

Ex Ante Investment, Ex Post Adaptation, and Asset Ownership*

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Abstract

This paper introduces ex post adaptation to unanticipated changes in trade circumstances into the well-known hold-up model developed in the literature on the property rights theory (PRT). We show that this simple extension may overturn the prominent result of PRT: it may be optimal to let the party who makes no investment own an asset. Specifically, we point out that assigning the asset to the party that makes an important investment may create a trade-off between ex ante investment and ex post adaptation. Our result is consistent with other theories of the firm, such as those involving transaction cost economics and multi-task incentive problems, and provides a formal explanation as to why some firms that are successful in existing technologies fail in adopting new technologies.

**Keywords:** Hold-up problems; ex post adaptation; firm boundaries; property rights

**JEL Classification:** D23

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1 Introduction

Property rights theory (hereafter, PRT), which was pioneered by Grossman and Hart (1986), has focused on ex ante under-investment problems called hold-up problems. In a world of incomplete contracts, ex post contract renegotiation is inevitable, and those who invest are forced to give up some portion of the benefit from their investments, which weakens their investment incentives and causes the under-investment problem. PRT asserts that this problem is reduced by allocating physical assets to those who make important investments. This follows because asset ownership brings them a larger portion of the benefit from their investments by improving their default payoffs, which makes their incentives to invest higher. PRT includes the first formal model of firm boundaries, which can deal with both the costs and the benefits of integration, and which clearly illustrates who should own physical assets. PRT has become so influential that a number of studies have applied a PRT framework or focused on hold-up problems (e.g., Chung [1991] and Aghion and Tirole [1997]). However, PRT misses the important issue on which the existing theory of firm boundaries (i.e., transaction cost economics [TCE]) mainly focuses: ex post problems such as haggling (i.e., costly bargaining over trade value) and maladaptation (i.e., failure in making efficient decisions ex post).

This study introduces ex post adaptation, which is one of TCE’s main topics, into the hold-up model, which has been developed in the PRT literature (e.g., Hart [1995]). The sequence of events is as follows. First, a buyer (B) and a seller (S) negotiate over who owns an asset. Second, S makes a non-contractible investment. Third, changes in trade circumstances occur with positive probability, in which case S decides whether to implement adaptation. Lastly, renegotiation occurs. It is clear that the only departure from the original PRT framework is the presence of the adaptation stage. However, we show that this simple extension may overturn
PRT’s prominent result: S’s ownership may create a trade-off between *ex ante* investment and *ex post* adaptation, and hence B’s ownership may be optimal even if B does not have any investment.

The logic behind the result is explained as follows. S’s ownership improves his payoff in the renegotiation, which makes S’s incentive for both *ex ante* investment and *ex post* adaptation higher. Nevertheless, higher *ex ante* investment degrades S’s incentive for *ex post* adaptation because S can receive a higher payoff from *ex ante* investment even without *ex post* adaptation. This implies that S’s ownership has two opposite effects on his incentive for *ex post* adaptation and that the optimal ownership structure depends on which activity (i.e., *ex ante* investment or *ex post* adaptation) S’s ownership encourages more strongly. A number of studies have pointed out that the result of PRT may be overturned under certain conditions or assumptions (e.g., Chiu [1998], De Menza and Lockwood [1998], and Rajan and Zingales [1998]). Our study is particularly related to Schmitz (2006) in the sense that both his study and ours incorporate *ex post* inefficiencies into the Grossman-Hart-Moore framework. Schmitz (2006) points out that introducing private information about a default payoff into the PRT framework leads to rent-seeking costs and *ex post* inefficiencies due to bargaining under asymmetric information, which become severe when the party who has an investment owns an asset. Our study, however, does not assume information asymmetry and focuses on maladaptation rather than haggling.

Our trade-off is similar to the one pointed out in the literature on TCE and *ex post* adaptation (e.g., Bajari and Tadelis [2001], Tadelis [2002], Tadelis and Williamson [2012]): the trade-off between high-powered incentives in markets and the adaptive advantage of internal organizations. Our trade-off implies that the incentive for *ex ante* investment can crowd out that for

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1 *Ex post* problems have recently received theoretical attention. For example, Matouschek (2004) analyzes the optimal ownership structure that minimizes *ex post* maladaptation (i.e., failure in efficient transactions) and Hart
ex post adaptation, which is related to multi-task incentive problems (e.g., Holmstrom and Milgrom [1991, 1994]). Thus, to preserve S’s incentive for adaptation, it may be optimal not to assign S the asset at the cost of his incentive for ex ante investment, which is what the equal compensation principle implies. Furthermore, our result provides a formal explanation as to why some firms that are successful in existing technologies (ex ante investment in our model) fail in the adoption of new technologies (ex post adaptation in our model). Our study may thus provide a formal justification for the innovator’s dilemma in Christensen (1997).

