Logrolling under Fragmented Authoritarianism: Theory and Evidence from China *

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ABSTRACT

The phenomenon of logrolling among vertical bureaucratic systems has been prevalent in China but its consequence has been under researched. This paper develops a formal model to study the effect of logrolling on policy making. We find that policies under logrolling tend to be overreaching, but policies excluded from logrolling tend to fall short of input. We provide empirical evidences by studying the logrolling between Ministry of Civil Affairs (MCA) and Ministry of Health (MOH) in China. MCA supports MOH by paying insurance premium for poor households in rural areas; in exchange, MOH supports MCA by allowing “Dibao” recipients to be automatically eligible to access healthcare services under medical assistance programs. The consequences of the logrolling are: 1) the benefit tied to “Dibao” becomes too high such that it even crowds out unemployment insurance enrollment; 2) too many people enrolled in rural health insurance but too few really use the health service;

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3) the supply of mental health care service is insufficient, because mental health care, which is not the priority issue for neither MOH nor MCA, is excluded from the logrolling.

Key Words: Authoritarianism, Logrolling, Fragmented Authoritarianism, Policy Making, China.

"The Chinese government makes policy according to a decision rule of delegation by consensus . . . . If the agents reach consensus, the decision is automatically ratified by the higher level; if the agents cannot agree, then the authorities step in to make the decision, or the matter is dropped or tabled until consensus can be achieved". — Shirk 1993, p.116

1. Introduction

Originated from the old custom in the lumber regions of Maine, where lumber-jacks assisted one another in rolling the logs to the river after they were felled and trimmed, the word logrolling is widely used to describe the process of vote-trading in democratic politics (Buchanan and Tullock 1962, p. 92). For instance, a vote on behalf of a tariff of textile may be traded by a congressman for a vote from another congressman on behalf of a subsidy to the steel interests to ensure that both acts will gain a majority and pass through the legislature (Stratmann 1995). But logrolling occurs not only just in democratic political systems, it is also common in authoritarian political systems. For instance, the ruling coalition in Germany before the First World War was the nationalist "marriage of iron and rye", in which aristocratic landowners supported a fleet-building program that industrial interests desired, and in exchange, big business supported high agriculture tariffs (Snyder 1991). In this paper, we make the first try to study logrolling in policy making in China, which has long been treated as a black-box where decisions are made in smoke-filled rooms. Using a simple analytical framework, we find that inefficient policies which are detrimental for the national good will be adopted as a result of logrolling among well-organized parochial interest groups. We find that policies under logrolling tend to be overreaching, but policies excluded from logrolling tend to fall short of input. We further study the logrolling among two vertical bureaucratic organizations in China — the Ministry of Civil Affairs (MCA) and the Ministry of Health (MOH). We illustrate how the two ministries logroll with each other, and test the policy inefficiencies as a result of logrolling.

The paper proceeds as follows. Section 2 reviews the existing literature on logrolling and policy making in China. Section 3 is the theoretical model of logrolling and our finding. In section 4, we present the empirical evidence of logrolling between MCA
and MOH. Conclusions are drawn in the final section.

2. Literature Review

2.1. Literature on logrolling

Most of the existing studies of logrolling are theoretical. After the early studies of Buchanan and Tullock, many formal models were constructed to study the welfare loss associated with logrolling in the form of vote trading (Wilson 1969; Riker and Brahms, 1973; Tullock 1970; Haefele 1971; Browning and Browning, 1979; Koford, 1982). But whether logrolling leads to welfare loss is highly disputable, since some argued that vote trading might be socially efficient, because it allowed legislators to express different intensities of preferences, making every legislator better off (Coleman 1966; Schwartz 1975). More recent works shift focus from welfare issues to the existence of logrolling under different voting rules (Miller 1977; Enelow, 1986; Carrubba and Volden 2000). The limitation of these theoretical works on logrolling is that they rely too heavily on spatial modeling approach. This approach has been commonly used to study electoral competition or social choice, but to a large extend, ignores the crucial strategic aspect of how agents interact in political and market environments. Different from the existing literature, our paper takes a different approach. We study logrolling by incorporating the bargaining theory.

In its broad sense, logrolling can be treated as a special form of bargaining game. For example, in organizational analysis, logrolling is described as one of the ways of bargaining that negotiators can reach integrative agreements (Pruitt, 1981, 1983). Logrolling is a bargaining process that allows the parties to trade off their low-priority concerns to achieve high-priority concerns (ibid.). But in its narrow sense, logrolling is different from the bargaining models described in the existing literature (see Rubinstein, 1982; Austen-Smith and Banks 1988; Baron and Ferejohn 1989; Osborne and Rubinstein, 1990; Alesina and Rosenthal 1996). First, in bargaining models, no matter the bargaining is bilateral or multilateral, players cannot trade favors with each other, which is considered to be the essence of logrolling. Second, in legislative bargaining literature, the agenda setter gets more benefits than the others in the sense that there exists agenda-setting power (Baron and Ferejohn 1989). But in logrolling, the agenda-setting power disappears. In order to see the differences more clearly, we will distinguish the logrolling game from the bargaining game, and we will compare the equilibrium outcomes under both games.

There is not a rich empirical literature on logrolling. Perhaps because to testify the existence of logrolling depends on the record of trading of favors, which may not exist when the trade is implicit (Evans, 1994), or may difficult to acquire as such trade
is usually considered morally reprehensible behavior (Buchanan and Tullock 1962). Stratmann (1992, 1995) made some progress in empirically test the phenomenon of logrolling in the US congressional voting in the 1960s and 1980s. Later, Irwin and Kroszner (1996) provide evidence on how interest groups traded favors with each other in the passage of Smoot-Hawley Tariff Act of 1930 by calculating the votes. The most recent literature tries to apply the theory of logrolling to explain the equilibrium EU policies (Crombez 2000) and political economy of IMF lending (Copelovitch, 2010). However, in the context of Chinese politics, it is almost impossible to find such voting records. The existing empirical approach seems to be not feasible in this paper. In this paper, we take an indirect approach by looking at the overreaching policy outcomes first, and then, use them to identify who are involved in logrolling and how they log roll with each other.

2.2. Literature on policy making in China

Making policy is the core function of all nation-states. Understanding the policy making process helps to open up the black box of China’s domestic politics. During Mao Zedong’s era, China has been treated as a near-totalitarian system (Richard Walker, 1955). There have been dramatic changes in China’s polity in the post-Mao reform era. The Chinese leaders, who have become progressively less dominant, have transformed toward more constrained figures who are primus inter pares within a collective group (Lampton, 2014, p.59).

Beginning from the end of the 1980s, a group of China Study scholars developed the "Fragmented Authoritarianism" framework. In their view, the authority below the very peak of the Chinese political system is vertically fragmented (stove-piped), reaching down from Beijing to various levels near the bottom. These separate functional vertical organizations, such as various ministries, have equal rank according to China’s bureaucratic ranking. Therefore, they cannot command each other. As no single organization is superior over another and voting has been avoided, the system falls back on bargain where decisions are made by “rule of consensus” or mutual accommodation (Lieberthal and Lampton 1992; Shirk 1993, p.116; Lampton 2014, p.86). The positive side of the fragmentation of authority is it prevents over-concentration of power; but the negative side is it makes achieving consensus very difficult (Shirk 1993, p.127). Each vertical organization is supposed to represent its constituents and pushes for policies in their own interests, but there are inadequate

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horizontal mechanisms of coordination, so they often find themselves at loggerheads or gridlock. If some organizations refuse to compromise and agreement cannot be reached, the issue is either dropped or is referred to a higher level for resolution (Shirk 1993, p.116). Usually, the excessive amounts of issues to be solved are over the higher level authorities' capacity limit. Therefore, in Chinese public organs, too many problems remain unsolved for a long time simply because of the objection from a minority (Chen 1987, Lampton 1992, p.73). However, the "Fragmented Authoritarianism" framework is mainly a descriptive narration of the domestic politics in China. It is still far from a theoretically and empirically grounded analytical framework, based on which we can make predictions. This paper tries to fill this gap by developing a theoretically and empirically sound framework to analyze policy making in China.

There do exist plenty of theoretical studies of China’s governance, but most of these studies focus on the vertical relation between the upper level governments and the lower level governments. Based on the theories of multidivisional structure of large corporations, the governance structure of China is modeled as a multiregional governance form (M-form), in which every region is controlled by the central government politically (e.g. Qian and Xu 1993; Eric Maskin, Qian, and Xu 2000; and Qian, Roland, and Xu 2006, 2007). Furthermore, a performance evaluation system is applied in the Chinese bureaucracy system. Appointment, promotion and demotion of lower level bureaucrats are decided by whether they have fulfilled the upper level government’s requirements for various policy targets (Li and Zhou 2005; Landry 2008). Different from the above studies which focus on the vertical central-local relation in China, our research addresses the horizontal coordination (logrolling) among the units within Chinese government, which has been under-researched in the existing theoretical literature.

3. The Theoretical Model

Consider a society with distinct but homogeneous interest groups $I \in \{\alpha, \beta\}$, each interest group can be seen as a single player $\alpha$ and $\beta$. Group $\alpha$ has preferences,

$$U^\alpha(h, x) = h + C(x),$$

and group $\beta$ has preferences,

$$U^\beta(h, y) = h + C(y),$$
where $h$ is general public good. The increasing and concave function $C(.)$, with $C(0) = 0$, is defined over the spending on group specific good $x \in [0,1]$ or $y \in [0,1]$. As a special example, suppose $C(\cdot) = \sqrt{\cdot}$.

$$x + y + h \leq 1.$$ 

3.1. Decision Made By Benevolent Social Planner

First, we derive the efficient benchmark. Suppose the allocation decision is made by a utilitarian social planner whose goal is to maximize social welfare $W$, then it maximizes the utilitarian social welfare function subject to the resource constraint $x + y + h \leq 1$:

$$\max_{(h,x,y)} U^\alpha(h, x) + U^\beta(h, y) = \max_{(h,x,y)} (2h + \sqrt{x} + \sqrt{y})$$

(3)

$$s.t. \ x + y + h \leq 1; x \geq 0; y \geq 0; h \geq 0.$$  

(4)

Since both utility functions are strictly increasing in $x$, $y$ and $h$, the public budget constraint is binding which implies

$$h = 1 - x - y.$$  

(5)

Hence, the social welfare problem is

$$\max_{(x,y)} (2 - 2x - 2y + \sqrt{x} + \sqrt{y})$$

(6)

$$s.t. \ x + y \leq 1, x \geq 0; y \geq 0.$$  

(7)

The objective function is concave and the constraint is linear, therefore the Kuhn-Tucker conditions are both necessary and sufficient.\(^3\)

The solution of the problem is,

$$(h^*, x^*, y^*) = \left( \frac{7}{8}, \frac{1}{16}, \frac{1}{16} \right).$$

This allocation is efficient as it maximizes the utilitarian social welfare.

\(^3\)The full set of the Kuhn-Tucker conditions is given in the appendix.
3.2. Decision Made By One Interest Group

Then suppose the policy decision is made by either one of the interest groups. If group \( \alpha \) has the decision making power, it will simply maximize its group welfare,

\[
\max_{(h,x,y)} U^\alpha = h + \sqrt{x}
\]  
\( (8) \)

\[\text{s.t. } h + x + y \leq 1; \ h \geq 0, \ y \geq 0, \ x \geq 0.\]  
\( (9) \)

As \( \alpha \)’s objective function does not depend on \( y \), then

\[y^\alpha = 0.\]

Moreover, since \( \alpha \)’s utility functions is strictly increasing in \( x \) and \( h \), the public budget constraint is binding which implies

\[h = 1 - x.\]  
\( (10) \)

Hence, the maximization problem is

\[
\max_x (1 - x + \sqrt{x}) \quad \text{s.t. } x \geq 0, \ x \leq 1.
\]  
\( (11) \)

The objective function is concave and the constraint is linear, therefore the Kuhn-Tucker conditions are both necessary and sufficient\(^4\):

The solution of the problem is, if \( \alpha \) has full power to choose its preferred allocation, then it would implement

\[(h^\alpha, x^\alpha, y^\alpha) = \left( \frac{3}{4}, \frac{1}{4}, 0 \right).\]

Symmetrically, if \( \beta \) has the decision making power, it will choose

\[(h^\beta, x^\beta, y^\beta) = \left( \frac{3}{4}, 0, \frac{1}{4} \right).\]

Compare the result with the policy made by the benevolent social planner, we can see

\[h^{\alpha/\beta} < h^*, x^\alpha > x^*, y^\alpha < y^*, x^\beta < x^*, y^\beta > y^*.\]

It means there is suboptimal amount of public good because of the excessive spending on its preferred issue by the group who has the decision making power.

\(^4\)The full set of the Kuhn-Tucker conditions is given in the appendix.
3.3. Decision Made By Bargaining

Next we focus on the bargaining process, adapting the simplest legislative bargaining process discussed in the seminal work by Baron and Ferejohn (1989), Persson (1998), Persson and Tabellini (2000). The bargaining follows the following sequence of events:

1. one of the interest groups, say $\alpha$ (we can also assume $\beta$, but the analysis is totally symmetric), is chosen to be the agenda setter;
2. the agenda setter makes a policy proposal $(h, x, y)$;
3. the other group $\beta$ chooses to accept the proposal or not. If it accepts the proposal made by the agenda setter then the proposal is implemented; if not, a default outcome $(h, x, y) = (1, 0, 0)$ gets implemented, which makes it to receive a reserve utility of $U^S = 1$.

From a formal point of view, this is equivalent to an ultimatum game, with usual standard properties. We can work out this simple game backward. The other group will reject the proposal made by the agenda setter if it not getting at least as high a payoff from the policy proposal as from the default policy (we assume throughout that the other group will accept a proposal when indifferent between it and the alternative).

Hence, the other group will accept the proposal if and only if

$U^\beta - U^S = 1 - x - y + \sqrt{y} - 1 \geq 0 \iff x \leq \sqrt{y} - y. \quad (12)$

Knowing this, the agenda setter $\alpha$ will maximize its own utility (1) subject to the “incentive constraint” (12). Thus, the agenda setter $\alpha$ chooses to make a policy proposal according to the following programming.

$max_{(x, y)} U^\alpha = 1 - x - y + \sqrt{x} \quad (13)$

$s.t. \quad x \leq \sqrt{y} - y; \quad x + y \leq 1; \quad x \geq 0; \quad y \geq 0. \quad (14)$

The objective function is concave and the constraints are convex, therefore the Kuhn-Tucker conditions are both necessary and sufficient$^5$.

