Strategic Determination of Renegotiation Costs*

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Abstract

Recently, some literature on incomplete contracts studies the cases where renegotiations take place inefficiently. We extend the incomplete contract model in Hart (2009) by assuming that one party chooses an action which affects renegotiation costs. In our model, renegotiation costs are determined endogenously. We characterize the condition that she can get higher payoff by manipulating renegotiation costs than when she cannot manipulate renegotiation costs and renegotiations take place efficiently. Whereas she chooses positive renegotiation costs, renegotiations never occur on the equilibrium paths. They work just as "credible threat". Her equilibrium share ratio of the *ex ante* bargaining surplus is higher than her bargaining power. As an application, we discuss underinvestment problem by using a variant of our basic model. We show that the agents mitigate underinvestment problem by setting some positive renegotiation costs and increasing a high skilled agent's share ratio of the *ex ante* bargaining surplus to give her larger incentive of investment.

JEL Classification

D23, D86, C78

1 Introduction

Over the past few years, some literature of incomplete contract theory and property right theory discusses optimal contracts and asset ownerships in environments where renegotiations of initial contracts take place inefficiently.[1] Hart (2009) discusses asset ownerships in a model where the size of renegotiation costs are exogenous. We extend Hart (2009) by assuming that either one contracting party chooses an action which affects renegotiation costs (hereafter, we simply say she chooses renegotiation costs). Thus, renegotiation costs are determined endogenously in our model. We study how she increases her payoff through manipulation of renegotiation costs.

We treat a situation where two agents collaborate. Each agent's benefit from the collaboration and her *ex post* disagreement point is uncertain *ex ante*. The timing is as follows. First, one agent chooses renegotiation costs. We assume that the other agent cannot take such an action. After the renegotiation costs were determined and observed by the agents, the agents sign a contract, which specifies non-contingent transfers, in a way to be described later. Then, the benefits of the collaboration and the disagreement points are realized. The state of nature is binary. After the realization, each agent observes it and decides to make a renegotiation offer or not. We say an agent holds up when she makes a

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[1]See Segal and Whinston (2013).

renegotiation offer according to the usage in Hart (2009). If either one of the agents holds up, a renegotiation occurs and they split the surplus reduced by the renegotiation costs in the generalized Nash bargaining. Otherwise, the contracted transfers are enforced.

How do the agents sign a contract? For each possible contract, the agents correctly anticipate by backward induction the states where renegotiations occur and their payoffs realized in each state after the contract is signed. Through the reasoning, the agents know the pair of expected payoffs which is attained by each contract. They sign a contract which gives the expected payoff pair maximizing the generalized Nash product in the feasible payoff set. The bargaining powers in the *ex ante* bargaining equal to those in the *ex post* bargaining.

In an equilibrium, the agent described above correctly anticipates the contract to be signed, when renegotiations occur and her *ex post* payoff in each state, given the renegotiation costs she chose. Then, she chooses renegotiation costs which maximize her *ex ante* payoff. An equilibrium is described as a triple, consisting of renegotiation costs chosen by the agent described above, transfers contracted in *ex ante* bargaining and a contingent occurrence of renegotiations.

Our main results are as follows. Each of the agents has a state in which her incentive of hold up is larger than in the other state. If the sum of one agent's bargaining power and the probability of the state in which she has a stronger incentive to hold up is smaller than that of the other agent, we say she is inactive in bargaining. We show that if and only if the agent who chooses renegotiation costs is inactive in bargaining, she chooses some positive renegotiation costs and obtains a greater payoff than in the case where renegotiations take place efficiently. Whereas she chooses some positive renegotiation costs, renegotiations do not occur on the equilibrium. The renegotiation costs work as a credible threat. Therefore, the renegotiation costs do not reduce the *ex ante* surplus but only changes each agent's share ratio of the *ex ante* bargaining surplus. We can interpret that to mean that the bargaining power of the agent who chooses renegotiation costs effectively increases when she is inactive in bargaining.

As an application, we also discuss underinvestment problem. It is a main focus on the traditional property right theory, represented by Grossman and Hart (1986) and Hart and Moore (1990), but not formally treated by Hart (2009).[2] We consider a variant of our basic model, in which renegotiation costs are also bargained and an investment stage follows. The result is that if the investment skill of the inactive agent is higher than the other agent, they choose positive renegotiation costs and effectively increase the inactive agent's bargaining power. Whereas underinvestment problems are mitigated through affecting disagreement points by asset ownership allocations in the traditional theory, the inactive agent's underinvestment is mitigated through improvement of her effective bargaining power by renegotiation costs in our model.

What is the source of renegotiation costs? Hart (2009) explains that when initial contracts are broken by a renegotiation offer, agents' relationship becomes hostile and a dead weight loss arises. This follows the view of Hart and Moore (2008).[3] Hart and Moore (2008) suppose that agents regard an initial contract as a reference point and they feel "badly treated" when they cannot receive what they feel entitled to under the initial contract. If an agent feels badly treated, she shades on performance. Fehr et al. (2008) provide an experimental test of some of the key predictions of the theory of Hart and Moore (2008).

If we hold the view that behavioral preferences cause renegotiation costs, it may seem

[2]However, Hart (2009) suggests that it may be necessary to introduce (noncontractible) *ex ante* investments into his model in order to understand the costs of vertical integration.

[3]Although Hart and Moore (2008) mainly assume that there are no chance of renegotiations, shading on performance may occur if initial flexible contracts are enforced.

impossible to manipulate renegotiation costs. However, the agents may be able to control renegotiation costs by delegation. Suppose that you are some firm's owner and have a profit maximizing preference. Although you are not behavioral, you can hire a manager with a behavioral preference. To what extent the manager is behavioral determines the size of renegotiation costs. If you hire a person who easily gets angry when the initial agreement is objected, renegotiation costs are large because the renegotiation process is perhaps troublesome.

Furthermore, there are many other non behavioral explanations in which agents have a room for manipulation of renegotiation costs. For example, renegotiation costs can be interpreted as a delay to agreements in the bargaining or some legal costs. If agents have hired a good lawyer in advance, the *ex post* bargaining process is smooth and a delay to agreement is very short.[4] Maintaining a firm's transparency may also increase renegotiation costs. Information disclosure about renegotiation takes costs (increment of litigation risk, additional paper work for PR activities, for example) and reduces a flexibility of negotiations.

Even if renegotiation costs should be considered as an exogenous parameter or a variable which is determined by a policy maker, we can use our results as a comparative analysis with renegotiation costs or a evaluation of policies about environments of renegotiation. Suppose that a government conducts a policy which facilitates smooth judiciary proceedings. From our results, such a policy may decrease the welfare levels of workers who have very small bargaining powers and so are inactive in bargaining, because their bosses may more frequently hold up them and their wages never increase but may decrease after such a policy.

The paper is organized as follows. Section 2 shows an illustrative example where a buyer and a seller trade a unit of good with uncertain value. We study the price determination mechanism, their *ex ante* payoffs and contingent occurrences of renegotiations. In Section 3, we describe the formal model and study the general results. In Section 4, we show an application of the basic model. We compute the feasible effective bargaining power set and study underinvestment problem.

2 Illustrative Example: Asymmetric Probability Distribution

Suppose that a buyer and a seller trade a unit of good. v denotes the value of the good. The costs of the good and the both agents' disagreement points are zero. Then, the trade generates the surplus equal to the value of the good v. There are a low state and a high state. The low state is realized with probability $\alpha \in \left(\frac{1}{2}, 1\right)$. The high state is realized with probability $1-\alpha$. Then, the low state more frequently happens than the high state. $v = v^L$ and $v = v^H (> v^L)$ are realized in the low state and in the high state, respectively.

The timing is as follows. First, the seller chooses renegotiation cost size $\lambda \in [0, \overline{\lambda}]$ without disutility. We assume that $\overline{\lambda} < v^L$. This guarantees that the trade is better than no trade even if the value of the good is low and a renegotiation takes costs λ. Then renegotiations never break down if they arise. For our purpose, we also assume that the seller can choose sufficiently large renegotiation cost size. Especially, we assume that $\overline{\lambda} > \alpha(v^H - v^L)$. After λ is determined and observed by the both agents, they bargain a price of good, $p \in \mathbb{R}$, and contract it. We discuss it in detail in the last of this section.

After contracting a price, the high or low state is realized and observed by the both

[4] Perhaps, the agents can hire a lawyer just before an *ex post* bargaining. However, the lawyer may not be able to perform as well as when she had been hired from the beginning, because she doesn't have enough knowledge about the both agents and their collaboration.

agents. Each of them decides whether she makes a renegotiation offer or not. When she makes the renegotiation offer, we say that she holds up. If either one of them holds up, they proceed to a renegotiation. In this sense, each of them has a veto power.

We denote the *ex post* payoff of the buyer when the value of the good is v by u_B and denote that of the seller by u_S. After a renegotiation, the *ex post* surplus is reduced by the amount of fixed renegotiation cost λ. They split the reduced surplus fifty-fifty. Therefore, after the renegotiation,

$$u_B = u_S = \frac{1}{2}(v - \lambda).$$

When no renegotiation occurs, they trade under the initial price and their *ex post* payoffs are realized as follows.

$$u_B = v - p, \qquad\qquad u_S = p.$$

We assume that each agent plays an optimal pure strategy which is not weakly dominated. The buyer chooses no hold up, if $p < \frac{1}{2}(v + \lambda)$. This is because her *ex post* payoff is higher in no renegotiation than in a renegotiation and so no hold up is her weakly dominant strategy. If $p = \frac{1}{2}(v + \lambda)$, she may choose either hold up or no hold up because they are best responses regardless of the seller's action and never weakly dominated. If $p > \frac{1}{2}(v + \lambda)$, she chooses hold up because it is her weakly dominant strategy. Similarly, the seller chooses no hold up if $p > \frac{1}{2}(v - \lambda)$. If $p = \frac{1}{2}(v - \lambda)$, she may choose either hold up or no hold up. If $p < \frac{1}{2}(v - \lambda)$, she chooses hold up.

Therefore, the condition that there is an equilibrium in which no renegotiation occurs when the value of the good is v and the price is p is

$$\frac{1}{2}(v - \lambda) \leq p \leq \frac{1}{2}(v + \lambda). \tag{1}$$

The condition that there is an equilibrium in which a renegotiation occur when the value of the good is v and the price is p is

$$p \leq \frac{1}{2}(v - \lambda) \qquad \text{or} \qquad \frac{1}{2}(v + \lambda) \leq p. \tag{2}$$

As described above, the two states exist. Thus, there are four possible contingent occurrences of renegotiation.

$$(x^L, x^H) = (N, N),\ (N, R),\ (R, N),\ (R, R).$$

x^L and x^H indicate that whether renegotiations occur in the low state and in the high state, respectively. N means no renegotiation and R means renegotiation. For example, $(x^L, x^H) = (N, R)$ means that no renegotiation occurs in the low state but a renegotiation occurs in the high state.

Given the renegotiation costs, the agents sign a contract in the Nash bargaining. As in the standard moral hazard model, we assume that the agents specify not only a price, which is contractible, but also a contingent plan of renegotiation through a contingent plan of hold up, which are not contractible. For enforcing hold up actions, an incentive compatibility condition about the agents' hold up actions must be satisfied. That is, given the renegotiation costs, the agents sign a price p and choose (x^L, x^H) which maximizes the Nash product (hereafter, we denote it by NP) subject to the constraint that there is some equilibrium in which renegotiations occur as (x^L, x^H) for p. The set of the all prices under which renegotiations occur as (x^L, x^H) in an equilibrium is denoted by $P(x^L, x^H)$. We describe the buyer's expected payoff and the seller's expected payoff as $U_B(p|x^L, x^H)$ and $U_S(p|x^L, x^H)$, respectively, when the price is p and renegotiations occur as (x^L, x^H).