The rest of the paper is organized as follows. Section 2 presents the model and our main result. Section 3 contains concluding remarks.

2 The Model

Consider two risk-neutral trading parties, a buyer (B) and a seller (S), who are to trade one unit of a good. The production of the good requires a physical asset, which is owned by either party. At date 0, the parties negotiate over an ownership structure: non-integration (i.e., S owns the asset) or integration (i.e., B owns the asset). Let n denote the number of assets S owns; n = 1 (resp. n = 0) then represents non-integration (resp. integration).

At date 1, S makes a non-contractible relationship-specific investment $a \in \mathbb{R}_+$. The investment $a$ creates trade value $R(a)$ and entails cost $C(a)$. We assume that $R' > 0$, $R'' \leq 0$, $R(0) = 0$, $C' > 0$, $C'' \geq 0$, and $C(0) = 0$ hold (the case in which $R'' = C'' = 0$ is ruled out). Both $R(a)$ and $C(a)$ are non-contractible. Following PRT, $a$ is the investment in S’s human capital, without which production cannot be accomplished; hence, B cannot produce the good.

and Moore (2008) have developed a behavioral theory of the firm, in which ex ante contracts serve as parties’ reference points and those who obtain smaller payoffs than their reference points undertake shading.

2 We refer to B as “she” and S as “he” for the purpose of identification only.

3 As in Schmitz (2006), we define non-integration and integration with regard to who owns the physical asset.
without $S$'s human capital. For simplicity, we assume that $B$’s default payoff is 0 regardless of the ownership structure $n$.\textsuperscript{4} $S$, on the other hand, can utilize his human capital and receive default payoff $r_n(a)$ without $B$’s collaboration. We employ the following standard assumption with respect to the marginal effect of $a$ on the trade value and $S$’s default payoff:

$$R'(a) > r'_1(a) \geq r'_0(a) \geq 0 \quad \forall a.$$  

This implies that implementing the transaction is always efficient: $R(a) > r_n(a)$ for all $n$ and $a \in \mathbb{R}_{++}$.

At date 2, changes in trade circumstances occur with probability $\tau$ (e.g., a new skill or method of production is found or developed), in which case $S$ decides whether to implement adaptation (e.g., whether to adopt the new skill or method). It is worth mentioning that this adaptation stage is the only departure from the original PRT framework. $S$’s non-contractible choice of adaptation is binary (as in Bajari and Tadelis [2001]), and is denoted by $p = \{0, 1\}$. $p = 0$ (resp. $p = 1$) corresponds to no adaptation (resp. implementing adaptation). Adaptation (i.e., choosing $p = 1$) creates value $V$ (non-contractible), but requires $S$ to make an additional investment whose non-contractible cost is $I$. To focus on interesting cases, we assume that the adaptation is efficient:

$$V - I \geq R(a^*),$$

where

$$a^* = \arg\max_{a} R(a) - C(a).$$

We employ an important assumption here: once the adaptation is implemented (i.e., $p = 1$ is chosen), the \textit{ex ante} investment level that $S$ has chosen at date 1 does not matter. More specifically, $R(a)$ and $r_n(a)$ are lost after choosing $p = 1$. For example, the skill or method of

\textsuperscript{4} Assuming $B$ has a positive default payoff does not affect our result.
production found or developed at date 2 is so different from the one in which $S$ has invested at date 1 that the \textit{ex ante} investment in the old method has no effect on the new one. Let $w_n$ denote $S$’s default payoff after adaptation under ownership structure $n$ ($w_1 \geq w_0$).

At date 3, renegotiation takes place, and the production is then accomplished. Following PRT, we assume that the parties agree to the Nash bargaining solution. The timing of the game is summarized in Figure 1.  

- INSERT FIGURE 1 HERE -

2.1 \textit{Ex Ante} Investment

We begin by examining $S$’s optimal \textit{ex ante} investment under each ownership structure $n$. Since our model is a simple extension of the PRT framework, its well-known result continues to hold in our model.