The solution of the problem is, if $\alpha$ is first-mover, then she would implement

$$(h^{\alpha B}, x^{\alpha B}, y^{\alpha B}) = \left(\frac{1}{2} + \sqrt{\frac{1}{8}}, \frac{1}{8}, \frac{3}{8} - \sqrt{\frac{1}{8}}\right).$$

$^5$The full set of the Kuhn-Tucker conditions is given in the appendix.
Symmetrically, in a bargaining setting if $\beta$ is the first-mover, then he will choose

$$(h^{\beta B}, x^{\beta B}, y^{\beta B}) = \left(\frac{1}{2} + \sqrt{\frac{1}{8} \cdot \frac{3}{8} - \sqrt{\frac{1}{8} \cdot \frac{1}{8}}} \right).$$

Therefore, when decision is made by bargaining, the agenda setter has the first-mover advantage.

Compare the result with the policy made by the benevolent social planner and the interest group itself, we can see

$$h^{\alpha/\beta} < h^{\alpha/\beta B} < h^*, x^\alpha > x^{\alpha B} > x^*, y^\alpha < y^{\alpha B} < y^*,$$  

$$x^\beta < x^{\beta B} < x^*, y^\beta > y^{\beta B} > y^*.$$  

It implies a suboptimal allocation of $x, y$ and $h$, because of an excessive spending on its preferred issue by the group having the agenda setting power. However, the distortion and overreaching are reduced with respect to the case when the interest group has the monopoly power on policy making.

### 3.4. Decision Made By Logrolling

Next, we consider the simplest logrolling process:

1. $\alpha^6$ proposes a motion $x^{\alpha L} \in [0, 1]$;
2. $\beta$ chooses whether to support or not $\alpha$’s proposal, i.e. $c^\beta \in \{Y, N\}$;
3. $\beta$ proposes a motion $y^{\beta L} \in [0, 1 - x^{\alpha L}]$;
4. $\alpha$ chooses whether to support or not $\beta$’s proposal, i.e. $c^\alpha \in \{Y, N\}$;
5. If the two players supported each other’s proposal, i.e. $c^\beta = c^\alpha = Y$, we say a logroll is forged, and the proposals $(1 - x^{\alpha L} - y^{\alpha L}, x^{\alpha L}, y^{\alpha L})$ in the logroll are implemented;
6. otherwise, if any player rejected the other, we say the logrolling failed. Then a default outcome $(h, x, y) = (1, 0, 0)$ gets implemented, hence both players will get the reserve utility

$$U^S = 1. \quad (15)$$

The following picture represents the game tree:

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\*We can let $\beta$ moves first, but the analysis is totally symmetric.
Figure 1. The game of logrolling

By working out this game from backward, we can find the equilibrium policy outcome under logrolling which is,

\[ (h^{\alpha L}, x^{\alpha L}, y^{\alpha L}) = (h^{\beta L}, x^{\beta L}, y^{\beta L}) = \left( \frac{1}{2}, \frac{1}{4}, \frac{1}{4} \right). \]

The deduction of the equilibrium policy outcome can be found in the appendix.

3.4.1. Comments:

We can put policy outcomes based on difference decision-making rules in order to make comparison.

- The efficient allocation is \((h^*, x^*, y^*) = \left( \frac{7}{8}, \frac{1}{16}, \frac{1}{16} \right)\);

- When decision making is by only one group (when one group dominates the other), the allocation is \((h^\alpha, x^\alpha, y^\alpha) = \left( \frac{3}{4}, \frac{1}{4}, 0 \right)\) or \((h^\beta, x^\beta, y^\beta) = \left( \frac{3}{4}, 0, \frac{1}{4} \right)\);

- When decision making is by bargaining, \((h^{\alpha B}, x^{\alpha B}, y^{\alpha B}) = \left(\frac{1}{2} + \sqrt{\frac{1}{8}}, \frac{3}{8} - \sqrt{\frac{1}{8}}\right)\) or \((h^{\beta B}, x^{\beta B}, y^{\beta B}) = \left(\frac{1}{2} + \sqrt{\frac{1}{8}}, \frac{3}{8} - \sqrt{\frac{1}{8}}\right)\). There is first-mover advantage;
- When decision making is by logrolling, \((h^{\alpha L}, x^{\alpha L}, y^{\alpha L}) = (h^{\beta L}, x^{\beta L}, y^{\beta L}) = (\frac{1}{2}, \frac{1}{4}, \frac{1}{4})\). There is no first-mover advantage.

Compare the allocation of different issues under logrolling with the previous allocations, we can see
\[
\begin{align*}
h^{\alpha L} &= h^{\beta L} < h^{\alpha/\beta} < h^{\alpha/\beta B} < h^*, \\
x^\alpha &= x^{\alpha L} > x^{\alpha B} > x^*, \\
y^\alpha &= y^{\alpha B} < y^* < y^{\alpha L}, \\
x^{\beta} &= x^{\beta B} < x^* < x^{\alpha L}, \\
y^{\beta} &= y^{\alpha L} > y^{\beta B} > y^*.
\end{align*}
\]
There are several interest points worth to mention. First, the allocation of \(x, y\) and \(h\) is suboptimal, but the distortion caused by logrolling is the most serious, as public good provision is the smallest under logrolling. The direction of policy distortion is different from other cases. Under logrolling, there is excessive spending on both \(x\) and \(y\), the two issues through which the interest groups trading favor with each other. It means the problem of policy overreaching is more serious under logrolling. Second, the first-mover advantage we observed in the case of bargaining no longer exists in the case of logrolling. The first-mover advantage in the existing legislative bargaining literature is treated as the agenda setting power (Persson, 1998). But when decision making is by logrolling, there is no first-mover advantage. This finding helps to understand the fundamental difference between logrolling and bargain. The game under bargaining is a strictly competitive game, in which one player’s gain is at the cost of the other player’s welfare. Therefore, the player who is chosen to move first will exploit all the advantages. However, the game under logrolling is not a strictly competitive game, where the sum of the players’ utility changes with their strategy. Coordinating well, they can increase their joint benefit at the cost of society’s welfare, as there is under-provision of public goods. This is the element of cooperation in logrolling which does not exist in bargaining. Therefore, under logrolling, each player’s welfare is internalized, and hence, it doesn’t matter which player moves first.

3.5. Robustness check

In the above calculations, we assume when logrolling fails a default outcome \((h, x, y) = (1, 0, 0)\) will be implemented. In order to check whether the outcome of logrolling depends on the default policy, we will generalize the default outcome. We can assume when logrolling failed both players will get a reserve utility \(U^D = \theta \in [0, 1]\).
The “incentive logrolling (IL) constraint” for $\alpha$ then becomes

$$U^\alpha(Y|h^3 = (x, Y, y)) \geq U^D$$

which implies

$$1 - x - y + \sqrt{x} \geq \theta$$

i.e. the other group will reject the logrolling proposal if it is not getting at least as high a payoff from the default policy. In equilibrium the player’s payoff from logrolling is either greater or equal than the default policy, therefore the players always have incentive to forge a logrolling deal and the value of the default policy is irrelevant from the outcome path.

**Proposition 1** In the logrolling game, assuming that both agents’ utility when the logrolling fails is

$$U^D = \theta \in [0, 1],$$

the outcome path is independent from $\theta$ and is

$$(h^{\alpha L}, x^{\alpha L}, y^{\alpha L}) = (h^{\beta L}, x^{\beta L}, y^{\beta L}) = \left(\frac{1}{2}, \frac{1}{4}, \frac{1}{4}\right).$$

4. The Empirical Evidence

In this section, we illustrate the consequence of logrolling between two vertical organizations in China: the Ministry of Civil Affairs (MCA) and the Ministry of Health (MOH). The logrolling involves three policies: **Dibao**, **Rural health insurance**, and **Mental Health Care**. **Dibao** (or "Minimum Livelihood Guarantee Scheme") was initiated in the late 1990s in urban China, and is the core responsibility of MCA. MCA’s objective is to provide as much financial and other assistance programs for households tagged as "Dibao” households as possible. **Rural health insurance** (or “New Cooperative Medical Scheme”) was initiated in 2004, and is the core responsibility of MOH. MOH’s objective is to expand the coverage of rural health insurance, and the ideal position for MOH is to have universal coverage. Both ministries are responsible for providing mental healthcare. However, **Mental healthcare**, although

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7The detailed prove is given in the appendix.

8Both Dibao and Rural health insurance require that a household rather than an individual as a basic statistical unit
<table>
<thead>
<tr>
<th>MCA</th>
<th>High Priority</th>
<th>Low Priority</th>
<th>Public Good</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dibao</td>
<td>Providing as much financial and other assistances for households tagged as “Dibao” households as possible.</td>
<td>Rural Health Insurance*</td>
<td>Mental Health Care</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Regarding to poor rural households, MCA has incentive to only insure high risk people with rural health insurance but not the low risk people.</td>
<td>Treating patients in MCA managed hospitals</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>MOH</th>
<th>Rural health insurance</th>
<th>Dibao</th>
<th>Mental Health Care</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Expanding coverage of health insurance. Increasing the number of enrollees.</td>
<td>MOH sets the standard for receiving medical assistance. Whether a recipient under Dibao or not is not very important for MOH</td>
<td>Treating patients in MOH managed hospitals</td>
</tr>
</tbody>
</table>

*Note: Different from social health insurance in developed counties, rural health insurance in China is a voluntary health insurance.

Figure 2. Preference of the ministries

an important public health issue, is not the core responsibility for neither MCA nor MOH. Nonetheless, both ministries have duties to operate mental hospitals.

The two ministries have different priorities on different issues, which makes logrolling between them possible. As is shown in Figure 2, MCA’s top priority is Dibao and MOH’s top priority is rural health insurance. If both ministries stick to their own preferences, they can never reach a consensus. According to the "rule of delegation by consensus" described in the citation at the beginning of the paper, these issues will be either “tabled” or referred to a higher level for resolution. Then, a higher level authority will step in to make decision, and the default policy will be implemented.9

The logrolling takes place in the following form. These two ministries will exchange

9Note that the Ministry of Finance is not likely to be involved in managing these social programs, even though it makes decisions about the allocation of budget. This is because the function of each government department in the post-Mao era has been increasingly differentiated and professional knowledge is required for policy making (Huang 2013: 10).
favors on each other’s prior policy at the expense of the policy that is not essential for either of them. In reality, MOH supports the Dibao program by allowing Dibao recipients to be automatically eligible for free healthcare services in urban hospitals under the urban medical assistance programs\textsuperscript{10}, in order to support MCA’s core interest in managing Dibao. In exchange, MCA, pays for the insurance premium for poor households\textsuperscript{11} in rural areas from the medical assistance fund managed by MCA. In this case, MCA supports MOH’s core interest in expanding coverage of health insurance. On the other hand, mental health care, which is not the priority for either of the two ministries, receives the least attention and insufficient input. Figure 3 shows how MOH and MCA exchange interests via various policies.

Note that logrolling agreement may be problematic if there is no external institution to guarantee the enforcement of the agreement (Shirk 1993, p.127). With regard to this strategic question, it’s essential to know the institutional arrangements for ministries to exchange their support. Weingast and Marshal (1998) demonstrate that the committee system in congress serves as enforcement. We think, in China, this point is also relevant in enforcing the logrolling deals, and it takes the form of inter-ministerial joint conferences (\textit{Lianxi Huiyi}) among representatives from different ministries. Joint conferences have been held regularly for Dibao, rural health insurance, and mental health care among MOH, MCA and other different ministries since 2003\textsuperscript{12}. Moreover, some of the future logrolling deals are institutionalized in the present through policy documents that planned ahead many years. This is similar to transaction in an economic market, in which each ministry may use policy documents as a kind of contract to establish and protect their “political property rights” away from the discretion of bureaucrats (Moe 1990). For example, in the guideline for social assistance released by Chinese government in May 2014, the role and responsibility of different government departments are stated explicitly for the forthcoming expansion of social assistance programs. However, the process of logrolling rather than enforcement of logrolling agreement is the major issue addressed by the current paper.

\textsuperscript{10}The free treatments Dibao recipients receive include a basic package of services and drugs according to the decision of MOH.

\textsuperscript{11}Note that these poor households are defined as poor households who have difficulties to afford medical fees and these poor households are not necessary to be dibao recipients.

\textsuperscript{12}For rural health insurance, eleven ministries including MOH and MCA have been involved. Representatives from MCA and MOH are appointed as deputy coordinators of this joint conference. For medical assistance program, MOH, MCA and other ministries hold regular working meetings to coordinate their policies for the recipients of medical assistance program. For mental healthcare, the joint conferences have been held regularly since 2006, where both MCA and MOH sit in the conference.
According to our theory, logrolling among MOH and MCA would result in inefficient policy outcomes due to overreaching in urban medical assistance and rural health insurance programs, and undersupply of government input in mental health care. The following subsections will demonstrate these inefficiencies.

### 4.1. Inefficiency in the high benefit associated with "Dibao"

In this subsection, we argue that, as a result of logrolling among the ministries, the benefit associated with urban medical assistance even crowds out unemployment insurance. The mechanism is that, after MCA and MOH exchanging their support, a Dibao recipient is also entitled to receive benefit from the MOH-supported complementary social assistance programs (e.g. urban medical assistance program). Thus, while the benefit level of Dibao itself is modest enough not to crowd out unemployment insurance, the aggregate benefit from Dibao (the direct benefit from Dibao and urban medial assistance program) may be higher than the benefit from unemployment insurance for an urban household.\(^{13}\) Therefore, some workers in the urban areas may want to opt out of the unemployment insurance but to enroll in Dibao.

\(^{13}\)Also the enrollees have to pay a premium of the unemployment insurance (about 1% of their salary), but they do not need to pay for any premium for Dibao.
In 1999, “Dibao” was initiated nationwide to provide a safety net for the urban poor. By the end of 2002, over 19 million people were included in the “Dibao” program. A total of 9.8 million former SOE employees and 5.5 million of their family members accounted for about 80% of total beneficiaries of “Dibao”\(^{14}\). Dibao is managed by MCA. Dibao benefit, which is in the form of cash transfer, is allocated to urban households whose income is lower than a threshold value (i.e. means tested benefit). Unemployment insurance also protects urban residents financially. Since 1990’s, all workers in the urban areas have been required to be covered by unemployment insurance. However, for workers working in the informal sector (i.e. workers who are not registered by their employers or workers hired by unregistered employers, see Park and Cai 2011), they can choose to enroll with the unemployment insurance or not. The number of workers in urban China in the informal sector amounted to over 160 million in 2012 while the total number of urban labor force was about 320 million in 2012. In other words, about 50% of urban labor force may choose to not contribute to unemployment insurance. While Dibao benefit is still modest (RMB 4,000 in average annually per recipient in 2012), the benefit level of unemployment insurance is also modest and in particular not earning-related. The annual benefit level for the unemployment insurance was about RMB 8,800 in average annually per recipient in 2012.

