The Structure of Negotiations: Incomplete Agreements and the Focusing Effect.*

Andrea Canidio[†] *and Heiko Karle*[‡]

This Version: May 20, 2015.

Abstract

We study the use of incomplete agreements in a deterministic environment. We show that, if preferences are context dependent, the negotiating players may negotiate in stages: first signing an incomplete agreement and then finalizing the outcome of the negotiation. Furthermore, if preferences are context dependent because of the focusing effect, incomplete agreements are used to eliminate extreme, off-equilibrium outcomes from the possible bargaining solutions. Our framework also justifies the existence of a number of pre-bargaining actions. For example, a seller may enter the negotiation over the sale of a good having already announced a maximum price. Similarly, a seller may prefer to produce a good and later bargain over the price of the good (i.e. may prefer to be held up), rather than simultaneously bargain over price and quality.

JEL classification: C78, D03, D86, F51.

Keywords: Incomplete Agreements, Context-dependent Preferences, Salience, Focusing Effect, Bargaining, Negotiations.

*We are grateful to Micael Castanheira, Markus Fels, Nicola Gennaioli, Dominik Grafenhofer, Paul Heidhues, Martin Hellwig, Fabian Herweg, Martin Kocher, Georg Kirchsteiger, Botond Kőszegi, Patrick Legros, Guido Maretto, Marc Möller, Nick Netzer, Georg Nöldeke, Klaus Schmidt, Armin Schmutzler, Heiner Schumacher, Adam Szeidl as well as seminar audiences at Central European University, Corvinus University, ETH Zurich, University of Zurich, LMU Munich, ECARES (ULB), Stony Brook International Conference on Game Theory, University of Oslo, Nordic Conference on Behavioral and Experimental Economics at Aarhus, Copenhagen Workshop on Limited Attention, UECE Lisbon Meetings on Game Theory, University of Bern, MPI for Research on Public Goods at Bonn, and MACCI Workshop on Behavioral IO for their valuable comments and suggestions.

†Department of Economics, Central European University, Nádor u. 9, 1051 Budapest, Hungary; email: canidioa@ceu.hu.

‡Center for Law and Economics, ETH Zurich, Haldeneggsteig 4 8092 Zurich, Switzerland; email: hkarle@ethz.ch.

1 Introduction

In the context of negotiations, many agreements are incomplete agreements, in the sense that they specify only some aspects of the final outcome and rely on future bargaining rounds to define the missing provisions. A case in point is the use of *framework agreements* in procurement, which may, for example, define a set of prices and a set of quality levels at which a transaction may occur, with the understanding that the details of this transaction will be established in one or several future agreements. Similarly, complex negotiations often happen in stages, with framework agreements preceding a final, comprehensive agreement. For example, the current Doha round of trade negotiations is structured in several negotiation tables and follows a set of principles,[1] the first of which is:

> Single Undertaking: Virtually every item of the negotiation is part of a whole and indivisible package and cannot be agreed separately. "*Nothing is agreed until everything is agreed*".

Hence, any agreement reached at a specific table is neither final nor binding but provides the framework for a later round of negotiation.

Several practitioners believe that the specific sequence of incomplete agreements that can be signed (i.e. the *negotiation structure*) has an effect on the *outcome* of a negotiation.[2] In addition, negotiating parties often bargain extensively over the structure of the negotiation, which implies that the parties have different preferences over the specific sequence of incomplete agreements to discuss. A case in point is the current peace negotiation between the FARC and the Colombian government, which is structured into 6 framework agreements each corresponding to a specific issue, which are discussed sequentially and are followed by a final bargaining round.[3] A preliminary, secret negotiation round was required to reach an agreement on how to structure the peace negotiation.[4]

In this paper, we introduce *context-dependent* preferences into a simple bargaining problem. Our goal is to explain the use of incomplete agreements, which we model as restrictions on the bargaining set of future bargaining rounds. We contrast our analysis to existing theories of incomplete agreements (and, more specifically, of incomplete contracts) by considering the case of perfect information. We show that, in a deterministic environment where every aspect of the agreement is contractible and preferences are context dependent, the bargaining parties may negotiate in stages, by first signing an incomplete agreement and later finalizing the outcome of the negotiation. Fur-

[1] See http://www.wto.org/english/tratop_e/dda_e/work_organi_e.htm (accessed on the 22nd of October 2014).

[2] For example, in his textbook "the art and science of negotiations", pages 207 to 209, Raiffa (1982) recommends that the bargaining parties start the negotiation by agreeing on the structure of the negotiation (which he calls the platform of the negotiation). He argues that the bargaining parties may try to strategically manipulate the structure of the negotiation to improve their final bargaining outcome.

[3] See http://www.bbc.com/news/world-latin-america-19875363 (accessed on 22nd of October 2014).

[4] See http://www.elespectador.com/noticias/paz/negociando-farc-cuba-articulo-394489 (accessed on 22nd of October 2014 - in Spanish).

thermore, we show that when the source of context dependence is the *focusing effect*, the bargaining parties may use incomplete agreements to eliminate from the set of possible bargaining solutions off-equilibrium outcomes which are particularly negative for one of the bargaining party.

There is ample evidence showing that individual preferences are context dependent, in the sense that a decision maker's evaluation of a good depends on the specific choice set faced. As a preliminary step to our analysis, we consider the decision problem of a single decision maker with context-dependent preferences who can eliminate some options from her choice set before consuming. This situation corresponds, for example, to a shopper deciding what shop to enter, where one shop (say, a large supermarket) contains all options available in all other shops. Our main assumption is that the decision maker is *consequential* in the sense that she reasons backward and evaluates each shop by the item she will consume in case she enters that shop. The key observation is that under this assumption, the context (and the preferences) when choosing over shops is generically different from the context (and the preferences) when choosing over individual items from the largest possible choice set. In other words, a decision maker with the *option* to restrict her choice set has different preferences compared to the same decision maker who does not have such an option. As a consequence, the decision maker may eliminate some options from her choice set when given the opportunity to do so.

The main result of this paper is that, when the same decision maker is in a bargaining context, the bargaining parties may restrict the future bargaining set via an incomplete agreement, which never happens when all players are rational. Rational players can never do better than bargaining in one period, because if incomplete agreements affect the final outcome of the negotiation, then the players will have opposite preferences over what incomplete agreement to sign. It follows that bargaining over incomplete agreements becomes equivalent to bargaining directly over the final outcome (see the discussion in Section 6.1). Instead, if the bargaining parties have context-dependent preferences and are consequential, their preferences will depend on whether they have the option to restrict the bargaining set before reaching an agreement. When they do have such option, the bargaining parties may restrict the bargaining set by signing an incomplete agreement. This way, they can align the preferences of their present and future selves, so that their future selves will agree on the bargaining solution preferred by their present selves.

To make further predictions, we restrict our attention to one specific source of context dependence, namely the focusing effect. The focusing effect (or focusing illusion) occurs whenever a person places too much importance on certain aspects of her choice set (i.e. certain elements are more *salient* than others). Intuitively, an agent's attention is unconsciously and automatically drawn toward certain attributes, which are therefore overvalued when making a choice. Kőszegi and Szeidl (2013) formalize this concept by assuming that agents maximize a *focus-weighted utility*

$$\tilde{U}(x_1, x_2, .., x_n) = \sum_{s=1}^{n} h_s u_s(x_s)$$

where $x = \{x_1, x_2, ...x_n\}$ is a given good with n attributes. The *focus weights* h_s are defined as:

$$h_s = h\left(\max_{x\in C} u_s(x_s) - \min_{x\in C} u_s(x_s)\right),$$

where C is the choice set and $h()$ is the *focusing function,* assumed strictly increasing. In this formalization, an agent overweights the utility generated by the attributes in which her options differ more, where these differences are measured in utility terms.[5] Note that the focus-weighted utility is a *decision utility*, because it describes the decision maker's choice. We will often contrast the agent's decision utility with her *material utility* corresponding to a rational benchmark in which all the focus weights are equal to one.

In a bargaining game in which the players preferences are distorted by the focusing effect, incomplete agreements may be used by the bargaining parties to eliminate from the bargaining set extremely bad outcomes with respect to a specific attribute or issue, and therefore reduce the salience of that specific attribute or issue. Furthermore, we argue that incomplete agreements may be used in equilibrium even when they can be renegotiated or ignored. Crucially, we assume that incomplete agreement can be ignored only by jointly waiting one period (at no cost), and then negotiating over the entire bargaining set.[6] Hence, previous incomplete agreements do not constrain the set of outcomes achievable within the negotiation. Despite this, incomplete agreements affect the set of outcomes achievable during a given bargaining stage, and therefore determine the players' preferences at a given bargaining stage. This result is relevant for two reasons. First, when facing a sequence of agreements between two parties, in case a later agreement contradicts an earlier agreement, courts typically enforce the most recent one. Hence, when parties negotiate, previous incomplete agreements signed by the same parties are never binding. Nonetheless, incomplete agreements are extensively used, which can be rationalized under our assumptions. Second, we show that the class of renegotiation-proof incomplete agreements is characterized using minimum utility guarantees to the two players.

Finally, within our framework, the players can improve their bargaining outcome by taking pre-bargaining actions which are commonly observed but difficult to justify using a standard bargaining framework. Unlike incomplete agreements, pre-bargaining actions are *unilateral* restrictions on the bargaining set, imposed to manipulate the other party's focus weights. For example, a seller may enter the negotiation having already announced a *maximum price*—or announcing a price, with the understanding that this price can be negotiated downward. The effect of this announcement is to

[5] Schkade and Kahneman (1998) show that, when asked about comparing life in California and in the Midwest, most people report California as the best place to live and cite the weather - i.e. the dimension in which the two choices differ the most - as the main reason. Despite this, actual measures of life satisfactions in the two regions are similar. Similarly, Kahneman, Krueger, Schkade, Schwarz, and Stone (2006) show that people place too much weight on differences in monetary compensation when asked to compare job offers, which is the dimension in which these offers differ the most.

[6] Therefore, jointly ignoring a previous incomplete agreement amounts to renegotiating the negotiation structure.

reduce the salience of the price dimension, to decrease the buyer's price sensitivity, and to increase the equilibrium transaction price.[7] Therefore, according to our model the seller of, for example, an expensive house may choose to constrain her bargaining position and announce a price before bargaining. Similarly, we show that a seller may choose to be held up by a focused buyer. If the focusing effect is particularly severe, rather than bargaining over price and quality simultaneously, a seller may prefer to invest in quality, announce a maximum price and then bargain over the final price. Doing so, the seller can decrease the price sensitivity of the buyer, which cannot be done simply by announcing a maximum price because this announcement can be renegotiated. According to our model, therefore, the strength of a buyer's focusing effect determines, for example, whether a construction company should build a house and then bargain with the buyer over the price, or simultaneously bargain with the buyer over the characteristics of the house and its price.

We develop our argument through two examples. In the first example, we consider the case of a buyer and a seller exchanging a good of known quality. We assume that the seller is rational but the buyer's utility is distorted by the focusing effect. We show that the final transaction price depends on whether the parties bargain in one period, or whether the parties agree first on the maximum price that can be charged, and then on the final price. When the negotiation happens in two steps, in the last period the set of possible transaction prices is bounded by the previous agreement. Therefore, the buyer will consider the price dimension as less salient, and is willing to accept a final price which is higher than the final price in case of no upper bound. More interestingly, when bargaining over the maximum price, the salience of the price dimension is low, because the buyer anticipates that extreme prices will be ruled out. Hence, the fact that a maximum price *can* be imposed—which is exogenously given in the moment in which the players bargain—is enough to modify the buyer's preferences, and make her less price sensitive. As a consequence, the buyer agrees on imposing a maximum price in period 1 which leads to a higher final transaction price than when no maximum price is imposed. We also allow the players to bargain over the negotiation structure to adopt, and derive conditions under which the players may adopt the two-step procedure.

In the second example, the players bargain over the price and the quality of a good. We show that the set of renegotiation-proof incomplete agreements can be fully characterized by a maximum transaction price, a maximum transaction quality, a minimum utility level guaranteed to the buyer, and a minimum utility level guaranteed to the seller. We compare four negotiations structures: the one-step negotiation, the two-step negotiation when players can sign a renegotiation-proof incomplete agreement specifying only a maximum price and maximum quality, the two-step negotiation when players can also specify a minimum utility guarantee to the buyer in their incomplete agreement, and finally the two-step negotiation in which the players can sign any renegotiation-proof

[7] In equilibrium, the maximum price announced by the seller is also the transaction price, which is higher compared to the case of no announcement. Hence, in equilibrium the announcement seems to provide an *anchor* for the price. However, in general, anchoring and the focusing effect are different behavioral biases, with the latter being more relevant for multi-dimensional problems as often considered in bargaining games.

incomplete agreement. We show that the negotiation structure affects the final quality agreed upon. In addition, we show that bargaining over a minimum utility guarantee to the buyer in period 1 leads to higher profits to the seller, while bargaining over a minimum utility guarantee to the seller in period 1 leads to lower profits to the seller.[8]

We structure the paper in the following way. In the reminder of this section, we discuss the relevant literature. In Section 2, as a preliminary step to our analysis, we argue that a decision maker with context-dependent preferences may progressively eliminate options from her choice set. In Section 3, we consider the case of a focused buyer and a rational seller trading a good of known quality. This example is solved under the most tractable assumptions. For example, we consider only one type of incomplete agreements (a maximum price), and we assume that incomplete agreements are binding. In Section 4, we consider the case of a focused buyer and a rational seller bargaining over the price and the quality of a good, and we introduce non-binding incomplete agreements. In this section, we also characterize the full set of renegotiation-proof incomplete agreements and consider the case of endogenous hold up by the seller. In Section 5, we consider a general bargaining game with context-dependent preferences for both players and we show that incomplete agreements may be used in equilibrium. In Section 6, we discuss the robustness of our results to changes in our main assumptions. For example, we show that rational players will never use incomplete agreements, and we argue that our results hold under both a cooperative and a non-cooperative bargaining solution. Section 7 concludes. In Appendix A, we present further robustness checks and extensions. Unless indicated otherwise, proofs are relegated to Appendix B.

Relevant Literature.

In their seminal work, Grossman and Hart (1986) define the concept of ownership as the residual right of control: the right to dispose of an object in case a contingency that was not specified in a previous agreement occurs. Hence, ownership is well defined only if contracts are incomplete: two parties cannot specify all details of a transaction in one contract, but need to negotiate in two stages. Grossman and Hart (1986) justify the assumption of contract incompleteness with uncertainty: the negotiating parties cannot write agreements contingent on all possible states of the world, and need to wait for the uncertainty to be resolved to complete the contract. However, the subsequent literature argued that rational players should always be able to write complete contracts. In particular, Maskin and Tirole (1999) show that, if parties have trouble specifying physical contingencies in a contract, they should nonetheless be able to specify payoff contingencies using a mechanism.

As a consequence, several authors argued that behavioral biases and cognitive limitations may explain the existence of incomplete contracts. The contracting parties may leave some contingencies

[8] Also here, we also derive the structure of the negotiation endogenously, by allowing the players to bargain over the set of incomplete agreements over which they will bargain.

unspecified and rely on future negotiation when the ex-ante contracting environment is complex (see Segal, 1999), when becoming aware or thinking about future contingencies is costly (see Tirole, 2009, and Bolton and Faure-Grimaud, 2010), or when there is a cost of specifying contingencies in a contract (see Battigalli and Maggi, 2002). More recently, Hart and Moore (2008) showed that, if the parties cannot write a complete contract, they may rely on ex-post negotiation to determine the bargaining outcome also in states of the world that are contractible ex ante. When a state of the world that is not specified in a contract occurs, the two parties have to negotiate a bargaining outcome. Hart and Moore (2008) crucially assume that, in the ex-post negotiation, each party takes the ex-ante contract as a reference point, and evaluates the outcome of the bargaining with respect to the best possible outcome specified in the contract. In addition, if a player is dissatisfied with the outcome of the bargaining, she can impose a cost on the other player. It follows that, when ex-post negotiation occurs with positive probability, the two parties may prefer to leave some provisions of the contract unspecified to avoid creating a reference point in future negotiations.[9]

Our paper is related to the above literature because, also here, agreements may be reached in steps. The parties may sign an incomplete agreement in period 1, and rely on a future bargaining round to complete the agreement. However, all the papers mentioned above rely on some uncertainty being resolved after completing a bargaining round, before moving to a new bargaining round. We think, however, that the arrival of specific pieces of information is often not a first-order determinant of the structure of negotiations. For example, in many negotiations we do not observe parties waiting for a new piece of information to arrive before starting a new bargaining round. Our paper considers exactly these situations, and provides an explanation for the existence of incomplete agreements in contexts with perfect information.[10] On the other hand, we do not consider here the issue of ownership, which is the central question of the literature mentioned above.

To the best of our knowledge, our paper provides the first justification for the use of incomplete agreements in a context with perfect information and where every aspect of an agreement is contractible. However, some authors argued that, if information is perfect but players are prevented from writing a complete contract, then the players may decide to leave some potentially contractible aspects of an agreements unspecified (see Bernheim and Whinston, 1998). Closest to our model, Battaglini and Harstad (2014) study international environmental agreements, and assume that countries cannot perform side payments. They argue that environmental agreements may be left incomplete, as a way of inducing more countries to sign the agreement (see also Harstad, 2007).

Several existing papers demonstrate that if the player's outside option are determined endoge-

[9] For a similar argument, but based on the agent's loss aversion, see Herweg and Schmidt (2014) and Herweg, Karle, and Müller (2014).

[10] The fact that the information set does not change between bargaining rounds does not imply that there is no uncertainty. However, an environment with uncertainty, perfect contracting, and no change in the information set across bargaining rounds is equivalent to an environment with perfect information. The reason is that the parties can bargain conditionally on the realization of a given state of the world.

nously, agreements may be reached gradually rather than in a single period. For example, the literature on gradualism in bargaining (see Compte and Jehiel, 2004) proposes models of alternating offers in which the disagreement payoffs depend on the history of offers. Related to our work, Compte and Jehiel (2003) introduce reference-dependent utility in a game of alternating offers and show that the history of offers affect the players' preferences and the final outcome. Whereas in Compte and Jehiel (2003) players exert pressure on each other in order to extract concessions, in our model each bargaining party has the incentive to eliminate from the bargaining set options which are particularly bad for the *opposing* party. Furthermore, we solve each bargaining stage via Nash bargaining, and show that multiple bargaining stages are possible. Hence, our model is a model of sequential agreements, and not of sequential offers.

With this respect, we are close to Esteban and Sákovics (2008), who assume that the disagreement outcomes of a bargaining game depends on the shape of the bargaining set. The intuition is that the shape of the bargaining set affects the players' position in the non-cooperative game played in case of a disagreement. Esteban and Sákovics (2008) allow players to disagree on how to split a fraction of the surplus (and agree on how to split the rest). Whereas in Esteban and Sákovics (2008) eliminating irrelevant alternatives from the bargaining set affects the bargaining solution via the outside options, in our model, eliminating irrelevant alternatives from the bargaining set affects the bargaining solution via the players' preferences. In addition, the goal of Esteban and Sákovics (2008) is to derive axiomatically the unique efficient bargaining solution in this context, while we are interested in understanding the use of incomplete agreements.

