Public–Private Monopoly

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Abstract

This paper presents comparative statics of organizational forms of natural monopoly in public utilities with a focus on co-ownership and co-governance. Private monopoly lowers output and increases price to maximize profit. Public monopoly incurs higher costs due to the lack of know-how. A regulated monopoly results in regulation costs to overcome informational asymmetries. A public–private partnership arises as an efficient organization mode when it enables the internalization of private know-how and saves regulation costs due to correspondingly sufficient private and public ownership and control. Public–private monopoly supports higher prices than marginal costs due to rent sharing, with its upper price frontier decreasing in private ownership.

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*Keywords: Natural Monopolies, Operational Efficiency, Public–Private Partnerships, Ownership Structure, Regulation*
Milton Friedman stated that, “when technical conditions make a monopoly the natural outcome of competitive market forces, there are only three alternatives that seem available: private monopoly, public monopoly, or public regulation. All three are bad so we must choose among evils” (Friedman 1962, 28). This paper presents comparative statics of a fourth, arguably also bad, alternative to organize natural monopolies in the utilities sector: public–private partnerships.


Notable works of economic modeling of public–private partnership include Engel, Fischer, and Galetovic (2013), Grout (1997, 2005), Hart (2003), and Iossa and Martimort (2012), who emphasize is on bundling versus unbundling, contract completeness, and mechanism design to auction projects, incentivize investments, and avoid costly renegotiations.

As the organization of natural monopoly is an enormously complicated subject, this paper brings another—different but nonetheless complementary—lens to bear. I focus on hybrid ownership structures. In an analogous manner to Modigliani and Miller (1958) on debt–equity capital structure and firm value, I maximize welfare by allowing mixed public–private ownership and profit sharing. The upshot is that equity co-ownership, which affords more intrusive oversight and involvement through the board of directors (Williamson 1988), is the preferred governance instrument for public utilities where asset specificity (infrastructure non-redeployability and know-how) is significant.

At the risk of oversimplifying, I model specific key drivers of PPP—namely, regulation cost and managerial expertise as functions of public–private ownership. In particular institutional settings, the relevance of these drivers is subdued to governance, contractual, and political constraints. In general, however, they do matter and hence should be factored in.
1 Natural Monopolies

A natural monopoly is defined as a state when market conditions make it unprofitable to support more than one company\(^1\) For example, many natural monopolies are network utilities (water, gas, electricity). Due to the specificity of these assets—namely, spatial immobility—it is not profitable to build more than one supply network in a given area.

The theory of natural monopoly is based on the following premises:

(a) For a single-product single-plant company, economies of scale which grant the incumbent provider an unbiased (i.e., natural) advantage. Baumol and Willig’s (1981) sustainability theory was subject to criticism regarding high entry costs. Economies of scale are not a necessary, but a sufficient condition for a natural monopoly (Viscusi, Vernon, and Harrington 2000)

(b) For a multi-plant company, global cost subadditivity—that is,

\[
TC(\sum x^i) < \sum TC(x^i),
\]

where \(TC\) is total costs and \(x^i\) is the output vector for company \(i\) (Newbery 2000; Sharkey 1982)

(c) For a multi-product company\(^2\) economies of scope—that is,

\[
TC(x_1, x_2, \ldots, x_n) < TC(x_1) + TC(x_2) + \cdots + TC(x_n),
\]

where \(x_1, x_2, \ldots, x_n\) are cost-related products.

The empirical literature is ambiguous on whether economies of scale take place in bundled multi-product public utilities, in which case they should be banned to avoid the rise of natural monopolies. Economies of scale in one division of a multi-product utility company is not a sufficient condition for a natural monopoly (i.e., its cost function subadditivity)\(^3\)

Sharkey (1982) gives the example of a cost function characterized by economies of scale, but not subadditivity:

\[
TC(x_1, x_2) = x_1 + x_2 + (x_1x_2)^\frac{3}{2}
\]  

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\(^2\) For example, supplying electricity to industrial customers and households might be regarded as two separate products, if it is possible to price differentiate between markets.

Increasing the output of product by 10%, yields a total cost equal to:

\[ TC(1.1 \cdot x_1, 1.1 \cdot x_2) = 1.1 \cdot x_1 + 1.1 \cdot x_2 + 1.1^{\frac{2}{3}} \cdot (x_1x_2)^{\frac{1}{3}} \]  \hspace{1cm} (2)

Increasing costs by 10% equals:

\[ 1.1 \cdot TC(x_1, x_2) = 1.1 \cdot x_1 + 1.1 \cdot x_2 + 1.1 \cdot (x_1x_2)^{\frac{1}{3}} \]  \hspace{1cm} (3)

As \( TC \) in equation (3) is greater than in equation (2), economies of scale occur. Increasing the output of any product, however, increases the total cost of all products: \( TC(x_1, x_2) > TC(x_1) + TC(x_2) \). Thus, it is better to set the production of \( x_1 \) and \( x_2 \) in separate plants, meaning that subadditivity does not occur.

The existence of natural monopoly was first questioned by Evans and Heckman (1982) hereafter, abbreviated as ‘E-H’), who developed an innovative model to test subadditivity in telecommunications. Diewert and Wales (1992) criticized the E-H model, as the cost functions did not meet a basic assumption of economic theory—namely, that costs cannot decrease in output. In E-H, marginal costs are negative in 21 out of 31 observations. In the remaining 10 cases, when production is split into two companies to carry out the subadditivity test, marginal costs are also negative. Shin and Ying’s (1992) model does not have this flaw. This model, however, has also been subject to criticism. First, the identified subadditivity is weak (from 1.62% to 3.81% savings compared to the monopoly). Second, these are average savings: in approximately one-third of cases, a monopoly is more effective. Assuming the existence of natural monopolies in one-third of the cases, can we prove that in the remaining two-thirds of cases with modest profit breaking the monopoly is efficient for all cases? Third, the cost of capital was identical for all theoretical companies irrespective of their size, but data show that the cost of capital is usually negatively correlated with size. Furthermore, overhead costs were divided by two. Why would overhead costs of the two smaller companies be proportionally lower? Shin and Ying (1992) acknowledge the importance of technological changes, but this is captured only in the time variable, while such changes can completely alter the function of costs and interrelations between products (Jamison 1997). In the time-span of their data, demand rose more than tenfold, which might shift the demand curve to a completely different position in terms of the marginal cost curve.

The occurrence of a natural monopoly depends on the relation of entry costs (investment)
to demand. The higher the relation, the more probable the occurrence of a natural monopoly. Analyzing railroads, Friedlaender, Berndt, Chiang, Showalter, and Vellturo (1991) show that in only 9 (4.97%) out of 181 observations (17 companies over a span of 11 years) were there no economies of scale. They conclude that the “calculations suggest that the economy of scale is an inherent feature of railway technology” (p. 20). Salvanes and Tjøtta (1998) arrived at the same conclusion concerning energy transmission in Norway.

A sector or company identified as a natural monopoly in the past might subsequently cease to be so. Many studies that have uncovered a natural monopoly focus in telecommunications. In the last decades, entry costs into telecommunication markets have fallen substantially due to technological advances. Furthermore, demand for telecommunication services has increased significantly. Both factors render telecommunications as a natural monopoly questionable.

In this paper, I limit my analysis to single-product natural and administrative monopolies. These monopolies often produce necessity goods, indispensable for living at minimum accepted standards. Water supply, sewage, electricity, public transportation, and roads are undoubtedly classified as necessity goods. The basic and elementary character of these services and egalitarian access to them support the argument for keeping them public or subject to regulation.

1.1 Model Setup

The model is a partial equilibrium setup with a representative consumer, a good produced by a natural monopoly $x$ and the set of other goods produced in a competitive market.

The consumer maximizes utility under budget constraints. Consumer utility is a function of quantity and quality of good $x$ and quantity of the other goods produced on the competitive market. The private investor maximizes profit $\pi$. The public agent (planner) maximizes welfare given by joint consumer and producer surpluses.

Quality $q$ of good $x$ is desirable and increasingly costly: $\partial u/\partial q > 0$, $\partial TC/\partial q > 0$, and $\partial^2 TC/\partial^2 q$ (Varian 1992). Quality is correlated with the technical infrastructure and

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4 For example, 30 years ago one cable was required per subscriber; now one optical fiber cable can handle hundreds of conversations simultaneously thanks to digital technologies.

