}} \{ & \eta\left[u(c^D(x^D,x^F,\theta))+w(g^D)\right]\\
& +(1-\eta)\left[u(c^F(x^F,x^D,\theta))+w(g^F)\right]\\
& +\beta\mathbb{E}_{\theta'}\left[S(b',\theta')\right]\} \qquad (95)
\end{aligned}
$$

subject to

$$V^D(b,\theta,\tau,\underline{x}) \geq V^{D0}(b,\theta), \qquad (96)$$

$$V^F(b,\theta,\tau,\underline{x}) \geq V^{F0}(b,\theta). \qquad (97)$$

Constraint (96) represents the participation constraint for policymaker D, where the outside option is described above. Constraint (97) is the participation constraint for policymaker F, given the outside option described in (93).

In order to characterize the politician's problem, the analysis restricts attention to the cases in which the value functions for the politician and the supranational authority are concave. The existence of functions $v(\cdot)$, $u(\cdot)$, and $w(\cdot)$ that satisfy the conditions necessary for the value functions to be concave and differentiable is established in the following Lemma.

Lemma 6 *There exist concave functions $v(r)$, $u(c)$, and $w(g)$ such that policymaker D's value function $V^i(b,\theta,\tau(b,\theta),\underline{x}(b,\theta))$ is concave and differentiable for $b^i \in \left(\underline{b}^i, b^{i,MAX}\right)$ given the equilibrium policy functions $\tau(b,\theta)$ and $\underline{x}(b,\theta)$,* *and the supranational authority's value function $S(b,\theta)$ is concave and differentiable in b^i for $b^i \in \left(\underline{b}^i, b^{i,MAX}\right)$ given the equilibrium policies chosen by the policymakers, $i \in \{D,F\}$.*

Proof. In Section D.4.1. ■

Lemma 6 allows for a characterization of the policymaker's problem in each country. While the conditions of the Lemma restrict the set of possible utility functions, this approach helps provide a tractable framework under which the problem can be analyzed.

Consider policymaker D's problem. Denote by $\lambda(b,\theta)$, $\varkappa(b,\theta)$, and $\varphi(b,\theta)$ the Lagrange multipliers on constraints (88a), (88b), and (88d), respectively. The first-order conditions for an internal solution with respect to r^D, g^D, and $b^{D\prime}$ are:

$$\lambda(b,\theta) - \varkappa(b,\theta) = (1-\gamma^D)v'(r^D), \tag{98}$$

$$\lambda(b,\theta) - \varkappa(b,\theta) + \varphi(b,\theta) = \gamma^D \sigma^D R u'(c^D(x^D, x^F, \theta)), \tag{99}$$

$$\lambda(b,\theta) = \gamma^D w'(g^D), \tag{100}$$

$$\lambda(b,\theta) = \mathbb{E}\left[-\frac{\partial V^D(b^{D\prime}, \theta', \tau', \underline{x}')}{\partial b^{D\prime}}\right]. \tag{101}$$

The above conditions show that the main driver of the results discussed in the two-period are also present in the dynamic environment. A binding intervention constraint, $\varkappa(b,\theta) > 0$, increases both rents and recapitalizations. This additional spending is financed by a reduction in public good provision in the current period, as well as by higher public debt.

We begin the analysis of the dynamic model by considering the change in household welfare under a partial banking union compared to no banking union. We obtain the dynamic equivalent of Proposition 2.

Proposition 9 *A partial banking union does not achieve a Pareto improvement if $\eta \leq \eta^{*d}$ where $\eta^{*d} \in (0,1)$: it increases household welfare in the country providing transfers, but it lowers household welfare in the country receiving transfers.*

Proof. In Section D.4.2. ■

Higher recapitalizations are beneficial for households in both countries; however, a banking union also leads to increased rent seeking, which could make country D households worse off. The intuition for why welfare might decrease is similar to the one presented in the two-period model: the rent seeking incentives of policymaker D together with the supranational decision over how the cost of bailouts is split between the two countries. Still, the dynamic model introduces another element in the decision problem of the supranational authority. Now the supranational authority places a different weight on decreases in the public good in country D in the current period versus decreases in the public good in country D in future periods. This happens because a decrease in country D public good today only affects the utility of country D consumers, while a decrease in future public good provision in country D also implies a decrease in future recapitalizations, through the effect of higher outstanding debt. This latter effect leads to $\eta^{*d} < \eta^{*}$.

D.3 The Role of Fiscal Rules

As in the two-period model, we consider the role of two types of fiscal rules: domestic and supranational.

D.3.1 Domestic fiscal rules.

The domestic fiscal rules are set by the households in each country to maximize their expected utility. Consider the type of rules that are contingent on the current period shock θ. This allows for the debt limit to be conditioned on the current state of the economy, but not on the entire history of shocks or on the outstanding debt that must be repaid each period. This assumption is in line with fiscal rules observed in practice, which are not made contingent on the history of economic shocks hitting the economy.[25]

[25] A survey of fiscal rules implemented since the 1980s across the world is given in Budina et al. (2012).

Consider a set of fiscal rules $\{\bar{b}^D(\theta)\}_{\theta\in\Theta}$ imposed in country D. The problem of setting $\{\bar{b}^D(\theta)\}_{\theta\in\Theta}$ becomes more complex in the dynamic environment, since the rule cannot be made contingent on outstanding debt. The fiscal rules must therefore be set given the distribution of θ and the invariant distribution of debt resulting from the policymaker's problem. The following Lemma establishes that an invariant distribution exists for public debt in country D.

Lemma 7 *The equilibrium distribution of debt in country D converges to a unique nondegenerate invariant distribution over $\left[\underline{b}^D, b^{D,MAX}\right]$.*

Proof. In Section D.4.3. ■

Without a partial banking union, policymaker D chooses policies $\alpha^{D0} = \{x^{D0}, r^{D0}, g^{D0}, b^{D0\prime}\}$ to solve

$$\begin{aligned}\overline{V^{D0}}(b,\theta,\bar{b}^D(\theta)) = \max_{\alpha^{D0}}\{&(1-\gamma^D)v(r^{D0}) \\ &+\gamma^D u(c^D(x^{D0},x^{F0},\theta)) + \gamma^D w(g^{D0}) \\ &+\beta\mathbb{E}_{\theta'}[\overline{V^{D0}}(b^{0\prime},\theta',\bar{b}^D(\theta'))]\}\end{aligned} \tag{102}$$

subject to

$$r^{D0} + x^{D0} + g^{D0} \leq e^D + \tau + \beta b^{D0\prime} - b^D, \tag{103a}$$

$$b^{D0\prime} \leq \bar{b}^D(\theta), \tag{103b}$$

$$x^{D0} \leq \theta I^D. \tag{103c}$$

Given the distributions of θ and b^D, the households of country D set $\{\bar{b}^D(\theta)\}_{\theta\in\Theta}$ in order to maximize

$$\overline{U^{D0}}(b,\theta) = \max_{\{\bar{b}^D(\theta)\}_{\theta\in\Theta}} \mathbb{E}_{\theta_t,b_t^D}\left[\sum_{t=0}^{\infty} u(c^D(x_t^{D0}(\bar{b}^D(\theta)),x_t^{F0},\theta_t)) + w(g_t^{D0}(\bar{b}^D(\theta)))\right],$$

subject to the policymaker's problem.

In order to obtain a concave problem for households, we make the following Assumption about the utility functions:

Assumption 4 *For the set of policies* $\{x^D, g^D, r^D\}$ *and* $\theta \in \Theta$ *that satisfy*

$$\gamma^D \sigma^D R u'(c^D(x^D, x^F, \theta)) = \left(1-\gamma^D\right) v'(r^D),$$
$$\gamma^D w'(g^D) = \left(1-\gamma^D\right) v'(r^D),$$

the following conditions are also satisfied:

$$\frac{u'''(c^D)}{(\sigma^D R)\, u''(c^D)^2} \geq \frac{\gamma^D}{(1-\gamma^D)} \frac{v'''(r^D)}{v''(r^D)^2}, \tag{104}$$
$$\frac{w'''(g^D)}{w''(g^D)^2} \geq \frac{\gamma^D}{(1-\gamma^D)} \frac{v'''(r^D)}{v''(r^D)^2}, \tag{105}$$

where $u'''(c^D)$, $w'''(g^D)$, *and* $v'''(r^D)$ *denote third derivatives.*

First, we establish that the problem for the households in country D is concave.

Proposition 10 *The value function* $\overline{U^{D0}}$ *is concave in* $\overline{b}^D(\theta)$ *for* $\overline{b}^D(\theta) \in (\underline{b}^D, b^{D,MAX})$, $\forall \theta \in \Theta$.

Proof. In Section D.4.4. ■

The problem is analogous in the F country. For simplicity, consider the case in which fiscal rules are only implemented in country D.[26] Denote by $\overline{V^D}(b, \theta, \tau, \underline{x}, \overline{b}^D)$ the utility of policymaker D when the outstanding debt is b, the shock is θ, the terms of the partial banking union are $(\tau, \underline{x})$, and the fiscal

[26] Having fiscal rules in country F as well would not change the main result of this section, as the assumption through the analysis is that e^F is sufficiently high for full recapitalizations to be achieved in country F. The simplification allows us to keep the focus on domestic fiscal rules in country D and their effects on household welfare.

rule $\bar{b}^D$. The supranational authority's problem in this case is then given by

$$\begin{aligned}\overline{S}(b,\theta,\bar{b}^D) \quad = \quad & \max_{\underline{x},\tau} \left\{ \eta \left[u(c^D(x^D, x^F, \theta)) + w(g^D) \right] \right. \\ & + (1-\eta) \left[u(c^F(x^F, x^D, \theta)) + w(g^F) \right] \\ & \left. + \beta \mathbb{E}_{\theta'} \left[\overline{S}(b', \theta', \bar{b}^D) \right] \right\} \qquad (106) \\ & \text{subject to} \\ & \overline{V^D}(b,\theta,\tau,\underline{x},\bar{b}^D) \geq \overline{V^{D0}}(b,\theta,\bar{b}^D), \qquad (107) \\ & V^F(b,\theta,\tau,\underline{x}) \geq V^{F0}(b,\theta). \qquad (108)\end{aligned}$$

As before, the supranational authority maximizes a weighted sum of household utilities in the two countries, subject to the participation constraints of the two policymakers. Policymaker D faces fiscal rules $\{\bar{b}^D(\theta)\}_{\theta\in\Theta}$ both with and without the partial banking union.