The literature on agenda setting in negotiation has long argued that, when players bargain over multiple issues, the order in which agreements are reached matters for the outcome of the bargaining process (see Lang and Rosenthal, 2001, Bac and Raff, 1996, Inderst, 2000, Busch and Horstmann, 1999b). Extending the bargaining model of Rubinstein (1982), these papers assume that each player can make an offer about one or more issues on the table. Once an agreement is reached on one issue, this agreement is binding for both parties. Hence the parties strategically choose whether to make offers about all the issues on the table, or only on some of them. In particular, Busch and Horstmann (1999a) and Flamini (2007) argue that the players may agree on the order in which the issues are resolved (i.e. the offers are made). In our paper, we are interested in negotiations that entail a unique final agreement reached via several incomplete agreements, rather than several issue-specific agreements.

We employ here the model of focusing in economic choice proposed by Kőszegi and Szeidl (2013). Bordalo, Gennaioli, and Shleifer (2013) also develop a model of salience. They assume that agents overvalue the attributes that differ the most with respect to a reference point. Hence, whereas the framework developed by Kőszegi and Szeidl (2013) can be directly used to describe how preferences change with the bargaining set, performing the same analysis using the framework developed by Bordalo, Gennaioli, and Shleifer (2013) would require to establish how changes in the bargaining set affect the reference point. For this reason, we develop our argument using Kőszegi and Szeidl (2013), but we show in Section 5 that the central prediction of our model—that the players may use incomplete agreements to restrict the bargaining set over time—is robust to using other models of context-dependent preferences such as Bordalo, Gennaioli, and Shleifer (2013).

A separate strand of literature assumes that the allocation of attention is a conscious decision made by the decision maker (also called *rational inattention*, see Sims, 2003). Within this literature, Gabaix (2014) develops a model in which agents consciously neglect some pieces of information in their decision making. Similarly to our model, a piece of information is more likely to be neglected when the variance of this piece of information is low. However, in our model salience is defined over utility-relevant outcomes rather than signals. Also, in our model, dimensions are never fully neglected or fully considered, but rather change their salience as the set of possible outcomes expands or shrinks.

2 Preliminary: sequential choice and context-dependent preferences

As a preliminary step to our analysis, we consider here an individual decision maker with context-dependent preferences who can choose her consumption bundle sequentially, by first restricting her choice set and then consuming. As a motivating example, consider a decision maker choosing over three possible snacks: an apple, a croissant and a chocolate bar. It is reasonable to assume that, when the choice is limited to either an apple or a croissant, the decision maker chooses the apple. Despite this, it is possible that when a chocolate bar is added to the choice set, the decision maker chooses the croissant, maybe because the chocolate bar reminds her of how delicious sweet snacks are. If this is the case, the decision maker displays context-dependent preferences, because her preferences over apple and croissant (as revealed by her choice) depend on the presence of the chocolate bar in her choice set.

Now, assume that the decision maker chooses her snack sequentially. First, she chooses whether to enter a small shop offering only apples and croissants or a supermarket offering all three items. Then, she consumes one of the snacks offered by the shop of her choice. Our main assumption is that the decision maker is *consequential*, in the sense that she reasons backward and evaluates each possible future choice set by the consumption bundle chosen in case this choice set is reached. Hence, in this example the decision maker anticipates that by going to the supermarket she will choose a croissant, while by going to the small shop she will end up eating the apple. Therefore, we can think of the ex-ante choice of where to shop as, again, a choice over apple or croissant, which implies that ex-ante the agent prefers the apple to the croissant. Consequently, to eat the apple the agent has to visit the small shop, where her choice is constrained to only two options.

The above example highlights two key insights that, as we will see, generalize to any sequential choice problem and any bargaining problem in which the decision maker has context-dependent preferences and is consequential. First of all, preferences when choosing over shops can, in general, be very different from preferences when choosing a given item within the large supermarket. Hence, exogenously giving the decision maker the option to eliminate some consumption bundles from the choice set causes a change in the decision maker's preferences. Second, when choosing over shops the agent anticipates that by entering a specific shop her preferences will suddenly change, which implies that her future self may choose options that are not optimal for her present self. The reason is that once a shop is chosen, options that were not relevant ex ante (because not chosen on the equilibrium path and therefore not relevant before entering the shop) become part of the choice set and therefore relevant in determining the agent's preferences. Hence, when possible, the agent chooses the shop that eliminates this change in preference and makes her future-self preferences aligned with her present-self preferences.

More formally, consider the choice over a set X. Call $C(X)$ the choice from X, i.e.

$$C(X) = \text{argmax}_{x \in X} u(x, X),$$

where we used the fact that preferences depend on the choice set. Suppose now that the decision maker is given the option to choose in two steps: in period 1, the decision maker can choose a subset $S \subset X$, and in period 2, she can choose an element of S. Call $\mathbb{S}$ the set of subsets S which can be chosen in period 1, and assume that $X \in \mathbb{S}$ so that the decision maker has the option not to restrict her choice set and choose from the entire X in period 2. Because the decision maker is consequential, in period 1, she evaluates each S by the $C(S)$ that will be chosen in case S is reached. Therefore, the period-1 choice is equivalent to choosing one of the consumption bundles that are achievable via a specific S, i.e. choose a $x \in \mathbb{C}$ where $\mathbb{C} = \{C(S)|S \in \mathbb{S}\}$. The solution to the period-1 problem is therefore

$$C(\mathbb{C}) = \text{argmax}_{x \in \mathbb{C}} u(x, \mathbb{C}).$$

Note that $C(X) \in \mathbb{C}$, meaning that the solution to the unconstrained utility maximization problem can be achieved by choosing X in period 1. However, the choice set faced by the agent in period 1 is $\mathbb{C}$ which may be different than X. A sufficient condition is that at least one element of X is never chosen under any S. In this case, the decision maker's preferences in period 1 are different from her preferences when she chooses from the entire choice set, and it is possible that $C(\mathbb{C}) \neq C(X)$.

Thus, moving from a one-step decision over the entire X to a two-step decision process *by itself* may change the decision maker's preferences and the outcome of the choice. Note also that, in case $C(\mathbb{C}) \neq C(X)$, in period 1 the decision maker will restrict her own future choice set so that her future self will choose $C(\mathbb{C})$ instead of $C(X)$, i.e. she must choose an $S \neq X$ in period 1. Finally, if the set $\mathbb{C}$ can itself be chosen in period 1 (i.e. if $\mathbb{C} \in \mathbb{S}$), then in period 1 the decision maker cannot do better than choosing $\mathbb{C}$. By doing so, her future self will face the same choice set as her present self, will have the same preferences as her present self, and will choose the consumption bundle which is optimal from period-1 point of view.

Two additional features of this simple choice problem are worthwhile discussing. First of all, all results continue to hold when the decision maker is partially naïve or not fully sophisticated, in the sense that she fails to correctly anticipate the period-2 choice when a given subset S is chosen in period 1. In this case, the period-1 context is $\tilde{\mathbb{C}} \neq \mathbb{C}$, which is the set of all the consumption bundles the agent thinks she will choose if each $S \in \mathbb{S}$ is reached. Nonetheless, in general $\tilde{\mathbb{C}} \neq X$ so that, also here, the decision maker's preferences when choosing over S are different from the preferences when choosing over the entire X, and she may want to restrict her future choice set. Second, the decision maker's preferences when choosing over subsets depend on $\mathbb{S}$, the set of possible subsets that can be chosen, so that when $\mathbb{S}$ changes, also the period-1 preferences (and choice) may change. This is a reflection of the fact that preferences are context dependent in period 1 as well, where the context is a function of $\mathbb{S}$. Therefore, moving from a static choice over X to a sequential choice simply substitutes a context with another, with no presumption that the final outcome will be "better" in any sense. Going back to our example, imagine that the decision maker is diabetic, and than when choosing over the 3 items the presence of the chocolate bar reminds her of the danger of sugar. However, when choosing only over apple or croissant, the agent is tempted and chooses the croissant. In this case, offering the agent the option to choose her choice set in the way we previously described leads to a worse outcome (from the health point of view) compared to the case in which the choice happen in only one step.

To conclude this section, note that the choice problem and the bargaining problem differ in a few important ways. For example, here we assumed that a decision maker must choose from S in period 2, but it is unclear why she must comply with her previous choice instead of changing her mind. As we will see, in a bargaining context the presence of a second player creates a degree of commitment toward previously-established incomplete agreements (see, especially, the discussion about renegotiation in Section 4). Finally, in a bargaining game the set of possible bargaining outcomes (i.e. the bargaining set) depends on the players' preferences. Hence, preferences are a function of the bargaining set, which itself is a function of preferences. Whereas in this section we assumed that choice sets are exogenously given, finding the bargaining set of a bargaining game requires solving for a fixed point problem.

3 Example 1: Focusing Effect and the Sale of a Good

Let us start by considering the simplest possible bargaining problem: the sale of a good of given quality.[11] Call $q \in \{0, v\}$ the quality exchanged, that can be either v (if there is a sale) or zero

[11] Any situation in which two players bargain over a two-dimensional issue and have opposite preferences over the two dimensions can fit our framework. The simplification in considering buyer and seller (as opposed to a more general set up) is that both players' preferences are assumed linear with respect to one of the dimensions (the price). This assumption will greatly simplify our derivations.

(if there is no sale).[12] Call p the transaction price, call $\{q,p\}$ a *bargaining outcome* and call $X = \{0, v\} \times \mathbb{R}_0^+$ the *unconstrained bargaining set*, or the set of all possible bargaining outcomes.

The players bargain in two periods. In the first period they agree on an incomplete agreement and in the second period they agree on a specific bargaining outcome. There is no time discounting.

Definition 1. An *incomplete agreement* is a set $S \subset X$. A *negotiation structure* $\mathbb{S}$ is the collection of convex S that can be chosen in period 1.

We interpret the case $\mathbb{S} = X$ as a one-step negotiation. Because there is no time discounting, negotiating in one step is equivalent to negotiating in two steps, under the constraint that the only S the players can choose in period 1 is the entire bargaining set. Also, in what follows we always assume that $X \in \mathbb{S}$, so that in period 1 the players can always choose to bargain over the entire bargaining set in period 2, i.e. they can always choose not to sign an incomplete agreement.

We solve this example for two specific negotiation structures: a one-step negotiation, and a two-step negotiation in which players first can impose a maximum transaction price and then agree on a final transaction price. In the latter case, the set $\mathbb{S}$ contains all the sets that can be written as $S = \{0, v\} \times [0, \hat{p}]$ for some $\hat{p} \geq 0$.[13] After finding the equilibrium bargaining outcome under the two negotiation structures, we introduce a period 0 in which buyer and seller bargain over the negotiation structure to adopt.

Assumption 1 (Binding incomplete agreements)**.** *A bargaining outcome* $\{q,p\}$ *is a* feasible solution *to the bargaining problem in period 2 if and only if* $\{q,p\} \in S \cup \{0,0\}$.

Under the above assumption, during the last stage of a negotiation the players can either agree on a bargaining outcome in compliance with a previous S, or they can disagree and not trade. Therefore, here, if a maximum price was established in period 1, in period 2 the players must respect this maximum price if they want to trade. In the second example we consider, we will relax this assumption and allow the players to both renegotiate and ignore a previous incomplete agreement (see Assumption 5 in Section 4).

Here we restrict our attention to a specific form of context dependence, the focusing effect as modeled by Kőszegi and Szeidl (2013). We introduce the focusing effect into the problem by assuming that the buyer's (decision) utility is:

$$U^b(q,p) = h(\bar{q} - \underline{q})q - h(\bar{p} - \underline{p})p$$

where $h(\bar{q}-\underline{q})$ and $h(\bar{p}-\underline{p})$ are the *focus weights*, and $h()$ is a strictly increasing function. The values of $\bar{q}$, $\underline{q}$, $\bar{p}$, $\underline{p}$ depend on the set of bargaining outcomes that the buyer considers possible, which we

[12] In this formulation, not transacting is equivalent to transacting at quality 0. Formulating the problem this way allows us to maintain the same notation also in our next example, where we assume that q is a continuous variable. See Section 4.

[13] Similarly to what discussed in the preliminary section, we will show that also here the specific $\mathbb{S}$ determines the players' preferences when bargaining over incomplete agreements.

call the *consideration set*. In particular, $\bar{q}$ and $\underline{q}$ are the largest and smallest q in the consideration set; similarly $\bar{p}$ and $\underline{p}$ are the largest and smallest p in the consideration set. Therefore, the buyer's preferences are context dependent, because they are affected by the set of possible bargaining outcomes. More specifically, the focusing effect causes the buyer to focus more on, and hence overweight, the dimension of the bargaining problem with the largest difference in terms of possible bargaining outcomes.

The seller is, instead, fully rational. His utility function is:

$$U^s(q,p) = \begin{cases} p - c & \text{if } q = v \\ p & \text{if } q = 0 \end{cases}$$

or

$$U^s(q,p) = p - \frac{c \cdot q}{v}.$$

Finally, we assume that both players have the same outside option equal to zero.

Assumption 2 (Consideration set)**.** *A bargaining outcome $\{q,p\}$ is in the consideration set if and only if it is feasible and both players satisfy their rationality constraint at $\{q,p\}$.*

In other words, the consideration set coincides with the bargaining set, and is composed of all the feasible bargaining outcomes which are preferred by both players to no agreement at all.[14] Note that finding the consideration set is a fixed point problem, because the focus weights determine the buyer's preferences, the buyer's rationality constraint and the shape of the consideration set. At the same time the shape of the consideration set determines the focus weights and the buyer's preferences.

Assumption 3. *If at any negotiation stage the parties disagree, the outcome is no trade.*

Assumption 3 implies a form of commitment with respect to the structure of the negotiation. In this example, players cannot meet again and bargain over the final price after having failed to agree either on a maximum price or on a transaction price. Furthermore, it implies that, at any stage of the negotiation, the lower bounds of the consideration set are always given by the option of not trading, so that $\underline{p} = \underline{q} = 0$.[15]

Assumption 4. *Each bargaining round is solved by Nash bargaining.*

This assumption implies that, within each bargaining round, once we fix the players' preferences irrelevant alternatives do not affect the bargaining outcome. However, in our model, preferences

[14] Clearly, other definition of the consideration set are possible. However, we believe that Assumption 2 is the most reasonable way to define the consideration set. See Section 6.2 and, especially, Section A.1 for a discussion.

[15] In Section A.2 in Appendix we discuss what happens if, after disagreeing, the players have the option to initiate a new round of negotiations.

are a function of the entire bargaining set, and therefore the solution to the bargaining problem is affected by irrelevant alternatives. We show in Section 6.1 that whenever preferences are rational and the bargaining solution satisfies independence of irrelevant alternatives, the structure of the negotiation does not affect the final bargaining solution. Hence, our approach isolates a single channel through which incomplete agreements affect the outcome of the negotiation: the players' preferences.[16]

Solving this example allows us to easily illustrate the main point of the paper: that the negotiation structure affects the final outcome of the negotiation. In particular, we will show that the negotiation structure determines the price paid by the buyer (which is never lower than the price paid by a rational buyer) but is not relevant in determining whether trade occurs. In Section 4, we present a second example in which quality is endogenously determined, and the structure of the negotiation affects the transaction quality.

3.1 One-Step Negotiation

Because the players' outside options are zero, Assumption 2 implies that $\{q, p\}$ is in the consideration set if and only if:

$$p \geq \frac{c \cdot q}{v}$$
$$h(\bar{q})q \geq h(\bar{p})p,$$

where we used the fact that not buying is always in the consideration set, so that $\underline{q} = \underline{p} = 0$. The only alternative to not buying is to buy v, which is in the consideration set if and only if there exists a price p such that both players satisfy their rationality constraints at $\{v, p\}$.

The value of $\bar{q}$ is v if there is a price that satisfies both players' rationality constraints, and is equal to zero otherwise. However, for any value of $\bar{q}$, the largest possible transaction price $\bar{p}$ is given by $\bar{p} : \bar{p}h(\bar{p}) = \bar{q}h(\bar{q})$. Therefore, $\bar{p} = \bar{q}$ always holds, and the buyer is equally focused on the price dimension as on the quality dimension. As a consequence, trade occurs if and only if $v \geq c$. Also, if $v \geq c$ then $\bar{p} = v$, meaning that the largest possible transaction price in the consideration set is v.

Lemma 1. *When bargaining occurs in one period, the two parties trade at price $p = \frac{v+c}{2}$ if $v \geq c$, and do not trade if $v < c$.*

Proof. In the text. □

Therefore, in the one-step negotiation, the focused buyer puts equal weights on price and quality and behaves like a rational buyer. This feature of the model is convenient as it reduces the cases we need to discuss, but is not a generic property of bargaining problems with focusing effect.[17]

[16] In Section 6.1 we assume that players are rational but that the bargaining solution violates the independence of irrelevant alternatives. In Section 6.2 we consider a non-cooperative version of this bargaining problem.

[17] For more details, see Section 6.3.

3.2 Two-Step Negotiation

Suppose now that the negotiation happens in two periods. In the first period, the parties bargain over $\hat{p}$: a maximum price the seller can charge. In the second period, the two parties agree on whether to trade and at what exact price.

