Bargaining Under Institutional Challenges

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Abstract

Standard legislative bargaining models assume that an agreed-upon allocation is final, whereas in practice, there exist mechanisms for challenging passed legislation when there is lack of sufficient consensus. Such mechanisms include popular vote requirements following insufficient majorities in the legislation. This paper analyzes a legislative bargaining game whose outcome can be challenged through a referendum. I study the effects of this institution on the bills passed in the legislature and analyze the incentives they provide for reaching grand bargains. The proposer party's trade-off between an expensive partner and a threatening opponent in the referendum summarizes the bargaining problem. The results indicate that it is possible to observe surplus coalitions formed in equilibrium even though smaller coalitions are sufficient for the passage of a bill and that measures of post-bargaining power do not necessarily translate into higher equilibrium payoffs. Moreover, disparities in campaigning resources incentivize challenge procedures. These results carry policy implications for various forms of post-bargaining power, such as caps on campaign contributions.

Keywords : Legislative Bargaining, Referenda, Political Campaigns.

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1 Introduction

Most existing legislative bargaining models assume that the agreed-upon allocation is final, whereas in practice, there exist mechanisms for challenging passed legislation when there is lack of sufficient consensus. Such mechanisms include popular vote requirements following insufficient majorities in the legislature. In most parliamentary systems, a bill (most often concerning a constitutional change) that fails to win a certain majority of votes in the legislature can be presented to a public vote as the final arbiter.[1] For example, in a referendum in May 2011, Britain rejected a proposal to switch from a first-past-the-post election system to an alternative vote system. In March 2011, shortly after the fall of the Mubarek regime, Egypt approved in a widely contested referendum a series of constitutional reforms including presidential term limits and election supervision mechanisms. Motivated by these examples, I analyze the effect of institutional mechanisms to challenge agreed-upon legislation on the formation of these bills and the equilibrium payoffs to the parties.

A large literature, including Matsusaka (2005a) and (2005b), documents the surge in spending on referendum campaigns, examples of which include advertising, media coverage, or political rallies. Moreover, there exists growing evidence that the public is influenced by these campaigns, as documented in de Figueiredo, Ji and Kousser (2011).[2] Given the growing prevalence of referendum campaigns, to what extent do the proposals introduced in a parliament reflect the parties' public vote calculus? For instance, would the Egyptian constitutional reform package include more liberal propositions if the liberal faction were considered a more powerful player in the subsequent referendum? Specifically, how does a referendum process in which parties campaign to influence its outcome affect the contents of a legislative proposal? Which conditions facilitate agreement on a grand bargain that would obviate a referendum?

In order to address these questions, I build a one-period legislative bargaining model

[1] I do not consider referenda that are constitutionally-mandated regardless of the level of consensus in the parliament. For example, many US states require constitutional amendments to be approved in a referendum regardless of the level of congressional majority.

[2] The authors find that spending both for and against a proposal influences the probability of its passage in the campaigners' intended direction. Earlier studies of the impact of campaign spending on referenda or citizen initiative outcomes include Gerber (1999) and Broder (2000). Some empirical studies such as Bowler and Donovan (1998) find asymmetric effects of money on referendum outcomes: While spending against a proposal decreases its chances of passage, a similar effect does not exist when spending supports the proposal. Lupia and Matsusaka (2004) provides an overview and discussion of these results.

in which parties bargain over a bill composed of a single-dimensional policy and a rent allocation. After the party with the most number of seats proposes a bill, other parties simultaneously vote on it. If the proposal fails to win a simple majority, it is rejected and the game ends. Otherwise, the proposal passes and immediately becomes the law if it gathers at least a supermajority of the votes. However, if the bill passes short of a supermajority, the post-bargaining stage begins in which the parties can challenge the approved bill by campaigning for or against it. The parties' exogenous campaigning budgets characterize their post-bargaining power. Whether the bill becomes the law is then determined by the public using a simple majority rule.

I analyze the political equilibrium of this model by focusing on two and three-party parliaments.[3] The results indicate that surplus coalitions are possible in the presence of looming institutional challenges. Moreover, measures of post-bargaining power do not necessarily translate into higher equilibrium payoffs as the proposer party faces a trade-off between a higher probability of having its bill upheld in a post-bargaining challenge by including a "powerful" party in its coalition and proposing a bill that captures a higher share of benefits for itself. In a two-party parliament, a grand bargain is more likely to be observed in equilibrium if the minority party commands a low status-quo payoff or if the proposer has a large campaigning budget. Similarly, parties reach a grand bargain more easily in a three-party parliament when the smaller parties do not command high status quo payoffs or if all the parties are ideologically close. In addition, I find that the chances of a referendum become higher as the campaigning budgets of the smaller parties diverge. More generally, a more asymmetric distribution of post-bargaining powers within a parliament incentivizes challenge procedures.

Having analyzed the factors that lead a dominant party to risk a subsequent challenge instead of inducing unanimity, I then study the composition of simple majority coalitions in three-party parliaments. The results show that the proposer party is more likely to partner in a referendum with the party that has a lower status quo payoff and a closer ideology. On the other hand, whether a large campaigning budget makes a party the preferred coalition partner depends on the type of political equilibrium being played. This ambiguity result is a consequence of the proposer party's main trade-off: Although a richer partner is desirable for increasing the probability that its bill is upheld, it comes at the expense of higher concessions during the bargaining stage. Which one of these

[3] A two-party parliament can be considered as representing the outcome of a first-past-the-post election system and a three-party parliament as the outcome of a proportional representation system.

effects dominates in equilibrium depends on the parameters of the model.

2 Related Literature

Building on the seminal work of Baron and Ferejohn (1989) and the uniqueness of equilibrium payoffs result proved in Eraslan (2002), the equilibrium consequences of different sources of bargaining power have been widely studied by treating the agreed-upon allocation as the final outcome.[4] Starting with Kalandrakis (2004), more recent dynamic bargaining models such as Duggan and Kalandrakis (2012) and Bowen, Chen and Eraslan (2014) have considered situations in which the agreed-upon allocation becomes the new status-quo in the next bargaining period. However, these papers do not study institutions outside of the bargaining environment through which the agreed-upon outcome can be challenged. Veto-player models such as Winter (1996) are an exception for incorporating a post-bargaining stage in which bargaining outcomes can be overturned.[5] This paper contributes to the existing literature by introducing a new source of bargaining power that is generated from post-bargaining behavior.

The institutions of direct democracy, represented here by the post-bargaining referendum, have been studied by both economists and political scientists from different angles. Romer and Rosenthal (1979) is one of the first models to study the voters' choice between a status quo policy and an alternative proposed by a bureaucrat with agenda-setting power. Lupia and Matsusaka (2004) provide an overview of the political science literature with a focus on the effects of campaign money.

The rest of the paper is organized as follows: Section 3 introduces the model. Sections 4 and 5 respectively analyze the equilibrium of two and three-party parliaments. Section 6 concludes.

[4]Some of these papers include Kalandrakis (2006) who studies proposal rights, McCarty (2000) who jointly studies proposal and veto rights, Snyder, Ting and Ansolabehere (2005) who study weighted voting, and Yildirim (2007) who studies endogenous proposal power. A branch of this literature that includes Eraslan and Merlo (2002) and Diermeier, Eraslan and Merlo (2002) studies bargaining models in which the surplus to be divided is stochastic.

[5]Powell (1996) considers a bargaining model in which players can impose outside settlements to capture the whole pie, but this happens with pre-determined probabilities. The vote of confidence mechanism in legislatures, studied in Diermeier and Feddersen (1998), is another example of a post-bargaining institution that affects the bargaining equilibrium. The authors show that the existence of such a mechanism decreases the price of building coalitions in the legislature and results in equilibrium coalitions that are more cohesive and rewarded more handsomely.

3 The Model

I consider a situation of one-period legislative bargaining over a bill that consists of ideology and distributive components, followed by a referendum if the number of votes in the parliament falls within an institutionally designated interval.

Let N denote the set of parties and $|N|$ the number of parties in the parliament. In this paper, only parliaments of two and three parties will be considered. The model consists of two stages: the bargaining stage and the challenge stage. In the bargaining stage, the party with the most number of seats proposes a bill and the other party (or parties) votes on it. In a three-party parliament, I assume that the two non-proposer parties vote simultaneously on the bill. Let $x \in [0,1]$ represent the ideological component of the proposal and let $\hat{x}_k$ denote party k's ideal ideological point. In addition, let y represent the proposed allocation of rents from the feasible set

$$Y = \{y : \sum_{k=1}^{|N|} y_k \leq 1 \text{ and } y_k \geq 0 \ \forall \ k\}, \tag{1}$$

where the fixed sum of rents is given by unity and y_k denotes party k's share. Hence, a proposal can be represented by $z \equiv (x, y) \in [0,1] \times Y$. When the proposal is introduced to the parliament, there exists a status-quo bill $s \equiv (q, y^q)$, where $q \in [0,1]$ denotes its ideological component and $y^q \in Y$ its rent allocation. I assume that party k's preferences over a bill are represented by the quasi-linear utility function

$$u_k(z) = -(x - \hat{x}_k)^2 + \alpha y_k, \tag{2}$$

where $\alpha \in (0,1)$ is some fixed weight.

After the proposer party makes an offer $z \in [0,1] \times Y$ and the other party (or parties) votes on it, the proposal is accepted or rejected according to the following criteria: Let $\bar{k}(z)$ denote the number of parties other than the proposer who support the bill z. If $\bar{k}(z) = |N| - 1$, the proposal z is unanimously accepted and becomes the law with no subsequent challenges. If $\bar{k}(z) = 0$, the bill is automatically rejected in a three-party parliament. On the other hand, rejection without a challenge is not feasible in two-party parliaments, since the proposer party always commands a simple majority. Finally, if $\bar{k}(z) = |N| - 2$, the proposal is temporarily accepted in the parliament to be challenged in a referendum. Any proposal that survives the challenge becomes the law.[6] [7]

If the proposal passes in the parliament without unanimous support, the dissenting party takes the bill to a referendum. I describe this challenge stage as a two-candidate competition in which the candidates are the proposal z and the status quo s. Before the referendum takes place, each party k simultaneously chooses a position $t \in \{Z, S\}$ and an irreversible campaign spending amount $c \geq 0$ to influence the voters (who will not be explicitly modeled). Position Z indicates a preference for the public acceptance of the proposal (yes vote on the referendum) and position S indicates a preference for its failure (no vote on the referendum).

Each party k is allocated an exogenously given campaigning budget $w_k \in [0, 1]$.[8] Upon observing the campaigns of each group, the public votes on the proposal in a referendum. If the proposal wins a simple majority of the public vote, it becomes the law. Otherwise, the status-quo bill prevails and all parties receive their status-quo payoffs. I assume that all the parameters of the model are common knowledge.

I model the referendum as a contest between the positions Z and S in which their winning prizes are given respectively by z and s. Hence, the winning prize constitutes a public good within the group that has more than one member. Let $C_t(z)$ denote the total campaign spending of parties aligned with position t when the proposed bill is z and let $p_t(C_Z(z), C_S(z))$ denote the probability that position t wins the referendum. I assume that the contest success function takes the Tullock lottery form so that the probability of winning for a party aligned with position t is given by

$$p_t(C_Z(z), C_S(z)) = \begin{cases} \frac{C_t(z)}{C_Z(z)+C_S(z)} & \text{if} \quad C_Z(z) + C_S(z) > 0 \\ \frac{1}{2} & \text{if} \quad C_Z(z) = C_S(z) = 0 \end{cases} \tag{3}$$

for $t = Z, S$ and proposal z. The above Tullock specification assumes that neither party has an inherent advantage in the contest. Moreover, it implies that a position's winning

[6] This acceptance criteria represents the following general rule in parliamentary systems for important legislation or constitutional amendment proposals. Let $\bar{k}$ denote the number of supportive legislators. If $\bar{k} \leq \frac{|N|-1}{2}$, where $|N|$ is odd, the proposal fails to win a simple majority and fails. If $\bar{k} \geq \lambda(|N| - 1)$, where $\lambda \in (\frac{1}{2}, 1)$, it is accepted without a referendum. Finally, for all $\bar{k} \in (\frac{|N|-1}{2}, \lambda(|N|-1))$, the proposal becomes law only if it is accepted in a referendum. Here, λ represents the supermajority parameter for the parliament.

[7] In a three party parliament, I assume without loss of generality that no party commands a majority of the seats and that two parties together cannot control a supermajority.

[8] Although private interest groups play an important role in financing referendum campaigns, I do not model them here in the interest of keeping the analysis tractable.

probability is increasing in the spending of the parties aligned with it and decreasing in the spending for the other position.

A pure bargaining strategy for party k consists of a proposal $z \in [0,1] \times Y$ if k is the proposer party, and an acceptance rule $a_k : [0,1] \times Y \to \{0,1\}$ for the non-proposer parties k such that $a_k(z) = 0$ indicates rejection of the proposal z and $a_k(z) = 1$ indicates its acceptance.[9] In addition, a pure challenge strategy for party k consists of the following: a position rule $\rho_k : [0,1] \times Y \to \{Z, S\}$ such that $\rho_k(z) = t$ indicates that party k has aligned itself with position t for the referendum, and a campaign spending rule $\zeta_k : [0,1] \times Y \to [0, w_k]$ such that $\zeta_k(z) = c$ yields the amount party k spends on its chosen position's campaign. Specifically, $\rho_k(z) = t$ indicates that party k spends an amount $c = \zeta_k(z)$ for position t. A party jointly chooses its position and campaign spending.

Without loss of generality, fix party 1 as the proposer party. Let $\sigma \equiv (\sigma_1, \{\sigma_k\}_{k=2}^{|N|})$ denote a strategy profile, where $\sigma_1 = (z, \rho_1, \zeta_1)$ for the proposer party and $\sigma_k = (a_k, \rho_k, \zeta_k)$ for the non-proposer party (or parties) $k \neq 1$.