5 See Appendix A for a glossary.
implemented technology.

Entering (or contesting) the natural monopoly requires sizable investments in specific non-liquid assets (i.e., “sunk costs”). These costs are shown in cost accounting as fixed costs: financial costs and depreciation.\footnote{The difference between “fixed costs” and “sunk costs,” according to Tirole (2001, 307–308), is one of degree, not of nature. In both cases their value depends on the output level. Fixed costs concern short periods of time, while sunk costs concern investment, which generates benefit in the long run, but they can never be recovered. Tirole (2001) goes on to explain that the rigid differentiation of the notions of fixed and sunk costs is a simplification for a number of reasons. First, there is a clear sequence of levels of engagement in time and the shift from extremes—one period and “forever”—is not direct. Second, both notions assume that these costs cannot be recovered during the time the assets are utilized, irrespectively of how long this period is. To simplify, in this paper all investment costs are sunk costs and fixed costs are investment costs spread over time.}

A pricing tariff is set in two parts: a fixed part covering fixed costs and a variable part $p(x)$ covering variable costs (Coase 1946). Fixed and variable costs are orthogonal.\footnote{An illustrative example is fresh water, purified by means of “pumps and filters.” Once a filter is installed, variable costs depend on the amount of water pumped, not on the filter. The correlation between the quantity and fixed costs (contradictory \textit{ex definitionis}) occurs only at the margin of production capacity. If the production capacity is set with a safety cushion, it allows the satisfaction of every demand and avoids unexpected supply interruptions (e.g., blackouts). Thus, safety capacity is a part of the quality, not quantity output.}

Total price equals the fixed part plus the variable part $p \cdot x$. Due to entry investments, fixed cost is high relative to variable cost. I assume that the fixed part is fully covered by the fixed fee and further deal with the variable part.

Average variable cost $c(x,q)/x$ reaches its minimum at a relatively low quantity output $x$.\footnote{Bator (1958) ascribes cases of increasing economies of scale to market failure in which a monopoly can prove to be the most effective form of market organization. In other words, increasing economies of scale are considered a good externality: The purchase by one consumer lowers the cost for the next consumer. Viner (1931) describes this case as a “financial externality.” For a detailed discussion on increasing economies of scale and natural monopolies see, for example, Baumol and Oates (1988) and Kahn (1970).}

Average total cost $TC(x,q)/x = [F(q) + c(x,q)]/x$ decreases in the region of its intersection with the demand curve, which is a sufficient condition for the occurrence of a natural monopoly in a one-product company.\footnote{In an empirical study on costs and the technological structure of water supply systems in Italy, Fabbri and Fraquelli (2000, 65–82) define economies of scale as the inverse of the cost elasticity of the production function, where cost elasticity equals $\epsilon_{x,TC} = \frac{x \cdot \partial TC}{\partial x} = \frac{ATC}{MC}$. If $\epsilon_{x,TC} > 1$, then there are economies of scale; if $\epsilon_{x,TC} = 1$, then there are no economies of scale; and if $\epsilon_{x,TC} < 1$, then there are diseconomies of scale. The study shows that the inverse of cost elasticity equals 2.38 for small companies, 0.99 for median companies, and 0.68 for large companies. In so doing, the authors shed light on the general description of the cost function in natural monopolies. If economies of scale disappear with increasing market size, it means that $ATC$ has a minimum at the intersection with $MC$ (F.O.C.: $\frac{TC}{x} = \frac{MC}{x} - \frac{ATC}{x}$; $MC = ATC = 0 \rightarrow MC = ATC$), which requires an upward slope of $MC$. Before intersecting $ATC$, $MC$ intersects $AVC$, also at its minimum (proof is analogous to $ATC$), which then increases, albeit more slowly than $MC$. Should $AVC$ be monotonically decreasing, the average cost pricing would be the optimal pricing scheme.}

Figure 1 illustrates the relationships among average

\begin{equation}
\frac{\partial}{\partial x} \left[ \frac{TC}{x} \right] = \frac{MC}{x} - \frac{ATC}{x} = 0 \rightarrow MC = ATC,
\end{equation}
Figure 1: Average total cost \( ATC(x) \), average variable cost \( AVC(x) \), marginal cost \( MC(x) \), and demand curve \( p(x) \) in natural monopolies. The set \( x^*, p^* \) is a first-best benchmark state. \( MC(x) \) cuts \( AVC(x) \) at a relatively low output; \( ATC(x) \) falls along a wide range output due to high sunk (fixed) cost.

total cost \( ATC \), average variable cost \( AVC \), marginal cost \( MC \), and demand \( p(x) \) in natural monopolies.

Fixed fee equal to fixed cost and price equal to marginal cost guarantee profit in increasing economies of scale and increasing marginal cost (Coase 1946).

1.2 Efficient Supply and Pricing

Price equal to marginal cost is Pareto-optimal (Coase 1946; Mas-Colell, Whinston, and Green 1995). Let \( p(x, q) \) be the inverse demand function after covering the fixed fee, differentiable,

\[ p(x^*) + f(q) \]

\[ \text{min } ATC(x) \]

\[ x^* \]

\[ p(x) \]

\[ AVC(x) \]

\[ p(x^*) \]

\[ ATC(x) \]

\[ MC(x) \]

\[ x \]

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1. I do not claim that there are no industries in which the marginal cost is monotonically decreasing (cfr., e.g., Viscusi, Vernon, and Harrington 2000; Tirole 2001); rather, I perceive this to be a special case of small markets. I sustain, however, that a typical natural monopoly’s demand is located to the right of its minimum \( AVC \) and to the left of its minimum \( ATC \). The higher the fixed costs (i.e., sunk investment) are, the bigger the distance between minima \( AVC \) and \( ATC \) will be.

Other empirical studies on network utilities (Clark and Stevie 1981; Crain and Zardkoohi 1978; Feigenbaum and Teeples 1983; Hines 1969; Visc and Giardina 1983) confirm increasing economies of scale. Evidence for constant economies of scale was found by Battiano and Giardina (1983) and Pola and Visc and Comandini (1989). No studies reported monotonically increasing economies of scale.

10 Should an industry show a decreasing marginal cost (\( \forall x \ MC < AVC \)), the pricing policy should be changed to average cost pricing. Such a pricing policy would not be, however, optimal (Viscusi, Vernon, and Harrington 2000). 347–348.)
and non-increasing: \( p'(x) \leq 0 \). Assuming constant quality at \( q^S \), price depends on quantity demand: \( p(x) \).

Optimal quantity output \( x^* \) will therefore occur at the level at which \( p(x^*) = MC(x^*) \). At this quantity consumer surplus equals:

\[
CS^* = \int_0^{x^*} p(x) \, dx - p(x^*) \cdot x^*
\]

and producer surplus is:

\[
PS^* = p(x^*) \cdot x^* - \int_0^{x^*} MC(x) \, dx
\]

with profit equal to:

\[
\pi^* = p(x^*) \cdot x^* - c(x^*)
\]

This set of output \( x^* \) and price \( p^* \) is the first-best benchmark state for further comparative statics.

Any pricing scheme different from fixed fee plus marginal cost pricing is socially suboptimal. If there is no fixed fee, then the loss incurred by the utilities company would equal the fixed cost. The only “correct” solution in this case is charging a lump-sum tax and subsidizing the utilities; however, this tax would thrust a “wedge” between the pricing and marginal costs (Viscusi, Vernon, and Harrington 2000). A wide range of arguments against lump-sum tax plus subsidy is presented by Viscusi, Vernon, and Harrington (2000, 346–347):

1. If consumer surplus is lower than total variable cost \( \left( \int_0^{x^*} p(x) \, dx < \int_0^{x^*} MC(x) \, dx \right) \), the good should not be produced at all

2. Moral hazard arises (lack of cost monitoring) as the operator knows a loss will be covered with subsidy

3. Regarding welfare distribution, it implies cross-subsiding from non-consumer to consumer taxpayers

A two-part tariff allows for an efficient welfare allocation. It encourages the producer to supply any quantity, as the marginal unit cost will be covered.\[11\]

\[11\] Since the beginning of the 1930s, economists have claimed that regulators, especially in the energy sector, should apply marginal cost pricing. Only then does the consumer pay for any additional quantity. If the price
Hereinafter, I assume that the quality level is constant and exogenously given by the planner and that the fixed cost is covered by the fixed fee, and focus on quantity output.

2 Comparative Statics of Public Utilities’ Organizational Forms

There are three classic forms of organizing natural monopolies—private monopoly, public monopoly, and regulated monopoly—as well as a fourth that is a hybrid extension of the former three: public–private partnership.

As a first-best benchmark state I utilize an efficient non-bureaucratic non-regulated two-part tariff marginal cost pricing utility. The public–private partnership (hereafter, often abbreviated as ‘PPP’) is modeled as a public–private joint-venture vehicle.