We start by solving the game in period 2: for given maximum price $\hat{p}$, the two players bargain over p. If $\hat{p} < c$, it is not possible to satisfy the rationality constraint of the seller (remember that the maximum price is binding), so that the upper bounds of the consideration set are $\bar{q} = \bar{p} = 0$. Similarly, if $\hat{p} > c$ but $c > v$, there will be no transaction, because there is no $\bar{p} \in [c, \hat{p}]$ such that $h(v)v \geq h(\bar{p})\bar{p}$. Also in this case the upper bounds of the consideration set are $\bar{q} = \bar{p} = 0$

Hence, a transaction is possible only if $v > c$ and $\hat{p} > c$. In this case, the maximum possible transaction price is $\bar{p} = \min\{v, \hat{p}\}$, where v is the largest possible transaction price in case no maximum price is imposed in period 1. Therefore, setting $\hat{p} > v$ is equivalent to not setting a maximum price and solving the one-step negotiation problem in period 2.[18] In this case, the buyer's utility is:

$$u^b(q,p) = h(v)v - h(\min\{v, \hat{p}\})p.$$

Note that at the largest possible transaction price $\bar{p} = \min\{v, \hat{p}\}$, both players satisfy their rationality constraint, and trade is therefore possible. Hence, when $v > c$ and $\hat{p} > c$ the Nash bargaining solution of the problem is:

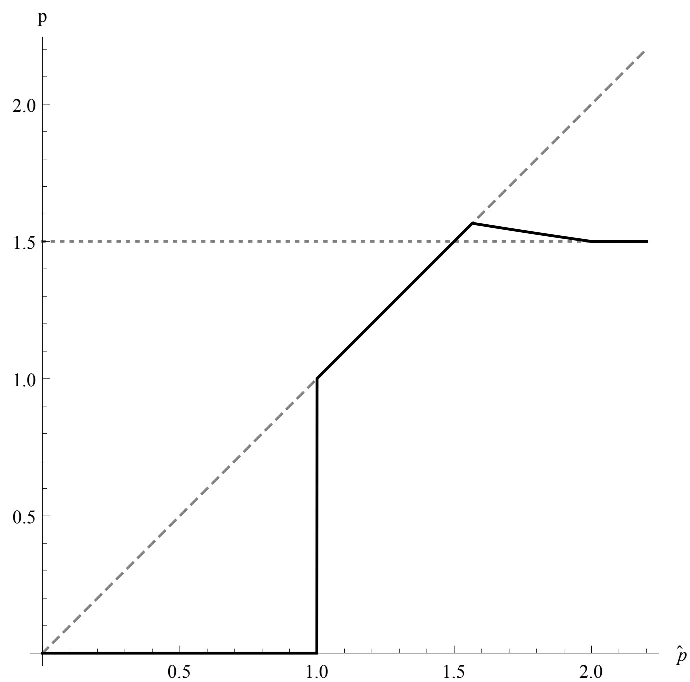
$$p(\hat{p}) = \min\left\{\frac{1}{2}\left(\frac{h(v)}{h(\min\{v, \hat{p}\})}v + c\right), \hat{p}\right\}. \tag{3.1}$$

Figure 3.1 plots the final price as a function of the maximum price. Note that $p(\hat{p})$ is first increasing and then decreasing in $\hat{p}$. At first, $p(\hat{p})$ is increasing because $\hat{p}$ is binding, meaning that the unconstrained solution to the Nash bargaining problem would be above the maximum price $\hat{p}$. Then $p(\hat{p})$ is decreasing because as $\hat{p}$ becomes large the salience of the price dimension increases. In other words, starting from a very large maximum price and decreasing the maximum price leads to a *higher* final transaction price, because the buyer becomes less price sensitive as $\hat{p}$ decreases. Intuitively, we can call this a "peace of mind" effect of decision utility, because knowing that high prices are excluded from the set of possible outcomes decreases the salience of the price dimension and the overall price sensitivity. As a consequence, the highest possible price achievable in period 2 is

$$\hat{p}^\star : \frac{1}{2}\left(\frac{h(v)}{h(\hat{p}^\star)}v + c\right) = \hat{p}^\star. \tag{3.2}$$

For future references, note that $\hat{p}^\star < v$: the largest possible transaction price achievable via a

[18] More formally, suppose that the maximum possible transaction price is $\bar{p} > v$. Trading at price $\bar{p}$ satisfies the rationality constraint of the buyer if $h(v)v \geq h(\bar{p})\bar{p}$, leading to a contradiction.



The period–2 price p (solid black line) as a function of the maximum price $\hat{p}$ set in period 1. In addition, the equilibrium price level in the one-step negotiation case $(v+c)/2$ (dotted gray line) and the maximum price $\hat{p}$ set in period 1 itself (dashed gray line). Parameter values are $v = 2$, $c = 1$ and $h(x) = x/5 + 1$; $p^\star = \hat{p}^\star = 1.5661$.

Fig. 3.1: Period–2 Price as a Function of the Maximum Price

maximum price is below the largest possible transaction price when the players bargain in one step.

Consider now period 1. Suppose that trade can occur in period 2 for some $\hat{p}$ (i.e. suppose $v > c$). Players are fully consequential and they understand that to every maximum price they may impose there is a corresponding final price given by equation (3.1). Define the transaction quality as a function of $\hat{q}$ as:

$$q(\hat{p}) = \begin{cases} v & \text{if } \hat{p} \geq c \\ 0 & \text{if } \hat{p} < c \end{cases}. \tag{3.3}$$

In case trade is possible, the buyer's utility in period 1 depends on the maximum price in the following way:

$$u^b(q(\hat{p}), p(\hat{p})) = h(v)q(\hat{p}) - h(\hat{p}^\star)p(\hat{p}),$$

where we used the fact that $\hat{p}^\star = \max_{\hat{p}}\{p(\hat{p})\}$, i.e. $\hat{p}^\star$ is the largest final price achievable by setting a maximum price in period 1.

The key observation is that, because $\hat{p}^\star < v$, when bargaining over the maximum price the buyer is less price sensitive than in the one-step negotiation case. Bargaining over a maximum price *by itself* eliminates from the set of possible bargaining outcomes extremely high prices and reduces the period-1 buyer's price sensitivity, even before a maximum price is actually imposed. Note the parallel with the discussion in Section 2, where we argued that giving the decision maker the possibility of restricting her choice set by itself changes the decision maker's preferences. Here, the possibility of restricting the future bargaining set (which is, at this point, exogenously given) distorts the buyer's preferences in period 1.

Because the maximum price matters only to the extent that it delivers a specific final price, bargaining in period 1 over $\hat{p}$ is equivalent to bargaining over p under the restriction that p can be achieved by setting a specific $\hat{p}$, i.e. under the restriction $p \in [c, \hat{p}^\star]$.[19] Therefore, the solution to period 1 bargaining problem is:

$$p^\star = \min\left\{\frac{1}{2}\left(\frac{h(v)}{h(\hat{p}^\star)}v + c\right), \hat{p}^\star\right\},$$

Note that, by equation (3.2), $p^\star = \hat{p}^\star$, which implies that the final transaction price equals the maximum price chosen in period 1.

Proposition 1. *Assume that the parties bargain first over the maximum price and then agree on whether to trade and at what final price. Whenever $v > c$, then the final price $\hat{p}^\star$ is such that $v > \hat{p}^\star > c$, and the final price is greater than in the one-step negotiation case. Whenever $v = c$, $\hat{p}^\star = c$ and the final price is equal to the one-step negotiation case. Whenever $v < c$, there is no trade.*

The above proposition contains three results. First, incomplete agreements are used in equilibrium whenever $v > c$. In period 1, buyer and seller could set $\hat{p} > v$ so that in period 2 the bargaining set is unconstrained. Instead, they agree to restrict the future bargaining set by setting a maximum price $\hat{p} = \hat{p}^\star < v$. Second, imposing a maximum price leads to a transaction price which is higher than the transaction price in case no maximum price is imposed. This result may sound surprising because we would expect that, by giving the buyer the opportunity to establish a maximum price before bargaining on the final price, she should be able to (weakly) improve her material payoff of the negotiation. Instead, the buyer ends up paying a higher price. This result follows from the fact that the buyer is less price sensitive when bargaining over the maximum price than in the one-step negotiation. Importantly, the fact that the period-1 discussion is about a maximum price is exogenously given (the next section will endogenize the negotiation structure), but by itself rules out very high transaction prices, provides "peace of mind" also in period 1. Third, in period 1, the buyer and the seller agree on the maximum price that delivers $\hat{p}^\star$, the highest transaction price

[19] Again, note the parallel between the set $\mathbb{C}$ in Section 2 and the set $[c, \hat{p}^\star]$. Both sets contain all final outcomes achievable by restricting the set of possible outcomes in period 1, and therefore determine the preferences in period 1.

achievable via a maximum price. Similarly to the intuition discussed in Section 2, here players want to align preferences between period 1 and period 2. In particular, by imposing a maximum price equal to $\hat{p}^{\star}$, the preferences of period-2 buyer become identical to the preferences of period-1 buyer. Hence, incomplete agreements are used in equilibrium to align the buyer's preferences across periods. Note that in this simple example period-1 buyer and period-2 buyer achieve a full alignment of preferences. Instead, in the next example the alignment of preferences will not be perfect because we will assume that incomplete agreements can be renegotiated, which imposes a constraint on set of incomplete agreements that can be signed.

Note also that this result is robust to the introduction of a buyer who is not sophisticated and fails to correctly anticipate how the future price will depend on $\hat{p}$ (i.e. the mapping between p and $\hat{p}$ is different from equation (3.1)). As long as period-1 buyer believes that the largest transaction price that is achievable by imposing a maximum price is below v, when bargaining over a maximum price he will be less price sensitive than when negotiating in one period. As a consequence, he is willing to restrict the future bargaining set by imposing a maximum price. In addition, Proposition 1 holds also whenever the buyer is fully *naïve*, and thinks that the final price does not depend on $\hat{p}$. In this case, in period 1 the buyer is indifferent between all possible $\hat{p}$, while the seller strictly prefers $\hat{p}^{\star}$.

Finally, note that, here, bargaining in 2 steps moves the bargaining outcome away from the bargaining outcome with a rational buyer, which coincides with the one-step negotiation. This result is specific to the model considered here and does not generalize to other contexts. For example, we show in Section 6.3 that when the good has two quality dimensions, in the one-step negotiation the agent is too price sensitive (relative to the rational benchmark). It follows that bargaining in two steps and imposing a maximum price reduces the agent's price sensitivity and moves the bargaining outcome closer to the rational benchmark.

3.3 Which Negotiation Structure?

In certain cases, the negotiation structure available to the two parties will be determined by exogenous circumstances, such as whether the good is perishable and the parties have no time to negotiate in two steps. However, in other cases the two parties may have the option to bargain over the negotiation structure to use. In this section, we introduce this possibility by assuming that, in period 0, buyer and seller bargain over what negotiation structure to adopt.

The negotiation over what bargaining structure to adopt can be conducted in several ways. For example, the players may bargain in period 0 over a minimum maximum-price. To every possible minimum maximum-price will correspond a final outcome, and the set of final outcomes achievable via a minimum maximum-price determines the players' context (and preferences) in period 0. Establishing a minimum maximum-price equal to zero will lead to the two-step negotiation described above and a transaction price equal to $\hat{p}^{\star}$. On the other hand, establishing a very high minimum maximum-price will cause the players to, effectively, bargain in one period and reach the transaction price $p = \frac{v+c}{2}$. Intermediate minimum maximum-prices will deliver intermediate transaction prices. Because, from period 0 point of view, the largest transaction price achievable is $\hat{p}^\star$, the buyer's focus weight on price in period 0 is $h(\hat{p}^\star)$. This is the same focus weight on price in period 1 in case the players negotiate in two steps as described above. Because the players want to avoid changes of preferences across periods, they will set the minimum maximum-price to zero and negotiate in two steps.

Alternatively, the players may negotiate in period 0 using side payments. Assume that, in period 0, the players bargain over whether to bargain in one or two steps, and over a period-0 monetary transfer. Call $m > 0$ a period-0 payment from the buyer to the seller. We assume that payments made in period 0 are also discounted by an appropriate focus weight, which depends on $\bar{m}$ and $\underline{m}$: the largest and smallest m in the consideration set. We also assume that, in case of disagreement in period 0, no trade occurs, so that $\underline{p} = \underline{q} = 0$. In addition, if trade is materially efficient (i.e. $v > c$), the highest price is achieved when the two players bargain in two steps.[20] Hence, the upper bounds of the consideration set with respect to q and p are $\bar{q} = v$ and $\bar{p} = \hat{p}^\star$. The buyer's utility is therefore

$$U^b(p, q, m) = h(v)q - h(\hat{p}^\star)p - h(\bar{m} - \underline{m})m$$

and the seller's utility is

$$U^s(p, q, m) = p + m - c \cdot \frac{q}{v}.$$

where $q \in \{0, v\}$ depending on whether there is a transaction.

It is easy to see that the smallest m in the consideration set is given by largest possible payment from the seller to the buyer, which is defined as:

$$-\underline{m} = \max_{p \in \{\hat{p}^\star, \frac{c+v}{2}\}} [p - c] = \hat{p}^\star - c \tag{3.4}$$

Similarly, the largest m in the consideration set is given by largest possible payment from the buyer to the seller, which is implicitly defined as:

$$\bar{m}: \; h(\bar{m} + \hat{p}^\star - c)\bar{m} = \max_{p \in \{\hat{p}^\star, \frac{c+v}{2}\}} [h(v)v - h(\hat{p}^\star)p] = h(v)v - h(\hat{p}^\star)\frac{c+v}{2} \tag{3.5}$$

where we used the expression for $\underline{m}$ derived in equation (3.4).

Note that the utility function of the buyer becomes linear in m if divided by the focus weight on m. If both utilities are linear in m, we can solve for the negotiation structure simply by maximizing

[20] If $v < c$, there is no trade independently on the negotiation structure.

the sum of the two utilities:

$$\max_{p \in \{\hat{p}^\star, \frac{c+v}{2}\}} \left[\frac{h(v)}{h(\bar{m} + \hat{p}^\star - c)} v - \frac{h(\hat{p}^\star)}{h(\bar{m} + \hat{p}^\star - c)} p + p - c \right],$$

where $\bar{m}$ is implicitly defined by (3.5). The key observation is that the expression is linear in p. Hence, whenever $h(\hat{p}^\star) > h(\bar{m} + \hat{p}^\star - c)$ monetary payments performed ex post (as price) are more salient than monetary payments performed ex ante (as m). The buyer is willing to pay some money ex ante in order to bargain in one shot and achieve the low price. If instead $h(\hat{p}^\star) < h(\bar{m} + \hat{p}^\star - c)$ the opposite is true, and the buyer will accept to bargain in two steps.

Therefore, the one-step negotiation is preferred if $\bar{m} > c$, and the two-step negotiation is preferred otherwise. By expression (3.5), it is easy to see that for c arbitrarily low, the negotiation will happen in two steps. The reason is that the total surplus to be split between buyer and seller is large, making the range of possible ex-ante payments wide and, as a consequence, very salient. It follows that the buyer is willing to accept a higher price provided that he receives an ex-ante transfer from the seller. If instead c is large (i.e. close to v), $\bar{m}$ is close to zero, and the players agree to negotiate in one step. The reason is that the total surplus to be split is small, which makes the ex-ante payment m less salient than the ex-post payment p. Hence, the buyer is willing to make an ex-ante payment in order to receive a lower price ex post.

Lemma 2. *If monetary transfers can be used, the two parties bargain in one step if c is large, and bargain in two steps if c is small. If they decide to negotiate in one step, in period 0 the buyer makes a positive monetary transfer to the seller. If they decide to negotiate in two steps, in period 0 the seller makes a positive monetary transfer to the buyer.*

Proof. In the text, and by noting that (3.5) is continuous in c. □

Note that, compared to the one-step bargaining case (and the rational-buyer case), the seller can extract a higher overall payment from the buyer whenever the two parties are given the option to decide on the negotiation structure to adopt.

3.4 Upper Bound on Price announced by the Seller

As we just showed, the existence of incomplete agreements that restrict the bargaining set may affect the final bargaining outcome and the player's payoffs. In the example just discussed, these incomplete agreements are the outcome of a previous negotiation stage. However, players may be able to affect the shape of the bargaining set also by unilaterally taking certain pre-bargaining actions.

In this section, we assume that negotiations always happen in one step, but the seller can credibly announce a maximum price $\hat{p}$.[21] Without much repetition, it is easy to see that the final price depends on $\hat{p}$ as described in equation (3.1), and that the seller can extract the highest price from the buyer by announcing a maximum price equal to $\hat{p} = \hat{p}^{\star}$. Therefore, the final transaction price is the same, independently of whether the maximum price is chosen by the seller or it is bargained upon by buyer and seller. Clearly, if the maximum price was announced by the buyer—and assuming that this announcement cannot be violated—the buyer will choose a maximum price equal to c. Nevertheless, when the seller and period-1 buyer are choosing over a maximum price, their preferences are strongly aligned because allowing the buyer to influence the choice of the maximum price does not affect the result. The reason is that, when bargaining over the maximum price, period-1 buyer has one main objective: to avoid changes in her own preferences (see Section 2 for an explanation). At the same time, the seller's objective is to maximize profits. In this example, these two objectives happen to be perfectly aligned, as preferences of period-1 buyer and period-2 buyer coincide at $\hat{p} = \hat{p}^{\star}$, which is also the maximum price generating the highest profits to the seller.

4 Example 2: Focusing Effect and the Negotiation over Price and Quality

Consider now a second example, in which the buyer and the seller negotiate over the quality of the good and the price to pay. Utilities are:

$$U^b(q, p) = h(\bar{q} - \underline{q})q - h(\bar{p} - \underline{p})p \tag{4.1}$$

$$U^s(q, p) = p - \frac{1}{2}q^2 \tag{4.2}$$

for $q \geq 0$ and $p \geq 0$, so that the set of unconstrained bargaining outcomes is $X = \mathbb{R}^2_+$, with $\{0, 0\}$ corresponding to no trade. We maintain Assumptions 2, 3 and 4. Among other things, these assumptions imply that, also here, $\underline{p} = \underline{q} = 0$.

Similarly to the previous example, we will derive the solution to the negotiation game under different negotiation structures. However, here the issue of whether—and to what extent—incomplete agreements are binding is particularly important. For example, suppose that a previous incomplete agreement imposes an upper bound on the transaction price. It is possible that, in period 2, both players are willing to ignore this upper bound provided that the transaction quality is sufficiently high. In other words, it is possible that an incomplete agreement may be renegotiated. In addition, in case of contradicting agreements among the same parties, courts typically enforce the most recent

[21] Alternatively, the seller announces a price with the understanding that this price can be negotiated downward. This is usually the case when selling, for example, houses.

one. Hence, in period 2 the players could agree to violate a period-1 agreement without any type of repercussion.

In this section, we assume that previous agreements can be renegotiated or even ignored. Crucially, we assume that ignoring a previous incomplete agreement is possible only by waiting one period (at no cost) and bargaining over the entire bargaining set in a third period. Hence, incomplete agreements are not binding in the sense that they do not affect the set of possible bargaining outcomes achievable within the negotiation. However, incomplete agreements affect the set of possible bargaining outcomes achievable in period 2 of the negotiation. The following assumption formalizes this intuition and replaces Assumption 1.

Assumption 5 (Renegotiation)**.** *For given S, a bargaining outcome $x \equiv \{q, p\} \in X$ is feasible if and only if at least one of the following conditions holds:*

- $x \in S \cup \{0, 0\}$,
- $U^b(x) \geq U^b(x')$ *and* $U^s(x) \geq U^s(x')$ *for some* x' *element of the Pareto frontier of* S,
- x *is the solution of the bargaining problem over the entire* X.

As earlier, in period 2, players can choose a bargaining solution that is in compliance with period-1 agreement or disagree and decide not to trade. Novel to Assumption 5, now the players can also renegotiate over period-1 agreement by implementing a bargaining outcome that dominates the Pareto frontier of S, or ignore completely the period-1 agreement and go to a fresh round of negotiation. This last possibility amounts renegotiating the actual structure of the negotiation, and move to the one-step negotiation even if the players are currently engaged in a two-step negotiation. The possibility of ignoring the period-1 agreement implies that, from period-2 point of view, one of the outcomes of the negotiation is the solution to the bargaining problem in case a third round of negotiations is triggered. In other words, independently of the period-1 incomplete agreement, the players can always choose in period 2 to implement the bargaining outcome x, the solution of the bargaining problem over the entire X.[22]

Note that in the example discussed in the previous section (the sale of a good of fixed quality), the possibility of renegotiating does not play any role because players have opposite preferences over the price, and all results derived there continue to hold under Assumption 5 as well. Instead, when price and quality are jointly determined renegotiation may happen. In the following, we restrict our attention to period-1 agreements that are closed, convex and renegotiation proof: for every $S \in X$, no element of X that is not in S can Pareto improve upon the Pareto frontier of S. We

[22] The fact that the third round of negotiation can be triggered only by mutual agreement is not essential to our results, but simplifies our derivations because the outside option of the players is always no trade. In case each player can trigger a third round of negotiation, the outside option when bargaining in period 2 is the outcome of the one-step negotiation. In Section A.2 we consider this problem, and show that incomplete agreements may nonetheless be used in equilibrium.

will characterize the set of renegotiation-proof incomplete agreements, and show that they may involve establishing minimum utility guarantees to the buyer or seller. In addition, we show that the structure of the negotiation here is relevant for material efficiency, as it affects the quality of the good exchanged.