Let $N_Z = \{k \in N : \rho_k(z) = Z\}$ and $N_S = \{k \in N : \rho_k(z) = S\}$ respectively denote the set of parties that align themselves with positions Z and S. Then, given a proposal z, the total campaign spending of group N_t for $t = Z, S$ can be written as

$$C_t(z) = \sum_{k \in N_t} \zeta_k(z). \tag{4}$$

Given the equilibrium behavior of every other player, a political equilibrium to this game consists of optimal party strategies during both the bargaining and the challenge stages. Through backward induction, I solve for the Subgame-Perfect Nash equilibrium of this model, which is defined below:

Definition 1. *A strategy profile $(\sigma_1, \{\sigma_k\}_{k=2}^{|N|})$ constitutes a political equilibrium if and only if the following conditions are satisfied:*

(E1) Given z and $a_k(z)$ for $k \neq 1$ from the bargaining stage, and other parties' challenge strategies ρ_{-k} and ζ_{-k}, party k's position rule $\rho_k(z) = t$ and campaign spending

[9] I assume that a party votes to accept a proposal when indifferent.

rule $\zeta_k(z) = c$ *solve*

$$\max_{t\in\{Z,S\},c\in[0,w_k]} u_k(s) + p_Z\left(\sum_{k\in N_t} c + \zeta_{-k}(z), C_{-t}(z)\right)[u_k(z) - u_k(s)] - c. \qquad (5)$$

(E2) For any given proposal z*, let*

$$V_k(z;\sigma) = u_k(s) + p_Z(C_{\rho_k(z)}(z), C_{-\rho_k(z)}(z))[u_k(z) - u_k(s)] - \zeta_k(z) \qquad (6)$$

denote party k*'s maximized expected payoff from the referendum when each party would be following its equilibrium challenge strategies. Then,*

- *If* $|N| = 2$*, or if* $|N| = 3$ *and* $a_{-k}(z) = 1$*,* $a_k(z) = 1$ *if and only if* $u_k(z) \geq V_k(z;\sigma)$*;*
- *If* $|N| = 3$ *and* $a_{-k}(z) = 0$*,* $a_k(z) = 1$ *if and only if* $V_k(z;\sigma) \geq u_k(s)$*.*

(E3) Party 1's proposal z *solves*

$$\max_{z\in[0,1]\times Y} u_1(s) + p_Z(C_Z(z), C_S(z)) \cdot [u_1(z) - u_1(s)] - \zeta_1(z). \qquad (7)$$

Condition (E1) requires that each party's position and campaign spending rules maximize its expected payoff from the referendum. Condition (E2) rules out the use of weakly dominated strategies by the non-proposer party (or parties) during legislative voting. It requires that an acceptance vote is given to a proposal if and only if it is weakly preferred to voting to reject it. Finally, condition (E3) requires that given the subsequent optimal acceptance, position, and campaign spending rules of all the parties, the proposer party makes an offer that maximizes its expected payoff.

Given the existence of equilibria in contests that describe the challenge stage of this model and the existence of a bargaining equilibrium for any profile of challenge strategies, a political equilibrium exists. In the following sections, I characterize the pure strategy Subgame Perfect Nash equilibria of this model respectively for two and three-party parliaments.

4 Two-Party Parliaments

To characterize equilibrium in a two-party parliament, I assume that the proposer party controls a simple majority but not a supermajority of the seats so that it needs the approval of party 2 in order to avoid a challenge stage.

First, consider the parties' equilibrium challenge strategies. If the game reaches this stage, the position choices for the referendum are trivial: On the equilibrium path, party 1 never campaigns against its own proposal so that $\rho_1(z) = Z$ for any given proposal z. Similarly, if party 2 preferred a yes vote on the referendum, it would have accepted the proposal z during bargaining in order to secure a sure outcome and not incur campaigning costs. Hence, $\rho_2(z) = S$ always holds as well on the equilibrium path.

Given these equilibrium position rules, the optimal campaign spending of the two parties for any given proposal z are given by

$$\zeta_1(z) \in \arg\max_{c \in [0, w_1]} \frac{c}{c + \zeta_2(z)} u_1(z) + \frac{\zeta_2(z)}{c + \zeta_2(z)} u_1(s) - c; \tag{8}$$

$$\zeta_2(z) \in \arg\max_{c \in [0, w_2]} \frac{\zeta_1(z)}{\zeta_1(z) + c} u_2(z) + \frac{c}{\zeta_1(z) + c} u_2(s) - c. \tag{9}$$

For a more concise exposition in the following analysis, let

$$\epsilon_k(z) = |u_k(z) - u_k(s)| \tag{10}$$

represent party k's stake from the challenge stage for any given proposal z, given by the difference in its payoff from the two potential outcomes z and s. Based on (8) and (9), the following lemma describes how a party's equilibrium campaign spending responds to the bargaining outcome:[10]

Lemma 1. *Let z and z' be two proposals such that $\epsilon_k(z) \geq \epsilon_k(z')$ for party $k = 1, 2$. Then, $\zeta_k(z) \geq \zeta_k(z')$.*

Lemma 1 states that each party's equilibrium campaign spending increases with its stake from the referendum. For example, if one of two proposals implies a lower payoff relative to the status-quo for party 2, then party 2 would fight harder for the failure of this proposal in the referendum. The larger difference between the winning and the

[10] All proofs are in the Appendix.

losing prizes justifies a higher amount of spending compared to the proposal with the lower stakes.

In the rest of this section, I first present the general characteristics of a political equilibrium in Proposition 1. Then, in Proposition 2, I focus on the parameter values that make a political equilibrium in which the challenge stage is reached on the equilibrium path more likely to be observed than one in which the parties settle in the parliament.

Proposition 1. *In the political equilibrium of a two-party parliament,*

1. *If the parties agree to settle, the proposer party captures a surplus equal to w_1 from the smaller party. In this unanimity-inducing proposal, the two parties compromise equally on ideology and the difference between their rent shares, $y_1 - y_2$, increases as $u_2(s)$ decreases or w_1 increases.*

2. *If the proposer party chooses to induce a referendum, it becomes more likely to do so by offering $z = (\hat{x}_1, 1, 0)$ as opposed to any other proposal that yields a higher utility for party 2 as the two parties diverge ideologically.*

The first part of Proposition 1 indicates that the threat of a challenge allows the proposer to extract a surplus from party 2's status-quo payoff in a settlement. Specifically, whenever the offer on the table involves sufficiently high stakes for the proposer that it would fight with its entire campaigning budget during a subsequent challenge, party 2's meager winning prize would not justify its counter campaign spending to defend the status-quo. Therefore, party 2 is willing to accept a cut from its status-quo payoff in order to avoid this expensive challenge.

The second part of the proposition focuses on the type of challenge equilibrium that would be preferred by the proposer. The analysis indicates that the optimal proposal to induce a challenge in which party 2 exhausts its campaigning budget is the one that maximizes party 1's winning prize, given by $z = (\hat{x}_1, 1, 0)$. This is due to the fact that the probability of winning for party 1 is not affected by how much further party 2's stake increases if party 2 is already spending its entire budget. For all other types of challenge stage equilibria that involve a lower spending by party 2, the proposer faces the following trade-off: Even though a more favorable proposal for itself increases party 1's winning prize, this comes at the expense of decreasing its winning probability as party 2 fights more aggressively by spending more. As $\hat{x}_1$ and $\hat{x}_2$ diverge, party 1's expected payoff from this challenge may decrease sufficiently that a proposal short of $z = (\hat{x}_1, 1, 0)$ is no longer justified. More specifically, as the value of $\epsilon_2(z)$ increases due to this divergence, leading to higher spending by party 2, the proposal compromise that was made in the hopes of putting a check on party 2's spending no longer pays off. In this situation, party 1 would be better-off offering $x = \hat{x}_1$ with all the rent allocated to itself, thereby provoking an all-out fight with $\zeta_1(z) = w_1$ and $\zeta_2(z) = w_2$.

Having described party 1's incentives in choosing how best to realize a unanimity outcome in the parliament or to induce a challenge, it remains an open question which option party 1 will prefer. The following proposition takes up this task:

Proposition 2. *Party 1 is more likely to prefer the unanimity outcome over a challenge for lower values of $u_2(s)$ and higher w_1. A lower w_2 incentivizes unanimity only if $w_2 > \alpha - u_1(s)$.*

The intuition for why a smaller status-quo payoff for party 2 unambiguously contributes to a higher likelihood of observing unanimity is straight-forward: Since party 1 offers $u_2(z) = u_2(s) - w_1$ to party 2 in order to get its acceptance, a lower $u_2(s)$ increases its sure payoff from the settlement. On the other hand, while a higher w_1 may contribute to a higher probability of winning for party 1 in a particular challenge, it also increases its unanimity payoff as w_1 is extracted from party 2. In equilibrium, the effect of w_1 on its unanimity payoff dominates the challenge stage effect, yielding the result in Proposition 2.

The conditional result in Proposition 2 on how w_2 affects party 1's incentives between a settlement and a challenge illustrates another interesting trade-off. In a challenge stage equilibrium with $(\zeta_1(z), \zeta_2(z)) = (w_1, w_2)$, changes in w_2 only affect party 1's probability of winning in the referendum. On the other hand, if the equilibrium is such that $\zeta_1(z) < w_1$ and $\zeta_2(z) = w_2$, changes in w_2 affect not only party 1's probability of winning, but also its campaign spending. Specifically, a higher w_2 unambiguously decreases party 1's winning probability in this equilibrium, while it also decreases $\zeta_1(z)$ when the condition in Proposition 2 holds. When this is true, the marginal effect of lower campaign spending on party 1's expected payoff from this challenge dominates the marginal effect of a lower winning probability, resulting in an increase in party 1's expected challenge payoff. Thus, in this challenge stage equilibrium, sufficiently high values of w_2 do not act as threat instruments due to their indirect effect on party 1's campaign spending.

Based on this analysis, we would expect to observe a proposer party with a high campaigning budget work towards achieving unanimity by buying the smaller party out. In contrast, the smaller party would act more aggressively by shunning a settlement if the stakes from the proposed bill are high enough. As its budget grows, this can initially act as a threat and therefore encourage unanimity. However, this effect may be reversed once a threshold is crossed. At this point, the smaller party's budget starts to constitute an impediment to settlement.

5 Three-Party Parliaments

Studying a three-party parliament offers richer dynamics on coalition formation and incentives for a grand bargain in a non-cooperative framework than the two-party setting allowed. In this section, I assume that neither party controls a simple majority of the seats and that two parties together do not command a supermajority. Therefore, at least two parties must agree in order for a bill to pass in the parliament. A bill that has passed in the parliament with votes short of unanimity moves to the challenge stage. To abstract away from potential informational advantages, I assume that after party 1 makes an offer, the other two parties vote on it simultaneously.

When making an offer, party 1 can induce one of the following four general outcomes: A grand bargain with unanimous agreement; rejection in the parliament; a challenge stage with party 2 as its partner and party 3 in the opposition; or a challenge stage with party 3 as its partner and party 2 in the opposition. Looking for a political equilibrium in a three-party parliament involves solving for the optimal offers that would induce each of the alternative outcomes and comparing party 1's maximum expected payoffs from those outcomes.

Baik (2008) characterizes the pure-strategy Nash equilibrium of group contests in which the winning prize is a public good within each group. Since the winning probability in the referendum is a function of each party group's *total* campaign spending, this characterization applies to the equilibrium of the challenge stage in this model. Specifically, since there are always two parties aligned with position Z in a challenge, the proposal z, which is the winning prize for members of group N_Z, constitutes a public good within this group.

To start characterizing the equilibrium of the challenge stage, first consider the parties' position choices for a given proposal z. As in the two-party case, it can never be optimal for the proposer to take a stand against its own bill so that we have $\rho_1(z) = Z$ on the equilibrium path for any given z. In order to have reached the challenge stage, it must have been the case that one party voted for the bill and one against it in the parliament. Let h and j denote these two non-proposer parties such that $a_h(z) = 1$ and $a_j(z) = 0$. If party h preferred a no vote on the referendum, it would have voted to reject the proposal in the bargaining stage, leading to its defeat and thereby avoiding a costly and risky referendum. Therefore, $\rho_h(z) = Z$ on the equilibrium path. Similarly, if party j preferred a yes vote on the referendum, it would have voted to accept the proposal during bargaining, leading to a unanimous agreement on z. Hence, it must be the case that $\rho_j(z) = S$ on the equilibrium path. Therefore, party h for whom $a_h(z) = 1$ becomes party 1's partner in the challenge stage and party j for whom $a_j(z) = 0$ becomes its opponent.

In the challenge stage, each group N_t for $t \in \{Z, S\}$ decides on a total campaign spending $C = C_t(z)$, where $C_t(z)$ is as defined in (4). The members of a group do not act cooperatively; instead, campaign spending choices are made independently. For a given proposal z and the total campaign spending of group N_S given by $C_S(z) = \zeta_j(z)$, let $C_Z^1(z)$ denote the best response total campaign spending of group N_Z to $C_S(z)$ from the perspective of party 1 and let $C_Z^h(z)$ denote the same best response from the perspective of its partner party h. Specifically, define $C_Z^1(z)$ and $C_Z^h(z)$ such that

$$C_Z^1(z) \in \underset{C \in [0, w_1 + w_h]}{\arg\max} \frac{C}{C + C_S(z)} u_1(z) + \frac{C_S(z)}{C + C_S(z)} u_1(s) - \zeta_1(z); \tag{11}$$

$$C_Z^h(z) \in \underset{C \in [0, w_1 + w_h]}{\arg\max} \frac{C}{C + C_S(z)} u_h(z) + \frac{C_S(z)}{C + C_S(z)} u_h(s) - \zeta_h(z). \tag{12}$$

As long as the proposal z is such that $\epsilon_1(z) \neq \epsilon_h(z)$, party 1 and its partner h have different opinions as to how they should best respond to $C_S(z)$. Moreover, since the winning prize z is a public good for them, the decision on how the burden of the total spending will be shared in equilibrium is not trivial.

The following lemma, based on Baik (2008), characterizes how the total campaign spending $C_Z(z)$ of group N_Z is determined and its burden is shared among parties 1 and h in a Nash equilibrium. This lemma will then be used to characterize the equilibrium of the challenge stage.

Lemma 2. *Suppose the proposal z is such that $\epsilon_1(z) \geq \epsilon_h(z) > 0$. Then, taking the total* *campaign spending* $C_S(z) = \zeta_j(z)$ *of group* N_S *as given, parties 1 and h choose their total campaign spending* $C_Z(z)$ *and its allocation between* $\zeta_1(z)$ *and* $\zeta_h(z)$ *as follows:*

1. *If* $C_Z^1(z) \leq w_1$, *then* $C_Z(z) = \zeta_1(z) = C_Z^1(z)$ *and* $\zeta_h(z) = 0$.

2. *If* $C_Z^h(z) \geq w_1 + w_h$, *then* $C_Z(z) = w_1 + w_h$, $\zeta_1(z) = w_1$, *and* $\zeta_h(z) = w_h$.

3. *If* $C_Z^1(z) > w_1$ *and* $C_Z^h(z) \leq w_1 + w_h$, *then* $C_Z(z) = max\{C_Z^h(z), w_1\}$, $\zeta_1(z) = w_1$, *and* $\zeta_h(z) = max\{0, C_Z^h(z) - w_1\}$.

Lemma 2 provides a full characterization of the equilibrium campaign spending of the members of group N_Z. To gain some intuition, first note that the party with the higher stake from a challenge, determined by the proposal z from the bargaining stage, will have a higher total campaign spending best response to group N_S than its partner. Part 1 of the lemma indicates that if the party with the higher stake can afford its best response total campaign spending using only its own resources, then it is the only member of group N_Z that contributes to the campaign in equilibrium; its partner free-rides on its spending. This campaign more than meets the partner's needs, obviating any spending on the partner's part. On the other hand, if the total resources of the group cannot cover even the lower best response of the partner, then part 2 of the lemma indicates that each member exhausts its budget in equilibrium. There exists no free-riding in this situation. Finally, if the party with the higher stake cannot afford its best response total campaign spending with its own resources but the partner's lower best response can be met with the total group budget, then the higher-stake party spends its entire budget on the campaign while its partner contributes the difference (if the difference is positive). In this scenario, the partner is at best a partial free-rider on the higher-stake party's campaign spending.