Given the decision on \textit{ex post} adaptation $p_n$, $S$’s problem is given as follows:

$$\max_a \frac{R(a) + r_n(a)}{2} - C(a) + \tau p_n \left( \frac{V + w_n}{2} - I - \frac{R(a) + r_n(a)}{2} \right).$$

Note that the parties agree to the Nash bargaining solution and $R(a)$ and $r_n(a)$ are lost once the adaptation is implemented (i.e., $p_n = 1$ is chosen). The first-order condition then becomes

$$C'(a_n) = \begin{cases} \frac{(1-\tau)(R'(a_n) + r_n'(a_n))}{2} & \text{if } p_n = 1, \\ \frac{R'(a_n) + r_n'(a_n)}{2} & \text{if } p_n = 0. \end{cases}$$

For convenience, we refer to $a_n$ given $p_n = 1$ (resp. $p_n = 0$) as $a^1_n$ (resp. $a^0_n$).

The following lemma is then evident:

\begin{lemma} \textsc{(Hart and Moore [1990] Proposition 2)} For given $p = \{0, 1\}$, $S$’s ownership... \end{lemma}

\footnote{All figures are located at the end of the main text. Figure 1 is based on Schmitz's (2006) Figure 1.}
improves his incentive for ex ante investment:

\[ a_0^p \leq a_1^p < a^*, \]

where

\[ a^* = \arg \max_a R(a) - C(a). \]

This implies that if only S makes an investment, he should own the asset, which is PRT’s prominent implication. Nevertheless, our primary focus is on how the incentive for the ex ante investment affects the ex post adaptive decision. The next subsection shows that S’s ownership may make S’s incentive for ex post efficient adaptation weaker.

2.2 Ex Post Adaptation

This subsection examines which ownership structure facilitates efficient adaptation given S’s incentive for ex ante investment under each ownership structure \( n \), which was derived in the last subsection.

Given that the changes in trade circumstances occur and the ex ante investment level \( a_n \) is chosen at date 1, S’s optimal adaptive decision under ownership structure \( n \) solves

\[
\max_{p_n} \left( \frac{V + w_n}{2} - I - \frac{R(a_n) + r_n(a_n)}{2} \right).
\]

We thus find that \( p_n = 1 \) is actually chosen if the following condition holds:\(^6\)

\[
\frac{V + w_n}{2} - I \geq \frac{R(a_n^1) + r_n(a_n^1)}{2}.
\]

\(^6\) When this condition fails, it is obvious that the following condition also fails because \( a_n^1 < a_n^0 \):

\[
\frac{V + w_n}{2} - I \geq \frac{R(a_n^0) + r_n(a_n^0)}{2}.
\]
This condition implies that $S$’s ownership encourages adaptation if

$$\{R(a_1^1) + r_1(a_1^1)\} - \{R(a_0^1) + r_0(a_0^1)\} \leq w_1 - w_0.$$  \hspace{1cm} (1)

The LHS (resp. RHS) of condition (1) represents the effect of $S$’s ownership on his incentive for \textit{ex ante} investment (resp. \textit{ex post} adaptation). We then have the following proposition:

\textbf{Proposition 1} \hspace{5pt} The ownership structure that is most likely to realize efficient adaptation is summarized as follows:

$$\begin{cases} n = 1 \ (\text{non-integration}) & \text{if condition (1) holds,} \\ n = 0 \ (\text{integration}) & \text{if condition (1) fails.} \end{cases}$$

\textbf{2.3 A Trade-Off between \textit{Ex Ante} Investment and \textit{Ex Post} Adaptation}

Proposition 1 implies that $S$’s ownership has two opposite effects on his incentive for \textit{ex post} adaptation. $S$’s ownership improves his default payoff (i.e., $r_1(a) \geq r_0(a)$ for all $a$ and $w_1 \geq w_0$), and hence, it encourages both \textit{ex ante} and \textit{ex post} investments. Nevertheless, encouraging \textit{ex ante} investment makes $S$’s incentive for \textit{ex post} adaptation weaker. This follows because the higher the level of $a$ $S$ chooses, the more unwilling he is to give up the benefit from $a$, which is lost if he implements the adaptation. The ownership structure that facilitates efficient adaptation thus depends on which investment (\textit{ex ante} or \textit{ex post}) $S$’s ownership encourages more strongly.