Then, the agents solve the next problem given the renegotiation costs.

$$\begin{aligned} \max_{p\in\mathbb{R},\ (x^L,x^H)\in\{N,R\}\times\{N,R\}} &\quad U_B(p|x^L,x^H)\cdot U_S(p|x^L,x^H) \\ \text{subject to} &\quad p\in P(x^L,x^H). \end{aligned}$$

The following decomposition of NP is convenient for the later analysis. Let $S \equiv U_B+U_S$ and $\Delta U \equiv U_B-U_S$. By using the fact that $U_B = \frac{1}{2}(S+\Delta U)$ and $U_S = \frac{1}{2}(S-\Delta U)$, we can represent NP as

$$U_B\cdot U_S = \frac{1}{4}(S^2-|\Delta U|^2). \tag{3}$$

For the NP maximizer, the more *ex ante* surplus and the less difference in the agent's *ex ante* payoffs are more desirable.

Unlike many previous studies, especially Hart (2009), we assume that *ex ante* transfers are not feasible. Only an *ex post* non-contingent transfer, a price, is contractible. Then, the *ex ante* bargaining is a non-transferable payoff game. It is the reason why the Nash solution does not necessarily maximize the *ex ante* surplus.

In an equilibrium, the seller correctly anticipates the contract to be signed, when renegotiations occur and her *ex post* payoff in each state, given the renegotiation costs she chose. Then, she chooses renegotiation costs which maximize her *ex ante* payoff. An equilibrium is described as a triple, consisting of renegotiation costs chosen by the seller, a price contracted in *ex ante* bargaining and a contingent occurrence of renegotiations.

2.1 Price Selection Mechanism

There are three difficulties in solving directly the constrained NP maximization problem defined above. First, for some price p, multiple (x^L,x^H) satisfy $p\in P(x^L,x^H)$. For example, when $p=\frac{1}{2}(v^L-\lambda)$, each of (N,R) and (R,R) occurs in an equilibrium. Second, the way prices affect the *ex ante* payoff pairs depends on (x^L,x^H). For example, when $(x^L,x^H)=(N,N)$, prices work just as *ex ante* transfers. The reason is that prices are never renegotiated. This is one extreme. When $(x^L,x^H)=(R,R)$, prices do not affect the *ex ante* payoffs. The reason is that renegotiations always scrap prices. This is the other extreme. Third, the feasible *ex ante* payoff set is not connected, although given each (x^L,x^H) the set of the all *ex ante* payoff pairs attained by the prices in $P(x^L,x^H)$ is connected.

To avoid these difficulties, we divide the problem to two pieces. The way is very similar to the moral hazard model in Grossman and Hart (1983). First, we fix each (x^L,x^H) and find the prices which maximize NP subject to $p\in P(x^L,x^H)$. We call it (x^L,x^H) implementation problem. When the solution price uniquely exists, we denote it by $p^{x^Lx^H}$. We also denote the *ex ante* surplus S, the difference in *ex ante* utilities ΔU, and NP attained under the price $p^{x^Lx^H}$ by $S^{x^Lx^H}$, $\Delta U^{x^Lx^H}$, and $NP^{x^Lx^H}$, respectively. Second, we find the implementation which gives the highest NP.

Now, go to the first piece, (x^L,x^H) implementation problems. $\Delta U^{x^Lx^H}(p)$ denotes the ΔU under a price p when renegotiations occur as (x^L,x^H). In each implementation problem, the probability distribution of renegotiation occurrence is fixed. It means $S^{x^Lx^H}$ is fixed there. Therefore, a price p minimizing $|\Delta U^{x^Lx^H}(p)|$ is a solution in each problem, because the Nash product is decreasing in $|\Delta U|$ as we have seen in (3). (x^L,x^H) implementation problem can be written as

$$\min_{p\in P(x^L,x^H)} |\Delta U^{x^Lx^H}(p)|.$$

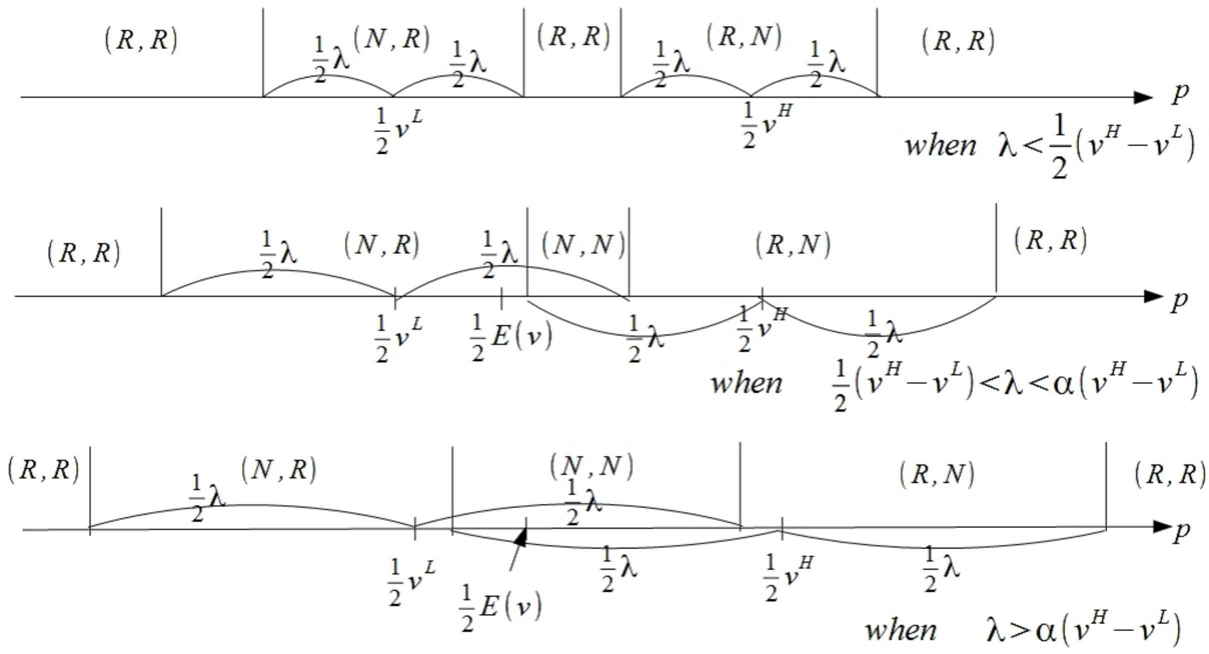

Figure I. Relation between prices and contingent renegotiation patterns.

Each $P(x^L, x^H)$ can be easily computed by (1) and (2). Figure I illustrates the relation between prices and each contingent renegotiation pattern (x^L, x^H) for three typical values of renegotiation costs.

(N,N) implementation problem First, we consider (N, N) implementation problem.

$$P(N,N) = \begin{cases} \left[\ \frac{1}{2}(v^H - \lambda),\ \frac{1}{2}(v^L + \lambda)\ \right] & \text{if } \lambda \geq \frac{1}{2}(v^H - v^L) \\ \quad \emptyset & \text{otherwise.} \end{cases}$$

Note that $\lambda \geq \frac{1}{2}(v^H - v^L)$ is equivalent to $\frac{1}{2}(v^H - \lambda) \leq \frac{1}{2}(v^L + \lambda)$. Because the initial price p is always enforced, the agents' *ex ante* payoffs are

$$U_B(p|N,N) = E(v) - p, \qquad U_S(p|N,N) = p.$$

Therefore, $S^{NN} = E(v)$ and $|\Delta U^{NN}(p)| = |E(v) - 2p|$ hold. Thus, the price nearest to $\frac{1}{2}E(v)$ in $P(N,N)$ minimizes $|\Delta U|$ if $P(N,N)$ is not empty. Thus, if $\lambda \geq \frac{1}{2}(v^H - v^L)$, p^{NN} exists and

$$\begin{aligned} p^{NN} &= \begin{cases} \frac{1}{2}(v^H - \lambda) & \text{if } \lambda \in \left[\frac{1}{2}(v^H - v^L), \alpha(v^H - v^L)\right] \\ \frac{1}{2}E(v) & \text{if } \lambda > \alpha(v^H - v^L), \end{cases} \\ NP^{NN} &= \frac{1}{4}\left\{\left(S^{NN}\right)^2 - |\Delta U^{NN}|^2\right\} \qquad (4) \\ &= \begin{cases} \frac{1}{4}\left\{E(v)^2 - \left(\ \alpha(v^H - v^L) - \lambda\right)^2\right\} & \text{if } \lambda \in \left[\frac{1}{2}(v^H - v^L), \alpha(v^H - v^L)\right] \\ \frac{1}{4}E(v)^2 & \text{if } \lambda > \alpha(v^H - v^L). \end{cases} \end{aligned}$$

Equation (3) is used for computing NP^{NN}. Note that $\lambda > \alpha(v^H - v^L)$ is equivalent to $\frac{1}{2}E(v) > \frac{1}{2}(v^H - \lambda)$.

(N,R) implementation problem

$$P(N,R) = \begin{cases} [\ \frac{1}{2}(v^L - \lambda),\ \ \frac{1}{2}(v^L + \lambda)\ ] & \text{if } \lambda \leq \frac{1}{2}(v^H - v^L), \\ [\ \frac{1}{2}(v^L - \lambda),\ \ \frac{1}{2}(v^H - \lambda)\ ] & \text{otherwise.} \end{cases}$$

In (N,R) implementation, no renegotiation occurs in the low state and a renegotiation occurs in the high state. Therefore, the both agents' *ex ante* payoffs are realized as

$$\begin{aligned} U_B(p|N,R) &= \alpha(v^L - p) + (1-\alpha) \cdot \frac{1}{2}(v^H - \lambda), \\ U_S(p|N,R) &= \alpha p + (1-\alpha) \cdot \frac{1}{2}(v^H - \lambda). \end{aligned}$$

Then, $S^{NR} = E(v) - (1-\alpha)\lambda$ and $|\Delta U^{NR}(p)| = |\alpha(v^L - 2p)|$ hold. Therefore, the solution price is the price nearest to $\frac{1}{2}v^L$ in $P(N,R)$. $\frac{1}{2}v^L \notin P(N,R)$ is equivalent to $\frac{1}{2}(v^H - \lambda) < \frac{1}{2}v^L$, or $\lambda > v^H - v^L$. Therefore, we conclude that p^{NR} exists for any λ and

$$\begin{aligned} p^{NR} &= \begin{cases} \frac{1}{2}v^L & \text{if } \lambda \leq v^H - v^L \\ \frac{1}{2}(v^H - \lambda) & \text{otherwise,} \end{cases} \\ NP^{NR} &= \frac{1}{4}\left\{\left(S^{NR}\right)^2 - |\Delta U^{NR}|^2\right\} \\ &= \begin{cases} \frac{1}{4}\{E(v) - (1-\alpha)\lambda\}^2 & \text{if } \lambda \leq v^H - v^L \\ \frac{1}{4}\left\{\left(E(v) - (1-\alpha)\lambda\right)^2 - \alpha^2(v^H - v^L - \lambda)^2\right\} & \text{otherwise.} \end{cases} \end{aligned} \tag{5}$$

(R,N) implementation problem We can discuss (R,N) implementation similarly.

$$\begin{aligned} P(R,N) &= \begin{cases} [\ \frac{1}{2}(v^H - \lambda),\ \ \frac{1}{2}(v^H + \lambda)\ ] & \text{if } \lambda \leq \frac{1}{2}(v^H - v^L) \\ [\ \frac{1}{2}(v^L + \lambda),\ \ \frac{1}{2}(v^H + \lambda)\ ] & \text{otherwise,} \end{cases} \\ U_B(p|R,N) &= \alpha \cdot \frac{1}{2}(v^L - \lambda) + (1-\alpha)(v^H - p), \\ U_S(p|R,N) &= \alpha \cdot \frac{1}{2}(v^L - \lambda) + (1-\alpha)p. \end{aligned}$$