Even if the fiscal rule is not binding for the policymaker for certain values of b, the problem for the supranational authority leads to a similar result as Proposition 4:

Proposition 11 *There exists threshold $\eta^{**d} > 0$ such that a partial banking union with domestic fiscal rules does not achieve a Pareto improvement compared to no banking union whenever $\eta < \eta^{**d}$.*

Proof. In Section D.4.5. ■

As in the two-period model, domestic fiscal rules are not, by themselves, sufficient in order to restore efficiency. The main result is the product of two distortions – one domestic due to rent-seeking and one supranational due to the lack of direct household participation when the supranational agreement is decided. The fiscal rules cannot fully remove both of these distortions. Hence, the Pareto inefficiency can still exist.

D.3.2 Supranational Fiscal Rules.

Consider the case in which fiscal rules are decided by the supranational authority at the same time as the transfer and reinvestment level. These fiscal rules are denoted $\left\{\overline{B}_t^i(\theta)\right\}_{t=0,..,\infty;\theta\in\Theta}$. Unlike the domestic fiscal rules, the supranational fiscal rules can be made contingent on outstanding debt every period, since they are decided at the same time as the terms of the partial banking union. Faced with fiscal rules $\{\overline{B}^D(\theta)\}_{\theta\in\Theta}$ imposed in country D, policymaker D solves the equivalent of problem (102). The problem for the supranational authority is given by:

$$\begin{aligned} S^{SR}(b,\theta) \quad = \quad & \max_{\underline{x},\tau,\overline{B}^D,\overline{B}^F} \{\eta\left[u(c^D(x^D,x^F,\theta))+w(g^D)\right] \\ & +(1-\eta)\left[u(c^F(x^F,x^D,\theta))+w(g^F)\right] \\ & +\beta\mathbb{E}\left[S^{SR}(b',\theta')\right]\} \end{aligned} \tag{109}$$

subject to

$$\overline{V^D}(b,\theta,\tau,\underline{x},\overline{B}^D) \geq V^{D0}(b,\theta), \tag{110a}$$

$$\overline{V^F}(b,\theta,\tau,\underline{x},\overline{B}^F) \geq V^{F0}(b,\theta), \tag{110b}$$

$$\overline{B}^i \in [\underline{b}^i, b^{i,MAX}]. \tag{110c}$$

Constraint (110a) represents the participation constraint for policymaker D, given the outside option of no partial banking union and no debt limit. Constraint (110b) is the participation constraint for policymaker F, who similarly faces an outside option of no partial banking union and no debt limit. Finally, constraint (110c) gives the feasible bounds on debt.

The policymakers retain some discretion in choosing current period policies. The supranational authority still limits debt increases, but it cannot offer incentives for increasing recapitalizations without these incentives also acting towards increasing rents. This leads us to the dynamic equivalent of Proposition 5:

Proposition 12 *With supranational fiscal rules, there exits $\overline{\eta^{B*}} > 0$ such that a partial banking union does not achieve a Pareto improvement if $\eta < \overline{\eta^{B*}}$; it increases F household welfare, but it lowers D household welfare.*

Proof. In Section D.4.6. ■

Proposition 12 shows that supranational fiscal rules cannot be used to generate a Pareto improvement, even when the supranational authority has the incentive to set binding fiscal rules. The supranational requirement for more recapitalizations leads to less spending on the public good in both countries, and to more rent seeking in country D. The increase in rents cannot be stopped through fiscal rules, since they only affect the inter-period allocation of resources, not the intra-period policy decision.

D.3.3 Fiscal rules together with political reforms.

As in the two-period model, consider the case in which fiscal rules can be set domestically conditional on the participation of the policymakers in the partial banking union, and the supranational authority can condition transfers on changes in γ^D. Each period, the supranational authority can propose the transfer τ_t, the minimum reinvestment $\underline{x}_t$ and a value $\gamma_t^D \leq \gamma^{D*}$, where γ^{D*} represents the maximum achievable γ^D. The households in country D set a debt limit $\bar{b}_t^D \in [\underline{b}^D, b^{D,MAX}]$ conditional of the country joining the partial banking union. A key assumption is that the outside option includes no fiscal rule and no change in γ^D.

We require the condition that fiscal rules are sufficiently constraining to policymaker D:

Lemma 8 *There exist functions $w(g)$ such that a partial banking union is not formed whenever $\bar{b}_t^D = \underline{b}^D$, $\forall t \geq 0$.*

Proof. In Section D.4.7. ∎

Under Lemma 8, a partial banking union with the most restrictive fiscal rule would not be implementable, as it would be too costly for the policymakers. We can then derive the following result:

Proposition 13 *Under Condition (8), a partial banking union achieves a Pareto improvement in household welfare if country D households set fiscal rules each period and supranational transfers each period can be conditioned on an increase in γ_t^D to some $\gamma^{D*} < 1$, $t = 0, ..., \infty$.*

Proof. In Section D.4.8. ■

Proposition 13 shows that the results from the two-period model can be extended to an infinite horizon model, but under much stricter conditions. Conditional fiscal rules must be set each period, and therefore they are also a function of the outstanding debt in country D, not only on the realization of the shock θ_t. This shows that in the dynamic environment, it becomes more difficult to provide incentives against rent seeking. In the dynamic setting, a closer integration of fiscal and banking policies is necessary in order to obtain a Pareto improvement.

D.4 Proofs for the Dynamic Model

D.4.1 Proof of Lemma 6

Below, I derive the conditions on the equilibrium policy functions $\tau(b,\theta)$ and $\underline{x}(b,\theta)$ under which $V(b,\theta)$ is concave and differentiable. I then derive the conditions on $b'(\tau,\underline{x}|\theta)$ under which concavity and differentiability of V implies concavity and differentiability of S. Finally, I derive the conditions on the utility functions that allow for these properties of the policy functions. This shows that an equilibrium exists in which the value functions are concave and differentiable.

Consider first the problem with respect to for $b^D \in (\underline{b}^D, b^{D,MAX})$.

Step 1

A feasible set $\{\tau, \underline{x}, b^D, b^{D\prime}\}$ given θ is an allocation that satisfies the conditions that $\underline{x} \leq e^D - b^D + \tau + \beta b^{D\prime}$ and $b^{D\prime} \in \left[\underline{b}^D, b^{D,MAX}\right]$. A feasible set of current-period policies for policymaker D, $\{r^{Df}, g^{Df}, x^{Df}\}$ associated with an allocation $\{\tau, \underline{x}, b^D, b^{D\prime}\}$ and θ must satisfy the budget constraint, the intervention constraint, and the constraints imposed by the upper/lower bounds on policies:

$$
\begin{aligned}
r^{Df} + x^{Df} + g^{Df} &\leq e^D - b^D + \tau + \beta b^{D\prime}, && (111)\\
r^{Df} + x^{Df} &\geq \underline{x}, && (112)\\
x^{Df} &\leq \theta I^D, && (113)\\
g^{Df} &\geq 0, x^{Df} \geq 0, r^{Df} \geq 0. && (114)
\end{aligned}
$$

Let $\Lambda(\tau, \underline{x}, b^D, b^{D\prime}, \theta)$ denote the set of feasible current-period policies given $\left\{\tau, \underline{x}, b^D, b^{D\prime}, \theta\right\}$. Let $\left\{r^D, g^D, x^D\right\} \in \Lambda$ be the solution to the intra-period maximization problem faced by policymaker D. Let r^{D0} and x^{D0} be the policies chosen by policymaker D when $\underline{x} = 0$ and $\tau = 0$. Then $r^{D0} + x^{D0} \leq r^D + x^D$, because it is a weakly dominated strategy for the supranational authority to set the intervention bound $\underline{x}$ to at least what the politician would choose without the bound. Therefore, constraint (112) holds with equality in equilibrium. Then, $g^D = e^D - b^D + \tau - \underline{x} + \beta b^{D\prime}$, so g^D is a concave function of debt b^D if $\tau - \underline{x}$ is also a concave function of debt b^D. Then, $w(g^D(b^D, b^{D\prime}))$ is a concave, non-decreasing function of a concave function, and therefore it is also a concave function of debt.

Condition 1 *The function $\tau - \underline{x}$ is a concave of debt b^D.*

The first-order conditions to the policymaker's problem lead to

$$\gamma^D \sigma^D R u'(c^D(x^D, x^F, \theta)) = \left(1 - \gamma^D\right) v'(r^D, \theta).$$

Also, $\gamma^D u(c^D(x^D, x^F, \theta)) + \left(1 - \gamma^D\right) v(r^D)$ is a concave function of $\underline{x}$, since the following conditions emerge from the policymaker's problem:

$$\frac{\partial^2 x^D}{\partial \underline{x}^2} + \frac{\partial^2 r^D}{\partial \underline{x}^2} = 0$$

and

$$\gamma^D \left(\sigma^D R\right)^2 u''(c^D) \left(\frac{\partial x^D}{\partial \underline{x}}\right)^2 + \gamma^D \sigma^D R u'(c^D) \frac{\partial^2 x^D}{\partial \underline{x}^2}$$
$$+ \left(1-\gamma^D\right) v''(r^D) \left(\frac{\partial r^D}{\partial \underline{x}}\right)^2 + \left(1-\gamma^D\right) v'(r^D) \frac{\partial^2 r^D}{\partial \underline{x}^2} < 0, \qquad (115)$$

which given the concavity of $u(c^D(x^D, x^F, \theta))$ and $v(r)$ implies

$$\frac{\partial^2 \left[\gamma^D u(c^D(x^D, x^F, \theta)) + (1-\gamma^D) v(r^D)\right]}{\partial \underline{x}^2} < 0.$$

Then, if $\underline{x}(b^D)$ is concave and increasing, $\gamma^D u(c^D(x^D, x^F, \theta)) + (1-\gamma^D) v(r^D)$ is also concave.

