Auctions vs. Negotiations: The Effects of Inefficient Renegotiation

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Abstract: For the procurement of complex goods the early exchange of information is important to avoid costly renegotiation ex post. We show that this is achieved by bilateral negotiations but not by auctions. Negotiations strictly outperforms auctions if sellers are likely to have superior information about possible design improvements, if renegotiation is costly, and if the buyer’s bargaining position is sufficiently strong. Moreover, we show that negotiations provide stronger incentives for sellers to investigate possible design improvements than auctions. This provides an explanation for the widespread use of negotiations as a procurement mechanism in private industry.

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I. Introduction

A company or a government agency that wants to buy a customized good has to decide which mechanism to use to select a contractor and to determine the price. The two most popular procurement mechanisms are auctions and direct negotiations. The advantages of auctions are well understood. An auction selects the bidder with the lowest cost, it achieves low prices by in-

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ducing strong competition between bidders, and it safeguards against corruption and favoritism because of its transparency and strict rules. For these reasons the Agreement on Government Procurement (GPA), which came into force in 1996 under the auspices of the World Trade Organization, requires transparent, nondiscriminatory, and open competitive tendering for the award of public procurement orders that exceed certain thresholds. Due to this agreement, nowadays, the large majority of public procurement in developed countries is conducted by competitive tendering. In the private sector, however, auctions are chosen far less frequently. In a recent empirical study Bajari, McMillan, and Tadelis (2009, p. 373) report that almost half of private sector non-residential building construction projects in Northern California from 1995 to 2000 were procured using negotiations. In this paper, we offer a new theoretical explanation for the common use of negotiations that focuses on the exchange of information and the inefficiencies of renegotiation.

When customized goods or services such as a building, a custom-made software or consultancy services are to be procured, the exchange of information between the buyer and the contractor is of crucial importance. Ex ante there are many possible designs of the project. Often the buyer is not aware of all design possibilities and she may lack important technical information. Sellers have complementary skills and information that can be very useful to specify the project efficiently. If the buyer decides to run an auction, a potential bidder has little incentive to communicate his ideas before winning the auction. In fact if a seller has an idea for a design improvement he will strategically hold back this information because it gives him an advantage over his rivals in the bidding process. If he wins the auction, he can use his information for making additional profits by offering to renegotiate the design. Thus, when the buyer prepares the auction she has to fix a specification of the good that is likely to be suboptimal. Not surprisingly, procurement contracts are frequently renegotiated giving rise to substantial design changes and cost increases.

If renegotiation is costless and yields an efficient outcome, there is no problem: The buyer and the contractor always renegotiate to the efficient design, no matter what the initial contract specifies. Furthermore, the seller with the best idea for a design improvement is likely to win the auction. He would gain most from renegotiating the contract and therefore bid most aggressively. However, there is substantial evidence that contract renegotiation is often costly.

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1 An agreement on government procurement was first negotiated during the Tokyo Round, in the context of the GATT Code, and entered into force on 1 January 1981. The GPA is a plurilateral agreement signed by, among others, Canada, the member states of the European Union and the European Commission, Japan, Norway, South Korea, Switzerland, and the United States (Audet, 2002). In the United States public procurement in accordance with the GPA is regulated by the Federal Acquisition Regulation. In the European Union three directives prescribe the rules for public procurement orders, which are often stricter than the GPA rules; Directive 2014/24/EU (Public Sector), Directive 2014/25/EU (Utilities), and Directive 2014/23/EU (concession contracts). For more details regarding public procurement in the EU see Drijber and Stergiou (2009).

2 This was first pointed out by Goldberg (1977). Sweet (1994) describes the problem as follows: “Separation of design and construction deprives the owner of contractor skill during the design process, such as sensitivity to the labor and material markets, knowledge of construction techniques, and their advantages, disadvantages and costs. A contractor would also have the ability to evaluate the coherence and completeness of the design, and, most importantly, the costs of any design proposed.”
and inefficient. In an empirical analysis of highway procurement contracts in California, Bajari, Houghton, and Tadelis (2014, p. 1317) estimate that the costs of renegotiation “range from 55 cents to around two dollars for every dollar in change”. Renegotiation is costly for two reasons: First, it disrupts the originally planned work and affects the contractual obligations of the buyer and the seller to other parties. Second, it gives rise to conflicts over who should bear the additional costs. These disputes gobble up additional resources for lawyers and arbitrators and slow down the completion of the project.

In contrast, if the buyer negotiates the contract with a selected seller, the two parties typically spend a lot of time discussing the optimal design of the project before the contract is signed. Thus, there is less need for costly renegotiation ex post. This is why practitioners and handbooks of procurement often recommend using negotiations for the procurement of complex projects.

In this paper, we develop a theoretical model with costly and inefficient renegotiation that allows us to analyze the costs and benefits of auctions and negotiations as procurement mechanisms. With an auction a seller will never reveal possible project improvements to the buyer before the contract is signed because the contract turns a highly competitive situation (the auction) into a bilateral monopoly (the renegotiation game). In contrast, with negotiations the seller will share his information with the buyer before the contract is written. Here the buyer and the seller are in a bilateral monopoly position from the start. If the seller withholds information in order to renegotiate later, he reduces the social surplus but his share of the surplus is unchanged. Thus, negotiations avoid costly renegotiation, but at the price of less ex ante competition. This is the first tradeoff that we identify: If renegotiation is costless auctions outperform negotiations because they induce more competition and lower prices. However, if renegotiation is costly, negotiations can be superior. This is more likely to be the case if design improvements are important, if renegotiation costs are large, and if the buyer’s bargaining position is strong.

In the second part of the paper we consider the incentives of (potential) sellers to invest into finding design improvements. This gives rise to a second tradeoff. We show that a seller who negotiates with a buyer always has a stronger incentive to invest than any seller participating in the auction. This is due to the fact that the return of this investment is diminished in the auction because renegotiation is inefficient. Furthermore there is a discouragement effect of the auction: the more sellers are participating in the auction, the smaller is the incentive for each bidder to invest. On the other hand, there is also a sampling effect. The more bidders there are, the more likely it is that at least one of them finds the design improvement (keeping the investment of each bidder constant). Negotiations are more likely to give rise to a higher probability of finding the project improvement if the cost of renegotiation is high.

Related literature: There is an extensive literature on the optimal design of procurement contracts, see e.g. McAfee and McMillan (1986) and Laffont and Tirole (1993). These mechanism
design approaches do not consider how the optimal procurement contract is allocated, i.e. they assume that the allocation procedure does not affect the performance of the contract. In contrast, our paper shows that the performance of a contract may depend on how it is allocated.

The seminal contribution comparing auctions and negotiations is Bulow and Klemperer (1996). They show that an open English auction with \( n + 1 \) bidders yields higher revenues than an optimally designed auction with \( n \) bidders. This result implies that a simple auction with at least two bidders is better than optimally structured bilateral negotiations—i.e., the value of negotiation skill is small relative to the value of additional competition.\(^4\) We show that this argument extends to the case of an incomplete contract (that needs to be renegotiated) if renegotiation is efficient. However, if renegotiation is costly and inefficient, the buyer can be better off to forgo competition and to negotiate with one seller.

\(^4\) Bulow and Klemperer (2009) directly compare a simple simultaneous auction to sequential negotiations when participation is costly. The auction generates higher expected revenues but is less desirable from a welfare point of view. A similar finding is obtained by Pagnozzi and Rosato (2014) in the context of firm takeovers.

Manelli and Vincent (1995) point to a potential disadvantage of auctions that arises if quality is unobservable ex ante and not verifiable by a third party ex post. If quality and production costs are positively correlated the seller with the lowest costs (who is going to win the auction) is likely to offer low quality. Manelli and Vincent (1995) show that there are conditions under which the optimal mechanism is a bargaining process, where the buyer can make take-it-or-leave it offers sequentially to each seller. In our setup there is no unobservable quality, so this problem does not arise.

The paper most closely related to ours is Bajari and Tadelis (2001). In their model, as well as in ours, the procurement contract is incomplete and renegotiation is costly. The focus of Bajari and Tadelis (2001) is on problems of ex post adaptation, when the initial design is endogenously incomplete. They do not consider different award procedures but compare the performance of two types of contracts, fixed-price and cost-plus contracts. If the good that the buyer wants to procure is a standardized or rather simple good, the buyer should use a fixed-price contract that gives strong cost saving incentives to the seller. If the good is complex, however, the procurement contract is likely to be renegotiated. In their model renegotiation is plagued by asymmetric information. Thus, with a fixed-price contract renegotiation fails with positive probability giving rise to an inefficient outcome. With a cost-plus contract, on the other hand, renegotiation is always efficient because the seller is automatically reimbursed for any cost increase. The cost-plus contract, however, gives no cost saving incentives to the seller because all additional costs are borne by the buyer. The authors show that if the cost of renegotiation is sufficiently large, a cost-plus contract outperforms a fixed-price contract. Our model differs in three important respects from Bajari and Tadelis (2001). First, we investigate how the procurement contract should be awarded, which is not an issue in their paper. Second, Bajari and Tadelis (2001) focus on ex post asymmetries of information arising during the execution of the project, while we are interested in the efficient design of the project ex ante. Finally, in their model the information
structure is exogenously given while we endogenize the incentives of the sellers to find project improvements.

Our paper is also related to some strands of the literature on incomplete contracts and renegotiation. Tirole (2009) derives contracts that are endogenously incomplete. In his model (as in ours) the contracting parties are unaware of the ex post optimal design, but they are aware that they are unaware. They can invest mental resources ex ante in order to figure out early what the ex post optimal design is. If one party discovers the optimal design, the enunciation of the design is an ‘eye-opener’ to the other party. Thus, if a party suggests contracting on this design, it gives the information away and cannot fully benefit from it, in the same way as an inventor in Arrow (1962) cannot fully benefit from his invention because he has to reveal it in order to contract on it. In Tirole (2009) the optimal design of the project becomes common knowledge automatically ex post, while in our model the seller has to find and reveal the optimal design to the buyer. This is why in Tirole (2009) there can be inefficiently high incentives to discover the ex post optimal design (contracts are “too complete”), which is not the case in our model. Moreover, Tirole only considers negotiations between one buyer and one seller. We compare negotiations to auctions and focus on the incentives of sellers to reveal their information early rather than late. In particular, we investigate the inefficiencies caused by deferred information revelation due to costly renegotiation, which does not play a role in Tirole’s model.

Moreover, our paper builds on the literature on contracts as reference points and ex post inefficiencies. Hart and Moore (2008) argue that a contract provides a reference point. The contracting parties have self-serving biases in the interpretation of the contract that give rise to conflicts and inefficient behavior ex post. Building on the idea that contracts are reference points, Herweg and Schmidt (2015) propose a theory of inefficient renegotiation. If two parties renegotiate a contract they compare the renegotiation proposal to the initial contract. This comparison is distorted by loss aversion which gives rise to an inefficient renegotiation outcome. In Herweg and Schmidt (2015) we ask how the initial contract should be structured in order to minimize the inefficiencies of the renegotiation process. In contrast, in the current paper we keep the contract fixed and show that the cost of renegotiation is affected by how the contract is allocated (by auction or by negotiation).

There are a few empirical papers comparing auctions and negotiations in procurement. Bajari, McMillan, and Tadelis (2009) look at a comprehensive data set of private sector building contracts in Northern California. They report that the more complex the project is the more likely it is to be awarded by negotiation than by auction. On the other hand, auctions are more likely to be used if there are more potential sellers. Similar findings are obtained by Leffler, Rucker, and Munn (2003) for private sales of timber in North Carolina and by Chong, Staropoli, and Yvrande-Billon (2014) for public building contracts in France. Lalive and Schmutzler
consider the procurement of public transport in Germany and report that auctions yield significantly lower prices than negotiations. Finally, Bajari, Houghton, and Tadelis (2014) study highway paving contracts in Northern California that are awarded by auction. They report that these contracts are often renegotiated and that renegotiation yields significant adaptation costs of 7.5 to 14 percent of the winning bid. All of these findings are consistent with our theoretical results and our modeling approach.

The remainder of the paper is organized as follow. Section II sets up the baseline model and discusses our modeling assumptions. In Section III the baseline model is solved by backwards induction, i.e., we first characterize the outcome of renegotiation, then sellers’ incentives for information disclosure are analyzed, and finally we compare the performance of auction and negotiation. In Section IV we augment the baseline model by allowing sellers to invest into finding project improvements. The final Section V concludes. All proofs are relegated to the Appendix A.