Thus, $S^{RN} = E(v) - \alpha\lambda$ and $|\Delta U^{RN}(p)| = |(1-\alpha)(v^H - 2p)|$. Then, p^{RN} exists for any λ and

$$\begin{aligned} p^{RN} &= \begin{cases} \frac{1}{2}v^H & \text{if } \lambda \leq v^H - v^L \\ \frac{1}{2}(v^L + \lambda) & \text{otherwise,} \end{cases} \\ NP^{RN} &= \frac{1}{4}\left\{(S^{RN})^2 - |\Delta U^{RN}|^2\right\} \\ &= \begin{cases} \frac{1}{4}\{\ E(v) - \alpha\lambda\ \}^2 & \text{if } \lambda \leq v^H - v^L \\ \frac{1}{4}\left\{\left(E(v) - \alpha\lambda\right)^2 - (1-\alpha)^2(v^H - v^L - \lambda)^2\right\} & \text{otherwise.} \end{cases} \end{aligned}$$

(R,R) implementation problem

$$P(R,R) = \mathbb{R} \setminus \left(\left(\frac{1}{2}(v^L - \lambda),\ \frac{1}{2}(v^L + \lambda)\right) \cup \left(\frac{1}{2}(v^H - \lambda),\ \frac{1}{2}(v^H + \lambda)\right) \right).$$

Renegotiations occur in the both states and so the agents' *ex ante* payoffs are realized as

$$U_B(p|R,R) = U_S(p|R,R) = \frac{1}{2}(\ E(v) - \lambda\ ).$$

Thus, $S^{RR} = E(v) - \lambda$ and $|\Delta U| = 0$ holds for any $p \in P(R,R)$. Therefore, any price $p \in P(R,R)$ is a solution of (R,R) implementation and

$$NP^{RR} = \frac{1}{4}(\ E(v) - \lambda)^2.$$

Implementation selection problem Now, proceed to the implementation selection problem. Which implementation should be selected depends on λ. As we have seen in (3), larger *ex ante* surplus S and smaller difference in the *ex ante* payoffs $|\Delta U|$ give greater NP. If $\lambda \geq \alpha(v^H - v^L)$, $S = E(v)$ and $|\Delta U| = 0$ are attained by (N,N) implementation. It gives the highest feasible *ex ante* surplus and the lowest difference in *ex ante* payoffs. All the other implementations give strictly smaller surplus and so strictly lower NP. Therefore, (N,N) implementation uniquely gives the highest value of NP.

Consider the case $\lambda < \frac{1}{2}(v^H - v^L)$. (N,N) is infeasible there. If $0 < \lambda < \frac{1}{2}(v^H - v^L)$, (N,R) implementation uniquely gives the largest *ex ante* surplus in the all feasible implementation problems and gives $|\Delta U| = 0$. Therefore, (N,R) implementation uniquely maximizes NP. If $\lambda = 0$, the all (x^L, x^H) implementations except for (N,N) implementation gives $S = E(v)$ and $|\Delta U| = 0$. Then, all of them are the solutions of the implementation selection problem. We summarize the results as the next lemma.

Lemma 2.1. *For renegotiation costs λ, the contracted price p, the ex ante payoffs U_B and U_S, and the contingent occurrence of renegotiations are realized as follows.*

(i) If $\lambda \geq \alpha(v^H - v^L)$, $p = \frac{1}{2}E(v)$, $U_B = U_S = \frac{1}{2}E(v)$ and no renegotiation occurs in the both states.

(ii) If $0 < \lambda < \frac{1}{2}(v^H - v^L)$, $p = \frac{1}{2}v^L$, $U_B = U_S = \frac{1}{2}\{E(v) - (1-\alpha)\lambda\}$ and a renegotiation occurs only in the high state.

(iii) If renegotiations take place efficiently, $\lambda = 0$, either one of (a), (b) and (c) holds.

(a) $p = \frac{1}{2}v^H$ and a renegotiation occurs only in the low state.

(b) $p = \frac{1}{2}v^L$ and a renegotiation occurs only in the high state.

(c) p is arbitrary and renegotiations occur in the both states.

In all $(a), (b)$ and (c), $U_B = U_S = \frac{1}{2}E(v)$.

Consider the last case, $\lambda \in \left[\ \frac{1}{2}(v^H - v^L),\ \alpha(v^H - v^L)\ \right)$. (N,N) implementation gives the highest S but $|\Delta U| > 0$. On the other hand, (N,R) implementation gives the second highest S and $|\Delta U| = 0$. (R,N) implementation and (R,R) implementation are strictly dominated by (N,R) implementation, because each of them gives S smaller than (N,R) implementation and $|\Delta U| = 0$. Therefore, (N,N) implementation or (N,R) implementation maximizes NP. There is a trade-off between increasing *ex ante* surplus and decreasing difference in *ex ante* payoffs.

We define $f : \left[\frac{1}{2}(v^H - v^L),\ \alpha(v^H - v^L)\right] \to \mathbb{R}$ as $f(\lambda) \equiv NP^{NN} - NP^{NR}$. Therefore, $f(\lambda) \gtreqless 0$ is equivalent to $NP^{NN} \gtreqless NP^{NR}$. By (4) and (5),

$$f(\lambda) \equiv \frac{1}{4}\left\{E(v)^2 - \left(\ \alpha(v^H - v^L) - \lambda\right)^2\right\} - \frac{1}{4}\left(E(v) - (1-\alpha)\lambda\right)^2.$$

Because $\lambda \leq \alpha(v^H - v^L)$ and $v^H > v^L > \overline{\lambda}$ hold, $\alpha(v^H - v^L) - \lambda \geq 0$ and $E(v) - (1 - \alpha)\lambda > v^L - (1-\alpha)\lambda > 0$ are satisfied. Therefore, $f(\lambda)$ is strictly increasing with λ. It reflects the fact that NP^{NN} is increasing with λ and NP^{NR} is decreasing with λ in $[\frac{1}{2}(v^H - v^L), \alpha(v^H - v^L)]$. Furthermore, $f(\alpha(v^H - v^L))$ is positive. When $\lambda = \alpha(v^H - v^L)$, (N, N) implementation gives $|\Delta U| = 0$ and the strictly higher surplus than in (N, R) implementation. That means $NP^{NN} > NP^{NR}$ at $\lambda = \alpha(v^H - v^L)$.

Therefore, there is some $\tilde{\lambda} < \alpha(v^H - v^L)$ such that

$$f(\lambda) \geq 0 \qquad \forall \lambda \in [\ \tilde{\lambda},\ \alpha(v^H - v^L)\ ].$$

If some λ' exists and satisfies $f(\lambda') = 0$, $\tilde{\lambda} = \lambda'$. Otherwise, $f(\lambda) > 0$ for all λ in $[\frac{1}{2}(v^H - v^L), \alpha(v^H - v^L)]$ and so $\tilde{\lambda} = \frac{1}{2}(v^H - v^L)$. The existence of λ' depends on α. For α sufficiently close to $\frac{1}{2}$, $\tilde{\lambda} = \frac{1}{2}(v^H - v^L)$. That means (N, N) implementation is always best for such a small α if it is feasible.

To see that whether $NP^{NN} > NP^{NR}$ or $NP^{NN} = NP^{NR}$ for $\lambda = \frac{1}{2}(v^H - v^L)$ depends on parameters, consider the following numerical examples. By substituting $\lambda = \frac{1}{2}(v^H - v^L)$, we get $NP^{NN} = \frac{1}{4}\{E(v)^2 - (\alpha - \frac{1}{2})^2(v^H - v^L)^2\}$ and $NP^{NR} = \frac{1}{4}\left\{E(v) - \frac{1-\alpha}{2}(v^H - v^L)\right\}^2$. Let $v^L = 96$ and $v^H = 144$. When $\alpha = \frac{2}{3}$, $E(v) = 112$ and $NP^{NN} = 56^2 - 4^2 > 52^2 = NP^{NR}$. In this case, $f(\lambda) = 0$ does not have the solution and $\tilde{\lambda} = \frac{1}{2}(v^H - v^L)$. If $\alpha = \frac{11}{12}$, $E(v) = 100$ and $NP^{NN} = 40 \times 60 < 49^2 = NP^{NR}$. Then, $f(\lambda) = 0$ has the solution and the solution λ is strictly larger than $\frac{1}{2}(v^H - v^L)$.

As a summary of the results, we get the next lemma.

Lemma 2.2. *Some $\tilde{\lambda} \in \left[\frac{1}{2}(v^H - v^L),\ \alpha(v^H - v^L)\right)$ exists and the contracted price p, the agents' ex ante payoffs U_B and U_S, and the contingent occurrence of renegotiations are realized as follows.*

(i) If $\tilde{\lambda} < \lambda < \alpha(v^H - v^L)$, $p = \frac{1}{2}(v^H - \lambda)$, $U_B = E(v) - \frac{1}{2}(v^H - \lambda) < U_S = \frac{1}{2}(v^H - \lambda)$ and renegotiations do not occur on the both states.

(ii) If $\lambda = \tilde{\lambda}$, either one of (a) and (b) holds.

(a) $p = \frac{1}{2}(v^H - \tilde{\lambda})$, $U_B = E(v) - \frac{1}{2}(v^H - \tilde{\lambda}) < U_S = \frac{1}{2}(v^H - \tilde{\lambda})$, and renegotiations do not occur in the both states.

(b) $p = \frac{1}{2}v^L$, $U_B = U_S = \frac{1}{2}(E(v) - (1-\alpha)\tilde{\lambda})$, and a renegotiation occurs only in the high state.

(iii) If $\frac{1}{2}(v^H - v^L) \leq \lambda < \tilde{\lambda}$, $p = \frac{1}{2}v^L$, $U_S = U_B = \frac{1}{2}(E(v) - (1-\alpha)\lambda)$ and a renegotiation occurs only in the high state.

Proof. We have already shown the contingent occurrence of renegotiations for each λ. The contracted prices and *ex ante* utilities are easily computed by substituting λ and the contingent occurrences of renegotiations. □

$\frac{1}{2}E(v)$ is a benchmark payoff level, because it is each agent's payoff gotten when *ex post* renegotiations are efficient. By Lemmas 2.1 and 2.2, the buyer's payoff never exceeds this level. Only the seller gets a payoff more than $\frac{1}{2}E(v)$ for some renegotiation costs.

2.2 Seller's Positive Gain by Manipulation of Renegotiation Costs

Now, we study how the seller increases her payoff by manipulating renegotiation costs. By Lemmas 2.1 and 2.2, we get the next theorem.