Condition 2 *The policy function $\underline{x}$ is a concave of debt b^D.*

Step 2

a) Concavity of the value function:

Assuming concavity of $\mathbb{E}[V^D(b', \theta', \tau'(b', \theta'), \underline{x}(b', \theta'))]$ we can show concavity of $V^D(b, \theta, \tau, \underline{x})$ by induction.

Consider two feasible values $b_1^D, b_2^D \in [\underline{b}^D, b^{D,MAX}]$, and $b_3^D = \vartheta b_1^D + (1-\vartheta) b_2^D$, $\vartheta \in (0,1)$. Then, the supranational policies are given by functions $\tau_1 = \tau(b_1, \theta)$, $\underline{x}_1 = \underline{x}(b_1, \theta)$, $\tau_2 = \tau(b_2, \theta)$, $\underline{x}_2 = \underline{x}(b_2, \theta)$, $\tau_3 = \tau(b_3, \theta)$, $\underline{x}_3 = \underline{x}(b_3, \theta)$, where $b_1 = (b_1^D, b^F)$, $b_2 = (b_2^D, b^F)$, and $b_3 = (b_3^D, b^F)$. Let

$$\begin{aligned}\left\{x_1^D, r_1^D, g_1^D, b_1^{D\prime}\right\} &= \arg\max V^D(b_1, \theta, \tau_1, \underline{x}_1),\\ \left\{x_2^D, r_2^D, g_2^D, b_2^{D\prime}\right\} &= \arg\max V^D(b_2, \theta, \tau_2, \underline{x}_2).\end{aligned}$$

Let $b_3^{D\prime} = \vartheta b_1^{D\prime} + (1-\vartheta) b_2^{D\prime}$, and $\left\{x_3^D, r_3^D, g_3^D\right\} = \arg\max (1-\gamma^D) v(r^D) + \gamma^D u(c^D(x^D, x^F, \theta)) + \gamma^D w(g^D)$, subject to constraints (111)-(114), given b_3^D, $b_3^{D\prime}$, τ_3, $\underline{x}_3$.

Value $b_3^{D\prime}$ is feasible given that the set $\Gamma \equiv [\underline{b}^D, b^{D,MAX}]$ is compact, and $\left\{x_3^D, r_3^D, g_3^D\right\}$ is feasible given the above maximization problem. Since

$u^P(b^D, b^{D\prime}, \theta) \equiv \gamma^D w(g^D) + \gamma^D u(c^D(x^D, x^F, \theta)) + \left(1 - \gamma^D\right) v(r^D)$ is concave under the conditions from Step 1:

$$\begin{aligned} V^D(b_3, \theta) &\geq u^P(b_3^D, b_3^{D\prime}, \theta) + \beta \mathbb{E}[V^D(b_3', \theta', \tau', \underline{x}')] \\ &\geq \vartheta \left[u^P(b_1^D, b_1^{D\prime}, \theta) + \beta \mathbb{E}[V^D(b_1^{D\prime}, \theta', \tau', \underline{x}')]\right] \\ &\quad + (1 - \vartheta) \left[u^P(b_2^D, b_2^{D\prime}, \theta) + \beta \mathbb{E}[V^D(b_2', \theta', \tau', \underline{x}')]\right]. \end{aligned}$$

By induction, the value function $V^D(b_3, \theta, \tau, \underline{x})$ is therefore concave.

b) Differentiability of the politician's value function:

The policy function is continuous, given the compact set Γ. The implicit utility function

$$\begin{aligned} u^p(b^D, b^{D\prime}, \theta) &= (1 - \gamma^D) v(r^D(b^D, b^{D\prime}, \theta)) \\ &\quad + \gamma^D u(c^D(x^D(b^D, b^{D\prime}, \theta), x^F, \theta)) + \gamma^D w(g^D(b^D, b^{D\prime}, \theta)) \end{aligned}$$

is concave and differentiable in b^D. It then follows by Lemma 1 of Benveniste and Scheinkman (1979) that $V(b, \theta, \tau, \underline{x})$ is differentiable with respect to b^D over $(\underline{b}^D, b^{D,MAX})$.

Part 3

Consider now the value function for the supranational authority. Denote the instantaneous utility function for the supranational authority as

$$\begin{aligned} u^S(b, b', \theta, \tau, \underline{x}) &\equiv \eta u(c^D(x^D(b, b', \theta, \tau, \underline{x}), x^F, \theta)) + \eta w(g^D(b, b', \theta, \tau, \underline{x})) \\ &\quad + (1 - \eta) u(c^F(x^F, x^D(b, b', \theta, \tau, \underline{x}), \theta)) \\ &\quad + (1 - \eta) w(g^F(\theta, \tau, \underline{x})). \end{aligned}$$

Given Condition 2, a sufficient condition for $\eta u(c^D) + (1 - \eta) u(c^F)$ to be a concave function of debt is that $x^D(\underline{x})$ is weakly concave. This requires:

$$\frac{\partial^2 x^D(\underline{x})}{\partial \underline{x}^2} < 0,$$

so, using the first-order conditions to policymaker D's problem

$$\begin{aligned}\frac{\partial^2 x^D(\underline{x})}{\partial \underline{x}^2} &= -\left(1+\frac{\gamma^D(\sigma^D R)^2 u''(c^D)}{(1-\gamma^D)v''(r^D)}\right)^{-2} \cdot \\ &\cdot \frac{\gamma^D(\sigma^D R)^2 u''(c^D)}{(1-\gamma^D)v''(r^D)}\left[\left(\frac{\sigma^D R u'''(c^D)}{u''(c^D)}\right.\right. \\ &+ \left.\frac{v'''(r^D)}{v''(r^D)}\right)\left(1+\frac{\gamma^D(\sigma^D R)^2 u''(c^D)}{(1-\gamma^D)v''(r^D)}\right)^{-1} \left. -\frac{v'''(r^D)}{v''(r^D)}\right].\end{aligned}$$

A sufficient condition for $\frac{\partial^2 x^D(\underline{x})}{\partial \underline{x}^2} < 0$ is that

Condition 3

$$\frac{u'''(c^D)}{\gamma^D \sigma^D R u''(c^D)^2} < \frac{v'''(r^D)}{(1-\gamma^D)v''(r^D)^2}.$$

Also, the maximization problem for the supranational authority is a concave function of τ, so a sufficient condition for concavity with respect to debt is for τ to be concave. Then, $u^S(b^D, b^{D\prime})$ is concave.

Condition 4 *The function τ is a concave function of b.*

Step 4

Consider feasible values $\left\{b_1^D, \tau_1, \underline{x}_1\right\}$ and $\left\{b_2^D, \tau_2, \underline{x}_2\right\}$. Let

$$\left\{b_3^D, \tau_3, \underline{x}_3\right\} = \vartheta \left\{b_1^D, \tau_1, \underline{x}_1\right\} + (1-\vartheta)\left\{b_2^D, \tau_2, \underline{x}_2\right\},$$

$\forall \vartheta \in (0,1)$. Then, $\{b_3^D, \tau_3, \underline{x}_3\}$ is feasible and satisfies all constraints. Due to the concavity of $u^S(b, b', \theta, \tau(b, b'), \underline{x}(b, b'))$, the concavity of $S(b, \theta)$ follows by induction, analogous to the proof in Step 2 : $S(b_3, \theta) \geq \vartheta S(b_1, \theta) + (1-\vartheta)S(b_2, \theta)$. Therefore, $S(b, \theta)$ is concave.

Part 5

Consider the sequence of feasible values b^{Dj} such that $b^{Dj} \to b^D$; then there is also a corresponding sequence $\{\tau^j, \underline{x}^j\}$ which converges to $\{\tau, \underline{x}\}$, since the instantaneous utility $u^S(b^D, b^{D\prime}, \tau, \underline{x})$ is continuous in $\{\tau, \underline{x}\}$. Given the policy correspondence $G(b^{Dj}, \tau^j, \underline{x}^j)$, we want to show that if $b^{Dj\prime} \in G(b^{Dj}, \tau^j, \underline{x}^j)$, then $\exists$ a convergent subsequence $b^{Dnj\prime} \to b^{D\prime}$ with $b^{D\prime} \in G(b^D, \tau, \underline{x})$. Since $\{\tau^j, \underline{x}^j\}$ are defined over compact sets, $\{\tau, \underline{x}\}$ is feasible. Moreover, it implies a convergent subsequence $\left\{b^{Dnj\prime}\right\}$ must exist. Then, by the Dominated Convergence Theorem, $b^{D\prime} = G(b^D, \tau, \underline{x})$. Therefore, the policy function is continuous.

Part 6

Consider the sequence $\{\tau, \underline{x}\}$ associated with the debt $b = \left(b^D, b^F\right), \left(b^D, b^F\right) \in (\underline{b}^D, b^{D,MAX}) \times (\underline{b}^F, b^{F,MAX})$. Then, with $S(b, \theta)$ concave and a continuous policy function, by the argument of Lemma 1 in Benveniste and Scheinkman (1979), $S(b, \theta)$ is differentiable in b^D over $(\underline{b}^D, b^{D,MAX})$.