In short, Lemma 2 shows that unless the stakes from a challenge are sufficiently high for both members of group N_Z, the party with the lower stake free-rides on its partner's campaign spending that contributes positively to its probability of winning in the referendum. The following lemma uses the results of Lemma 2 in order to describe the general properties of a challenge stage equilibrium, which requires that group N_Z is in equilibrium and that both groups are best-responding to each other:

Lemma 3. *Let* z *and* z' *be two proposals such that* $\epsilon_k(z) \geq \epsilon_k(z')$ *for party* $k = 1, 2, 3$. *Then,* $\zeta_k(z) \geq \zeta_k(z')$ *in equilibrium. Moreover, for any given proposal* z, *the condition*

$\epsilon_1(z) \geq \epsilon_h(z)$ needs to hold in order for party $h \in N_Z$ to free-ride on $\zeta_1(z)$ in a challenge stage equilibrium.

Lemmas 2 and 3 together describe the properties of a challenge equilibrium for any proposal z from the bargaining stage. Based on this challenge equilibrium, the political equilibrium of the model can be solved for via backward induction. The following propositions present general results on a political equilibrium. Following the same order of analysis as in the previous section, I first study the structure of proposals that would induce a grand bargain or a subsequent challenge. Then, I focus in the remainder of the section on the conditions that make a grand bargain among the three parties more likely to be observed on the equilibrium path than a challenge.

Proposition 3. *The following are true of inducing a grand bargain in the political equilibrium of a three-party parliament:*

1. *In a unanimous agreement on a proposal z that would otherwise lead to a challenge with free-riding in group N_Z, the party who would have been the free-rider partner is punished.*
2. *In party 1's optimal unanimity-inducing offer z, its rent share y_1 increases in y_1^q, w_1, and $(q - \hat{x}_k)$ for $k = 2, 3$. Furthermore, its unanimity payoff $u_1(z)$ increases as the three parties get ideologically closer.*

Proposition 3 discusses the structure of proposals on which a grand bargain is achievable. The first part indicates that a proposal z on which a grand bargain is possible reflects the division of $C_Z(z)$ among parties 1 and $h \in N_Z$ that would be observed if z was instead rejected. For example, the proof shows that if an offer z implies a challenge stage equilibrium in which $\zeta_1(z) = w_1$ and $\zeta_h(z) = 0$, party 1 extracts a premium from party $h \in \{2, 3\}$ equal to w_1 in a grand bargain. Likewise, if the opposite is true, party 1 needs to offer party h a premium of w_h in order to persuade it to join in the agreement.

The second part of Proposition 3 characterizes the properties of the optimal offer for party 1 that would induce unanimity. Not surprisingly, we observe that party 1 captures a higher share of the surplus as it becomes a more powerful player, either due to a higher status-quo or a higher campaigning budget. The intuition for these effects is as follows: A higher status-quo rent share for party 1 means that the other parties command less, thereby decreasing the amount they need to be compensated for in a grand bargain.

Likewise, the more non-proposer parties are away from their ideal ideological points in the status-quo, the lower the compensation they require. On the effect of w_1 on y_1, the proof shows that party 1's optimal unanimity-inducing offer z is such that if rejected, it would lead to a challenge equilibrium with $\zeta_1(z) = w_1$. Thus, w_1 can be interpreted as party 1's reward for making an offer that "saves" the non-proposer parties the spending on their groups' campaigns. Nonetheless, party 1 needs to compensate them for their ideological loss in the form of higher rent shares in proposal z. Therefore, the results indicate that an ideologically-divided parliament always hurts party 1 in a grand bargain.

Having studied the structure of a unanimous agreement in a three-party parliament, the following proposition focuses on the same questions for a referendum:

Proposition 4. *The following are true of inducing a challenge with party h as the partner and party j as the opponent of party 1 in the political equilibrium of a three-party parliament:*

1. *For any challenge-inducing proposal z, party 1's expected payoff from a challenge increases as y_h^q decreases, $(q - \hat{x}_h)^2$ increases, and $\hat{x}_1$ and $\hat{x}_h$ get closer.*
2. *For any challenge-inducing proposal z for which $\zeta_h(z) > 0$, a higher w_h decreases party 1's expected payoff from the challenge if w_h and $u_1(s)$ are sufficiently high.*

The results in Proposition 4 illustrate party 1's incentives when deciding on the identity of its partner in a challenge. First, the proposition states that it necessarily increases party 1's expected payoff from a challenge if its partner has a lower status-quo payoff. This is due to the fact that a party always requires at least its status-quo payoff in order to become party 1's partner regardless of whether it will contribute to group N_Z's campaign spending or become a free-rider in equilibrium. Before discussing these incentives in more detail, specifically part 2 of Proposition 4, the following corollary presents some results on the proposer's choice of partner in a challenge.

Corollary 1. *All else constant, party 1 prefers to partner with party 2 instead of party 3 if*

- $u_2(s) \leq u_3(s)$;
- $w_2 > w_3$ *for any proposal that implies a challenge stage equilibrium with* $\zeta_h(z) = 0$;

- $w_2 \leq w_3$ *whenever* w_h *and* $u_1(s)$ *are sufficiently high, and* $w_2 > w_3$ *otherwise, for any proposal* z *that implies a challenge stage equilibrium with* $\zeta_h(z) > 0$.

The fact that a lower status-quo payoff makes it more likely for a party to be designated as party 1's partner in a challenge-inducing proposal follows directly from part 1 of Proposition 4. To gain an intuition for why party 1's decision on whether to partner with the high or the low-budget party depends on the type of challenge stage equilibrium considered and on the level of resources, note that the amount of a partner's campaigning resources have two opposing effects on party 1's expected challenge payoff: In an equilibrium with positive contributions from the partner, a higher w_h weakly increases the proposal's winning probability. However, a party also demands a premium over its status-quo payoff from party 1 for agreeing to become an active partner. The analysis indicates that for proposals that imply a challenge with an active partner, the positive effect of a higher w_h on party 1's expected challenge payoff due to a higher probability of winning is dominated by its negative effect due to a higher payment to the partner whenever w_h is too high or party 1's stakes from the challenge are too low. In this case, a higher w_h overall decreases party 1's expected payoff from such a challenge, because the high payment needed to persuade a rich party to become a partner does not justify the increase in party 1's winning probability. On the other hand, for lower values of w_h and $u_1(s)$ that imply high stakes from the challenge, the payment to the partner is justified. In this situation, party 1 would prefer the richer party as its partner.

However, Corollary 1 also indicates that this trade-off between a higher winning probability and a higher partner premium disappears once an equilibrium with a free-rider partner is considered. In these cases, a party can no longer demand a premium for agreeing to become a partner and its budget no longer affects the proposal's probability of winning. However, the opponent's budget w_j negatively affects party 1's expected challenge payoff, giving party 1 the incentive to designate the low-budget party as its opponent.

Given the previous results in Proposition 3 on inducing a grand bargain and the above results on possible challenges, the following proposition presents the main result of this section on party 1's preference between a grand bargain and a challenge:

Proposition 5. *In the political equilibrium of a three-party parliament, party 1 becomes more likely to prefer a grand bargain over a challenge as*

- *The non-proposer parties command lower status-quo payoffs;*
- *The three parties get ideologically closer;*
- *The non-proposer parties' campaigning budgets become more similar.*

The first and the second parts of Proposition 5 are a direct implication of party 1's unanimity payoff. To see why similar campaigning budgets between the non-proposer parties incentivizes a grand bargain, note that w_2 and w_3 do not affect party 1's unanimity payoff, but they determine the proposal required to induce a given challenge equilibrium. In a challenge stage equilibrium in which the partner also contributes, the premium it demands increases as its resources become more similar to the opponent's, because this increases the competitiveness of the referendum. Since this decreases party 1's expected payoff from this challenge, it will be more likely to prefer a grand bargain.

The results on the proposer's incentives between a grand bargain and a challenge in a three-party parliament mirror those in a two-party parliament. Specifically, the results for both types of parliaments indicate that lower status-quo payoffs of the non-proposer parties always incentivize unanimity. Moreover, both sections suggest that a partner's higher budget can be a blessing for the proposer in a challenge as long as it is not too high. However, due to the presence of an additional party that the proposer can play against the other, the results on non-cooperative coalition formation are richer in the three-party parliament setting.

6 Concluding Remarks

This paper developed a model of legislative bargaining over a bill consisting of both an ideology and a distributive component followed by a challenge stage. I addressed the question of how an institutional challenge mechanism such as a referendum affects the parties' optimal behavior in a parliament. The analysis of a proposer's incentives between a grand bargain and a challenge indicates that post-bargaining power does not necessarily translate into higher equilibrium payoffs. Although the focus of the model is on legislative bargaining over proposals that can be subsequently challenged, its insights are applicable to other settings, including private sector organizational models. For example, the players in the model can be chosen to represent the board of directors of a corporation, with the chairman as the proposer and shareholders as the voters on proposals not approved with sufficient majority in the board room.

The results here have implications for campaign finance policies. Even though referenda can be both publicly and privately financed in most countries, this model is silent on this issue. The results for both two and three-party parliaments indicate that whether high or low campaigning budgets incentivize grand bargains depend on the parameters of the model. Therefore, if a planner's goal is to propagate unanimously-approved deals in the parliament over costly challenge procedures, the appropriate campaign finance policy will depend on the status-quo commanded by each party and their private financing options.

There exists a number of directions in which the model can be extended. For example, while I assumed that all the parameters on campaigning budgets, ideal ideological points and status-quo payoffs are common knowledge, incorporating uncertainty with regards to either one of these parameters can be a natural extension. Although I believe that complete information is a more realistic setting in this model of a public interaction, incomplete information might be a better depiction of reality in private interaction models such as the corporate board example. Extending the model to N players for a more general setting or specifically modeling voters with ideological preferences may also yield interesting results on the dynamics of non-cooperative coalition formation.

Finally, this model does not entertain the possibility of new rounds of bargaining following a challenge stage. However, in reality, political processes might reconsider the same measures. Therefore, this might be a useful endeavor for the purpose of capturing the dynamic aspects of similar political processes. Similarly, an additional stage of legislative elections would make voters strategic by giving them control over the identity of the proposer.

It is important to stress that I do not make any efficiency arguments in favor of one policy over another. For example, if the results suggest caps on campaign financing to incentivize grand bargains for certain ranges of parameters, this study can still not answer the question of how this policy would affect voter welfare. Any attempt to answer this question would require a normative exercise this paper does not perform.

7 Appendix

Proof of Lemma 1. Based on (8) and (9), the first-order conditions for the parties' optimal campaign spending choices are given by

$$\epsilon_1(z)\left[\frac{\zeta_2(z)}{(\zeta_1(z)+\zeta_2(z))^2}\right]-1\begin{cases}\geq 0 & \text{if } \zeta_1(z)>w_1\\ =0 & \text{if } \zeta_1(z)\in[0,w_1]\\ \leq 0 & \text{if } \zeta_1(z)=0;\end{cases} \tag{13}$$

and

$$\epsilon_2(z)\left[\frac{\zeta_1(z)}{(\zeta_1(z)+\zeta_2(z))^2}\right]-1\begin{cases}\geq 0 & \text{if } \zeta_2(z)>w_2\\ =0 & \text{if } \zeta_2(z)\in[0,w_2]\\ \leq 0 & \text{if } \zeta_2(z)=0.\end{cases} \tag{14}$$

Solving (13) and (14) implies that the unique pair of equilibrium campaign spending rules $(\zeta_1(z),\zeta_2(z))$ is given by one of the following four equilibrium candidates, depending on the outcome of the bargaining stage:

1. $(\zeta_1(z),\zeta_2(z))=(w_1,w_2)$ if $\epsilon_1(z)\geq\frac{(w_1+w_2)^2}{w_2}$ and $\epsilon_2(z)\geq\frac{(w_1+w_2)^2}{w_1}$.
2. $(\zeta_1(z),\zeta_2(z))=(w_1,\sqrt{w_1\epsilon_2(z)}-w_1)$ if $\epsilon_1(z)\geq\frac{w_1\epsilon_2(z)}{\sqrt{w_1\epsilon_2(z)}-w_1}$ and $\epsilon_2(z)\leq\frac{(w_1+w_2)^2}{w_1}$.
3. $(\zeta_1(z),\zeta_2(z))=(\sqrt{w_2\epsilon_1(z)}-w_2,w_2)$ if $\epsilon_1(z)\leq\frac{(w_1+w_2)^2}{w_2}$ and $\epsilon_2(z)\geq\frac{w_2\epsilon_1(z)}{\sqrt{w_2\epsilon_1(z)}-w_2}$.
4. $(\zeta_1(z),\zeta_2(z))=(\frac{\epsilon_1(z)^2\epsilon_2(z)}{[\epsilon_1(z)+\epsilon_2(z)]^2},\frac{\epsilon_1(z)\epsilon_2(z)^2}{[\epsilon_1(z)+\epsilon_2(z)]^2})$ if $\frac{\epsilon_1(z)^2\epsilon_2(z)}{[\epsilon_1(z)+\epsilon_2(z)]^2}<w_1$ and $\frac{\epsilon_1(z)\epsilon_2(z)^2}{[\epsilon_1(z)+\epsilon_2(z)]^2}<w_2$.

If the challenge stage equilibrium involves $\zeta_k(z)=w_k$ for some k, then $\zeta_k(z)$ is constant in the value of $\epsilon_k(z)$. On the other hand, if $\zeta_k(z)=\sqrt{w_{-k}\epsilon_k(z)}-w_{-k}$ or if the equilibrium is interior as characterized in item four above, then $\zeta_k(z)$ is increasing in the value of $\epsilon_k(z)$. This is straightforward to see for the first case. To see that $\zeta_k(z)$ is increasing in the value of $\epsilon_k(z)$ when the challenge stage equilibrium is interior, differentiate $\zeta_k(z)$ as characterized in item four with respect to the value of $\epsilon_k(z)\equiv\bar{\epsilon}_k$ to get

$$\frac{(2\bar{\epsilon}_k\bar{\epsilon}_{-k})(\bar{\epsilon}_k+\bar{\epsilon}_{-k})^2-2(\bar{\epsilon}_k^2\bar{\epsilon}_{-k})(\bar{\epsilon}_k+\bar{\epsilon}_{-k})}{(\bar{\epsilon}_k+\bar{\epsilon}_{-k})^4}. \tag{15}$$

The denominator of (15) is clearly positive and it can be shown with a simplification that its numerator is also positive. Hence, the equilibrium campaign spending of each party k as characterized in item four is increasing in the value of $\epsilon_k(z)$. This completes the proof of Lemma 1.