The following descriptive figures provide the overall glimpse of the inefficiencies associated with "Dibao". One stylized fact is that there are much more urban labor forces enrolling with another social insurance program — the Basic Pension Scheme (BPS) — than with the unemployment insurance (UI). From Figure 4, we can see there are only 40% of urban labor forces registered with unemployment insurance while over 60% of urban labor forces registered with the Basic Pension Scheme. Both BPS and UI are compulsory for urban labor forces and in principal the enrollment rate should be similar. However, the enrollment of UI is much lower, so we can infer that many urban labor forces chose to quit UI. And most of those who quit UI, join Dibao because of its higher benefit. From figure 5, we can see most of the people who enrolled in Dibao are actually unemployed or flexibly employed.

**4.1.1. Data and Estimation Methods**

In order to test whether the benefit from urban medical assistance programs crowds out unemployment insurance. We estimate the following model:

\[
UI_{i,t} = \beta Dibao_{i,t-1} + \gamma Med_{Ass,i,t-1} + \delta X_{i,t} + \omega_t + e_{i,t}.
\]

Figure 4. Share of enrollees of unemployment insurance and basic pension scheme in urban labor forces.

Source: China Civil Affairs Statistical Yearbook, China Labor and Social Security Statistical Yearbook & China Statistical Yearbook, various years.
Figure 5. Composition of Beneficiaries under Dibao in urban areas
$UI_{i,t}$ is the dependent variable which is the ratio of the number of enrollees of unemployment insurance and number of employees in city $i$ during year $t$. $Dibao_{i,t-1}$ is the amount of cash benefit per recipient in the city $i$ during year $t - 1$, which measures the direct benefit of Dibao. Our aim of including this variable is to test whether the increasing direct benefit of Dibao crowds out unemployment insurance. $Med\_Ass_{i,t-1}$ is the amount of government health expenditure per person in the city $i$ during year $t - 1$, which is the proxy to measure the complementary benefit of Dibao from urban medical assistance programs. We use lagged variable here because if enrollees are crowded out by increasing benefit of urban medical assistance, it takes time that people’s enrollment status is reflected in the statistical number. Medical assistance expenditure is counted as a sub-category of government health expenditure since 2007. $\beta$, $\gamma$ and $\delta$ are parameters for the corresponding variable(s) in the model. $X_{i,t}$ corresponds to covariates including average income, fiscal expenditure per capita, unemployment rate and gross product per capita of the city. $\omega_t$ corresponds to year dummy variables, with $e_{i,t}$ defined as the error term. If there is a crowding out effect between urban medical assistance programs and unemployment insurance, we will expect the number of enrollees of unemployment insurance decreases with the benefit level of urban medical assistance.

Many research on crowding-out effect use individual level data. However, we do not have the luxury to access such data in China. Two data sources are used in this section. The first data source is China City Statistical Yearbook (NBS: various years). This dataset includes observations of 282 cities (prefecture level) in 26 out of 31 provinces (i.e. 4 provincial level city and Tibet are not included). The data for enrollees for unemployment insurance is only available for the year 2011, 2012 and 2013. The second dataset is data reported by MCA about Dibao data at the prefecture level, which was collected from the website of MCA. Both of our datasets include data for 282 prefecture cities out of 332 prefecture cities in total in China. City statistical yearbook only reports data in 282 cities. Excluding the observations with missing data, we end up with 268 city level observations in each year. A prefecture city usually has both urban (i.e. city district) and rural areas (i.e. county). Since we are interested in urban social programs, data in this study are city district level data in these prefecture cities. Given the data limitation that the city level unemployment insurance data is only available for recent two years, we just pool all the observations together. A detailed description of variables and descriptive statistics can be found in the appendix.

Omitted variable bias and simultaneity are concerns for the estimation. We use two ways to cross-check our results. First, we use government education expenditure level as a placebo to check whether the urban education assistance, which is managed
by ministry of education and also target for poor urban households, can have a similar effect as urban medical assistance. The ministry of education, unlike the ministry of health, has little overlapping business with ministry of civil affairs. Our model, in this case, will predict that the level of education assistance, which is measured by the proxy variable government education expenditure, will not crowd out the enrollees of unemployment insurance. Second, we use the enrollment rate of basic pension scheme for urban employees (BPS) as a reference point to measure the crowding out effect. BPS, same as unemployment insurance, is compulsory for employees in urban formal sectors. For urban employees working in the informal sector, similar to the case of unemployment insurance, they can opt out of the BPS. However, unlike unemployment insurance in which the benefit level is flat and not earning related, the benefit level of BPS is earning-related. In other words, the more people contribute to the BPS, the benefit level of BPS will be higher. For urban informal workers with a reasonable income, benefit of BPS may be much larger than the benefit of Dibao and urban medical assistance. In this case, an increasing benefit level of urban medial assistance is less likely to crowd out the enrollees of BPS. The enrollment rate of BPS can also serve as a reference point to check the scale of the crowding out effect between urban medical assistance and unemployment insurance.

Another caveat in interpreting the result is that we use the proxy variable of government health expenditure to measure the benefit level of urban medical assistance. The main reason to use the proxy is that there is no public data available for the city level urban medical assistance data. Besides, government expenditure for the urban medical assistance is an important component of government health expenditure in China, since the main goal of the government health expenditure is to provide basic healthcare for all households\textsuperscript{15}. Therefore, the benefit level of the urban medical assistance must be positively correlated with government health expenditure. However, when an imprecise measure of a variable is used in a regression model, the model inevitably contains measurement error. Under the classical errors-in-variables (CEV) assumption, the estimated effect will be attenuated. This kind of attenuation bias is not likely to cause serious problems to the result, because if a significant effect of government health expenditure is observed, it means the real effect of urban medical assistance has an even greater magnitude. However, if the CEV assumption is violated, for example, in the case where the measurement error is negatively correlated with government health expenditure and the weight is larger than for government health expenditure, it is possible to observe a negative effect of government health

\textsuperscript{15}For example, a recent report about the government plan for health reform between 2011 and 2015 explicitly states that the role of government is to provide basic health care service. http://finance.china.com.cn/industry/medicine/20120726/904196.shtml, accessed Jan 28, 2015.
expenditure even when the true effect of urban medial assistance is positive. Due to the data limitations, it is not possible to test the validity of the CEV assumption. This is a caveat of the current analysis.

4.1.2. Results

We find that higher direct benefit of Dibao does not associate with less enrollment of unemployment insurance, but higher benefit of urban medical assistance associates with less enrollment of unemployment insurance. It implies that many urban residents have evaded from unemployment insurance because the aggregate benefit from Dibao (but not the direct benefit from Dibao) is higher than the benefit from unemployment insurance. Our finding suggests that the crowding out effect is a result of logrolling, since the aggregate benefit from Dibao has been pulled up by the complementary social assistance programs from other ministries such as urban medical assistance (i.e. MOH-supported social assistance). The regression results are shown in the Table1:

Column (1) shows the results regressing the number of enrollees of unemployment insurance on the benefit level of Dibao. The coefficient for direct Dibao benefit is not statistically significant, which shows that the direct benefit level of Dibao is not correlated with the number of enrollees under the unemployment insurance. The coefficient for the level of GDP is significant and positive. The coefficient for the size of the informal sector is negative and significant. These results are consistent with our expectation that a more developed and formal economy will have more people enrolled with unemployment insurance.

In column (2), MOH supported social assistance (i.e. medical assistance) is included as an independent variable. The coefficient for Dibao benefit remains to be not statistically significant. It confirms the earlier result that Dibao is not crowding out enrollee of unemployment insurance. The level of urban medical assistance is negative and significant. One standard deviation above the mean of the government health expenditure per capita in the previous year, the number of enrollees under unemployment insurance will be decreased by 2.2 percent of labor force. This result suggests that the urban medical social assistance may actually crowding out the enrollees under unemployment insurance. Unemployment rate, gross product and fiscal expenditure are positively associated with the enrollment of unemployment insurance and the size of the informal sector is negatively associated with the enrollment rate of enrollment rate of unemployment insurance.

To check the robustness of our result, in column (3) and (4), we add government education expenditure as a regressor measuring the level of education assistance.
Table 1
Unemployment Insurance and Medical assistance Regression result

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>ui</td>
<td>ui</td>
<td>ui</td>
<td>ui</td>
<td>bps/ui</td>
</tr>
<tr>
<td>direct benefit of Dibao (lagged)</td>
<td>-0.00761 (0.00548)</td>
<td>-0.00686 (0.00556)</td>
<td>-0.00651 (0.00543)</td>
<td>-0.00785 (0.00551)</td>
<td>0.000140 (0.000244)</td>
</tr>
<tr>
<td>MOH-supported social assistance (lagged)</td>
<td>-13.48* (7.346)</td>
<td>-23.13** (9.497)</td>
<td>0.952* (0.497)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>education assistance (lagged)</td>
<td>7.579** (3.730)</td>
<td>1.728 (2.850)</td>
<td>-0.391* (0.200)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>fiscal expenditure</td>
<td>0.505** (0.257)</td>
<td>0.289 (0.259)</td>
<td>0.525** (0.255)</td>
<td>0.506** (0.257)</td>
<td>-0.0136 (0.0132)</td>
</tr>
<tr>
<td>average income</td>
<td>0.390*** (0.126)</td>
<td>0.501*** (0.134)</td>
<td>0.388*** (0.130)</td>
<td>0.372*** (0.130)</td>
<td>-0.00598 (0.00689)</td>
</tr>
<tr>
<td>informal sector size</td>
<td>-0.278 (5.896)</td>
<td>-10.01** (4.765)</td>
<td>-0.991 (5.866)</td>
<td>-0.0304 (5.911)</td>
<td>0.00494 (0.299)</td>
</tr>
<tr>
<td>gross product</td>
<td>0.0639* (0.0372)</td>
<td>0.0955** (0.0380)</td>
<td>0.0551 (0.0378)</td>
<td>0.0579 (0.0385)</td>
<td>-0.000373 (0.00179)</td>
</tr>
<tr>
<td>unemployment</td>
<td>42.08 (30.84)</td>
<td>56.39* (31.38)</td>
<td>41.74 (30.97)</td>
<td>41.95 (30.85)</td>
<td>2.368 (1.793)</td>
</tr>
<tr>
<td>constant</td>
<td>17.02*** (6.413)</td>
<td>22.79*** (6.645)</td>
<td>19.38*** (6.624)</td>
<td>16.11** (6.584)</td>
<td>2.484*** (0.335)</td>
</tr>
<tr>
<td>N</td>
<td>796</td>
<td>798</td>
<td>796</td>
<td>796</td>
<td>796</td>
</tr>
<tr>
<td>Year Dummy</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>overall. $R^2$</td>
<td>0.151</td>
<td>0.181</td>
<td>0.184</td>
<td>0.150</td>
<td>0.053</td>
</tr>
</tbody>
</table>

Standard errors in parentheses
* p < .1, ** p < 0.05, *** p < 0.01
The coefficient for direct Dibao benefit is still not significant, which suggests there is no crowding out effect from Dibao. Education expenditure, as a proxy measurement for the benefit of education assistance, is not significant in both column (3) and (4). However, government health expenditure remains to be significant and negatively associated with the number of enrollees under unemployment insurance.

To cross check the validity of our results, we change the dependent variable as the ratio of the enrollees of basic pension scheme for urban employees and unemployment insurance. The result is shown in column (5). As in previous models, The direct benefit of Dibao is not statistically significant. The result in column (5) also shows that the level of urban medical assistance is positively and significantly correlated with the ratio between the enrollment number of basic pension scheme and unemployment insurance, which implies that a higher benefit level of urban medical assistance is associated with a higher ratio of the number of enrollees under basic pension program and unemployment insurance. In other words, the gap of enrollee number between the basic pension scheme and unemployment insurance is increasing with the level of medical assistance. This result implies that many workers in the informal sector may opt out of the unemployment insurance.

4.2. Inefficiency in the enrollment of rural health insurance

In this subsection, we argue that, as a result of logrolling among the ministries, the overreaching in the policy area of the rural health insurance causes inefficient policy outcomes. Rural health insurance is recognized as the prior policy area for MOH. Because of logrolling MCA supports MOH by subsidizing enrollees of rural health insurance using its own medical assistance fund.

The inefficiency is shown by too high level of enrollment of rural health insurance and too low utilization of health care services. It is estimated that there are over 100 million people are being covered by more than one social health insurance programs in China. People can only claim benefit from one of these social health insurance programs if they register with more than one social health insurances. It is not efficient that enrollees pay premium and government pay subsidy for an insurance plan for which those enrollees may never claim benefit from. Figure 6 shows that the coverage of health insurance is universal. In 2013, the total number of enrollees under three social health insurance plans is over 1.37 billion which exceed the number of total population in China (1.36 billion, National Bureau of Statistics 2013). A large amount of people are covered by more than one social insurance plans. However, poor family may still have difficulties to pay for healthcare expenditure. The out-of-pocket expenditure is over RMB 1 trillion in 2013 (total health expenditure is about

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4.2.1. Data and Estimation Methods

Our hypothesis is that rural health insurance is managed in an inefficient way when the MCA subsidizes some rural residents to enroll with the rural health insurance. With an increasing number of enrollees under the rural health insurance, if utilization of healthcare services are not increasing, it indicates some inefficiency, since accessing to healthcare is not improved with better financial coverage. We also use the number of those enrollees under urban health insurance, whose premiums are financed by urban medical assistance programs, as a control group. Since health insurance in the urban areas is managed by the ministry of human resources and social security, if there is exchange of interests between MCA and MOH in the context of rural health insurance, the urban medical assistance will have a different impact over health service utilization compared to rural medical
assistance. In order to test the above hypothesis. We estimate the following model:

$$Utilization_{i,t} = \beta Asstnce_{i,t} + \delta X_{i,t} + \mu_i + \omega_t + \epsilon_{i,t}. $$

(20)

We use provincial level data to explore the relation between medical assistance fund and rural health insurance plan. The data is collected from China Health Statistical Yearbook as well as China Civil Affairs Statistical Yearbook, various years. The time span of our data is between 2009 and 2013. We study this time period for two reasons. First, the recent round of health reform stated in 2009. Second, the urban medical assistance data is only available since 2009. $Utilization_{i,t}$ denotes outcomes for health service utilization including inpatient and outpatient service volume as well as the revenue of service providers in province $i$ in year $t$. $Asstnce_{i,t}$ refers to the number of recipients of medical assistance which covers their insurance premium at rural and urban areas in province $i$ in year $t$. Control variables $X_{i,t}$ include the supply side determinants: the number of beds in the hospitals and the number of doctors. Demand side determinants are also included as regressors: coverage of major health insurances, average income level and the proportion of population whose is 65 years or elder in province $i$ in year $t$. $\omega_t$ and $\mu_i$ corresponds to year and provincial dummy variables. All variables are weighted by local population. Supply side determinants are important for health service utilization since the number of doctors and hospital beds are major inputs for the health care service production. Health insurance and average level of income determine the demand for health services. People with insurance coverage or with higher income is more likely to utilize health care service compared to people without health insurance or lower income. Also, the share of population with 65 years or elder is also important for healthcare service demand since people over 65 years old consume much more health services compared to other groups of population.