Our result points out that if \textit{ex ante} investment is more strongly encouraged than \textit{ex post} adaptation by $S$’s ownership (i.e., if condition (1) fails), the well-known result of PRT may fail: it may be optimal to allocate the asset to the party that does not have an investment, namely, $B$.\footnote{It is worth mentioning that even if condition (1) holds, PRT’s implication may be overturned: $S$’s ownership still encourages \textit{ex ante} investment (see Lemma 1) and a larger trade value than integration. If some disturbance occurs, on the other}
hand, integration is more likely to realize efficient adaptation than non-integration. This trade-off is consistent with the assertion of TCE: market transactions feature high-powered incentives and internal organizations enjoy adaptive advantages.

Our result is also related to the multi-task incentive problem analyzed in Holmstrom and Milgrom (1991, 1994). If condition (1) fails, S’s ownership makes him pay too much attention to \textit{ex ante} investment and crowds out his incentive for efficient adaptation. It may then be optimal not to assign him any asset to preserve his incentive to adapt at the cost of his incentive for \textit{ex ante} investment. This is consistent with what the equal compensation principle implies.

Furthermore, our result may provide a formal justification for the innovator’s dilemma in Christensen (1997). We admit that the skill or method that may become available at date 2 in our model is not completely consistent with the "disruptive technology" in Christensen’s (1997) sense. Nevertheless, our result provides a formal explanation as to why some firms that are successful in existing technologies may not necessarily be successful in adopting new technologies.

It is also worth noting that in contrast to PRT, joint ownership may be optimal in our model. This follows because our main result is that it may be optimal not to allocate the physical asset to S, and \( n = 0 \) can be interpreted as joint ownership as well as integration.

We conclude this section by presenting an illustration. To illustrate the result clearly, we does not necessarily improve his incentive for \textit{ex ante} investment. Suppose the following conditions hold:

\[
\frac{V + w_1}{2} - I \geq \frac{R(a_{11}^1) + r_1(a_{11}^1)}{2} \quad \text{and} \quad \frac{V + w_0}{2} - I \leq \frac{R(a_{00}^1) + r_n(a_{00}^1)}{2}.
\]

These imply that only S’s ownership can achieve adaptation. We thus find that S’s choice of \textit{ex ante} investment under each ownership structure satisfies

\[
C'(a_{11}^1) = \frac{(1 - \tau)(R(a_{11}^1) + r_1'(a_{11}^1))}{2} \quad \text{and} \quad C'(a_{00}^1) = \frac{R'(a_{00}^1) + r_0'(a_{00}^1)}{2}.
\]

These may result in \( a_{11}^1 < a_{00}^1 \): S’s ownership weakens his incentive for \textit{ex ante} investment.
focus on the situation in which $R(a) = a$, $C(a) = a^2/2$, $r_1(a) = a/2$, $r_0(a) = a/4$, $V = 3/2$, $I = 1/2$, $w_1 = 1/8$, and $w_0 = 0$. Let $W_n$ denote the expected total surplus under ownership structure $n$. Figure 2 depicts the effect of $\tau$ (i.e., the probability with which changes in trade circumstances occur at date 2) on the expected total surplus.

- INSERT FIGURE 2 HERE -

Figure 2 implies that while $B$’s ownership ($W_0$) is optimal when $\tau$ is intermediate (i.e., $7/15 \leq \tau < 2/3$ in Figure 2), $S$’s ownership ($W_1$) maximizes the expected total surplus when $\tau$ is sufficiently high (i.e., $2/3 \leq \tau < 1$ in Figure 2) or low (i.e., $0 < \tau < 7/15$ in Figure 2). This clearly illustrates our main result: given the choice of ex post adaptation $p_0 = p_1$, $S$’s ownership enjoys higher ex ante investment and dominates $B$’s ownership, but the efficient adaptation is more likely to be implemented under $B$’s ownership than under $S$’s ownership. Figure 2 also implies that higher uncertainty makes integration (i.e., $B$’s ownership) more likely to be chosen, which is consistent with one of the main assertions of TCE. If there is a low level of uncertainty about whether changes in trade circumstances occur at date 2 (i.e., $\tau$ is sufficiently high or low), the precision of each party’s expectation about whether adaptation is required at date 2 is high, in which case maladaptation (i.e., the failure of ex post efficient adaptation) is less likely to be problematic, and $S$’s ownership, which features a higher ex ante incentive than $B$’s ownership, should be chosen. If $\tau$ is intermediate, on the other hand, it is quite uncertain whether changes in trade circumstances occur at date 2, and maladaptation is more likely to occur. In such a case, $B$’s ownership, which has an adaptive advantage over $S$’s ownership, becomes optimal.