Part 7

We now show that Conditions 1-4 can be satisfied in equilibrium. First, from policymaker D's problem, $b^{D\prime}$ is a decreasing function of $\tau - \underline{x}$, which, by the inverse function properties, means that $(\tau - \underline{x})$ being concave (Condition 1) requires that $b^{D'}$ is a concave function of $\tau - \underline{x}$. This then implies that $g^H(\tau - \underline{x})$ is also concave, given the policymaker's budget constraint and the first-order conditions to the policymaker's problem, (98)-(101). Moreover, since $b^{D\prime}$ is a decreasing function of $\tau - \underline{x}$, if $b^{D\prime}$ is a concave function of $\tau - \underline{x}$, then $\tau - \underline{x}(\tau)$ must be a convex function of τ, which requires that $\underline{x}(\tau)$ is concave.

- If the participation constraint for policymaker D binds in equilibrium, then $\forall i \in \{1, 2, 3\}$:

$$\begin{aligned}
&\left(1-\gamma^D\right) v(r_i^D) + \gamma^D u(c^D(x_i^D, x^F, \theta)) + \gamma^D w(g_i^D) \\
&\quad + \beta \mathbb{E} V^D(b_i', \theta', \tau'(b', \theta'), \underline{x}'(b', \theta')) = \\
&\quad \left(1-\gamma^D\right) v(r^{D0}) + \gamma^D u(c^D(x^{D0}, x^{F0}, \theta)) + \gamma^D w(g^{H0}) \\
&\qquad + \beta \mathbb{E} V^D(b'^0, \theta', \tau'(b'^0, \theta'), \underline{x}'(b'^0, \theta')). \quad (116)
\end{aligned}$$

Condition (116) allows us to derive $\underline{x}(\tau)$ given that the policies α_i^D are chosen according to first-order conditions (98)-(101). Let $\tau_3 = \vartheta\tau_1 + (1-\vartheta)\tau_2$, and consider the case in which $b'(\tau, \underline{x}(\tau))$ is concave. Then, $b'_3 \geq \vartheta b'_1 + (1-\vartheta)b'_2$, where $b'_i \equiv b'(\tau_i)$, $\forall i = 1,2,3$. Since $\mathbb{E}\left[V^D(b^{D\prime})\right]$ is decreasing in $b^{D\prime}$, $\mathbb{E}\left[V^D(b_3^{D\prime})\right] < \mathbb{E}\left[V^D\left(\vartheta b_1^{D\prime} + (1-\vartheta)b_2^{D\prime}\right)\right]$ and since it is concave, $\mathbb{E}\left[V'(b_3^{D\prime})\right] < \mathbb{E}\left[V'\left(\vartheta b_1^{D\prime} + (1-\vartheta)b_2^{D\prime}\right)\right]$.

Changes in τ in the current period do not change the outside option of the politician, so condition (116) implies:

$$\begin{aligned}
&\left(1-\gamma^D\right)v(r_3^D) + \gamma^D u(c^D(x_3^D, x^F, \theta) + \gamma^D w(g_3^D) \\
&+\beta\mathbb{E}\left[V^D(b'_3, \theta', \tau', \underline{x}')\right] = \vartheta\left[\left(1-\gamma^D\right)v(r_1^D)\right. \\
&+\gamma^D u(c^D(x_1^D, x^F, \theta)) + \gamma^D w(g_1^D) + \beta\mathbb{E}\left[V^D(b_1^{D\prime}, \theta')\right]\Big] \\
&\quad +(1-\vartheta)\left[\left(1-\gamma^D\right)v(r_2^D) + \gamma^D u(c^D(x_2^D, x^F, \theta))\right. \\
&\qquad\qquad + \gamma^D w(g_2^D) + \beta\mathbb{E}\left[V^D(b'_2, \theta', \tau', \underline{x}')\right]\Big],
\end{aligned} \tag{117}$$

with $x_i^D \equiv x(\tau_i)$, $r_i^D = r(\tau_i)$, $g_i^D = g(\tau_i)$, $\forall i = 1,2,3$ and $x^F = \theta I^F$.

From the first-order conditions, $\mathbb{E}\left[-V^{D\prime}(b_i^{D\prime})\right] = \gamma^D w'(g_i^D)$, so $w'(g_3^D) > w'\left(\vartheta g_1^D + (1-\vartheta)g_2^D\right)$. Since $w(g^D)$ is a concave function, this requires $g_3^D(\tau_3) < \vartheta g_1^D(\tau_1) + (1-\vartheta)g_2^D(\tau_2)$, so $g^D(\tau)$ is convex. Yet, given the budget constraint (88a), this implies:

$$g^D(\tau) - \beta b^{D\prime}(\tau) = \tau - \underline{x}(\tau), \tag{118}$$

so

$$\frac{\partial^2 g^D(\tau)}{\partial^2\tau} - \beta\frac{\partial^2 b^{D\prime}(\tau)}{\partial^2\tau} = -\frac{\partial^2\underline{x}(\tau)}{\partial\tau^2}. \tag{119}$$

Therefore,

$$-\frac{\partial^2\underline{x}(\tau)}{\partial\tau^2} \geq 0, \tag{120}$$

and $\underline{x}(\tau)$ is concave. Then, from the supranational authority's maximization problem, $\tau(b)$ is the solution to the maximization of a concave function over a convex set, so $\tau(b)$ is concave. Conditions 1, 2 and 4 follow.

- If the participation constraint for policymaker D does not bind in equilibrium, then the first-order conditions to the supranational authority's problem yield:

$$\left[\eta\sigma^D Ru'(c^D) + (1-\eta)\left(1-\sigma^D\right) Ru'(c^F)\right]\frac{\partial x^D}{\partial \underline{x}} = \eta w'(g^D) \\ -\beta\mathbb{E}\left[\frac{\partial S'(b',\theta',\tau,\underline{x})}{\partial b^{D\prime}}\right]\frac{\partial b^{D\prime}}{\partial \underline{x}}.$$

  Applying the Envelope Theorem in the above condition, we obtain the expression for $\frac{\partial^2 \underline{x}(\tau)}{\partial \tau^2}$ as a function of $v(r^D)$, $u(c^D)$, and $w(g^D)$ consistent with

$$-\frac{\partial^2 \underline{x}(\tau)}{\partial \tau^2} \geq 0.$$

  Thus, there exist concave utility functions $v(r^D)$, $u(c^D)$, and $w(g^D)$ such that conditions 1-4 hold, and $V(b,\theta,\tau,\underline{x})$ and $S(b,\theta,\tau,\underline{x})$ are concave and differentiable over $b^D \in (\underline{b}^D, b^{D,MAX})$.

The proof for concavity and differentiation with respect to $b^F \in (\underline{b}^F, b^{F,MAX})$ is analogous to the above.

D.4.2 Proof of Proposition 9

Given Lemma 6, the proof is similar to the proof to Proposition 2.

Assumption 3 ensures that full recapitalizations ($x^F = \theta I^F$) are provided in the F country every period even if transfers are made to the D country.

Step 1. *The policymakers' problem*

Consider a partial banking union with terms τ and $\underline{x}$ in the current period. Denote by $\lambda(b,\theta)$, $\varkappa(b,\theta)$, and $\varphi(b,\theta)$ the Lagrange multipliers on constraints (88a), (88b), and (88d), respectively. The first-order conditions for an internal solution with respect to r^D, g^D, and $b^{D\prime}$ are:

$$\begin{aligned}
\lambda(b,\theta) - \varkappa(b,\theta) &= (1-\gamma^D)v'(r^D), && (121a)\\
\lambda(b,\theta) - \varkappa(b,\theta) + \varphi(b,\theta) &= \gamma^D\sigma^D R u'(c^D(x^D, x^F, \theta)), && (121b)\\
\lambda(b,\theta) &= \gamma^D w'(g^D), && (121c)\\
\lambda(b,\theta) &= \mathbb{E}\left[-\frac{\partial V^D(b',\theta',\tau',\underline{x}')}{\partial b^{D\prime}}\right]. && (121d)
\end{aligned}$$

The maximization problem for policymaker F given $\{\tau, \underline{x}\}$ and Assumption 3 leads to

$$\begin{aligned}
\gamma^F w'(g^F) &= (1-\gamma^F)v'(r^F), && (122a)\\
x^F &= \theta I^F, && (122b)\\
\gamma^F w'(g^F) &= \mathbb{E}\left[-\frac{\partial V^F(b',\theta',\tau',\underline{x}')}{\partial b^{F\prime}}\right]. && (122c)
\end{aligned}$$

Step 2. *The supranational authority's problem*

The supranational authority sets $\tau \geq 0$ and $\underline{x} \geq 0$ in the current period in order to maximize (95) given (96) and (97). The minimum reinvestment requirement is $r^D + x^D \geq \underline{x}$. Setting $\underline{x}$ at least equal to the policymaker's unconstrained choices is a weakly dominant strategy, as shown in Lemma 6. Then, so $\forall t$, $r_t^D \geq r_t^{D0}$ and $v(r_t^D) \geq v(r_t^{D0})$ Then, $\forall s \geq 0$,

$$\mathbb{E}\sum_{t=s}^{\infty}\beta^{t-s}(1-\gamma^D)v(r_t^D) \geq \mathbb{E}\sum_{t=s}^{\infty}\beta^{t-s}(1-\gamma^D)v(r_t^{D0}). \qquad (123)$$

The expected policymaker utility under a partial banking union:

$$\begin{aligned}
V_s^D = \max_{\{r_t^D, g_t^D, x_t^D\}} \mathbb{E}_\theta \Bigg\{ \sum_{t=s}^{\infty}\beta^{t-s}\big[(1-\gamma^D)v(r_t^D) \\
+\gamma^D w(g_t^D) + \gamma^D u(c^D(x_t^D, x_t^F, \theta))\big]\Bigg\},
\end{aligned}$$

and the policymaker utility under no partial banking union in any period is:

$$
\begin{aligned}
V_s^{D0} \;=\; & \max_{\{r_t^{D0}, g_t^{D0}, x_t^{D0}\}} \mathbb{E}_\theta \left\{ \sum_{t=s}^{\infty} \beta^{t-s} \left[(1-\gamma^D) v(r_t^{D0}) \right.\right. \\
& \left.\left. + \gamma^D w(g_t^{D0}) + \; \gamma^D u(c^D(x_t^{D0}, x_t^{F0}, \theta)) \right] \right\}.
\end{aligned}
$$

Constraint (96) holds if $V_s^D \geq V_s^{D0}$. This implies

$$
\begin{aligned}
& \gamma^D \mathbb{E} \sum_{t=s}^{\infty} \beta^{t-s} \left[w(g_t^D) + u(c^D(x_t^D, x_t^F, \theta)) \right] \\
& -\gamma^D \mathbb{E} \sum_{t=s}^{\infty} \beta^{t-s} \left[w(g_t^{D0}) + u(c^{D0}(x_t^{D0}, x_t^{F0}, \theta)) \right] \geq \\
& \qquad (1-\gamma^D) \mathbb{E} \sum_{t=s}^{\infty} \beta^{t-s} (1-\gamma^D) v(r_t^{D0}) \\
& \qquad -(1-\gamma^D) \mathbb{E} \sum_{t=s}^{\infty} \beta^{t-s} (1-\gamma^D) v(r_t^{D}). \qquad (124)
\end{aligned}
$$

Step 3. *We show that if for some* $\eta^C \in (0,1)$, *constraint (96) does not bind, then it does not bind* $\forall \eta \geq \eta^C$.