$\square$

Proof of Proposition 1. Using backward induction, I first characterize the equilibrium acceptance strategy $a_2(z)$ of party 2 for any given proposal z. If party 2 accepts the proposal z, its payoff would be $u_2(z)$ with certainty. Since it is risk-neutral, party 2 will accept any offer that yields a sure payoff of $u_2(z)$ that is at least as great as its expected payoff from the challenge stage that would be observed based on $\epsilon_1(z)$ and $\epsilon_2(z)$.

Given w_1, w_2, and the status-quo bill s, suppose party 1 makes an offer z such that $\epsilon_k(z) \geq \frac{(w_1+w_2)^2}{w_{-k}}$ for both k. If rejected, this offer would imply $(\zeta_1(z), \zeta_2(z)) = (w_1, w_2)$. Therefore, since $\rho_1(z) = Z$ and $\rho_2(z) = S$ in any challenge stage equilibrium, this proposal z implies an expected payoff for party 2 from the challenge stage given by

$$u_2(s) + \left(\frac{w_1}{w_1 + w_2}\right)[u_2(z) - u_2(s)] - w_2. \tag{16}$$

Comparing the sure payoff $u_2(z)$ with (16) implies that party 2 accepts z if and only if $u_2(z) \geq u_2(s) - (w_1 + w_2)$, which can also be written as $\epsilon_2(z) \leq w_1 + w_2$. However, since this proposal z is such that $\epsilon_2(z) \geq \frac{(w_1+w_2)^2}{w_1}$ and $\frac{(w_1+w_2)^2}{w_1} > w_1 + w_2$, the acceptance criteria can never be satisfied. Hence, any proposal z that points to a challenge stage in which $(\zeta_1(z), \zeta_2(z)) = (w_1, w_2)$ if rejected will be rejected by party 2.

Second, suppose party 1 makes an offer z such that the conditions for the challenge stage equilibrium in which $(\zeta_1(z), \zeta_2(z)) = (w_1, \sqrt{w_1\epsilon_2(z)} - w_1)$ as listed in item two in the proof of Lemma 1 are satisfied. This offer implies the following expected payoff for party 2 from the challenge stage:

$$u_2(s) + \sqrt{\frac{w_1}{\epsilon_2(z)}}[u_2(z) - u_2(s)] - \sqrt{w_1\epsilon_2(z)} + w_1. \tag{17}$$

Comparing the sure payoff $u_2(z)$ with (17) implies that party 2 accepts z if and only if

$$u_2(z) \geq u_2(s) + \frac{w_1\sqrt{\epsilon_2(z)} - \sqrt{w_1}\epsilon_2(z)}{\sqrt{\epsilon_2(z)} - \sqrt{w_1}}, \tag{18}$$

where the last term is negative. Simplifying (18) yields the condition that $\epsilon_2(z) \leq w_1$. Therefore, party 2 will accept any proposal z that would imply a subsequent challenge stage with $(\zeta_1(z), \zeta_2(z)) = (w_1, \sqrt{w_1\epsilon_2(z)} - w_1)$ as long as $\epsilon_2(z) \leq w_1$.

Third, suppose party 1 makes an offer z such that the resulting challenge stage equilibrium if z were rejected would be characterized by $(\zeta_1(z), \zeta_2(z)) = (\sqrt{w_2\epsilon_1(z)} - w_2, w_2)$. The expected challenge stage payoff for party 2 is given in this case by

$$u_2(s) + \frac{\sqrt{w_2\epsilon_1(z)} - w_2}{\sqrt{w_2\epsilon_1(z)}}[u_2(z) - u_2(s)] - w_2. \tag{19}$$

Based on (19), party 2 accepts any offer z that satisfies $\epsilon_2(z) \leq \sqrt{w_2\epsilon_1(z)}$. However, since this proposal z is such that $\epsilon_2(z) \geq \frac{w_2\epsilon_1(z)}{\sqrt{w_2\epsilon_1(z)}-w_2}$, the acceptance criteria can never be satisfied, because $\sqrt{w_2\epsilon_1(z)} < \frac{w_2\epsilon_1(z)}{\sqrt{w_2\epsilon_1(z)}-w_2}$. Therefore, party 2 will reject all offers that would subsequently lead to a challenge stage with $(\zeta_1(z), \zeta_2(z)) = (\sqrt{w_2\epsilon_1(z)} - w_2, w_2)$.

Finally, suppose party 1's offer z is such that the challenge stage equilibrium would be characterized by the interior equilibrium as listed in item four in the proof of Lemma 1. Constructing the expected payoff from the challenge stage as in the above three cases yields the condition that z must satisfy $\epsilon_2(z) \leq \zeta_1(z) + \zeta_2(z)$ in order to be accepted by party 2. Plugging in the equilibrium values of $\zeta_1(z)$ and $\zeta_2(z)$ into this condition yields

$$\epsilon_2(z) \leq \frac{\epsilon_1(z)\epsilon_2(z)}{\epsilon_1(z) + \epsilon_2(z)}, \tag{20}$$

which reduces to $u_2(z) \geq u_2(s)$.

Bringing together the above analysis of party 2's acceptance criteria for each possible challenge stage equilibrium, we observe that any proposal z for which $\zeta_2(z) = w_2$ is rejected (although these are not the only offers that will be rejected). In addition, whenever z is such that $\zeta_2(z) < w_2$, party 2 accepts any offer for which $\epsilon_2(z) \leq w_1$ if $\zeta_1(z) = w_1$ and any offer for which $\epsilon_2(z) \leq 0$ (i.e. $u_2(z) \geq u_2(s)$) if $\zeta_1(z) < w_1$.

Given the above characterization of party 2's equilibrium acceptance strategy, I now solve for party 1's equilibrium proposal strategy. Suppose that party 1 makes an offer that will be accepted by party 2, thereby avoiding a challenge stage. The above analysis indicated that there exist two methods by which party 1 can induce unanimity: By offering z such that a) $\epsilon_1(z) \geq \frac{w_1\epsilon_2(z)}{\sqrt{w_1\epsilon_2(z)}-w_1}$ and $\epsilon_2(z) \leq w_1$; or b) $\frac{\epsilon_k(z)^2\epsilon_{-k}(z)}{(\epsilon_1(z)+\epsilon_2(z))^2} < w_k$ for $k = 1, 2$ and $\epsilon_2(z) \leq 0$. Since method a) requires party 1 to only propose a z such that

$u_2(z) = u_2(s) - w_1$ whereas method b) requires $u_2(z) = u_2(s)$ for acceptance, party 1 would choose the first method if it wanted to induce unanimity.

For the optimal proposal, party 1 maximizes $u_1(z)$ subject to party 2's acceptance constraint $u_2(z) \geq u_2(s) - w_1$ and the technical constraint $z \in [0,1] \times Y$. The Lagrangian of this problem can be written as follows:

$$L = -(x - \hat{x}_1)^2 + \alpha y_1 + \lambda[-(x - \hat{x}_2)^2 + \alpha(1 - y_1) - u_2(s) + w_1] \tag{21}$$

$$+\mu_1 x - \mu_2(x - 1) + \gamma_1 y_1 - \gamma_2(y - 1).$$

The first-order conditions for (21) are $x \in [0,1]$, $y_1 \in [0,1]$, $\lambda \geq 0$, $\mu_1 \geq 0$, $\mu_2 \geq 0$, $\gamma_1 \geq 0$, $\gamma_2 \geq 0$,

$$-2(x - \hat{x}_1) - 2\lambda(x - \hat{x}_2) + (\mu_1 - \mu_2) \leq 0; \tag{22}$$

$$[-2(x - \hat{x}_1) - 2\lambda(x - \hat{x}_2) + (\mu_1 - \mu_2)]x = 0; \tag{23}$$

$$\alpha - \lambda\alpha + (\gamma_1 - \gamma_2) \leq 0; \tag{24}$$

$$[\alpha - \lambda\alpha + (\gamma_1 - \gamma_2)]y_1 = 0; \tag{25}$$

$$-(x - \hat{x}_2)^2 + \alpha(1 - y_1) - u_2(s) + w_1 \geq 0; \tag{26}$$

$$[-(x - \hat{x}_2)^2 + \alpha(1 - y_1) - u_2(s) + w_1]\lambda = 0; \tag{27}$$

along with $\mu_1 x = 0$; $\mu_2(1 - x) = 0$; $\gamma_1 y_1 = 0$; and $\gamma_2(1 - y_1) = 0$. An interior solution to this problem entails $\mu_1 = \mu_2 = \gamma_1 = \gamma_2 = 0$ and $\lambda = 1$ based on (25), yielding

$$x = \frac{\hat{x}_1 + \hat{x}_2}{2}. \tag{28}$$

Solving for y_2 using the fact that party 1 will never make an offer z that gives party 2 any higher utility than is needed for acceptance, $u_2(z) = u_2(s) - w_1$ implies

$$y_2 = \alpha^{-1}\left[\left(\frac{\hat{x}_1 + \hat{x}_2}{2}\right)^2 - (q - \hat{x}_2)^2 + \alpha y_2^q - w_1\right]. \tag{29}$$

Therefore, an equilibrium proposal z characterized by the ideology as given in (28) and the rent allocation with y_2 as given in (29) and $y_1 = 1 - y_2$ induces an optimal unanimity outcome for party 1. Specifically, $y_1 = 1 - y_2$ is given by

$$y_1 = \alpha^{-1}\left[-\left(\frac{\hat{x}_1 + \hat{x}_2}{2}\right)^2 + (q - \hat{x}_2)^2 + \alpha y_1^q + w_1\right]. \tag{30}$$

Therefore, the difference between the rent shares of the two parties is given by

$$y_1 - y_2 = \alpha^{-1}\left[-\frac{(\hat{x}_1 + \hat{x}_2)^2}{2} + 2(q - \hat{x}_2)^2 + \alpha(y_1^q - y_2^q) + 2w_1\right]. \tag{31}$$

Notice that this difference increases as party 2's status-quo payoff $u_2(s)$ decreases and w_1 increases. This proves part 1 of Proposition 1.

Before proceeding to the proof of part 2, consider possible corner solutions to this maximization problem. First, I claim that there exists no solution with $x = 0$ or $x = 1$. To see this, first let $\mu_1 > 0$ and $\mu_2 = \gamma_1 = \gamma_2 = 0$. This yields $\lambda = 1$ as before, resulting in the equality $2\hat{x}_1 + 2\hat{x}_2 + \mu_1 = 0$. Since this would imply $\mu_1 < 0$, the desired result is achieved. Second, let $\mu_2 > 0$ and $\mu_1 = \gamma_1 = \gamma_2 = 0$. This situation yields the equality $-4 + 2\hat{x}_1 + 2\hat{x}_2 - \mu_2 = 0$, implying that μ_2 must be negative. Hence, we can conclude that the optimal ideology component of z must be such that $x \in (0, 1)$.

Second, I claim that solutions with $y_1 = 0$ or $y_1 = 1$ are possible for certain values of α. Suppose $\gamma_1 > 0$ and $\gamma_2 = 0$. With $\mu_1 = \mu_2 = 0$, this yields $\alpha - \lambda\alpha + \gamma_1 < 0$, or $\lambda > \frac{\alpha+\gamma_1}{\alpha}$. Then, the condition $(x - \hat{x}_1) + \lambda(x - \hat{x}_2) = 0$ implies

$$\frac{x - \hat{x}_1}{x - \hat{x}_2} = -\lambda < -\left(1 + \frac{\gamma_1}{\alpha}\right), \tag{32}$$

which can hold for small values of α, yielding $y_1 = 0$. In this situation, party 1 chooses x closer to $\hat{x}_1$. Likewise, letting $\gamma_2 > 0$ implies

$$\frac{x - \hat{x}_1}{x - \hat{x}_2} = \lambda < -\left(1 - \frac{\gamma_2}{\alpha}\right), \tag{33}$$

which can hold for larger values of the parameter α, yielding $y_1 = 1$. Here, party 1 chooses x closer to $\hat{x}_2$ in order to secure party 2's acceptance. Since these solutions arise only under extreme parameter values for α, they are not included in the main proposition in the interest of space.

To prove the second part of the proposition, suppose that party 1 makes an offer that will be rejected by party 2, paving the way for a challenge. Of the four scenarios with which party 1 can push the bill z into a challenge as analyzed in the proof of part 1, two involve proposals that would imply $\zeta_2(z) = w_2$ (items one and three in the proof of Lemma 1). In these cases, the optimal z is such that $x = \hat{x}_1$, $y_1 = 1$, and $y_2 = 0$, because the exact proposal z no longer affects the probability of winning for party 1 once party 2 exceeds spending a constant sum of w_2 in the referendum. Therefore, party 1 maximizes its expected payoff from the challenge by maximizing the value of $\epsilon_1(z)$.

To see when inducing a challenge in which $\zeta_2(z) < w_2$ would be preferred to one with $\zeta_2(z) = w_2$, I focus on the challenge stage equilibrium in which $(\zeta_1(z), \zeta_2(z)) = (w_1, \sqrt{w_1\epsilon_2(z)} - w_1)$, which arises if the rejected proposal z is such that $\epsilon_1(z) \geq \frac{w_1\epsilon_2(z)}{\sqrt{w_1\epsilon_2(z)} - w_1}$ and $\epsilon_2(z) \in \left(w_1, \frac{(w_1+w_2)^2}{w_1}\right)$. For any proposal z that satisfies these conditions, the expected payoff to party 1 from this challenge is given by

$$u_1(s) + \sqrt{\frac{w_1}{\epsilon_2(z)}}\epsilon_1(z) - w_1, \tag{34}$$

maximizing which subject to the above conditions yields $x = \frac{\hat{x}_1 + \hat{x}_2}{2}$.

Party 1 prefers to induce the above challenge over the one in which $(\zeta_1(z), \zeta_2(z)) = (w_1, w_2)$ whenever

$$\sqrt{\frac{w_1}{\epsilon_2(z)}}\epsilon_1(z) \geq \frac{w_1}{w_1 + w_2}(\alpha - u_1(s)). \tag{35}$$

Note that since the proposal z in (35) satisfies $\epsilon_2(z) \in \left(w_1, \frac{(w_1+w_2)^2}{w_1}\right)$, the probability of winning is at least as high in the challenge represented on the left-hand side of (35) as on the right-hand side of it. Therefore, in order for (35) to hold, the proposal z that would induce the challenge stage equilibrium with $(\zeta_1(z), \zeta_2(z)) = (w_1, \sqrt{w_1\epsilon_2(z)} - w_1)$ must be such that

$$\epsilon_1(z) \in \left[\frac{w_1}{w_1 + w_2}(\alpha - u_1(s)), (\alpha - u_1(s))\right]. \tag{36}$$

Therefore, if the optimal proposal that would induce this challenge implies $\epsilon_1(z) < \frac{w_1}{w_1+w_2}(\alpha - u_1(s))$, party 1 prefers the challenge stage equilibrium with $\zeta_2(z) = w_2$. Since the optimal proposal to induce a challenge with $\zeta_2(z) < w_2$ involves equal compromise on ideology, $\epsilon_1(z)$ decreases as $(\hat{x}_1 - \hat{x}_2)^2$ increases. This proves part 2 of Proposition 1 and hence completes its proof.[11] □

[11] Similar comparisons between other types of challenge stage equilibria yield the same result and hence are not repeated here.