4.1 One-Step Negotiation

In absence of a previous incomplete agreement, the consideration set is given by the $\{q, p\}$ that satisfy the players' rationality constraints. Therefore, the upper bound of the consideration set is given by the $\{q, p\}$ that satisfies the two rationality constraints with equality:

$$h(\bar{q})\bar{q} = h(\bar{p})\bar{p} \tag{4.3}$$

$$\bar{q}^2 = 2\bar{p}, \tag{4.4}$$

which implies $\bar{q} = \bar{p} = 2$. Given the identical focus weights, also here, the one-shot bargaining outcome is equivalent to the outcome when the buyer is rational: $p^\star = \frac{3}{4}$, $q^\star = 1$.

4.2 Two-Step Negotiation

Consider a closed, convex incomplete agreement S, such that all elements of S satisfy the players' rationality constraints.[23] There exist a $\underline{u}^b \geq 0$, $\underline{u}^s \geq 0$, $\hat{p} \geq 0$, and a $\hat{q} \geq 0$ such that $\forall \{p, q\} \in S$

$$h(\bar{q})q - h(\bar{p})p \geq \underline{u}^b \tag{4.5}$$

$$p - \frac{1}{2}q^2 \geq \underline{u}^s \tag{4.6}$$

$$\hat{p} = \max\{p | \{p, q\} \in S \text{ for some } q\} \tag{4.7}$$

$$\hat{q} = \max\{q | \{p, q\} \in S \text{ for some } p\} \tag{4.8}$$

where (4.5) and (4.6) hold with equality for some $\{p, q\} \in S$. Assuming that S will not be renegotiated, the upper bounds of the consideration set are $\bar{q} = \max\{\hat{q}, 1\}$ and $\bar{p} = \max\{\hat{p}, \frac{3}{4}\}$, where we use the fact that the solution to the one-step negotiation can always be reached by waiting one period.

Remember that a given S can be renegotiated if a bargaining outcome on the Pareto frontier of S can be renegotiated, independently on whether this outcome is chosen in equilibrium. It follows that an S satisfying (4.5) to (4.8) will not be renegotiated if the $\{p, q\}$ on the Pareto frontier of the unconstrained bargaining set $X = \mathbb{R}^2_+$ giving at least utility $\underline{u}^s$ to the seller and utility $\underline{u}^b$ to the

[23] Considering only this class of incomplete agreements is without loss of generality, because the elements of S that do not satisfy one of the players rationality constraint are not part of period-2 consideration set and are therefore irrelevant.

buyer are all elements of S. More formally, the agreement S will not be renegotiated if all $\{p', q'\}$ such that

$$\frac{h(\max\{\hat{p}, \frac{3}{4}\})}{h(\max\{\hat{q}, 1\})} = \frac{1}{q'} \tag{4.9}$$

$$u^s = p' - \frac{q'^2}{2} \tag{4.10}$$

for

$$\underline{u}^s \leq u^s \leq \frac{h(\max\{\hat{q}, 1\})q'^2 - \underline{u}^b}{h(\max\{\hat{p}, \frac{3}{4}\})} - \frac{q'^2}{2} \tag{4.11}$$

are elements of S. In other words, a renegotiation-proof S contains the tangency points between indifference curves of buyer and seller (pinned down by equations (4.9) and (4.10) as a function of u^s), giving utility greater or equal than $\underline{u}^s$ to the seller and $\underline{u}^b$ to the buyer (expressed in equation (4.11) as a range of possible u^s). Figure 4.1(a) represents a renegotiation-proof agreement, because the Pareto optimal elements of S (the tangency points) are also Pareto optimal within $\mathbb{R}^2_+$. On the other hand, Figure 4.1(b) represents a period-1 agreement that can be renegotiated. The set of Pareto optimal elements of S includes some $\{p, q\}$ on the top-right corner of S (for example, the point of tangency between the set S and the indifference curve corresponding to zero utility to the seller). These elements are Pareto dominated by the points in bold on the line corresponding to $q = \frac{h(\max\{\hat{q},1\})}{h(\max\{\hat{p},\frac{3}{4}\})}$, which are not elements of S.

Conditions (4.9), (4.10), and (4.11) can be used to derive necessary conditions for a S to be renegotiation proof.

Lemma 3. *If a convex S is renegotiation proof, then the following conditions must hold:*

$$\hat{p} + \frac{\underline{u}^b}{h(\max\{\hat{p}, \frac{3}{4}\})} \geq \left(\frac{h(\max\{\hat{q}, 1\})}{h(\max\{\hat{p}, \frac{3}{4}\})}\right)^2 \tag{4.12}$$

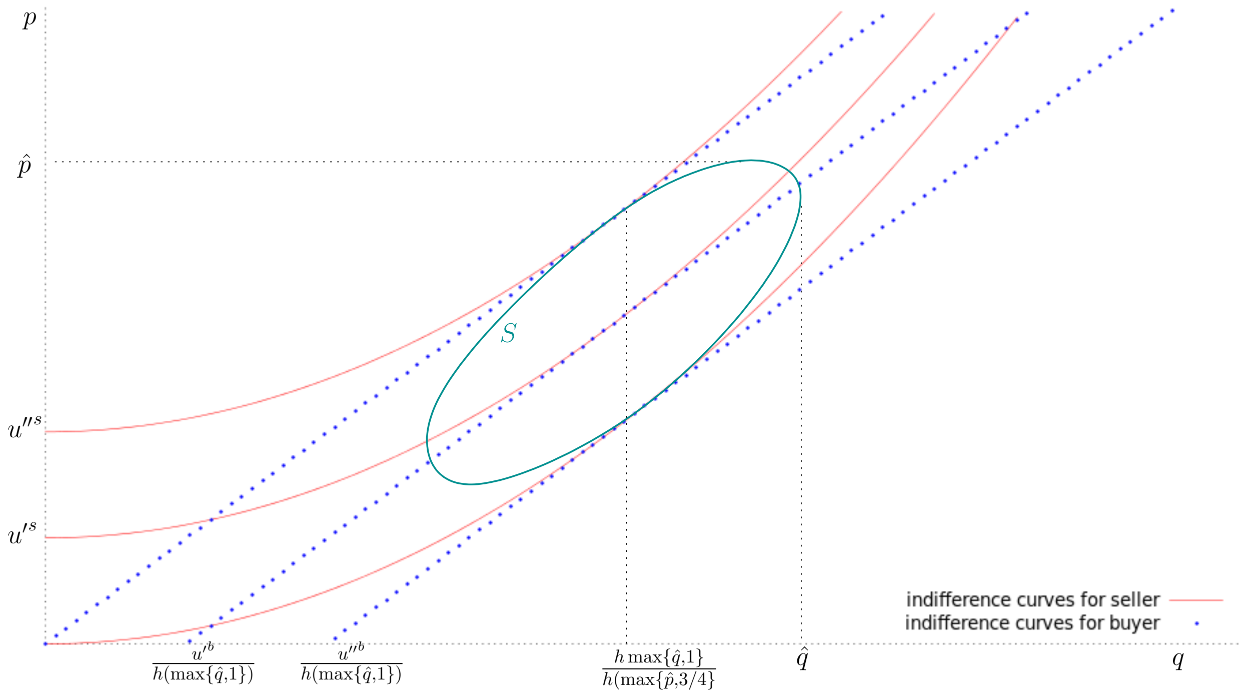
$$\hat{q} \geq \frac{h(\max\{\hat{q}, 1\})}{h(\max\{\hat{p}, \frac{3}{4}\})} \tag{4.13}$$

$$\hat{p} + \frac{\underline{u}^b}{h(\max\{\hat{p}, \frac{3}{4}\})} \leq \frac{h(\max\{\hat{q}, 1\})}{h(\max\{\hat{p}, \frac{3}{4}\})}\hat{q} \tag{4.14}$$

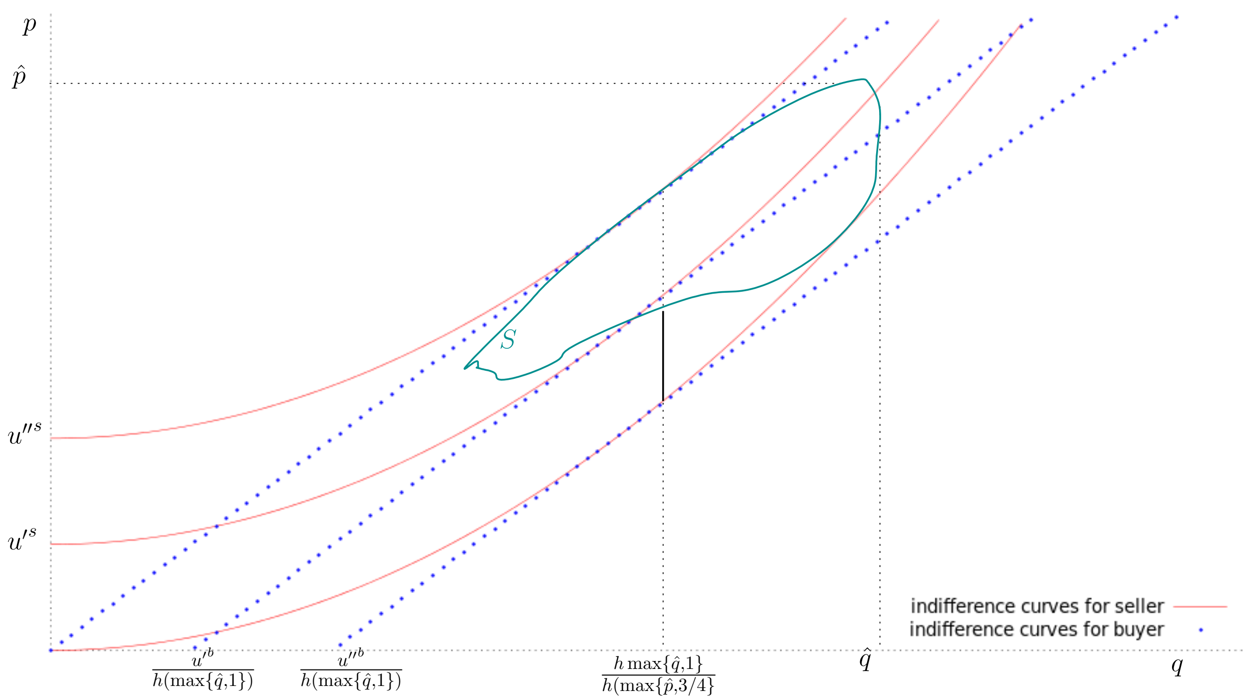
$$\hat{p} - \frac{1}{2}\hat{q}^2 \geq \underline{u}^s \tag{4.15}$$

In the above lemma, the first two conditions state that $\hat{p}, \hat{q}$ should be larger than the largest $\{p, q\}$ on the Pareto frontier of the set S. The last two conditions are the rationality constraints of the players at $\{\hat{p}, \hat{q}\}$.

Lemma 4. *For every $\underline{u}^b$, $\underline{u}^s$, $\hat{p}$, $\hat{q}$ that satisfy conditions* (4.12), (4.13), (4.14), *and* (4.15), *there exist a renegotiation-proof S satisfying conditions* (4.5) *to* (4.8).



(a) Renegotiation-proof period-1 agreement.



(b) Non renegotiation-proof period-1 agreement.

Fig. 4.1: Period-1 agreements.

Proof. For example, define S as the set of $x \in \mathbb{R}^2$ that satisfy equations (4.5) to (4.8), with $\{p, q\} \leq \{\hat{p}, \hat{q}\}$ for all $\{p, q\} \in S$. $\square$

It follows that conditions (4.12), (4.13), (4.14), and (4.15) are also sufficient to characterize some renegotiation-proof S.

Finally, note that starting from any closed and convex S satisfying conditions (4.5), (4.6), (4.7) and (4.8) (i.e. a renegotiation-proof S) for the same $\underline{u}^b$, $\underline{u}^s$, $\hat{p}$, $\hat{q}$, buyer and seller will reach the same period-2 solution. The reason is that the buyer's preferences and the set of possible solutions (the Pareto frontier of the set S) are the same across all renegotiation-proof, closed and convex S having the same $\underline{u}^b$, $\underline{u}^s$, $\hat{p}$, $\hat{q}$. Therefore, in order to characterize the set of possible renegotiation-proof, closed and convex incomplete agreements, we only need to characterize the set of $\underline{u}^b$, $\underline{u}^s$, $\hat{p}$, $\hat{q}$ that satisfy conditions (4.12), (4.13), (4.14), and (4.15). The proposition follows.

Proposition 2. *The set of $\underline{u}^b$, $\underline{u}^s$, $\hat{p}$, $\hat{q}$ that satisfy conditions* (4.12), (4.13), (4.14), *and* (4.15), *fully characterizes the set of final bargaining outcomes achievable via a renegotiation-proof, convex incomplete agreement.*

Proof. In the text. $\square$

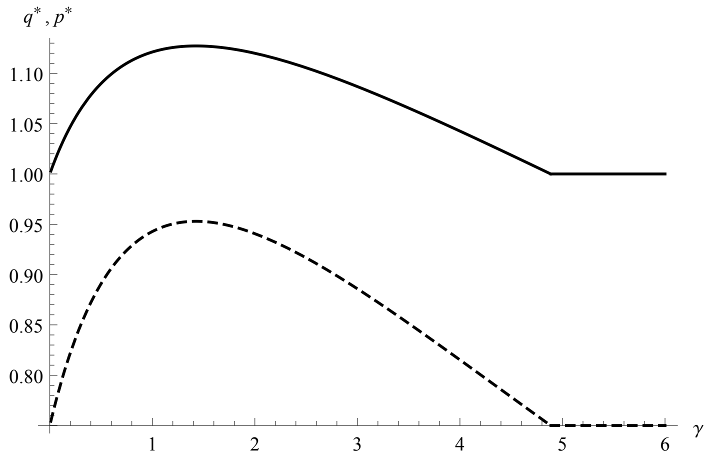
We derive the solution to the two-step negotiation game under three negotiation structures. First, we assume that, in period 1, the players cannot use minimum utility guarantees, so that the set of possible period-1 agreements is the set of renegotiation-proof agreements with $\underline{u}^b = \underline{u}^s = 0$. Second, we allow the players to bargain also over the minimum utility guaranteed to the buyer. Finally, we allow the players to choose in period 1 any S that is renegotiation-proof and convex. In order to simplify our derivations, we make the following functional-form assumption:

Assumption 6. *The focusing function has an exponential form: $h(x) = x^\gamma$ for some $\gamma > 0$.*

The main point of this section is to show that the structure of the negotiation is relevant for material efficiency, because the type of good exchanged depends not only on whether the negotiation happens in one step or two steps, but also on the set of incomplete agreements can be signed in period 1. Similarly to the case of a single decision maker discussed in Section 2, also here the buyer's period-1 preferences depend on the set of possible incomplete agreements that can be signed. Therefore, different two-step negotiation structures will lead to different negotiation outcomes.

No minimum utility guarantees

We start by assuming that the players cannot use minimum utility guarantees, and therefore bargain in period 1 over $\hat{p}$ and $\hat{q}$, under the constraints (4.12) to (4.15). The equilibrium bargaining outcome for this negotiation structure will provide the benchmark against which to compare the effect of introducing minimum-utilities guarantees.



$q^\star(\gamma)$ (solid line), $p^\star(\gamma)$ (dashed line) as a function of γ.

Fig. 4.2: No Minimum Utility: Period–2 Quality and Price

Lemma 5. *Suppose that, in period-1, the players can sign only renegotiation-proof agreements with* $\underline{u}^s = \underline{u}^b = 0$. *The solution to the bargaining problem is*

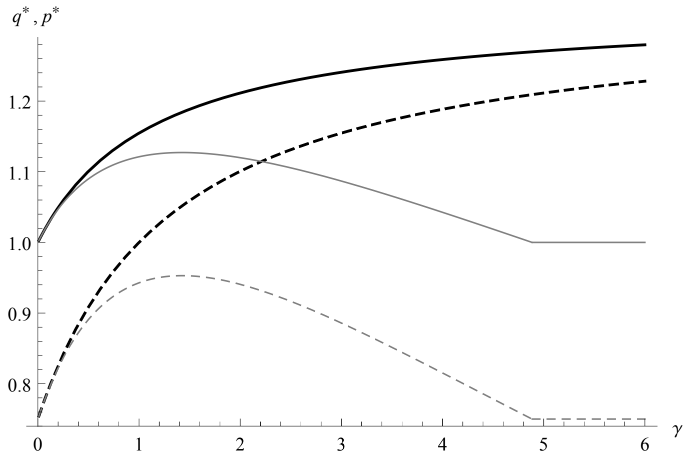
$$q^\star = \max\left\{\left(\frac{2^{\frac{3\gamma+4}{2(\gamma+1)}}}{3}\right)^\gamma, 1\right\}$$

$$p^\star = \frac{3}{4}(q^\star)^2.$$

When the parties are restricted to bargain over $\hat{p}$ and $\hat{q}$ (no minimum utilities), for γ sufficiently low, bargaining in two periods will lead to a bargaining outcome that is different from the bargaining outcome reached in the one-shot case. This result is illustrated in Figure 4.2. Therefore, for γ low, the structure of the negotiation is relevant for material efficiency, because it affects the quality of the item exchanged.

The way the parameter γ affects the outcome of the bargaining is quite complex. It is possible to show that the largest period-2 $\frac{h(\max\{\hat{q},1\})}{h(\max\{\hat{p},\frac{3}{4}\})}$ achievable via a period-1 agreement is increasing in γ. In other words, the stronger the focusing effect, the more quality sensitive the buyer can be made in period-2 via an appropriate period-1 agreement. In addition, when bargaining in period 2, final quality and final price increase with $\frac{h(\max\{\hat{q},1\})}{h(\max\{\hat{p},\frac{3}{4}\})}$. The key observation is that for low period-2 $\frac{h(\max\{\hat{q},1\})}{h(\max\{\hat{p},\frac{3}{4}\})}$ the transaction quality will be above the transaction price, while for large period-2 $\frac{h(\max\{\hat{q},1\})}{h(\max\{\hat{p},\frac{3}{4}\})}$ the transaction price will be above the transaction quality.