Theorem 2.3. *In an equilibrium, the seller chooses renegotiation costs* $\lambda = \tilde{\lambda}$*, the price* $p = \frac{1}{2}(v^H - \tilde{\lambda})$ *is contracted, the ex ante payoffs are realized as* $U_B = E(v) - \frac{1}{2}(v^H - \tilde{\lambda}) < \frac{1}{2}E(v)$ *and* $U_S = \frac{1}{2}(v^H - \tilde{\lambda}) > \frac{1}{2}E(v)$*, and renegotiations do not occur in the both states.*

Proof. By Lemmas 2.1 and 2.2, $U_S \leq \frac{1}{2}E(v)$ if $\lambda \notin [\tilde{\lambda},\ \alpha(v^H - v^L))$, and $U_S > \frac{1}{2}E(v)$ if $\lambda \in (\tilde{\lambda}, \alpha(v^H - v^L))$. Then, the seller's best responses belong to $[\tilde{\lambda},\ \alpha(v^H - v^L))$ if they exist. If renegotiations do not occur on the both states for $\lambda = \tilde{\lambda}$, $U_S = \frac{1}{2}(v^H - \lambda)$ holds for $\lambda \in [\tilde{\lambda},\ \alpha(v^H - v^L)]$ and so $\lambda = \tilde{\lambda}$ is her best response. If a renegotiation occurs only in the high state for $\lambda = \tilde{\lambda}$, $U_S = \frac{1}{2}(E(v) - (1-\alpha)\tilde{\lambda}) < \frac{1}{2}E(v)$ for $\lambda = \tilde{\lambda}$ and $U_S = \frac{1}{2}(v^H - \lambda) > \frac{1}{2}E(v)$ for $\lambda \in (\tilde{\lambda}, \alpha(v^H - v^L)]$. U_S is not continuous with λ at $\lambda = \tilde{\lambda}$ and so her best response does not exist. Then, she chooses $\lambda = \tilde{\lambda}$ and renegotiations never occur on the equilibrium paths. U_B, U_S and p are easily computed by Lemma 2.2. □

This theorem says that the seller sets renegotiation costs at some appropriate positive level to prevent renegotiations in the both states and gets a strictly higher *ex ante* payoff than that attained when renegotiation is efficient. Why does she prevent renegotiations in the both states to improve her *ex ante* payoff? The reasons are as follows. First, she cannot get higher *ex ante* payoff in any other contingent occurrence of renegotiations than that she gets when renegotiations are efficient. If a renegotiation occurs in some state, the renegotiation splits the *ex post* surplus in the state fifty- fifty and so the parties contract the price which splits the *ex post* surplus fifty-fifty in the other state. Thus, the seller gets only a half of the *ex ante* surplus reduced by the renegotiation costs. Second, the price of (N, N) implementation is higher than $\frac{1}{2}E(v)$ when renegotiation costs are not too large. To prevent renegotiations in the both states, the price p^* which solves $\min_{p \in \mathbb{R}} \max\left\{|\frac{1}{2}v^L - p|, |\frac{1}{2}v^H - p|\right\}$ is the most desirable in the sense that p^* is the unique price which always belongs to $P(N, N)$ if $P(N, N)$ is not empty. p^* is the mid point of $\frac{1}{2}v^L$ and $\frac{1}{2}v^H$, $\frac{1}{4}(v^L + v^H)$, whereas $\frac{1}{2}E(v)$ is smaller than the mid point because the low state occurs with higher probability than the high state.

Although the price of (N, N) implementation gives the buyer less than a half of share ratio of the *ex ante* surplus, they agree to contract the high price and avoid the dead weight loss of renegotiations in the *ex ante* bargaining if renegotiation costs are not too small. However, the seller should choose the minimum of the renegotiation costs which prevent renegotiations in the both states if the minimum exists, because larger renegotiation costs gives bigger $P(N, N)$ and so the price of (N, N) implementation is closer to $\frac{1}{2}E(v)$.

3 General Results: Asymmetric Bargaining Powers and General Probability Distribution of Benefits and Disagreement Points

3.1 The Model

Now, we give a formal description of a model which allows asymmetric bargaining powers and more general probability distribution of benefits and disagreement points.[5] Agent 1 and 2 make a collaboration in the future. Each agent i's benefit from the collaboration, denoted by b_i, and her disagreement point, denoted by d_i are uncertain *ex ante.*

The timing is as follows. First, agent 2 chooses renegotiation costs $\lambda \in [0, \overline{\lambda}]$ without disutility. After the renegotiation costs are determined and observed by the agents, they contract non-contingent transfers (t_1, t_2). The budget balance condition $t_1 + t_2 = 0$ must

[5] The illustrative example in Section 2 is a special case. Let agents 1 and 2 be the buyer and the seller. Then, $b_1 = v$, $t_1 = -p$, $b_2 = 0$, $t_2 = p$, and $d_1 = d_2 = 0$.

be satisfied. The transfers are contracted in a way to be described later. After the transfers are contracted, a state ω is realized and observed by the both agents. $\omega = \omega^L$ and $\omega = \omega^H$ are realized with probability $\alpha_1 \in (0,1)$ and $\alpha_2 = 1-\alpha_1$, respectively. The values of b_i and d_i realized when $\omega = \omega^K$ are denoted by b_i^K and d_i^K. After the state is realized, each agent decides whether to hold up or not. When at least one agent holds up, a renegotiation occurs. When a renegotiation occurs in ω^K, they split the *ex post* surplus reduced by λ in the generalized Nash bargaining and each agent i's *ex post* payoff u_i is realized as $u_i = \rho_i(b_1^K + b_2^K - \lambda - d_1^K - d_2^K) + d_i^K$, where ρ_i is agent i's bargaining power and so $\rho_1 + \rho_2 = 1$. We assume that $\rho_1, \rho_2 > 0$. If no renegotiation occurs in ω^K, $u_i = b_i^K + t_i$.

We assume that each agent plays an optimal pure action which is not weakly dominated. In the same way with Section 2, each agent i chooses an action as follows. She chooses no hold up if $b_i + t_i > \rho_i(b_1 + b_2 - \lambda - d_1 - d_2) + d_i$. She may choose either no hold up or hold up if $b_i + t_i = \rho_i(b_1 + b_2 - \lambda - d_1 - d_2) + d_i$. She chooses hold up if $b_i + t_i < \rho_i(b_1 + b_2 - \lambda - d_1 - d_2) + d_i$.

By these conditions and the budget balance condition, the condition that no renegotiation occurs at ω^K in some equilibrium is represented as

$$e^K - \rho_2\lambda \le t_2 \le e^K + \rho_1\lambda, \tag{6}$$

where

$$e^K \equiv \{\rho_2(b_1^K + b_2^K - d_1^K - d_2^K) + d_2^K\} - b_2^K.$$

e^K is the value of agent 2's transfer under which each of $b_1^K - e^K$ and $b_2^K + e^K$ equals each agent's *ex post* payoff level attained through the *ex post* bargaining at ω^K when $\lambda = 0$. In (6), the lower bound is agent 2's minimum transfer level for which no hold up is not weakly dominated and the upper bound is from that of agent 1. We assume $e^H > e^L$.[6] Then, the lower bound in (6) is tighter in ω^H and the upper bound in it is tighter in ω^L. It means that agent 2 has a stronger incentive of hold up at ω^H, whereas agent 1 has a stronger incentive of hold up at ω^L.

We can also compute the condition that a renegotiation occurs at ω^K in some equilibrium. It is represented as

$$t_2 \le e^K - \rho_2\lambda \quad \text{or} \quad e^K + \rho_1\lambda \le t_2. \tag{7}$$

We assume that

$$\max\left\{\frac{\alpha_1}{\rho_2}(e^H - e^L), \frac{\alpha_2}{\rho_1}(e^H - e^L)\right\} < \bar{\lambda} < \min\left\{\sum_i (b_i^L - d_i^L),\ \sum_i (b_i^H - d_i^H)\right\}. \tag{8}$$

The right inequality guarantees that renegotiations never break down because a collaboration is always better than no collaboration. The left inequality guarantees that agent 2 can choose sufficiently high renegotiation costs λ.

As before, there are four possible contingent occurrences of renegotiations.

$$(x^L, x^H) = (N,N), (N,R), (R,N), (R,R).$$

We denote the set of t_2 for which renegotiations occur as (x^L, x^H) in some equilibrium by $\hat{T}_2(x^L, x^H)$. By (6) and (7),

[6] By relabeling the names of the states suitably, $e^H \ge e^L$ is always satisfied. When $e^H = e^L$, the agents' *ex ante* payoffs are realized as if there was no uncertainty. This is the reason why we exclude it.

$$\hat{T}_2(N,N) = \begin{cases} \emptyset & \text{if } \lambda < e^H - e^L \\ \left[e^H - \rho_2\lambda,\ e^L + \rho_1\lambda\right] & \text{otherwise} \end{cases} \tag{9}$$

$$\hat{T}_2(N,R) = \begin{cases} [\ e^L - \rho_2\lambda,\ \ e^L + \rho_1\lambda\ ] & \text{if } \lambda < e^H - e^L \\ [\ e^L - \rho_2\lambda,\ \ e^H - \rho_2\lambda\ ] & \text{otherwise} \end{cases} \tag{10}$$

$$\hat{T}_2(R,N) = \begin{cases} [\ e^H - \rho_2\lambda,\ \ e^H + \rho_1\lambda\ ] & \text{if } \lambda < e^H - e^L \\ [\ \ e^L + \rho_1\lambda,\ \ e^H + \rho_1\lambda\ ] & \text{otherwise} \end{cases} \tag{11}$$

$$\hat{T}_2(R,R) = \mathbb{R} \setminus \left((e^L - \rho_2\lambda,\ e^L + \rho_1\lambda) \cup (e^H - \rho_2\lambda,\ e^H + \rho_1\lambda) \right). \tag{12}$$

Figure II illustrates the relation between each (x^L, x^H) and t_2 for two typical λ.

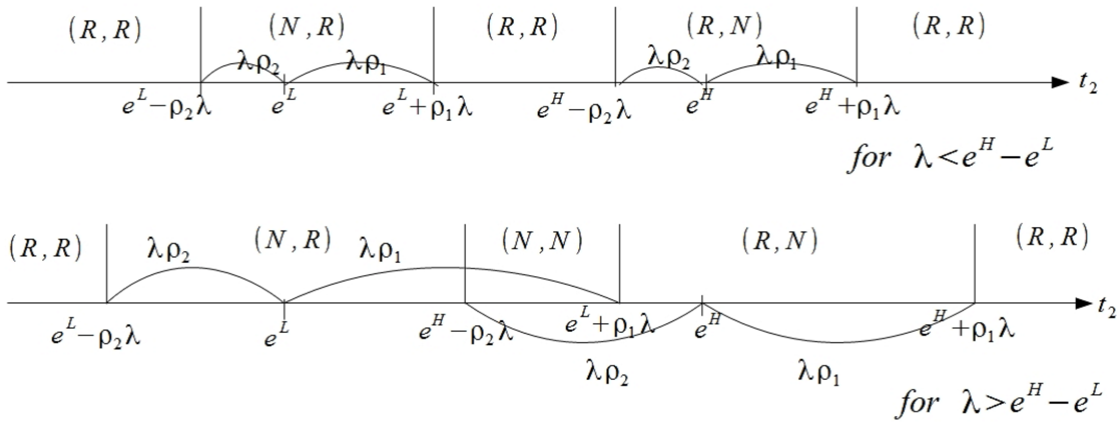

Figure II. Relation between agent 2's transfer and contingent occurrences of renegotiation.

Given the renegotiation costs, the agents sign a contract in the generalized Nash bargaining. We assume that each agent i's disagreement point in the *ex ante* bargaining is $E(d_i)$. We can interpret that to mean that each agent i cannot immediately find another partner and she must wait and will receive disagreement point d_i after the value is realized *ex post* if the *ex ante* bargaining breaks down. As in the illustrative example, we assume that the agents solve the next problem given the renegotiation costs.

$$\max_{t_2 \in \mathbb{R},\ (x^L,x^H)\in\{N,R\}\times\{N,R\}} \left(\ U_1(t_2|x^L,x^H) - E(d_1)\ \right)^{\rho_1} \cdot \left(U_2(t_2|x^L,x^H) - E(d_2)\right)^{\rho_2}$$
$$\text{subject to } t_2 \in \hat{T}_2(x^L,x^H),$$

where $U_i(t_2|x^L,x^H)$ denotes agent i's *ex ante* payoff when t_2 is contracted and renegotiations occur as (x^L,x^H).

We denote that each agent i's *ex ante* bargaining rent by $\hat{U}_i$ and the *ex ante* bargaining surplus by $\hat{S}$, that is $\hat{U}_i \equiv U_i - E(d_i)$ and $\hat{S} \equiv \hat{U}_1 + \hat{U}_2$. Furthermore, we define $\hat{\rho}_i \equiv \frac{\hat{U}_i}{\hat{S}}$, which is agent i's share ratio of the *ex ante* bargaining surplus.