2.1 Private Monopoly

The first historical form of the organization of a natural monopoly was a private (unregulated) monopoly (Newbery 2000). A private monopoly maximizes profit:

\[ \pi_m = p(x_m) \cdot x_m - c(x_m) \]  

Profit maximization is achieved when marginal revenue \( MR \) equals marginal cost (F.O.C.):

\[ MR(x_m) = MC(x_m) \]  

where \( x_m \leq x^* \), with equality for perfectly inelastic (stiff) demand.

Consumer surplus equals:

\[ CS_m = \int_0^{x_m} p(x) \, dx - p(x_m) \cdot x_m \]  

The difference between consumer surplus and the benchmark state \( CS_m - CS^* \) equals:

\[ CS_m - CS^* = \int_0^{x_m} p(x) \, dx - p(x_m) \cdot x_m - \int_0^{x^*} p(x) \, dx + p(x^*) \cdot x^* \]  

\[ CS_m - CS^* = p(x^*) \cdot x^* - p(x_m) \cdot x_m - \int_{x_m}^{x^*} p(x) \, dx \]  

is set at the average cost, the consumer bears a price higher or lower than actual cost.

Marginal cost pricing allows for profit. Regulators opposed these arguments for a long time, due to difficulties in its computation and maybe because consumers perceived profit from utilities as unfair. In the 1980s, regulators began to apply marginal cost pricing. Currently, most electricity tariffs are adjusted depending on the season and time of the day, reflecting the changes in the marginal cost (Fischer, Dornbusch, and Schmalensee 1990, 335–336).
The consumer incurs welfare loss when price increases \( (p(x_m) > p(x^*)) \) and quantity decreases \( (x_m < x^*) \).

Private monopoly surplus equals:

\[
P_{Sm} = p(x_m) \cdot x_m - \int_0^{x_m} MC(x) \, dx  \tag{11}
\]

The difference between private monopoly surplus \( P_{Sm} \) and the benchmark state \( P^* \) equals:

\[
P_{Sm} - P^* = p(x_m) \cdot x_m - \int_0^{x_m} MC(x) \, dx - p(x^*) \cdot x^* + \int_0^{x^*} MC(x) \, dx
\]

\[
P_{Sm} - P^* = p(x_m) \cdot x_m - p(x^*) \cdot x^* + \int_{x_m}^{x^*} MC(x) \, dx  \tag{12}
\]

and is positive (otherwise private monopoly would not change price or quantity to \( p_m, x_m \)).

Total welfare change equals:

\[
C_{Sm} - C^* + P_{Sm} - P^* =
\]

\[
= p(x^*) \cdot x^* - p(x_m) \cdot x_m - \int_{x_m}^{x^*} p(x) \, dx + p(x_m) \cdot x_m - p(x^*) \cdot x^* + \int_{x_m}^{x^*} MC(x) \, dx =
\]

\[
= \int_{x_m}^{x^*} MC(x) \, dx - \int_{x_m}^{x_m} p(x) \, dx  \tag{13}
\]

Figure 2 illustrates welfare change from the benchmark state to private monopoly.

Given that \( MC(x) < p(x) \) for \( x \in (x_m, x^*) \), there is deadweight loss. In the industrial organization literature (Tirole 2001; Varian 1992; Mas-Colell, Whinston, and Green 1995; Martin 2001), transfers from consumer surplus to producer surplus are not considered deadweight loss if the company owners and consumers are members of the same society and the weights ascribed to their payoffs are the same. For inelastic demand, there is no deadweight loss \( \left( \int_{x_m}^{x^*} MC(x) \, dx = 0; \int_{x_m}^{x^*} p(x) \, dx = 0 \right) \).

It is a common practice to consider that only part of a private monopoly’s profit constitutes social welfare (Cook and Fabella 2002). First, redistribution might be detrimental for welfare. Second, if the external sector is involved, part of the profit might be expatriated and contribute to an increase in welfare, but only to the external sector. Hereinafter, unless otherwise specified, I assume that profit fully accounts for welfare.

\[
_{12} \text{An empirical analysis of price-elasticity demand concerning public utilities goods in Poland can be found in Moszoro (2010, Appendix D).}
\]

\[
_{13} \text{Due to taxation, transfers can take the form of off-market transfer prices between entities connected with the investor (e.g., more expensive purchase of raw materials or sales at a discount).}
\]
Figure 2: Welfare change from the benchmark state to private monopoly. A private monopoly maximizes profit at output $x_m$, at which point marginal revenue $MR$ equals marginal cost $MC$. As quantity drops from $x^*$ to $x_m$ and price from $p(x^*)$ to $p(x_m)$, deadweight loss is incurred.

Monopoly pricing shortens consumption by $\int_{x^*}^{x_m} MC(x) \, dx$, which is next reallocated to other goods. Whereas monopoly pricing and demand curbing significantly decrease welfare, public monopoly and regulated monopoly are called into existence.  

2.2 Public Monopoly

Public monopoly aims to maximize welfare. Its objective function is:

$$W_{pu} = \max_{x_{pu}} (CS_{pu} + PS_{pu}) =$$

$$= \max_{x_{pu}} \left[ \int_0^{x_{pu}} p(x) \, dx - p(x_{pu}) \cdot x_{pu} + p(x_{pu}) \cdot x_{pu} - \int_0^{x_{pu}} MC(x) \, dx \right] =$$

$$= \max_{x_{pu}} \left[ \int_0^{x_{pu}} p(x) \, dx - \int_0^{x_{pu}} MC(x) \, dx \right]$$

(14)

From the F.O.C. of equation (14), the optimal output for the public monopoly $x_{pu}$ is at $p(x_{pu}) = MC(x_{pu})$.

14 Natural monopoly does not always call for an intervention. Not all natural monopolies decrease welfare so as to be nationalized/communalized or become subject to regulation, such as cable television (Viscusi, Vernon, and Harrington 2000; Williamson 1976).
The cost function of the public monopoly is higher than the cost function of the first-best benchmark utility company described in equations (5) and (6). Besides quantity, variable cost depends on technology, administrative procedures, and management skills. These features—hereinafter referred in short as “know how”—correspond to average variable cost:

\[ AVC = \begin{cases} 
  \frac{c(x)}{x} + k & \text{when know-how is lacking} \\ 
  \frac{c(x)}{x} & \text{when know-how is available}
\end{cases} \quad (15) \]

Due to the lack of know-how, public monopoly’s production of each unit of \( x \) is more costly by \( k \) than that of private monopoly.

The public monopoly produces at

\[ p(x_{pu}) = MC(x_{pu}) + k \]

that is, equilibrium occurs at a lower output and higher price than the benchmark state. Deadweight loss from public monopoly compared to the benchmark state is:

\[ CS_{pu} - CS^* = \int_0^{x_{pu}} p(x) \, dx - p(x_{pu}) \cdot x_{pu} - \int_0^{x^*} p(x) \, dx + p(x^*) \cdot x^* = 
\]

\[ = p(x^*) \cdot x^* - p(x_{pu}) \cdot x_{pu} - \int_{x_{pu}}^{x^*} p(x) \, dx \quad (16) \]

and

\[ PS_{pu} - PS^* = p(x_{pu}) \cdot x_{pu} - p(x^*) \cdot x^* + \int_0^{x^*} MC(x) \, dx - \int_0^{x_{pu}} [MC(x) + k] \, dx \quad (17) \]

Total welfare change results from the sum of equations (16) and (17):

\[ CS_{pu} - CS^* + PS_{pu} - PS^* = \int_0^{x^*} MC(x) \, dx - \int_0^{x_{pu}} [MC(x) + k] \, dx - \int_{x_{pu}}^{x^*} p(x) \, dx \quad (18) \]

Replacing

\[ \int_0^{x^*} MC(x) \, dx - \int_0^{x_{pu}} [MC(x) + k] \, dx = -k \cdot x_{pu} \quad (19) \]

in equation (18) we obtain:

\[ CS_{pu} - CS^* + PS_{pu} - PS^* = 
\]

\[ = \int_0^{x_{pu}} MC(x) \, dx - \int_0^{x_{pu}} [MC(x) + k] \, dx - \int_{x_{pu}}^{x^*} p(x) \, dx + \int_{x_{pu}}^{x^*} MC(x) \, dx \quad (20) \]

\[ = - \int_{x_{pu}}^{x^*} p(x) \, dx + \int_{x_{pu}}^{x^*} MC(x) \, dx - k \cdot x_{pu} \]

\[ \text{An example of how private know-how can be conducive to lowering operating costs was presented by Dalkia Termika (Vivendi group) to Polish municipalities in the early 2000s. The company offered to upgrade electricity generation and heating infrastructure in return for average historical revenue for a set period of time. The company’s strategy consisted of reducing variable costs from 75% to 62% by means of cogeneration technology, and administrative costs from 25% to 20%. Savings of 18% would constitute the company’s return on investment.} \]
Figure 3: Welfare change from the benchmark state to public monopoly. Due to the lack of know-
how, public monopoly produces at a higher average and marginal cost $k$, setting equilibrium output
at $x_{pu}$. As quantity drops from $x^*$ to $x_{pu}$ and price increases from $p(x^*)$ to $p(x_{pu})$, deadweight loss is incurred.