It follows that, from period-1 point of view, when the focusing effect is very strong, the largest final price achievable by means of a period-1 agreement is above the corresponding largest transaction quality. Hence, in period 1, the buyer is *more* price sensitive than in the one-shot case. Because



$q^\star(\gamma)$ (black solid line), $p^\star(\gamma)$ (black dashed line) as a function of the focusing intensity γ. Gray lines represent the corresponding equilibrium outcomes under no minimum utility.

Fig. 4.3: Minimum Utility to the Buyer: Period–2 Quality and Price

the smallest quality which is achievable via a renegotiation-proof agreement is equal to $q = 1$, the players agree on this quality. When the focusing effect is not strong, the opposite is true and, in period 1, the buyer is less price sensitive than in the one-shot case. As a consequences, the players agree on $\{q, p\}$ that are larger than the one-step case.

Intuitively, when the focusing effect is strong, the buyer anticipates that he may accept prices above quality in the future, and this expectation makes him very price sensitive in period 1. When instead the focusing effect is not very strong, the buyer knows that he won't accept prices above quality in period 2, and therefore is less price sensitive in period 1. Interestingly, when the behavioral distortion is sufficiently severe, the solution is identical to the case of a rational agent.

Minimum utility to the buyer

Lemma 6. *Suppose that, in period-1, the players can sign any renegotiation-proof agreement with* $\underline{u}^b \geq 0$, $\underline{u}^s = 0$. *The solution to the bargaining problem is*

$$q^* = \left(\frac{4}{3}\right)^{\frac{\gamma}{\gamma+1}} \tag{4.16}$$

$$p^* = \left(\frac{4}{3}\right)^{\frac{\gamma-1}{\gamma+1}}. \tag{4.17}$$

By comparing the result of Lemma 5 and 6, we see that the equilibrium price and quality are greater when the players bargain also over $\underline{u}^b$ compared to when the players bargain only over $\hat{p}$

and $\hat{q}$. This result is illustrated in Figure 4.3. Compared with Lemma 5, here if a very large utility level is promised to the buyer in period 1, achieving this utility level in period 2 implies transacting at very high quality level and very low price. From period 1 point of view, the possibility of such a transaction makes the buyer particularly focused on the quality dimension rather than on the price dimension.

Interestingly, in period 1, the parties agree on $\underline{u}^b = 0$: on the equilibrium path there is no minimum-utility guaranteed to the buyer. However, because by manipulating $\underline{u}^b$ very high quality and very low prices can be achieved in period 2, the fact that players could impose a minimum utility guarantee makes the buyer more quality sensitive, and willing to accept a greater $q^\star$ and $p^\star$. This leads to higher profit to seller compared to the solution in Lemma 5.

Minimum utility for buyer and seller

Lemma 7. *Suppose that, in period 1, the players can sign any renegotiation-proof agreement with $\underline{u}^b \geq 0$, $\underline{u}^s \geq 0$. There exist a $\overline{\gamma}$ such that*

- *for $\gamma > \overline{\gamma}$ the solution to the bargaining problem is*

$$q^* = \left(\frac{4}{3}\right)^{\frac{\gamma}{\gamma+1}} \tag{4.18}$$

$$p^* = \left(\frac{4}{3}\right)^{\frac{\gamma-1}{\gamma+1}}. \tag{4.19}$$

which is the same solution derived in Lemma 6.

- *for $\gamma < \overline{\gamma}$ the solution to the bargaining problem is*

$$q^* < \left(\frac{4}{3}\right)^{\frac{\gamma}{\gamma+1}} \tag{4.20}$$

$$p^* = \frac{3}{4}(q^\star)^2. \tag{4.21}$$

Imposing a minimum utility guarantee to the seller has two effects on period-2 prices. For given transaction quality, when the minimum utility guarantee to the seller is binding the final transaction price is increasing with $\underline{u}^s$. As a consequence, the possibility of imposing a minimum utility guarantee in period 1 makes the buyer more price sensitive, which tends to decrease the transaction price and transaction quality.

The previous lemma shows that for γ large, the second effect dominates: increasing $\underline{u}^s$ above zero lowers the transaction price in period 2, so that the highest period-2 transaction price and quality are achieved at $\underline{u}^s = 0$. As a consequence, when γ is large, in period 1, the buyer is as price sensitive in case $\underline{u}^s \geq 0$ as in case $\underline{u}^s = 0$. Instead, when γ is small, the second effect dominates:

by setting $\underline{u}^s$ positive, it is possible to achieve some large period-2 prices that are not achievable when $\underline{u}^s = 0$ (period-2 transaction quality is independent of $\underline{u}^s$), which implies that in period 1, the buyer is particularly price sensitive when $\underline{u}^s > 0$ is possible.

Overall, when the parties bargain over a minimum utility guarantee to the seller, seller's profits are weakly lower compared to the case in which only a minimum utility guarantee to the buyer is discussed. Hence, the seller is better off by bargaining over a minimum utility guarantee to the buyer but avoiding discussing over a minimum utility to the seller. Finally, we show in the proof that, also here, the minimum utilities are relevant off-equilibrium but, in equilibrium, the players will set $\underline{u}^b = \underline{u}^s = 0$.

4.3 Which Negotiation Structure?

Also here, we can introduce a period-0 when the two players bargain over the negotiation structure to adopt. Assume that the two players can choose between:

- negotiating in one period, leading to a transaction quality $q_1^\star = 1$.
- negotiating in two periods choosing any renegotiation-proof agreement with $\underline{u}^s = \underline{u}^b = 0$ in period 1, leading to a transaction quality $q_2^\star = \max\left\{\left(\frac{2^{\frac{3\gamma+4}{2(\gamma+1)}}}{3}\right)^\gamma, 1\right\}$,
- negotiating in two periods choosing any renegotiation-proof agreement with $\underline{u}^s = 0$ and $\underline{u}^b \geq 0$ in period 1, leading to a transaction quality $q_3^\star = \left(\frac{4}{3}\right)^{\frac{\gamma}{\gamma+1}}$.
- negotiating in two periods choosing any renegotiation-proof agreement, leading to a transaction quality $q_4^\star \leq q_3^\star$

with corresponding prices given by $p^\star = \frac{3}{4}(q^\star)^2$.

Again, we assume that in case of disagreement in period 0 the two parties will not transact, and that monetary payments are possible. The buyer's utility function is

$$U^b(q, p, m) = h\left(q_3^\star\right) q - h\left(\frac{3}{4}\left(q_3^\star\right)^2\right) p + h(\bar{m} - \underline{m})m,$$

where we used the fact that the largest transaction quality and price are, respectively, $q_3^\star$ and $\frac{3}{4}\left(q_3^\star\right)^2$, which therefore determine the focus weights on price and quality.

The lower bound on m is given by the largest transfer the seller is willing to make to the buyer, i.e.

$$-\underline{m} = \max_{q \in \{q_1^\star, q_2^\star, q_3^\star, q_4^\star\}} \left[\frac{3}{4}q^2 - \frac{1}{2}q^2\right] = \frac{1}{4}\left(q_3^\star\right)^2 \tag{4.22}$$

and the upper bound on m is given by the largest transfer the buyer is willing to make to the seller, i.e.

$$\bar{m}: \; h\left(\bar{m} + \frac{1}{4}\left(q_3^\star\right)^2\right)\bar{m} = \max_{q\in\{q_1^\star,q_2^\star,q_3^\star,q_4^\star\}}\left[h(q_3^\star)q - h\left(\frac{3}{4}\left(q_3^\star\right)^2\right)\left(\frac{3}{4}q^2\right)\right]. \tag{4.23}$$

Following the same argument made in the previous example, the negotiation structure agreed upon by buyer and seller solves:

$$\max_{q\in\{q_1^\star,q_2^\star,q_3^\star,q_4^\star\}}\left(\frac{h\left(q_3^\star\right)}{h\left(\bar{m} + \frac{1}{4}\left(q_3^\star\right)^2\right)}q - \frac{h\left(\frac{3}{4}\left(q_3^\star\right)^2\right)}{h\left(\bar{m} + \frac{1}{4}\left(q_3^\star\right)^2\right)}\frac{3}{4}q^2 + \frac{1}{4}q^2\right). \tag{4.24}$$

Solving numerically the above problem reveals that for γ sufficiently large buyer and seller choose to negotiate in two steps. The reason is that the seller's profits in the two step negotiations are increasing in γ (see Figure 4.4). Hence, from period 0 point of view, as γ increases the range of possible ex-ante payments m expands, and with it its salience. Similarly to the previous example, as the salience of ex-ante payments increases, the parties becomes more likely to agree to negotiate in two steps while compensating the buyer with a transfer in period 0.

4.4 Endogenous Hold up

We previously argued that introducing the focusing effect into a model of bargaining expands the set of possible pre-bargaining actions players may take. The same is true in this example as well, and we illustrate this fact by allowing the seller to enter the negotiation having already invested in quality.

Suppose that the players can only bargain in one step. However, the seller can decide to bargain having already set the quality and announced a maximum price, or to bargain over price and quality simultaneously.[24] In this section, we show that the seller may prefer to be held up: by fixing the quality in advance and bargaining from a worse position, the seller is able to manipulate the buyer's focus weights and may earn higher profits.

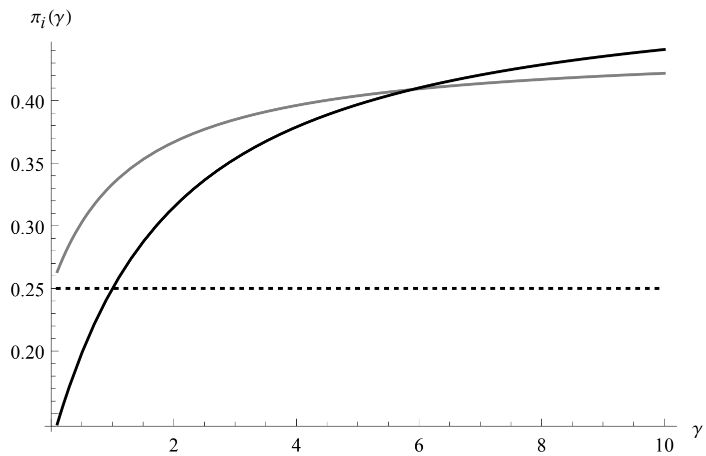
Assume that, in period 2, quality q has already been decided, and a maximum transaction price $\hat{p} \leq q$ has been established. By the Nash bargaining solution, the final transaction price is

$$p^\star = \max\left\{\left(\frac{q}{\hat{p}}\right)^\gamma \frac{q}{2}, \hat{p}\right\},$$

which, as a function of q, reaches its maximum at

$$p^\star = \hat{p} = \frac{q}{2^{\frac{1}{1+\gamma}}}.$$

[24] This example corresponds to, for example, a construction company that can either build a house, announce a price (up to negotiation) and bargain with the buyer over the final price, or bargain with the buyer before the house is build.



$\pi_i(\gamma)$: two-step bargaining (gray line), hold up (black line) and one-shot bargaining (dotted line) as a function of the focusing intensity γ.

Fig. 4.4: Profits as a Function of the Focusing Intensity

When choosing q and $\hat{p}$ in period 1, the seller solves

$$\max_q \left\{ \frac{q}{2^{\frac{1}{1+\gamma}}} - \frac{1}{2}q^2 \right\},$$

which is maximized at $q = 2^{-\frac{1}{1+\gamma}}$, yielding profits equal to $\pi = 2^{-\frac{3+\gamma}{1+\gamma}}$.

Given the solution to the one-step negotiation derived in Section 4.1, we can compute profits in case price and quality are jointly determined, which are equal to 1/4. Simple algebra shows that whenever $\gamma > 1$ the seller chooses to fix quality before bargaining.

Lemma 8. *Whenever $\gamma > 1$, the seller prefers to be held up rather than to negotiate in one period.*

Proof. In the text. □

Hence, when the focusing effect is particularly strong, the possibility of manipulating the buyer's focus weights outweighs the cost of being held up.

It is also possible to compare profits under hold up with profits when the players negotiate in two steps. Among the cases considered above, the seller achieves the maximum profits of $2^{\frac{2(\gamma-1)}{\gamma+1}} 3^{\frac{-2\gamma}{\gamma+1}}$ when in period 1 the players can sign any renegotiation-proof agreement with $\underline{u}^b \geq 0$, $\underline{u}^s = 0$. Simple algebra shows that, again, for γ sufficiently large the seller prefers to be held up rather than bargaining in two steps. Negotiating in two steps allows the seller to manipulate the focus weight of the buyer, but only under a no-renegotiation constraint. By fixing the quality in advance instead the seller can manipulate the focus weights of the buyer without constraints. Depending on the strength of the focusing effect, the seller may prefer to be held up. Figure 4.4 illustrates how profits under different negotiation structures vary with the strength of the focusing effect.

5 General bargaining problem with context-dependent preferences

In this section, we consider a general bargaining problem with context-dependent preferences, in which both players' evaluations of a bargaining outcome depend on the consideration set of the bargaining problem. We show that, under some mild conditions, having context-dependent preferences is a sufficient condition for the use of incomplete agreements in equilibrium. Therefore, our main result is robust both to the use of models of context-dependent preferences different from Kőszegi and Szeidl (2013), and to the introduction of a focused seller.[25] Because we leave preferences completely unspecified, we abstract away from any issue related to the existence and uniqueness of the bargaining solution within each bargaining round.

Assume that two players b and s bargain over an outcome $x \in C$, where C is the bargaining set, i.e. the set of feasible bargaining outcomes. Both players have the same outside option normalized to zero, and have context-dependent preferences of the form $U^b(x, \hat{C})$, $U^s(x, \hat{C})$, where $\hat{C}$ is the players' consideration set given the bargaining set C, defined as

$$\hat{C} = \left\{ x \in C | \ U^b(x, \hat{C}) \geq 0, \ U^s(x, \hat{C}) \geq 0 \right\}. \tag{5.1}$$

We assume that the consideration set $\hat{C}$ always exists and that it is always unique for any bargaining set C. The solution to the bargaining problem over C is therefore:

$$x(C) = \text{argmax}_{x \in C} U^b(x, \hat{C}) U^s(x, \hat{C})$$

which we also assume to exist and to be unique. Finally, note that two bargaining sets C and C' will give rise to the same consideration set $\hat{C}$ whenever C and C' differ at most by some bargaining outcome which violates one of the players' rationality constraint (for preferences given by $\hat{C}$). It follows that two bargaining sets giving rise to the same consideration set will also deliver the same bargaining solution.

Suppose now that the two players are given the option to bargain in two steps: first, choose a set $S \in \mathbb{S}$ with $S \subseteq C$; then, choose a bargaining outcome. In case of disagreement during either period 1 or period 2, the outcome is no agreement. We assume that period-1 agreements are binding, so that the bargaining set in period 2 coincides with the set S chosen in period 1. In addition, we assume that $C \in \mathbb{S}$, so that the players can always decide not to restrict their bargaining set and bargain over the entire C in period 2. Call the set of anticipated bargaining outcomes $\mathbb{C}$, defined as:

$$\mathbb{C} \equiv \{x(S) | S \in \mathbb{S}\}.$$

[25] Note that when each of the bargaining parties is a group or committee, context dependence may emerge due to the way preferences are aggregated (this would be the case if the group evaluates each option using, for example, Borda count).

The set $\mathbb{C}$ represents the bargaining set in period 1, which determines the period-1 consideration set as defined by equation (5.1). Under two-step negotiation, we will refer to the period-1 consideration set as $\hat{\mathbb{C}}$. It follows that the period-1 bargaining problem has a solution given by:

$$x(\mathbb{C}) = \text{argmax}_{x \in \mathbb{C}} U^b(x, \hat{\mathbb{C}}) U^s(x, \hat{\mathbb{C}}).$$

The goal of this section is to establish under what conditions $x(\mathbb{C}) \neq x(C)$, so that in period 1 the two players choose to restrict their future bargaining set.

First, note that whenever $C = \mathbb{C}$ the bargaining set in period 1 is equal to the unconstrained bargaining set, and the period-1 problem is the same as the unconstrained problem. It follows that $C \neq \mathbb{C}$ is a necessary condition for $x(\mathbb{C}) \neq x(C)$, which holds whenever there are some bargaining outcomes in C that cannot be achieved as a solution to the bargaining problem under any $S \in \mathbb{S}$.[26] Second, whenever $\hat{\mathbb{C}} = \hat{C}$, again, the solutions to the one-step and two-step bargaining problem are the same. Hence, a necessary condition for $x(\mathbb{C}) \neq x(C)$ is that C and $\mathbb{C}$ differ in some bargaining outcome delivering positive utility to both players. Third, the fact that $\hat{\mathbb{C}} \neq \hat{C}$ does not guarantee that the players' preferences will be different under the two consideration sets. This will depend on the specific form of context-dependent preferences.

Hence, we assume here that $\hat{\mathbb{C}} \neq \hat{C}$ and that the players' preferences under $\hat{\mathbb{C}}$ are different than under $\hat{C}$. In this case, if $\mathbb{C}$ itself is available in period 1 (i.e. $\mathbb{C} \in \mathbb{S}$) and is chosen, period-2 players' context (and preferences) will be identical to period-1 players' context (and preferences). Hence, by choosing $\mathbb{C}$ in period 1, the period-2 problem becomes identical to the period-1 problem, and whatever bargaining outcome is chosen in period 2 also solves the period-1 bargaining problem. By revealed preferences it follows that, in period 1, if $\mathbb{C}$ is available, it will be chosen over all other available options (including the unconstrained bargaining set C). In such cases, an incomplete agreement will be used in equilibrium to shrink the bargaining set from C to $\mathbb{C}$.[27]

Finally, note that, similarly to what was discussed in our second example, the set of incomplete agreements which can be signed in period 1, $\mathbb{S}$, affects the players' preferences in period 1, the choice of incomplete agreements and the final bargaining outcome. Hence, even if for some $\mathbb{S}$ the players will not want to progressively restrict their bargaining set, for some other $\mathbb{S}'$ they may decide to do so.

[26] Going back to our first example, if buyer and seller can bargain over both a maximum and a minimum price in period 1, then they can effectively choose the transaction price for period 2. It follows that $C = \mathbb{C}$, and a two-step negotiation when players discuss in period 1 about a maximum and minimum price is equivalent to a one-step negotiation. However, the question remains as to whether such a bargaining structure will be chosen in period 0, or instead the players will decide to discuss only about a maximum price.

[27] See page 11 in Section 2 for a similar argument applied to a choice problem.

6 Discussion

In this section, we present some robustness checks of our results. First we show that our results cannot be replicated in a bargaining game with rational agents, even considering different possible bargaining solutions. We then discuss the non-cooperative implementation of the bargaining game with focusing effect. Finally, we argue that, in general, the solution to the one-step negotiation may be different from the solution under rational preferences. Interpreting the rational benchmark as an efficient benchmark, this implies that, in some cases, the two-step negotiation may be more efficient than the one-step negotiation.

6.1 Rational players

Our main result is that, in equilibrium, players may choose to restrict their future bargaining possibilities via an incomplete agreement. Here we consider whether a similar result can be obtained with rational players under different bargaining solutions.