Proof of Proposition 2. If party 1 induces unanimity by offering $u_2(z) = u_2(s) - w_1$, its payoff is given by

$$u_1(z) = -\left(\frac{\hat{x}_2 - \hat{x}_1}{2}\right)^2 - \left(\frac{\hat{x}_1 + \hat{x}_2}{2}\right)^2 + (q - \hat{x}_2)^2 + \alpha y_1^q + w_1. \tag{37}$$

Suppose the parties are sufficiently distant ideologically that party 1 prefers a challenge stage equilibrium with $\zeta_2(z) = w_2$. With the optimal proposal given by $z = (\hat{x}_1, 1, 0)$, party 1's maximum expected payoff from this challenge becomes

$$u_1(s) + \frac{w_1}{w_1 + w_2}[\alpha - u_1(s)] - w_1 \tag{38}$$

if $\zeta_1(z) = w_1$, and

$$u_1(s) + \left(1 - \sqrt{\frac{w_2}{\alpha - u_1(s)}}\right)[\alpha - u_1(s)] - \sqrt{w_2(\alpha - u_1(s))} + w_2 \tag{39}$$

if $\zeta_1(z) = \sqrt{w_2 \epsilon_1(z)} - w_2$.

Comparing (37) first with (38) suggests that party 1 is more likely to prefer a settlement over a challenge for low values of $u_2(s)$, and high values of w_1 and w_2. Comparing (37) with (39) confirms the relationship with respect to $u_2(s)$ and w_1. However, differentiating (39) with respect to w_2 indicates that higher values of w_2 make settlement more likely to be preferred only if $w_2 < \alpha - u_1(s)$.

To complete the proof, now suppose that the parties are ideologically closer so that party 1 would prefer a challenge stage equilibrium with $\zeta_2(z) < w_2$. Focusing on the equilibrium with $(\zeta_1(z), \zeta_2(z)) = (w_1, \sqrt{w_1 \epsilon_2(z)} - w_1)$, party 1's expected payoff from this referendum is as given in (34), where z is such that $\epsilon_1(z) \geq \frac{w_1 \epsilon_2(z)}{\sqrt{w_1 \epsilon_2(z)} - w_1}$ and $\epsilon_2(z) \in \left(w_1, \frac{(w_1+w_2)^2}{w_1}\right)$. Comparing (37) with (34) confirms the above results on $u_2(s)$ and w_1. Therefore, we can conclude that a lower $u_2(s)$ and a higher w_1 unambiguously make settlement more likely to be observed. This completes the proof of Proposition 2. □

Proof of Lemma 2. The proof is an application of the main result in Baik (2008) for players with a budget constraint.

Based on (11), $C_Z^1(z)$ satisfies

$$\epsilon_1(z)\left[\frac{C_S(z)}{(C_Z^1(z)+C_S(z))^2}\right]-1\begin{cases}\geq 0 & \text{if } C_Z^1(z) > w_1+w_h\\ =0 & \text{if } C_Z^1(z)\in[0,w_1+w_h]\\ \leq 0 & \text{if } C_Z^1(z)=0.\end{cases} \tag{40}$$

Similarly, based on (12), $C_Z^h(z)$ satisfies

$$\epsilon_h(z)\left[\frac{C_S(z)}{(C_Z^h(z)+C_S(z))^2}\right]-1\begin{cases}\geq 0 & \text{if } C_Z^h(z) > w_1+w_h\\ =0 & \text{if } C_Z^h(z)\in[0,w_1+w_h]\\ \leq 0 & \text{if } C_Z^h(z)=0.\end{cases} \tag{41}$$

Accordingly, the individual campaign spending of parties 1 and h must satisfy the following first-order conditions in equilibrium for any given $C_S(z)=\zeta_j(z)$:

$$\epsilon_1(z)\left[\frac{C_S(z)}{(\zeta_1(z)+\zeta_h(z)+C_S(z))^2}\right]-1\begin{cases}\geq 0 & \text{if } \zeta_1(z) > w_1\\ =0 & \text{if } \zeta_1(z)\in[0,w_1]\\ \leq 0 & \text{if } \zeta_1(z)=0;\end{cases} \tag{42}$$

$$\epsilon_h(z)\left[\frac{C_S(z)}{(\zeta_1(z)+\zeta_h(z)+C_S(z))^2}\right]-1\begin{cases}\geq 0 & \text{if } \zeta_h(z) > w_h\\ =0 & \text{if } \zeta_h(z)\in[0,w_h]\\ \leq 0 & \text{if } \zeta_h(z)=0.\end{cases} \tag{43}$$

First, suppose that $C_Z^1(z)\leq w_1$ so that solving (40) yields $C_Z^1(z)=\sqrt{\epsilon_1(z)C_S(z)}-C_S(z)$. Then, the individual best response of party 1 to its partner must also be less than or equal to w_1. Furthermore, it must equal $C_Z^1(z)$. By the assumption in Lemma 2 that $\epsilon_1(z)\geq\epsilon_h(z)$, it must be true that $C_Z^1(z)\geq C_Z^h(z)$. Therefore, the best response of party h to party 1's best response of choosing $C_Z^1(z)$ for any given $\zeta_h(z)$ is to spend a zero amount on the group's campaign. In equilibrium, simultaneous best responding implies $\zeta_1(z)=C_Z^1(z)$ and $\zeta_2(z)=0$ for a total equilibrium campaign spending of $C_Z(z)=C_Z^1(z)$. This proves part 1 of Lemma 2.

For part 2, suppose that $C_Z^h(z)\geq w_1+w_h$, which implies $C_Z^1(z)\geq w_1+w_h$. Then, the individual best response of each party to the other must be greater than its respective budget. This implies that we must have $\zeta_1(z) = w_1$ and $\zeta_h(z) = w_h$ in equilibrium, for a total campaign spending of $C_Z(z) = w_1 + w_h$. This proves part 2 of Lemma 2.

For the final part of the lemma, suppose the proposal z is such that $C_Z^1(z) > w_1$ and $C_Z^h(z) \leq w_1 + w_h$. First, consider the case in which $C_Z^h(z) \leq w_1$. For any $\zeta_1(z) \in [C_Z^h(z), w_1]$, party h's individual best response to party 1 is to choose a zero amount of campaign spending since $\zeta_1(z) \geq C_Z^h(z)$. This would imply a total campaign spending of $C_Z(z) \in [C_Z^h(z), w_1]$. However, since $C_Z^1(z) > w_1$, this cannot be optimal for party 1. Specifically, party 1 would have an incentive to increase its spending up to w_1. Similarly, for any $\zeta_1(z) < C_Z^h(z)$, party h best responds by choosing $\zeta_h(z) = C_Z^h(z) - \zeta_1(z)$, resulting in a total campaign spending of $C_Z(z) = C_Z^h(z)$. However, since $C_Z^1(z) > C_Z^h(z)$, this also cannot be optimal for party 1. Therefore, the only equilibrium occurs at $\zeta_1(z) = w_1$ and $\zeta_2(z) = 0$, yielding $C_Z(z) = w_1$. In this case, party 1 cannot increase its individual spending since it is already exhausting its budget and does not have an incentive to decrease it since $C_Z^1(z) > w_1$. Party h does not have an incentive to increase its spending either since $C_Z^h(z) \leq w_1$. Therefore, if $C_Z^1(z) > w_1$ and $C_Z^h(z) \leq w_1$, the equilibrium is such that $\zeta_1(z) = w_1$ and $\zeta_2(z) = 0$.

Second, consider the case in which $w_1 < C_Z^h(z) \leq w_1 + w_2$. We know that any $\zeta_1(z) < w_1$ cannot be an equilibrium, since party h would best respond to it by choosing $\zeta_h(z) = C_Z^h(z) - \zeta_1(z)$ and the resulting total campaign spending $C_Z(z) = C_Z^h(z)$ would be suboptimal from party 1's point of view. Specifically, party 1 would have an incentive to increase its spending from $\zeta_1(z) < w_1$ to w_1. Therefore, the only equilibrium is such that $\zeta_1(z) = w_1$ and $\zeta_h(z) = C_Z^h(z) - w_1$, yielding the same $C_Z(z) = C_Z^h(z)$. This completes the proof of Lemma 2. $\square$

Proof of Lemma 3. Lemma 2 characterized the optimal campaign spending of parties 1 and h within the group N_Z. For any given $C_Z(z)$, the optimal campaign spending of the only member of group N_S, party j, is such that

$$\zeta_j(z) \in \underset{C \in [0, w_j]}{\arg\max} \frac{C_Z(z)}{C + C_Z(z)} u_j(z) + \frac{C}{C + C_Z(z)} u_j(s) - C. \tag{44}$$

The first-order conditions that the optimal $\zeta_j(z)$ needs to satisfy are given by

$$\epsilon_j(z)\left(\frac{C_Z(z)}{(C_Z(z)+\zeta_j(z))^2}\right)-1\begin{cases}\geq 0 & \text{if } \zeta_j(z)>w_j\\ =0 & \text{if } \zeta_j(z)\in[0,w_j]\\ \leq 0 & \text{if } \zeta_j(z)=0,\end{cases}\tag{45}$$

where $C_Z(z) = \zeta_1(z) + \zeta_h(z)$. Note that party j cares only about $C_Z(z)$ and not about how its burden is shared among the members of group N_Z. Thus, for any given $C_Z(z)$, party j best responds by choosing a campaign spending equal to either w_j or $\sqrt{\epsilon_j(z)C_Z(z)} - C_Z(z)$, whichever is smaller.

To solve for the best response of group N_Z to any given amount of $C_S(z)$, first suppose the proposal z is such that $\epsilon_1(z) \geq \epsilon_h(z)$. Based on (40) and (41), the best response $C_Z(z)$ in this case is given by

$$C_Z(z)=\begin{cases}\sqrt{\epsilon_1(z)C_S(z)}-C_S(z) & \text{if } C_Z^1(z)<w_1\\ w_1+w_h & \text{if } C_Z^h(z)\geq w_1+w_h\\ \max\{w_1,C_Z^h(z)\} & \text{if } C_Z^1(z)>w_1 \text{ and } C_Z^h(z)\leq w_1+w_h.\end{cases}\tag{46}$$

On the other hand, if the proposal z is such that $\epsilon_h(z) \geq \epsilon_1(z)$, then the best response $C_Z(z)$ to any given $C_S(z)$ becomes

$$C_Z(z)=\begin{cases}\sqrt{\epsilon_h(z)C_S(z)}-C_S(z) & \text{if } C_Z^h(z)<w_h\\ w_1+w_h & \text{if } C_Z^1(z)\geq w_1+w_h\\ \max\{w_h,C_Z^1(z)\} & \text{if } C_Z^h(z)>w_h \text{ and } C_Z^1(z)\leq w_1+w_h.\end{cases}\tag{47}$$

Thus, solving for the unique pure-strategy equilibrium of the challenge stage in which both groups are simultaneously best responding to each other yields the following candidates for the equilibrium triplet $(\zeta_1(z), \zeta_h(z), \zeta_j(z))$:

1. $(\zeta_1(z), \zeta_h(z), \zeta_j(z)) = (\sqrt{\epsilon_1(z)w_j} - w_j, 0, w_j)$ if and only if $\epsilon_1(z) \geq \epsilon_h(z)$; $\epsilon_1(z) \leq \frac{(w_1+w_j)^2}{w_j}$; and $\epsilon_j(z) \geq \frac{\epsilon_1(z)w_j}{\sqrt{\epsilon_1(z)w_j}-w_j}$.
2. $(\zeta_1(z), \zeta_h(z), \zeta_j(z)) = (w_1, w_h, w_j)$ if and only if $\epsilon_1(z) \geq \frac{(\sum_k w_k)^2}{w_j}$; $\epsilon_h(z) \geq \frac{(\sum_k w_k)^2}{w_j}$; and $\epsilon_j(z) \geq \frac{(\sum_k w_k)^2}{w_1+w_h}$.

3. $(\zeta_1(z), \zeta_h(z), \zeta_j(z)) = (w_1, \max\{\sqrt{\epsilon_h(z)w_j} - w_j - w_1, 0\}, w_j)$ if and only if $\epsilon_1(z) \geq \epsilon_h(z)$; $\epsilon_1(z) \geq \frac{(w_1+w_j)^2}{w_j}$; $\epsilon_h(z) \leq \frac{(\sum_k w_k)^2}{w_j}$; and $\epsilon_j(z) \geq \max\{\frac{\epsilon_h(z)w_j}{\sqrt{\epsilon_h(z)w_j}-w_j}, \frac{(w_1+w_j)^2}{w_1}\}$.

4. $(\zeta_1(z), \zeta_h(z), \zeta_j(z)) = (0, \sqrt{\epsilon_h(z)w_j} - w_j, w_j)$ if and only if $\epsilon_h(z) \geq \epsilon_1(z)$; $\epsilon_h(z) \leq \frac{(w_h+w_j)^2}{w_h}$; and $\epsilon_j(z) \geq \frac{\epsilon_h(z)w_j}{\sqrt{\epsilon_h(z)w_j}-w_j}$.

5. $(\zeta_1(z), \zeta_h(z), \zeta_j(z)) = (\max\{\sqrt{\epsilon_1(z)w_j} - w_j - w_h, 0\}, w_h, w_j)$ if and only if $\epsilon_h(z) \geq \epsilon_1(z)$; $\epsilon_1(z) \leq \frac{(\sum_k w_k)^2}{w_j}$; $\epsilon_h(z) \geq \frac{(w_h+w_j)^2}{w_j}$; and $\epsilon_j(z) \geq \max\{\frac{\epsilon_1(z)w_j}{\sqrt{\epsilon_1(z)w_j}-w_j}, \frac{(w_h+w_j)^2}{w_h}\}$.

6. $(\zeta_1(z), \zeta_h(z), \zeta_j(z)) = \left(\frac{\epsilon_1(z)^2\epsilon_j(z)}{(\epsilon_1(z)+\epsilon_j(z))^2}, 0, \frac{\epsilon_1(z)\epsilon_j(z)^2}{(\epsilon_1(z)+\epsilon_j(z))^2}\right)$ if and only if $\epsilon_1(z) \geq \epsilon_h(z)$; $\left(\frac{\epsilon_1(z)^2\epsilon_j(z)}{(\epsilon_1(z)+\epsilon_j(z))^2}\right) < w_1$; and $\left(\frac{\epsilon_1(z)\epsilon_j(z)^2}{(\epsilon_1(z)+\epsilon_j(z))^2}\right) < w_j$.