A detailed description of variables and descriptive statistics can be found in the appendix.

4.2.2. Results

The following table 2 show that the number of people subsidized for paying pre- sumption of rural health insurance is not positively associated with the revenue, volume of inpatient services in hospitals as well as outpatients services. Column (1) shows that the number of those enrollees of poor households under rural health insurance, whose insurance premium is covered by medical assistance fund, is not associated with the utilization of inpatient services. However, the number of enrollees financed by urban medical assistance is positively associated with the utilization of inpatient
services. Column (2) shows that the number of enrollees covered by medical assistance fund is not associated with the revenue of service provides. From column (3) and (4), it shows that the number of enrollees financed by rural medical assistance is also not significantly associated with the volume outpatient service provided by hospitals and primary care clinics. However, these regression results suggest that the effectiveness of rural medical assistance is in question. Rural health insurance plan is managed by the Minister of Health. The number of rural health insurance enrollees financed by rural medical assistance program is not associated with utilization of services as well as revenue generated by providing services. Also, from these regression results, urban medical assistance is likely to be more effective since urban medical assistance encourages poor households to access inpatient services. Note that the urban insurance plan is managed by the Ministry of Human resource and social security. Therefore, our hypothesis is supported and there is inefficiency in the policy area of rural health insurance.

4.3. Inefficiency in the supply of infrastructure in mental health care

While mental healthcare services are provided by hospitals under MOH and MCA, mental health care are not the priority issue for both MCA and MOH to invest on. The supply of infrastructure includes both physical inputs, such as the number of beds in the hospitals, and nonphysical inputs, such as the number of medical professionals working in the hospitals. From the theoretical model, we predict that there will be deficiency in government input and the supply of infrastructure in mental health care. The beds occupancy rates were 96.5% and 80% in MOH and MCA mental hospitals in 2013. Figure 7 also shows that the share of government input in mental hospitals in total government health expenditure is decreasing in both types of hospitals under ministry of Health (MOH) and Ministry of Civil Affairs (MCA). World Health Organization (WHO)’s threshold level is at least 2% of total health expenditure should be allocated for mental healthcare, whereas the share of mental hospital expenditure in China is less than this threshold.

In the following we show that underupply of infrastructure rather than the demand side reasons (e.g. income, education, insurance status) is the major constraint for mental health care, as we have predicted by the theoretical model.

4.3.1. Data and Estimation Methods

Our hypothesis is that the level of mental healthcare infrastructure is positively associated with the utilization rate of mental healthcare services. We use two groups of hospitals to test this hypothesis. One group is those mental hospitals managed by
Table 2  
Volume of inpatient services and outpatients service hospital revenue

<table>
<thead>
<tr>
<th></th>
<th>(1) inpatients revenue</th>
<th>(2) hospital outpatient revenue</th>
<th>(3) clinic outpatient revenue</th>
<th>(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>rural insurance (assisted)</td>
<td>0.0269 (0.293)</td>
<td>6.334 (25.60)</td>
<td>38.08 (32.64)</td>
<td></td>
</tr>
<tr>
<td>urban insurance (assisted)</td>
<td>1.124* (0.612)</td>
<td>-55.87 (53.74)</td>
<td>-42.77 (68.53)</td>
<td></td>
</tr>
<tr>
<td>doctors</td>
<td>0.138 (1.334)</td>
<td>127.0 (116.1)</td>
<td>83.19 (148.1)</td>
<td></td>
</tr>
<tr>
<td>beds</td>
<td>19.38*** (1.469)</td>
<td>-0.386 (4.477)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>age65</td>
<td>-28.87 (56.32)</td>
<td>3762.6 (4749.9)</td>
<td>-2406.2 (6057.1)</td>
<td></td>
</tr>
<tr>
<td>bhi</td>
<td>0.329 (0.262)</td>
<td>134.2*** (23.03)</td>
<td>-77.27*** (29.37)</td>
<td></td>
</tr>
<tr>
<td>cms</td>
<td>0.141* (0.0750)</td>
<td>1.942 (6.532)</td>
<td>-0.363 (8.330)</td>
<td></td>
</tr>
<tr>
<td>urbmi</td>
<td>0.0213 (0.0315)</td>
<td>-3.179 (2.751)</td>
<td>2.249 (3.509)</td>
<td></td>
</tr>
<tr>
<td>urban disposable income</td>
<td>1.622** (0.697)</td>
<td>-51.80 (60.26)</td>
<td>118.6 (76.85)</td>
<td></td>
</tr>
<tr>
<td>rural disposable income</td>
<td>-1.627 (1.207)</td>
<td>-111.4 (101.4)</td>
<td>126.6 (129.2)</td>
<td></td>
</tr>
<tr>
<td>constant</td>
<td>-31.87*** (11.34)</td>
<td>78.47 (956.3)</td>
<td>1242.6 (1219.5)</td>
<td></td>
</tr>
</tbody>
</table>

| Provincial Dummy | Yes | Yes | Yes | Yes |
| Year Dummy       | Yes | Yes | Yes | Yes |

| N                | 149 | 149 | 149 | 149 |
| adj. $R^2$       | 0.966 | 0.890 | 0.638 | 0.704 |

Standard errors in parentheses
* p<.1, ** p<0.05, *** p<0.01
Figure 7. Government inputs in mental hospitals as a share of total government health expenditure
MOH and the other group is mental hospitals managed by MCA. This is a provincial level data analysis. The time span for the data is between 2007 and 2013 (i.e. 7 years). We collect the data from China Health Statistical Yearbook and China Civil Affairs Statistical Yearbook, various years. If there is under-investment on the infrastructure, it will show that the marginal effect of infrastructure is positive and magnitude will be high. The limitation of data is that we do not have the number of psychiatrists at the provincial level. However, we can use the number of doctors in MOH hospitals and the number of medical staffs in MCA mental hospitals as a proxy measurement for the number of psychiatrists in MOH and MCA mental hospitals.

\[
\text{Mentalcare\_utilization}_{i,t} = \beta \text{Infrastructure}_{i,t} + \delta X_{i,t} + \mu_i + \omega_t + \epsilon_{i,t}
\]

\text{Mentalcare\_utilization}_{i,t} denotes outcomes for utilization of mental healthcare services (inpatient) in MOH and MCA hospitals in province \(i\) in year \(t\). \text{Infrastructure}_{i,t} denotes the number of beds as well as the number of medical staffs in the MOH and MCA hospitals in province \(i\) in year \(t\). \(\omega_t\) and \(\mu_i\) are year and provincial dummy variables. The control variables include education level, insurance coverage, local average income level and the share of industry in local economy. All of these variables are demand side determinants for mental healthcare services. All variables are weighted by population size. As in the previous section, a higher level of income and better insurance, which are included as regressors, may imply a higher demand of mental healthcare. The control variable "education\_ratio" denotes the share of tertiary education, which in the literature implies a higher demand of mental healthcare since the educated people is more informed. The share of industry in the local economy, denoted by "industry\_ratio", is also positively associated with the utilization of mental healthcare according to the literature (Chen, et al 2014).

A detailed description of variables and descriptive statistics can be found in the appendix.

4.3.2. Results

Table 3 shows the regression results.

Column (1) and (2) are the benchmark results about the level of infrastructure and utilization of health services. In column (1), the coefficients for the number of MOH hospital beds is positive and statistically significant at 1% level. According to column (1), one more bed in million people will increase utilization of mental healthcare inpatient services by about 4.7 per million people in MOH hospitals. Column (2) shows that the number of beds in MCA hospitals are not statistically significant. However, the number of medical doctors is another significant determinant.
Table 3
Regression results showing the determinants of utilization of mental healthcare

<table>
<thead>
<tr>
<th></th>
<th>(1) MOH inpatients</th>
<th>(2) MCA inpatients</th>
<th>(3) MOH inpatients</th>
<th>(4) MCA inpatients</th>
</tr>
</thead>
<tbody>
<tr>
<td>doctors</td>
<td>0.279** (0.116)</td>
<td>4.595** (2.144)</td>
<td>0.0872 (0.0886)</td>
<td>5.279*** (1.789)</td>
</tr>
<tr>
<td>beds</td>
<td>4.737*** (1.137)</td>
<td>1.657 (1.097)</td>
<td>5.097*** (0.847)</td>
<td>1.280 (1.043)</td>
</tr>
<tr>
<td>education ratio</td>
<td></td>
<td></td>
<td>-5.262 (3.287)</td>
<td>1.931 (1.433)</td>
</tr>
<tr>
<td>urban insurance</td>
<td>0.601 (1.215)</td>
<td>0.779 (0.537)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>industry ratio</td>
<td>3.227 (2.199)</td>
<td>-0.571 (1.819)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>disposable income</td>
<td>15.29 (9.531)</td>
<td>0.542 (4.653)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>constant</td>
<td>-414.6*** (84.51)</td>
<td>110.0** (44.29)</td>
<td>-476.6*** (168.6)</td>
<td>77.59 (109.0)</td>
</tr>
<tr>
<td>Year Dummy</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Provincial Dummy</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>N</td>
<td>212</td>
<td>200</td>
<td>212</td>
<td>200</td>
</tr>
<tr>
<td>adj. $R^2$</td>
<td>0.794</td>
<td>0.133</td>
<td>0.811</td>
<td>0.122</td>
</tr>
</tbody>
</table>

Standard errors in parentheses
* p<.1, ** p<0.05, *** p<0.01
In column (1), the coefficients for the number of medical staffs in both MCA and MOH is positive and statistically significant at 5% level. One more medical staff in a thousands people will increase utilization of mental healthcare inpatient services by about 279 and 4,595 per million people in MOH and MCA hospitals. Demand side determinants are added in column (3) and (4). It turns out that in column (3) and (4), none of these demand side determinants such as income, urbanization, the size of manufacture sector, education level is statistically significant. The numbers of beds medical staffs remain to be significant determinants for MOH and MCA mental hospitals. MOH hospitals are more likely to be constrained by the number of beds and MCA hospitals are more likely to be constraints by the number of staffs. It implies that in MOH and MCA mental hospitals, with under-supplied physical and nonphysical infrastructures, demand side reasons are not significant. Therefore, our hypothesis is supported. The regression results in this table, therefore, show that the supply side constraint is major reason for under treatment of patients with mental diseases in both MCA and MOH hospitals.

5. Conclusion

This paper develops a formal model to study the effect of logrolling on policy making. We compare the policy outcome under logrolling with policy outcome under other decision-making rules. We find that, policies under logrolling tend to be inefficiently high and policies excluded from logrolling tend to be inefficiently low. Besides, the first-mover advantage we observed in the case of bargaining no longer exists in the case of logrolling. We provide empirical evidence by studying the logrolling between Ministry of Civil Affairs and Ministry of Health. The preliminary results show that there is inefficiency due to policy overreaching in "Dibao" and rural health insurance, and there is insufficient input in infrastructure building in mental health care.

References


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6. Appendix

6.1. Kuhn-Tucker conditions when decision is made by a benevolent social planner

The Lagrangian is

\[ L(x, y) = 2 - 2x - 2y + \sqrt{x} + \sqrt{y} + \lambda(1 - x - y) \]  

and the Kuhn-Tucker conditions for this Lagrangian are:

\[
\frac{\partial L}{\partial x} = -2 + \frac{1}{2}x^{-\frac{1}{2}} - \lambda \leq 0, \quad x \geq 0, \quad \text{and} \quad x(-2 + \frac{1}{2}x^{-\frac{1}{2}} - \lambda) = 0
\]  

\[
\frac{\partial L}{\partial y} = -2 + \frac{1}{2}y^{-\frac{1}{2}} - \lambda \leq 0, \quad y \geq 0, \quad \text{and} \quad y(-2 + \frac{1}{2}y^{-\frac{1}{2}} - \lambda) = 0
\]  

\[
\lambda \geq 0, \quad x + y - 1 \leq 0, \quad \text{and} \quad \lambda(x + y - 1) = 0
\]

We can find solutions of these conditions as follows:

- \(-2 + \frac{1}{2}x^{-\frac{1}{2}} - \lambda \leq 0 \Rightarrow x > 0;
- \(-2 + \frac{1}{2}y^{-\frac{1}{2}} - \lambda \leq 0 \Rightarrow y > 0;
- \text{hence } -2 + \frac{1}{2}x^{-\frac{1}{2}} = \lambda = -2 + \frac{1}{2}y^{-\frac{1}{2}} \Rightarrow x = y;
- \text{if } \lambda > 0 \text{ then } x + y - 1 = 0 \text{ which implies } x = y = \frac{1}{2}. \text{ Then } \lambda = -2 + \frac{\sqrt{2}}{2} < 0, \text{ contradicting } \lambda > 0;
- \text{hence } \lambda = 0, \text{ which in turn implies } x = y = \frac{1}{16}.

6.2. Kuhn-Tucker conditions when decision is made by one interest group

The Kuhn-Tucker conditions are:

\[-1 + \frac{1}{2}x^{-\frac{1}{2}} - \lambda \leq 0, \quad x \geq 0, \quad \text{and} \quad x\left(-1 + \frac{1}{2}x^{-\frac{1}{2}} - \lambda\right) = 0
\]  

\[
\lambda \geq 0, \quad x - 1 \leq 0, \quad \text{and} \quad \lambda(x - 1) = 0
\]

We can find solutions of these conditions as follows:

- \(-1 + \frac{1}{2}x^{-\frac{1}{2}} - \lambda \leq 0 \Rightarrow x > 0;
- \text{if } \lambda > 0 \text{ then } x - 1 = 0 \text{ which implies } \lambda = -\frac{1}{2} < 0, \text{ contradicting } \lambda > 0;
- \text{hence } \lambda = 0, \text{ which in turn implies } x = \frac{1}{4}.
6.3. Deduction of equilibrium policy outcome under logrolling

We solve the game by backward induction.