For the later analysis, we decompose the generalized Nash product denoted by NP as follows.

$$NP \equiv (\ U_1 - E(d_1)\ )^{\rho_1}(\ U_2 - E(d_2)\ )^{\rho_2} = \hat{S} \cdot (1-\hat{\rho}_2)^{1-\rho_2}\hat{\rho}_2^{\rho_2}$$

Its log derivative with $\hat{\rho}_2$ is

$$-\frac{1-\rho_2}{1-\hat{\rho}_2} + \frac{\rho_2}{\hat{\rho}_2}.$$

It is positive, zero and negative for $\hat{\rho}_2 < \rho_2$, $\hat{\rho}_2 = \rho_2$ and $\hat{\rho}_2 > \rho_2$ respectively. Then, the next lemma holds.

Lemma 3.1. *Suppose that NP' is the value of NP when $\hat{S} = \hat{S}'$ and $\hat{\rho}_2 = \hat{\rho}_2'$, NP'' is the value of NP when $\hat{S} = \hat{S}'' > 0$ and $\hat{\rho}_2 = \hat{\rho}_2''$. Assume that either $\rho_2 \leq \hat{\rho}_2' \leq \hat{\rho}_2''$ or $\hat{\rho}_2'' \leq \hat{\rho}_2' \leq \rho_2$ holds. Then, $NP' > NP''$ if $\hat{S}' \geq \hat{S}''$, $|\rho_2 - \hat{\rho}_2'| \leq |\rho_2 - \hat{\rho}_2''|$ and at least one of the two inequalities is strict.*

Note that $|\rho_1 - \hat{\rho}_1| = |\rho_2 - \hat{\rho}_2|$. This lemma says that the more *ex ante* surplus and each agent's effective bargaining power nearer to her bargaining power are preferred in the *ex ante* bargaining.

In an equilibrium, agent 2 correctly anticipates the contract to be signed, when renegotiations occur and her *ex post* payoff in each state, given the renegotiation costs she chose. Then, she chooses renegotiation costs which maximize her *ex ante* payoff. An equilibrium is described as a triple, consisting of renegotiation costs chosen by agent 2, transfers contracted in *ex ante* bargaining and a contingent occurrence of renegotiations.

3.2 Transfer Determination Mechanism

As in the illustrative example, we separate the NP maximization problem. First, we fix each (x^L, x^H) and find the set of t_2 which maximizes NP subject to $t_2 \in \hat{T}_2(x^L, x^H)$. We call it (x^L, x^H) implementation problem. In each problem, $\hat{S}$ is fixed because expected renegotiation costs size is fixed. Because $\hat{T}_2(x^L, x^H)$ is an interval and Lemma 3.1 holds, the set of the all solutions of (x^L, x^H) implementation is the set of the all solutions of

$$\min_{t_2 \in \hat{T}_2(x^L, x^H)} \left| \rho_2 - \hat{\rho}_2(t_2 | x^L, x^H) \right|,$$

where $\hat{\rho}_2(t_2|x^L, x^H)$ is the value of $\hat{\rho}_2$ when a transfer t_2 is contracted and renegotiations occur as (x^L, x^H). Second, we find the (x^L, x^H) implementation problems which give the highest NP.

Now discuss each (x^L, x^H) implementation problem. The next lemma holds.

Lemma 3.2. *The set of the all solutions of (R, R) implementation is $\hat{T}_2(R, R)$. If $\lambda < e^H - e^L$, $\hat{T}_2(N, N)$ is empty. Otherwise, (N, N) implementation has a unique solution t_2^{NN} and*

$$t_2^{NN} = \begin{cases} \min\{E(e), e^L + \lambda\rho_1\} & \text{if } (x^L, x^H) = (N, N) \text{ and } \rho_2 + \alpha_2 \geq 1 \\ \max\{E(e), e^H - \lambda\rho_2\} & \text{if } (x^L, x^H) = (N, N) \text{ and } \rho_2 + \alpha_2 < 1. \end{cases}$$

Each of (N, R) and (R, N) implementation always has a unique solution t_2^{NR} and t_2^{RN}, respectively and

$$t_2^{x^L x^H} = \begin{cases} \min\{e^L, e^H - \rho_2\lambda\} & \text{if } (x^L, x^H) = (N, R) \\ \max\{e^H, e^L + \rho_1\lambda\} & \text{if } (x^L, x^H) = (R, N). \end{cases}$$

Proof. See the Appendix. □

A sketch of the proof is as follows. As mentioned above, in each (x^L, x^H) implementation problem, the solution minimizes the distance between ρ_2 and $\hat{\rho}_2$ if $\hat{T}_2(x^L, x^H)$ is not empty. In any state in which a renegotiation occurs, t_2 is not enforced and the renegotiation splits the *ex post* surplus in the state according to the agents' bargaining powers. Especially, in (R, R) implementation, any t_2 is indifferent and the $\hat{\rho}_2$ is equal to ρ_2, because renegotiations occur in the both states.

In the all other implementation problems, t_2 affects the NP through the *ex post* payoffs in the states where no renegotiation occurs. If t_2 equals the expectation of e conditional on no renegotiation, the agents' expected payoffs conditional on no renegotiation is realized as if efficient renegotiations occurred in the states where $x = N$. Therefore, in each $(x^L, x^H) \neq (R, R)$ implementation, the solution is the nearest t_2 to the expectation of e conditional on no renegotiation. The value is $E(e)$, e^L and e^H in $(N, N), (N, R)$ and (R, N), respectively.

When the expectation of e conditional on no renegotiation does not belong to $\hat{T}_2(x^L, x^H)$, (x^L, x^H) implementation problems has a corner solution. Whether (x^L, x^H) implementation problem has a corner solution or not depends on λ. In (N, N) implementation, $E(e)$ does not belong to $\hat{T}_2(N, N)$ for small λ. On the other hand, each of e^L and e^H is not in $\hat{T}_2(N, R)$ and $\hat{T}_2(R, N)$, respectively, for large λ. By finding the condition that each of $(N, N), (N, R)$ and (R, N) implementation has a corner solution and the value of the corner solution, we get Lemma 3.2.

As we will see later, agent 2 can increase her effective bargaining power and *ex ante* payoff if and only if some condition is satisfied. The condition is that the solution of (N, N) implementation is higher than $E(e)$ if it has a corner solution. By Lemma 3.2, the condition is $\rho_2 + \alpha_2 < 1$. By the fact that $\rho_1 + \rho_2 = \alpha_1 + \alpha_2 = 1$, it is equivalent to $\rho_2 + \alpha_2 < \rho_1 + \alpha_1$. As we have seen in Section 3.1,[7] agents 1 and 2 have stronger incentives to hold up in ω^L and ω^H, respectively. Then α_i is the probability of the state in which each agent i has a stronger incentive to hold up. We can interpret the condition, $\rho_2 + \alpha_2 < \rho_1 + \alpha_1$, to mean that agent 2 is less aggressive in bargaining than agent 1.

Definition 3.3. *If $\rho_i + \alpha_i < 1$, agent i is* ***inactive in bargaining****.*

Figure III explains why t_2^{NN} is higher than $E(e)$ when agent 2 is inactive in bargaining.

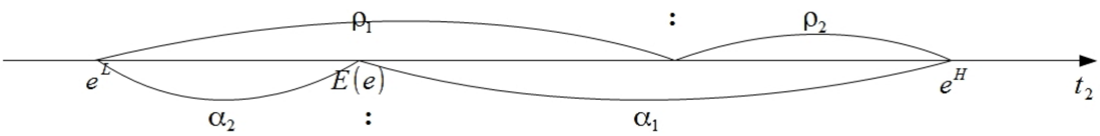

Figure III. $\rho_2 e^L + \rho_1 e^H$ is greater than $E(e)$ when agent 2 is inactive in bargaining.

When $\lambda = e^H - e^L$, $\hat{T}_2(N, N)$ is the singleton $\{\ \rho_2 e^L + \rho_1 e^H\ \}$. If agent 2 is inactive in bargaining, $\rho_2 e^L + \rho_1 e^H > E(e)$. As λ gets larger, $\hat{T}_2(N, N)$ expands to right and left and the minimum of $\hat{T}_2(N, N)$ comes close to $E(e)$ from the right.

Now, we proceed to the implementation selection problem, given renegotiation costs λ. The next lemma holds.

Lemma 3.4. *Suppose that agent 2 is inactive in bargaining. Then, some $\hat{\lambda} \geq e^H - e^L$ exists and the contingent occurrence of renegotiations and agent 2's share ratio of the ex ante bargaining surplus $\hat{\rho}_2$ satisfy the following properties.*

(i) If $\lambda > \hat{\lambda}$, renegotiations do not occur in the both states.

(ii) If $\lambda = \hat{\lambda}$, either (a) or (b) holds.

(a) Renegotiations do not occur in the both states and $\hat{\rho}_2 > \rho_2$.

(b) A renegotiation occurs in some state and $\hat{\rho}_2 = \rho_2$.

(iii) If $\lambda < \hat{\lambda}$, a renegotiation occurs in some state and $\hat{\rho}_2 = \rho_2$.

Proof. See the Appendix. □

[7] See the paragraph after (6).

Lemma 3.4 says that (N, N) implementation is chosen when λ is sufficiently large. The reason is as follows. NP attained through (N, N) is non-decreasing with λ, because increment of λ just looses the constraints but do nothing worse for NP maximization. Each of NP attained through all the other implementation problems are strictly decreasing with λ because $\hat{S}$ is strictly decreasing with λ and the distance between $\hat{\rho}_1$ and ρ_1 is weakly increasing with λ in the implementation. Lemma 3.4 also says that agent 2's effective bargaining power differs from her bargaining power only in the equilibria where (N, N) implementation is chosen. It may seem inconsistent with Lemma 3.2, because Lemma 3.2 says each of t_2^{NR} and t_2^{RN} differs from e^L and e^H for some large λ and so $\hat{\rho}_2$ differs from ρ_2. However, (N, N) implementation gives the largest NP for such λ as we will see in the Appendix.

3.3 Inactiveness in Bargaining and Renegotiation Costs

Now we study what amount of λ agent 2 chooses. She does not choose any positive λ which does not leads to (N, N), because any *ex ante* payoff pair induced by such λ is Pareto dominated by that induced by $\lambda = 0$.

If agent 2 is inactive in bargaining, she chooses $\lambda = \hat{\lambda}$, renegotiations do not occur in the both states in the equilibrium and $\hat{\rho}_2 > \rho_2$ holds. The reason is as follows. When $\lambda = \hat{\lambda}$, $t_2^{NN} = e^H - \hat{\lambda}\rho_2 > E(e)$ because (N, N) implementation attains $\hat{\rho}_2 > \rho_2$. When $\lambda \in (\hat{\lambda}, \hat{\lambda} + \epsilon)$ for sufficiently small $\epsilon > 0$, renegotiations do not occur in the both states and $t_2 = e^H - \lambda\rho_2 > E(e)$ is contracted because t_2^{NN} is continuous with λ and $t_2^{NN} > E(e)$ for $\lambda = \hat{\lambda}$. Then, U_2 is decreasing with λ for $\lambda \in (\hat{\lambda}, \hat{\lambda} + \epsilon)$ and nonincreasing with λ for $\lambda > \hat{\lambda}$. If renegotiations occur as any (x^L, x^H) except for (N, N) when $\lambda = \hat{\lambda}$, there is no λ which is her best response. Therefore, renegotiations do not occur in the both states when $\lambda = \hat{\lambda}$ in the equilibrium and her best response is $\lambda = \hat{\lambda}$ there.

If agent 2 is not inactive in bargaining, her share ratio of the *ex ante* surplus never exceeds her bargaining power. There are two ways of best responses she chooses in the equilibrium paths. The first is $\lambda = 0$, for which a renegotiation occurs in some state but the expected loss from renegotiation is zero. The second is to choose λ which is so large that renegotiations do not occurs in the both state and $t_2^{NN} = E(e)$ is contracted. The existence of such λ is guaranteed by (8), We summarize the results as the next theorem.