Assume there exists a value $\eta^C \in (0,1)$ at which (96) does not bind.

Case A. If $\underline{x} = \underline{x}^* = \theta I^D + r^{D*}$, with r^{D*} defined implicitly by $\left(1-\gamma^D\right) v'(r^{D*}) = \gamma^D \sigma^D R u'(\theta I^D, \theta I^F)$. In this case, there is full recapitalization in country D: $x^D = \theta I^D$.

Let ι denote the Lagrange multiplier on constraint (97). Then, the first order-condition that determines τ is

$$
\begin{aligned}
& \eta w'(g^D) \frac{\partial g^D}{\partial \tau} + (1-\eta) w'(g^F) \left(-\frac{\partial g^F}{\partial \tau} \right) (1 + \gamma^F \iota) \\
& + \iota (1-\gamma^F) v'(r^F) \left(-\frac{\partial r^F}{\partial \tau} \right) + \beta \mathbb{E} \left[\frac{\partial S(b^{D\prime}, b^{F\prime}, \theta)}{\partial b^{D\prime}} \right] \frac{\partial b^{D\prime}}{\partial \tau} \\
& + \beta \mathbb{E} \left[\frac{\partial S(b^{D\prime}, b^{F\prime}, \theta)}{\partial b^{F\prime}} \right] \frac{\partial b^{F\prime}}{\partial \tau} = 0. \qquad (125)
\end{aligned}
$$

Given this condition, an increase in η would increase τ. For country D, this is equivalent to increasing e^D, so it implies that

$$\frac{\partial V_s^D}{\partial \eta} > 0. \tag{126}$$

Policymaker D's utility is increasing; hence, (96) does not bind $\forall \eta \geq \eta^C$.

Case B: there is an interior solution for $\underline{x}$.

The first-order conditions to the supranational authority's maximization problem, in case of an internal solution, are:

$$\left[(1-\eta)\left(1-\sigma^D\right) Ru'(c^F)(1+\gamma^F \iota) + \eta\sigma^D Ru'(c^D)\right] \frac{\partial x^D}{\partial \underline{x}} = \eta w'(g^D)\frac{\partial g^D}{\partial \underline{x}} + \beta \mathbb{E}\left[\frac{\partial S(b^{D\prime}, b^{F\prime}, \theta)}{\partial b^{D\prime}}\right] \frac{\partial b^{D\prime}}{\partial \underline{x}}, \tag{127}$$

$$\eta w'(g^D)\frac{\partial g^D}{\partial \tau} + (1-\eta) w'(g^F)\left(\frac{\partial g^F}{\partial \tau}\right)(1+\gamma^F\iota) + \iota(1-\gamma^F)v'(r^F)\frac{\partial r^F}{\partial \tau} + \beta\frac{\partial S(b^{D\prime}, b^{F\prime}, \theta)}{\partial b^{D\prime}}\frac{\partial b^{D\prime}}{\partial \tau} + \beta\mathbb{E}\left[\frac{\partial S(b^{D\prime}, b^{F\prime}, \theta)}{\partial b^{F\prime}}\right]\frac{\partial b^{F\prime}}{\partial \tau} = 0. \tag{128}$$

From (121a)-(121d), $\frac{\partial x^D}{\partial \underline{x}} > 0$, $\frac{\partial r^D}{\partial \underline{x}} > 0$, $\frac{\partial g^D}{\partial \underline{x}} - \beta\frac{\partial b^{D\prime}}{\partial \underline{x}} = -1$, $\frac{\partial g^D}{\partial \tau} - \beta\frac{\partial b^{D\prime}}{\partial \tau} = 1$. From (122a)-(122c), $0 < -\frac{\partial g^F}{\partial \tau} < 1$ and $0 < -\frac{\partial r^F}{\partial \tau} < 1$. Then, from (127) and (128) an increase in η implies $\frac{\partial \underline{x}}{\partial \eta} < 0$ and $\frac{\partial \tau}{\partial \eta} > 0$. But $\underline{x}$ and τ come into the policymaker's maximization problem through constraints (88a) and (88b), so

$$\frac{\partial \mathbb{E}\sum_{t=s}^{\infty}\beta^{t-s}\left[w(g_t^D) + u(c^D(x_t^D, x_t^F, \theta))\right]}{\partial \eta} > 0. \tag{129}$$

It follows that if constraint (96) does not bind at η^C, then it does not bind $\forall \eta \geq \eta^C$.

Step 4. *We show that if for some $\eta^B \in (0,1)$ constraint (96) binds, then it binds $\forall \eta \leq \eta^B$.*

Since $\underline{x}$ is at least as high as policymaker $D's$ policy choices, $r^D \geq r^{D0}$. If

(96) binds, then τ is inferred implicitly from this constraint:

$$\begin{aligned}&\left(1-\gamma^{D}\right) v(r^{D})+\gamma^{D} u(c^{D}(x^{D}, x^{F}, \theta))+\gamma^{D} w(g^{D}) \\ &+\beta \mathbb{E}\left[V^{D}(b', \theta', \tau'(b', \theta'), \underline{x}'(b', \theta'))\right]=V^{D0}(b, \theta)\end{aligned} \tag{130}$$

*Case A.*There is a corner solution for $\underline{x}=\underline{x}^{*}$, as defined in Step 3. Then, a decrease in η would not change the value of $\underline{x}$ or the value of τ. Hence, (2) binds $\forall \eta<\eta^{B}$.

Case B. If $\underline{x}<\underline{x}^{*}$. Constraint (96) binding implies that the first-order conditions (127) and (128) become

$$\begin{aligned}&\left[(1-\eta)\left(1-\sigma^{D}\right) R u'(c^{F})+\eta \sigma^{D} R u'(c^{D})\right] \frac{\partial x^{D}}{\partial \underline{x}} \\ &-\eta w'(g^{D}) \frac{\partial g^{D}}{\partial \underline{x}}-\beta \mathbb{E}\left[\frac{\partial S(b^{D\prime}, b^{F\prime}, \theta)}{\partial b^{D\prime}}\right] \frac{\partial b^{D\prime}}{\partial \underline{x}}>0,\end{aligned} \tag{131}$$

and

$$\begin{aligned}&\eta w'(g^{D}) \frac{\partial g^{D}}{\partial \tau}+(1-\eta) w'(g^{F})\left(-\frac{\partial g^{F}}{\partial \tau}\right)(1+\gamma^{F} \iota) \\ &+\beta \frac{\partial S(b^{D\prime}, b^{F\prime}, \theta)}{\partial b^{D\prime}} \frac{\partial b^{D\prime}}{\partial \tau}+\beta \mathbb{E}\left[\frac{\partial S(b^{D\prime}, b^{F\prime}, \theta)}{\partial b^{F\prime}}\right] \frac{\partial b^{F\prime}}{\partial \tau}<0.\end{aligned} \tag{132}$$

Then, a decrease in η keeps the constraint (96) binding.

Step 5 *We show that there exists $\eta^{B*} \in (0,1)$ such that constraint (96) binds for $\eta<\eta^{B*}$ and it does not bind for $\eta>\eta^{B*}$.*

If $\eta=0$, the supranational authority maximizes the utility of the F households only, so τ is minimized and $\underline{x}$ is maximized given constraint (96). At $\eta=0$, the first-order conditions to the supranational authority's problem are given by (131) and (132). The left-hand side of first-order condition (131) is strictly decreasing in η, and the left-hand side of condition (132) is strictly increasing in η. By the continuity of the utility functions it then follows that $\exists \eta^{B*}>0$ such that (131) and (132) hold with equality.