7. $(\zeta_1(z), \zeta_h(z), \zeta_j(z)) = (w_1, w_h, \sqrt{\epsilon_j(z)(w_1 + w_h)} - w_1 - w_h)$ if and only if $\sqrt{\bar{\epsilon}(z)} \geq \sqrt{\frac{(w_1+w_h)\epsilon_j(z)}{\bar{\epsilon}(z)}}+\sqrt{\frac{(w_1+w_h)\bar{\epsilon}(z)}{\epsilon_j(z)}}$; and $\epsilon_j(z) \leq \frac{(\sum_k w_k)^2}{w_1+w_h}$, where $\bar{\epsilon}(z) \equiv \max\{\epsilon_1(z), \epsilon_h(z)\}$.

8. $(\zeta_1(z), \zeta_h(z), \zeta_j(z)) = \left(w_1, \max\{\frac{\epsilon_h(z)^2\epsilon_j(z)}{(\epsilon_h(z)+\epsilon_j(z))^2} - w_1, 0\}, \max\{\frac{\epsilon_h(z)\epsilon_j(z)^2}{(\epsilon_h(z)+\epsilon_j(z))^2}, \sqrt{\epsilon_j(z)w_1} - w_1\}\right)$ if and only if $\epsilon_1(z) \geq \epsilon_h(z)$; $\frac{\epsilon_1(z)^2\epsilon_j(z)}{(\epsilon_1(z)+\epsilon_j(z))^2} > w_1$; $\frac{\epsilon_h(z)^2\epsilon_j(z)}{(\epsilon_h(z)+\epsilon_j(z))^2} < w_1 + w_h$; and $\max\{\frac{\epsilon_h(z)\epsilon_j(z)^2}{(\epsilon_h(z)+\epsilon_j(z))^2}, \sqrt{\epsilon_j(z)w_1} - w_1\} < w_j$.

9. $(\zeta_1(z), \zeta_h(z), \zeta_j(z)) = \left(0, \frac{\epsilon_h(z)^2\epsilon_j(z)}{(\epsilon_h(z)+\epsilon_j(z))^2}, \frac{\epsilon_h(z)\epsilon_j(z)^2}{(\epsilon_h(z)+\epsilon_j(z))^2}\right)$ if and only if $\epsilon_h(z) \geq \epsilon_1(z)$; $\frac{\epsilon_h(z)^2\epsilon_j(z)}{(\epsilon_h(z)+\epsilon_j(z))^2} < w_h$; and $\frac{\epsilon_h(z)\epsilon_j(z)^2}{(\epsilon_h(z)+\epsilon_j(z))^2} < w_j$.

10. $(\zeta_1(z), \zeta_h(z), \zeta_j(z)) = \left(\max\{\frac{\epsilon_1(z)^2\epsilon_j(z)}{(\epsilon_1(z)+\epsilon_j(z))^2} - w_h, 0\}, w_h, \max\{\frac{\epsilon_1(z)\epsilon_j(z)^2}{(\epsilon_1(z)+\epsilon_j(z))^2}, \sqrt{\epsilon_j(z)w_h} - w_h\}\right)$ if and only if $\epsilon_h(z) \geq \epsilon_1(z)$; $\frac{\epsilon_h(z)^2\epsilon_j(z)}{(\epsilon_h(z)+\epsilon_j(z))^2} > w_h$; $\frac{\epsilon_1(z)^2\epsilon_j(z)}{(\epsilon_1(z)+\epsilon_j(z))^2} < w_1 + w_h$; and $\max\{\frac{\epsilon_1(z)\epsilon_j(z)^2}{(\epsilon_1(z)+\epsilon_j(z))^2}, \sqrt{\epsilon_j(z)w_h} - w_h\} < w_j$.

In each of these equilibrium candidates, it can be observed that $\zeta_k(z)$ is increasing as the value of $\epsilon_k(z)$ increases. In addition, party h free-rides on party 1's campaign spending only if the proposal z is such that $\epsilon_1(z) \geq \epsilon_h(z)$. This can be seen by inspecting the above candidates in which $\zeta_h(z) = 0$ or $\zeta_h(z) = C_Z^h(z) - w_1$. This completes the proof of Lemma 3. $\square$

Proof of Proposition 3. I first characterize the acceptance strategies $a_2(z)$ and $a_3(z)$ of the non-proposer parties for any given proposal z. Each party's payoff from voting to accept or reject the proposal depends on the other party's vote. First, for any given proposal z, if $a_k(z) = 1$ for both k, then each party k gets a sure payoff of $u_k(z)$. Second, if $a_2(z) = 1$ and $a_3(z) = 0$, then the bill moves to a challenge stage in which $\rho_1(z) = \rho_2(z) = Z$ and $\rho_3(z) = S$, with the associated equilibrium campaign spending of each party given by $(\zeta_1(z), \zeta_2(z), \zeta_3(z))$. In this case, each party's receives an expected payoff determined by the specific challenge being played. Third, if $a_2(z) = 0$ and $a_3(z) = 1$, the challenge stage features $\rho_1(z) = \rho_3(z) = Z$ and $\rho_2(z) = S$, along with each party's associated equilibrium campaign spending. Finally, if $a_k(z) = 0$ for both parties, then each party k receives its status-quo payoff $u_k(s)$.

For any given proposal z, $a_2(z) = 1$ is a dominant strategy for party 2 if a) $u_2(z)$ is at least as great as its expected payoff from a challenge with $\rho_1(z) = \rho_3(z) = Z$ and $\rho_2(z) = S$; and b) its expected payoff from a challenge with $\rho_1(z) = \rho_2(z) = Z$ and $\rho_3(z) = S$ is at least as great as $u_2(s)$. Similarly, $a_3(z) = 1$ is a dominant strategy for party 3 if a) $u_3(z)$ is at least as great as its expected payoff from a challenge with $\rho_1(z) = \rho_2(z) = Z$ and $\rho_3(z) = S$; and b) its expected payoff from a challenge with $\rho_1(z) = \rho_3(z) = Z$ and $\rho_2(z) = S$ is at least as great as $u_3(s)$.

In order for party 1 to induce unanimity in equilibrium, the proposal z must be such that the following conditions based on the non-proposer parties' acceptance strategies hold:[12]

- $u_k(z)$ is at least as great as party k's expected payoff from a challenge with $\rho_k(z) = S$ for $k = 2, 3$;
- The following does not simultaneously hold for $k = 2, 3$: $u_k(s)$ is at least as great as party k's expected payoff from a challenge with $\rho_k(z) = Z$.

To solve for party 1's optimal proposal z that satisfies the above conditions, we need to find the unanimity-inducing offer z for each of the possible challenge stage equilibrium candidates identified in Lemma 3 and compare party 1's unanimity payoff for all such offers.

I start by focusing on the first five equilibrium candidates listed in the proof of Lemma 3 in which $\zeta_j(z) = w_j$. Consider a proposal z such that $\epsilon_1(z) \leq \frac{(w_1+w_j)^2}{w_j}$, $\epsilon_1(z) \geq \epsilon_h(z)$, and $\epsilon_j(z) \geq \frac{\epsilon_1(z)w_j}{\sqrt{\epsilon_1(z)w_j - w_j}}$, which would lead to the challenge stage equilibrium listed in item one if rejected. Suppose party 1 chooses $h = 2$ and $j = 3$ so that if rejected, this proposal would imply a challenge stage equilibrium with $\rho_2(z) = Z$, $\rho_3(z) = S$,

[12] I again assume that parties vote to accept a proposal when indifferent.

and $(\zeta_1(z), \zeta_2(z), \zeta_3(z)) = (\sqrt{\epsilon_1(z)w_3} - w_3, 0, w_3)$. Party 3's expected payoff from this challenge is given by

$$\left(\frac{\sqrt{\epsilon_1(z)w_3} - w_3}{\sqrt{\epsilon_1(z)w_3}}\right) u_3(z) + \left(\frac{w_3}{\sqrt{\epsilon_1(z)w_3}}\right) u_3(s) - w_3. \tag{48}$$

Then, party 3 accepts this offer if and only if $u_3(z)$ is at least as great as (48), which implies that we must have $\epsilon_3(z) \leq \sqrt{\epsilon_1(z)w_3}$.

To derive party 2's acceptance condition, suppose that party 1 now chooses $h = 3$ and $j = 2$ so that this offer goes to a challenge in which $\rho_2(z) = S$ and $\rho_3(z) = Z$. In this scenario, the equilibrium levels of campaign spending are given by $(\zeta_1(z), \zeta_2(z), \zeta_3(z)) = (\sqrt{\epsilon_1(z)w_2} - w_2, w_2, 0)$, yielding the following expected payoff for party 2:

$$\left(\frac{\sqrt{\epsilon_1(z)w_2} - w_2}{\sqrt{\epsilon_1(z)w_2}}\right) u_2(z) + \left(\frac{w_2}{\sqrt{\epsilon_1(z)w_2}}\right) u_2(s) - w_2. \tag{49}$$

Then, party 2 accepts this offer if and only if $\epsilon_2(z) \leq \sqrt{\epsilon_1(z)w_2}$.

In order for unanimity to be realized, the additional condition that $u_k(s)$ is not at least as great as party k's expected payoff from a challenge with $\rho_k(z) = Z$ for both $k = 2, 3$ needs to be met. To check for this, construct party k's expected payoff from a challenge with $\rho_k(z) = Z$ for $k = 2, 3$:

$$\left(\frac{\sqrt{\epsilon_1(z)w_{-k}} - w_{-k}}{\sqrt{\epsilon_1(z)w_{-k}}}\right) u_k(z) + \left(\frac{w_{-k}}{\sqrt{\epsilon_1(z)w_{-k}}}\right) u_k(s). \tag{50}$$

The condition that $u_k(s)$ is at least as great as (50) reduces to $u_k(s) \geq u_k(z)$ for $k = 2, 3$. Thus, unanimity requires that $u_2(s) \geq u_2(z)$ and $u_3(s) \geq u_3(z)$ are not simultaneously true for proposal z.

First, suppose without loss of generality that the proposal z is such that $u_2(s) < u_2(z)$. Then, the conditions that need to hold for unanimity are $u_2(z) \geq u_2(s)$ and $u_3(z) \geq u_3(s) - \sqrt{\epsilon_1(z)w_3}$. Since the challenge stage equilibrium under consideration requires that $\epsilon_2(z) \leq \epsilon_1(z) \leq \frac{(w_1+w_3)^2}{w_3}$ and $\epsilon_3(z) \geq \frac{\epsilon_1(z)w_3}{\sqrt{\epsilon_1(z)w_3} - w_3}$, bringing these conditions together with the parties' acceptance criteria implies the following: Party 2 accepts any proposal z such that $u_2(z) \in [u_2(s), u_2(s) + \frac{(w_1+w_3)^2}{w_3}]$. However, there exists no proposal z that simultaneously satisfies $\epsilon_3(z) \geq \frac{\epsilon_1(z)w_3}{\sqrt{\epsilon_1(z)w_3} - w_3}$ and party 3's acceptance criteria. Second, suppose the proposal z is such that $u_3(s) < u_3(z)$. Carrying out the same analysis as above this time yields the result that party 2's acceptance criteria cannot be reconciled with the equilibrium conditions on z. Therefore, any proposal z that would imply a subsequent challenge stage equilibrium with $(\zeta_1(z), \zeta_h(z), \zeta_j(z)) = (\sqrt{\epsilon_1(z)w_j} - w_j, 0, w_j)$ if rejected cannot induce unanimity in the parliament.

Carrying out the same analysis for equilibrium candidates numbered two through five in the proof of Lemma 3 yields the same result as the first equilibrium candidate above. In the interest of space, each of these analyses will not be presented separately. Overall, we can conclude that any proposal z for which $\zeta_j(z) = w_j$ will be rejected by party $j \in N_S$.

Consider the sixth equilibrium candidate listed in the proof of Lemma 3 in which the proposal z leads to an interior challenge equilibrium characterized by $(\zeta_1(z), \zeta_h(z), \zeta_j(z)) = \left(\frac{\epsilon_1(z)^2\epsilon_j(z)}{(\epsilon_1(z)+\epsilon_j(z))^2}, 0, \frac{\epsilon_1(z)\epsilon_j(z)^2}{(\epsilon_1(z)+\epsilon_j(z))^2}\right)$. Based on their expected payoffs, the acceptance criteria for parties $k = 2, 3$ become $\epsilon_k(z) \leq 0$. Moreover, if a proposal z meets either one of these acceptance criteria, then the final condition for achieving unanimity is also met. Therefore, if party 1 wanted to induce unanimity with a proposal z that would lead to the challenge stage equilibrium in item six if rejected, it chooses z in order to maximize $u_1(z)$ subject to the parties' acceptance criteria and the equilibrium conditions. Solving this program yields the following two optimal unanimity-inducing offers: First, party 1 can choose $x = \frac{\hat{x}_1+\hat{x}_3}{2}$, thereby compromising ideologically with party 3. In addition, it can offer the following rent shares:

$$y_1 = \alpha^{-1}\left[\alpha y_1^q + (q-\hat{x}_2)^2 + (q-\hat{x}_3)^2 - \left(\frac{\hat{x}_1+\hat{x}_3-2\hat{x}_2}{2}\right)^2 - \left(\frac{\hat{x}_1-\hat{x}_3}{2}\right)^2\right], \quad (51)$$

$$y_2 = \alpha^{-1}\left[\alpha y_2^q - (q-\hat{x}_2)^2 + \left(\frac{\hat{x}_1+\hat{x}_3-2\hat{x}_2}{2}\right)^2\right], \quad (52)$$

$$y_3 = \alpha^{-1}\left[\alpha y_3^q - (q-\hat{x}_3)^2 + \left(\frac{\hat{x}_1-\hat{x}_3}{2}\right)^2\right]. \quad (53)$$

Second, it can choose $x = \frac{\hat{x}_1+\hat{x}_2}{2}$, compromising ideologically with party 2, and offer

$$y_1 = \alpha^{-1}\left[\alpha y_1^q + (q-\hat{x}_2)^2 + (q-\hat{x}_3)^2 - \left(\frac{\hat{x}_1+\hat{x}_2-2\hat{x}_3}{2}\right)^2 - \left(\frac{\hat{x}_1-\hat{x}_2}{2}\right)^2\right], \quad (54)$$

$$y_2 = \alpha^{-1}\left[\alpha y_2^q - (q-\hat{x}_2)^2 + \left(\frac{\hat{x}_1 - \hat{x}_2}{2}\right)^2\right], \tag{55}$$

$$y_3 = \alpha^{-1}\left[\alpha y_3^q - (q-\hat{x}_3)^2 + \left(\frac{\hat{x}_1 + \hat{x}_2 - 2\hat{x}_3}{2}\right)^2\right]. \tag{56}$$