6.3.1. STEP 4:

In the last decision step, $\alpha$ chooses between to accept the policy proposed by $\beta$ and the default policy. We should distinguish two different set of subgames, whether in step 2 $\beta$ rejected or accepted $\alpha$’s proposal. If $c^\beta = N$, then $\alpha$’s choice is irrelevant as both players will get the default outcome. If $c^\beta = Y$, then $c^\alpha = Y$ if and only if

$$U^\alpha (Y | h^3 = (x, Y, y)) \geq U^S \iff 1 - x - y + \sqrt{x} \geq 1 \iff y \leq \sqrt{x} - x$$

(27)

The condition defined by inequality (27) is $\alpha$’s “incentive logrolling (IL) constraint”. Hence, we get the following sequential best reply for $\alpha$:

$$SBR^\alpha (h^3) = \begin{cases} S, & \text{if } h^3 = (x, N, y) \\ N, & \text{if } h^3 = (x, Y, y) \text{ s.t. } y \geq \sqrt{x} - x \\ Y, & \text{if } h^3 = (x, Y, y) \text{ s.t. } y \leq \sqrt{x} - x \end{cases}$$

(28)

6.3.2. STEP 3:

In step 3, $\beta$ will choose $y^\beta_L$ to maximize his utility subject to $SBR^\alpha = Y$. Again, we should distinguish two different set of subgames, i.e. whether in step 2 $\beta$ rejected or accepted $\alpha$’s proposal. If $c^\beta = N$, then $\beta$’s choice is irrelevant as both players will get the default outcome. Hence

1. if $h^2 = (x, N)$, then $SBR^\beta (h^2) \in [0, 1]$;
2. if $h^2 = (x, Y)$, then $SBR^\beta (h^2) \in \text{argmax}_y U^\beta (y | h^2 = (x, Y), SBR^\alpha (h^3))$.

If $y^\beta_L \geq \sqrt{x} - x$, then $\alpha$ will not choose to logroll with $\beta$, and $\beta$ can only get the reserve utility, i.e. $U^\beta (y^\beta_L | h^2 = (x, Y), SBR^\alpha (h^3)) = U^S = 1$.

Otherwise, $\beta$ will solve following maximization problem:

$$\max_y (1 - x - y + \sqrt{y})$$

$$\text{s.t. } y \leq \sqrt{x} - x$$

(IL constraint)

$$y \leq 1 - x$$

(Resource constraint)

$$y \geq 0$$

(30)

Note that in the above problem IL constraint is more restrictive than the resource constraint, hence we can omit it. The objective function $1 - x - y + \sqrt{y}$ is greater than 1 if and only if $x \leq \frac{1}{4}$. Moreover, $\beta$’s objective function has a maximum when...
\( y = \frac{1}{4}, \) which is great or equal than \( \sqrt{x} - x \) for any \( x \). Therefore, the solution of the sequential best response of \( \beta \) is

\[
SBR^\beta(h^2) = \begin{cases} 
\sqrt{x} - x & \text{if } h^2 = (x, Y) \& x \in [0, \frac{1}{4}] \\
\sqrt{x} - x & \text{if } h^2 = (x, Y) \& x \in [\frac{1}{4}, 1] \\
0 & \text{if } h^2 = (x, N).
\end{cases}
\]

6.3.3. STEP 2

In step 2, \( \beta \) will choose whether to support \( \alpha \) or not. In particular, \( c^\beta = Y \) if and only if

\[
U^\beta(Y|x, SBR^\beta(h^2), SBR^\alpha(h^3)) \geq U^S
\]

which implies \( x \leq \sqrt{SBR^\beta(h^2)} - SBR^\beta(h^2) \).

Hence, we need to distinguish two cases, \( x \in [0, \frac{1}{4}] \) and \( x \in [\frac{1}{4}, 1] \).

**Case** \( x \in [0, \frac{1}{4}] \)

In this case \( \beta \) will choose \( c^\beta = Y \) if and only if

\[
x \leq \sqrt{SBR^\beta(h^2)} - SBR^\beta(h^2)
\]

which implies

\[
x \leq \sqrt{\sqrt{x} - x} - (\sqrt{x} - x).
\]

(33) is always satisfied for any \( x \in [0, \frac{1}{4}] \).

**Case** \( x \in [\frac{1}{4}, 1] \)

In this case \( U^\beta(Y|x, SBR^\beta(h^2), SBR^\alpha(h^3)) = U^S \), hence \( c^\beta \in \{Y, N\} \).

From both cases we get the following sequential best reply for \( \beta \):

\[
SBR^\beta(x) = \begin{cases} 
Y & \text{if } x \in [0, \frac{1}{4}] \\
\{Y, N\} & \text{if } x \in [\frac{1}{4}, 1].
\end{cases}
\]

6.3.4. STEP 1

Moving backward, in step 1, \( \alpha \) will choose \( x \in [0, 1] \) to maximize \( U^\alpha(x|SBR^\beta(x), SBR^\beta(h^2), SBR^\alpha(h^3)) \), which is

\[
U^\alpha(x|SBR^\beta(x), SBR^\beta(h^2), SBR^\alpha(h^3)) = \begin{cases} 
1 - x - \sqrt{x} + x + \sqrt{x} = 1 & \text{if } x \in [0, \frac{1}{4}] \\
1 & \text{if } x \in [\frac{1}{4}, 1].
\end{cases}
\]
Thus $U^\alpha(x|SBR^\alpha(x), SBR^\beta(h^2), SBR^\alpha(h^3))$ is constant and equal to 1, hence $SBR^\alpha \in [0, 1]$. To find a solution, we assume that there is a lexicographic preference for forging an agreement, hence $SBR^\alpha = \frac{1}{4}$.

$$(h^\alpha_L, x^\alpha_L, y^\alpha_L) = (h^\beta_L, x^\beta_L, y^\beta_L) = \left(\frac{1}{2}, \frac{1}{4}, \frac{1}{4}\right).$$

### 6.4. Proof of proposition 1

In this appendix, we will prove proposition 1. We can work out this game from backward.

#### 6.4.1. STEP 4:

In the last decision step, $\alpha$ chooses between to accept the policy proposed by $\beta$ and the default policy. We should distinguish two different set of subgames, whether in step 2 $\beta$ rejected or accepted $\alpha$’s proposal. If $c^\beta = N$, then $\alpha$’s choice is irrelevant as both players will get the default outcome. If $c^\beta = Y$, then $c^\alpha = Y$ if and only if

$$U^\alpha(Y|h^3 = (x, Y, y)) \geq U^D \iff 1 - x - y + \sqrt{x} \geq \theta \iff y \leq 1 + \sqrt{x} - x - \theta \quad (35)$$

The condition defined by inequality (35) is $\alpha$’s “incentive logrolling constraint”.

Hence, we get the following sequential best reply for $\alpha$:

$$SBR^\alpha(h^3) = \begin{cases} N, & \text{if } h^3 = (x, N, y) \\ \{Y\}, & \text{if } h^3 = (x, Y, y) \text{ s.t. } y \geq 1 + \sqrt{x} - x - \theta \end{cases}.$$  \hspace{1cm} (36)

#### 6.4.2. STEP 3:

In step 3, $\beta$ will choose $y^\alpha_l$ to maximize his utility subject to $SBR^\alpha$. Again, we should distinguish two different set of subgames, i.e. whether in step 2 $\beta$ rejected or accepted $\alpha$’s proposal. If $c^\beta = N$, then $\beta$’s choice is irrelevant as both players will anyway get the default outcome. Hence

1. if $h^2 = (x, N)$, then $SBR^\beta(h^2) \in [0, 1]$;
2. if $h^2 = (x, Y)$, then $SBR^\beta(h^2) \in \arg\max_y U^\beta(y|h^2 = (x, Y), SBR^\alpha(h^3))$.

Note that $y^\alpha_l > 1 + \sqrt{x} - x - \theta \implies U^\beta(y^\alpha_l|h^2 = (x, S), SBR^\alpha(h^3)) = \theta$. Otherwise, we have the following maximization problem:

$$\begin{align*} & \max_y (1 - x - y + \sqrt{y}) \\ & \text{s.t. } y \leq 1 + \sqrt{x} - x - \theta \quad \text{(IL constraint)} \end{align*}$$
\[ y \leq 1 - x \]  
\[ y \in [0, 1] \]  
(38)  

First note that the IL constraint is more or restrictive than the resource constraint according to the possible values of \( \theta \). In particular  
\[ 1 + \sqrt{x} - x - \theta \leq 1 - x \Leftrightarrow x \leq \theta^2. \]

Hence we distinguish two maximization problems:

1. when \( x \leq \theta^2 \),  
\[ \max_y (1 - x - y + \sqrt{y}) \]  
\[ s.t. \; y \leq 1 + \sqrt{x} - x - \theta \]  
\[ y \geq 0 \]  
(39)  
(40)  

2. when \( x \geq \theta^2 \),  
\[ \max_y (1 - x - y + \sqrt{y}) \]  
\[ s.t. \; y \leq 1 - x \]  
\[ y \geq 0 \]  
(41)  
(42)  

Moreover, \( \beta \) objective function has an unconstrained maximum in \( y = \frac{1}{4} \geq (1 + \sqrt{x} - x - \theta) \) or \( (1 - x) \) depending on \( x \) and \( \theta \). In particular  
\[ \frac{1}{4} \leq 1 + \sqrt{x} - x - \theta \Leftrightarrow \sqrt{x} - x \geq \theta - \frac{3}{4} \Leftrightarrow \begin{cases} x \in [0, 1] & \text{if } \theta \leq \frac{3}{4} \\ x \in \left[ \frac{5}{4} - \theta - \sqrt{1 - \theta}, \frac{5}{4} - \theta + \sqrt{1 - \theta} \right] & \text{if } \theta \geq \frac{3}{4} \end{cases} \]

and  
\[ \frac{1}{4} \leq 1 - x \Leftrightarrow x \in \left[ 0, \frac{3}{4} \right]. \]

Hence

1. when \( x \leq \theta^2 \; \& \; \theta \in [0, \frac{3}{4}] \),  
\[ SBR^\beta(h^2) = \begin{cases} \frac{1}{4} & \text{if } h^2 = (x, S) \; \& \; x \in [0, 1] \\ \in [0, 1] & \text{if } h^2 = (x, N) \; \& \; x \in [0, 1] \end{cases} \]
2. when \( x \leq \theta^2 \) & \( \theta \in \left[ \frac{3}{4}, 1 \right] \),

\[
SBR^\beta(h^2) = \begin{cases} 
\frac{1}{4} & \text{if } h^2 = (x, S) \& x \in \left[ \frac{5}{4} - \theta - \sqrt{1 - \theta}, \frac{5}{4} - \theta + \sqrt{1 - \theta} \right] \\
1 + \sqrt{x - x - \theta} & \text{if } h^2 = (x, S) \& x \in \left[ \frac{5}{4} - \theta - \sqrt{1 - \theta} \right] \cup \left[ \frac{5}{4} - \theta + \sqrt{1 - \theta}, 1 \right] \\
\in [0, 1] & \text{if } h^2 = (x, N) \& x \in [0, 1];
\end{cases}
\]

3. when \( x \geq \theta^2 \),

\[
SBR^\beta(h^2) = \begin{cases} 
\frac{1}{4} & \text{if } h^2 = (x, S) \& x \in \left[ 0, \frac{3}{4} \right] \\
1 - x & \text{if } h^2 = (x, S) \& x \in \left[ \frac{3}{4}, 1 \right] \\
\in [0, 1] & \text{if } h^2 = (x, N).
\end{cases}
\]

In case 2 when \( x \leq \theta^2 \) & \( \theta \in \left[ \frac{3}{4}, 1 \right] \), consider

\[
x \in \left[ 0, \theta^2 \right] \cap \left[ \frac{5}{4} - \theta - \sqrt{1 - \theta}, \frac{5}{4} - \theta + \sqrt{1 - \theta} \right] \iff
\]

\[
\iff x \in \begin{cases} 
\left[ \frac{5}{4} - \theta - \sqrt{1 - \theta}, \theta^2 \right] & \theta \in \left[ \frac{3}{4}, \frac{\sqrt{3}}{2} \right] \\
\left[ \frac{5}{4} - \theta - \sqrt{1 - \theta}, \frac{5}{4} - \theta + \sqrt{1 - \theta} \right] & \theta \in \left[ \frac{\sqrt{3}}{2}, 1 \right]
\end{cases}
\]

since

\[
\frac{5}{4} - \theta - \sqrt{1 - \theta} < 0 \iff \theta \in \emptyset
\]

\[
\frac{5}{4} - \theta + \sqrt{1 - \theta} \geq \theta^2 \iff \theta \in \left[ 0, \frac{\sqrt{3}}{2} \right]
\]

Hence we can summarize the sequential best response of \( \beta \) when \( y^\alpha L \leq 1 + \sqrt{x - x - \theta} \) as follows:

1. \( \theta \in \left[ 0, \frac{3}{4} \right] \)

\[
SBR^\beta(h^2) = \begin{cases} 
\frac{1}{4} & \text{if } h^2 = (x, S) \& x \in \left[ 0, \frac{3}{4} \right] \\
1 - x & \text{if } h^2 = (x, S) \& x \in \left[ \frac{3}{4}, 1 \right] \\
\in [0, 1] & \text{if } h^2 = (x, N).
\end{cases}
\]
2. $\theta \in \left[\frac{3}{4}, \frac{\sqrt{3}}{2}\right]$

$$SBR^3(h^2) = \begin{cases} 
1 + \sqrt{x} - x - \theta & \text{if } h^2 = (x, S) \& x \in \left[0, \frac{5}{4} - \theta - \sqrt{1 - \theta}\right] \\
\frac{1}{2} & \text{if } h^2 = (x, S) \& x \in \left[\frac{5}{4} - \theta - \sqrt{1 - \theta}, \frac{5}{4}\right] \\
1 - x & \text{if } h^2 = (x, S) \& x \in \left[\frac{5}{4}, 1\right] \\
\in [0, 1] & \text{if } h^2 = (x, N).
\end{cases}$$

3. $\theta \in \left[\frac{\sqrt{3}}{2}, 1\right]$

$$SBR^3(h^2) = \begin{cases} 
1 + \sqrt{x} - x - \theta & \text{if } h^2 = (x, S) \& x \in \left[0, \frac{5}{4} - \theta - \sqrt{1 - \theta}\right] \\
\frac{1}{2} & \text{if } h^2 = (x, S) \& x \in \left[\frac{5}{4} - \theta - \sqrt{1 - \theta}, \frac{5}{4}\right] \\
1 + \sqrt{x} - x - \theta & \text{if } h^2 = (x, S) \& x \in \left[\frac{5}{4} - \theta + \sqrt{1 - \theta}, 1\right] \\
\in [0, 1] & \text{if } h^2 = (x, N).
\end{cases}$$

Following these rules, the maximum utility would be:

1. $\theta \in \left[0, \frac{3}{4}\right]$

$$U^3(SBR^3(h^2)) = \begin{cases} 
\frac{5}{4} - x & \text{if } h^2 = (x, S) \& x \in \left[0, \frac{3}{4}\right] \\
\sqrt{1 - x} & \text{if } h^2 = (x, S) \& x \in \left[\frac{3}{4}, 1\right] \\
\theta & \text{if } h^2 = (x, N).
\end{cases}$$

2. $\theta \in \left[\frac{3}{4}, \frac{\sqrt{3}}{2}\right]$

$$U^3(SBR^3(h^2)) = \begin{cases} 
\theta - \sqrt{x} + \sqrt{1 + \sqrt{x} - x - \theta} & \text{if } h^2 = (x, S) \& x \in \left[0, \frac{5}{4} - \theta - \sqrt{1 - \theta}\right] \\
\frac{5}{4} - x & \text{if } h^2 = (x, S) \& x \in \left[\frac{5}{4} - \theta - \sqrt{1 - \theta}, \frac{5}{4}\right] \\
\sqrt{1 - x} & \text{if } h^2 = (x, S) \& x \in \left[\frac{5}{4}, 1\right] \\
\theta & \text{if } h^2 = (x, N).
\end{cases}$$

3. $\theta \in \left[\frac{\sqrt{3}}{2}, 1\right]$

$$U^3(SBR^3(h^2)) = \begin{cases} 
\theta - \sqrt{x} + \sqrt{1 + \sqrt{x} - x - \theta} & \text{if } h^2 = (x, S) \& x \in \left[0, \frac{5}{4} - \theta - \sqrt{1 - \theta}\right] \\
\frac{5}{4} - x & \text{if } h^2 = (x, S) \& x \in \left[\frac{5}{4} - \theta - \sqrt{1 - \theta}, \frac{5}{4}\right] \\
\theta - \sqrt{x} + \sqrt{1 + \sqrt{x} - x - \theta} & \text{if } h^2 = (x, S) \& x \in \left[\frac{5}{4} - \theta + \sqrt{1 - \theta}, 1\right] \\
\theta - \sqrt{x} + \sqrt{1 + \sqrt{x} - x - \theta} & \text{if } h^2 = (x, N).
\end{cases}$$
Otherwise, if \( y^{\alpha L} > 1 + \sqrt{x} - x - \theta \), then \( \beta \) can guarantee himself \( U^\beta (y^{\alpha L} | h^2 = (x, S), SBR^\alpha (h^3)) = \theta \), hence we should check that

\[ U^\beta (SBR^\beta (h^2)) \geq \theta \]

i.e.