II. The Baseline Model

II.1. Procurement Operator and Potential Contractors

A buyer (B) wants to procure a complex project that is tailored to her specific needs, such as a particular building, a tailor made software program, or a custom made component needed in production. The project can be executed by \( n \geq 2 \) potential sellers, denoted by \( S_i \) with \( i = 1, \ldots, n \). We assume that sellers are symmetric and have the same cost function \( c(\cdot) \), which depends on the implemented specification of the project.\(^6\) The seller who is selected to carry out the project is called the contractor (he). The buyer’s (she) gross benefit is \( v(\cdot) \), which also depends on the specification of the project. Without receiving additional information, the buyer believes that specification \( \bar{y} \) maximizes the social surplus, i.e., the buyer wants to procure \( \bar{y} \) in this case. The specification \( \bar{y} \) gives rise to gross benefit \( v(\bar{y}) = \bar{v} \), cost \( c(\bar{y}) = \bar{c} \), and social surplus \( \bar{S} = \bar{v} - \bar{c} \). The outside option utilities of all parties are normalized to zero.

Sellers often have additional skills and knowledge and may be able to come up with a more efficient project than project \( \bar{y} \). The buyer lacks these skills and knowledge and therefore is unaware of these alternative specifications, which might also be a reason for why the project is procured from an outside firm and not produced in-house. We model this as follows: Each seller may be aware of a set of superior specifications \( Y \), where all specifications \( y \in Y \) yield a higher benefit for the buyer but are also more costly to produce for the seller than product \( \bar{y} \).

\(^6\) The assumption that all sellers are symmetric is made for simplicity only. It strengthens the case for auctions because in this case an auction gives all the surplus to the buyer. We briefly discuss the case of asymmetric sellers in the conclusions.

\(^7\) It is straightforward to extend the analysis to the case where \( y \) reduces the buyer’s benefit but reduces the seller’s cost even more, or where \( \bar{y} \) increases the buyer’s benefit and decreases the seller’s cost. See Herweg and Schmidt (2015) for a formal analysis of costly renegotiation in these cases.
Assumption 1. For all project specifications in the superior set, \( y \in Y \), it holds that:

(i) \( v(y) - c(y) > \bar{v} - \bar{c} \);
(ii) \( c(y) > \bar{c} \).

Assumption 1 implies that for all \( y \in Y \) we have \( v(y) > \bar{v} \). To begin with, we posit that each seller is aware of the set of superior specifications with exogenous probability \( q \in (0, 1) \). Later, we endogenize the probabilities with which sellers are aware of superior project specifications. For each seller \( S_i \) an independent draw by nature determines whether a seller knows \( Y \) or not. The knowledge of \( Y \) is private information of a seller.

Suppose that a seller did find superior projects \( Y \) and revealed them to the buyer ex ante. Now, if the buyer wants to procure a project \( y \in Y \), all sellers can produce this project at the same cost \( c(y) \)—even sellers who initially were not aware of the set of projects \( Y \). A project specification \( y \) corresponds to an innovative idea on how the project could be specified. Once the innovative specification is explained to an unaware seller, the seller understands the new idea (‘his eyes are opened’) and he can execute this specification at the same costs as a seller who initially came up with the idea. A crucial question is whether an informed seller has an incentive to reveal the superior set \( Y \) to the buyer before concluding a contract or only afterwards. If he informs the buyer about the set of superior projects \( Y \) before the initial contract is signed, the buyer wants to procure project

\[
y^* \in \arg \max_{y \in Y} \{v(y) - c(y)\}
\]

that gives rise to social surplus \( S^* = v^* - c^* > \bar{S} \), where \( v(y^*) = v^* \) and \( c(y^*) = c^* \). Define

\[
\Delta S^* = S^* - \bar{S}
\]

as the maximal additional surplus that can be generated due to the superior project. If none of the sellers informed the buyer about the existence of possible superior projects ex ante, the buyer procures project \( \bar{y} \). In this case, after the contract on \( \bar{y} \) has been concluded, the contractor can still reveal \( Y \) and propose to renegotiate. A crucial assumption we impose is that the buyer is unaware of the specifications that the superior projects may take.\(^8\) Nevertheless, the buyer is aware that she is unaware, i.e., she knows that sellers may be aware of superior specifications, but she does not know how these superior specifications may look like.\(^9\)

II.2. Procurement Mechanisms

The buyer can use one of two mechanisms to conclude the procurement contract: Either an auction (\( A \)) or bilateral negotiations (\( N \)).

\(^8\)The buyer faces radical uncertainty in the sense of Knight (1921).

\(^9\)In the language of former U.S. Secretary of Defense Donald Rumsfeld, the potential superior project is a ‘known unknown’; “We also know there are known unknowns; that is to say we know there are some things we do not know.” See Defense.gov News Transcript, DoD News Briefing — Secretary Rumsfeld and Gen. Myers, United States Department of Defense (defense.gov), http://www.defense.gov/transcripts/transcript.aspx?transcriptid=2636.
If $B$ uses an auction, she runs a sealed-bid second-price auction for a project specification $\tilde{y}$—i.e., the seller who offers the lowest price is awarded the contract and receives the price offered by the second lowest bidder. If several bidders make the same lowest bid, one of them is selected at random. In general, a cost-minimizing auction involves setting a maximum bid. However, in our model all sellers have the same cost function. Thus, if the surplus from good $\tilde{y}$ is sufficiently high, the optimal maximum bid for the procurement of good $\tilde{y}$ is simply $c(\tilde{y}) = \tilde{c}$, which never precludes a seller from participating in the auction. Therefore, a second-price auction without a maximum bid is without loss of generality. Crucially, in our setting, the buyer cannot use a scoring auction. In a scoring auction the seller who places the bid $(y, p)$ with the highest score $f(y, p)$ wins the auction, where $f(y, p)$ is the scoring function. For example, if the buyer could set $f(y, p) = v(y) - p$, then the seller offering the highest surplus to the buyer would win the auction. However, we are looking at a situation where the buyer is not aware of all possible design specifications. She cannot specify a scoring function $f(y, p)$ for $y \in Y$ ex ante because she is unaware of $y \in Y$.\footnote{There are several additional problems with the implementation of scoring auctions. Scoring auctions can be used for dimensions of quality that are contractible ex ante and verifiable ex post, e.g. the size or resolution of a computer screen or the fuel efficiency of an engine. It cannot be used for quality dimensions that are non-contractible and/or non-verifiable, such as the aesthetics of a building, the appeal of a marketing strategy, or the quality of a consulting project. Furthermore, scoring auctions are prone to corruption and favoritism. See Din, Picini, and Valletti (2006) for a more extensive discussion of scoring auctions.}\footnote{\text{Din, Picini, and Valletti (2006)} for a more extensive discussion of scoring auctions.}

On the other hand, if the buyer decides to negotiate the procurement contract, she picks one seller at random, i.e., each seller is selected with probability $1/n$. We employ the Generalized Nash Bargaining Solution (GNBS) to characterize the bargaining outcome. Let the buyer’s relative bargaining power be $\alpha \in (0, 1)$.\footnote{\text{In Appendix B we show that the GNBS gives rise to the same bargaining outcome as a non-cooperative bargaining game in which the buyer (seller) makes a take-it-or-leave-it offer with probability $\alpha (1 - \alpha)$, respectively.}}

II.3. Time Structure

The time structure of the model is as follows:

0) At stage 0 the buyer decides which procurement mechanism to use, $A$ or $N$. The choice is publicly announced, but the exact procurement contract is not yet specified. If mechanism $N$ is used, $B$ randomly selects one seller $i, i \in \{1, \ldots, n\}$, with whom to negotiate a contract. In this case the buyer commits to negotiate the procurement contract solely with the selected contractor. The other sellers exit the game. If the buyer chooses to run an auction, all sellers can participate in the auction and the buyer commits to awarding the contract to the bidder offering the lowest price.\footnote{The assumption that the buyer can commit to a mechanism is standard in the literature on auctions and mechanism design. In practice this commitment can be achieved if the buyer is afraid to lose her reputation as a trustworthy contracting partner or if she signs a pre-contract that debars her from legally entering into a similar contract at a later time with any other party.}\footnote{\text{The assumption that the buyer can commit to a mechanism is standard in the literature on auctions and mechanism design. In practice this commitment can be achieved if the buyer is afraid to lose her reputation as a trustworthy contracting partner or if she signs a pre-contract that debars her from legally entering into a similar contract at a later time with any other party.}}

1) At stage 1, nature determines by an independent draw for each potential seller whether
this seller becomes aware of the set of superior specifications \(Y\) or not. If a seller knows a project improvement he decides whether or not to tell the buyer about it.

(2) At stage 2 the procurement mechanism is executed. In case of an auction the buyer auctions the procurement order for project \(\bar{y}\) if she is uninformed and for project \(y^*\) if she is informed about the set \(Y\) by at least one of the sellers. Each seller \(i\) places a bid \(p_i\) and the seller who placed the lowest bid wins the auction, and the price is determined by the second lowest bid. In case of negotiation the buyer and the selected seller (the contractor) negotiate a specific performance contract \((\hat{y}, p)\). If \(B\) is uninformed the two parties agree to trade specification \(\bar{y}\), while if \(B\) is informed they agree to trade specification \(y^*\).

(3) At stage 3 the parties may renegotiate. If the initial contract specifies project \(y^*\), there is no need for renegotiation and project \(y^*\) is executed. If the contract specifies \(\bar{y}\), there may be scope for renegotiation. If the contractor is aware of possible project improvements, but the buyer was not informed at stage 1, the contractor may now inform the buyer about \(Y\). If the buyer learns \(Y\), the parties renegotiate.

Note that the buyer commits to the procurement mechanism. If she chooses to negotiate with one selected seller she cannot turn to another seller, if she chooses an auction she cannot allocate the contract to a seller who is not the lowest bidder.

II.4. Renegotiation

If the parties renegotiate, they split the surplus from renegotiation according to the Generalized Nash Bargaining Solution (GNBS). We assume that the relative bargaining power of the buyer in the renegotiation game does not depend on whether the initial contract was allocated via an auction or via negotiations, and that the buyer’s bargaining power is the same in the initial negotiation game and in the renegotiation game. Thus, the buyer's relative bargaining power is always \(\alpha \in (0, 1)\). Notice that with negotiation the buyer’s relative bargaining power is the same when she negotiates at stage 2 and when she renegotiates at stage 3. In contrast, with an auction there is a fundamental transformation in the sense of [Williamson (1985)] —i.e., a highly competitive situation at stage 2 turns into a bilateral monopoly at stage 3.

A crucial point of our modeling approach, next to sellers’ superior information regarding the optimal specification, is that we posit renegotiation to be plagued by imperfections. In other words, renegotiating a contract is costly. There are at least two reasons for this assumption, (1) physical adjustment costs and (2) psychological costs, e.g. caused by loss aversion.

**Physical adjustment costs:** Suppose that the parties signed an initial contract on implementing project \(\bar{y}\) at price \(\bar{p}\). After some time the seller approaches the buyer and informs her that he found some other, more efficient project \(y \in Y\). When the parties change \(\bar{y}\) to \(y\) they have to incur adjustment costs. For example, in preparation for \(\bar{y}\) the buyer and the seller had to
make plans how to use their resources to implement \( \bar{y} \), they had to commit to a time table that is based on \( \bar{y} \) and that affects other projects they are involved in, and they had to write additional contracts with subcontractors, investors, and/or clients that are all conditional on \( \bar{y} \). Switching to a new project \( y \) and a new price \( p \) implies that the parties have to undo some of these commitments which is costly and disruptive.