This is merely an illustration; there can, of course, be other cases (e.g., $S$’s ownership cannot realize ex post adaptation for all $\tau$ for some parameter values). We leave complete welfare analysis for future research.
3 Conclusion

We introduce *ex post* adaptation to changes in trade circumstances into a well-known PRT framework (i.e., the hold-up model) and find that this simple extension may overturn its prominent result: it may be optimal to let the party that makes no investment own the asset. More specifically, we show that assigning the asset to the party that makes important investments may create a trade-off between *ex ante* investment and *ex post* adaptation. We derive the implications for make-or-buy decisions, which are consistent with the assertion of existing theories, such as TCE and multi-task problems. Furthermore, this model provides a formal explanation as to why firms that are successful in existing technologies sometimes fail in the adoption of new technology; this may provide a formal justification for the innovator’s dilemma in Christensen (1997).

There are some topics left unaddressed (e.g., introducing B’s investment, including investment in physical assets, and employing a more sophisticated bargaining model to depict haggling), and one of the important extensions is the continuous choice of *ex post* adaptive effort. For example, we assume that adaptive choice $p \in \mathbb{R}_+$ at date 2 creates trade value $V(p)$ ($V' > 0$, $V'' < 0$, and $V(0) = 0$) and costs $D(p)$ ($D' > 0$, $D'' \geq 0$, and $D(0) = 0$). We also assume that $S$’s default payoff after adaptation (i.e., $p > 0$) under ownership structure $n$ is given by $w_n(p)$ ($w'_1 \geq w'_0 \geq 0$ and $w''_n < 0$). $S$’s problem then changes as follows:

$$\max_{a, p \in \mathbb{R}_+} \frac{R(a) + r_n(a)}{2} - C(a) + \tau \left( \frac{V(p) + w_n(p)}{2} - D(p) - \alpha \frac{R(a) + r_n(a)}{2} \right),$$

where

$$\alpha = \begin{cases} 1 & \text{if } p > 0, \\ 0 & \text{if } p = 0. \end{cases}$$

We then easily check that given $p_1 > 0$ and $p_0 > 0$, $p_1 \geq p_0$ holds. We thus need to carefully
reexamine the trade-off between investment and adaptive incentives, but we can check that there exist some specifications under which our trade-off continues to hold.

Consider, for example, the following specification: \( R(a) = Ra^{1/2}, \ C(a) = a, \ r_n(a) = r_n a^{1/2}, \ V(p) = Vp^{1/2}, \ D(p) = kp, \) and \( w_n(p) = r_np^{1/2} \) (\( V > R > r_1 \geq r_0 \geq 0 \) and \( k > 0 \)). We easily find that \( S \)'s optimal \textit{ex ante} investment under ownership structure \( n \) is given by

\[
a_n = \begin{cases} 
\frac{(1-\tau)(R+r_n)^2}{16} & \text{if } p > 0, \\
\frac{(R+r_n)^2}{16} & \text{if } p = 0.
\end{cases}
\]

Furthermore, given that \textit{ex post} adaptation is implemented (i.e., \( p_n > 0 \)), \( S \)'s optimal \textit{ex post} adaptive effort under ownership structure \( n \) is

\[
p_n = \frac{(V+r_n)^2}{16k^2}.
\]

It is obvious that \( a_1 \geq a_0 \) and \( p_1 \geq p_0 \) hold. Given \( a_n \) and \( p_n \), \( S \) actually chooses \( p_n \) if the following condition holds:

\[
\frac{V(p_n) + r_n(p_n)}{2} - D(p_n) \geq \frac{R(a_n) + r_n(a_n)}{2} \iff k \leq \frac{(V + r_n)^2}{2(1 - \tau)(R + r_n)^2} \equiv k_n.
\]

We can check that

\[
k_1 \leq k_0.
\]

This implies that for given \( k \), \( B \)'s ownership is more likely to implement \textit{ex post} adaptation than \( S \)'s ownership. We thus have the trade-off between the incentive for the \textit{ex ante} and \textit{ex post} investments and the implementation of \textit{ex post} adaptation.

While continuous adaptive choice is an important extension, we believe that discrete adaptive choice, which is employed in our model, is most suitable to illustrate the main result clearly. We thus leave the further analysis for future research.
$B$ and $S$ choose $S$ invests a disturbance may occur renegotiation

$n \in \{0,1\}$ and $S$ chooses $p \in \{0,1\}$

Figure 1: The Sequence of Events
Figure 2: The Effect of $\tau$ on the Expected Total Surplus
References