Consider a bargaining solution that satisfies independence of irrelevant alternatives. For given incomplete agreement S, in period 2 the bargaining solution solves:

$$\max_{x \in S} f\left(U^b(x), U^b(x)\right).$$

Because the players' preferences do not change across period, in period 1 the bargaining solution solves

$$\max_{S \in \mathbb{S}} \left[\max_{x \in S} f\left(U^b(x), U^b(x)\right)\right],$$

which is maximized for the largest possible S.

Lemma 9. *Suppose both players are rational. Suppose, furthermore, that the players have the option to restrict their bargaining set by signing an incomplete agreement. For any bargaining solution satisfying independence of irrelevant alternatives, the players cannot do better than bargaining in one period.*

Proof. In the text. □

In case the bargaining solution does not satisfy the independence of irrelevant alternatives, for a given incomplete agreement S, the solution to the bargaining problem is:

$$\max_{x \in S} f\left(U^b(x), U^b(x), S\right),$$

which may depend on previous agreements through S. However if players are rational, bargaining over S becomes equivalent to bargaining over the final bargaining outcome, and no player has an incentive to sign an incomplete agreement.

Lemma 10. *Suppose both players are rational, have utility linear in money, and monetary payments are possible (i.e. the players can freely exchange cash unless a previous agreement restrict its use). In period 0, the players cannot do better than agreeing to bargain in one period.*

Note that when the bargaining solution does not satisfy the independence of irrelevant alternatives, the bargaining strength of a player depends both on her outside option and on the shape of the bargaining set. The above lemma says that when both players have equal bargaining power in the sense that the bargaining frontier is linear, then incomplete agreements will not be used in equilibrium.

6.2 Non-cooperative bargaining game

In the body of the paper we solved each bargaining round using the Nash bargaining solution. This modeling choice allowed us to focus on the sequence of agreements rather than on how each agreement was reached. However, in this section we consider whether our results can be extended to a non-cooperative bargaining game.

Assume that the bargaining parties play Rubinstein (1982)'s game of alternating offers during each stage of the negotiation. Clearly, taking the player's preferences as given, the Rubinstein (1982) result applies: if the players are arbitrarily patient the only subgame-perfect equilibrium of the game is the Nash bargaining solution (i.e. equal split of surplus). Hence, the main question of interest is how to define the players' consideration set (and preferences) in the game of alternating offers.

In choice problems with context-dependent preferences, it is common to assume that the consideration set coincides with the choice set. Similarly, we think that also in a situation of strategic interaction, the players' consideration set should be defined as the set of outcomes which are possible but will not arise in equilibrium.[28] In the game of alternating offers, this implies that the consideration set of a player is given by:

1. The set of Nash equilibria of the game, which in Rubinstein (1982) game of alternating offers coincides with the Pareto frontier of the bargaining set.

2. All allocations that are achievable from a Nash equilibrium via a unilateral deviation of this player.

Similarly to a decision problem, whenever an outcome is achievable via a unilateral decision of a player, this outcome should be in his/her consideration set. This implies that, for example, the disagreement outcome is always in the consideration set of both players. Similarly, each player can always propose a bargaining solution to her opposer that is (weakly) better for the opposer

[28] To the best of our knowledge, we are the first one addressing the issue of how to define the consideration set in strategic situations.

compared to the equilibrium offer, and hence any such deviation should be in the consideration set. More interestingly, here we assume that the set of possible equilibria in the consideration set is the set of Nash equilibria of the game. Intuitively, among the set of possible equilibria only one is subgame perfect and will be played. However, the players know that they could coordinate on another equilibrium, and this awareness affects the players' preferences.

Finally, note that as long as the bargaining set is closed and convex, the set of bargaining outcomes that can be achieved starting from the Pareto frontier and making a unilateral concession to the other player coincides with the entire bargaining set. Hence, for both players the consideration set of the game of alternating offers coincides with the bargaining set, which is what we assumed in the body of the paper (Assumption 2).[29] As a consequence, all our results are robust to a non-cooperative implementation of the bargaining game.

6.3 When the two-step negotiation is materially efficient.

In both our examples, the outcome of the one-step negotiation coincides with the outcome under a rational buyer. We chose to construct our examples in this way to maintain the number of possible cases to the minimum. However, our choice had one important consequence, namely that the two-step negotiation always moves the outcome away from the rational benchmark.

However, this fact is specific to the cases considered, and is not a general feature of a bargaining problem with the focusing effect. For example, consider a bargaining problem over an object with two quality dimensions v_1 and v_2. The utility function of the buyer is now:

$$h(\bar{v}_1)v_1 + h(\bar{v}_2)v_2 - h(\bar{p})p,$$

where v_1 and v_2 can be interpreted as a basic good plus a possible add-on.[30] Assuming that v_1 and v_2 are exogenously given, we can solve for the focus weights in case of a one-step negotiation, which are given by:

$$h(\bar{p})p = h(v_1)v_1 + h(v_2)v_2.$$

Therefore, compared to the rational case, in the one-step negotiation the buyer puts more weight on the price dimension than on the two quality dimensions. Similarly to what discussed earlier, switching to a two-step negotiation and imposing a maximum price in period 1 reduces the buyer's price sensitivity. In this case, however, the outcome of the two-step negotiation may be closer to the rational benchmark than the outcome of the one-step negotiation. Similarly, if v_1 and v_2 are endogenous, when bargaining in one step focusing will cause the players to agree on v_1 and v_2 which are smaller than what a rational player would achieve. A two-step negotiation instead pushes the outcome closer to the outcome with rational players.

[29] See also Section A.1 in Appendix for further discussions on how to define the consideration set.

[30] Whether this add-on can be purchased independently on the main good is not relevant for our argument.

7 Conclusion

We provide a theory of incomplete agreements in the absence of uncertainty. Our key assumption is that the players' preferences are context dependent, in the sense that they depend on the set of possible bargaining outcomes. We use our theory to understand the way negotiations are structured.

When the player's preferences are context dependent, the presence of previous incomplete agreements which restrict the set of possible bargaining outcomes may affect the bargaining solution. More interestingly, when bargaining over an incomplete agreement, the type of incomplete agreements which can be signed determines the preferences of the players during that bargaining round. At this stage, a player with context dependent preferences values the possibility of aligning her present- and future-self preferences, so that the future self will decide according to the present self preferences. Achieving this alignment of preferences may require the players to sign an incomplete agreement restricting the future bargaining possibilities.

Similarly, players may manipulate the other players' preferences by entering a negotiation having already excluded some bargaining solutions. As an example, we consider the case of a seller bargaining with a buyer with preferences distorted by the focusing effect. We show that the seller may want to enter the negotiation over the price of a good having already announced a maximum transaction price. We also show that the seller may prefer to enter a negotiation having already invested in quality, rather than bargaining over quality and price simultaneously.

We assumed that time has no value, and therefore reaching a given agreement in one period is equivalent to reaching the same agreement in two periods. Introducing a cost of waiting would not affect our main results. If the cost of waiting is sufficiently small, the players may still prefer to negotiate in two steps. However, with a positive cost of waiting, the players prefer reaching the same bargaining outcome earlier than later. As a consequence, there is scope for introducing a mediator (an intermediary, an authority, or a platform), who can restrict the bargaining possibilities available to the agents in period 1. By restricting the bargaining set in an appropriate way, the mediator may induce the players to reach in one step the solution to the two-step negotiation without mediator. Understanding the incentives of a mediator, and her ability to restrict the bargaining possibility of the players, is left for future work.

Finally, incomplete agreements have been shown to be relevant in many contexts. We focus on negotiations because negotiations are often structured as a sequence of incomplete agreements, with no new information expected to arrive between bargaining rounds. Hence, this context is well suited to illustrate our main point: when preferences are distorted by the focusing effect, incomplete agreements may be used even in the absence of uncertainty. Introducing the focusing effect in other contexts where incomplete agreements are used (such as, for example, the allocation of ownership) is also left for future work.

A Appendix: further robustness checks.

A.1 The consideration set

So far, we assumed that the consideration set is equivalent to the entire bargaining set. Other alternative assumptions are possible. For example, the consideration set could be equal to the Pareto frontier of the bargaining set. In this case, the lower bounds of the consideration set are the minimum possible transaction price and the minimum possible transaction quality.[31] It follows that period-1 agreements imposing a lower bound on prices and quality may be used in order to manipulate the final bargaining outcome. In other words, under some alternative assumptions on the shape of the consideration set, the basic logic we derived goes through but the space of possible period-1 agreements changes.

However, we think that the consideration set should be equal to the bargaining set. Consider a generic bargaining problem with reference-dependent preferences (such as the one discussed in Section 5), solved using a Generalized Nash Bargaining (GNB) solution:

$$\max_{p,q}\left\{\left(\tilde{U}^b(p,q,\bar{p},\bar{q})\right)^{\nu_s}\left(\tilde{U}^s(p,q,\bar{p},\bar{q})\right)^{\nu_b}\right\},$$

subject to the players' rationality constraint, where the weights ν_b and ν_s represent the player's ability to influence the bargaining process. The GNB solution is equivalent to the Nash bargaining solution whenever $\nu_b = \nu_s$. The GNB solution is obtained starting from the same axioms underlying the Nash bargaining solution, and weakening the symmetry axiom (see Roth, 1979).

At the limit case $\nu_s = 0$, the bargaining problem is equivalent to a choice problem: choose the p, q preferred by the buyer subject to the players rationality constraints. Hence, the players' rationality constraint define the choice set of the problem. As discussed in the introduction, in a choice problem the consideration set is equivalent to the choice set. Therefore, by consistency with the choice problem, when $\nu_s = 0$ the consideration set should be equal to the bargaining set.

We make the following assumption

Assumption 7. *The consideration set is independent on ν_s and ν_b.*

The above assumption implies that the buyer's *preferences* over each bargaining outcome do not depend on ν_s and ν_b (on the other hand, the solution to the bargaining problem will clearly depend on ν_s and ν_b). If the consideration set is independent of ν_s and ν_b, by consistency with the limit case $\nu_s = 0$ the consideration set should be equivalent to the entire bargaining set for all ν_s and ν_b, including for $\nu_s = \nu_b$.

[31] Note that when players can choose any maximum price, maximum quality, minimum price, and minimum quality, they can effectively choose the terms of the transaction in period 1. Hence, this situation is equivalent to a one-step negotiation. The question is whether the players will decide in period 0 to adopt this specific bargaining structure, or instead restrict the minimum and maximum quality and price which can be chosen in period 1. See Footnote 26.

Finally, note that a similar argument holds when we modify the bargaining problem by adding another seller. Due to Bertrand competition among sellers, the resulting bargaining problem is equivalent to a choice problem for the buyer. Again, if we assume that preferences over bargaining outcomes do not change with the number of buyers or sellers, then we should also assume that, for any number of buyers and sellers, the consideration set is always equivalent to the bargaining set.

A.2 One-step negotiation as outside option

Up to now, we assumed that when players disagree, the outcome is no trade. This assumption is relevant in case the players bargain in two steps, because it implies that the seller can commit herself not to trade with the buyer outside the agreed negotiation structure. This level of commitment is beneficial to the seller (in both examples, profits when negotiating in two steps are larger than when negotiating in one step), but it may be problematic in many contexts.

Here we consider the opposite assumption: the disagreement outcome of the two-step negotiation is the one-step negotiation. It is easy to see that, under this alternative assumption, in example 1 the two-step negotiation will never be employed. During the last round of negotiation, the buyer anticipates that, by disagreeing, he will be able to purchase the object at a lower price compared to the transaction price in case an agreement is reached. Independently on his price sensitivity, the buyer always prefers lower prices, and hence in the last period he will disagree and trigger a new round of negotiation.

In our second example, however, negotiations may occur in two steps also when the disagreement outcome is the one-step negotiation. In the one-step negotiation, the buyer is equally focused on price and quality, and behave as a rational buyer. However, at the various stages of a two-step negotiation, the buyer may be relative more quality sensitive or relative more price sensitive. In these cases, the buyer may prefer to reach a solution that is different from the one-step negotiation solution.

More formally, suppose that the buyer and seller bargain in two steps over q and p. Assume that the negotiation already reached its final stage, and that the boundaries of the consideration set are $\bar{p}$, $\underline{p}$, $\bar{q}$, $\underline{q}$. Because the solution to the one-step negotiation is $q = 1$ and $p = \frac{3}{4}$, a given bargaining outcome is in the consideration set if:

$$h(\bar{q} - \underline{q})(q - 1) - h(\bar{p} - \underline{p})\left(p - \frac{3}{4}\right) \geq 0,$$

$$p - \frac{1}{2}q^2 \geq \frac{1}{4}.$$

In this bargaining round, surplus is positive (and the players will not disagree) if there exists a q

such that when $p = \frac{1}{4} + \frac{1}{2}q^2$, the buyer strictly prefers to transact, i.e.

$$h(\bar{q} - \underline{q})(q-1) - h(\bar{p} - \underline{p})\frac{(q^2-1)}{2} > 0,$$

or

$$\frac{h(\bar{q} - \underline{q})}{h(\bar{p} - \underline{p})} \begin{cases} > \frac{q+1}{2} & \text{if } q > 1 \\ < \frac{q+1}{2} & \text{if } q < 1. \end{cases} \tag{A.1}$$

Therefore, as long as $\frac{h(\bar{q}-\underline{q})}{h(\bar{p}-\underline{p})} \neq 1$, it is always possible to find a $q \neq 1$ such that the two players strictly prefer to transact at q rather than disagreeing. If $\frac{h(\bar{q}-\underline{q})}{h(\bar{p}-\underline{p})} > 1$, the buyer would prefer to trade at a higher quality and price compared to the outside option. If instead $\frac{h(\bar{q}-\underline{q})}{h(\bar{p}-\underline{p})} < 1$, then the buyer prefers to trade at a lower quality and price compared to the outside option. Hence, as long as the focus weights are not equal, the players strictly prefer to reach an agreement within the current negotiation round rather than going to a fresh round of negotiations. If instead $\frac{h(\bar{q}-\underline{q})}{h(\bar{p}-\underline{p})} = 1$, the buyer strictly prefers the outside option to any $q \neq 1$, and is indifferent between the outside option and transacting if the transaction quality is $q = 1$. The reason is that the buyer's preferences in the last period of negotiations are identical to her preferences in case of a one-period negotiation. Hence, the buyer is indifferent between reaching the same agreement at the end of the two-step negotiation, or by triggering a fresh round of negotiation. Overall, without loss of generality we can assume that the players will not disagree and conclude the negotiation at the end of period 2.

B Appendix: mathematical derivations.

Proof of Lemma 1. To conclude the proof of Lemma 1, we need to show that $v > \hat{p}^\star > c$ whenever $v > c$ and that $\hat{p}^\star$ is larger than the price agreed in the one-step negotiation case. It is easy to see that $\hat{p}^\star = v$ whenever $v = c$. Assume therefore that $v > c$. By simple algebra, $p(\hat{p} = v)$ in (3.1) is equal to $(v + c)/2$, and therefore $p(\hat{p} = v) > c$. Moreover, since $p'(\hat{p}) < 0$ for all $\hat{p} \in [\hat{p}^\star, v]$, it must be that $p(\hat{p}) \geq c$ for all $\hat{p} \in [\hat{p}^\star, v]$. Since $p(\hat{p}^\star) = \hat{p}^\star$, we conclude that $\hat{p}^\star > c$ whenever $v > c$. In addition, $\hat{p}^\star < v$ for $v > c$ follows directly from (3.2). Finally, as $p(\hat{p} = v) = (v + c)/2$ is equal to the price in the one-shot bargaining case, and $p'(\hat{p}) < 0$ for $\hat{p} \in [\hat{p}^\star, v]$, we conclude that $\hat{p}^\star > (v + c)/2$. □

Proof of Lemma 3. We want to show that whenever one of conditions (4.12) - (4.15) is violated, then S is not renegotiation proof.

The fact that whenever (4.12) or (4.13) are violated S cannot be renegotiation proof follows immediately from the fact that, for S to be renegotiation proof, the Pareto frontier of $\mathbb{R}^2_+$ giving utility at least $\underline{u}^b$ to the buyer and $\underline{u}^s$ to the seller should be an element of S.

Suppose that $\{\hat{p}, \hat{q}\} \in S$ (i.e. the maximum price and quality are achieved at the same bargaining outcome), then again it is easy to see that (4.14) and (4.15) should hold, because otherwise one of the rationality constraints will be violated.

Finally, suppose that $\{\hat{p},\hat{q}\} \notin S$. Call the two "extreme" bundles $\{p',\hat{q}\} \in S$ and $\{\hat{p},q'\} \in S$ for $q' < \hat{p}$ and $p' < \hat{p}$. It is easy to see that, whenever (4.14) is violated, it must be the case that the buyer's rationality constraint is violated at $\{\hat{p},q'\}$. Similarly, whenever (4.15) is violated, it must be the case that the seller's rationality constraint is violated at $\{p',\hat{q}\}$.

□

Proof of Lemma 5. $\underline{u}^s = \underline{u}^b = 0$, $\hat{p},\hat{q}$ are renegotiation proof if

$$\frac{h(\max\{\hat{q},1\})}{h(\max\{\hat{p},\frac{3}{4}\})} \geq \frac{\hat{p}}{\hat{q}}, \tag{B.1}$$

$$\frac{\hat{q}}{2} \leq \frac{\hat{p}}{\hat{q}}, \tag{B.2}$$

$$\frac{h(\max\{\hat{q},1\})}{h(\max\{\hat{p},\frac{3}{4}\})} \leq \hat{q}, \tag{B.3}$$

$$\frac{h(\max\{\hat{q},1\})}{h(\max\{\hat{p},\frac{3}{4}\})} \leq \sqrt{\hat{p}}. \tag{B.4}$$

By Nash bargaining, for every $\frac{h(\max\{\hat{q},1\})}{h(\max\{\hat{p},\frac{3}{4}\})}$ that satisfy conditions (B.1), (B.2), (B.3) and (B.4) the period-2 outcome is

$$q^\star = \frac{h(\max\{\hat{q},1\})}{h(\max\{\hat{p},\frac{3}{4}\})},$$

$$p^\star = \frac{3}{4}\left(\frac{h(\max\{\hat{q},1\})}{h(\max\{\hat{p},\frac{3}{4}\})}\right)^2,$$

Hence, the set of possible bargaining outcomes achievable in period-2 depends on the set of $\frac{h(\max\{\hat{q},1\})}{h(\max\{\hat{p},\frac{3}{4}\})}$ that satisfy conditions (B.1), (B.2), (B.3) and (B.4).