Having solved for the optimal way to induce unanimity with a proposal that would induce a challenge stage equilibrium as listed in item six if rejected, now consider the seventh equilibrium candidate characterized by $(\zeta_1(z), \zeta_h(z), \zeta_j(z)) = (w_1, w_h, \sqrt{\epsilon_j(z)(w_1+w_h)} - w_1 - w_h)$. Based on their expected payoffs, the acceptance criteria of parties $k = 2, 3$ become $\epsilon_k(z) \leq w_1 + w_{-k}$. Moreover, the final unanimity condition implies that we must have $\epsilon_k(z) \geq w_k\sqrt{\frac{\epsilon_{-k}(z)}{w_1+w_k}}$ for at least one $k \in \{2,3\}$. Without loss of generality, suppose that this condition holds for party 2. Then, the unanimity conditions yield $\epsilon_3(z) \leq w_1 + w_2$ and $\epsilon_2(z) \leq w_2$. Confirming that there exist proposals z that can simultaneously satisfy these and the equilibrium conditions for item seven, party 1 maximizes $u_1(z)$ by choosing z subject to the above two constraints. Solving this program yields the following two alternative unanimity-inducing offers: First, party 1 can choose $x = \frac{\hat{x}_1+\hat{x}_3}{2}$ and offer the following rent shares:

$$y_1 = \alpha^{-1}\left[\alpha y_1^q + w_1 + (q-\hat{x}_2)^2 + (q-\hat{x}_3)^2 - \left(\frac{\hat{x}_1 + \hat{x}_3 - 2\hat{x}_2}{2}\right)^2 - \left(\frac{\hat{x}_1 - \hat{x}_3}{2}\right)^2\right], \tag{57}$$

$$y_2 = \alpha^{-1}\left[\alpha y_2^q + w_2 - (q-\hat{x}_2)^2 + \left(\frac{\hat{x}_1 + \hat{x}_3 - 2\hat{x}_2}{2}\right)^2\right], \tag{58}$$

$$y_3 = \alpha^{-1}\left[\alpha y_3^q - (w_1 + w_2) - (q-\hat{x}_3)^2 + \left(\frac{\hat{x}_1 - \hat{x}_3}{2}\right)^2\right]. \tag{59}$$

Second, party 1 can choose $x = \frac{\hat{x}_1+\hat{x}_2}{2}$ and offer

$$y_1 = \alpha^{-1}\left[\alpha y_1^q + w_1 + (q-\hat{x}_2)^2 + (q-\hat{x}_3)^2 - \left(\frac{\hat{x}_1 + \hat{x}_2 - 2\hat{x}_3}{2}\right)^2 - \left(\frac{\hat{x}_1 - \hat{x}_2}{2}\right)^2\right], \tag{60}$$

$$y_2 = \alpha^{-1}\left[\alpha y_2^q - (w_1 + w_3) - (q-\hat{x}_2)^2 + \left(\frac{\hat{x}_1 - \hat{x}_2}{2}\right)^2\right], \tag{61}$$

$$y_3 = \alpha^{-1}\left[\alpha y_3^q + w_3 - (q-\hat{x}_3)^2 + \left(\frac{\hat{x}_1 + \hat{x}_2 - 2\hat{x}_3}{2}\right)^2\right]. \tag{62}$$

Now consider the eighth equilibrium candidate characterized by $(\zeta_1(z), \zeta_h(z), \zeta_j(z)) = \Big(w_1, \max\{\frac{\epsilon_h(z)^2\epsilon_j(z)}{(\epsilon_h(z)+\epsilon_j(z))^2} - w_1, 0\}, \max\{\frac{\epsilon_h(z)\epsilon_j(z)^2}{(\epsilon_h(z)+\epsilon_j(z))^2}, \sqrt{\epsilon_j(z)w_1} - w_1\}\Big)$. A similar analysis suggests that there again exist two unanimity-inducing offers corresponding to two different acceptance criteria: First, $u_2(z) \geq u_2(s) - w_1$ and $u_3(z) \geq u_3(s)$; and second $u_2(z) \geq u_2(s)$ and $u_3(z) \geq u_3(s) - w_1$. Checking that there exist proposals z that satisfy both the acceptance criteria and the equilibrium conditions, we can proceed with party 1's maximization problem. If party 1 chooses a unanimity-inducing offer z based on the first acceptance criteria, the offer z involves $x = \frac{\hat{x}_1+\hat{x}_3}{2}$, y_1 as given in (57), and

$$y_2 = \alpha^{-1}\left[\alpha y_2^q - w_1 - (q - \hat{x}_2)^2 + \left(\frac{\hat{x}_1 + \hat{x}_3 - 2\hat{x}_2}{2}\right)^2\right], \tag{63}$$

$$y_3 = \alpha^{-1}\left[\alpha y_3^q - (q - \hat{x}_3)^2 + \left(\frac{\hat{x}_1 - \hat{x}_3}{2}\right)^2\right]. \tag{64}$$

On the other hand, if it chooses the offer based on the second acceptance criteria, the offer z now involves $x = \frac{\hat{x}_1+\hat{x}_2}{2}$, y_1 as given in (60), and

$$y_2 = \alpha^{-1}\left[\alpha y_2^q - (q - \hat{x}_2)^2 + \left(\frac{\hat{x}_1 - \hat{x}_2}{2}\right)^2\right], \tag{65}$$

$$y_3 = \alpha^{-1}\left[\alpha y_3^q - w_1 - (q - \hat{x}_3)^2 + \left(\frac{\hat{x}_1 + \hat{x}_2 - 2\hat{x}_3}{2}\right)^2\right]. \tag{66}$$

The ninth equilibrium candidate is similar to the sixth candidate in the sense that the partners completely free-ride in both cases and the groups fight unconstrained against each other. The only difference is the identity of the partner. Thus, partner party h's acceptance criteria is stricter, requiring a higher premium from party 1. Thus, this way of inducing unanimity will always be dominated.

Finally, consider the tenth equilibrium candidate, which implies the following alternative acceptance criteria: First, $u_3(z) \geq u_3(s)$ and $u_2(z) \geq u_2(s) + w_2$, and second $u_3(z) \geq u_3(s) + w_3$ and $u_2(z) \geq u_2(s)$. If party 1 chooses a unanimity-inducing offer z based on the first acceptance criteria, the offer z involves $x = \frac{\hat{x}_1+\hat{x}_3}{2}$,

$$y_1 = \alpha^{-1}\left[\alpha y_1^q - w_2 + (q - \hat{x}_2)^2 + (q - \hat{x}_3)^2 - \left(\frac{\hat{x}_1 + \hat{x}_3 - 2\hat{x}_2}{2}\right)^2 - \left(\frac{\hat{x}_1 - \hat{x}_3}{2}\right)^2\right], \tag{67}$$

$$y_2 = \alpha^{-1}\left[\alpha y_2^q + w_2 - (q-\hat{x}_2)^2 + \left(\frac{\hat{x}_1+\hat{x}_3-2\hat{x}_2}{2}\right)^2\right], \tag{68}$$

$$y_3 = \alpha^{-1}\left[\alpha y_3^q - (q-\hat{x}_3)^2 + \left(\frac{\hat{x}_1-\hat{x}_3}{2}\right)^2\right]. \tag{69}$$

On the other hand, if it chooses this offer based on the second acceptance criteria, then the offer z involves $x = \frac{\hat{x}_1+\hat{x}_2}{2}$,

$$y_1 = \alpha^{-1}\left[\alpha y_1^q - w_3 + (q-\hat{x}_2)^2 + (q-\hat{x}_3)^2 - \left(\frac{\hat{x}_1+\hat{x}_2-2\hat{x}_3}{2}\right)^2 - \left(\frac{\hat{x}_1-\hat{x}_2}{2}\right)^2\right], \tag{70}$$

$$y_2 = \alpha^{-1}\left[\alpha y_2^q - (q-\hat{x}_2)^2 + \left(\frac{\hat{x}_1-\hat{x}_2}{2}\right)^2\right], \tag{71}$$

$$y_3 = \alpha^{-1}\left[\alpha y_3^q + w_3 - (q-\hat{x}_3)^2 + \left(\frac{\hat{x}_1+\hat{x}_2-2\hat{x}_3}{2}\right)^2\right]. \tag{72}$$

The above analysis indicates that the utility each party settles for in a grand bargain reflects its strength in the post-bargaining stage. To see this, first consider equilibrium candidate seven in which both members of group N_Z fight against N_S with all their resources. In this case, the two parties who would belong to N_Z if the proposal z is rejected can each extract a premium equal to their campaigning budgets from the party that would belong to N_S in a grand bargain. In the equilibrium candidate eight, the non-proposer partner party is at least partially free-riding on party 1's campaign spending. Thus, party 1 is able to extract from its partner an amount equal to its campaigning budget when inducing a settlement. Equilibrium candidate ten demonstrates the reverse of this situation with party 1 free-riding on its partner's campaign spending. This proves part 1 of Proposition 3.

Part 2 of the proposition describes the optimal proposal with which to induce unanimity. Comparing the maximum values of $u_1(z)$ from a unanimous agreement in each of the cases considered above, it can be observed that party 1 can secure the maximum payoff from unanimity with a proposal z that satisfies the equilibrium conditions of items seven or eight. Although the optimal z that induces unanimity in these two cases is different, they both imply the same sure-payoff for party 1. Specifically, for each of these cases, party 1 can induce unanimity by proposing either $x = \frac{\hat{x}_1+\hat{x}_3}{2}$ and (57) for itself, or $x = \frac{\hat{x}_1+\hat{x}_2}{2}$ and (60) for itself. Its rent share in either of these cases indicates that it is increasing in y_1^q, w_1, $(q - \hat{x}_2)$, and $(q - \hat{x}_3)$. Moreover, since each party gets compensated for their ideological utility loss in the grand bargain through its rent share as can be observed in (58), (59), (61), and (62), party 1's unanimity payoff strictly increases as the three parties get ideologically closer. This completes the proof of Proposition 3. $\square$

Proof of Proposition 4. In order to analyze the optimal proposals to get to a given challenge stage equilibrium for party 1, I first focus on the non-proposer parties' voting strategies. In order for a proposal z to induce a unique challenge stage equilibrium with $\rho_h(z) = Z$ and $\rho_j(z) = S$ for $h, j \in \{2, 3\}$ and $h \neq j$, the following conditions must hold:

- Party h's expected payoff from a challenge with $\rho_h(z) = Z$ is at least as great as $u_h(s)$;
- Party j's expected payoff from a challenge with $\rho_j(z) = S$ is at least as great as $u_j(z)$;
- The following conditions do not simultaneously hold: Party h's expected payoff from a challenge with $\rho_h(z) = S$ is at least as great as $u_h(z)$; and party j's expected payoff from a challenge with $\rho_j(z) = Z$ is at least as great as $u_j(s)$.

Consider the challenge stage equilibrium candidate listed in item one in the proof of Lemma 3. In order to induce a challenge stage equilibrium with $\rho_h(z) = Z$, $\rho_j(z) = S$, and $(\zeta_1(z), \zeta_h(z), \zeta_j(z)) = (\sqrt{\epsilon_1(z) w_j} - w_j, 0, w_j)$, party 1's proposal z must meet the corresponding equilibrium conditions, satisfy party h's acceptance criteria, and violate party j's acceptance criteria. The analysis in Proposition 3 indicated that party j will reject any offer z that would give rise to this equilibrium if rejected. In addition, among the range of proposals that would give rise to this challenge if rejected in the parliament, party h will accept any z such that $u_h(z) \in [u_h(s), u_h(s) + \frac{(w_1 + w_j)^2}{w_j}]$.

In equilibrium, party 1 will not offer any higher surplus to party h than is required to get its acceptance. Thus, the optimal z to induce this challenge will be such that $u_h(z) = u_h(s)$. Moreover, since $\zeta_j(z) = w_j$ for any proposal z in this range, party 1 cannot influence the amount of $C_S(z)$. Thus, the proposal z need not worry about party j's rejection as long as it satisfies the equilibrium conditions. The Lagrangian of this problem can be written as

$$L = -(x-\hat{x}_1)^2+y_1-2\sqrt{w_j\epsilon_1(z)}+w_j+(\lambda_1-\lambda_2)[-(x-\hat{x}_h)^2+1-y_1-u_h(s)]+\lambda_2\frac{(w_1+w_j)^2}{w_j}$$
$$+\lambda_3\left[-(x-\hat{x}_j)^2-\frac{\epsilon_1(z)w_j}{\sqrt{\epsilon_1(z)w_j}-w_j}\right]. \tag{73}$$

With $y_j = 0$, solving this program for x, y_1, and $y_h = 1 - y_1$ yields $x = \frac{\hat{x}_1+\hat{x}_h}{2}$,

$$y_1 = y_1^q + y_j^q + (q - \hat{x}_h)^2 - \left(\frac{\hat{x}_1 - \hat{x}_h}{2}\right)^2, \tag{74}$$

$$y_h = y_h^q - (q - \hat{x}_h)^2 + \left(\frac{\hat{x}_1 - \hat{x}_h}{2}\right)^2. \tag{75}$$

Proceeding with a similar analysis for the remaining equilibrium candidates yields the result that the optimal proposal z to induce any challenge stage equilibrium involves offering $x = \frac{\hat{x}_1+\hat{x}_h}{2}$. Since party h requires at least $u_h(s)$ in order to become party 1's partner in a challenge regardless of how much it will spend, it can be observed from (74) and (75) that party 1's winning prize increases as y_h^q decreases, $(q - \hat{x}_h)^2$ increases, and it gets ideologically closer to party h. Moreover, since $\epsilon_h(z)$ increases as $u_h(s)$ decreases for any proposal z, Lemma 3 indicates that $\zeta_h(z)$ would be weakly higher, thus weakly increasing the proposal's winning probability. Therefore, party 1's expected payoff would increase. This proves part 1 of Proposition 4.

Part 2 of Proposition 4 focuses on how party 1's expected payoff from a challenge is affected by its partner's campaigning budget. In the interest of brevity, I do not present here the solutions for the optimal proposals that would induce each possible challenge stage equilibrium. Instead, I focus on two examples that demonstrate party 1's different incentives with regards to the other parties' campaigning budgets.