Theorem 3.5. *The renegotiation costs λ chosen by agent 2, her ex ante surplus U_2, and her share ratio of the ex ante bargaining surplus $\hat{\rho}_2$ satisfy the following properties.*

(i) If agent 2 is inactive in bargaining, $\lambda = \hat{\lambda}$, $\hat{\rho}_2 > \rho_2$, and U_2 is larger than that attained by efficient renegotiation, $\rho_2 E\big(\sum_i (b_i - d_i)\big) + E(d_2)$.

(ii) Otherwise, $\hat{\rho}_2 = \rho_2$, $U_2 = \rho_2 E\big(\sum_i (b_i - d_i)\big) + E(d_2)$, and the expected renegotiation costs are zero.

Theorem 2.3 is a special case of Theorem 3.5. In the illustrative example, the symmetry of their bargaining powers is assumed. Then, the seller is inactive in bargaining if and only if the high state is realized with a probability less than a half. On the other hand, Theorem 3.5 gives another implication that the renegotiation inefficiency tends to improve the person who is not good at bargaining. When renegotiations take place inefficiently, renegotiation costs reduce the agents' payoffs in the ratio of their bargaining powers. Then, the agent with larger bargaining power has a smaller incentive of hold up. To prevent renegotiations in the both states, her payoff tends to be regarded as less important. An extreme is the case where one agent's bargaining power is zero. Strictly speaking, it violates the assumption that the bargaining powers are positive. But almost the same result holds when her bargaining power is sufficiently small. For example, suppose that you are an employee and your bargaining power is zero. Your boss keeps your wage as low as possible in every states. If renegotiations take place efficiently, you always receive your reservation wage. However, you can get a higher wage if renegotiation take place inefficiently to some extent. Your boss is afraid of your hold up and keep your wage as high as your reservation wage in the state where it takes a higher value.

4 Mitigation of Investments Problem thorough the Effective Bargaining Power Manipulation

4.1 The Effective Bargaining Power Set

We have assumed that renegotiation costs are chosen by either one agent until now. But sometimes we are interested in the cases where the renegotiation costs are parameter or determined by both agents. The parts of results gotten in the previous sections, especially Lemma 2.1, Lemma 2.2 and Lemma 3.2 are useful even in such cases. This is because these results are independent of the way how renegotiation costs are determined. These results can be straightforwardly used for a comparative statics for renegotiation costs when renegotiation costs are regarded as a parameter.

We focus on the illustrative example for simplicity. We describe the seller's share ratio of the *ex ante* bargaining surplus as $\hat{\rho}_S$. Now, we compute the set of all $\hat{\rho}_S$ each of which is realized in an equilibrium for some renegotiation costs. We call it the effective bargaining power set of the seller, or simply the effective bargaining power set. Then, the next theorem holds.

Theorem 4.1. *The effective bargaining power set of the seller is* $[\frac{1}{2}, \overline{\rho}_S]$*, where*

$$\overline{\rho}_S \equiv \frac{\frac{1}{2}(v^H - \tilde{\lambda})}{E(v)} > \frac{1}{2}.$$

$\tilde{\lambda}$ is the value of λ which is described in Lemma 2.2. We can also compute the set of the all pairs of *ex ante* surplus S and $\hat{\rho}_S$ which can be attained for some λ. We call it the feasible set of $(S, \hat{\rho}_S)$. By Lemmas 2.1 and 2.2, no renegotiation occurs in the both states when $\hat{\rho}_S > \frac{1}{2}$ and so $S = E(v)$. On the other hand, $\hat{\rho}_S = \frac{1}{2}$ is attained not only by a λ for which renegotiations do not occurs in the both states and the price $p = \frac{1}{2}E(v)$ is contracted, but also by a positive λ for which renegotiations occur as either (N, R), (R, N) or (R, R). The feasible pair set of $(S, \hat{\rho}_S)$ is

$$\left[E(v) - \overline{\lambda}, E(v)\right] \times \left\{\frac{1}{2}\right\} \cup \{E(v)\} \times \left[\frac{1}{2},\ \overline{\rho}_S\right].$$

For simplicity, however, we assume that $S = E(v)$ is realized in the equilibria and the parties choose an effective bargaining power $\hat{\rho}_S \in [\frac{1}{2}, \overline{\rho}_S]$. That is, they choose a feasible pair $(S, \hat{\rho}_S)$ from the feasible set subject to $S = E(v)$. In many cases, this assumption is not restrictive. This is because if a feasible pair $(S', \hat{\rho}'_S)$ satisfies $S' < E(v)$ and so $\hat{\rho}'_S = \frac{1}{2}$, then $(S', \hat{\rho}'_S)$ is Pareto dominated by the feasible pair $(E(v), \frac{1}{2})$. Especially, we can easily show this assumption is not restrictive in the analysis in Section 4.2.

4.2 Mitigation of Underinvestment Problem

We show an application of the effective bargaining power set. We consider a variant of the illustrative model where an *ex ante* investment stage exists. First, renegotiation costs are chosen by the parties in a way to be described later. After the renegotiation costs are chosen and observed by the both parties, the buyer and the seller make investment I_B and I_S for searching a project, respectively. The present values of the good and renegotiation costs in the following stages are multiplied by $(1 + 2\beta_B\sqrt{I_B} + 2\beta_S\sqrt{I_S})$. We can interpret that to mean that their investments make them find a project sooner and these present values are less time-discounted. Each β_i represents the skill of investments of agent i. When a price p is contracted, the high state is realized and a renegotiation occurs, $u_B = u_S = \frac{1}{2}(1+2\beta_B\sqrt{I_B}+2\beta_S\sqrt{I_S})(v_H - \lambda)$. When a price p is contracted, the low state is realized and no renegotiation occurs, $u_B = (1 + 2\beta_B\sqrt{I_B} + 2\beta_S\sqrt{I_S})v_L - p$ and $u_S = p$. Note that the price p realized in the equilibrium is equal to $(1+2\beta_B\sqrt{I_B}+2\beta_S\sqrt{I_S})$ times the equilibrium price in the illustrative example. Therefore, the investments does not affect the relation between λ and $\hat{\rho}_S$ and the effective bargaining power set is independent of them. Figure IV illustrates the time line.

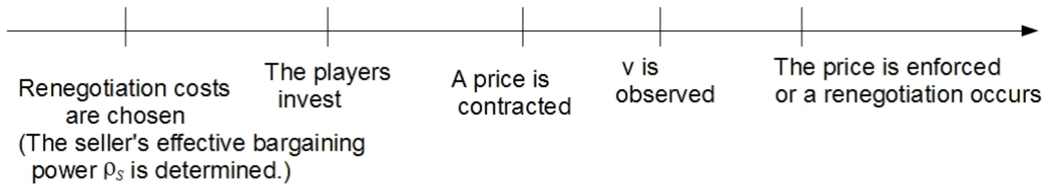

Figure IV. Time line of the model in this section.

$\grave{U}_i$ denotes agent i's *ex ante* payoffs multiplied by $(1+2\beta_B\sqrt{I_B}+2\beta_S\sqrt{I_S})$ and reduced by the costs of investment I_i. Investment size I_i is measured by its disutility. When each agent i's effective bargaining power is $\hat{\rho}_i$ under the renegotiation costs chosen by the agents,

$$\grave{U}_i = \hat{\rho}_i \left(1 + \sum_k 2\beta_k\sqrt{I_k}\right) E(v) - I_i \qquad \text{for } i = B, S.$$

$\grave{S}$ denotes the sum of $\grave{U}_B$ and $\grave{U}_S$. That is,

$$\grave{S} = \left(1 + \sum_k 2\beta_k\sqrt{I_k}\right) E(v) - I_B - I_S.$$

The investment pair (I_B^*, I_S^*) which maximize $\grave{S}$ is characterized by the next first order condition.

$$\frac{\beta_i E(v)}{\sqrt{I_i^*}} - 1 = 0 \qquad i = B, S.$$

Then, $I_i^* = \beta_i^2 E(v)^2$ holds for $i = B, S$. On the other hand, the investment level I_i^{**} chosen by the agent i independently to maximize $\grave{U}_i$ is characterized by

$$\frac{\hat{\rho}_i\beta_i}{\sqrt{I_i^{**}}} E(v) - 1 = 0 \qquad i = B, S.$$

Then, $I_i^{**} = \hat{\rho}_i^2\beta_i^2 E(v)^2 \leq I_i^*$ and the equality holds if and only if $\hat{\rho}_i = 1$. In fact, the both agents underinvest, because $\hat{\rho}_S \in [\frac{1}{2}, \overline{\rho}_S]$.

Therefore, the agents cannot perfectly remove this inefficiency but they can mitigate it by affecting their effective bargaining powers through the manipulation of renegotiation costs. We assume that λ is also determined in the Nash bargaining. We denote the $\hat{\rho}_S$ realized in the bargaining by $\hat{\rho}_S^*$. Then, $\hat{\rho}_S^*$ solves the next problem.

$$\max_{\hat{\rho}_S \in [\frac{1}{2}, \overline{\rho}_S]} \grave{U}_B \grave{U}_S, \tag{13}$$

where

$$\grave{U}_i = \hat{\rho}_i \left(1 + \sum_k 2\beta_k \sqrt{I_k(\hat{\rho}_k)}\right) E(v) - I_i(\hat{\rho}_i) \qquad i = B, S$$
$$I_i(\hat{\rho}_i) = \hat{\rho}_i^2 \beta_i^2 E(v)^2$$
$$\hat{\rho}_B = 1 - \hat{\rho}_S. \tag{14}$$

The next theorem holds.

Theorem 4.2. *Suppose that the seller's investment skill β_S is higher than that of the buyer β_B. Then, some positive renegotiation costs λ are chosen and the seller's effective bargaining power $\hat{\rho}_S$ is larger than her bargaining power $\frac{1}{2}$. Furthermore, the present value of the ex ante surplus reduced by the investment costs $\grave{S}$ is larger than that attained when $\lambda = 0$ is chosen.*

Proof. See the Appendix. □

The result is intuitive. The seller should have higher effective bargaining power because her investment is more important. In our model, underinvestment problem are mitigated differently from the traditional property right theory. In the traditional property right theory, asset allocations mitigate underinvestment problems through affecting agents' disagreement points. In our model, the agents mitigate the problem not by asset allocations but by controlling renegotiation costs. Furthermore, renegotiation costs do not change the parties' disagreement points but changes their effective bargaining powers.

5 Conclusion

In this paper, we discuss the situations where two agents contract a non-contingent transfer for a future collaboration with uncertain private benefits and disagreement points. Some of the two can manipulate the size of renegotiation costs before the contract is signed. In Section 2, we study the illustrative example. A buyer and a seller trade a unit of good there. Their bargaining powers are symmetric but the probability distribution of the value of the good is asymmetric in the sense that the low quality is likely to be realized. The seller is the agent who can choose renegotiation costs. Although her share ratio of the *ex ante* surplus is a half when renegotiations take place efficiently, she sets the renegotiation costs at some positive amount and obtains the more share of the *ex ante* surplus in the equilibrium. We regard that she improves her effective bargaining power.

However, the ability of manipulating renegotiation costs is not suffice to increase her payoff. As we see in the general results, she can improve her effective bargaining power $\hat{\rho}_i$ if and only if she is inactive in bargaining. We call she is inactive in bargaining when the sum of two factors about her is larger than that about her opponent. One is the probability of the state in which her hold up incentive is larger than in the other state. In the illustrative example, the seller's hold up incentive is larger when the value of the good is high. The second is her bargaining power ρ_i. Consider an extreme example in which a boss contract a wage with an employee with almost zero bargaining power. In this case, an employee is generically inactive in bargaining.