1. when \( \theta \in \left[ 0, \frac{3}{4} \right] \) & \( x \in \left[ 0, \frac{3}{4} \right] \)
   \[
   \frac{5}{4} - x \geq \theta \iff x \leq \frac{5}{4} - \theta
   \]

2. when \( \theta \in \left[ 0, \frac{3}{4} \right] \) & \( x \in \left[ \frac{3}{4}, 1 \right] \)
   \[
   \sqrt{1-x} \geq \theta \iff 1-x \geq \theta^2 \iff x \leq 1-\theta^2
   \]

3. \( \theta \in \left[ \frac{3}{4}, \sqrt{\frac{5}{2}} \right] \) & \( x \in \left[ 0, \frac{5}{4}-\theta-\sqrt{1-\theta} \right] \)
   \[
   \theta - \sqrt{x} + \sqrt{1+\sqrt{x}-x-\theta} \geq \theta \iff \sqrt{1+\sqrt{x}-x-\theta} \geq \sqrt{x} \iff 1+\sqrt{x}-x-\theta \geq x \iff \\
   2x - \sqrt{x} \leq 1-\theta \iff x \in \left[ \frac{5}{8} - \frac{1}{2} \theta - \frac{1}{8} \sqrt{9-8\theta}, \frac{5}{8} - \frac{1}{2} \theta + \frac{1}{8} \sqrt{9-8\theta} \right]
   \]
   since \( 2x+\theta-1-\sqrt{x} = 0 \), Solution is:\( \left\{ -\frac{1}{2} \theta - \frac{1}{8} \sqrt{-8\theta + 9} + \frac{5}{8}, -\frac{1}{2} \theta + \frac{1}{8} \sqrt{-8\theta + 9} + \frac{5}{8} \right\} \)

4. \( \theta \in \left[ \frac{3}{4}, \sqrt{\frac{5}{2}} \right] \) & \( x \in \left[ \frac{5}{4}-\theta-\sqrt{1-\theta}, \frac{3}{4} \right] \)
   \[
   \frac{5}{4} - x \geq \theta \iff x \leq \frac{5}{4} - \theta
   \]

5. \( \theta \in \left[ \frac{3}{4}, \sqrt{\frac{5}{2}} \right] \) & \( x \in \left[ \frac{3}{4}, 1 \right] \)
   \[
   \sqrt{1-x} \geq \theta \iff 1-x \geq \theta^2 \iff x \leq 1-\theta^2
   \]

6. \( \theta \in \left[ \sqrt{\frac{5}{2}}, 1 \right] \) & \( x \in \left[ 0, \frac{5}{4}-\theta-\sqrt{1-\theta} \right] \)
   \[
   \theta - \sqrt{x} + \sqrt{1+\sqrt{x}-x-\theta} \geq \theta \iff \sqrt{1+\sqrt{x}-x-\theta} \geq \sqrt{x} \iff 1+\sqrt{x}-x-\theta \geq x \iff \\
   2x - \sqrt{x} \leq 1-\theta \iff x \in \left[ \frac{5}{8} - \frac{1}{2} \theta - \frac{1}{8} \sqrt{9-8\theta}, \frac{5}{8} - \frac{1}{2} \theta + \frac{1}{8} \sqrt{9-8\theta} \right]
   \]
   since \( 2x+\theta-1-\sqrt{x} = 0 \), Solution is:\( \left\{ -\frac{1}{2} \theta - \frac{1}{8} \sqrt{-8\theta + 9} + \frac{5}{8}, -\frac{1}{2} \theta + \frac{1}{8} \sqrt{-8\theta + 9} + \frac{5}{8} \right\} \)
7. \( \theta \in \left[ \frac{\sqrt{3}}{2}, 1 \right] \) & \( x \in \left[ \frac{5}{4} - \theta - \sqrt{1 - \theta}, \frac{5}{4} - \theta + \sqrt{1 - \theta} \right] \)

\[
\frac{5}{4} - x \geq \theta \iff x \leq \frac{5}{4} - \theta
\]

8. \( \theta \in \left[ \frac{\sqrt{3}}{2}, 1 \right] \) & \( x \in \left[ \frac{5}{4} - \theta + \sqrt{1 - \theta}, 1 \right] \)

\[
\theta - \sqrt{x + \sqrt{1 + \sqrt{x - x - \theta}}} \geq \theta \iff \sqrt{1 + \sqrt{x - x - \theta}} \geq \sqrt{x} \iff 1 + \sqrt{x - x - \theta} \geq x \iff
\]

\[
2x - \sqrt{x} \leq 1 - \theta \iff x \in \left[ \frac{5}{8} - \frac{1}{2} \theta - \frac{1}{8} \sqrt{9 - 8 \theta}, \frac{5}{8} - \frac{1}{2} \theta + \frac{1}{8} \sqrt{9 - 8 \theta} \right]
\]

since \( 2x + \theta - 1 - \sqrt{x} = 0 \), Solution is: \( \left\{ \frac{5}{8} - \frac{1}{2} \theta - \frac{1}{8} \sqrt{9 - 8 \theta} + \frac{5}{8}, \frac{5}{8} - \frac{1}{2} \theta + \frac{1}{8} \sqrt{9 - 8 \theta} + \frac{5}{8} \right\} \).

Hence the sequential best response of \( \beta \) is:

1. \( \theta \in \left[ 0, \frac{3}{4} \right] \)

\[
SBR^{\beta}(h^2) = \begin{cases} 
\frac{1}{4} & \text{if } h^2 = (x, S) \& x \in \left[ 0, \frac{3}{4} \right] \cap \left[ 0, \frac{5}{4} - \theta \right] \\
1 - x & \text{if } h^2 = (x, S) \& x \in \left[ \frac{3}{4}, 1 \right] \cap \left[ 0, 1 - \theta^2 \right] \\
\in [0, 1] & \text{if } h^2 = (x, N) \\
\in [1 + \sqrt{x} - x - \theta, 1] & \text{otherwise}
\end{cases}
\]

2. \( \theta \in \left[ \frac{3}{4}, \frac{\sqrt{3}}{2} \right] \)

\[
SBR^{\beta}(h^2) = \begin{cases} 
1 + \sqrt{x} - x - \theta & x \in \left[ 0, \frac{5}{4} - \theta - \sqrt{1 - \theta} \right] \cap \left[ \frac{5}{8} - \frac{1}{2} \theta - \frac{1}{8} \sqrt{9 - 8 \theta}, \frac{5}{8} - \frac{1}{2} \theta + \frac{1}{8} \sqrt{9 - 8 \theta} \right] \\
\frac{1}{4} & \text{if } h^2 = (x, S) \& x \in \left[ \frac{5}{4} - \theta - \sqrt{1 - \theta}, \frac{3}{4} \right] \cap \left[ 0, \frac{5}{4} - \theta \right] \\
1 - x & \text{if } h^2 = (x, S) \& x \in \left[ \frac{3}{4}, 1 \right] \cap \left[ 0, 1 - \theta^2 \right] \\
\in [0, 1] & \text{if } h^2 = (x, N) \\
\in [1 + \sqrt{x} - x - \theta, 1] & \text{otherwise}
\end{cases}
\]
3. $\theta \in \left[\frac{\sqrt{3}}{2}, 1\right]$

$$SBR^3(h^2) = \begin{cases} 
1 + \sqrt{x} - x - \theta & \text{if } h^2 = (x, S) \land x \in [0, \frac{5}{4} - \theta - \sqrt{1 - \theta}] \cap \left[\frac{5}{8} - \frac{1}{2}\theta - \frac{1}{8}\sqrt{9 - 8\theta} \cap \frac{5}{8} - \frac{1}{2}\theta + \frac{1}{8}\sqrt{9 - 8\theta}\right] \\
\frac{1}{4} & \text{if } h^2 = (x, S) \land x \in \left[\frac{5}{4} - \theta - \sqrt{1 - \theta}, \frac{5}{4} - \theta + \sqrt{1 - \theta}\right] \cap [0, \frac{5}{4} - \frac{1}{2}\theta - \frac{1}{8}\sqrt{9 - 8\theta}] \\
1 + \sqrt{x} - x - \theta & \text{if } h^2 = (x, S) \land x \in \left[\frac{5}{4} - \theta + \sqrt{1 - \theta}, 1\right] \cap \left[\frac{5}{8} - \frac{1}{2}\theta - \frac{1}{8}\sqrt{9 - 8\theta}, \frac{5}{8} - \frac{1}{2}\theta + \frac{1}{8}\sqrt{9 - 8\theta}\right] \\
\in [0, 1] & \text{if } h^2 = (x, N) \\
\in [1 + \sqrt{x} - x - \theta, 1] & \text{otherwise}
\end{cases}$$

This sequential best replies can be simplified since:

1. when $\theta \in [0, \frac{3}{4}]$

$$x \in \left[0, \frac{3}{4}\right] \cap \left[0, \frac{5}{4} - \theta\right] = \left\{ \begin{array}{ll} \left[0, \frac{3}{4}\right] & \theta \in \left[0, \frac{1}{2}\right] \\ \left[0, \frac{5}{4} - \theta\right] & \theta \in \left[\frac{1}{2}, \frac{3}{4}\right] \end{array} \right.$$

2. when $\theta \in \left[0, \frac{3}{4}\right]$

$$x \in \left[\frac{3}{4}, 1\right] \cap [0, 1 - \theta^2] = \left\{ \begin{array}{ll} \left[\frac{3}{4}, 1 - \theta^2\right] & \theta \in \left[0, \frac{1}{2}\right] \\ \emptyset & \theta \in \left[\frac{1}{2}, \frac{3}{4}\right] \end{array} \right.$$

3. when $\theta \in \left[\frac{3}{4}, \frac{\sqrt{3}}{2}\right]$

$$x \in \left[0, \frac{5}{4} - \theta - \sqrt{1 - \theta}\right] \cap \left[\frac{5}{8} - \frac{1}{2}\theta - \frac{1}{8}\sqrt{9 - 8\theta}, \frac{5}{8} - \frac{1}{2}\theta + \frac{1}{8}\sqrt{9 - 8\theta}\right] =$$

$$= \left\{ \begin{array}{ll} \emptyset & \theta \in \left[\frac{3}{4}, \frac{3\sqrt{5}}{2} - \frac{5}{2}\right] \\ \left[\frac{5}{8} - \frac{1}{2}\theta - \frac{1}{8}\sqrt{9 - 8\theta}, \frac{5}{4} - \theta - \sqrt{1 - \theta}\right] & \theta \in \left[\frac{3\sqrt{5}}{2} - \frac{5}{2}, \frac{\sqrt{3}}{2}\right] \end{array} \right.$$

since $5 - 4x + \sqrt{9 - 8x} - 8\sqrt{1 - x} = 0$, Solution is: $0, \frac{3}{2}\sqrt{5} - \frac{5}{2}$

4. when $\theta \in \left[\frac{3}{4}, \frac{\sqrt{3}}{2}\right]$

$$x \in \left[\frac{5}{4} - \theta - \sqrt{1 - \theta}, \frac{3}{4}\right] \cap \left[0, \frac{5}{4} - \theta\right] = \left[\frac{5}{4} - \theta - \sqrt{1 - \theta}, \frac{5}{4} - \theta\right]$$
5. when $\theta \in \left[\frac{3}{4} \frac{3}{2}\right]$

\[ x \in \left[\frac{3}{4}, 1\right] \cap \left[0, 1 - \theta^2\right] = \emptyset \]

6. when $\theta \in \left[\frac{\sqrt{3}}{2}, 1\right]$

\[ x \in \left[0, \frac{5}{4} - \theta - \sqrt{1 - \theta}\right] \cap \left[\frac{5}{8} - \frac{1}{2} \theta - \frac{1}{8} \sqrt{9 - 8\theta}, \frac{5}{8} - \frac{1}{2} \theta + \frac{1}{8} \sqrt{9 - 8\theta}\right] = \]

\[ = \left[\frac{5}{8} - \frac{1}{2} \theta - \frac{1}{8} \sqrt{9 - 8\theta}, \frac{5}{4} - \theta - \sqrt{1 - \theta}\right] \]

7. when $\theta \in \left[\frac{\sqrt{3}}{2}, 1\right]$

\[ x \in \left[\frac{5}{4} - \theta - \sqrt{1 - \theta}, \frac{5}{4} - \theta + \sqrt{1 - \theta}\right] \cap \left[0, \frac{5}{4} - \theta\right] = \]

\[ = \left[\frac{5}{4} - \theta - \sqrt{1 - \theta}, \frac{5}{4} - \theta\right] \]

8. when $\theta \in \left[\frac{\sqrt{3}}{2}, 1\right]$

\[ x \in \left[\frac{5}{4} - \theta + \sqrt{1 - \theta}, 1\right] \cap \left[\frac{5}{8} - \frac{1}{2} \theta - \frac{1}{8} \sqrt{9 - 8\theta}, \frac{5}{8} - \frac{1}{2} \theta + \frac{1}{8} \sqrt{9 - 8\theta}\right] = \left\{ \begin{array}{ll}
\emptyset & \theta \in \left[\frac{\sqrt{3}}{2}, 1\right) \\
\left\{\frac{1}{4}\right\} & \theta = 1.
\end{array} \right. \]