Figure 3 shows the change in welfare from the benchmark state to public monopoly.

For price-inelastic demand $x^S$, deadweight loss compared to the benchmark state equals $k \cdot x^S$.

Comparing deadweight loss incurred due to a lower output at a higher price in the case of a private monopoly and due to higher variable costs in the case of public monopolies, the latter is preferred when:

$$CS_{pu} - CS^* + PS_{pu} - PS^* - (CS_m - CS^* + PS_m - PS^*) =$$

$$= CS_{pu} - CS_m - PS_{pu} + PS_m$$

$$= \int_{x_{pu}}^{x^*} MC(x) \, dx - \int_{x_{pu}}^{x^*} p(x) \, dx - k \cdot x_{pu} - \int_{x_m}^{x^*} MC(x) \, dx + \int_{x_m}^{x^*} p(x) \, dx =$$

$$= \int_{x_m}^{x_{pu}} p(x) \, dx - \int_{x_m}^{x_{pu}} MC(x) \, dx - k \cdot x_{pu} > 0$$

When

$$k \cdot x_{pu} = \int_{x_m}^{x_{pu}} p(x) \, dx - \int_{x_m}^{x_{pu}} MC(x) \, dx,$$  \hspace{1cm} (22)

$^{16}$ The case of inelastic demand is normal for first need goods.
it is welfare indifferent whether the monopoly is private or public. For price-inelastic demand, a private monopoly will always prove to be welfare superior.

Adjusting for the weight $\alpha$ of profit in welfare and dividing by $x_{pu}$, we obtain:

$$k = \frac{\int_{x_m}^{x_{pu}} p(x) \, dx - \int_{x_m}^{x_{pu}} MC(x) \, dx + (1 - \alpha)[p(x_m) \cdot x_m - c(x_m)]}{x_{pu}}.$$  

(23)

Thus, the more elastic the demand (i.e., the greater the difference is between $x_{pu}$ and $x_m$), the lower the weight of profit in welfare (i.e., the lower $\alpha$ is), and the lower the additional marginal cost resulting from a lack of specific know-how (i.e., the lower $k$ is) are, the stronger the incentives for public monopoly will be. Conversely, the higher demand inelasticity, the larger the weight of profit in welfare (e.g., by dispersed stock ownership), and the lower the know-how cost advantage from private management are, then the less detrimental the private monopoly will be.

For price-inelastic demand (i.e. $x_{pu} = x_m = x^S$), a private monopoly is welfare superior when:

$$k > (1 - \alpha) \left[ p(x^S) - \frac{c(x^S)}{x^S} \right]$$  

(24)

that is, when the increased public monopoly variable cost is higher than the unit profit, which does not constitute welfare.

2.3 Regulated Monopoly

The literature on the regulation of natural monopoly is ample and well established.\(^{17}\) There are two basic regulating mechanisms to curb monopoly power: rate of return regulation and price cap regulation.

Rate of return regulation—developed and widely used in the US—allows an increasing price if the rate of return does not exceed the set level. As a form of price control, it requires constant monitoring by the regulator. Generally, it provides the operator with a certain degree of certainty on the expected return on investment (Pongsiri 2002).

Price cap regulation bounds maximum price depending on other price indicators (e.g., retail price index—RPI). This method of price regulation is common in the UK and Western Europe (Dobbs and Elson 1999). According to the British RPI-X approach, prices for public

\(^{17}\) I refer to Joskow (2007) and the literature therein.
services are updated according to the difference between RPI and a given productivity growth (‘X’). In some cases, the variable Y is added to the formula to account for input costs growth beyond the influence of the monopoly or regulated elsewhere.

The aim of regulation is setting a price close to marginal cost—that is, drawing the private monopoly close to the first-best benchmark state. The producer knows its cost and quality output, but the regulator does not. The regulator bears the costs of overcoming information asymmetry in terms of operations, compliance with quality standards, cash flows, and cost of capital (Newbery 2000). These costs are deadweight loss.

Suppose arguendo, that the regulator in order to identify the marginal cost has to incur cost $g$ for each unit of output $x_{re}$. This regulation cost is next passed to the consumer through higher taxation or higher price.

Deadweight loss from the regulated monopoly compared to the benchmark state is:

$$CS_{re} - CS^* = \int_0^{x_{re}} p(x) \, dx - [p(x_{re}) + g] \cdot x_{re} - \int_0^{x^*} p(x) \, dx + p(x^*) \cdot x^* =$$

$$= p(x^*) \cdot x^* - p(x_{re}) \cdot x_{re} - g \cdot x_{re} - \int_{x_{re}}^{x^*} p(x) \, dx$$

and

$$PS_{re} - PS^* = p(x_{re}) \cdot x_{re} - p(x^*) \cdot x^* + \int_{x_{re}}^{x^*} MC(x) \, dx$$

Total welfare change results from the sum of equations (25) and (26):

$$CS_{re} - CS^* + PS_{re} - PS^* = - \int_{x_{re}}^{x^*} p(x) \, dx + \int_{x_{re}}^{x^*} MC(x) \, dx - g \cdot x_{re}$$

The regulated monopoly’s price is $MC(x_{re}) = p(x_{re})$, but the consumer pays $p(x_{re}) + g$ for each unit of $x$. Taxation yields an analogous result to an increase in the marginal cost.

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18 Loeb and Magat (1979) presented an interesting suggestion for equating the price to the marginal cost. They assume that the monopolist has correct information about his costs and demand, while the regulator has information about only the demand, and propose to pay the profit maximizer monopolist a subsidy equal to consumer surplus. In this mechanism, the monopolist maximizes profit when price equals marginal cost. This is an optimal solution, but the monopoly would internalize all consumer surplus. Loeb and Magat (1979) go further to propose bidding for the monopoly franchise. No bidder will bid below the marginal cost. The higher the price bid, the larger the consumer surplus internalized by the regulator, from which she will be able to pay out the subsidy in accordance with the L-M model.

19 Regulation cost exceeds the budgetary cost of running a regulatory agency. Regulation cost comprises cost of compliance, lower flexibility, and hence higher risk, which results in higher price.

20 X-type inefficiency (Leibenstein 1966), which refers to inefficiency resulting from the monopoly’s lack of incentives to reduce cost, especially when it adopts marginal cost pricing, has been disregarded.
Adjusting for the weight $\alpha$ of profit in welfare, we obtain:

$$CS_{re} - CS^* + PS_{re} - PS^* - (1 - \alpha)\pi_{re} = $$

$$= -\int_{x_{re}}^{x^*} p(x) \, dx + \int_{x_{re}}^{x^*} MC(x) \, dx - g \cdot x_{re} - (1 - \alpha) [p(x_{re}) \cdot x_{re} - c(x_{re})]$$  \hspace{1cm} (28)

Regulated monopoly is preferred to private monopoly when the welfare change is positive:

$$CS_{re} + PS_{re} - CS_m - PS_m - (1 - \alpha)(\pi_{re} - \pi_m) = $$

$$= \int_{x_m}^{x_{re}} p(x) \, dx - g \cdot x_{re} - \int_{x_m}^{x_{re}} MC(x) \, dx - \int_{x_m}^{x_{re}} p(x) \, dx + \int_{x_m}^{x_{re}} MC(x) \, dx + $$

$$-(1 - \alpha) \{[p(x_{re}) \cdot x_{re} - c(x_{re})] - [p(x_m) \cdot x_m - c(x_m)]\} =$$

$$= \int_{x_m}^{x_{re}} p(x) \, dx - \int_{x_m}^{x_{re}} MC(x) \, dx - g \cdot x_{re} +$$

$$-(1 - \alpha) \{[p(x_{re}) \cdot x_{re} - c(x_{re})] - [p(x_m) \cdot x_m - c(x_m)]\} > 0$$  \hspace{1cm} (29)

Figure 4 shows the change in welfare from the benchmark state to the regulated monopoly.

Figure 4: Welfare change from the benchmark state to regulated monopoly. The public agent incurs regulation cost $g$ to overcome information asymmetry on marginal cost, which is borne by the consumer. As quantity drops from $x^*$ to $x_{re}$ and price increases from $p(x^*)$ to $p(x_{re})$, deadweight loss is incurred.