Step 6. *We show there exists $\eta^{*} \in (0,1)$ such that D household utility is the same as without a partial banking union.*

If $\eta = 0$, the supranational authority maximizes the utility of the F households only, so τ is minimized and $\underline{x}$ is maximized under the constraint that the participation constraint for the D government binds. If $\tau > 0$, then $r^D > r^{D0}$, so from (124),

$$\mathbb{E}\sum_{t=0}^{\infty}\beta^t\left[w(g_t^D)+u(c^D(x_t^D,x_t^F,\theta))\right] < \mathbb{E}\sum_{t=s}^{\infty}\beta^{t-s}\left[w(g_t^{D0})+u(c^{D0}(x_t^{D0},x_t^{F0},\theta))\right].$$

If $\eta = 1$, the supranational authority maximizes the utility of the D households, so the transfer τ will be at the maximum level at which the participation constraint for the F government holds. It then follows that

$$\mathbb{E}\sum_{t=0}^{\infty}\beta^t\left[w(g_t^D)+u(c^D(x_t^D,x_t^F,\theta))\right] > \mathbb{E}\sum_{t=0}^{\infty}\beta^{t}\left[w(g_t^{D0})+u(c^{D0}(x_t^{D0},x_t^{F0},\theta))\right].$$

Given the continuity of $U^D(x^D, x^F, g^D, g_1^D)$ and (129), there exists $\eta^* \in (0,1)$ such that at η^*,

$$\mathbb{E}\sum_{t=0}^{\infty}\beta^t\left[w(g_t^D)+u(c^D(x_t^D,x_t^F,\theta))\right] = \mathbb{E}\sum_{t=0}^{\infty}\beta^{t}\left[w(g_t^{D0})+u(c^{D0}(x_t^{D0},x_t^{F0},\theta))\right].$$

Then, for $\eta < \eta^*$,

$$\mathbb{E}\sum_{t=0}^{\infty}\beta^t\left[w(g_t^D)+u(c^D(x_t^D,x_t^F,\theta))\right] < \mathbb{E}\sum_{t=0}^{\infty}\beta^{t}\left[w(g_t^{D0})+u(c^{D0}(x_t^{D0},x_t^{F0},\theta))\right].$$

D.4.3 Proof of Lemma 7

The proof follows the same approach as the proof of Proposition 3 in Battaglini and Coate (2008). For ease of notation, I omit the country D superscript on debt b.

Let $\psi_t(b')$ denote the distribution function of the current level of debt at the beginning of period t. The distribution function $\psi_1(b')$ is exogenous and determined by the initial level of debt b_0. Let P denote the cumulative distribution of θ.

The correspondence implied by the politician's equilibrium choices and the supranational authority's equilibrium policy choices is given by $T : [\underline{b}^D, b^{D,MAX}] \times [\underline{b}^D, b^{D,MAX}] \longrightarrow \widehat{\Theta}$:

$$T(b, b') = \begin{cases} \underline{\theta} & \text{if } b' < b'^{\min}(b) \\ \min\left\{\theta \in \widehat{\Theta} : b'(b, \theta, \tau, \underline{x}) = b'\right\} & \text{if } b' \in [b'^{\min}(b), b'^{\max}(b)] \\ \overline{\theta} & \text{if } b' > b'^{\max}(b) \end{cases}$$

where

$$\begin{aligned} b'^{\min}(b) &= b'(b, \underline{\theta}, \tau(b, \underline{\theta}), \underline{x}(b, \underline{\theta})), \\ b'^{\max}(b) &= b'(b, \overline{\theta}, \tau(b, \overline{\theta}), \underline{x}(b, \overline{\theta})). \end{aligned}$$

The correspondence $T(b, b')$ gives the minimum combination of shocks under which the equilibrium new debt level would be b', given outstanding debt b. Then, the transition function is given by

$$H(b, b') = P(T(b, b')).$$

The function $H(b, b')$ gives the probability that next period's debt will be less than or equal to b' given the current outstanding debt b. Then, the distribution of debt at the beginning of any period $t \geq 2$ is defined inductively by

$$\psi_t(b') = \int_b H(b, b') d\psi_{t-1}(b).$$

The sequence of distributions $\psi_t(b')$ converges to distribution $\psi(b')$ if $\forall b \in [\underline{b}^D, b^{D,MAX}]$,

$\lim_{t\to\infty} \psi_t(b') = \psi(b')$. The limiting distribution is invariant if $\psi^*(b') = \int_b H(b, b') d\psi^*(b)$.

To prove that the sequence of distributions converges to a unique invariant distribution, we must first prove that $H(b, b')$ has the Feller Property, and that it is monotonic in b. By Theorem 12.12 in Lucas, Stokey and Prescott (1989), the following mixing condition must be satisfied: $\exists \epsilon > 0$ and $m \geq 1$, such that for any $b^* \in (\underline{b}^D, b^{D,MAX})$, $H^m(b^{D,MAX}, b^*) \geq \epsilon$ and $1 - H^m(\underline{b}^D, b^*) \geq \epsilon$, where the function $H^m(b, b')$ is defined inductively by $H^1(b, b') = H(b, b')$, and $H^m(b, b') = \int_z H(z, b') dH^{m-1}(b, z)$. This condition requires that starting from the highest level of debt $b^{D,MAX}$, we will end up at or below debt b^* with probability greater than ϵ after m periods, and if we start with the lowest level of debt, we will end up at or above b^* with probability greater than ϵ in m periods.

We use the monotonicity properties of the equilibrium policy functions, with respect to both b and the shock θ to show that the mixing condition is satisfied.

For any $b \in [\underline{b}^D, b^{D,MAX}]$ and $\theta \in \Theta$ define the sequence $\langle \phi_m(b, \theta) \rangle$ as follows: $\phi_0(b, \theta) = b$, $\phi_{m+1}(b, \theta) = b'(\phi_m(b, \theta), \theta)$, assuming that the supranational authority is following the equilibrium policies $\tau(b, \theta)$ and $\underline{x}(b, \theta)$. This means that $\phi_m(b, \theta)$ is the level of new debt starting from outstanding debt b, and assuming the same shock θ is repeated in periods 1 through m. By the setup of the model, there is a positive probability on each θ, therefore $P(\theta') - P(\theta) > 0$ for $\theta' > \theta$. This implies that, for a small λ^m, $H^m(b, \phi_m(b, \theta')) - H^m(b, \phi_m(b, \underline{\theta})) = (\xi \lambda^m)^{m-1} > 0$.

Using the above, it can be shown that $H^m(b^{D,MAX}, b^*) > 0$, for m sufficiently large. It suffices to show that, for m sufficiently large, $T(\phi_m(b^{D,MAX}, \underline{\theta}), b^*) > \underline{\theta}$. Then, for any such m, by continuity, there exits a small λ^m such that

$T(\phi_m(b^{D,MAX}, \underline{\theta} + \lambda^m), b^*) > \underline{\theta}$. So,

$$\begin{aligned}
H^m(b^{D,MAX}, b^*) &= \int_z H(z, b^*) dH^{m-1}(b^{D,MAX}, z) \\
&= \int_z P(T(z, b^*)) dH^{m-1}(b^{D,MAX}, z) \\
&\geq \int_{\phi_m(b^{D,MAX}, \underline{\theta})}^{\phi_m(b^{D,MAX}, \underline{\theta}+\lambda^m)} P(T(z, b^*)) dH^{m-1}(b^{D,MAX}, z) \\
&\geq \widehat{P}(T(\phi_m(b^{D.MAX}, \underline{\theta} + \lambda^m), b^*)) \cdot \\
&\quad \big[H^{m-1}(b^{D,MAX}, \phi_{m-1}(b^{D,MAX}, \underline{\theta} + \lambda^m)) \\
&\quad - H^{m-1}(b^{D,MAX}, \phi_{m-1}(b^{D,MAX}, \underline{\theta}))\big] \\
&\geq \widehat{P}(T(\phi_m(b^{D,MAX}, \underline{\theta} + \lambda^m), b^*))(\xi\lambda^m)^{m-1} > 0.
\end{aligned}$$

Suppose, to the contrary, that $T(\phi_m(b^{D,MAX}, \underline{\theta}), b^*) \leq \underline{\theta}$. Then, from the politician's first-order conditions, the realization of shock $\underline{\theta}$ implies that we obtain a decreasing sequence $\{\phi_m(b^{D,MAX}, \underline{\theta})\}_m$. Suppose that $\phi_m(b^{D,MAX}, \underline{\theta})$ converged to some $b^{**} > \underline{b}^D$. Then, in the limit, by the continuity of the policy functions, $\lim_{m\to\infty} g'(\phi_m(b^{D,MAX}, \underline{\theta}), \chi, \theta) = g'(b^\infty, \theta)$, for all θ. However, the policy g is strictly decreasing in θ, and by (100) and (101), b' must be decreasing, which contradicts the convergence assumption. The analogous argument can be made starting from $\underline{b}^D$, given repeated $\overline{\theta}$ shocks, to show that $1 - H^m(\underline{b}^D, b^*) \geq \epsilon$. Thus, the necessary conditions are satisfied for a unique invariant distribution.

D.4.4 Proof of Proposition 10

The following Lemma is first established:

Lemma 9 *There exists $b^{D*} \in [\underline{b}^D, b^{D,MAX}]$ such that if $\overline{b^D}(\theta)$ does not bind in policymaker D's problem $\forall b^D < \widetilde{b^D}$, and $\overline{b}^D(\theta)$ binds $\forall b^D > \widetilde{b^D}$.*

Proof. In Section D.4.9. ■

Denote by $U^{D0}(b^D, \theta)$ the value of the D household utility given the solution to policymaker D's maximization problem without the partial banking union and without the fiscal rule. Also, denote by $\overline{U^{D0}}(b^D, \theta, \bar{b}^D)$ the value of D household utility given the solution to policymaker D's maximization problem without a partial banking union, but with a fiscal rule $\bar{b}^D$. Let $f(b^D)$ denote the p.d.f. for the stationary distribution of b^D. The expected household utility is is

$$EU(\bar{b}^D) = \int_{\underline{b}^D}^{\widetilde{b^D}(\bar{b}^D)} U^{D0}(b^D, \theta) f(b^D) db^D + \int_{\widetilde{b^D}(\bar{b}^D)}^{b^{D,MAX}} \overline{U^{D0}}(b^D, \theta, \bar{b}^D) f(b^D) db^D.$$