Hence the sequential best response of $\beta$ is:

1. $\theta \in \left[0, \frac{1}{2}\right]$

\[ SBR^\beta(h^2) = \left\{ \begin{array}{ll}
\frac{1}{4} & \text{if } h^2 = (x, S) \text{ & } x \in \left[0, \frac{3}{4}\right] \\
1 - x & \text{if } h^2 = (x, S) \text{ & } x \in \left[\frac{3}{4}, 1 - \theta^2\right] \\
& \text{if } h^2 = (x, N) & \text{otherwise}
\end{array} \right. \]

2. $\theta \in \left[\frac{1}{2}, \frac{3}{4}\right]$

\[ SBR^\beta(h^2) = \left\{ \begin{array}{ll}
\frac{1}{4} & \text{if } h^2 = (x, S) \text{ & } x \in \left[0, \frac{5}{4} - \theta\right] \\
& \text{if } h^2 = (x, N) & \text{otherwise}
\end{array} \right. \]
3. $\theta \in \left[\frac{3}{4}, \frac{3\sqrt{3}}{2} - \frac{5}{2}\right]$

$$SBR^\theta(h^2) = \begin{cases} 
\frac{1}{4} & \text{if } h^2 = (x, S) \land x \in \left[\frac{5}{4} - \theta - \sqrt{1 - \theta}, \frac{5}{4} - \theta\right]
\in [0, 1] \\
\in [1 + \sqrt{x} - x - \theta, 1] & \text{if } h^2 = (x, N)
\end{cases}$$

4. $\theta \in \left[\frac{3\sqrt{3}}{2} - \frac{5}{2}, \frac{\sqrt{3}}{2}\right]$

$$SBR^\theta(h^2) = \begin{cases} 
1 + \sqrt{x} - x - \theta & \text{if } h^2 = (x, S) \land x \in \left[\frac{5}{4} - \frac{1}{8} \sqrt{9 - 8\theta}, \frac{5}{4} - \theta - \sqrt{1 - \theta}\right]
\frac{1}{4} & \text{if } h^2 = (x, S) \land x \in \left[\frac{5}{4} - \theta - \sqrt{1 - \theta}, \frac{5}{4} - \theta\right]
\in [0, 1] \\
\in [1 + \sqrt{x} - x - \theta, 1] & \text{if } h^2 = (x, N)
\end{cases}$$

5. $\theta \in \left[\frac{\sqrt{3}}{2}, 1\right]$

$$SBR^\theta(h^2) = \begin{cases} 
1 + \sqrt{x} - x - \theta & \text{if } h^2 = (x, S) \land x \in \left[\frac{5}{4} - \frac{1}{8} \sqrt{9 - 8\theta}, \frac{5}{4} - \theta - \sqrt{1 - \theta}\right]
\frac{1}{4} & \text{if } h^2 = (x, S) \land x \in \left[\frac{5}{4} - \theta - \sqrt{1 - \theta}, \frac{5}{4} - \theta\right]
\in [0, 1] \\
\in [1 + \sqrt{x} - x - \theta, 1] & \text{if } h^2 = (x, N)
\end{cases}$$

6.4.3. **STEP 2**

In step 2, $\beta$ will choose whether to support $\alpha$ or not. In particular, $c^\beta = Y$ if and only if

$$U^\beta(Y|x, SBR^\beta(h^2), SBR^\alpha(h^3)) \geq U^S \iff 1 - x - SBR^\beta(h^2) + \sqrt{SBR^\beta(h^2)} \geq \theta \iff (43)$$

$$\iff x \leq 1 - \theta + \sqrt{SBR^\beta(h^2)} - SBR^\beta(h^2).$$

Hence, again we need to distinguish five cases for $\theta$:

1. Suppose $\theta \in \left[0, \frac{1}{2}\right]$, then $\beta$ will choose $c^\beta(x) = Y$ if and only if

$$x \leq 1 - \theta + \sqrt{SBR^\beta(h^2) - SBR^\beta(h^2)} \iff$$

$$\iff x \in \begin{cases} 
[0, \frac{5}{4} - \theta] & \text{if } x \in [0, \frac{3}{4}]
[0, x - \theta + \sqrt{1 - x}] & \text{if } x \in [\frac{3}{4}, 1 - \theta^2]
\end{cases}$$
i.e. if and only if
\[ x \in \left\{ \left[ \frac{3}{4}, 1 - \theta^2 \right] \cap \left[ 0, x - \theta + \sqrt{1-x} \right] = \left[ 0, \frac{3}{4} \right] \right\}. \]

Therefore when \( \theta \in \left[ 0, \frac{1}{2} \right] \)
\[ c^\beta(x) = \begin{cases} 
Y & \text{if } x \in \left[ 0, 1 - \theta^2 \right] \\
N & \text{otherwise.} 
\end{cases} \]

2. suppose \( \theta \in \left[ \frac{1}{2}, \frac{3}{4} \right] \), then \( \beta \) will choose \( c^\beta(x) = Y \) if and only if
\[ x \leq 1 - \theta + \sqrt{SBR^3(h^2) - SBR^3(h^2)} \]
\[ \iff x \in \left\{ \left[ 0, \frac{5}{4} - \theta \right] \text{ if } x \in \left[ 0, \frac{5}{4} - \theta \right] \right\} \]
i.e. if and only if
\[ x \in \left[ 0, \frac{5}{4} - \theta \right] \]

Therefore when \( \theta \in \left[ \frac{1}{2}, \frac{3}{4} \right] \)
\[ c^\beta(x) = \begin{cases} 
Y & \text{if } x \in \left[ 0, \frac{5}{4} - \theta \right] \\
N & \text{otherwise.} 
\end{cases} \]

3. suppose \( \theta \in \left[ \frac{3}{4}, \frac{3\sqrt{5}}{2} - \frac{5}{2} \right] \approx [0.75, 0.854] \), then \( \beta \) will choose \( c^\beta(x) = Y \) if and only if
\[ x \leq 1 - \theta + \sqrt{SBR^3(h^2) - SBR^3(h^2)} \]
\[ \iff x \in \left\{ \left[ 0, \frac{5}{4} - \theta \right] \text{ if } x \in \left[ \frac{5}{4} - \theta - \sqrt{1 - \theta}, \frac{5}{4} - \theta \right] \right\} \]
i.e. if and only if
\[ x \in \left[ \frac{5}{4} - \theta - \sqrt{1 - \theta}, \frac{5}{4} - \theta \right] \]

Therefore when \( \theta \in \left[ \frac{3}{4}, \frac{3\sqrt{5}}{2} - \frac{5}{2} \right] \approx [0.75, 0.854] \)
\[ c^\beta(x) = \begin{cases} 
Y & \text{if } x \in \left[ \frac{5}{4} - \theta - \sqrt{1 - \theta}, \frac{5}{4} - \theta \right] \\
N & \text{otherwise.} 
\end{cases} \]
4. suppose \( \theta \in \left[ \frac{3\sqrt{5}}{2} - \frac{5}{2}, \frac{\sqrt{3}}{2} \right] \simeq [0.854, 0.866] \), then \( \beta \) will choose \( c^\beta (x) = Y \) if and only if

\[
x \leq 1 - \theta + \sqrt{SBR^3(h^2)} - SBR^3(h^2) \Leftrightarrow
\]

\[
\Leftrightarrow x \in \begin{cases} 
0, \sqrt[3]{x-x} & \text{if } x \in \left[ \frac{5}{8} - \frac{1}{2}\theta - \frac{1}{8}\sqrt{9-8\theta}, \frac{5}{4} - \theta - \sqrt{1-\theta} \right] \\
\left[ \frac{5}{4} - \theta - \sqrt{1-\theta}, \frac{5}{4} - \theta \right] & \text{if } x \in \left[ \frac{5}{4} - \theta - \sqrt{1-\theta}, \frac{5}{4} - \theta \right]
\end{cases}
\]

i.e. if and only if

\[
x \in \left\{ \left[ \frac{5}{8} - \frac{1}{2}\theta - \frac{1}{8}\sqrt{9-8\theta}, \frac{5}{4} - \theta - \sqrt{1-\theta} \right] \right\} = \left[ \frac{5}{8} - \frac{1}{2}\theta - \frac{1}{8}\sqrt{9-8\theta}, \frac{5}{4} - \theta \right]
\]

Therefore when \( \theta \in \left[ \frac{3\sqrt{5}}{2} - \frac{5}{2}, \frac{\sqrt{3}}{2} \right] \simeq [0.854, 0.866]

\( c^\beta (x) = \begin{cases} Y & \text{if } x \in \left[ \frac{5}{8} - \frac{1}{2}\theta - \frac{1}{8}\sqrt{9-8\theta}, \frac{5}{4} - \theta \right] \\
N & \text{otherwise.}
\end{cases} \)

5. suppose \( \theta \in \left[ \frac{\sqrt{3}}{2}, 1 \right] \simeq [0.866, 1] \), then \( \beta \) will choose \( c^\beta (x) = Y \) if and only if

\[
x \leq 1 - \theta + \sqrt{SBR^3(h^2)} - SBR^3(h^2) \Leftrightarrow
\]

\[
\Leftrightarrow x \in \begin{cases} 
0, \sqrt[3]{x-x} & \text{if } x \in \left[ \frac{5}{8} - \frac{1}{2}\theta - \frac{1}{8}\sqrt{9-8\theta}, \frac{5}{4} - \theta - \sqrt{1-\theta} \right] \\
\left[ \frac{5}{4} - \theta - \sqrt{1-\theta}, \frac{5}{4} - \theta \right] & \text{if } x \in \left[ \frac{5}{4} - \theta - \sqrt{1-\theta}, \frac{5}{4} - \theta \right]
\end{cases}
\]

i.e. if and only if

\[
x \in \left\{ \left[ \frac{5}{8} - \frac{1}{2}\theta - \frac{1}{8}\sqrt{9-8\theta}, \frac{5}{4} - \theta - \sqrt{1-\theta} \right] \right\} = \left[ \frac{5}{8} - \frac{1}{2}\theta - \frac{1}{8}\sqrt{9-8\theta}, \frac{5}{4} - \theta \right]
\]

Therefore when \( \theta \in \left[ \frac{\sqrt{3}}{2}, 1 \right] \simeq [0.866, 1]

\( c^\beta (x) = \begin{cases} Y & \text{if } x \in \left[ \frac{5}{8} - \frac{1}{2}\theta - \frac{1}{8}\sqrt{9-8\theta}, \frac{5}{4} - \theta \right] \\
N & \text{otherwise.}
\end{cases} \)

6.4.4. STEP 1

Moving backward, in step 1, \( \alpha \) will choose \( x \in [0, 1] \) to maximize

\[
U^\alpha (x|SBR^3(x), SBR^3(h^2), SBR^3(h^3))
\]

that we will denote by \( U^\alpha (x|\theta) \) since it depends on the possible values of \( \theta \)
Case 1: $\theta \in \left[0, \frac{1}{2}\right]$

$$U^\alpha (x|\theta) = \begin{cases} 1 - \frac{1}{4} - x + \sqrt{x} = \frac{3}{4} - x + \sqrt{x} & \text{if } x \in \left[0, \frac{3}{4}\right] \\ -SBR^B(h^2) - x + \sqrt{x} = \sqrt{x} & \text{if } x \in \left[\frac{3}{4}, 1 - \theta^2\right] \\ -SBR^B(h^2) \theta & \text{otherwise.} \end{cases}$$

Hence

$$\max_{x \in [0,1]} U^\alpha (x|\theta) = \begin{cases} 1 - \frac{1}{4} \sqrt{1 - \theta^2} & \text{if } x = \frac{1}{4} \\ \frac{\sqrt{1 - \theta^2}}{\theta} & \text{if } x = 1 - \theta^2 \\ \theta & \text{otherwise.} \end{cases}$$

Thus when $\theta \in \left[0, \frac{1}{2}\right]$, $U^\alpha(x, SBR^\alpha(x), SBR^B(h^2), SBR^\alpha(h^3))$ is maximized for $x = \frac{1}{4}$ and the solution does not depend on the default utility $\theta$

$$\left(h^{\alpha_L}, x^{\alpha_L}, y^{\alpha_L}\right) = \left(h^{\beta_L}, x^{\beta_L}, y^{\beta_L}\right) = \left(\frac{1}{2}, \frac{1}{4}, \frac{1}{4}\right).$$

Case 2: $\theta \in \left[\frac{1}{2}, \frac{3}{4}\right]$

$$U^\alpha (x|\theta) = \begin{cases} 1 - \frac{1}{4} - x + \sqrt{x} = \frac{3}{4} - x + \sqrt{x} & \text{if } x \in \left[0, \frac{5}{4} - \theta\right] \\ -SBR^B(h^2) \theta & \text{otherwise.} \end{cases}$$

Hence

$$\max_{x \in [0,1]} U^\alpha (x|\theta) = \begin{cases} 1 - \frac{1}{4} \theta & \text{if } x = \frac{1}{4} \\ \theta & \text{otherwise.} \end{cases}$$

Thus when $\theta \in \left[\frac{1}{2}, \frac{3}{4}\right]$, $U^\alpha(x, SBR^\alpha(x), SBR^B(h^2), SBR^\alpha(h^3))$ is maximized for $x = \frac{1}{4}$ and the solution does not depend on the default utility $\theta$

$$\left(h^{\alpha_L}, x^{\alpha_L}, y^{\alpha_L}\right) = \left(h^{\beta_L}, x^{\beta_L}, y^{\beta_L}\right) = \left(\frac{1}{2}, \frac{1}{4}, \frac{1}{4}\right).$$
**Case 3:** $\theta \in \left[ \frac{3}{4}, \frac{3\sqrt{5}}{2} - \frac{5}{2} \right] \simeq [0.75, 0.854]

$$U^\alpha (x|\theta) = \begin{cases} 
1 - \frac{1}{4} & -x + \sqrt{x} = \frac{3}{4} - x + \sqrt{x} \text{ if } x \in \left[ \frac{5}{4} - \theta - \sqrt{1-\theta}, \frac{5}{4} - \theta \right] \\
\theta & \text{otherwise}
\end{cases}$$

Hence

$$\max_{x \in [0,1]} U^\alpha (x|\theta) = \begin{cases} 
1 & \text{if } x = \frac{1}{4} \\
\theta & \text{otherwise}
\end{cases}$$

Thus when $\theta \in \left[ \frac{3}{4}, \frac{3\sqrt{5}}{2} - \frac{5}{2} \right] \simeq [0.75, 0.854]$, $U^\alpha (x, SBR^\alpha (x), SBR^\beta (h^2), SBR^\alpha (h^3))$ is maximized for $x = \frac{1}{4}$ and the solution does not depend on the default utility $\theta$.