In some cases, it seems to be impossible to manipulate the inefficiency of renegotiation. However, parts of our results can be used for a comparative analysis with renegotiation costs. For example, we can compute the feasible set of all *ex ante* surplus and effective bargaining powers which can be attained for some renegotiation costs.

We can also discuss the case where renegotiation costs are also determined in bargaining. In Section 4.2, we discuss a variant of the illustrative model in which renegotiation costs are determined in such a way. The seller and the buyer have an investment stage before they contract a price. They have a symmetric bargaining power but the seller has a better investment skill. The effective bargaining power is chosen from the effective bargaining power set in Nash bargaining. Theorem 4.2 shows that the agents increase the seller's effective bargaining power as the result of the bargaining.

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Appendix

Proof of Lemma 3.2

In (R, R) implementation, $\hat{U}_B, \hat{U}_S$ and $\hat{S}$ are independent of t_2. Then, $\hat{\rho}_2$ is independent of t_2 by its definition. Therefore, any t_2 in $\hat{T}_2(R, R)$ is a solution of (R, R) implementation. In (N, N) implementation, $\hat{T}_2(N, N)$ is empty if and only if $\lambda < e^H - e^L$. In this proof, we assume that $(x^L, x^H) \neq (R, R)$. We also assume that $\lambda \geq e^H - e^L$ if $(x^L, x^H) = (N, N)$.

(i) The solutions when the constraints are not restrictive. Suppose that the constraint is not restrictive in some (x^L, x^H) implementation problem. It means that the solution of the problem does not change even if we remove the constraint $t_2 \in \hat{T}_2(x_L, x_H)$. Then, a transfer t_2 is a solution if and only if $\rho_2 = \hat{\rho}_2(t_2|x^L x^H)$. It is equivalent to $\rho_2 \hat{S} + E(d_2) = U_2$. Then,

$$\rho_2 \left[E\left(\sum_i (b_i - d_i) \right) - Prob(R|x^L x^H)\lambda \right] + E(d_2) = Prob(N|x^L x^H) \left\{ t_2 + E(b_2|N, x^L x^H) \right\}$$
$$+Prob(R|x^L x^H) E\left[\rho_2 \left(\sum_i (b_i - d_i) - \lambda \right) + d_2 \;\middle|\; R, x^L x^H \right], \qquad (15)$$

where $Prob(K|x^L, x^H)$ denotes the probability of $x = K$ given renegotiations occur as (x^L, x^H),[8] and $E(z|K, (x^L, x^H))$ denotes the expectation of z given that renegotiations occur as (x^L, x^H) and $x = K$ is realized.[9] When $(x^L, x^H) = (N, N)$, (15) is

$$E\left[\rho_2 \sum_i (b_i - d_i) + d_2\right] = t_2 + E(b_2).$$

By solving it, we have $t_2^{NN} = E(e)$. When $(x^L, x^H) = (N, R)$, (15) is

$$E\left[\rho_2 \left(\sum_i (b_i - d_i) - \alpha_2\lambda\right) + d_2\right] = \alpha_1(t_2 + b_2^L) + \alpha_2\left[\rho_2\left(\sum_i (b_i^H - d_i^H) - \lambda\right) + d_2^H\right].$$

By solving it, we have $t_2^{NR} = e^L$. Similarly, we have $t_2^{RN} = e^H$.

Therefore, if the constraint is not restrictive,

$$t_2^{x^L x^H} = E\left(\ e \mid N, (x^L, x^H)\ \right) = \begin{cases} E(e) & \text{if } (x^L, x^H) = (N, N), \\ e^L & \text{if } (x^L, x^H) = (N, R), \\ e^H & \text{if } (x^L, x^H) = (R, N). \end{cases} \tag{16}$$

(ii) The solutions when the constraint is restrictive. Suppose that in some (x^L, x^H) implementation problem, $E(e|N, (x^L, x^H)) \notin \hat{T}_2(x^L, x^H)$. Since $\hat{T}_2(x^L, x^H)$ is an interval,

$$E(e|N, (x^L, x^H)) < \min \hat{T}_2(x^L, x^H) \quad \text{or} \quad \max \hat{T}_2(x^L, x^H) < E(e|N, (x^L, x^H)). \tag{17}$$

If the first inequality of (17) holds, $t_2^{x^L x^H} = \min \hat{T}_2(x^L, x^H)$. This is because $\hat{\rho}_2(t_2|x^L, x^H)$ is increasing with t_2 and $\rho_2 = \hat{\rho}_2\left(E(e|N, (x^L, x^H)) \mid x^L, x^H\right)$ by Equation (16). Similarly, if the second inequality of (17) holds, $t_2^{x^L x^H} = \max \hat{T}_2(x^L, x^H)$.

By (9), the constraint $t_2 \in \hat{T}_2(N, N)$ binds in (N, N) implementation if and only if

$$\lambda < \frac{\alpha_1}{\rho_2}(e^H - e^L) \qquad \text{or} \qquad \lambda < \frac{\alpha_2}{\rho_1}(e^H - e^L).$$

The first and second inequality correspond to the first and second inequality in (17), respectively. By using the assumption that $\lambda \geq e^H - e^L$ if $(x^L, x^H) = (N, N)$, we have the followings. When the first inequality holds, $1 > \rho_2 + \alpha_2$ and $t_2^{NN} = \min \hat{T}_2(N, N) = e^H - \rho_2\lambda$. When the second inequality holds, $1 < \rho_2 + \alpha_2$ and $t_2^{NN} = \max \hat{T}_2(N, N) = e^L + \rho_1\lambda$. The constraint $t_2 \in \hat{T}_2(N, R)$ is restrictive in (N, R) implementation if and only if

$$\frac{e^H - e^L}{\rho_2} < \lambda \qquad \text{and} \qquad t_2^{NR} = \max \hat{T}_2(N, R) = e^H - \rho_2\lambda < E\left(e|\ N, (N, R)\right).$$

The constraint $t_2 \in \hat{T}_2(R, N)$ is restrictive in (R, N) implementation if and only if

$$\frac{e^H - e^L}{\rho_1} < \lambda \qquad \text{and} \qquad t_2^{RN} = \min \hat{T}_2(R, N) = e^L + \rho_1\lambda > E\left(e|\ N, (R, N)\right).$$

(iii) The solution By the results in (i) and (ii), we get Lemma 3.3. □

[8] For example, $P(N|N, N) = 1, P(N|N, R) = \alpha_1$, and $P(R|N, R) = \alpha_2$.

[9] For example, $E(\sum_i b_i|N, (N, N)) = \sum_i E(b_i), E(\sum_i b_i|N, (N, R)) = \sum_i b_i^L$ and $E(d_1|R, (N, R)) = d_1^H$.

Proof of Lemma 3.4

We introduce the following notations. $t_2^{x^Lx^H}(\lambda)$, $U_2(t_2|x^L,x^H,\lambda)$, $\hat{\rho}_2(t_2|x^L,x^H,\lambda)$, $\hat{S}^{x^Lx^H}(\lambda)$ and $NP^{x^Lx^H}(\lambda)$ denote the values of $t_2^{x^Lx^H}$, $U_2(t_2|x^L,x^H)$, $\hat{\rho}_2(t_2|x^L,x^H)$, $\hat{S}^{x^Lx^H}$ and $NP^{x^Lx^H}$ when renegotiation costs are λ, respectively. We define

$$D^{x^Lx^H}(\lambda) \equiv \left|\rho_2 - \hat{\rho}_2(t_2^{x^Lx^H}(\lambda)|x^L,x^H,\lambda)\right|.$$

By the results of (i) in Proof of Lemma 3.2, $\hat{U}_2(E(e|N,x^L,x^H)\ |x^L,x^H,\lambda) = \rho_2\hat{S}^{x^Lx^H}(\lambda)$ and so

$$\begin{aligned} D^{x^Lx^H}(\lambda) &= \frac{\left|U_2(E(e|N,x^L,x^H)|x^L,x^H,\lambda) - U_2(t_2^{x^Lx^H}(\lambda)|x^L,x^H,\lambda)\right|}{\hat{S}^{x^Lx^H}(\lambda)} \\ &= \frac{Prob(N|x^L,x^H)}{\hat{S}^{x^Lx^H}(\lambda)} \times \left|E(e|N,(x^L,x^H)) - t_2^{x^Lx^H}(\lambda)\right| \end{aligned} \tag{18}$$

We define a function $F : [0,\overline{\lambda}] \to \mathbb{R}$ as

$$F(\lambda) \equiv \begin{cases} NP^{NN}(\lambda) - \max\{\ NP^{NR}(\lambda),\ NP^{RN}(\lambda),\ NP^{RR}(\lambda)\ \} & \text{if } \lambda \geq e^H - e^L \\ -1 & \text{otherwise.} \end{cases}$$

If $F(\lambda) > 0$, (N,N) uniquely maximizes $NP^{x^Lx^H}$ given λ. If $F(\lambda) = 0$, (N,N) and some another (x^L,x^H) maximizes $NP^{x^Lx^H}$, given λ. Otherwise, (N,N) does not maximize $NP^{x^Lx^H}$ or (N,N) is not feasible, given λ.

First, we show the existence of $\hat{\lambda} \in [0,\overline{\lambda}]$ such that $F(\lambda) \geq 0$ if and only if $\lambda \geq \hat{\lambda}$. By (18) and Lemma 3.2, we have (i) $\hat{S}^{NN}(\lambda)$ is constant, $D^{NN}(\lambda)$ is weakly decreasing and so $NP^{NN}(\lambda)$ is nondecreasing with λ, and (ii) for any $(x^L,x^H) \neq (N,N)$, $\hat{S}^{x^Lx^H}(\lambda)$ is decreasing, $D^{x^Lx^H}(\lambda)$ is nondecreasing, and so $NP^{x^Lx^H}(\lambda)$ is decreasing with λ. Therefore, $F(\lambda)$ is increasing with λ on $[e^H-e^L,\overline{\lambda}]$. If $F(e^H-e^L) \geq 0$, $\hat{\lambda} = e^H-e^L$. If $F(e^H-e^L) < 0$, we can show $\hat{\lambda}$ exists in (e^H-e^L,λ°) where $\lambda^\circ \equiv \frac{\alpha_1}{\rho_2}(e^H-e^L)$. Because of (8) and $\frac{\alpha_1}{\rho_2} > 1$,[10] $\lambda^\circ \in (e^H-e^L,\overline{\lambda})$ and $t_2^{NN}(\lambda^\circ) = E(e)$ holds. Then, $D^{NN}(\lambda^\circ) = 0$. Because $\hat{S}^{NN}$ is larger than any other $\hat{S}^{x^Lx^H}$, $NP^{NN}(\lambda^\circ)$ is larger than all the other $NP^{x^Lx^H}(\lambda)$ and so $F(\lambda^\circ) > 0$. Then, some $\lambda' < \lambda^\circ$ exists and $F(\lambda') = 0$. In this case, $\hat{\lambda} = \lambda'$. We have shown the existence of $\hat{\lambda}$. We can also see that $\hat{\rho}_2(t_2^{NN}|N,N,\hat{\lambda}) > \rho_2$ because $t_2^{NN} > E(e)$ holds for $\lambda \in [e^H-e^L,\lambda^\circ)$.