**Psychological costs:** There is also a psychological cost of renegotiation. Both parties have to make concessions. The buyer has to pay a higher price than planned initially and the seller has to incur higher production costs. Each party feels entitled to the concession of the other party but is reluctant to concede itself. This gives rise to haggling and conflicts. Following Tversky and Kahneman (1991) and Herweg and Schmidt (2015) we model this psychological cost as loss aversion.\(^{13}\) When the parties renegotiate the initial contract they compare the renegotiation proposal \((y, p)\) to the initial contract \((\bar{y}, \bar{p})\). From the perspective of the buyer the new contract offers a higher benefit \( v(y) > v(\bar{y}) \), which is considered a gain, but because \( y \) is more costly to produce for the seller it also requires a higher price \( p > \bar{p} \), which is considered a loss by the buyer. Similarly, from the perspective of the seller the higher price is considered a gain, while the higher production cost is considered a loss. For both parties losses loom larger than gains of equal size. This drives a wedge between the benefit of the buyer and the cost of the seller giving rise to an inefficient renegotiation outcome. There is ample experimental and field evidence showing that people evaluate outcomes not (only) in absolute terms but (also) relative to a reference point, and that losses (in comparison to this reference point) loom larger than gains of equal size.\(^{14}\)

**Empirical evidence:** Several empirical studies emphasize the importance of costly renegotiation, including Crocker and Reynolds (1993), Chakravarty and MacLeod (2009) and Bajari, Houghton, and Tadelis (2014). Bajari, Houghton, and Tadelis (2014) consider highway procurement contracts in California and report that renegotiation costs are substantial. They distinguish between ‘direct’ and ‘indirect adaptation costs’. Direct adaptation costs are due to disruption of the originally planned work and correspond to our interpretation of physical adjustment costs. Indirect adaptation costs are due to contract renegotiation and dispute resolution and are related to the psychological cost of renegotiation: “Each side may try to blame the other for any needed changes, and they may disagree over the best way to change the plans and specifications. Disputes over changes may generate a breakdown in cooperation on the project site and possible lawsuits” (Bajari, Houghton, and Tadelis, 2014, p.1294-95).

**Modeling renegotiation costs:** Following Herweg and Schmidt (2015) we model both types of adaptation costs as follows. If the parties renegotiate contract \((\bar{y}, \bar{p})\) to contract \((y, p)\) with

\(^{13}\)Tversky and Kahneman (1991, p. 1057) argue that “contracts define the reference levels for [...] bargaining; in the bargaining context the aversion to losses takes the form of an aversion to concessions”.

\(^{14}\)See Herweg and Schmidt (2015) for a detailed discussion and analysis of loss aversion in contract renegotiation.
v(y) > v(\tilde{y}), c(y) > c(\tilde{y}), and p > \tilde{p}, then the final utilities of the buyer and the contractor are given by

\begin{align*}
U^B(y, p) &= v(y) - p - \lambda^B[p - \tilde{p}], \quad \text{and} \\
U^S(y, p) &= p - c(y) - \lambda^S[c(y) - c(\tilde{y})],
\end{align*}

respectively. The parameter $\lambda^j \geq 0$, with $j \in \{B, S\}$, measures how costly renegotiation is to party $j$. In the first interpretation, $\lambda^j$ measures how costly adjustments are, while in the second interpretation it measures the degree of loss aversion. The buyer incurs a loss that is proportional to the price increase (e.g. because of higher costs to acquire additional financing for the new project or because she feels the loss of the price increase more strongly than an equally sized gain of the increase in benefits). The contractor incurs a loss that is proportional to the increase in production costs (e.g. because changing the project is disruptive or because he feels a loss of the cost increase more strongly than an equally sized gain of the price increase).

We choose this specification of renegotiation costs because it allows for a unified treatment of physical and psychological adaptation costs. Furthermore, the simplicity of the model due to its linear structure allows us to fully characterize the renegotiation outcome.

### III. Analysis of the Baseline Model

We analyze symmetric pure-strategy perfect Bayesian Nash equilibria and solve the game by backward induction. First, we characterize the outcome of renegotiation for an initial contract $(\tilde{y}, \tilde{p})$. Thereafter, we investigate sellers’ incentives to reveal design improvements already at stage 1 and how these incentives differ across the two types of procurement mechanisms, auction and negotiation. Finally, we analyze which mechanism maximizes the buyer’s expected utility.

#### III.1. The Outcome of Renegotiation

Suppose the buyer and the contractor concluded a contract $(\tilde{y}, \tilde{p})$ at stage 2. If the contractor is unaware of a superior specification, the initial contract is executed. If the contractor is aware of the set of superior projects $Y$, there is scope for renegotiation at stage 3—i.e., the contractor reveals $Y$ to the buyer and the parties consider renegotiating the initial contract. Renegotiation necessarily leads to a higher price because the contractor incurs higher production costs. By the Generalized Nash Bargaining Solution (GNBS) the renegotiation contract $(y^R, p^R)$ solves

\begin{equation}
\max_{y, p} \left\{ v(y) - \tilde{v} - (1 + \lambda^B)(p - \tilde{p}) \right\}^\alpha \left\{ p - \tilde{p} - (1 + \lambda^S)[c(y) - \tilde{c}] \right\}^{1-\alpha}.
\end{equation}

The solution to this problem is characterized by the following proposition.

---

\[\text{Our main findings do not rely on our specific modeling approach but hold (qualitatively) for any model of costly renegotiation. A convincing feature of our approach is that contracts are typically renegotiated in the light of new information and that the implemented adjustments are often too small compared to the adjustments necessary to implement the first-best specification.}\]
**Proposition 1 (Outcome of Renegotiation).** Let the initial contract be \((\bar{y}, \bar{p})\). At stage 3 the contract \((y^R, p^R)\) is implemented (potentially after renegotiation), with

\[
y^R \in \arg \max_{y \in \{Y \cup \{\bar{y}\}\}} \{ v(y) - \bar{v} - (1 + \lambda^B)(1 + \lambda^S)[c(y) - \bar{c}] \}
\]

and

\[
p^R = \bar{p} + \frac{1 - \alpha}{1 + \lambda^B} [v(y^R) - \bar{v}] + \alpha(1 + \lambda^S)[c(y^R) - \bar{c}].
\]

The final payoffs are given by

\[
U^B = \bar{v} - \bar{p} + \alpha \{ v(y^R) - \bar{v} - (1 + \lambda^B)(1 + \lambda^S)[c(y^R) - \bar{c}] \}
\]

\[
U^S = \bar{p} - \bar{c} + \frac{1 - \alpha}{1 + \lambda^B} \{ v(y^R) - \bar{v} - (1 + \lambda^B)(1 + \lambda^S)[c(y^R) - \bar{c}] \}.
\]

**Proof.** All proofs are relegated to the appendix. \(\square\)

Note that if there are no adjustment costs, \(\lambda^B = \lambda^S = 0\), the parties will always renegotiate and the renegotiation outcome is efficient, i.e. \(y^R = y^*\). If \(\lambda^B\) and/or \(\lambda^S\) are strictly positive, the renegotiation outcome is inefficient because the adjustment costs drive a wedge between the benefit of the buyer and the cost of the seller. More precisely, renegotiation takes place if and only if there is a \(y \in Y\) such that

\[
v(y) - \bar{v} > (1 + \lambda^B)(1 + \lambda^S)[c(y) - \bar{c}].
\]

When renegotiation takes place, the renegotiated project depends on the adjustment cost parameters \(\lambda^B\) and \(\lambda^S\), but it does not depend on \(\alpha\), the bargaining power of the buyer. However, the additional surplus that is generated through renegotiation does depend on \(\alpha\). If the buyer has all the bargaining power (\(\alpha = 1\)), the renegotiation surplus is given by

\[
\Delta S^R(\lambda^B, \lambda^S) \equiv v(y^R) - \bar{v} - (1 + \lambda^B)(1 + \lambda^S)(c(y^R) - \bar{c}) \geq 0.
\]

If the seller has some bargaining power (\(\alpha < 1\)), the surplus from renegotiation is reduced to \(\frac{1 + \alpha \lambda^B}{1 + \lambda^B} \Delta S^R\). The reason is that a higher bargaining power of the seller implies a higher renegotiation price for the buyer. Transfers, however, are costly. A price increase by \(\Delta p\) reduces the buyer’s utility by \((1 + \lambda^B)\Delta p\) and thus gives rise to a further welfare loss of \(\lambda^B \Delta p\).

**III.2. Information Revelation**

We now turn to stage 1 and investigate whether sellers have an incentive to reveal design improvements at stage 1 before the procurement contract is determined. In order to do so, we also have to analyze the outcome at stage 2 under the two procurement mechanisms.

\[\text{[16] See Proposition 1 by Herweg and Schmidt (2015).}\]
**Negotiation:** First, we consider the case where the buyer negotiates the contract with one (randomly selected) seller. In this case, the contractor is better off by revealing any project improvements early.

**Proposition 2** (Information Disclosure in Negotiations). Suppose the buyer negotiates the procurement contract with one seller, the contractor. Then, the contractor has a strict incentive to reveal any project improvements early to the buyer, i.e. before the contract is signed.

The intuition behind Proposition 2 is straightforward: If the contractor informs the buyer about possible project improvements the parties will agree to trade the efficient project $y^*$ and the contractor gets fraction $1 - \alpha$ of the surplus. If the seller does not inform the buyer, the parties will contract on $\bar{y}$ initially. Now the contractor waits until stage 3 and then reveals that there are possible project improvements and thus scope for renegotiation. However, renegotiation is inefficient. Therefore, the seller will get fraction $1 - \alpha$ of the renegotiation surplus, additionally to fraction $1 - \alpha$ of the initial surplus based on trade of $\bar{y}$. This, however, is smaller than the surplus he would have received if he had revealed $Y$ right away.

**Auction:** Consider now the case of an auction. With an auction, each seller is better off not revealing any information about possible project improvements until after the procurement contract has been signed.

**Proposition 3** (Information Disclosure in Auctions). Suppose the buyer runs an auction. Then, each seller strictly prefers not to reveal possible project improvements to the buyer before the contract has been signed.

If seller $i$ finds a project improvement and informs the buyer about it, the buyer auctions off project $y^*$. In this case each seller $i$ bids $b_i = c(y^*)$. Thus, each seller gets an expected payoff of zero from the auction. If seller $i$ does not reveal the project improvement immediately but waits until after the auction, he may get a strictly positive payoff in the renegotiation game. The reason is that after the auction the competitive situation turns into a bilateral monopoly in which the seller has some bargaining power and gets fraction $1 - \alpha$ of the renegotiation surplus. If he is the only seller who discovered possible project improvements, this ex post rent will not be competed away in the auction and he gets a strictly positive profit.

**III.3. Auction vs. Negotiation**

We now turn to the decision of the buyer whether to run an auction or to negotiate with one seller. Propositions 2 and 3 point at an important tradeoff. If the buyer uses an auction sellers will not reveal possible project improvements early. Thus, renegotiation is required to implement project improvements. If renegotiation is costless this is not an issue and an auction is always optimal. If, on the other hand, renegotiation yields an inefficient outcome, using the auction is costly. In this case negotiations may outperform auctions because they provide an incentive to
the contractor to reveal his information early, so costly renegotiation can be avoided. However, negotiation with one seller is expensive because the buyer has a weaker bargaining position in negotiation than in an auction.