If

$$g \cdot x_{re} < \int_{x_m}^{x_{re}} p(x) \, dx - \int_{x_m}^{x_{re}} MC(x) \, dx +$$

$$-(1 - \alpha) \{[p(x_{re}) \cdot x_{re} - c(x_{re})] - [p(x_m) \cdot x_m - c(x_m)]\}$$  \hspace{1cm} (30)
regulating the natural monopoly is welfare superior. When demand is price inelastic \((x = x_m = x^S)\), private monopoly price is \(p_m\), and regulated monopoly price equals marginal cost, then the condition for the preference of regulation boils down to:

\[
g \cdot x^S < (1 - \alpha) \{ [p_m \cdot x^S - c(x^S)] - [MC(x^S) \cdot x^S - c(x^S)] \}
\]

\[
g < (1 - \alpha) [p_m - MC(x^S)]
\] (31)

that is, when the unit regulatory cost is lower than the welfare-weighted \((1 - \alpha)\) difference between the monopolist’s price and the price of the regulated monopoly.

Likewise, a regulated monopoly is preferred to a public monopoly when welfare change is positive:

\[
CS_{re} + PS_{re} - CS_{pu} - PS_{pu} - (1 - \alpha) \pi_{re} =
\]

\[
\int_0^{x_{re}} p(x) \, dx - \int_0^{x_{re}} MC(x) \, dx - g \cdot x_{re} - \int_0^{x_{re}} p(x) \, dx + \int_0^{x_{pu}} [MC(x) + k] \, dx + (1 - \alpha) [p(x_{re}) \cdot x_{re} - c(x_{re})] > 0
\] (32)

As \(\int_0^{x_{pu}} [MC(x) + k] \, dx = \int_0^{x_{pu}} MC(x) \, dx + k \cdot x_{pu}\), equation (32) can be simplified to:

\[
CS_{re} + PS_{re} - CS_{pu} - PS_{pu} - (1 - \alpha) \pi_{re} =
\]

\[
\int_0^{x_{re}} p(x) \, dx - \int_0^{x_{re}} MC(x) \, dx - g \cdot x_{re} + k \cdot x_{pu} - (1 - \alpha) [p(x_{re}) \cdot x_{re} - c(x_{re})]
\] (33)

Thus regulated monopoly is welfare superior to public monopoly if:

\[
g \cdot x_{re} < k \cdot x_{pu} + \int_0^{x_{re}} p(x) \, dx - \int_0^{x_{re}} MC(x) \, dx - (1 - \alpha) [p(x_{re}) \cdot x_{re} - c(x_{re})]
\] (34)

that is, when regulation cost is lower than additional costs due to a lack of know-how and adjusted deadweight loss from lower output.

If demand is price inelastic \((x^S)\), condition (34) boils down to:

\[
g < k - (1 - \alpha) \left[ MC(x^S) - \frac{c(x^S)}{x^S} \right]
\] (35)

meaning the unit regulation cost is lower than the marginal additional cost due to a lack of know-how adjusted for the unit profit margin that is not part of welfare.

\[\text{21} \text{ The private monopoly can engage in a strategic game and establish such a price that will make regulation non-preferable, analogously to a monopolist that lowers price to deter entrance [Martin 2001]. Nonetheless, without perfect market contestability, self-regulation will prove insufficient to force the private monopoly to equate price to marginal cost.}\]
I have omitted the comparative statics of public regulated monopoly and corruption. The regulation of publicly owned and managed utilities is an extension of the public sector itself. Public monopoly as modeled in section is the best outcome the public sector can deliver, with or without self-regulation. Corruption leads to a forced payment to divert a public agent from his/her job prescription (capture) or to encourage a public agent to perform his/her job prescription (extortion). Capture can be embedded in $g$ and extortion in $k$.  

3 Public–Private Partnership

3.1 Institutional Public–Private Joint Ventures

Out of the nine combinations of public, semi-public, and private ownership and public, semi-public, and private management, seven (i.e., all but fully public and fully private schemes) have been regarded in different literatures as public–private partnerships. Hereafter, I limit my analysis to institutional PPP—also referred as “equity public–private joint ventures”—where the public agent and private investor co-share ownership and management (i.e., investment and operational risk).

Institutional PPP presents a series of advantages:

(a) Reduces information asymmetries between the investor and the public agent regarding output quality, actual investment, and operating cost.

(b) Offsets transaction costs concerning *ex ante* negotiation and regulation and *ex post* possible renegotiation of quality and price between private and public agents.

(c) Enables the internalization of private technology and specific know-how that lead to operating cost reduction and quality improvement without complex monitoring systems.

---

22 Examples of public regulated monopolies can be found in practice. The Polish energy grid PSE is a natural monopoly regulated by the Energy Regulatory Office, and most water supply and heating companies are owned municipalities and supervised by “independent” municipal departments. Tensions between the Telecommunications Regulatory Office and Telekomunikacja Polska were palpable long before the privatization of the company.

23 For formal models of corruption in public procurement I refer to Auriol (2006, Auriol (2013), and Shleifer and Vishny (1998).

24 Kogut (1988, 321) argues that “joint venture creates the best supervisory mechanism and stimulates to revealing information, sharing technologies and ensuring good practices.”
(d) Limits the social perception of opportunistic risk thanks to direct formal and informal audits (Balakrishnan and Koza 1993)

(e) In case of need, increases the acceptance for public aid—for example, guarantees, preferential loans, and direct subsidies (Trujillo, Cohen, Freixas, and Sheehy 1998)—compared to private or regulated monopoly

From a model standpoint, institutional PPP encapsulates all major trade-offs of public–private relations, rendering alternative public–private schemes as particular cases. Comparative statics of institutional PPP will, therefore, shed light on the whole spectrum of PPP and are not intended to vindicate mixed ownership PPP.

3.2 Objective Functions

Let $\theta \in (0,1)$ be the private and $1 - \theta$ be the public share in outlays and profit of the joint-venture public–private monopoly. The investor maximizes profit $\pi_{jv}$ and the public agent maximizes welfare $W$ given by:

$$\max_{x, \theta} W = \int_0^{x_{jv}} p(x) \, dx - \int_0^{x_{jv}} MC(x) \, dx - (1 - \alpha)\theta[p(x_{jv}) \cdot x_{jv} - c(x_{jv})]$$  \hspace{1cm} (36)

subject to $x_{jv} \geq 0$ and $0 \leq \theta \leq 1$.

The Lagrangian $Z$ of equation (36) can be formulated as:

$$Z = \int_0^{x_{jv}} p(x) \, dx - \int_0^{x_{jv}} MC(x) \, dx - (1 - \alpha)\theta[p(x_{jv}) \cdot x_{jv} - c(x_{jv})] + \lambda \theta$$  \hspace{1cm} (37)

with Kuhn-Tucker conditions:

\[\begin{align*}
\frac{\partial Z}{\partial x} &\leq 0, \quad x_{jv} \geq 0, \text{ and } x_{jv} \frac{\partial Z}{\partial x} = 0 \\
\frac{\partial Z}{\partial \theta} &\leq 0, \quad \theta \geq 0, \text{ and } \theta \frac{\partial Z}{\partial \theta} = 0, \text{ and } \\
\theta &\geq 0, \quad \lambda \geq 0, \text{ and } \lambda \theta = 0
\end{align*}\]  \hspace{1cm} (38)

where $\lambda$ is the Lagrange multiplier. Differentiating $Z$ with respect to $x$ and $\theta$ yields:

$$\frac{\partial Z}{\partial x} = p(x_{jv}) - MC(x_{jv}) - (1 - \alpha)\theta \left[ \frac{\partial p(x)}{\partial x} \cdot x_{jv} + p(x_{jv}) - MC(x_{jv}) \right]$$  \hspace{1cm} (39)

\[^{25}\text{Following Chiang and Wainwright (2005), Kuhn-Tucker conditions are presented after the simplification and elimination of ancillary variables.}\]
\[
\frac{\partial Z}{\partial \theta} = -(1 - \alpha)[p(x_{jv}) \cdot x_{jv} - c(x_{jv})] + \lambda \tag{40}
\]

For any output \(x_{jv} > 0\), optimization requires \(\frac{\partial Z}{\partial x} = 0\), i.e.:

\[
p(x_{jv}) - MC(x_{jv}) = (1 - \alpha)\left[\frac{\partial p(x)}{\partial x} \cdot x_{jv} + p(x_{jv}) - MC(x_{jv})\right] \tag{41}
\]

\(\theta\) and \(\alpha\) are negatively correlated, reasonably assuming that \(\alpha = 1\) only applies when \(\theta = 0\). If \(\theta = 0\) and \(\alpha = 1\) (public monopoly), market clearance is realized at a higher price \(p(x_{pu}) = MC(x_{pu}) + k\). For \(\alpha, \theta \in (0, 1)\), in equilibrium:

\[
\frac{\partial p(x)}{\partial x} = \frac{[1 - (1 - \alpha)\theta] \cdot [p(x_{jv}) - MC(x_{jv})]}{(1 - \alpha) \theta \cdot x_{jv}} \tag{42}
\]

As \(x\) is a normal good \((\frac{\partial p(x)}{\partial x} < 0)\), the welfare maximizing public agent would set \(p(x_{jv}) = AVC(x_{jv}) < MC(x_{jv})\), which means incremental expropriation (loss).