The function $\overline{U^{D0}}(\theta, \bar{b}^D)$ is a continuous and differentiable function of $\bar{b}^D$, since $u(c)$, $w(g)$ and $v(r)$ are continuously differentiable. Also, $\widetilde{\theta}(\bar{b}^D)$ is differentiable since it is a continuous function of $u(\cdot)$, $w(\cdot)$ and $v(\cdot)$, derived from the solution $\bar{b}^D$ to policymaker D's problem. Taking the first-derivative with respect to $\bar{b}^D$, we obtain

$$\begin{aligned}
\frac{\partial EU(\bar{b}^D)}{\partial \bar{b}^D} &= U^{D0}(\widetilde{b^D}(\bar{b}^D), \theta) f(\widetilde{b^D}) \frac{\partial \widetilde{b^D}(\bar{b}^D)}{\partial \bar{b}^D} \\
&+ \int_{\widetilde{b^D}(\bar{b}^D)}^{\bar{\theta}} \frac{\partial \overline{U^{D0}}(b^D, \theta, \bar{b}^D) f(b^D)}{\partial \bar{b}^D} db^D \\
&- \overline{U^{D0}}(\widetilde{b^D}(\bar{b}^D), \theta, \bar{b}^D) f(\widetilde{b^D}) \frac{\partial \widetilde{b^D}(\bar{b}^D)}{\partial \bar{b}^D}.
\end{aligned}$$

Notice that for $b^D = \widetilde{b^D}$, we have $U^{D0}(\widetilde{b^D}, \theta) = \overline{U^{D0}}(\widetilde{b^D}, \theta, \bar{b}^D)$, so

$$\frac{\partial EU(\bar{b}^D)}{\partial \bar{b}^D} = \int_{\widetilde{\theta}(\bar{b}^D)}^{\bar{\theta}} \frac{\partial \overline{U^{D0}}(b^D, \theta, \bar{b}^D) f(b^D)}{\partial \bar{b}^D} db^D.$$

Then,

$$\frac{\partial^2 EU(\bar{b}^D)}{\partial \bar{b}^{D^2}} = \int_{\widetilde{\theta}(\bar{b}^D)}^{\bar{\theta}} \frac{\partial^2 \overline{U^{D0}}(b^D, \theta, \bar{b}^D) f(b^D)}{\partial \bar{b}^{D^2}} db^D - \frac{\partial \overline{U^{D0}}(\widetilde{b^D}, \theta, \bar{b}^D) f(\widetilde{b^D})}{\partial \bar{b}^D} \frac{\partial \widetilde{b^D}(\bar{b}^D)}{\partial \bar{b}^D}.$$

But $\frac{\partial \overline{U^{D0}}(\widetilde{b^D},\theta,\bar{b}^D)f(\widetilde{b^D})}{\partial \bar{b}^D} = 0$ since any increase in $\bar{b}^D$ would make the debt constraint slack. Therefore,

$$\frac{\partial^2 EU(\bar{b}^D)}{\partial \bar{b}^{D^2}} = \int_{\widetilde{\theta}(\bar{b}^D)}^{\bar{\theta}} \frac{\partial^2 \overline{U^{D0}}(b^D,\theta,\bar{b}^D)f(b^D)}{\partial \bar{b}^{D^2}} db^D.$$

Then
$$\frac{\partial^2 \overline{U^{D0}}(b^D,\theta,\bar{b}^D)f(b^D)}{\partial \bar{b}^{D^2}} < 0 \Leftrightarrow \frac{\partial^2 EU(\bar{b}^D)}{\partial \bar{b}^{D^2}} < 0.$$

The change in instantaneous household utility due to the change in the binding debt limit $\bar{b}^D$ is given by

$$\Psi \equiv \sigma^D R u'(c^D)\frac{\partial x^D}{\partial \bar{b}^D} + w'(g^D)\frac{\partial g^D}{\partial \bar{b}^D} - \beta w'(e^D - \bar{b}^D).$$

Then,

$$\begin{aligned}\frac{\partial \Psi}{\partial \bar{b}^D} = & \left[\left(\sigma^D R\right)^2 u''(c^D(x^D,x^F,\theta))\left(\frac{\partial x^D}{\partial \bar{b}^D}\right)^2 \right. \\ & \left. + w''(g^D)\left(\frac{\partial g^D}{\partial \bar{b}^D}\right)^2 + \beta w''(e^D - \bar{b}^D)\right] \\ & + w'(g^D)\left(-\frac{\partial^2 r^D}{\partial \bar{b}^{D^2}}\right)\end{aligned}$$

The first-order conditions to the Home government's problem give

$$\begin{aligned}\gamma^D \sigma^D R u'(c^D(x^D,x^F,\theta)) &= (1-\gamma^D)v'(r^D), \\ \gamma^D w'(g^D) &= (1-\gamma^D)v'(r^D).\end{aligned}$$

Then,

$$\begin{aligned}\gamma^D \left(\sigma^D R\right)^2 u''(c^D(x^D,x^F,\theta))\frac{\partial x^D}{\partial \bar{b}^D} &= (1-\gamma^D)v''(r^D)\frac{\partial r^D}{\partial \bar{b}^D}, \\ \gamma^D w''(g^D)\frac{\partial g^D}{\partial \bar{b}^D} &= (1-\gamma^D)v''(r^D)\frac{\partial r^D}{\partial \bar{b}^D},\end{aligned}$$

and

$$\frac{\partial x^D}{\partial \bar{b}^D} + \frac{\partial r^D}{\partial \bar{b}^D} + \frac{\partial g^D}{\partial \bar{b}^D} = \beta.$$

Combining the above conditions,

$$\frac{\partial r^D}{\partial \bar{b}^D} = \beta \left[1 + \frac{\left(1-\gamma^D\right) v''(r^D)}{\gamma^D \left(\sigma^D R\right)^2 u''(c^D)} + \frac{\left(1-\gamma^D\right) v''(r^D)}{\gamma^D w''(g^D)} \right]^{-1}.$$

So

$$\begin{aligned} \frac{\partial^2 r^D}{\partial \bar{b}^{D^2}} &= -\left(\frac{\partial r^D}{\partial \bar{b}^D}\right)^3 \frac{1}{\beta} \left(\frac{1-\gamma^D}{\gamma^D} v''(r^D)\right)^2 \cdot \\ &\quad \cdot \left(\frac{\gamma^D}{1-\gamma^D} \frac{v'''(r^D)}{v''(r^D)^2} \left(\frac{1}{\left(\sigma^D R\right)^2 u''(c^D)} + \frac{1}{w''(g^D)} \right) \right. \\ &\quad \left. - \frac{u'''(c^D)}{\left(\sigma^D R\right) u''(c^D)^3} - \frac{w'''(g^D)}{w''(g^D)^3} \right). \end{aligned}$$

By Assumption 4,

$$\frac{\partial^2 r^D}{\partial \bar{b}^{D^2}} \geq 0.$$

This, together with the concave increasing functions $u(c^D)$, $w(g^D)$ implies

$$\frac{\partial \Psi}{\partial \bar{b}^D} < 0.$$

Then, by induction,

$$\frac{\partial^2 \overline{U^{D0}}(b^D, \theta, \bar{b}^D)}{\partial \bar{b}^{D^2}} < 0$$

and

$$\frac{\partial^2 EU(\bar{b}^D)}{\partial \bar{b}^{D^2}} < 0.$$

Given the strict concavity of the objective function, it follows that the maximization problem has a unique solution $\bar{b}^{D*} \in [\underline{b}^D, b^{D,MAX}]$.

D.4.5 Proof of Proposition 11

The utility of country D households each period is concave and differentiable in b^{D0}, given Lemma 6. By induction, the value function $S(b, \theta, \overline{b^D})$ is concave. Moreover, the policy functions are continuous, and, by the standard arguments,[27] $S(b, \theta)$ is differentiable in b^D over $(\underline{b}^D, b^{D,MAX})$. The proof of the existence of η^{**} is then analogous to the proof of Proposition 9.

D.4.6 Proof of Proposition 12

Consider first a different problem, in which the supranational authority can directly choose the debt in each country, $b^{D\prime}$ and $b^{F\prime}$. In this case, the problem for the policymaker would be to simply choose the intra-period spending, leading to a value function

$$\begin{aligned} V^{Dr}(b, \theta, \tau, \underline{x}) &= \max_{\{r^D, x^D, g^D\}} (1-\gamma^D)v(r^D) + \gamma^D u(c^D(x^D, x^F, \theta)) \\ &\quad + \gamma^D w(g^D) + \beta \mathbb{E}_\theta [V^{Dr}(b', \theta, \tau, \underline{x})], \end{aligned} \tag{133}$$

subject to

$$r^D + x^D + g^D \leq e^H + \beta b^{D\prime} - b^D.$$

Since the politician is only doing an intra-period maximization, the function $V^{Dr}(b^D, \theta, \tau, \underline{x})$ is concave and differentiable in b^D over $(\underline{b}^D, b^{D,MAX})$. The problem for policymaker F is analogous.

The problem for the supranational authority is

$$\begin{aligned} S^r(b, \theta) &= \max_{\underline{x}, \tau, b^{D\prime}, b^{F\prime}} \{ \eta \left[u(c^D(x^D, x^F, \theta)) + w(g^D) \right] \\ &\quad + (1-\eta) \left[u(c^F(x^F, x^D, \theta)) + w(g^F) \right] \\ &\quad + \beta \mathbb{E} \left[S^r(b', \theta') \right] \} \end{aligned} \tag{134}$$

[27] Lemma 1 of Benveniste and Scheinkman (1979).

subject to

$$
\begin{aligned}
&(1-\gamma^D)v(r^D)+\gamma^D u(c^D(x^D,x^F,\theta))+\gamma^D w(g^D)\\
&+\beta\mathbb{E}[V^{Dr}(b',\theta,\tau,\underline{x})]\geq V^{D0}(b,\theta),\\
&\quad V^F(b,\theta,\tau,\underline{x})\geq V^{F0}(b,\theta),\\
&\quad b^{i\prime}\in[\underline{b}^i,b^{i,MAX}].
\end{aligned}
$$

This problem is equivalent to problem (109) when the debt limit is binding in every period.