In order to balance these pros and cons of negotiations in comparison to auctions, the buyer needs to calculate her expected utility under the two mechanisms. Recall that the buyer is unaware of the potentially superior projects ex ante. Nevertheless, the buyer is aware that she is unaware and knows that the sellers might know project specifications that are more efficient than $\bar{y}$. For simplicity we assume that the buyer knows the additional surplus that can be achieved by implementing a superior specification, either directly or indirectly via renegotiation. Hence, even though the buyer is unaware of set $Y$ ex ante, she can calculate her expected payoff resulting from a particular procurement mechanism.\footnote{An experienced buyer may have a rough idea of the likelihood and the value of possible project improvements from previous procurement situations. It is straightforward to model this in a stochastic fashion. Suppose the set $Y$ is drawn stochastically and so are the gains from implementing design improvements ex ante ($\Delta S^*$) as well as the gains from implementing them ex post ($\Delta S^R$). If the buyer forms unbiased expectations about $\Delta S^*$ and $\Delta S^R$, then all our results regarding the optimal procurement mechanism still hold.}

Formally, if the buyer negotiates, her expected payoff is

$$EU_B^U(\alpha, q, \bar{S}, \Delta S^*) = (1 - q)\alpha(\bar{v} - \bar{c}) + q\alpha(\nu^* - \bar{c}^*) = \alpha\bar{S} + q\alpha\Delta S^*. \quad (12)$$

If the seller runs an auction, three cases have to be distinguished. With probability $(1 - q)^n$ no seller finds a project improvement. In this case all sellers bid $b_i = \bar{c}$ and the buyer’s payoff is $U^B = \bar{v} - \bar{c} = \bar{S}$. With probability $nq(1 - q)^{n-1}$ exactly one of the sellers finds an improvement. In this case the successful seller gets the contract at price $\bar{p} = \bar{c}$, but then there is renegotiation. Thus, the buyer’s payoff is $U^B = \bar{v} - \bar{c} + \alpha\Delta S^R = \bar{S} + \alpha\Delta S^R$. Finally, with probability $1 - (1 - q)^n - nq(1 - q)^{n-1}$ two or more sellers are successful. In this case competition in the auction drives down the price to $\bar{p} = \bar{c} - \frac{1 - \alpha}{1 + \lambda^B} [(v(y^R) - \bar{v}) - (1 + \lambda^B)(1 + \lambda^S)(c(y^R) - \bar{c})]$, so the buyer’s payoff is $U^B = \bar{S} + \alpha\Delta S^R + \frac{1 - \alpha}{1 + \lambda^B} \Delta S^R$.\footnote{The buyer’s ex post utilities are directly obtained from Proposition. If a seller knows the set $Y$, his price bid is obtained by solving $U^S = 0$.}

Thus, the expected payoff of the buyer if she runs an auction is

$$EU_A^U(n, \alpha, q, \bar{S}, \lambda^B, \lambda^S) = (1 - q)^n\bar{S} + nq(1 - q)^{n-1}[\bar{S} + \alpha\Delta S^R] + [1 - (1 - q)^n - nq(1 - q)^{n-1}] \left[\bar{S} + \alpha\Delta S^R + \frac{1 - \alpha}{1 + \lambda^B} \Delta S^R\right]$$

$$= \bar{S} + \Delta S^R \left\{\alpha[1 - (1 - q)^n] + \frac{1 - \alpha}{1 + \lambda^B} [1 - (1 - q)^n - nq(1 - q)^{n-1}]\right\}. \quad (13)$$

The following proposition, which is our first main result, shows that there are situations in which the buyer strictly prefers to negotiate with one seller and other circumstances in which she strictly prefers to run an auction.
Proposition 4 (Auction vs. Negotiation). The buyer strictly prefers to run an auction
(a) if the renegotiation costs are small ($\lambda^B$, $\lambda^S$ close to zero) and/or
(b) if she has little bargaining power ($\alpha$ close to 0).

The buyer strictly prefers to negotiate
(c) if renegotiation is highly inefficient ($\lambda^B$, $\lambda^S$ large) while $\Delta S^*$ is sufficiently large, and/or
(d) if her bargaining position is very strong ($\alpha$ close to 1) and the probability with which
sellers are aware of design improvements is large ($q$ close to 1).

Moreover, the payoff advantage of running an auction, $\Psi(n, \alpha, q, \bar{S}, \Delta S^*, \lambda^B, \lambda^S) = EU_A^B - EU_N^B$, is increasing in the number of potential sellers $n$.

The main advantage of negotiations are that they lead to early information revelation. How important early information revelation is depends on the inefficiencies of renegotiation. If renegotiation is efficient, running an auction with at least two competing sellers always outperforms negotiating with one seller. If renegotiation is plagued by high inefficiencies then bilateral negotiations outperform auctions even if there are many competing sellers in the auction. One advantage of running an auction is that it leads to a strong bargaining position of the buyer ex ante, i.e., the buyer can exploit the competition between sellers to get a larger share of the ex ante surplus. Therefore, if the buyer’s bargaining position is weak in bilateral negotiations, an auction is more likely to be superior. A second advantage of running an auction is that it increases the probability with which design improvements are implemented ex post. If there are sellers who know the set of superior projects, the auction always selects one of these sellers as the contractor. The more sellers there are the more likely it is that at least one of them is aware of the potential project improvements. This is the reason why the payoff advantage of running an auction is increasing in the number of sellers.

Auction vs. Negotiation: Extensions

Heterogeneous sellers: Suppose sellers are not equally likely to know about project improvements. For the sake of the argument consider the extreme case where only seller $S_1$ is aware of superior projects with positive probability. This is known to the buyer. Thus if the buyer decides to negotiate the procurement contract with one seller, she will select seller $S_1$ as the contractor. In this case, negotiation outperforms auction if the probability of knowing superior projects is sufficiently high, i.e., if

\[ q \geq \frac{1 - \alpha}{\alpha} \frac{\bar{S}}{\Delta S^* - \Delta S^R} > 0. \] (14)

In other words, if there is sufficient heterogeneity among sellers and this is known to the buyer, bilateral negotiations are more likely to be optimal. Companies often have a good idea which supplier has the most expertise in providing the required product and thus is most likely to come up with ideas for a superior project.

\[ q \geq \frac{1 - \alpha}{\alpha} \frac{\bar{S}}{\Delta S^* - \Delta S^R} > 0. \] (14)

In this case $EU_N^B = \alpha \bar{S} + \alpha q \Delta S^*$ while $EU_A^B = (1 - q)\bar{S} + q(\bar{S} + \alpha \Delta S^R) = \bar{S} + \alpha q \Delta S^R$. Note $EU_N^B$ is the same as in (12) while $EU_A^B$ is smaller than (13). Comparing $EU_N^B$ and $EU_A^B$ yields condition (14).
**Correlated success probabilities:** So far, we assumed that the probability with which a seller finds project improvements is independent of the probabilities with which the other sellers do so, i.e., the success probabilities are uncorrelated. While correlation does not affect the performance of bilateral negotiation it does affect the performance of the auction. The more strongly success probabilities are correlated, the smaller is the probability that at least one seller finds project improvements. This effect makes an auction less attractive. On the other hand, the more success probabilities are correlated the smaller is the probability that exactly one seller finds possible improvements. If exactly one seller is aware of superior specifications, this seller receives a rent in the auction. Thus, a reduction of the probability that exactly one seller is successful makes the auction more attractive. Which of the two effects dominates depends on the buyer’s bargaining power and the efficiency loss if the price is increased. If $\alpha$ and $\lambda^B$ are large, the successful seller does not get a high rent if he is the only one who is successful. In this case the first effect outweighs the second and thus the auction becomes less attractive as correlation increases. This is stated formally in the following proposition for the case of two sellers.

**Proposition 5** (Correlated Probabilities). Let $n = 2$. The payoff advantage of running an auction $\Psi$ is decreasing in the coefficient of correlation if and only if $(1 - \alpha)/\alpha \leq 1 + \lambda^B$.

**IV. Investments in Finding Project Improvements**

So far we took the information structure as exogenously given. The information structure may be very different, however, for different procurement mechanisms: bilateral negotiation may give different incentives to investigate possible project improvements than an auction. What are the incentives of the seller(s) to invest into finding project improvements? In order to answer this question, we replace stage 1 of the baseline model by an investment stage. At stage 1 each seller can invest into finding more efficient project specifications than $\bar{y}$. If a seller invests $q \geq 0$ at cost $k(q)$, he finds with probability $q$ the set of superior projects $Y$, where each $y \in Y$ satisfies Assumption 1. The investment cost function satisfies the following assumption.

**Assumption 2.** The investment cost function, $k(q)$, is strictly increasing and convex and satisfies the Inada conditions, i.e.,

1. $k(0) = 0$, and for all $q > 0$: $k'(q) > 0$ and $k''(q) > 0$;
2. $\lim_{q \to 0} k'(q) = 0$, and $\lim_{q \to 1} k(q) = \infty$.

In order to obtain a closed form solution and unambiguous comparative static results we will sometimes impose the assumption of a quadratic cost function, i.e., $k(q) = \kappa q^2$ with $\kappa > (1 - \alpha)\Delta S^*$.

\[\kappa > (1 - \alpha)\Delta S^*\]  \[20\]

\[20\] With a quadratic cost function, $\kappa > (1 - \alpha)\Delta S^*$ ensures that in equilibrium the probability of finding a project improvement is smaller than 1.
Note that the outcome of renegotiation and the sellers’ incentives for information disclosure are unaffected by how the probabilities are determined at stage 1. Thus we can directly start investigating sellers’ investment incentives under the two procurement mechanisms.

IV.1. Incentives for Finding Project Improvements

Negotiation: Suppose that the buyer decided to negotiate with one seller. In this case the contractor will reveal any possibilities for project improvements before the contract is signed and his expected utility is given by

\[ EU^S_N = q(1 - \alpha)[v^* - c^*] + (1 - q)(1 - \alpha)[\bar{v} - \bar{c}] - k(q) \]

\[ = (1 - \alpha)\bar{S} + q(1 - \alpha)\Delta S^* - k(q) \]  

(15)

The contractor’s optimal investment under negotiation, \( q^N \), is characterized in the next proposition.

Proposition 6 (Investment Incentives under Negotiation). The probability that the contractor who negotiates with the buyer finds possible project improvements is fully characterized by

\[ k'(q^N) = (1 - \alpha)\Delta S^*. \]  

(16)

Moreover, with bilateral negotiations the success probability \( q^N(\alpha, \Delta S^*) \) of the contractor is

(a) decreasing in the bargaining power of the buyer, \( \alpha \), i.e. \( \frac{\partial q^N}{\partial \alpha} < 0 \);

(b) increasing in the surplus generated by the investment, \( \Delta S^* \), i.e. \( \frac{\partial q^N}{\partial \Delta S^*} > 0 \).

These findings are highly intuitive. The larger the buyer’s bargaining power, the smaller is the contractor’s ex post share of the surplus generated by the investment, so he will invest less. Furthermore, the larger the surplus generated by the investment, the stronger are his investment incentives.

Auction: Now, we consider the case of an auction. Recall that a seller makes a positive profit if and only if he is the only seller who found the project improvement. In this case he wins the auction at \( \bar{p} = \bar{c} \). After the contract is signed he reveals the possible project improvement and renegotiates. So in this case his profit is

\[ U^S_A = \frac{1 - \alpha}{1 + \lambda^B}[v(g^R) - \bar{v}) - (1 + \lambda^B)(c(g^R) - \bar{c})] = \frac{1 - \alpha}{1 + \lambda^B} \Delta S^R. \]  

(17)

This happens with probability \( q_i \prod_{j \neq i}(1 - q_j) \). Thus, his expected profit is given by

\[ EU^S_A = q_i \prod_{j \neq i}(1 - q_j) \frac{1 - \alpha}{1 + \lambda^B} \Delta S^R - k(q). \]

For given investments of all sellers \( j \neq i \), seller \( i \)'s optimal investment \( q^A_i \) is determined by the first-order condition. In the symmetric equilibrium all sellers choose the same success probability \( q^A \), which is characterized in the following proposition.
Proposition 7 (Investment Incentives under the Auction). In the symmetric equilibrium of the investment game the probability that a seller finds possible project improvements is fully characterized by

\[ k'(q^A) = (1 - q^A)^{n-1} \frac{1 - \alpha}{1 + \lambda^{S} \Delta S^R}. \]  

(18)

Moreover, with an auction the success probability \( q^A(n, \alpha, \lambda^B, \lambda^S) \) of any given seller is

(a) decreasing in the number of bidders, i.e. \( \frac{\partial q^A}{\partial n} < 0 \);

(b) decreasing in the bargaining power of the buyer, \( \alpha \), i.e. \( \frac{\partial q^A}{\partial \alpha} < 0 \);

(c) decreasing in the adjustment cost parameters \( \lambda^B \) and \( \lambda^S \).

The findings are again highly intuitive. First, the more potential sellers there are, the lower is the probability that seller \( i \) is the only one who is successful in finding a project improvement and thus the less profitable is his investment. Second, the larger the bargaining power of the buyer, the smaller is the share of the renegotiation surplus that is going to the successful seller who wins the auction, which reduces sellers’ investment incentives. Finally, an increase of the adjustment costs \( \lambda^B \) and \( \lambda^S \) reduces the renegotiation surplus and thus the payoff going to the contractor, which in turn reduces sellers’ incentives to invest.

**Auction vs. negotiation:** Which of the two procurement mechanisms generates the higher individual incentives to invest in finding project improvements? A comparison of Propositions 6 and 7 shows that \( q^N > q^A \). This is implied by the fact that the surplus generated by the investment is larger with negotiations where the seller reveals any possible project improvements early while there is inefficient delay with an auction. Furthermore, with an auction a seller who is successful in finding project improvements is not guaranteed to benefit from his success. He benefits only if no other seller is also successful.

**Corollary 1.** The success probability of a seller with whom the buyer negotiates is always higher than the success probability of a seller who participates in an auction, no matter how many potential sellers there are, i.e. for all \( n \geq 2 \)

\[ q^N > q^A. \]  

(19)

Corollary 1 shows that there is a second tradeoff. The auction reduces the price that the buyer has to pay as compared to negotiations, but it also reduces the incentives of each seller to invest into finding project improvements.