From maximization conditions regarding \(\theta\):

\[
\lambda = (1 - \alpha)[p(x_{jv}) \cdot x_{jv} - c(x_{jv})] \tag{43}
\]

where \(\lambda\) is the dual price of increasing welfare in relation to the private share constraint \((\theta \leq 1)\), it follows that if profit equals zero, sufficient Kuhn-Tucker conditions are met for every \(\theta > 0\).

The private investor maximizes profit:

\[
\max_{x, \theta} \pi = \theta[p(x_{jv}) \cdot x_{jv} - c(x_{jv})] \tag{44}
\]

subject to: \(x_{jv} \geq 0\) and \(0 \leq \theta \leq 1\). As \(\theta\) is a linear multiplier, profit maximization results directly from the F.O.C. of function (44): \(\frac{\partial \pi}{\partial x} = 0\). Therefore, the private investor will aim to maximize \(\theta\).

### 3.3 Structure and Governance

The public agent requires a minimum ownership and profit share \(h\) to exercise internal regulation, so that:

\[
MC_{jv}(x) = \begin{cases} MC(x) + g & \text{if } 1 - \theta < h \text{ (information asymmetry)} \\ MC(x) & \text{if } 1 - \theta \geq h \text{ (internal regulation)} \end{cases} \tag{45}
\]

\(^{26}\) In particularly, if there are transfers to the external sector, \(\alpha < 1\) applies.
The private investor requires \( p_{jv} \geq MC(x_{jv}) \) and a minimum ownership and profit share \( e \) to transfer know-how, so that:

\[
MC_{jv}(x) \begin{cases} 
MC(x) + k & \text{if } \theta < e \text{ (lack of know-how)} \\
MC(x) & \text{if } \theta \geq e \text{ (know-how available)}
\end{cases}
\]  \hspace{1cm} (46)

PPP feasibility requires, hence, \( \theta \in [e, 1-h] \) to be not empty. This parameter space is the contracting (negotiable) area. \[27\]

Negotiations concern quality, quantity, and price. Assuming that quality is fixed, \[28\] the private investor will aim for monopoly output and the public agent will aim for the output at which the price equals average cost.

### 4 Comparative Statics of Public–Private Partnership

Welfare difference between public–private partnership \((jv)\) and private monopoly yields:

\[
W_{jv} - W_m = \int_0^{x_{jv}} p(x) \, dx - \int_0^{x_{jv}} MC(x) \, dx - (1-\alpha)\theta[p(x_{jv}) \cdot x_{jv} - c(x_{jv})] - \\
+ \int_0^{x_m} p(x) \, dx + \int_0^{x_m} MC(x) \, dx - (1-\alpha)[p(x_m) \cdot x_m - c(x_m)] = \\
= \int_{x_m}^{x_{jv}} p(x) \, dx - \int_{x_m}^{x_{jv}} MC(x) \, dx + \\
+ (1-\alpha)[p(x_m) \cdot x_m - c(x_m)] - \theta[p(x_{jv}) \cdot x_{jv} - c(x_{jv})]
\]  \hspace{1cm} (47)

PPP is welfare superior depending on \( x_{jv}, p_{jv}, \alpha, \) and \( \theta \).

For price-inelastic demand \( x_{jv} = x_m = x^S \),

\[
W_{jv} - W_m = (1-\alpha)[x^S(p_m - \theta \cdot p_{jv}) - (1-\theta)c(x^S)] > 0,
\]  \hspace{1cm} (48)

indicating that PPP is welfare superior to private monopoly for any \( \alpha \in (0,1), \theta \in [e, 1-h], \) and \( p_{jv} \in [p^*, p_m] \).

\[27\] For example, in Polish public–private partnerships in water supply and sewage presented in [Moszoro (2010) Table 1.8], \( \theta \) ranged between 33% and 64%, implying values of \( e \in [0.33, 0.64] \) and \( h \in [0.36, 0.67] \).

\[28\] Notwithstanding quality as a key contractual dimension, it is in general exogenously given in laws and regulation standards.
The welfare difference between public–private partnership and public monopoly yields:

\[ W_{jv} - W_{pu} = \int_{0}^{x_{jv}} p(x) \, dx - \int_{0}^{x_{jv}} MC(x) \, dx - (1 - \alpha)\theta[p(x_{jv}) \cdot x_{jv} - c(x_{jv})] - \]
\[ + \int_{0}^{x_{pu}} p(x) \, dx + \int_{0}^{x_{pu}} [MC(x) + k] \, dx \]
\[ = \int_{x_{pu}}^{x_{jv}} p(x) \, dx - \int_{x_{pu}}^{x_{jv}} MC(x) \, dx - (1 - \alpha)\theta[p(x_{jv}) \cdot x_{jv} - c(x_{jv})] + k \cdot x_{pu} \tag{49} \]

For price-inelastic demand \( x_{jv} = x_{pu} = x^S \) and \( p_{jv} \in [p^*, p_m] \), \( W_{jv} > W_{pu} \) if:

\[ (1 - \alpha)\theta \pi_{jv} < k \cdot x^S, \tag{50} \]

such as when part of the profit not included in welfare is lower than the additional cost due to a lack of know-how.

The welfare difference between public–private partnership and regulated monopoly yields:

\[ W_{jv} - W_{re} = \int_{0}^{x_{jv}} p(x) \, dx - \int_{0}^{x_{jv}} MC(x) \, dx - (1 - \alpha)\theta[p(x_{jv}) \cdot x_{jv} - c(x_{jv})] - \]
\[ + \int_{0}^{x_{re}} p(x) \, dx + \int_{0}^{x_{re}} [MC(x) + g] \, dx + (1 - \alpha)[p(x_{re}) \cdot x_{re} - c(x_{re})] = \]
\[ = \int_{x_{re}}^{x_{jv}} p(x) \, dx - \int_{x_{re}}^{x_{jv}} MC(x) \, dx + g \cdot x_{re} + \]
\[ + (1 - \alpha)\{[p(x_{re}) \cdot x_{re} - c(x_{re})] - \theta[p(x_{jv}) \cdot x_{jv} - c(x_{jv})]\} \tag{51} \]

For price-inelastic demand \( x_{jv} = x_{re} = x^S \), PPP is welfare superior when:

\[ W_{jv} - W_{re} = g \cdot x^S + (1 - \alpha)\left[ x^S(p_{re} - \theta \cdot p_{jv}) - (1 - \theta) \cdot c(x^S) \right] > 0 \tag{52} \]

As \( p_{re} = MC(x^S) \), condition \( 52 \) can be reduced to:

\[ \theta \cdot p_{jv} - MC(x^S) < \frac{g}{1 - \alpha} - \frac{(1 - \theta) \cdot c(x^S)}{x^S} \]
\[ p_{jv} - MC(x^S) < \frac{g}{1 - \alpha} + (1 - \theta) \left[ p_{jv} - \frac{c(x^S)}{x^S} \right] \tag{53} \]

where \( p_{jv} - c(x^S)/x^S \) is the PPP unit profit. The higher \( r \) and lower \( \theta \in [e, 1 - h] \) are, the more welfare efficient a PPP will be. For \( p_{jv} \) close to \( MC(x^S) \), PPP will generate profit \( (p_{jv} > c(x^S)/x^S) \) and will be more welfare efficient than regulation.