It is easy to see that the minimum $\frac{h(\max\{\hat{q},1\})}{h(\max\{\hat{p},\frac{3}{4}\})}$ is reached when both conditions (B.1), (B.2) are binding, so that

$$\min\left\{\frac{h(\max\{\hat{q},1\})}{h(\max\{\hat{p},\frac{3}{4}\})}\right\} = 1.$$

Consider a given $\hat{q}' \geq 1$. Condition (B.4) can be written as

$$\left(\max\{\hat{p},\frac{3}{4}\}\right)^\gamma \sqrt{\hat{p}} \geq (\hat{q}')^\gamma,$$

which implies that $\hat{p} > 1$. Hence, the smallest $\hat{p}$ that satisfies conditions (B.2), (B.3), and (B.4) is

$$\underline{\hat{p}}(\hat{q}') = \max\left\{\frac{(q')^2}{2}, (q')^{\frac{\gamma-1}{\gamma}}, (q')^{\frac{2\gamma}{2\gamma+1}}\right\},$$

so that

$$\max\left\{\frac{h(\hat{q}')}{h(\hat{p})}\right\} = \frac{h(\hat{q}')}{h(\underline{\hat{p}}(\hat{q}'))} = \max\left\{\min\left\{\hat{q}', (\hat{q}')^{\frac{\gamma}{2\gamma+1}}, \left(\frac{2}{\hat{q}'}\right)^{\gamma}\right\}\right\}.$$

Also, by varying $\hat{q}'$, $\frac{h(\hat{q}')}{h(\underline{\hat{p}}(\hat{q}'))}$ reaches its maximum for

$$\overline{\hat{q}'} : (\hat{q}')^{\frac{\gamma}{2\gamma+1}} = \left(\frac{2}{\hat{q}'}\right)^{\gamma},$$

or

$$\overline{\hat{q}'} = 2^{\frac{2\gamma+1}{2\gamma+2}},$$

which implies

$$\max_{q'}\left\{\frac{h(\hat{q}')}{h(\overline{\hat{p}}(\hat{q}'))}\right\} = \left(\frac{\overline{\overline{\hat{q}'}}}{2}\right)^{\gamma} = 2^{\frac{\gamma}{2\gamma+2}}.$$

It follows that the highest quality and the highest price achievable in period 2 by choosing a renegotiation-proof $\{\hat{p}, \hat{q}\}$ in period 1 are $2^{\frac{\gamma}{2\gamma+2}}$ and $\left(\frac{3}{4}\right) 2^{\frac{\gamma}{\gamma+1}}$ respectively. As a consequence, the period-1 bargaining problem is:

$$\max_{\alpha}\left\{\left(h\left(2^{\frac{\gamma}{2\gamma+2}}\right)\alpha - h\left(\frac{3}{4}\cdot 2^{\frac{\gamma}{\gamma+1}}\right)\frac{3}{4}\alpha^2\right)\left(\frac{3}{4}\alpha^2 - \frac{1}{2}\alpha^2\right) \text{ s.t. } 1 \leq \alpha \leq 2^{\frac{\gamma}{2\gamma+2}}\right\},$$

Where α is the ratio of period-2 focus weights $\frac{h(\max\{\hat{q},1\})}{h(\max\{\hat{p},\frac{3}{4}\})}$. Note that, in period 1, the quality dimension is more salient than the price dimension for small γ and the opposite is true for large γ. The solution is

$$\alpha^{\star} = \max\left\{h\left(2^{\frac{\gamma}{2\gamma+2}}\right)/h\left(\frac{3}{4}\cdot 2^{\frac{\gamma}{\gamma+1}}\right), 1\right\},$$

which corresponds to

$$q^{\star} = \max\left\{\left(\frac{2^{\frac{3\gamma+4}{2(\gamma+1)}}}{3}\right)^{\gamma}, 1\right\},$$

$$p^{\star} = \frac{3}{4}(q^{\star})^2.$$

□

Proof of Lemma 6. In this case, a renegotiation-proof agreement has to satisfy

$$\hat{p} + \frac{\underline{u}^b}{h(\max\{\hat{p}, \frac{3}{4}\})} \geq \left(\frac{h(\max\{\hat{q}, 1\})}{h(\max\{\hat{p}, \frac{3}{4}\})}\right)^2, \tag{B.5}$$

$$\hat{q} \geq \frac{h(\max\{\hat{q}, 1\})}{h(\max\{\hat{p}, \frac{3}{4}\})}, \tag{B.6}$$

$$\hat{p} + \frac{\underline{u}^b}{h(\max\{\hat{p}, \frac{3}{4}\})} \leq \frac{h(\max\{\hat{q}, 1\})}{h(\max\{\hat{p}, \frac{3}{4}\})}\hat{q}, \tag{B.7}$$

$$\frac{2}{\hat{q}} \geq \frac{\hat{q}}{\hat{p}}. \tag{B.8}$$

It is easy to see that the smallest possible renegotiation-proof $\frac{h(\max\{\hat{q},1\})}{h(\max\{\hat{p},\frac{3}{4}\})}$ is determined by condition (B.7) for $\underline{u}^b = 0$ and it is equal to 1. Regarding the largest possible $\frac{h(\max\{\hat{q},1\})}{h(\max\{\hat{p},\frac{3}{4}\})}$, we again fix $\hat{q}' \geq 1$ and compute the largest $\frac{h(\hat{q}')}{h(\max\{\hat{p},\frac{3}{4}\})}$ satisfying conditions (B.5), (B.6) and (B.8).

The smallest $\hat{p}$ satisfying conditions (B.5), (B.6) and (B.8) is given by:

$$\underline{\hat{p}}(\hat{q}') = \max\left\{\tilde{p}(\hat{q}', \underline{u}^b), (\hat{q}')^{\frac{\gamma-1}{\gamma}}, \frac{(q')^2}{2}\right\},$$

where $\tilde{p}(\hat{q}', \underline{u}^b)$ is implicitly defined using condition (B.5) as

$$\tilde{p}(\hat{q}', \underline{u}^b)^{2\gamma+1} + \underline{u}^b \tilde{p}(\hat{q}', \underline{u}^b)^{\gamma} = (\hat{q}')^{2\gamma}. \tag{B.9}$$

Note that $\tilde{p}(\hat{q}', \underline{u}^b)$ is increasing in $\hat{q}$ and decreasing in $\underline{u}$. It follows that

$$\max\left\{\frac{h(\max\{\hat{q}', 1\})}{h(\max\{\hat{p}, \frac{3}{4}\})}\right\} = \frac{h(\hat{q}')}{h(\underline{\hat{p}}(\hat{q}'))} = \max\left\{\min\left\{\hat{q}', \left(\frac{\hat{q}'}{p(\hat{q}', \underline{u}^b)}\right)^{\gamma}, \left(\frac{2}{\hat{q}'}\right)^{\gamma}\right\}\right\}.$$

Define $\hat{q}(\underline{u}^b)$ as $\hat{q}' : \left(\frac{\hat{q}'}{p(\hat{q}', \underline{u}^b)}\right)^{\gamma} = \left(\frac{2}{\hat{q}'}\right)^{\gamma}$, which by using (B.9), can be expressed as

$$\hat{q}(\underline{u}^b) = \left(\left(1 - \frac{\underline{u}^b}{2^{\gamma}}\right) 2^{2\gamma+1}\right)^{\frac{1}{2\gamma+2}}. \tag{B.10}$$

Note the following:

- $\hat{q}(\underline{u}^b)$ is unique for every $\underline{u}^b$, which implies that $\left(\frac{2}{\hat{q}'}\right)^{\gamma}$ and $\left(\frac{\hat{q}'}{p(\hat{q}', \underline{u}^b)}\right)^{\gamma}$ intercept only once.

- $\left(\frac{\hat{q}'}{p(\hat{q}',\underline{u}^b)}\right)^{\gamma}$ is strictly increasing in $\hat{q}'$ for $\hat{q}' > 1$. By the implicit function theorem:

$$\frac{dp(\hat{q}',\underline{u}^b)}{d\hat{q}'} = \frac{2\gamma \hat{q}'^{2\gamma-1}}{(2\gamma+1)p(\hat{q}',\underline{u}^b)^{2\gamma} + \gamma \underline{u}^b p(\hat{q}',\underline{u}^b)^{\gamma-1}}. \tag{B.11}$$

  Hence,

$$\frac{d\frac{\hat{q}'}{p(\hat{q}',\underline{u}^b)}}{d\hat{q}'} = \frac{p(\hat{q}',\underline{u}^b) - \frac{dp(\hat{q}',\underline{u}^b)}{d\hat{q}'}\hat{q}'}{p(\hat{q}',\underline{u}^b)^2} = \frac{p(\hat{q}',\underline{u}^b) - \frac{2\gamma \hat{q}'^{2\gamma}}{(2\gamma+1)p(\hat{q}',\underline{u}^b)^{2\gamma}+\gamma \underline{u}^b p(\hat{q}',\underline{u}^b)^{\gamma-1}}}{p(\hat{q}',\underline{u}^b)^2}. \tag{B.12}$$

  The above expression is positive at $\underline{u}^b = 0$ (and therefore it is positive for every $\underline{u}^b$) whenever

$$\tilde{p}(\hat{q}',\underline{u}^b)^{2\gamma+1} > \frac{2\gamma}{2\gamma+1}(\hat{q}')^{2\gamma},$$

  which is true by (B.9).

- $\left(\frac{\hat{q}'}{p(\hat{q}',\underline{u}^b)}\right)^{\gamma} < \hat{q}'$ at $\hat{q}' = \hat{q}(\underline{u}^b)$ for every $\underline{u}^b \leq 2^{\gamma-1}$. But, as we show later, $\underline{u}^b > 2^{\gamma-1}$ implies negative utility for the seller, so that we can ignore this case

Taken together, the three facts above imply that the focusing ratio achieves its maximum at $\hat{q}(\underline{u}^b)$, so that

$$\max_{\hat{q}'}\left\{\frac{h(\hat{q}')}{h(\underline{\hat{p}}(\hat{q}'))}\right\} = \frac{h(\hat{q}(\underline{u}^b))}{h(\hat{p}(\underline{u}^b))} = \left(\frac{2}{\hat{q}(\underline{u}^b)}\right)^{\gamma}, \tag{B.13}$$

where using (B.9) and (B.10) we can define

$$\hat{p}(\underline{u}^b) \equiv \frac{\hat{q}(\underline{u}^b)^2}{2}. \tag{B.14}$$

Finally, note that $\max_{\hat{q}'}\left\{\frac{h(\hat{q}')}{h(\underline{\hat{p}}(\hat{q}'))}\right\}$ is strictly increasing in $\underline{u}^b$.

We now consider the set of possible period-2 outcomes achievable in period-2 by mean of a renegotiation proof $\frac{h(\max\{\hat{q},1\})}{h(\max\{\hat{p},\frac{3}{4}\})}$. In particular, we want to determine the highest price and the highest quality achievable in period 2, as these quantities will determine preferences in period-1. Note that, in period-2, the minimum utility $\underline{u}^b$ affects the bargaining solution in two ways. On the one hand it affects the highest focusing ratio $\max_{\hat{q}'}\left\{\frac{h(\hat{q}')}{h(\underline{\hat{p}}(\hat{q}'))}\right\}$ achievable. On the other hand, it imposes a constraint on the period-2 bargaining problem.

It is quite straightforward to see that the highest quantity and price achievable in period 2 are achieved if the minimum utility constraint is binding. Let's first consider the case in which the constrained solution coincides with the unconstrained solution:

$$q^{\star}(\underline{u}^b) = \frac{h(\max\{\hat{q}(\underline{u}^b),1\})}{h(\max\{\hat{p}(\underline{u}^b),\frac{3}{4}\})} = \left(\frac{2}{\hat{q}(\underline{u}^b)}\right)^{\gamma}, \tag{B.15}$$

$$p^\star(\underline{u}^b) = \frac{3}{4}\left(q^\star(\underline{u}^b)\right)^2. \tag{B.16}$$

The corresponding minimum-utility constraint equals

$$U^b\left(q^\star(\underline{u}^b), p^\star(\underline{u}^b), \underline{u}^b\right) = \frac{1}{4}\frac{h(\max\{\hat{q}(\underline{u}^b), 1\})^2}{h(\max\{\hat{p}(\underline{u}^b), \frac{3}{4}\})} = \underline{u}^b. \tag{B.17}$$

Note that LHS of condition (B.17) simplifies to

$$\frac{1}{4}\frac{\hat{q}(\underline{u}^b)^{2\gamma}}{\frac{\hat{q}(\underline{u}^b)^{2\gamma}}{2^\gamma}} = \frac{2^\gamma}{4} = 2^{\gamma-2}.$$

Thus, the minimum-utility constraint is binding for $\underline{u}^b \geq \underline{u}^{b\star} \equiv 2^{\gamma-2}$.

For $\underline{u}^b > \underline{u}^{b\star} \equiv 2^{\gamma-2}$, the final outcome of the constrained Nash bargaining problem in period 2 equals

$$q^{\star\star}(\underline{u}^b) = \left(\frac{h(\max\{\hat{q}(\underline{u}^b), \frac{3}{4}\})}{h(\max\{\hat{p}(\underline{u}^b), 1\})}\right) = \left(\frac{2}{\hat{q}(\underline{u}^b)}\right)^\gamma, \tag{B.18}$$

$$p^{\star\star}(\underline{u}^b) = \left(q^{\star\star}(\underline{u}^b)\right)^2 - \left(\frac{\underline{u}^b}{h(\max\{\hat{p}(\underline{u}^b), \frac{3}{4}\})}\right) = \left(q^{\star\star}(\underline{u}^b)\right)^{-2} 2^\gamma, \tag{B.19}$$

where the last equality follows from equations (B.10) and (B.14). Therefore $q^{\star\star}(\underline{u}^b)$ is increasing in $\underline{u}^b$ while $p^{\star\star}(\underline{u}^b)$ is decreasing in $\underline{u}^b$. Call

$$U^s(\underline{u}^b) = p^{\star\star}(\underline{u}^b) - \frac{1}{2}q^{\star\star}(\underline{u}^b)^2$$

the utility of the seller as a function of the minimum utility for the buyer for $\underline{u}^b \geq \underline{u}^{b\star}$. The maximum level of $\underline{u}^b$ such that the seller will want to trade is $\underline{u}^{b\star\star} \equiv 2^{\gamma-1}$. It follows that highest period-2 quantity and price are equal to

$$q^{\star\star}(\underline{u}^{b\star\star}) = \max\left\{\frac{h(\max\{\hat{q}(\underline{u}^{b\star\star}), 1\})}{h(\max\{\hat{p}(\underline{u}^{b\star\star}), \frac{3}{4}\})}\right\} = \left(\frac{2}{\hat{q}(\underline{u}^{b\star\star})}\right)^\gamma = 2^{\frac{\gamma}{\gamma+1}},$$

$$p^\star(\underline{u}^{b\star}) = 3/4\left(\max\left\{\frac{h(\max\{\hat{q}(\underline{u}^{b\star}), 1\})}{h(\max\{\hat{p}(\underline{u}^{b\star}), \frac{3}{4}\})}\right\}\right)^2 = 3^{\frac{1}{\gamma+1}}2^{\frac{\gamma-2}{\gamma+1}}.$$

Having established this, the value function of the period-1 bargaining problem is:

$$V(\alpha) \equiv \left\{\left(h\left(2^{\frac{\gamma}{\gamma+1}}\right)\alpha - h\left(3^{\frac{1}{\gamma+1}}2^{\frac{\gamma-2}{\gamma+1}}\right)\beta(\alpha)\right)\left(\beta(\alpha) - \frac{1}{2}\alpha^2\right) \text{s.t. } 1 \leq \alpha \leq 2^{\frac{\gamma}{\gamma+1}}\right\},$$

for

$$\beta(\alpha) = \begin{cases} \frac{3}{4}\alpha^2 & \text{if } 1 \le \alpha \le 3^{-\frac{\gamma}{2\gamma+2}} 2^{\frac{3\gamma}{2\gamma+2}} \\ \frac{2^\gamma}{\alpha^2} & \text{if } 3^{-\frac{\gamma}{2\gamma+2}} 2^{\frac{3\gamma}{2\gamma+2}} \le \alpha \le 2^{\frac{\gamma}{\gamma+1}}. \end{cases}$$

In other words, in period-1 the players agree on a period-2 focusing ratio α which will determine period-2 transaction price and quality. Note that if α is high enough, the period-1 minimum utility will be binding, which will affect the final price.

It is easy to see that $V(\alpha)$ is always below

$$\tilde{V}(\alpha, \beta) \equiv \left\{ \left(h\left(2^{\frac{\gamma}{\gamma+1}}\right)\alpha - h\left(3^{\frac{1}{\gamma+1}} 2^{\frac{\gamma-2}{\gamma+1}}\right)\beta \right) \left(\beta - \frac{1}{2}\alpha^2 \right) \text{s.t. } 1 \le \alpha \le 2^{\frac{\gamma}{\gamma+1}} \right\},$$

where β can be any positive number. In practice, $\tilde{V}(\alpha, \beta)$ is equivalent to choosing a quality α and a price β, without worrying about whether this price and quality will be implementable in period 2 via a renegotiation-proof period-1 agreement. Maximizing $\tilde{V}(\alpha, \beta)$ with respect to α and β, the solution is

$$\alpha^\star = \left(\frac{4}{3}\right)^{\frac{\gamma}{\gamma+1}},$$

$$\beta^\star = \frac{3}{4}(\alpha^\star)^2,$$

which is always no lower than 1 and smaller than $3^{-\frac{\gamma}{2\gamma+2}} 2^{\frac{3\gamma}{2\gamma+2}}$, and therefore also maximizes $V(\alpha)$. The period-1 solution is

$$q^* = \left(\frac{4}{3}\right)^{\frac{\gamma}{\gamma+1}}, \tag{B.20}$$

$$p^* = \left(\frac{4}{3}\right)^{\frac{\gamma-1}{\gamma+1}}. \tag{B.21}$$

Interestingly, this result can be achieved by setting $\underline{u}^b = 0$ in period 1. To see this, note that highest focusing ratio that can be achieved in period 2 when $\underline{u}^b = 0$ is $2^{\frac{\gamma}{2\gamma+2}}$, which also corresponds to the highest possible quality achievable on period 1 by setting $\underline{u}^b = 0$. This quality is below the equilibrium quality. Therefore, when bargaining in period 1 buyer and seller will agree on $\underline{u}^b = 0$.

□

Proof of Lemma 7. Transaction quality in period 2 only depends on $\frac{h(\max\{\hat{q},1\})}{h(\max\{\hat{p},\frac{3}{4}\})}$, and is given by

$$q^\star = \frac{h(\max\{\hat{q}, 1\})}{h(\max\{\hat{p}, \frac{3}{4}\})} = \left(\frac{\max\{\hat{q}, 1\}}{\max\{\hat{p}, \frac{3}{4}\}}\right)^\gamma.$$

We show that the set of $\frac{h(\max\{\hat{q},1\})}{h(\max\{\hat{p},\frac{3}{4}\})}$ that is achievable whenever $\underline{u}^s \geq 0$ is the same set of $\frac{h(\max\{\hat{q},1\})}{h(\max\{\hat{p},\frac{3}{4}\})}$ that is achievable whenever $\underline{u}^s = 0$. Note that $\underline{u}^s$ matters only for constraint (4.15), which can be rewritten as:

$$\frac{1}{2}\left(\frac{\hat{q}}{\hat{p}}\right)^2 \leq \frac{1}{\hat{p}}\left(1 - \frac{\underline{u}^s}{\hat{p}}\right).$$

Suppose that, for some $\underline{u}^s > 0$, at $\max\left\{\frac{h(\max\{\hat{q},1\})}{h(\max\{\hat{p},\frac{3}{4}\})}\right\}$ the above constraint is binding. By lowering $\underline{u}^s$ and increasing $\hat{q}$, it is always possible to achieve an even higher $\frac{h(\max\{\hat{q},1\})}{h(\max\{\hat{p},\frac{3}{4}\})}$. Hence, whatever $\max\left\{\frac{h(\max\{\hat{q},1\})}{h(\max\{\hat{p},\frac{3}{4}\})}\right\}$ can be achieved by allowing $\underline{u}^s$ to be greater or equal to zero can also be achieved by fixing $\underline{u}^s = 0$. Compared with the case analyzed in the previous lemma, allowing for $\underline{u}^s \geq 0$ does not change the set of possible period-2 transaction quality levels.