The buyer is less interested in the investment incentives of each individual seller but more in the aggregate probability of implementing design improvements ex post. With negotiation the contractor’s individual investment, \( q^N \), is also the probability with which design improvements are implemented ex post, but this is not the case for the auction. With an auction, the probability of implementing design improvements ex post depends on the number of sellers \( n \) ex ante. How does the number of sellers affect the probability that at least one seller finds the project
improvement? Let $Q^A(n, \alpha, \lambda^B, \lambda^S) \equiv 1 - (1 - q^A)^n = 1 - \exp\{n \ln(1 - q^A)\}$ denote the probability that at least one seller finds $Y$. Then we have

$$\frac{dQ^A}{dn} = -(1 - q^A)^n \left[ \ln(1 - q^A) - n \frac{dq^A}{dn} \frac{1}{1 - q^A} \right]$$

$$= -(1 - q^A)^n \ln(1 - q^A) + (1 - q^A)^{n-1} n \frac{dq^A}{dn}$$

$$\begin{cases} > 0, & \text{sampling effect} \\ \leq 0, & \text{discouragement effect} \end{cases}$$

(20)

The total effect of an increase of the numbers of bidders on the probability that at least one bidder will find the project improvement can be split up in a discouragement effect and a sampling effect. Each additional bidder makes it less likely that bidder $i$ is the only bidder who is successful which discourages his investment. This effect is always negative. Each additional seller, however, increases the probability that at least one seller will be successful. This is the sampling effect which is always positive. The sum of the two effects can be positive or negative.

**Proposition 8** (Probability of Implementing Project Improvements). *The probability of implementing project improvements

(i) is larger with negotiations than with an auction, i.e $q^N > Q^A$, if $\lambda^B$ and/or $\lambda^S$ are sufficiently large,

(ii) is smaller with negotiations than with an auction, i.e $q^N < Q^A \forall n \geq 2$, if $\lambda^B = \lambda^S = 0$

and $k(q) = \frac{\kappa}{2} q^2$ with $\kappa > \frac{(1-\alpha)(1+\sqrt{5})}{2} \Delta S^*$.\]

If the buyer runs an auction, then the effect of an increase of the number of bidders on the probability that at least one seller will find project improvements is given by

$$\frac{dQ^A}{dn} = -\frac{1 + \lambda^B}{(1 - \alpha) \Delta S^R} \frac{dq^A}{dn} \left[ (1 - q^A(n)) k''(q^A(n)) - k'(q^A(n)) \right]$$

(21)

with $\frac{dQ^A}{dn} > 0$ if and only if the term in square brackets is positive, which is the case if $k(q) = \frac{\kappa}{2} q^2$.

The proposition shows that there are parameter values such the probability of implementing project improvements is larger if the buyer chooses to negotiate. This is easiest to see for the case of prohibitive cost of renegotiation, so that the parties do not renegotiate but are always stuck with the initial contract. In this case, if the buyer runs an auction, no seller has an incentive to investigate project improvements. However, if the buyer chooses to negotiate the procurement contract, the contractor has a strict incentive to invest in finding project improvements. In case renegotiation is highly efficient, on the other hand, the probability of implementing project improvements can be larger with an auction, which typically is the case if there are many potential sellers. More precisely, this probability is increasing in the number of bidders if the cost function is sufficiently convex, which implies that each seller’s probability of finding project improvements is small in equilibrium.
IV.2. The Optimal Procurement Mechanism

Now, we can compare the performance of the two mechanisms for the case with endogenous probabilities of finding project improvements. If the buyer negotiates with one seller, her expected payoff is

$$EU_B^N(\alpha, \bar{S}, \Delta S^*) = \alpha \bar{S} + q^N \alpha \Delta S^*. \quad (22)$$

If, on the other hand, the buyer runs an auction, her expected payoff is given by

$$EU_A^B(n, \alpha, \bar{S}, \lambda_B, \lambda_S) = \bar{S} + \alpha Q A \Delta S^R + \frac{1 - \alpha}{1 + \lambda_B} \left[ Q A - n q A (1 - q A)^n - 1 \right] \Delta S^R. \quad (23)$$

The following proposition shows that the main result from our baseline model, Proposition 4, carries over to the situation with endogenous investments.

**Proposition 9 (Auction vs. Negotiation).** The buyer strictly prefers to run an auction

(a) if the renegotiation cost is small ($\lambda_B, \lambda_S$ close to 0) and if the probability that at least one seller will find the project improvement is larger with an auction than with renegotiation and/or
(b) if she has little bargaining power ($\alpha$ close to 0).

The buyer strictly prefers to negotiate

(c) if renegotiation is very inefficient ($\lambda_B, \lambda_S$ large) while $\Delta S^*$ is sufficiently large and/or
(d) her bargaining power is very strong ($\alpha$ is close to 1) and sellers' cost function is not too convex ($k''(0)$ close to zero).

Auctions outperform negotiations if renegotiation is relatively efficient. In this case the fact that sellers will not reveal possible project improvements early if the buyer runs an auction is not too costly for the buyer. Furthermore, even though each seller participating in the auction has a smaller incentive to investigate possible project improvements than the one seller with whom the buyer negotiates, the probability that at least one seller will find a project improvement can be larger with an auction than with negotiations (by Proposition 8). Hence, in this case running an auction yields a strictly higher payoff for the buyer (and is more efficient). The buyer also prefers the auction if her bargaining position is weak. The auction makes sure that she gets at least $\bar{S} > 0$ no matter how small $\alpha$, while her payoff from negotiation (and renegotiation) goes to zero if $\alpha$ goes to zero.

On the other hand, if renegotiation is very inefficient, the parties are always stuck with $\bar{y}$ and no seller has an incentive to investigate project improvements if there is an auction. In this case the buyer’s payoff from the auction is restricted to $\bar{S}$, while she would get $\alpha \bar{S} + \alpha q^N \Delta S^*$ if she negotiates. Thus, if the potential for project improvements ($\Delta S^*$) is sufficiently large the buyer prefers to negotiate.

Finally, if the buyer has all the bargaining power, no seller is going to invest into finding project improvements. In this case there is no difference between the two procurement mechanisms. However, if, starting at $\alpha = 1$, the bargaining power of the buyer is reduced, then the
investment incentive of the seller with whom the buyer negotiates is differently effected than
the incentives of sellers in the auction. In particular, if the cost function is not too convex and \( \alpha \)
close to 1, negotiation is accompanied with higher investment incentives than the auction. More
precisely, the reduction in \( \alpha \) has a strictly positive first-order effect on the buyer’s payoff if she
negotiates with one seller, while the first-order effect is zero if she runs an auction.

The next proposition offers some additional comparative static results.

**Proposition 10** (Auction vs. Negotiation: Comparative Statics). *The payoff advantage of running an auction, \( \Psi(n, \alpha, \bar{S}, \Delta S^*, \lambda^B, \lambda^S) = EU_A^B - EU_N^B \), is
(a) weakly decreasing in the renegotiation costs \( \lambda^B \) and \( \lambda^S \);
(b) strictly decreasing in the maximal additional surplus \( \Delta S^* \);
(c) strictly increasing in surplus achieved without implementing project improvements \( \bar{S} \).*

The cost of renegotiation affect the performance of an auction but not the performance of
renegotiation. Hence running an auction becomes more attractive if the costs of renegotiation
 go down. Similarly, an increase of \( \Delta S^* \) (keeping everything else constant) makes negotiations
more attractive without directly affecting the performance of auctions. Finally, if \( \bar{S} \) increases
the buyer captures all of this increase with an auction, while she gets only fraction \( \alpha \) of this if
she negotiates.

**V. Conclusions**

The preceding analysis highlights two important benefits of using negotiations rather than auc-
tions in procurement. First, negotiations give an incentive to the seller to reveal possible design
improvements early. In contrast, in an auction all bidders prefer not to reveal this information
before the contract is signed. Thus, a contract that was allocated by an auction is more likely
to be renegotiated. There is ample empirical evidence that the renegotiation of procurement
contracts is often very costly and inefficient. Thus, if a complex good is to be procured where
the expertise of the seller is of crucial importance for its optimal design it may be better for the
buyer to negotiate with one preselected seller in order to reduce the cost of renegotiation.

Second, negotiations give stronger incentives to investigate potential project improvements.
An auction diminishes the return of this investment because the surplus of a project improve-
ment is reduced in the inefficient renegotiation process. Furthermore, each seller has a dimin-
ished incentive to invest because he benefits from his investment only if he is the only seller
finding the improvement. On the other hand, because there are several sellers participating in
the auction, there is also a sampling effect which may increase the probability that at least one
seller finds the project improvement. On balance, if renegotiation is very costly, then it is likely
that negotiations will implement project improvements with a higher probability. These argu-
ments may explain why negotiations are so often used to allocate private procurement contracts.

To keep the analysis simple our model abstracts from many real world complications that
affect the tradeoff between auctions and negotiations. For example, we assume that all sellers
are identical. If there are cost differences between different sellers or if different sellers have
different skills for finding project improvements, an auction has the advantage to select the
most efficient seller, but it will also leave a rent to this seller. On the other hand, if the buyer
knows who the most efficient seller is, she can select this seller and directly negotiate with him.
Thus, negotiations are more likely to be optimal if there are few competing sellers with large
efficiency differences and if the buyer knows the efficiency of the potential sellers well.

We also assumed that the probability that one seller finds a project improvement is indepen-
dent of the probabilities that all the other sellers are successful. If these successes are positively
correlated, the sampling effect is reduced, the probability that at least one seller finds the project
improvement goes down, and sellers in the auction have a lower incentive to invest. All of these
effects make negotiations more attractive.

Moreover, we ignored the possibility of favoritism and collusion. It is often argued that an
important benefit of auctions is that they make favoritism and collusion more difficult. In fact,
this is the reason why there are legal rules in many countries that require competitive tendering
in public procurement. However, in a recent paper Gretschko and Wambach (2014) show that
an auction may be more prone to favoritism than negotiations.21

Finally, the reader might wonder whether the buyer can benefit from using a two-stage pro-
cedure in order to award the procurement contract, i.e., a procedure similar to architectural
competitions. For example, at the first stage the buyer could award a fixed price for the best
design proposal. At the second stage the procurement contract for the best design is auctioned
off. Such a procedure may enhance information revelation compared to our one-stage auction
but does not implement fully efficient information exchange ex ante. To see this suppose that
there is one seller with a brilliant idea who is very confident to win the first stage competition,
i.e., to win the fixed price for the best design. This seller will also win the fixed price when
making a proposal which is slightly worse than the best design he is aware of. By doing so, he
not only receives the fixed price but can additionally benefit from renegotiating the contract ex
post. The analysis of multi-stage procurement procedures is an important and fascinating topic
for future research, but it is beyond the scope of this paper.

A. Appendix

Proof of Proposition[7] We introduce some notation first. Then we characterize under what
circumstances renegotiation takes place. Finally, we prove equations [6]–[9] of the proposition,
starting with the specification that is implemented at stage 3, \( y^R \).

Define
\[
\Delta S^R(y) \equiv v(y) - v(\bar{y}) - (1 + \lambda_B)(1 + \lambda_S)[c(y) - c(\bar{y})],
\]

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Define
\[
\Delta S^R(y) \equiv v(y) - v(\bar{y}) - (1 + \lambda_B)(1 + \lambda_S)[c(y) - c(\bar{y})],
\]
and let

\[ Y^R \equiv \{ y \in Y \cup \{ \bar{y} \} \mid y \in \arg \max \Delta S^R(y) \}. \tag{A.2} \]

In words, \( Y^R \) is the set containing all specifications \( y \in \{ Y \cup \{ \bar{y} \} \} \) that maximize the ‘surplus’ generated by renegotiation, \( \Delta S^R(\cdot) \).