Finally, we prove $\hat{\rho}_2(t_2^{x^Lx^H}(\lambda)|x^L,x^H,\lambda) = \rho_2$ for any $\lambda \leq \hat{\lambda}$ and any $(x^L,x^H) \neq (N,N)$, by contradiction. Assume that $\hat{\rho}_2(t_2^{x^Lx^H}(\lambda^*)|x^L,x^H,\lambda^*) \neq \rho_2$ for some $\lambda^* \leq \hat{\lambda}$ and some $(x^L,x^H) \neq (N,N)$. Because $D^{RR}(\lambda') = 0$ for any λ, $(x^L,x^H) \neq (R,R)$. By (18), $\hat{\rho}_2(t_2^{x^Lx^H}(\lambda)|x^L,x^H,\lambda) = \rho_2$ is equivalent to $E(e|N,(x^L,x^H)) = t_2^{x^Lx^H}(\lambda)$ for $(x^L,x^H) \neq (R,R)$. Suppose that $(x^L,x^H) = (N,R)$. By Lemma 3.2 and the results in the proof, $t_2^{NR}(\lambda^*) = e^H - \rho_2\lambda^* = \max\hat{T}_2(N,R) = \min\hat{T}_2(N,N)$. As Figure V illustrates, when $t_2 = t_2^{NR}(\lambda^*)$ and the high state is realized, a renegotiation and no renegotiation are indifferent for agent 2. Then, $\hat{U}_2(t_2^{NR}(\lambda^*)|N,R,\lambda^*) = \hat{U}_2(t_2^{NR}(\lambda^*)|N,N,\lambda^*)$, while $\hat{U}_1(t_2^{NR}(\lambda^*)|N,R,\lambda^*) < \hat{U}_1(t_2^{NR}(\lambda^*)|N,N,\lambda^*)$ holds because $\hat{S}^{NN}(\lambda^*) > \hat{S}^{NR}(\lambda^*)$. Then,

$$\begin{aligned} NP^{NR}(\lambda^*) &= \hat{U}_1(t_2^{NR}(\lambda^*)|N,R,\lambda^*)^{\rho_1}\hat{U}_2(t_2^{NR}(\lambda^*)|N,R,\lambda^*)^{\rho_2} \\ &< \hat{U}_1(t_2^{NR}(\lambda^*)|N,N,\lambda^*)^{\rho_1}\hat{U}_2(t_2^{NR}(\lambda^*)|N,N,\lambda^*)^{\rho_2} \leq NP^{NN}(\lambda^*). \end{aligned}$$

By Lemma 3.2, it is a contradiction. We can similarly show a contradiction when $(x^L,x^H) =$

[10] Figure III illustrates that $\frac{\alpha_1}{\rho_2} > 1$ when agent 2 is inactive in bargaining.

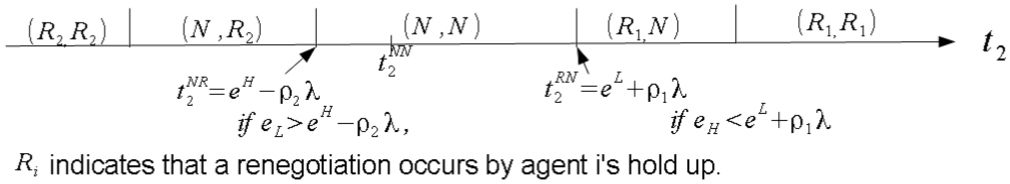

Figure V. Relation between agent 2' transfer and contingent occurrences of hold up actions.

(R,N). Then, $\hat{\rho}_2(t_2^{x^Lx^H}(\lambda)|x^L,x^H)=\rho_2$ for any $\lambda\leq\hat{\lambda}$, if $(x^L,x^H)\neq(N,N)$. □

Proof of Theorem 4.2

Since $2\beta_k\sqrt{I_k(\hat{\rho}_k)}=\frac{2I_k(\hat{\rho}_k)}{\hat{\rho}_kE(v)}$ holds,

$$\begin{aligned}\grave{U}_i=&\hat{\rho}_i\left[1+\sum_k\left\{\frac{2I_k(\hat{\rho}_k)}{\hat{\rho}_kE(v)}\right\}\right]E(v)-I_i(\hat{\rho}_i)=\hat{\rho}_iE(v)+I_i(\hat{\rho}_i)+2\left(\frac{\hat{\rho}_i}{\hat{\rho}_j}\right)I_j(\hat{\rho}_j)\\=&E(v)\left\{\hat{\rho}_i+\hat{\rho}_i^2\beta_i^2E(v)+2\hat{\rho}_i\hat{\rho}_j\beta_j^2E(v)\right\}>0\qquad\text{for } i\neq j.\end{aligned}\tag{19}$$

Because $\grave{U}_B\grave{U}_S=\frac{1}{4}\{\grave{S}^2-(\grave{U}_B-\grave{U}_S)^2\}$ and $\grave{S}>0$, the derivative of $\grave{U}_B\grave{U}_S$ is positive if that of $\grave{S}$ is positive and that of $(\grave{U}_B-\grave{U}_S)^2$ is negative, while the derivative of $\grave{U}_B\grave{U}_S$ is negative if that of $\grave{S}$ is negative and that of $(\grave{U}_B-\grave{U}_S)^2$ is positive.

Let $\grave{U}_i(\hat{\rho}_S,\hat{\rho}_B)\equiv E(v)\{\hat{\rho}_i+\hat{\rho}_i^2\beta_i^2E(v)+2\hat{\rho}_i\hat{\rho}_j\beta_j^2E(v)\}$ for $i\neq j$. Then,

$$\begin{aligned}\frac{d}{d\hat{\rho}_i}\grave{U}_i\Big|_{\hat{\rho}_S+\hat{\rho}_B=1}&=\frac{\partial}{\partial\hat{\rho}_i}\grave{U}_i(\hat{\rho}_S,\hat{\rho}_B)-\frac{\partial}{\partial\hat{\rho}_j}\grave{U}_i(\hat{\rho}_S,\hat{\rho}_B)\\&=E(v)\left[1+2E(v)\{\hat{\rho}_i\beta_i^2+(\hat{\rho}_j-\hat{\rho}_i)\beta_j^2\}\right]>0\qquad\text{for } i\neq j.\end{aligned}\tag{20}$$

The last inequality of (20) holds because (i) $\hat{\rho}_S-\hat{\rho}_B\geq0$ and so $\frac{d}{d\hat{\rho}_B}\grave{U}_B\Big|_{\hat{\rho}_S+\hat{\rho}_B=1}$ is positive, and (ii) $\frac{d}{d\hat{\rho}_S}\grave{U}_S\Big|_{\hat{\rho}_S+\hat{\rho}_B=1}$ is also positive by the equation,

$$\hat{\rho}_S\beta_S^2+(\hat{\rho}_B-\hat{\rho}_S)\beta_B^2\geq\hat{\rho}_S\beta_S^2+(\hat{\rho}_B-\hat{\rho}_S)\beta_S^2=\hat{\rho}_B\beta_S^2.$$

By (20),

$$\begin{aligned}&\frac{d}{d\hat{\rho}_S}\grave{S}\Big|_{\hat{\rho}_S+\hat{\rho}_B=1}=\frac{d}{d\hat{\rho}_S}\left(\grave{U}_S+\grave{U}_B\right)\Big|_{\hat{\rho}_S+\hat{\rho}_B=1}=\frac{d}{d\hat{\rho}_S}\grave{U}_S\Big|_{\hat{\rho}_S+\hat{\rho}_B=1}-\frac{d}{d\hat{\rho}_B}\grave{U}_B\Big|_{\hat{\rho}_S+\hat{\rho}_B=1}\\=&2E(v)^2\left(\hat{\rho}_B\beta_S^2-\hat{\rho}_S\beta_B^2\right)=2E(v)^2\left\{\beta_S^2-\hat{\rho}_S(\beta_S^2+\beta_B^2)\right\}.\end{aligned}$$

Therefore,

$$\frac{d}{d\hat{\rho}_S}\grave{S}\Big|_{\hat{\rho}_S+\hat{\rho}_B=1}\gtreqless0\quad\text{if}\quad\hat{\rho}_S\lesseqgtr\frac{\beta_S^2}{\beta_S^2+\beta_B^2}>\frac{1}{2}.\tag{21}$$

By (20), we have

$$\frac{d}{d\hat{\rho}_S}(\grave{U}_B-\grave{U}_S)\Big|_{\hat{\rho}_S+\hat{\rho}_B=1}=-\frac{d}{d\hat{\rho}_B}\grave{U}_B\Big|_{\hat{\rho}_S+\hat{\rho}_B=1}-\frac{d}{d\hat{\rho}_S}\grave{U}_S\Big|_{\hat{\rho}_S+\hat{\rho}_B=1}<0.\tag{22}$$

Because $\frac{d}{d\hat{\rho}_S}(\grave{U}_B - \grave{U}_S)^2|_{\hat{\rho}_S+\hat{\rho}_B=1} = 2(\grave{U}_B - \grave{U}_S)\frac{d}{d\hat{\rho}_S}(\grave{U}_B - \grave{U}_S)|_{\hat{\rho}_S+\hat{\rho}_B=1}$, we have

$$\left.\frac{d}{d\hat{\rho}_S}(\grave{U}_B - \grave{U}_S)^2\right|_{\hat{\rho}_S+\hat{\rho}_B=1} \lesseqgtr 0 \qquad \text{if} \qquad \grave{U}_B \gtreqless \grave{U}_S. \tag{23}$$

Now, we show that $\hat{\rho}_S^* > \frac{1}{2}$ and $\lambda > 0$ is chosen. By (19), $\grave{U}_B > \grave{U}_S$ for $\hat{\rho}_S = \frac{1}{2}$. By (21) and (23), $\frac{d}{d\hat{\rho}_S}\grave{U}_B\grave{U}_S|_{\hat{\rho}_S+\hat{\rho}_B=1} > 0$ for $\hat{\rho}_S = \frac{1}{2}$. Therefore, $\hat{\rho}_S^* > \frac{1}{2}$ and so $\lambda > 0$ is chosen, by Lemmas 2.1 and 2.2.

Finally, we show that $\grave{S}$ is greater when $\hat{\rho}_S = \hat{\rho}_S^*$ than when $\lambda = 0$ and so $\hat{\rho}_S = \frac{1}{2}$. Suppose that $\grave{U}_B < \grave{U}_S$ for $\hat{\rho}_S = \frac{\beta_S^2}{\beta_S^2+\beta_B^2}$. By (22), $\grave{U}_B < \grave{U}_S$ for $\hat{\rho}_S \geq \frac{\beta_S^2}{\beta_S^2+\beta_B^2}$. By (21) and (23), $\frac{d}{d\hat{\rho}_S}\grave{U}_B\grave{U}_S|_{\hat{\rho}_S+\hat{\rho}_B=1} < 0$ for $\hat{\rho}_S \geq \frac{\beta_S^2}{\beta_S^2+\beta_B^2}$, and so $\hat{\rho}_S^* < \frac{\beta_S^2}{\beta_S^2+\beta_B^2}$. By (21), we have that $\grave{S}$ is greater when $\hat{\rho}_S = \hat{\rho}_S^*$ than when $\lambda = 0$. Therefore, it suffices to show that $\grave{U}_B < \grave{U}_S$ for $\hat{\rho}_S = \frac{\beta_S^2}{\beta_S^2+\beta_B^2}$. By (19), we have

$$\begin{aligned}\grave{U}_S - \grave{U}_B &= \frac{E(v)}{(\beta_S^2+\beta_B^2)^2}\left[(\beta_S^2-\beta_B^2)(\beta_S^2+\beta_B^2) + E(v)(\beta_S^6-\beta_B^6) + 2E(v)\left\{\beta_S^2\beta_B^4 - \beta_B^2\beta_S^4\right\}\right] \\ &= \frac{E(v)}{(\beta_S^2+\beta_B^2)^2}\left[(\beta_S^4-\beta_B^4) + E(v)\left\{(\beta_S^2-\beta_B^2)^3 + \beta_S^2\beta_B^2(\beta_S^2-\beta_B^2)\right\}\right] > 0.\end{aligned}$$

Therefore, $\grave{U}_B < \grave{U}_S$ holds for $\hat{\rho}_S = \frac{\beta_S^2}{\beta_S^2+\beta_B^2}$. The proof is completed. □