Nonetheless, a minimum utility guaranteed to the seller may affect the final transaction price if it is binding in period 2. For given preferences, whenever the minimum utility guaranteed to the seller is binding the period-2 transaction price equals

$$p^\star = \underline{u}^s + \frac{1}{2}\left(\frac{\max\{\hat{q},1\}}{\max\{\hat{p},\frac{3}{4}\}}\right)^{2\gamma},$$

where $\underline{u}^s$ binding implies (using the structure in (B.15) and (B.16))

$$\underline{u}^s \geq \frac{1}{4}\left(\frac{\max\{\hat{q},1\}}{\max\{\hat{p},\frac{3}{4}\}}\right)^{2\gamma}.$$

It follows that there is a potential trade-off. A higher minimum utility guarantee to the seller increases final price because it shifts bargaining power to the seller. At the same time, a minimum utility guarantee to the seller affects the buyer's preferences and may make the buyer more price sensitive.

Following steps that are similar to the ones used in the previous lemma, we can derive $\left(\frac{\hat{q}(\underline{u}^b,\underline{u}^s)}{\hat{p}(\underline{u}^b,\underline{u}^s)}\right)^\gamma$, i.e. the highest renegotiation-proof focusing ratio for given $\underline{u}^b, \underline{u}^s$. Consider a given $\hat{q}' \geq 1$. The smallest renegotiation-proof $\hat{p}$ is given by:

$$\underline{\hat{p}}(\hat{q}') = \max\left\{\tilde{p}(\hat{q}', \underline{u}^b), (\hat{q}')^{\frac{\gamma-1}{\gamma}}, \frac{(q')^2}{2} + \underline{u}^s\right\},$$

where $\tilde{p}(\hat{q}', \underline{u}^b)$ is, again, implicitly defined by (B.9). It follows that, for given $\hat{q}'$

$$\max\left\{\frac{h(\hat{q}')}{h(\hat{p})}\right\} = \frac{h(\hat{q}')}{h(\underline{\hat{p}}(\hat{q}'))} = \max\left\{\min\left\{\hat{q}', \left(\frac{\hat{q}'}{p(\hat{q}', \underline{u}^b)}\right)^\gamma, \left(\frac{\hat{q}'}{2} + \frac{\underline{u}^s}{\hat{q}'}\right)^{-\gamma}\right\}\right\}.$$

The same argument discussed in the proof of the previous lemma (see the three bullet points) guarantees that the above expression reaches its maximum at

$$\hat{q}(\underline{u}^b, \underline{u}^s): \left(\frac{\hat{q}'}{p(\hat{q}', \underline{u}^b)}\right)^{\gamma} = \left(\frac{\hat{q}'}{2} + \frac{\underline{u}^s}{\hat{q}'}\right)^{-\gamma},$$

so that

$$\max_{\hat{q}'}\left\{\frac{h(\hat{q}')}{h(\hat{p}(\hat{q}'))}\right\} = \frac{\hat{q}(\underline{u}^b, \underline{u}^s)^{\gamma}}{\hat{p}(\underline{u}^b, \underline{u}^s)^{\gamma}} = \left(\frac{\hat{q}(\underline{u}^b, \underline{u}^s)}{2} + \frac{\underline{u}^s}{\hat{q}(\underline{u}^b, \underline{u}^s)}\right)^{-\gamma}.$$

Note that, using (B.9), $\hat{q}(\underline{u}^b, \underline{u}^s)$ can be implicitly defined as

$$\left(\frac{\hat{q}(\underline{u}^b, \underline{u}^s)^2}{2} + \underline{u}^s\right)^{2\gamma+1} + \underline{u}^b\left(\frac{\hat{q}(\underline{u}^b, \underline{u}^s)^2}{2} + \underline{u}^s\right)^{\gamma} = \hat{q}(\underline{u}^b, \underline{u}^s)^{2\gamma}. \tag{B.22}$$

By implicit differentiation, we can show that:

$$\frac{\partial\left[\left(\frac{\hat{q}(\underline{u}^b, \underline{u}^s)}{2} + \frac{\underline{u}^s}{\hat{q}(\underline{u}^b, \underline{u}^s)}\right)^{-\gamma}\right]}{\partial \underline{u}^b} = -\gamma\left(\frac{\hat{q}}{2} + \frac{\underline{u}^s}{\hat{q}}\right)^{-\gamma-1}\left(\left[\frac{1}{2} - \frac{\underline{u}^s}{\hat{q}(\underline{u}^b, \underline{u}^s)^2}\right]\frac{d\hat{q}(\underline{u}^b, \underline{u}^s)}{d\underline{u}^b}\right)$$

which is positive if $\underline{u}^s < \frac{\hat{q}(\underline{u}^b, \underline{u}^s)^2}{2}$, and negative otherwise.

Given this, there are two cases

1. if $\underline{u}^s > \frac{\hat{q}(\underline{u}^b = 0, \underline{u}^s)^2}{2} \equiv \frac{\hat{q}(\underline{u}^s)^2}{2}$, then the highest renegotiation-proof focusing ratio for given $\underline{u}^s$ binding is reached at $\underline{u}^b = 0$, and is equal to

   $$\left(\frac{\hat{q}(\underline{u}^s)}{\hat{p}(\underline{u}^s)}\right)^{\gamma} = \hat{q}(\underline{u}^s)^{\frac{\gamma}{2\gamma+1}},$$

   where, using equation (B.22), $\hat{q}(\underline{u}^s)$ is implicitly defined as

   $$\left(\frac{\hat{q}(\underline{u}^s)}{2} + \frac{\underline{u}^s}{\hat{q}(\underline{u}^s)}\right)^{2\gamma+1} \cdot \hat{q}(\underline{u}^s) = 1.$$

   Using the fact that $\underline{u}^s > \frac{\hat{q}(\underline{u}^s)^2}{2}$, the above expression implies

   $$\hat{q}(\underline{u}^s) < 1,$$

   which is not possible, as $\hat{q}(\underline{u}^s)$ is that $\hat{q}' \geq 1$ at which the focusing ratio reaches its maximum as a function of $\underline{u}^s$.

2. if $\underline{u}^s < \frac{\hat{q}(\underline{u}^b, \underline{u}^s)^2}{2}$, then the highest renegotiation-proof focus weight for given $\underline{u}^s$ binding is

reached at $\underline{u}^b$ that is also binding:

$$\underline{u}^b = \frac{\hat{q}(\underline{u}^s)^{2\gamma}}{\hat{p}(\underline{u}^s)^{\gamma}} - \hat{p}(\underline{u}^s)^{\gamma}\left(\underline{u}^s + \frac{1}{2}\left(\frac{\hat{q}(\underline{u}^s)}{\hat{p}(\underline{u}^s)}\right)^{2\gamma}\right). \tag{B.23}$$

Condition (B.23), together with the fact that $\underline{u}^s$ and $\underline{u}^b$ are binding (i.e. conditions (4.14) and (4.15) are binding), implies that conditions (4.12) and (4.13) are also binding: that is, the highest possible transaction quality is the actual quality exchanged, and the highest possible transaction price is the actual price exchanged so that:[32]

$$\hat{q}(\underline{u}^s) = \left(\frac{\hat{q}(\underline{u}^s)}{\hat{p}(\underline{u}^s)}\right)^{\gamma}, \tag{B.26}$$

$$\hat{p}(\underline{u}^s) = \underline{u}^s + \frac{1}{2}\left(\frac{\hat{q}(\underline{u}^s)}{\hat{p}(\underline{u}^s)}\right)^{2\gamma}. \tag{B.27}$$

Dividing the LHS of equation (B.26) with the LHS of equation (B.27), and the RHS of equation (B.26) with the RHS of equation (B.27) leads to:

$$\left(\frac{\hat{q}(\underline{u}^s)}{\hat{p}(\underline{u}^s)}\right)^{\gamma-1} = \underline{u}^s + \frac{1}{2}\left(\frac{\hat{q}(\underline{u}^s)}{\hat{p}(\underline{u}^s)}\right)^{2\gamma}.$$

Finally, we derive the maximum transaction price by solving for

$$\max_{\underline{u}^s}\left\{\underline{u}^s + \frac{1}{2}\left(\frac{\hat{q}(\underline{u}^s)}{\hat{p}(\underline{u}^s)}\right)^{2\gamma}\right\} \tag{B.28}$$

$$\text{s.t.}\ \frac{1}{4}\left(\frac{\hat{q}(\underline{u}^s)}{\hat{p}(\underline{u}^s)}\right)^{2\gamma} \le \underline{u}^s \le \frac{1}{2}\left(\frac{\hat{q}(\underline{u}^s)}{\hat{p}(\underline{u}^s)}\right)^{2\gamma}. \tag{B.29}$$

Where the constraint requires that $\underline{u}^s$ is binding, and that $\underline{u}^s$ is such that the buyer enjoys non-negative utility. We preform this last step numerically, and we compare the solution to the maximum price achievable when $\underline{u}^s = 0$ (see the previous lemma). The results of the computation are reported in Figure B.1. Note that, for low γ the maximum price is achieved for $\underline{u}^s > 0$, while for high γ the maximum price is achieved for $\underline{u}^s = 0$.

When γ is large, the maximum price that can be achieved in period 2 by setting $\underline{u}^s > 0$ is below

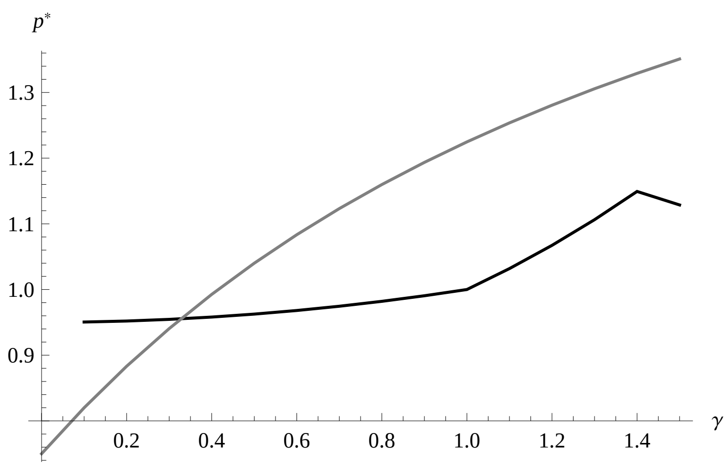
[32] Consider a $\hat{q}' \ge (\hat{q}'/\hat{p}')^{\gamma}$. Conditions (B.23) and (4.15) imply

$$\underline{u}^b = \frac{\hat{q}'^{2\gamma}}{\hat{p}'^{\gamma}} - \hat{p}'^{\gamma+1}, \tag{B.24}$$

while condition (4.14) implies

$$\underline{u}^b = \hat{q}'^{\gamma+1} - \hat{p}'^{\gamma+1}. \tag{B.25}$$

Taking the difference between these two equations leads to $\hat{q}'^{2\gamma}/\hat{p}'^{\gamma} = \hat{q}'^{\gamma+1}$, which is equivalent to condition (B.26). Condition (B.27) follows from (B.26) and the fact that $\underline{u}^s$ is binding.



$p^*(\gamma)$: maximum transaction price for $\underline{u}^s > 0$ (black line) and for $\underline{u}^s = 0$ (gray line) as a function of the focusing intensity γ.

Fig. B.1: Maximum Transaction Price

the maximum price that can be achieved in period 2 by setting $\underline{u}^s = 0$ (which is the maximum period-2 price derived in the Lemma 6). We argued earlier that the maximum quality achievable in period 2 is achieved at $\underline{u}^s = 0$. Hence, for γ large the period-1 focus weights on price and quality are equal to the period-1 focus weight on price and quality derived in the previous lemma, and hence the final negotiation outcome is the same as in Lemma 6.

If instead γ is low, the maximum price that can be achieved in period 2 by setting $\underline{u}^s > 0$ is *above* the maximum price that can be achieved in period 2 by setting $\underline{u}^s = 0$. Compared to the solution in Lemma 6, in period 1 the focus weight on quality is unchanged, but the focus weight on price is higher. Hence, in period 1 the agent is relative more price sensitive here compared to the case considered in Lemma 6, leading to lower transaction price and quality.

Finally, we showed in Lemma 6 that, in equilibrium, $\underline{u}^b = 0$: the possibility of setting a minimum utility to the buyer affects the buyer's focus weights but is not used in equilibrium. The reason is that all period-2 $\frac{h(\hat{q})}{h(\hat{p})}$ lower than some threshold can be achieved with $\underline{u}^b = 0$, while higher $\frac{h(\hat{q})}{h(\hat{p})}$ require to set $\underline{u}^b > 0$. The equilibrium price and quality derived in Lemma 6 can be implemented by choosing in period 1 a period-2 $\frac{h(\hat{q})}{h(\hat{p})}$ achievable with $\underline{u}^b = 0$. The same thing happens here: in equilibrium $\underline{u}^b = \underline{u}^s = 0$. This result is immediate when γ is large, because the solution to this case is the same as in Lemma 6. When γ is low, however, the final price and quality here are below the final price and quality derived in Lemma 6. Hence, the period-2 $\frac{h(\hat{q})}{h(\hat{p})}$ that implements the equilibrium price and quality here is below the period-2 $\frac{h(\hat{q})}{h(\hat{p})}$ that implements the equilibrium price and quality derived in Lemma 6. Hence, also the period-2 $\frac{h(\hat{q})}{h(\hat{p})}$ that implements the equilibrium price and quality here can be achieved by setting $\underline{u}^b = \underline{u}^s = 0$ in period 1.

$\square$

Proof of Lemma 10. Because in period 0 no previous agreement is in place, the players can freely exchange money and the Pareto frontier is a straight line. Similarly, if the players decide to bargain in one period, then in period 1 the players can freely exchange money and, again, the Pareto frontier is a straight line.

Hence, both the players' preference, and the shape of the bargaining set are identical in period 0 and in period 1 when the negotiation is in one step. It follows that whatever bargaining outcome solves the one-step negotiation problem, also solves the period-0 negotiation problem. □

References

Bac, M. and H. Raff (1996). Issue-by-issue negotiations: The role of information and time preference. *Games and Economic Behavior 13*(1), 125–134.

Battaglini, M. and B. Harstad (2014). Participation and duration of environmental agreements. *The Journal of Political Economy*, forthcoming.

Battigalli, P. and G. Maggi (2002). Rigidity, discretion, and the costs of writing contracts. *The American Economic Review*, 798–817.

Bernheim, B. D. and M. D. Whinston (1998). Incomplete contracts and strategic ambiguity. *The American Economic Review*, 902–932.

Bolton, P. and A. Faure-Grimaud (2010). Satisficing contracts. *The Review of Economic Studies 77*(3), 937–971.

Bordalo, P., N. Gennaioli, and A. Shleifer (2013). Salience and consumer choice. *The Journal of Political Economy 121*(5), 803–843.

Busch, L.-A. and I. J. Horstmann (1999a). Endogenous incomplete contracts: A bargaining approach. *Canadian Journal of Economics*, 956–975.

Busch, L.-A. and I. J. Horstmann (1999b). Signaling via an agenda in multi-issue bargaining with incomplete information. *Economic Theory 13*(3), 561–575.

Compte, O. and P. Jehiel (2003). Bargaining with reference dependent preferences. *Working Paper*.

Compte, O. and P. Jehiel (2004). Gradualism in bargaining and contribution games. *The Review of Economic Studies 71*(4), 975–1000.

Esteban, J. and J. Sákovics (2008). A theory of agreements in the shadow of conflict: The genesis of bargaining power. *Theory and Decision 65*(3), 227–252.

Flamini, F. (2007). First things first? the agenda formation problem for multi-issue committees. *Journal of Economic Behavior & Organization 63*(1), 138–157.

Gabaix, X. (2014). A sparsity-based model of bounded rationality. *The Quarterly Journal of Economics*.

Grossman, S. J. and O. D. Hart (1986). The costs and benefits of ownership: A theory of vertical and lateral integration. *The Journal of Political Economy*, 691–719.

Harstad, B. (2007). Harmonization and side payments in political cooperation. *The American Economic Review 97*(3), 871–889.

Hart, O. and J. Moore (2008). Contracts as reference points. *The Quarterly Journal of Economics 123*(1), 1–48.

Herweg, F., H. Karle, and D. Müller (2014). Incomplete contracting, renegotiation, and loss aversion. *CEPR Discussion Paper 9874*.

Herweg, F. and K. M. Schmidt (2014). Loss aversion and inefficient renegotiation. *The Review of Economic Studies*, forthcoming.

Inderst, R. (2000). Multi-issue bargaining with endogenous agenda. *Games and Economic Behavior 30*(1), 64–82.

Kahneman, D., A. B. Krueger, D. Schkade, N. Schwarz, and A. A. Stone (2006). Would you be happier if you were richer? a focusing illusion. *Science 312*(5782), 1908–1910.

Kőszegi, B. and A. Szeidl (2013). A model of focusing in economic choice. *The Quarterly Journal of Economics 128*(1), 53–104.

Lang, K. and R. W. Rosenthal (2001). Bargaining piecemeal or all at once? *The Economic Journal 111*(473), 526–540.

Maskin, E. and J. Tirole (1999). Unforeseen contingencies and incomplete contracts. *The Review of Economic Studies 66*(1), 83–114.

Raiffa, H. (1982). *The art and science of negotiation*. Harvard University Press.

Roth, A. E. (1979). *Axiomatic Models of Bargaining*. Number 170. Springer-Verlag Berlin.

Rubinstein, A. (1982). Perfect equilibrium in a bargaining model. *Econometrica: Journal of the Econometric Society*, 97–109.

Schkade, D. A. and D. Kahneman (1998). Does living in california make people happy? a focusing illusion in judgments of life satisfaction. *Psychological Science 9*(5), 340–346.

Segal, I. (1999). Complexity and renegotiation: A foundation for incomplete contracts. *The Review of Economic Studies 66*(1), 57–82.

Sims, C. A. (2003). Implications of rational inattention. *Journal of monetary Economics 50*(3), 665–690.

Tirole, J. (2009). Cognition and incomplete contracts. *The American Economic Review 99*(1), 265–294.