The renegotiation contract maximizes the generalized Nash product, which is given by

\[ GNP(y, p) \equiv \left\{ v(y) - v(\bar{y}) - (1 + \lambda^B)(p - \bar{p}) \right\}^\alpha \left\{ p - \bar{p} - (1 + \lambda^S)[c(y) - c(\bar{y})] \right\}^{1-\alpha}. \tag{A.3} \]

Thus, \((y^R, p^R)\) solves the following problem:

\[
\max_{(y, p) \in \{Y \cup \{\bar{y}\} \times \mathbb{R}} GNP(y, p) \tag{A.4}
\]

subject to:

\[ v(y) - v(\bar{x}) - (1 + \lambda^B)(p - \bar{p}) \geq 0 \tag{IR^B} \]
\[ p - \bar{p} - (1 + \lambda^S)[c(y) - c(\bar{y})] \geq 0. \tag{IR^S} \]

For a given \( y \) the constraints are easiest to satisfy if the price mark-up is as low as possible, i.e., if

\[ p - \bar{p} = (1 + \lambda^S)[c(y) - c(\bar{y})]. \tag{A.5} \]

At this mark-up the seller is indifferent between the new and the old contract. The buyer (weakly) prefers the new contract if

\[ \Delta S^R(y) = v(y) - v(\bar{y}) - (1 + \lambda^B)(1 + \lambda^S)[c(y) - c(\bar{y})] \geq 0. \tag{A.6} \]

Hence, only \( y^R \in \{ Y \cup \{ \bar{y} \} \} \) for which \( \Delta S^R(y^R) \geq 0 \) can be reached by renegotiation. This implies that renegotiation takes place—in a strict sense—only if there is a \( y \in Y \) so that \( \Delta S^R(y) > 0 \).

Now, we show that \( y^R \) has to maximize \( \Delta S^R \). Assume, in contradiction, that the parties agreed to trade \((y, p)\) with \( y \notin Y^R \). The generalized Nash product is given by

\[ GNP(y, p) = \left\{ v(y) - v(\bar{y}) - (1 + \lambda^B)(p - \bar{p}) \right\}^\alpha \left\{ p - \bar{p} - (1 + \lambda^S)[c(y) - c(\bar{y})] \right\}^{1-\alpha}. \tag{A.7} \]

Alternatively, the parties could trade \( \hat{y} \in Y^R \) at price

\[ \hat{p} = p + (1 + \lambda^S)[c(\hat{y}) - c(y)], \tag{A.8} \]

where \( \hat{p} \) is constructed so that the seller is indifferent between \((y, p)\) and \((\hat{y}, \hat{p})\). The generalized Nash product amounts to

\[
GNP(\hat{y}, \hat{p}) = \left\{ v(\hat{y}) - v(\bar{x}) - (1 + \lambda^B)(p - \bar{p}) - (1 + \lambda^B)(1 + \lambda^S)[c(\hat{y}) - c(\bar{y})] \right\}^\alpha \times \left\{ p - \bar{p} - (1 + \lambda^S)[c(y) - c(\bar{x})] \right\}^{1-\alpha}. \tag{A.9}
\]
Note that $GNP(\hat{y}, \hat{p}) > GNP(y, p)$ if and only if
\[
v(\hat{y}) - v(\bar{x}) - (1 + \lambda^B)(p - \bar{p}) - (1 + \lambda^B)(1 + \lambda^S)[c(\hat{y}) - c(y)] > v(y) - v(\bar{x}) - (1 + \lambda^B)(p - \bar{p}),
\] (A.10)
which is equivalent to
\[
\Delta S^R(\hat{y}, y) \equiv v(\hat{y}) - v(y) - (1 + \lambda^B)(1 + \lambda^S)[c(\hat{y}) - c(y)] > 0
\] (A.11)
Note that $\Delta S^R(y, y) = 0$ and that $\hat{y} \in \arg\max_z \Delta S^R(z, y)$. By assumption $y \notin Y^R$ and thus $\Delta S^R(\hat{y}, y) > 0$. Put differently, the specification implemented at stage 3 satisfies,
\[
y^R \in \arg\max_{y \in Y \cup \{\hat{y}\}} \Delta S^R(y).
\]
Note that $y^R = y^R(\lambda^B, \lambda^S, \hat{y})$.

Finally, from Proposition 2 by Herweg and Schmidt (2015), it is readily obtained that $y^R$ is implemented at price
\[
p^R = \bar{p} + \frac{1 - \alpha}{1 + \lambda^B}[v(y^R) - v(\bar{y})] + \alpha(1 + \lambda^S)[c(y^R) - c(\bar{y})]
\]
which gives rise to the expected payoffs (8) and (9).

**Proof of Proposition 2** If the seller reveals $Y$ to the buyer, the parties solve
\[
\max_{y, p} [v(y) - p]^\alpha \cdot [p - c(y)]^{1-\alpha}.
\] (A.12)
The solution to this problem is $y^* = \arg\max_y \{v(y) - c(y)\}$ and $p^* = c(y^*) + (1 - \alpha)[v(y^*) - c(y^*)]$, so the seller’s payoff in the negotiation game (N) if he knows $Y$ and informs (I) the buyer immediately is
\[
U^S(NI) = (1 - \alpha)(v^* - c^*).
\] (A.13)
If the seller does not inform the buyer about possible project improvements (either because he did not find them or because he chose not to reveal $Y$ to the buyer), then the GNBS implies that the parties will agree to trade project $\bar{y}$ at price $\bar{p} = \bar{c} + (1 - \alpha)[\bar{v} - \bar{c}]$.\(^{22}\) However, if the seller did find possible project improvements he will reveal them at stage 3 and renegotiate (R). In this case the parties renegotiate to the contract characterized by Proposition 1 and the seller’s payoff is
\[
U^S(NR) = \bar{p} - \bar{c} + (1 - \alpha)[v(y^R) - \bar{v}] - (1 + \lambda^B)(1 + \lambda^S)[c(y^R) - \bar{c}]
\] (A.14)
\(^{22}\)Strictly speaking, if the seller knows $Y$ but does not inform the buyer about it, initial negotiation takes place with asymmetric information. The GNBS is a concept for bargaining under symmetric information. The seller does not reveal his information, thus both parties behave as if they agree that trading specification $\hat{y}$ is optimal. This is exactly what is characterized by the GNBS in this case. In Appendix B we show that the identical result can be obtained for a bargaining game where the asymmetric information is taken explicitly into account.
Note that \( y^* \in \arg \max \{(v(y) - \bar{v}) - (c(y) - \bar{c})\} \) and \( y^R \in \arg \max \{(v(y) - \bar{v}) - (1 + \lambda^B)(1 + \lambda^S)(c(y) - \bar{c})\} \). Hence
\[
(v(y^*) - \bar{v}) - (c(y^*) - \bar{c}) \geq (v(y^R) - \bar{v}) - (c(y^R) - \bar{c})
\]
\[
> (v(y^R) - \bar{v}) - (1 + \lambda^B)(1 + \lambda^S)(c(y^R) - \bar{c}). \quad (A.15)
\]

Therefore, the seller’s utility if he renegotiates is smaller than his utility if he informs the buyer before the contract is signed:
\[
U^S(NR) = \bar{p} - \bar{c} + \frac{1 - \alpha}{1 + \lambda^B} [(v(y^R) - \bar{v}) - (1 + \lambda^B)(1 + \lambda^S)(c(y^R) - \bar{c})]
\]
\[
< \bar{p} - \bar{c} + \frac{1 - \alpha}{1 + \lambda^B} [(v(y^*) - \bar{v}) - (c(y^*) - \bar{c})]
\]
\[
< \bar{p} - \bar{c} + (1 - \alpha)[(v(y^*) - \bar{v}) - (c(y^*) - \bar{c})]
\]
\[
= \bar{c} + (1 - \alpha)(\bar{v} - \bar{c}) - \bar{c} + (1 - \alpha)(v^* - c^*) - (1 - \alpha)(\bar{v} - \bar{c})
\]
\[
= (1 - \alpha)(v^* - c^*) = U^S(NI).
\]

\[\square\]

**Proof of Proposition 2** If one of the sellers informs the buyer about \( Y \) the buyer will run the auction on project \( y^* \) and each seller makes a profit of 0. If no seller informs the buyer about \( Y \) the buyer will auction off project \( \bar{y} \). Suppose that in this case seller \( i \) wins the auction and knows about possible project improvements. He will then renegotiate at stage 3, and his payoff in the auction (\( A \)) after renegotiation (\( R \)) is
\[
U^S(AR) = \bar{p} - \bar{c} + \frac{1 - \alpha}{1 + \lambda^B} [(v(y^R) - \bar{v}) - (1 + \lambda^B)(1 + \lambda^S)(c(y^R) - \bar{c})]. \quad (A.16)
\]
Thus, in the second price auction it is a (weakly) dominant strategy for seller \( i \) to bid
\[
b_i = \bar{c} - \frac{1 - \alpha}{1 + \lambda^B} [(v(y^R) - \bar{v}) - (1 + \lambda^B)(1 + \lambda^S)(c(y^R) - \bar{c})]. \quad (A.17)
\]
If there are two or more sellers who found the project improvement, one of them wins the auction and all sellers make an expected profit of zero. Similarly, if no seller found the project improvement, all sellers will bid \( b_1 = \bar{c} \) and make an expected profit of zero. However, if seller \( i \) is the only seller who found the project improvement, then he wins the auction at price \( \bar{p} = \bar{c} \). In this case his profit is
\[
U^S(AR) = \frac{1 - \alpha}{1 + \lambda^B} [(v(y^R) - \bar{v}) - (1 + \lambda^B)(1 + \lambda^S)(c(y^R) - \bar{c})] > 0. \quad (A.18)
\]
Hence, it optimal for all sellers who found project improvements not to reveal this information before the auction takes place.

\[\square\]

**Proof of Proposition 3** The buyer strictly prefers to run an auction if and only if \( \Psi = EU_A^B - EU_R^B > 0 \), where
\[
\Psi = (1 - \alpha)\bar{S} + \Delta S^R \left\{ \alpha [1 - (1 - q)^n] + \frac{1 - \alpha}{1 + \lambda^B} \left[ 1 - (1 - q)^n - nq(1 - q)^{n-1} \right] \right\}
\]
\[
- \alpha q \Delta S^* \cdot \quad (A.19)
\]
(a) If renegotiation costs are small, i.e., $\lambda^B \to 0$ and $\lambda^S \to 0$, then $\Delta S^R \to \Delta S^*$. Hence, we have

$$\Psi = (1 - \alpha)\bar{S} + \Delta S^* \left\{ \alpha(1 - q) \left[ 1 - (1 - q)^{n-1} \right] + (1 - \alpha) \left[ 1 - (1 - q)^n - nq(1 - q)^{n-1} \right] \right\} > 0. \quad (A.20)$$

(b) For $\alpha \to 0$, we have

$$\Psi = \bar{S} + \Delta S^R \left\{ \frac{1}{1 + \lambda^B} \left[ 1 - (1 - q)^n - nq(1 - q)^{n-1} \right] \right\} > 0. \quad (A.21)$$

(c) If renegotiation costs are prohibitively large, $\Delta S^R = 0$. In this case we have

$$\Psi = (1 - \alpha)\bar{S} - q\alpha\Delta S^*, \quad (A.22)$$

which is negative for $\Delta S^*$ sufficiently large.

(d) For $\alpha \to 1$, we have

$$\Psi = \Delta S^R \left[ 1 - (1 - q)^n \right] - q\Delta S^*, \quad (A.23)$$

which is negative for $q$ sufficiently close to 1.