The bargaining power of each of the parties can be measured using the modified Lerner
Figure 5 presents a graphic representation of the PPP negotiating area. The higher the price above average cost and higher $\theta$ are (PR set), the higher the bargaining (monopolistic) power of the private investor is. Conversely, the closer the price to average cost (PU set) is, the higher the bargaining (regulating) power of the public agent is.

\[ L_{jv} = \theta \frac{p_{jv} - \frac{c(x_{jv})}{x_{jv}}}{p_{jv}} \]  

(54)

Figure 5 presents a graphic representation of the PPP negotiating area. The private investor requires minimum private ownership $\theta \geq e$ to transfer know-how; the public agent requires minimum public ownership $1 - \theta \geq h$ to exercise internal control rights and waive costly regulation. Price cannot exceed monopoly price $p_m$ nor be below variable average cost $p_{pu}$. The negotiating area is, therefore, bounded by $\theta \in [e, 1 - h]$ and $p_{jv} \in (p_{pu}, p_m)$.

Table 1 presents the results of comparative statics of the four analyzed forms of organization of natural monopoly.

---

Cfr. standard Lerner index: $\frac{p - MC}{p}$, that is, the higher the price above the marginal cost, the greater the company’s pricing power (Lerner 1934).
Table 1: Comparison of economic efficiency of institutional forms of organization of natural monopoly.

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Public monopoly</strong></td>
<td><strong>Public monopoly</strong></td>
</tr>
<tr>
<td>( k &lt; (1 - \alpha) \left[ p_m - \frac{c(x^S)}{x^S} \right] )</td>
<td>( g &lt; (1 - \alpha) \left[ p_m - MC(x^S) \right] )</td>
</tr>
<tr>
<td><strong>Regulated monopoly</strong></td>
<td></td>
</tr>
<tr>
<td>( g &lt; (1 - \alpha) \left[ p_m - MC(x^S) \right] )</td>
<td>( g + (1 - \alpha) \left[ MC(x^S) - \frac{c(x^S)}{x^S} \right] &lt; k )</td>
</tr>
<tr>
<td><strong>Public–Private Partnership</strong></td>
<td></td>
</tr>
<tr>
<td>( p_{jv} &lt; p_m )</td>
<td>( (1 - \alpha)\theta \left[ p_{jv} - \frac{c(x^S)}{x^S} \right] &lt; k )</td>
</tr>
<tr>
<td></td>
<td>( (1 - \alpha) \left[ \theta \cdot p_{jv} - MC(x^S) + \frac{(1-\theta)c(x^S)}{x^S} \right] &lt; g )</td>
</tr>
</tbody>
</table>

*Note:* Results are presented as conditions for welfare superiority of row A compared to column B (\( A > B \)), for \( x^S, q^S, \theta \in [e, 1 - h] \) for PPP, and \( \alpha \in [0, 1] \).
Overall, welfare superiority of PPP requires meeting the following conditions:

\[(1 - \alpha)\theta \left[ p_{jv} - \frac{c(x^S)}{x^S} \right] < k \]  

and

\[(1 - \alpha) \left[ \theta \cdot p_{jv} - MC(x^S) + \frac{(1 - \theta) \cdot c(x^S)}{x^S} \right] < g \]  

Summing equations (55) and (56) yields the necessary condition for efficient PPP:

\[(1 - \alpha)\theta \left[ p_{jv} - \frac{c(x^S)}{x^S} \right] + (1 - \alpha)\theta \cdot p_{jv} - MC(x^S) + \frac{(1 - \theta) \cdot c(x^S)}{x^S} \right] < k + g \]

\[(1 - \alpha) \left[ 2\theta \cdot p_{jv} + (1 - 2\theta) \frac{c(x^S)}{x^S} - MC(x^S) \right] < k + g \]  

PPP price \(p_{jv}\) is, therefore, bounded by:

\[p_{jv} < \frac{k + g}{2\theta} - \frac{(1 - 2\theta) \cdot c(x^S)}{x^S} + MC(x^S) \]  

Inequality (58) is a necessary but not sufficient condition for overall PPP welfare superiority.\(^{30}\) The necessary and sufficient condition is:

\[(1 - \alpha)\theta \left[ p_{jv} - \frac{c(x^S)}{x^S} \right] < \min \left\{ k, g + (1 - \alpha) \left[ MC(x^S) - \frac{c(x^S)}{x^S} \right] \right\} \]  

Thus, maximum negotiable \(p_{jv}\) is bounded by:

\[p_{jv} < \min \left\{ \frac{k}{(1 - \alpha)\theta} + \frac{c(x^S)}{x^S}, \frac{g}{(1 - \alpha)\theta} + \frac{MC(x^S) - \frac{c(x^S)}{x^S}}{\theta} + \frac{c(x^S)}{x^S} \right\} \]  

Analyzing \(p_{jv}\) as a function of \(\theta\) in its efficient frontier given by condition (60), the maximum \(p_{jv}(\theta)\) is decreasing and convex in \(\theta\).\(^{31}\) Its downward slope depends, ceteris paribus, on \(\alpha\): The higher \(\alpha\) is, the steeper the slope will be. Minimum \(p_{jv}\) is bounded by first-best marginal cost.

\(^{30}\) In other words, it is true that if \(f_1(p_{jv}, \theta) < k \land f_2(p_{jv}, \theta) < g\), then \(f_1(p_{jv}, \theta) + f_2(p_{jv}, \theta) < k + g\); but it is false that if \(f_1(p_{jv}, \theta) + f_2(p_{jv}, \theta) < k + g\), then \(f_1(p_{jv}, \theta) \land f_2(p_{jv}, \theta) < g\).

\(^{31}\) Proof: For \(p_{jv} = \frac{k}{(1 - \alpha)\theta} + \frac{c(x^S)}{x^S}\) and \(\alpha, \theta \in (0, 1)\), then \(\frac{\partial p_{jv}}{\partial \theta} = -\frac{k}{(1 - \alpha)\theta^2} < 0\) and \(\frac{\partial^2 p_{jv}}{\partial \theta^2} = 2\frac{k}{(1 - \alpha)\theta^3} > 0\). For \(p_{jv} = \frac{MC(x^S) - \frac{c(x^S)}{x^S}}{\theta} + \frac{c(x^S)}{x^S}\) and \(\alpha, \theta \in (0, 1)\), then \(\frac{\partial p_{jv}}{\partial \theta} = -\frac{g}{(1 - \alpha)\theta^2} - \frac{MC(x^S) - \frac{c(x^S)}{x^S}}{\theta^2} < 0\) and \(\frac{\partial^2 p_{jv}}{\partial \theta^2} = \frac{g}{(1 - \alpha)\theta^3} - 2\frac{MC(x^S) - \frac{c(x^S)}{x^S}}{\theta^3} > 0\).
Figure 6 presents a graphic representation of the PPP negotiating area upper and lower bounded by alternative organization modes and their welfare outcomes.

**Figure 6:** Negotiating area for efficient public–private partnership. The private investor requires minimum private ownership $\theta \geq e$ to transfer know-how; the public agent requires minimum public ownership $1 - \theta \geq h$ to exercise internal control rights and waive costly regulation. Price $p_{jv}$ is lower bounded by the best alternative for the private investor (i.e., first-best marginal cost pricing) and upper bounded by the best alternative for the public agent, either public or regulated monopoly pricing including $k$ and $g$. This upper bound decreases in private ownership $\theta$ as part of private profit $\alpha$ does not constitute welfare.

When factoring in alternative organization modes and their related costs $g$ and $k$, the PR set becomes unattainable for the private investor. As the negotiation area is bounded by the efficient $p_{jv}$ frontier, the private investor aims to minimize the distance to PR set.\footnote{This optimization problem can be reduced to minimizing the Cartesian distance $\sqrt{(p_{m} - p_{jv})^2 + (\theta - 1 + h)^2}$ in relation to $p_{jv}$ and $\theta$. Minimum public ownership $h$ is not known ex ante, but the private investor can estimate it quite precisely during negotiations.}

PPP efficiency is subject to saving regulation cost $g$ when $\theta \geq 1 - h$ and unskillfulness cost $k$ when $\theta \leq e$. If $e \leq 1 - h$. Figure 7 depicts regulatory and lack of know-how costs as

\footnote{According to Vaillancourt-Roseau (2000), PPP might not trigger a decrease in regulatory costs.}
discrete functions of private ownership share in PPP.

Figure 7: Regulatory and lack of know-how costs as discrete functions of private ownership share in a public–private partnership. When private ownership $\theta$ is above threshold $e$, private know-how is transferred and cost drops by $k$. Analogously, when public ownership $1 - \theta$ is above threshold $h$, regulation cost drops by $g$. PPP is welfare superior for $\theta \in [e, 1 - h]$. Depicted levels of $g$ and $k$ are exemplary ($k$ might be greater than $g$).