$$(h^{\alpha L}, x^{\alpha L}, y^{\alpha L}) = \left( h^{\beta L}, x^{\beta L}, y^{\beta L} \right) = \left( \frac{1}{2}, \frac{1}{4}, \frac{1}{4} \right).$$

**Case 4:** $\theta \in \left[ \frac{3\sqrt{5}}{2} - \frac{5}{2}, \frac{3\sqrt{3}}{2} \right] \simeq [0.854, 0.866]

$$U^\alpha (x|\theta) = \begin{cases} 
1 - \frac{1}{4} & -x + \sqrt{x} = \frac{3}{4} - x + \sqrt{x} \text{ if } x \in \left[ \frac{5}{4} - \theta - \sqrt{1-\theta}, \frac{5}{4} - \theta - \sqrt{1-\theta} \right] \\
\theta & \text{otherwise}
\end{cases}$$

Hence

$$\max_{x \in [0,1]} U^\alpha (x|\theta) = \begin{cases} 
\theta & \text{if } x \in \left[ \frac{5}{8} - \frac{1}{2} \theta - \frac{1}{8} \sqrt{9 - 8 \theta}, \frac{5}{4} - \theta - \sqrt{1-\theta} \right] \\
1 & \text{if } x = \frac{1}{4} \\
\theta & \text{otherwise}
\end{cases}$$

Thus when $\theta \in \left[ \frac{3\sqrt{5}}{2} - \frac{5}{2}, \frac{3\sqrt{3}}{2} \right] \simeq [0.854, 0.866]$, $U^\alpha (x, SBR^\alpha (x), SBR^\beta (h^2), SBR^\alpha (h^3))$ is maximized for $x = \frac{1}{4}$ and the solution does not depend on the default utility $\theta$. 
Case 5: \( \theta \in \left[ \frac{\sqrt{3}}{2}, 1 \right] \simeq [0.866, 1] \)

\[
U^\alpha (x|\theta) = \begin{cases} 
1 - \left(1 + \sqrt{x} - x - \theta \right) - x + \sqrt{x} = \theta & \text{if } x \in \left[ \frac{5}{8} - \frac{1}{2} \theta - \frac{1}{8} \sqrt{9 - 8\theta}, \frac{5}{4} - \theta - \sqrt{1 - \theta} \right] \\
1 - \frac{1}{4} - x + \sqrt{x} = \frac{3}{4} - x + \sqrt{x} & \text{if } x \in \left[ \frac{5}{4} - \theta - \sqrt{1 - \theta}, \frac{5}{4} - \theta \right] \\
\theta & \text{otherwise}
\end{cases}
\]

Hence

\[
\max_{x\in[0,1]} U^\alpha (x|\theta) = \begin{cases} 
\theta & \text{if } x \in \left[ \frac{5}{8} - \frac{1}{2} \theta - \frac{1}{8} \sqrt{9 - 8\theta}, \frac{5}{4} - \theta - \sqrt{1 - \theta} \right] \\
1 & \text{if } x = \frac{1}{4} \\
\theta & \text{otherwise}
\end{cases}
\]

Thus when \( \theta \in \left[ \frac{\sqrt{3}}{2}, 1 \right] \simeq [0.866, 1] \), \( U^\alpha(x, SBR^\alpha(x), SBR^\alpha(h^2), SBR^\alpha(h^3)) \) is maximized for \( x = \frac{1}{4} \) and the solution does not depend on the default utility \( \theta \).

\[
(h^\alphaL, x^\alphaL, y^\alphaL) = (h^\betaL, x^\betaL, y^\betaL) = \left( \frac{1}{2}, \frac{1}{4}, \frac{1}{4} \right).
\]

6.5. Variables and descriptive statistics for the test of inefficiency associated with Dibao

6.5.1. Variables

The dependent variable is the ratio of the number of enrollees of unemployment insurance and total number of employees in the city. Figure 2 shows the number of unemployed and flexible employed who are under Dibao program, who account for about 60% of beneficiaries who are under Dibao program.

Independent and control variables are listed as following:

- Direct Benefit of Dibao denotes the city level benefit of the Dibao program (RMB). (Figure ??).
- Health expenditure denotes the city level government health spending per resident (in 1,000 RMB), which indicates the level of medical assistance.
- Fiscal expenditure of a prefecture city per resident (in 1,000 RMB). Fiscal expenditure measures the scale of local fiscal policies.
- Gross product denotes the gross product in a prefecture city per resident (in 1,000 RMB), which measures the development stage of the local economy.
Table 4
Descriptive statistics of Dibao and unemployment insurance

<table>
<thead>
<tr>
<th></th>
<th>count</th>
<th>mean</th>
<th>sd</th>
<th>min</th>
<th>max</th>
</tr>
</thead>
<tbody>
<tr>
<td>Direct Benefit of Dibao</td>
<td>796</td>
<td>47.101</td>
<td>26.140</td>
<td>4.317</td>
<td>291.291</td>
</tr>
<tr>
<td>MOH supported social assistance</td>
<td>796</td>
<td>205.127</td>
<td>228.980</td>
<td>7.234</td>
<td>2924.629</td>
</tr>
<tr>
<td>education assistance</td>
<td>796</td>
<td>0.404</td>
<td>0.169</td>
<td>0.065</td>
<td>2.344</td>
</tr>
<tr>
<td>fiscal expenditure</td>
<td>796</td>
<td>8.931</td>
<td>5.806</td>
<td>1.185</td>
<td>74.648</td>
</tr>
<tr>
<td>education exp</td>
<td>796</td>
<td>0.972</td>
<td>0.458</td>
<td>0.060</td>
<td>5.727</td>
</tr>
<tr>
<td>gross product</td>
<td>796</td>
<td>64.023</td>
<td>50.205</td>
<td>8.4056</td>
<td>466.996</td>
</tr>
<tr>
<td>average income</td>
<td>796</td>
<td>42.812</td>
<td>9.346</td>
<td>19.267</td>
<td>92.357</td>
</tr>
<tr>
<td>informal sector size</td>
<td>796</td>
<td>.436</td>
<td>.159</td>
<td>.018</td>
<td>1.949</td>
</tr>
<tr>
<td>unemployment</td>
<td>796</td>
<td>0.030</td>
<td>0.024</td>
<td>0.002</td>
<td>0.411</td>
</tr>
<tr>
<td>bps</td>
<td>796</td>
<td>82.058</td>
<td>46.209</td>
<td>5.169</td>
<td>497.273</td>
</tr>
</tbody>
</table>

Average income: this variable refers to the average annual wage level in the city district of a city (in 1,000 RMB). The average wage is calculated on the basis of wage level in state owned, privately owned as well as foreign owned enterprises. This variable measures local conditions of economic development.

Informal sector size: This variable refers to the number of employees working in private owned enterprises and self employed as a share of total labor force the city district of a city. This variable is relevant since we expect a larger size of the informal sector may imply that there are more people who are not willing to register as urban unemployed (Park and Cai 2011).

Year dummies are also included.

6.5.2. Descriptive statistics

We have 796 observations for 268 prefecture level cities in three years. The variable "employees" denotes the total number of employees in a city. "bps" denotes the number of enrollees for the urban basic pension scheme (in 1,000). "education_exp" refers to government education expenditure per person. In some cities, the ratio between enrollees under unemployment insurance and employees is over 1, similarly, in some cities, the ratio between workers in the size of informal sector and the number of employees is larger than 1. The reason is that in some cities the size of the informal sector is very big and the number of workers may be much more than the number of workers hired by the registered companies (i.e. the variable "employees" and in some
Table 5
Descriptive statistics

<table>
<thead>
<tr>
<th></th>
<th>count</th>
<th>mean</th>
<th>sd</th>
<th>min</th>
<th>max</th>
</tr>
</thead>
<tbody>
<tr>
<td>cms</td>
<td>149</td>
<td>56.921</td>
<td>18.307</td>
<td>0.051</td>
<td>82.503</td>
</tr>
<tr>
<td>urban disposable income</td>
<td>149</td>
<td>19.359</td>
<td>5.630</td>
<td>11.929</td>
<td>40.188</td>
</tr>
<tr>
<td>inpatients volume</td>
<td>149</td>
<td>82.673</td>
<td>22.271</td>
<td>35.928</td>
<td>159.732</td>
</tr>
<tr>
<td>outpatients in hospitals</td>
<td>149</td>
<td>4430.867</td>
<td>1496.305</td>
<td>2511.529</td>
<td>9583.209</td>
</tr>
<tr>
<td>outpatients in clinics</td>
<td>149</td>
<td>2602.269</td>
<td>812.257</td>
<td>895.786</td>
<td>4687.936</td>
</tr>
<tr>
<td>doctors</td>
<td>149</td>
<td>1.757</td>
<td>.619</td>
<td>.868</td>
<td>5.850</td>
</tr>
<tr>
<td>rural insurance (assisted)</td>
<td>149</td>
<td>3.715</td>
<td>2.782</td>
<td>0.031</td>
<td>14.472</td>
</tr>
<tr>
<td>urban insurance (assisted)</td>
<td>149</td>
<td>1.329</td>
<td>1.211</td>
<td>0.010</td>
<td>5.808</td>
</tr>
<tr>
<td>age65</td>
<td>149</td>
<td>0.089</td>
<td>0.0174</td>
<td>0.0482</td>
<td>.132</td>
</tr>
<tr>
<td>beds</td>
<td>149</td>
<td>3.979</td>
<td>.736</td>
<td>2.568</td>
<td>6.06</td>
</tr>
<tr>
<td>bhi</td>
<td>149</td>
<td>19.062</td>
<td>12.001</td>
<td>7.359</td>
<td>64.057</td>
</tr>
<tr>
<td>urban disposable income</td>
<td>149</td>
<td>20.040</td>
<td>6.145</td>
<td>11.929</td>
<td>43.851</td>
</tr>
<tr>
<td>rural disposable income</td>
<td>149</td>
<td>7.259</td>
<td>3.216</td>
<td>2.980</td>
<td>19.595</td>
</tr>
<tr>
<td>urbmi</td>
<td>149</td>
<td>15.598</td>
<td>12.220</td>
<td>2.864</td>
<td>89.970</td>
</tr>
</tbody>
</table>

cases, the number of workers may be also higher than local residents (e.g. the variable "people"), which is defined as people who have registered with their residential status with the city and reside more than 6 months in that city. The variances across cities is big. In the richest city, the annual average income is RMB 70 thousands while in the annual average income is RMB 15 thousands the poorest city. The fiscal expenditure of richest city reached RMB 61,000 per person and fiscal expenditure in the poorest city had only RMB 1,184 per person.

6.6. Variables and descriptive statistics for the test of inefficiency associated with mental healthcare

6.6.1. Variables:
The dependent variable is defined as the volume of inpatient services in two types of mental hospitals: MOH and MCA mental hospitals in a province.

The independent variables include the number of beds in two types of hospitals in a province.

The control variables such as income, education, manufacturing sector as well as insurance measure the demand for mental health care. People residing in a more economic developed region, in a better educated region, in a region with a larger manufacturing sector and more generous insurance coverage are likely to demand
Table 6
Descriptions of variables

<table>
<thead>
<tr>
<th>Variable name</th>
<th>Variable description</th>
</tr>
</thead>
<tbody>
<tr>
<td>moh inpatients</td>
<td>number of inpatient visits in MOH hospital (per million people)</td>
</tr>
<tr>
<td>mca inpatient</td>
<td>number of inpatient visits in MCA hospital (per million people)</td>
</tr>
<tr>
<td>moh beds</td>
<td>Bed for patients with mental diseases in MOH hospitals (per million people)</td>
</tr>
<tr>
<td>mca beds</td>
<td>Bed for patients with mental diseases in MCA hospitals (per million people)</td>
</tr>
<tr>
<td>education ratio</td>
<td>Education(share of people with tertiary education or above)</td>
</tr>
<tr>
<td>industry ratio</td>
<td>Industry (The share of manufacturing sector in total employment)</td>
</tr>
<tr>
<td>disposable income</td>
<td>Average Disposable income in the province (1,000 RMB)</td>
</tr>
<tr>
<td>urban insurance</td>
<td>coverage for urban social health insurance(%)</td>
</tr>
<tr>
<td>moh doctors</td>
<td>no. of doctors in a million people</td>
</tr>
<tr>
<td>mca staffs</td>
<td>the number of staffs working in MCA mental hospital per million residents</td>
</tr>
</tbody>
</table>

People residing in a more economically developed and better educated region are more likely to be informed about the mental diseases. A province with a larger manufacturing sector is more likely to have more workers working in a mass production process and suffer mental diseases (thinking about migrant workers). People residing in a province with better insurance plan will be covered for the fees charged by the mental hospitals.

All variables are weighted by population and the description of variables is shown in the following table. Note that inpatient volume for MCA hospitals is calculated by average number of occupied beds per day *365. We use the average length of stay 45 days in all mental health providers (including MCA and MOH hospitals) in 2010 to calculate the inpatient volume in MCA hospitals (Ma, et al, 2012).
<table>
<thead>
<tr>
<th>Variable</th>
<th>Count</th>
<th>Mean</th>
<th>SD</th>
<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>mental inpatients</td>
<td>212</td>
<td>818.568</td>
<td>419.157</td>
<td>119.354</td>
<td>2029.798</td>
</tr>
<tr>
<td>mental beds</td>
<td>212</td>
<td>161.715</td>
<td>113.132</td>
<td>7.372</td>
<td>596.239</td>
</tr>
<tr>
<td>mca inpatients</td>
<td>200</td>
<td>276.238</td>
<td>190.537</td>
<td>0.0603</td>
<td>1213.583</td>
</tr>
<tr>
<td>mca mental beds</td>
<td>200</td>
<td>50.240</td>
<td>34.047</td>
<td>6.639</td>
<td>260.996</td>
</tr>
<tr>
<td>education ratio</td>
<td>212</td>
<td>8.036</td>
<td>6.259</td>
<td>0.024</td>
<td>37.350</td>
</tr>
<tr>
<td>urban insurance</td>
<td>212</td>
<td>63.220</td>
<td>23.582</td>
<td>20.915</td>
<td>191.835</td>
</tr>
<tr>
<td>industry ratio</td>
<td>212</td>
<td>20.835</td>
<td>12.706</td>
<td>0.0355</td>
<td>50.549</td>
</tr>
<tr>
<td>urban disposable income</td>
<td>212</td>
<td>18.909</td>
<td>6.468</td>
<td>10.012</td>
<td>43.851</td>
</tr>
<tr>
<td>mca staff</td>
<td>200</td>
<td>18.511</td>
<td>9.702</td>
<td>1.625</td>
<td>51.399</td>
</tr>
<tr>
<td>moh doctors</td>
<td>212</td>
<td>1662.005</td>
<td>522.959</td>
<td>791.999</td>
<td>3646.052</td>
</tr>
</tbody>
</table>

Table 7
Summary of variables