To complete the proof, we show that $\partial \Psi / \partial n \geq 0$ (with strict inequality if $\Delta S^R > 0$). Taking the partial derivative of $\Psi$ with respect to $n$ yields

$$\frac{\partial \Psi}{\partial n} = \Delta S^R \left\{ -\alpha \ln(1 - q)(1 - q)^n + \frac{1 - \alpha}{1 + \lambda^B} \left[ -\ln(1 - q)(1 - q)^n - q(1 - q)^{n-1} - nq(1 - q)^{n-1} \ln(1 - q) \right] \right\}. \quad (A.24)$$

Rearranging the above expression leads to

$$\frac{\partial \Psi}{\partial n} = -\Delta S^R (1 - q)^{n-1} \times \left\{ \alpha \ln(1 - q)(1 - q) + \frac{1 - \alpha}{1 + \lambda^B} [\ln(1 - q)(1 - q) + q + nq \ln(1 - q)] \right\}. \quad (A.25)$$

Thus, $\partial \Psi / \partial n \geq 0$ if

$$\ln(1 - q)(1 - q) + q + nq \ln(1 - q) \leq 0. \quad (A.26)$$

The above inequality is hardest to satisfy for $n = 2$ (lowest possible $n$) and thus $\partial \Psi / \partial n \geq 0$ if

$$\Gamma(q) \equiv \ln(1 - q)(1 + q) + q \leq 0. \quad (A.27)$$

Noting that $\Gamma(q)$ is strictly decreasing and approaches 0 for $q \to 1$ completes the proof.
Proof of Proposition 5. The information of supplier $S_i \in \{S_1, S_2\}$ is denoted by $I_i \in \{0, 1\}$, where $I_i = 1$ means that supplier $S_i$ is aware of the set $Y$ and $I_i = 0$ means that he is unaware. Let $\rho \in [0, 1]$ denote the Pearson correlation coefficient. The correlated probabilities are displayed in the following probability table:

<table>
<thead>
<tr>
<th>$I_1$</th>
<th>$I_2 = 0$</th>
<th>$I_2 = 1$</th>
<th>$\sum$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I_1 = 0$</td>
<td>$(1-q)[\rho + (1-\rho)(1-q)]$</td>
<td>$q(1-q)(1-\rho)$</td>
<td>$1-q$</td>
</tr>
<tr>
<td>$I_1 = 1$</td>
<td>$q(1-q)(1-\rho)$</td>
<td>$q[\rho + (1-\rho)q]$</td>
<td>$q$</td>
</tr>
<tr>
<td>$\sum$</td>
<td>$1-q$</td>
<td>$q$</td>
<td>$1$</td>
</tr>
</tbody>
</table>

Table 1: Correlated probability table for $n = 2$

For $\rho \to 0$, the success probabilities are uncorrelated, while for $\rho \to 1$ we have perfect correlation. The buyer’s expected utility from negotiation is independent of the degree of correlation $\rho$. The buyer’s expected utility from running an auction is

$$EU_B^R(\rho) = (1-q)[\rho + (1-\rho)(1-q)]\bar{S} + 2q(1-q)(1-\rho)\left[\bar{S} + \alpha \Delta S^R + \frac{1-\alpha}{1+\lambda^B}\Delta S^R\right].$$

Taking the partial derivative with respect to $\rho$ yields:

$$\frac{\partial EU_B^R}{\partial \rho} = (1-q)q \left[1-\frac{\alpha}{1+\lambda^B} - \alpha\right] \Delta S^R,$$

which completes the proof.

Proof of Proposition 6. The contractor’s expected payoff is strictly concave in $q$ and due to the imposed Inada conditions the optimal investment has to be interior. Thus, the success probability of the contractor, $q^N$, is fully characterized by

$$(1-\alpha)\Delta S^* - k'(q^N) = 0$$

Using the implicit function theorem we get:

$$\frac{dq^N}{d\alpha} = -\frac{-\Delta S^*}{-k''(q^N)} < 0,$$

$$\frac{dq^N}{d\Delta S^*} = -\frac{(1-\alpha)}{-k''(q^N)} > 0.$$

Proof of Proposition 7. If a symmetric pure-strategy equilibrium of the investment game exists, the equilibrium investment of each seller, $q^A$, is characterized by the following first-order condition:

$$(1-q^A)^{n-1} \frac{1-\alpha}{1+\lambda^B} \Delta S^R = k'(q^A).$$

$^23\rho = \text{cov}(I_1, I_2)/[\sigma(I_1)\sigma(I_2)]$, where $\text{cov}(\cdot)$ denotes the covariance and $\sigma(\cdot)$ the standard deviation.
The comparative statics results follow from the implicit function theorem. Let
\[ \Phi = (1 - q^A)n^{-1} \frac{1 - \alpha}{1 + \lambda^B} \Delta S^R - k'(q^A) = 0. \tag{A.33} \]

(a) Recall that \( \frac{d}{dx}[C^{ax}] = C^{ax} \ln C \cdot a \). Thus, we have
\[
\frac{dq^A}{dn} = -\frac{\partial \Phi}{\partial q^A} = -\frac{(1 - q^A)n^{-1} \ln(1 - q^A)\frac{1 - \alpha}{1 + \lambda^B} \Delta S^R}{-(n - 1)(1 - q^A)n^{-2} \frac{1 - \alpha}{1 + \lambda^B} \Delta S^R - k''(q^A)} < 0, \tag{A.34}
\]
where the strict inequality follows from \( \ln(1 - q^A) < 0 \).

(b) \[
\frac{dq^A}{d\alpha} = -\frac{\partial \Phi}{\partial \alpha} = -\frac{-\frac{1}{1 + \lambda^B} (1 - q^A)^n-1 \Delta S^R}{-(n - 1)(1 - q^A)n^{-2} \frac{1 - \alpha}{1 + \lambda^B} \Delta S^R - k''(q^A)} < 0, \tag{A.35}
\]

(c) Note that \( \Delta S^R \) depends on \( \lambda^j \) not only directly but also indirectly through \( y^R = y^R(\lambda^B, \lambda^S) \). Thus, \( \Delta S^R \) is not everywhere continuously differentiable with respect to \( \lambda^j \). From the definition of \( y^R \) it is readily obtained that \( \Delta S^R \) is continuous in \( \lambda^j \) and strictly decreasing in \( \lambda^j \) whenever \( \Delta S^R > 0 \). At points at which \( \Delta S^R \) is differentiable, we can apply the implicit function theorem and obtain:
\[
\frac{dq^A}{d\lambda^S} = -\frac{\partial \Phi}{\partial \lambda^S} = -\frac{\partial \Phi}{\partial \lambda^S} \frac{\partial \Delta S^R}{\partial \lambda^S} \frac{\partial \Delta S^R}{\partial q^A} = -\frac{-(1 + \lambda^B)(c(y^R) - c)}{-(n - 1)(1 - q^A)n^{-2} \frac{1 - \alpha}{1 + \lambda^B} \Delta S^R - k''(q^A)} < 0, \tag{A.36}
\]
\[
\frac{dq^A}{d\lambda^B} = -\frac{\partial \Phi}{\partial \lambda^B} = -\frac{\partial \Phi}{\partial \lambda^B} \frac{\partial \Delta S^R}{\partial \lambda^B} = -\frac{-(1 - q^A)n^{-1} \Delta S^R}{-(n - 1)(1 - q^A)n^{-2} \frac{1 - \alpha}{1 + \lambda^B} \Delta S^R - k''(q^A)} < 0. \tag{A.37}
\]

Note that \( q^A \) is continuous in \( \Delta S^R \) and thus continuous in \( \lambda^j \). Hence, we can conclude from equation (A.36) and (A.37) that \( q^A \) is decreasing in \( \lambda^S \) and \( \lambda^B \), respectively.

Finally, we argue that a symmetric pure-strategy equilibrium of the investment game always exists. Seller \( i \)'s optimal investment \( q_i^* \) maximizes
\[
q_i \Pi_{j \neq i}(1 - q_j) \frac{(1 - \alpha)\Delta S^R}{1 + \lambda^B} - k(q). \tag{A.38}
\]

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First, note that a seller $i$ never chooses $q_i = 1$ because $\lim_{q \to 1} k(q) = \infty$. This implies that $\Pi_{j \neq i}(1 - q_j) = X > 0$. With $X > 0$ and $k'(0) = 0$ it always pays off for a seller to invest a small amount, i.e., $q_i > 0$ which implies that $X \in (0,1)$. The reaction function of firm $i$ is implicitly characterized by the first-order condition:

$$X \frac{(1 - \alpha) \Delta S^R}{1 + \lambda^B} = k'(q^R(X)).$$  \hspace{1cm} (A.39)$$

Implicit differentiation with respect to $X$ yields

$$\frac{dq_i^R}{dX} = \frac{(1 - \alpha) \Delta S^R}{k''(q_i)(1 + \lambda^B)} > 0.$$  \hspace{1cm} (A.40)

Note that $X$ is decreasing in $q_j$ for all $j \neq i$, so that $i$ invests less if its rivals invest more. In the limit we have $\lim_{X \to 0} q_i^R(X) = 0$ and $\lim_{X \to 1} q_i^R(X) = \bar{q} > 0$, with $\frac{(1 - \alpha) \Delta S^R}{1 + \lambda^B} = k'(\bar{q})$.

The reaction functions are all symmetric and continuously decreasing and approach zero if $\Pi_{j \neq i}(1 - q_j) \to 1$. Thus, a symmetric $q^A$ exists at which all reaction functions intersect each other. \hfill $\square$

**Proof of Corollary**  \hspace{1cm} 7 It has to be shown that for all $n \geq 2$: $q^N > q^A(n)$. Note that

$$\begin{align*}
(1 - \alpha) \Delta S^* &= (1 - \alpha)[(v(y^*) - \bar{v}) - (c(y^*) - \bar{c})] \\
&\geq (1 - \alpha)[(v(y^*) - \bar{v}) - (1 + \lambda^B)(1 + \lambda^S)(c(y^*) - \bar{c})] \\
&\geq \frac{1 - \alpha}{1 + \lambda^R} \Delta S^R \\
&> (1 - q^A)^{n-1} \frac{1 - \alpha}{1 + \lambda^B} \Delta S^R. \hspace{1cm} (A.41)
\end{align*}$$

Thus, convexity of $k(\cdot)$ implies the result. \hfill $\square$

**Proof of Proposition**  \hspace{1cm} 8 First, we show that there are parameter values so that $q^N > q^A$ and so that $Q^A > q^N$.

(i) $q^N > q^A(n)$ for all $n \geq 2$: By \hspace{1cm} 11 we know that if $\lambda^B$ and/or $\lambda^S$ are sufficiently large then $\Delta S^R = 0$. Furthermore, $q^A(n)$ is fully characterized by FOC \hspace{1cm} 18, which requires

$$k'(q^A) = (1 - q^A)^{n-1} \frac{1 - \alpha}{1 + \lambda^B} \Delta S^R.$$  \hspace{1cm} (11)

$\Delta S^R = 0$ implies $q^A = 0$ for all $n \geq 2$ which implies $Q^A(n) = 0$ for all $n \geq 2$. On the other hand, $q^N$, which is fully characterized by \hspace{1cm} 16, is independent of $\lambda^B$ and $\lambda^S$. Thus, if $\lambda^B$ and/or $\lambda^S$ are sufficiently large then $Q^A(n) = 0 < q^N$ for all $n \geq 2$.

(ii) $Q^A(n) > q^N$ for all $n \geq 2$: By Proposition \hspace{1cm} 1 we know that if $\lambda^B = \lambda^S = 0$ then $\Delta S^R = \Delta S^*$. Suppose that $k(q) = \frac{1}{k} q^2$. In this case, using equation \hspace{1cm} 16, we obtain

$$q^N = \frac{1 - \alpha}{k} \Delta S^*.$$
Furthermore, by (18) we have for \( n = 2 \)
\[
\kappa q^A = (1 - q^A)(1 - \alpha)\Delta S^* \iff \quad q^A = \frac{(1 - \alpha)\Delta S^*}{\kappa + (1 - \alpha)\Delta S^*},
\]
which implies
\[
Q^A(2) = 1 - (1 - q^A)^2 = q^A(2 - q^A) = \frac{(1 - \alpha)^2(\Delta S^*)^2 + 2(1 - \alpha)\kappa\Delta S^*}{[\kappa + (1 - \alpha)\Delta S^*]^2}.
\]
Thus, \( Q^A(2) > q^N \) if and only if
\[
\frac{(1 - \alpha)^2(\Delta S^*)^2 + 2(1 - \alpha)\kappa\Delta S^*}{[\kappa + (1 - \alpha)\Delta S^*]^2} > \frac{1 - \alpha}{\kappa} \Delta S^*.
\]
The above inequality is satisfied if and only if
\[
\kappa > \frac{(1 - \alpha)(1 + \sqrt{5})}{2} \Delta S^*.
\]
It remains to be shown that \( Q^A(n) > q^N \) for all \( n \geq 2 \). By (21) (to be shown below) we have that \( \frac{dq}{dn} > 0 \) if \( (1 - q^A)k''(q^A) - k'(q^A) > 0 \). Using the quadratic cost function and the fact that \( \frac{dq}{dn} < 0 \), this is the case if and only if
\[
q^A(2) = \frac{(1 - \alpha)\Delta S^*}{\kappa + (1 - \alpha)\Delta S^*} < \frac{1}{2},
\]
which is equivalent to \( \kappa > (1 - \alpha)\Delta S^* \). Note that \( \frac{1 - \alpha}{2} \Delta S^* > (1 - \alpha)\Delta S^* \). Hence, if \( \kappa > \frac{1 - \alpha}{2} \Delta S^* \), then \( Q^A(n) > q^N \) for all \( n \geq 2 \).