Assuming differentiable and monotonic functions $g(\theta)$ and $k(\theta)$, so that $g'(\theta) > 0$ and $k'(\theta) < 0$, for $\theta \in [0, 1]$, the necessary and sufficient conditions for welfare superior PPP can be formulated as the minimization of $g + k$, such that for $\theta_{jv}$, that $g'(\theta_{jv}) + k'(\theta_{jv}) = 0$ and $g''(\theta_{jv}) + k''(\theta_{jv}) > 0$, where $\theta_{jv} \neq 0$ and $\theta_{jv} \neq 1$.

Paraphrasing Ronald Coase, the optimal size of a public enterprise is the ownership share at which marginal regulatory cost equals marginal cost due to the lack of know-how, where $k(1)$ can be interpreted as unit X-type inefficiency cost, and $g(0)$ as a unit cost of

\[^{34}\text{If } \forall \theta_{jv} \in (0, 1) g'(\theta_{jv}) + k'(\theta_{jv}) \neq 0 \text{ or } g'(\theta_{jv}) + k'(\theta_{jv}) = 0 \text{ and } g''(\theta_{jv}) + k''(\theta_{jv}) < 0, \text{ then the minimum (minima) is located on the set boundary.}
\[^{35}\text{Coase (1937) referred to the size of the company, which results from equating the company’s internal and external marginal costs.}

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Figure 8: Regulatory and lack of know-how costs as continuous functions of private ownership in PPP. Cost of lack of know-how $k$ decreases and regulation cost $g$ rises in private ownership $\theta$. PPP is welfare superior when regulation cost $g$ plus lack of know-how cost $k$ minimum is internal $\theta \in (0,1)$.

internal regulation, which is not necessarily equal to zero (Balakrishnan and Koza 1993).

5 Concluding Remarks

Public–private partnership is not a distinctive institutional organization mode, but a creative hybrid solution designed to internalize transaction costs (regulation and information asymmetry) and to harness private know-how.

Public–private monopoly is arguably closer to the first-best benchmark, albeit with a price higher than marginal cost. Managerial incentives and cost-saving know-how are highly subject to sufficient private ownership. Likewise, provided sufficient public control rights, information asymmetry vanishes and quality is set and audited internally; thus, regulation cost is minimized. With public and private ownership satisfying their minimum requirements,
the decision making and renegotiations are internalized. Although budget constraints are acknowledged, partial public ownership increases the acceptance for subsidies. The public opinion welcomes marrying welfare and efficiency, but is sensitive to corruption and favoritism that a close relationship between the sectors usually brings.

Table 2 presents a summary of public–private partnership compared to classic forms of organization of natural monopoly. Comparative statics, supported by marginal analysis, show that when public–private partnership enables the internalization of private know-how and saves regulation costs due to correspondingly sufficient private and public ownership, it is welfare superior to private, public, and regulated monopolies.

\footnote{When renegotiations are not internalized, at least renewal probability is increased. \textcite{Gautier and Yvrande-Billon (2013)} observe that when the incumbent is a mixed company, it is renewed in 95.5\% of the cases, whereas a private incumbent is renewed in 83.1\% of the cases.}
### Table 2: Comparison of classic forms of organization of natural monopoly and public–private partnership.

<table>
<thead>
<tr>
<th>Concept</th>
<th>Private monopoly</th>
<th>Public monopoly</th>
<th>Regulated monopoly</th>
<th>Public–private partnership</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ownership</td>
<td>100% private</td>
<td>100% public</td>
<td>100% private</td>
<td>Mixed public–private</td>
</tr>
<tr>
<td>Output</td>
<td>Low, profit maximizing</td>
<td>Welfare maximizing, but costly</td>
<td>Higher than private monopoly, depends on regulation</td>
<td>Arguably closer to first-best</td>
</tr>
<tr>
<td>Price</td>
<td>High</td>
<td>Equal to average cost</td>
<td>Equal to marginal cost</td>
<td>Higher than marginal cost and below monopolistic</td>
</tr>
<tr>
<td>Managerial incentives</td>
<td>High</td>
<td>Low: managers as administrative personnel</td>
<td>Possible X-inefficiency</td>
<td>High s.t. sufficient private ownership</td>
</tr>
<tr>
<td>Know-how</td>
<td>High</td>
<td>Low</td>
<td>High</td>
<td>High s.t. sufficient private ownership</td>
</tr>
<tr>
<td>Quality</td>
<td>Low</td>
<td>Set by public agent</td>
<td>Contracted <em>ex ante</em>, audited <em>ex post</em></td>
<td>Set and audited internally</td>
</tr>
<tr>
<td>Information asymmetry</td>
<td>Acknowledged</td>
<td>N.a.</td>
<td>Acknowledged</td>
<td>Internalized</td>
</tr>
<tr>
<td>Regulation cost</td>
<td>None</td>
<td>None</td>
<td>Depends on regulation complexity</td>
<td>Minimized s.t. sufficient public ownership</td>
</tr>
<tr>
<td>Risk of opportunism</td>
<td>Acknowledged</td>
<td>N.a.</td>
<td>Acknowledged</td>
<td>Limited</td>
</tr>
<tr>
<td>Decision making</td>
<td>Unilateral</td>
<td>Administrative</td>
<td>Consultative</td>
<td>Internal negotiations</td>
</tr>
<tr>
<td>Renegotiation</td>
<td>Opportunistic</td>
<td>N.a.</td>
<td>Possibly opportunistic</td>
<td>Internalized</td>
</tr>
<tr>
<td>Budget constraints</td>
<td>Acknowledged</td>
<td>“Deep pockets”: possible subsidies</td>
<td>Acknowledged</td>
<td>Acknowledged, but partial public ownership increases acceptance for subsidies</td>
</tr>
<tr>
<td>Public opinion</td>
<td>Opposed</td>
<td>Dissatisfied with low quality and bureaucracy</td>
<td>Demands regulation</td>
<td>Supports marrying welfare and efficiency; sensitive to corruption and favoritism</td>
</tr>
</tbody>
</table>
## Appendix A  Notation

<table>
<thead>
<tr>
<th>Variable</th>
<th>Formula</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>$ATC$</td>
<td>$TC(x, q)/x$</td>
<td>Average total cost</td>
</tr>
<tr>
<td>$AVC$</td>
<td>$c(x, q)/x$</td>
<td>Average variable cost</td>
</tr>
<tr>
<td>$c(x, q)$</td>
<td></td>
<td>Variable cost of producing $x$ at quality $q$</td>
</tr>
<tr>
<td>$CS$</td>
<td></td>
<td>Consumer surplus</td>
</tr>
<tr>
<td>$F(q)$</td>
<td></td>
<td>Fixed cost of natural monopoly producing at quality $q$</td>
</tr>
<tr>
<td>$e$</td>
<td></td>
<td>Ownership threshold above which the private investors transfer cost-saving know-how (i.e., $e \geq \theta$)</td>
</tr>
<tr>
<td>$g$</td>
<td></td>
<td>Marginal regulation cost</td>
</tr>
<tr>
<td>$h$</td>
<td></td>
<td>Ownership threshold above which the public agent foregoes costly external regulation (i.e., $h \geq 1 - \theta$)</td>
</tr>
<tr>
<td>$k$</td>
<td></td>
<td>Marginal operating cost increase due to lack of know-how</td>
</tr>
<tr>
<td>$MC$</td>
<td>$\partial TC(x, q)/\partial x$</td>
<td>Marginal cost</td>
</tr>
<tr>
<td>$p$</td>
<td></td>
<td>Variable part of a two-part tariff</td>
</tr>
<tr>
<td>$PS$</td>
<td></td>
<td>Producer surplus</td>
</tr>
<tr>
<td>$q$</td>
<td></td>
<td>Quality of the good produced by the public utility; quality is desirable ($\partial u/\partial q &gt; 0$) and costly ($\partial TC/\partial q &gt; 0$; $\partial^2 TC/\partial^2 q$)</td>
</tr>
<tr>
<td>$TC(x, q)$</td>
<td></td>
<td>Total cost of producing $x$ at quality $q$</td>
</tr>
<tr>
<td>$x$</td>
<td></td>
<td>Quantity output of the good produced by the natural monopoly</td>
</tr>
<tr>
<td>$Z$</td>
<td></td>
<td>Lagrangian</td>
</tr>
</tbody>
</table>

$\alpha$  Weight of profit in welfare  
$\lambda$ Lagrange multiplier  
$\pi_{jv}$ Public–private partnership profit  
$\pi_m$ Private monopoly profit  
$\pi_{pu}$ Public monopoly profit  
$\pi_{re}$ Regulated monopoly profit  
$\theta$ Private investor’s share in investment and profit  
$1 - \theta$ Public agent’s share in investment and profit

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>PPP</td>
<td>Public–Private Partnership</td>
</tr>
</tbody>
</table>
References