The following Lemma establishes that $S^r(b,\theta)$ is concave and differentiable, given the assumptions of the model.

Lemma 10 *The supranational authority's value function* $S^r(b,\theta)$ *is concave and differentiable in* $b^D\in\left(\underline{b}^D,b^{D,MAX}\right)$.

Proof. In Section D.4.10. ■

Following the same steps as in Proposition 9, it follows that there exists a value η^{B*r} such that, under problem (134), $\forall\eta<\eta^{B*r}$, the participation constraint of policymaker D binds:

$$
V^{Dr}(b,\theta)=V^{D0}(b,\theta),
$$

and

$$
\mathbb{E}\sum_{t=0}^{\infty}\beta^t\left[w(g_t^D)+u(c^D(x_t^D,x_t^F,\theta))\right]\leq\mathbb{E}\sum_{t=0}^{\infty}\beta^t\left[w(g_t^{D0})+u(c^{D0}(x_t^{D0},x_t^{F0},\theta))\right].
$$

The utility of the supranational authority when setting fiscal rules $\{\overline{B}^i\}$, given in (109) can then be bounded in the following way:

$$
S(b,\theta)\leq S^{SR}(b,\theta)\leq S^r(b,\theta),
$$

where $S(b,\theta)$ is given in (95) and $S^r(b,\theta)$ is given in (134). Then, for $\eta\leq\min\{\eta^{B*},\eta^{B*r}\}$, the participation constraint of policymaker D binds for both $S(b,\theta)$ and $S^r(b,\theta)$, so $S^{SR}(b,\theta)=S^r(b,\theta)=S(b,\theta)$.

Since $v(r_t) > v(r_t^0)$ for any binding intervention rule $\underline{x}$, it follows that $\forall \eta \leq \overline{\eta^{B*}} \equiv \min\{\eta^{B*}, \eta^{B*r}\}$,

$$\mathbb{E}\sum_{t=0}^{\infty}\beta^t\left[w(g_t^D)+u(c^D(x_t^D,x_t^F,\theta))\right] < \mathbb{E}\sum_{t=0}^{\infty}\beta^t\left[w(g_t^{D0})+u(c^{D0}(x_t^{D0},x_t^{F0},\theta))\right].$$

D.4.7 Proof of Lemma 8

A sufficient condition for no partial banking union is that the participation constraint for policymaker D is not satisfied when there are conditional fiscal rules, the weight placed by the policymaker on household utility outside of a partial banking union is γ^D, and the weight placed by the policymaker on household utility in a partial banking union is $\widehat{\gamma}^D \in [\gamma^D, \gamma^{D*}]$.

By Lemma 7, the stationary distribution of debt is non-degenerate. Then, given $\widehat{b}^D$ in the stationary distribution of debt, let $\widehat{g}^D = \min_{\widehat{b}^D}\{e^D - \widehat{b}^D + \beta\underline{b}^D, 0\}$. A the sufficient condition is that

$$w(\widehat{g}^D) \to -\infty.$$

D.4.8 Proof of Proposition 13

The proof is similar to that of Proposition 6. Denote the initial value of γ^D by γ^{D0}. First, since there is no commitment on either the side of the supranational authority or the side of the policymakers, the value of γ^D will be set to the maximum each period:

Claim 3 *The supranational authority either sets $\gamma_t^D = \gamma^{D*}$, or $\gamma_t^D = \gamma_t^{D**} < \gamma^{D*}$ and the participation constraint for policymaker D binds at γ_t^{D**}, $\forall t \geq 0$.*

Proof. Assume $\gamma^D < \gamma^{D*}$ and the participation constraint (96) does not bind at $\bar{b}^D = b^D$ (when the debt limit is non-binding). Then marginally increasing γ^D increases the utility of the supranational authority, while the participation constraints still holds. So $\gamma^D < \gamma^{D*}$ cannot be optimal. ■

Step 2. Country D's choice of $\bar{b}^D$

Case A: If without a debt limit, the following holds:

$$\mathbb{E}\sum_{t=0}^{\infty}\beta^t\left[w(g_t^D)+u(c^D(x_t^D,x_t^F,\theta))\right]\geq\mathbb{E}\sum_{t=0}^{\infty}\beta^t\left[w(g_t^{D0})+u(c^{D0}(x_t^{D0},x_t^{F0},\theta))\right].$$

In this case, household utility in country D under the partial banking union is already higher than in the outside option. Households of country D choose $\bar{b}_t^D$ in each period in order to maximize their expected utility, hence

$$\begin{aligned}\mathbb{E}\sum_{t=0}^{\infty}\beta^t\left[w(g_t^D|\bar{b}_t^D)+u(c^D(x_t^D,x_t^F,\theta|\bar{b}_t^D))\right] \quad \geq \quad & \mathbb{E}\sum_{t=0}^{\infty}\beta^t\left[u(c^{D0}(x_t^{D0},x_t^{F0},\theta))\right. \\ & \left.+\,w(g_t^{D0})\right]. \end{aligned} \tag{135}$$

Case B: If without a debt limit, the following holds:

$$\mathbb{E}\sum_{t=0}^{\infty}\beta^t\left[w(g_t^D)+u(c^D(x_t^D,x_t^F,\theta))\right]\leq\mathbb{E}\sum_{t=0}^{\infty}\beta^t\left[w(g_t^{D0})+u(c^{D0}(x_t^{D0},x_t^{F0},\theta))\right].$$

Let $\left(r^D,x^D,g^D,g_1^D\right)$ denote the policies chosen by the D government under a partial banking union with terms $\underline{x}$, τ and γ^D.

Setting a binding debt limit $\bar{b}^D$ decreases the utility of policymaker D (since b^D is the level at which the policymaker's expected utility is maximized).

Under Lemma 8, a partial banking union is not implementable when $\bar{b}^D=\underline{b}^D$, since the participation constraint (96) is not satisfied. The policymaker value function V^D is a sum of instantaneous policymaker utilities with weight γ_t^D on the instantaneous household utility. It is therefore an increasing function of $\bar{b}_t^D$. This and (96) not holding at $\bar{b}^D=\underline{b}^D$ imply there exists $\widehat{b}_t^D>\underline{b}^D$ such that the participation constraint (68) holds with equality at when $\bar{b}_t^D=\widehat{b}_t^D$.

If there exists $\bar{b}_t^D\in\left[\widehat{b}_t^D,b^{D,MAX}\right]$ such that condition (135) holds, then $\bar{b}_t^D$ can be set to maximize expected household utility above the outside option. If condition (135) does not hold $\forall\bar{b}_t^D\in\left[\widehat{b}_t^D,b^{D,MAX}\right]$, then under Lemma 8, setting $\bar{b}_t^D=\underline{b}^D$ is an equilibrium strategy[28], and it leads to no partial banking

[28] The households cannot take any action in period t after setting $\bar{b}^D$.

union in period t.

If a partial banking union is implemented, then the participation constraint for policymaker F is satisfied, so (97) holds.

Since $\tau_t \geq 0 \ \forall t \geq 0$, rents in country F satisfy $r_t^F < r_t^{F0} \ \forall t \geq 0$, and therefore

$$\mathbb{E}\sum_{t=0}^{\infty}\beta^t\left[w(g_t^F)+u(c^F(x_t^F,x_t^D,\theta))\right] \geq \mathbb{E}\sum_{t=0}^{\infty}\beta^t\left[w(g_t^{F0})+u(c^{F0}(x_t^{F0},x_t^{D0},\theta))\right].$$

Therefore, a partial banking union achieves a Pareto improvement whenever it is formed.

D.4.9 Proof of Lemma 9

The value function $\overline{V^{D0}}(b^0,\theta,\overline{b^D}(\theta))$ is concave and differentiable, by the standard arguments. Denote by λ^0, ψ^0 and φ^0 the Lagrange multipliers on constraints (103a)-(103c), respectively. The first-order conditions to the policymaker's problem lead to

$$(1-\gamma^D)v'(r^{D0}) = \lambda^0 \tag{136}$$
$$\gamma^D\sigma^D Ru'(c^D(x^{D0},x^{F0},\theta)) = \lambda^0+\varphi^0 \tag{137}$$
$$\gamma^D w'(g^{D0}) = \lambda^0 \tag{138}$$
$$\beta\lambda^0-\psi^0 = \beta\mathbb{E}_{\theta'}\left[\lambda^{0\prime}\right], \tag{139}$$

the last equality due to the Envelope condition

$$\mathbb{E}_{\theta'}\left[-\frac{\partial\overline{V^{D0}}'(b^{D0\prime},\theta',\overline{b^D}(\theta'))}{\partial b^{D0\prime}}\right] = \mathbb{E}_{\theta'}\left[\lambda^{0\prime}\right]. \tag{140}$$

Given (136)-(140) and (103a),

$$\frac{\partial b^{D0\prime}}{\partial b^{D0}} > 0.$$

Then, given this monotonicity, there exists $\widetilde{b^D}(\theta) \in [\underline{b}^D, b^{D,MAX}]$ such that if the fiscal rule $\overline{b^D}(\theta)$ binds for $b^{D0} < \widetilde{b^D}(\theta)$ and it does not bind for $b^{D0} > \widetilde{b^D}(\theta)$.

D.4.10 Proof of Lemma 10

Policymaker i's static choices of $x^i(b^i, \theta, \tau, \underline{x}, b^{i\prime})$, $g^i(b^i, \theta, \tau, \underline{x}, b^{i\prime})$, and for $r^i(b^i, \theta, \tau, \underline{x}, b^{i\prime})$, $i \in \{D, F\}$, lead to concave and differentiable functions $u(c^i)$, $v(r^i)$ and $w(g^i)$. Then, by induction, the value function $S^r(b, \theta)$ is concave. Moreover, the policy functions are continuous, and, by the standard arguments, $S^r(b, \theta)$ is differentiable in b^D over $(\underline{b}^D, b^{D,MAX})$.