Now we show (21). Let \( Q \equiv 1 - (1 - q^A(n))^n \). In the following we often suppress the superscript \( A \) and the dependence of \( n \), i.e., we write \( q \) instead of \( q^A(n) \). In the symmetric equilibrium, each seller’s probability of finding the project improvements is given by
\[
(1 - q)^{n-1} = \frac{1 + \lambda^B}{(1 - \alpha)\Delta S^R} k'(q).
\]
Thus, \( Q \) can be written as
\[
Q = 1 - (1 - q)k'(q) \frac{1 + \lambda^B}{(1 - \alpha)\Delta S^R}.
\]
Differentiating \( Q \) with respect to \( n \) yields
\[
\frac{dQ}{dn} = -\frac{1 + \lambda^B}{(1 - \alpha)\Delta S^R} \frac{dq}{dn} \left[ (1 - q)k''(q) - k'(q) \right].
\]
Hence, \( dQ/dn > 0 \) if and only if the term in square brackets is positive.

Consider the quadratic cost function \( k(q) = \frac{\alpha}{2} q^2 \). Then,
\[
\frac{dQ}{dn} = -\frac{1 + \lambda^B}{(1 - \alpha)\Delta S^R} \frac{dq}{dn} \kappa \left[ (1 - 2q) \right].
\]
Comparing (22) and (23) we have that

Thus, $q^A(2) < \frac{1}{2}$ iff $\kappa > \frac{1-\alpha}{1+\lambda^R} \Delta S^R$. This condition is always satisfied because $\kappa > (1-\alpha) \Delta S^*$. 

Proof of Proposition 9. Comparing (22) and (23) we have that $EU^B_A > EU^B_N$ if and only if

$$\Psi = \left(1 - \alpha\right)S + \alpha\left[Q\Delta S^R - q^N \Delta S^*\right] + \frac{1-\alpha}{1 + \lambda^B} \left[Q - nq^A(1 - q^A)^{n-1}\right] \Delta S^R > 0.$$  

(a) The first and the third term of this expression are clearly positive, so consider the second term. If $\lambda^B = \lambda^S = 0$, then $\Delta S^R = \Delta S^*$. Thus, $Q(n)\Delta S^R - q^N \Delta S^* = (Q(n) - q^N) \Delta S^*$. If $Q(n) > q^N$ this term is strictly positive and the auction outperforms negotiations. Only if the incentive effect of negotiations is very strong, i.e. if $q^N > Q(n)$, is it possible that the sum of the three terms becomes negative.

(b) If $\alpha$ goes to zero, the second term goes to 0 while the first term goes to $S$. Thus, $\Psi > 0$.

(c) If $\lambda^B$ and $\lambda^S$ are sufficiently large such that there does not exist a $y \in Y$ with $v(y) - \bar{v} - (1 + \lambda^B)(1 + \lambda^S)(c(y) - \bar{c}) > 0$, then there is no renegotiation. The parties will always trade $\bar{y}$, and thus $\Delta S^R = 0$. In this case the buyer prefers to negotiate if $(1 - \alpha)\bar{S} < q^N\alpha \Delta S^*$, which is equivalent to $\Delta S^* > \frac{1-\alpha}{\alpha} \frac{\bar{S}}{q^N}$.

(d) If $\alpha = 1$ then $q^N = q^A = 0$. Therefore, the buyer’s payoff is $\bar{v} - \bar{c}$ no matter whether he negotiates or runs an auction. If $\alpha$ is reduced (starting from $\alpha = 1$) the effect on the buyer’s payoff from running an auction is given by

$$\frac{\partial EU^B_A}{\partial \alpha} = \left[1 - (1 - q^A)^n\right] \Delta S^R + \alpha n(1 - q^A)^{n-1} \frac{dq^A}{d\alpha} \Delta S^R$$

$$- \frac{1}{1 + \lambda^B} \left[1 - (1 - q^A)^n - nq^A(1 - q^A)^{n-1}\right] \Delta S^R$$

$$+ \frac{1-\alpha}{1 + \lambda^B} \left[n(1 - q^A)^{n-1} \frac{dq^A}{d\alpha} - n \left[\frac{dq^A}{d\alpha}(1 - q^A)^{n-1} - (n-1)(1 - q^A)^{n-2} \frac{dq^A}{d\alpha}\right]\right] \Delta S^R.$$  

(A.43)

By evaluating this term at $\alpha = 1$, we obtain that $q^A(1) = 0$ and $\frac{dq^A(1)}{d\alpha} = 0$ because $\ln(1 - q^A(1)) = \ln 1 = 0$. Therefore,

$$\frac{\partial EU^B_A}{\partial \alpha} \bigg|_{\alpha = 1} = 0.$$  

(A.44)
Thus, the first-order effect from reducing $\alpha$ at $\alpha = 1$ is zero. On the other hand, the effect on the buyer’s payoff from negotiating is given by

$$\frac{\partial EU^B}{\partial \alpha} \bigg|_{\alpha=1} = S + q^N(\alpha) \Delta S^* + \alpha \frac{\partial q^N}{\partial \alpha} \Delta S^*$$

$$= S + q^N(1) \Delta S^* + \alpha \frac{-\Delta S^*}{k''(q^N(1))} \Delta S^*$$

$$= S + 0 \cdot \Delta S^* - \left(\frac{\Delta S^*}{k''(0)}\right)^2 \tag{A.45}$$

because $\lim_{\alpha \to 1} q^N(\alpha) = 0$. Thus, if $k''(0)$ is sufficiently close to zero, $\frac{\partial U^B_N}{\partial \alpha}|_{\alpha=1} < 0$. The buyer’s payoff increases, but now the first order effect of a reduction of $\alpha$ is strictly positive. Thus, for $\alpha$ close to 1, negotiations are better than auctions.

\[
\square
\]

Proof of Proposition 10. The function $\Psi$ is given in equation (A.42). Recall that $Q = 1 - (1 - q^A)^n$. Taking the partial derivative of $\Psi$ with respect to $S$ yields

$$\frac{\partial \Psi}{\partial S} = 1 - \alpha > 0 \tag{A.46}$$

The partial derivative of $\Psi$ with respect to $\Delta S^*$ is

$$\frac{\partial \Psi}{\partial \Delta S^*} = -\alpha \left[ q^N + \Delta S^* \frac{dq^N}{d\Delta S^*} \right] < 0. \tag{A.47}$$

The partial derivatives with respect to $\lambda^i$ are somewhat more complicated. Note that $\Delta S^R$ is a continuously decreasing function in $\lambda^i$ and thus differentiable almost everywhere. Thus, the partial derivative of $\Psi$ with respect to $\lambda^i$ exists almost everywhere. Differentiating $\Psi$ with respect to $\lambda^S$ yields

$$\frac{\partial \Psi}{\partial \lambda^S} = \alpha \left[ Q \frac{d\Delta S^R}{d\lambda^S} + \Delta S^R \frac{dQ}{d\lambda^S} \right] + \frac{1 - \alpha}{1 + \lambda^B} \left[ 1 - (1 - q^A)^n - nq^A(1 - q^A)^{n-1} \right] \frac{d\Delta S^R}{d\lambda^S}$$

$$+ \frac{1 - \alpha}{1 + \lambda^B} \Delta S^R \left[ n(1 - q^A)^{n-1} \frac{dq^A}{d\lambda^S} - n(1 - q^A)^{n-1} \frac{dq^A}{d\lambda^S} + n(n - 1)q^A(1 - q^A)^{n-2} \frac{dq^A}{d\lambda^S} \right] \leq 0, \tag{A.48}$$

because $dQ/d\lambda^S \leq 0$, $dq^A/d\lambda^S \leq 0$, and $d\Delta S^R/d\lambda^S \leq 0$.

Taking the partial derivative of $\Psi$ with respect to $\lambda^B$ yields

$$\frac{\partial \Psi}{\partial \lambda^B} = \alpha \left[ Q \frac{d\Delta S^R}{d\lambda^B} + \Delta S^R \frac{dQ}{d\lambda^B} \right]$$

$$+ (1 - \alpha) \left[ 1 - (1 - q^A)^n - nq^A(1 - q^A)^{n-1} \right] \frac{d\Delta S^R}{d\lambda^B} \frac{(1 + \lambda^B) - \Delta S^R}{(1 + \lambda^B)^2}$$

$$+ \frac{1 - \alpha}{1 + \lambda^B} \Delta S^R \left[ n(1 - q^A)^{n-1} \frac{dq^A}{d\lambda^B} - n(1 - q^A)^{n-1} \frac{dq^A}{d\lambda^B} + n(n - 1)q^A(1 - q^A)^{n-2} \frac{dq^A}{d\lambda^B} \right] \leq 0. \tag{A.49}$$
A final remark is in order. Strictly speaking, $\Delta S^*$ is not an exogenous variable of the model. The exogenous variable that affects $\Delta S^*$ is the set of superior projects $Y$. Enlarging the set $Y$ does not only change $\Delta S^*$ but probably also $\Delta S^R$. The effects of a larger set $Y$ on the payoff advantage of running an auction is thus more complicated and these effects are typically opposing.

B. Bargaining with Asymmetric Information

In the paper we employ the GNBS in order to determine the outcome of initial negotiation as well as ex post renegotiation. Initially negotiation takes place under asymmetric information if the contractor knows the set of superior projects $Y$ but has not informed the buyer about it at stage 1 of the game. The GNBS does not take this asymmetric information explicitly into account. In the following, we discuss an alternative bargaining game which takes the asymmetric information explicitly into account and show that it is isomorphic to the application of the GNBS.

Suppose the bargaining game at stage 2 and the renegotiation game at stage 3 proceeds as follows: At the beginning of stage 2, nature determines the party that can make take-it-or-leave-it (TIOLI) offers throughout the game (at stage 2 and stage 3). The buyer can make the TIOLI offer with probability $\alpha \in (0,1)$ and the contractor makes the TIOLI offer with the converse probability $1-\alpha$.

With asymmetric information being only an issue with bilateral negotiations, we will focus on negotiation as procurement mechanism in the following. First, suppose the draw by nature determined that the contractor can make the offers. If the initial contract specifies $\bar{y}$ at price $\bar{p}$ and the contractor is aware of $Y$, there is scope for renegotiation at stage 3. When the contractor proposes the specification $y$, the highest price he can demand is

$$p^R = \bar{p} + \frac{v(y) - \bar{v}}{1 + \lambda^B}.$$  

(B.1)

The contractor’s utility from this offer is

$$U^S = p^R - c(y) - \lambda^S[c(y) - \bar{c}]$$

$$= \bar{p} + \lambda^S \bar{c} + \frac{1}{1 + \lambda^B} [v(y) - \bar{v} - (1 + \lambda^S)(1 + \lambda^B)c(y)],$$  

(B.2)

which is maximized at $y = y^R$.

When making the initial contract offer and the contractor has not revealed $Y$, the optimal offer is specification $\bar{y}$ at price $\bar{p} = \bar{v}$. Thus, the contractor’s utility amounts to

$$U^S = \bar{v} + \lambda^S \bar{c} + \frac{1}{1 + \lambda^B} [v(y^R) - \bar{v} - (1 + \lambda^S)(1 + \lambda^B)c(y^R)].$$  

(B.3)

If, on the other hand, the buyer can make the TIOLI offer, the contractor receives a zero utility. Thus, the contractor’s expected utility from disclosing his private information at stage 1
is

\[ EU^S(NR) = (1 - \alpha)(\bar{v} - \bar{c}) + \frac{1 - \alpha}{1 + \lambda^S} \left[ v(y^R) - \bar{v} - (1 + \lambda^S)(1 + \lambda^B)[c(y^R) - \bar{c}] \right]. \quad (B.4) \]

Note that (B.4) is equal to (A.14).

Now, suppose the contractor revealed \( Y \) at stage 1. If the contractor makes the TIOLI offer, he offers \( y^* \) at price \( \bar{p} = v^* \). His payoff in this case is

\[ US = v^* - c^*. \quad (B.5) \]

If the buyer can make the offers, then \( US = 0 \). Thus, the contractor’s expected utility from revealing his information at stage 1 is

\[ EU^S(NI) = (1 - \alpha)(v^* - c^*). \quad (B.6) \]

Recall that (B.6) is equal to (A.13).

Under the alternative bargaining game, the contractor’s expected payoffs from information disclosure and information revelation are exactly the same as those obtained by applying the GNBS. Thus, the contractor here has a strict incentive to reveal his private information at stage 1, i.e., \( EU^S(NI) > EU^S(NR) \).

References