Consider the equilibrium candidate listed in item two in the proof of Lemma 3. Solving for the optimal proposal to induce this particular challenge equilibrium yields $x = \frac{\hat{x}_1+\hat{x}_h}{2}$, $y_j = 0$,

$$y_1 = y_1^q + y_j^q + (q - \hat{x}_h)^2 - \left(\frac{\hat{x}_1 - \hat{x}_h}{2}\right)^2 - \frac{(w_1 + w_h + w_j)^2}{w_j}, \tag{76}$$

$$y_h = y_h^q - (q - \hat{x}_h)^2 + \left(\frac{\hat{x}_1 - \hat{x}_h}{2}\right)^2 + \frac{(w_1 + w_h + w_j)^2}{w_j}. \tag{77}$$

As a result, party 1's maximum expected payoff from this type of challenge becomes

$$\left(\frac{w_1 + w_h}{w_1 + w_h + w_j}\right)\left[y_j^q + \sum_{k=1,h}(q - \hat{x}_k)^2 - 2\left(\frac{\hat{x}_1 - \hat{x}_h}{2}\right)^2 - \frac{(w_1 + w_h + w_j)^2}{w_j}\right] + u_1(s) - w_1. \tag{78}$$

Differentiating (78) with respect to w_h yields

$$\frac{(w_j)^2 \epsilon_1(z) - 2(w_1 + w_h)(w_1 + w_h + w_j)}{(w_1 + w_h + w_j)^2 w_j}, \tag{79}$$

where $\epsilon_1(z)$ is calculated using the optimal proposal and equals the expression in brackets in (78). The sign of this expression depends on the parameters of the model. Specifically, it is negative if

$$\epsilon_1(z) < \frac{2(w_1 + w_h)(w_1 + w_h + w_j)}{(w_j)^2}, \tag{80}$$

and positive otherwise. Thus, we conclude that a higher w_h decreases party 1's expected payoff from the type of challenge in item 2 of Lemma 3 if $\epsilon_1(z)$ is sufficiently small, which happens if $u_1(s)$ is large, or if w_1 or w_h are high. Analyzing other equilibrium candidates in which $\zeta_h(z) > 0$ indicates that this relationship holds more generally. This proves part 2 and hence completes the proof of Proposition 4. $\square$

Proof of Corollary 1. Since a lower $u_h(s)$ and higher $u_j(s)$ necessarily increase party 1's expected payoff in any challenge equilibrium, it follows that holding everything else constant, party 1 would prefer to partner with the party that commands the lower status-quo payoff.

To see how the partner decision is affected by the parties' campaigning budgets, consider an alternative challenge equilibrium in which the proposal z is such that $\zeta_h(z) = 0$. Since it has already been analyzed, I focus on the equilibrium given in the first item in Lemma 3. With the proposal z given by $x = \frac{\hat{x}_1 + \hat{x}_h}{2}$, $y_j = 0$, y_1 as in (74), and y_h as in (75), party 1's maximum expected payoff from this challenge becomes

$$\left[1 - \sqrt{\frac{w_j}{\epsilon_1(z)}}\right]\epsilon_1(z) + u_1(s), \tag{81}$$

where $\epsilon_1(z) = -2\left(\frac{\hat{x}_1-\hat{x}_h}{2}\right)^2 + y_j^q + (q-\hat{x}_1)^2 + (q-\hat{x}_h)^2$. It can be observed from (81) that it does not depend on w_h and depends negatively on w_j. Furthermore, this relationship holds in other challenge equilibrium candidates in which $\zeta_h(z) = 0$. Thus, party 1 would prefer to have as its opponent the party with the lower campaigning budget in such challenge equilibria. The rest of the corollary follows from the proof of Proposition 4. □

Proof of Proposition 5. Analyzing party 1's incentives between inducing a grand bargain and a challenge requires comparing its maximum payoff from each of the two outcomes. However, since the type of challenge equilibrium that will maximize party 1's expected payoff depends on different conditions on the parameters of the model, I only present here the relevant results from comparing party 1's maximum unanimity payoff with certain types of challenge stage equilibria in the interest of brevity.

First, consider the challenge stage equilibrium listed in item one in the proof of Lemma 3, where $(\zeta_1(z), \zeta_h(z), \zeta_j(z)) = (\sqrt{\epsilon_1(z)w_j} - w_j, 0, w_j)$. Given the optimal proposal z characterized in the proof of Proposition 4 that would give rise to this challenge equilibrium, party 1's maximized payoff from this challenge is as given in (81), where $\epsilon_1(z) = -2\left(\frac{\hat{x}_1-\hat{x}_h}{2}\right)^2 + y_j^q + (q-\hat{x}_1)^2 + (q-\hat{x}_h)^2$. The proof of Proposition 3 characterized the optimal proposal z to induce unanimity, which involves $x = \frac{\hat{x}_1+\hat{x}_j}{2}$ and

$$y_1 = \alpha^{-1}\left[\alpha y_1^q + w_1 + (q-\hat{x}_h)^2 + (q-\hat{x}_j)^2 - \left(\frac{\hat{x}_1+\hat{x}_j-2\hat{x}_h}{2}\right)^2 - \left(\frac{\hat{x}_1-\hat{x}_j}{2}\right)^2\right].$$

Therefore, party 1's maximum payoff from unanimity is given by

$$\alpha y_1^q + w_1 + \sum_{k=2,3}(q-\hat{x}_k)^2 - \left(\frac{\hat{x}_1+\hat{x}_j-2\hat{x}_h}{2}\right)^2 - 2\left(\frac{\hat{x}_1-\hat{x}_j}{2}\right)^2. \tag{82}$$

Comparing (82) with the maximum expected payoff from the considered challenge indicates that party 1 prefers a grand bargain over this challenge if

$$w_1 - \left(\frac{\hat{x}_1+\hat{x}_j-2\hat{x}_h}{2}\right)^2 - 2\left(\frac{\hat{x}_1-\hat{x}_j}{2}\right)^2 \geq -2\left(\frac{\hat{x}_1-\hat{x}_h}{2}\right)^2 + y_j^q - \sqrt{w_j\epsilon_1(z)} - (q-\hat{x}_1)^2, \tag{83}$$

where $\epsilon_1(z)$ is given as before. Condition (83) is more likely to hold if the non-proposer parties h and j each commands a lower status-quo payoff. Moreover, this relationship carries over to other types of challenge equilibria. This proves Part 1 of Proposition 5.

Notice that condition (83) becomes more likely to hold as

$$-\left(\frac{\hat{x}_1+\hat{x}_j-2\hat{x}_h}{2}\right)^2-2\left(\frac{\hat{x}_1-\hat{x}_j}{2}\right)^2+2\left(\frac{\hat{x}_1-\hat{x}_h}{2}\right)^2 \tag{84}$$

increases, which, when manipulated, suggests that all parties need to be ideologically close for unanimity to be preferred. This is also a relationship that carries over to other types of challenge equilibria. This proves Part 2 of Proposition 5.

Proposition 4 and Corollary 1 indicated that the individual roles w_h and w_j might play on party 1's incentives between a grand bargain and a challenge are ambiguous and depend on the particular challenge equilibrium considered. However, to see how party 1's incentives respond to the *relative* budgets of the non-proposer parties, consider a challenge equilibrium such as item five in the proof of Lemma 3 in which $(\zeta_1(z),\zeta_h(z),\zeta_j(z)) = (\max\{\sqrt{\epsilon_1(z)w_j}-w_j-w_h,0\},w_h,w_j)$. Solving for the optimal proposal z that would lead to this challenge yields $x=\frac{\hat{x}_1+\hat{x}_h}{2}$, $y_j=0$,

$$y_1=y_1^q+(q-\hat{x}_h)^2-\left(\frac{\hat{x}_1-\hat{x}_h}{2}\right)^2-\frac{(w_h+w_j)^2}{w_j}, \tag{85}$$

$$y_h=y_h^q-(q-\hat{x}_h)^2+\left(\frac{\hat{x}_1-\hat{x}_h}{2}\right)^2+\frac{(w_h+w_j)^2}{w_j}. \tag{86}$$

The condition obtained by comparing the maximum expected payoff from this challenge and (82) is more likely to hold as $\frac{w_h}{w_j}$ increases. Note that if $w_h>w_j$, this would require the two parameters to diverge, whereas if $w_h<w_j$, they must become more similar. However, since this is a challenge equilibrium in which the low-budget party is more likely to become the partner based on Corollary 1, it is more likely that $w_h<w_j$. Thus, more similar budgets decrease the payoff from this challenge. This completes the proof of Proposition 5. □

References

[1] Ashworth, S. (2006), "Campaign Finance and Voter Welfare with Entrenched Incumbents," *American Political Science Review*, vol. 100(1), 55-68.

[2] Austen-Smith, D. and J. Banks, (1988), "Elections, Coalitions, and Legislative Outcomes," *American Political Science Review*, vol. 82(2), 405-22.

[3] Baik, K. (2008), "Contests With Group-Specific Public Good Prizes," *Social Choice and Welfare*, vol. 30(1), 103-17.

[4] Banks, J. (2000), "Buying Supermajorities in Finite Legislatures," *American Political Science Review*, vol. 94(3), 677-81.

[5] Banks, J. and J. Duggan, (2000), "A Bargaining Model of Collective Choice," *American Political Science Review*, vol. 94(1), 73-88.

[6] Baron, D. and J. Ferejohn, (1989), "Bargaining in Legislatures," *American Political Science Review*, vol. 83(4), 1181-1206.

[7] Baron, D. (1994), "Electoral Competition with Informed and Uninformed Voters," *American Political Science Review*, vol. 88(1), 33-47.

[8] Besley, T. and S. Coate, (1997), "An Economic Model of Representative Democracy," *Quarterly Journal of Economics*, vol. 112(1), 85-114.

[9] Bowen, R., Y. Chen and H. Eraslan, (2014), "Mandatory versus Discretionary Spending: The Status Quo Effect," *American Economic Review*, vol. 104(10), 2941-74.

[10] Bowler, S. and T. Donovan, *Demanding Choices: Opinion, Voting, and Direct Democracy*, Ann Arbor, MI: University of Michigan Press, 1998.

[11] Broder, D. *Democracy Derailed: Initiative Campaigns and the Power of Money*, New York, NY: Harcourt, 2000.

[12] Coate, S. (2004), "Pareto-Improving Campaign Finance Policy," *American Economic Review*, vol. 94(3), 628-55.

[13] Diermeier, D. and T. Feddersen, (1998), "Cohesion in Legislatures and the Vote of Confidence Procedure," *American Political Science Review*, vol. 92(3), 611-21.

[14] Diermeier, D., H. Eraslan and A. Merlo, (2002), "Coalition Governments and Comparative Constitutional Design," European Economic Review, vol. 46, 893-907.

[15] Dixit, A. (1987), "Strategic Behavior in Contests," *American Economic Review*, vol. 77(5), 891-98.

[16] Downs, A. *An Economic Theory of Democracy*, New York, NY: Harper and Row, 1957.

[17] Duggan, J and T. Kalandrakis, (2012), "Dynamic Legislative Policy Making," *Journal of Economic Theory*, vol. 147(5), 1653-88.

[18] Eraslan, H. (2002), "Uniqueness of Stationary Equilibrium Payoffs in the Baron-Ferejohn Model," *Journal of Economic Theory*, vol. 103(1), 11-30.

[19] Eraslan, H. and A. McLennan, (2013), "Uniqueness of Stationary Equilibrium Payoffs in Coalitional Bargaining," *Journal of Economic Theory*, vol. 148(6), 2195-222.

[20] de Figueiredo, J., C. Ji and T. Kousser, (2010), "Financing Direct Democracy: Revisiting the Research on Campaign Spending and Citizen Initiatives," *Journal of Law, Economics, and Organization*, vol. 27(3), 485-514.

[21] Gerber, E., *The Populist Paradox: Interest Group Influence and the Promise of Direct Legislation*, Princeton, NJ: Princeton University Press, 1999.

[22] Goreclose, T. and J. Snyder, (1996), "Buying Supermajorities," *American Political Science Review*, vol. 90(2), 303-15.

[23] Haller, H. and S. Holden, (1997), "Ratification Requirement and Bargaining Power," *International Economic Review,* vol. 38(4), 825-51.

[24] Humphreys, M., (2007), "Strategic Ratification," *Public Choice*, vol. 132(1-2), 191-208.

[25] Harrington, J. (1990), "The Power of the Proposal Maker in a Model of Endogenous Agenda Formation," *Public Choice*, vol. 64(1), 1-20.

[26] Hillman, A. and J. Riley, (1989), "Politically Contestable Rents and Transfers," *Economics and Politics*, vol. 1(1), 17-39.

[27] Iida, K. (1996), "Involuntary Defection in Two-Level Games," *Public Choice*, vol. 89(3-4), 283-303.

[28] Ingberman, D. (1985), "Running Against the Status-quo: Institutions for Direct Democracy Referenda and Allocations Over Time," *Public Choice*, vol. 46(1), 19-43.

[29] Kalandrakis, A. (2004), "A Three-Player Dynamic Majoritarian Bargaining Game," *Journal of Economic Theory*, vol. 116(2), 294-322.

[30] Kalandrakis, A. (2006), "Proposal Rights and Political Power," *American Journal of Political Science*, vol. 50(2), 441-48.

[31] Lupia, A. (1992), "Busy Voters, Agenda Control, and the Power of Information," *The American Political Science Review*, vol. 86(2), 390-403.

[32] Lupia, A. and J. Matsusaka, (2004), "Direct Democracy: New Approaches to Old Questions," *Annual Review of Political Science*, vol. 7, 463-82.

[33] Matsusaka, J. (2005a), "Direct Democracy Works," *Journal of Economic Perspectives*, vol. 19(2), 185-206.

[34] Matsusaka, J. (2005b), "The Eclipse of Legislatures: Direct Democracy in the 21st Century," *Public Choice*, vol. 124(1), 157-77.

[35] McCarty, N. (2000), "Proposal Rights, Veto Rights and Political Bargaining," *American Journal of Political Science*, vol. 44(3), 506-22.

[36] Powell, R. (1996), "Bargaining in the Shadow of Power," *Games and Economic Behavior*, vol. 15, 255-89.

[37] Putnam, R. (1988), "Diplomacy and Domestic Politics: The Logic of Two-Level Games," *International Organization*, vol. 42(3), 427-60.

[38] Romer, T. and H. Rosenthal, (1978), "Political Resource Allocation, Controlled Agendas, and the Status-quo," *Public Choice*, vol. 33(4), 27-43.

[39] Romer, T. and H. Rosenthal, (1979), "Bureaucrats versus Voters: On the Political Economy of Resource Allocation by Direct Democracy," *Quarterly Journal of Economics*, vol. 93(4), 563-87.

[40] Skaperdas, S. (1996), Contest Success Functions," *Economic Theory*, vol. 7(2), 283-90.

[41] Skaperdas, S. and S. Vaidya, (2012), "Persuasion as a Contest," *Economic Theory*, vol. 51(2), 465-86.

[42] Snyder, J. (1989), "Election Goals and the Allocation of Campaign Resources," *Econometrica*, vol. 57(3), 637-60.

[43] Snyder, J., M. Ting and S. Ansolabehere, (2005), "Legislative Bargaining under Weighted Voting," *American Economic Review*, vol. 95(4), 981-1004.

[44] Szidarovszky, F. and K. Okuguchi, (1997), "On the Existence and Uniqueness of Pure Nash Equilibrium in Rent-Seeking Games," *Games and Economic Behavior*, vol. 18(1), 135-40.

[45] Tullock, G., "Efficient Rent-Seeking" in: J.M. Buchanan, R.D. Tollison and G. Tullock, Toward a Theory of the Rent-Seeking Society, College Station, TX: Texas A.&M. University Press, 97-112, 1980.

[46] Winter, E. (1996), "Voting and Vetoing," *American Political Science Review*, vol. 90(4), 813-23.

[47] Yildirim, H. (2007), "Proposal Power and Majority Rule in Multilateral Bargaining with Costly Recognition," *Journal of Economic Theory*, vol. 136(1), 167-96.