Institutions for spatially managing the harvest of wild forest products: 
Implications for welfare and ecology

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Abstract
Wild forests products benefit many rural communities in developing countries. Often these 
forests also contain ecosystem services that are globally valuable, such as biodiversity and 
carbon, which may not be as important to local communities. This paper develops a spatial 
model for harvesting non-timber forest products (NTFPs), like mushrooms and medicinal plants. 
It asks: how much can spatial management of NTFP labor simultaneously improve welfare and 
ecological outcomes? I develop a theoretical model that accounts for the shape of the forest, the 
size of the harvest community, and incorporates real-world constraints. The results first show 
that even under open access conditions (harvesters compete uncooperatively with one another), if 
the forest is large relative to the size of the community then harvesters still profit. This profit 
may reduce incentives for collective action, which could earn the community even more. Second, 
managing a forest to maximize NTFP value does not always protect other regionally or globally 
important ecosystem services like biodiversity or water storage capacity. Using a unique dataset 
of mushroom harvests in Yunnan, China, I test for characteristics associated with harvester’s 
foraging distance. The results support the theoretical model’s spatial foundation, suggesting 
harvesters travel farther to avoid competition. More experienced and less-wealthy households 
tend to rely on more distant harvests. There are livelihood benefits to cooperation but potential 
ecological costs in some contexts. Regardless, limiting access likely disproportionately affects 
the most vulnerable.
1. Introduction

Many populations in developing countries rely on harvests of wild products from local forests for livelihood support (FAO, 2008; World Bank, 2008), a local benefit. But these forests with valuable non-timber forest products (NTFPs) also often have high levels of biodiversity and provide other important regional and global ecosystem goods and services (e.g. Peters et al., 1989). Villagers’ travel and extraction activities during NTFP harvests can negatively impact these regional and global benefits: forest can be degraded (Shankar et al., 1998; Ashton et al., 2001; Kumar and Shahabuddin, 2005), species’ evolutionary trajectories can change (Ashley et al., 2003; Law and Salick, 2005; Mooney and McGraw, 2007) or altered ecosystem dynamics can decrease biodiversity (Moegenburg and Levey, 2002). Local NTFP management is widely promoted as a way to simultaneously improve both local livelihoods and promote sustainable use of resources (Bhattacharya and Hayat, 2004; Ticktin, 2004; Cooke et al., 2008; Laird et al., 2010; Guariguata et al., 2011). But what are the ecological and welfare implications of protecting ecosystems from people versus integrating local populations within ecosystems (Naughton-Treves et al., 2005; Adams and Hutton, 2007; Coad et al., 2008)?

As global forest rights devolve from public ownership toward more local control (Sunderlin et al., 2008), implicitly we require greater private provision of regional and global ecosystem services from forests. Local ownership is sometimes thought to induce stewardship and responsible use (e.g., Somanathan et al., 2009; Hvistendahl, 2012), but collective ownership does not imply collective action, which is often advocated for to ensure forest resources are sustained (Ostrom, 1990). Further, secure local forest tenure does not imply healthy or productive ecosystems (Brockerhoff et al., 2008; Robinson et al., 2013a). Constructing policies
that conserve forest ecosystems and also preserve local livelihoods requires a better understanding of resource extraction patterns, welfare outcomes and pressure on forests.

This paper first constructs a spatially explicit model of forest product extraction, generalizing some previous efforts and allowing for various configurations of forest geometry. I provide solutions for cooperative and non-cooperative harvesting and make land and labor constraints explicit. These aspects of a model are necessary to understand the effects of management on welfare and the spatial distribution of labor, which I use as a proxy for impact on ecosystems. I present a numerical simulation of the model using a renewable resource framework, exploring the effect of the size of the community, and the size of the forest. If the forest is large relative to the size of the community then harvesters can still earn profits, irrespective of management. This profit may reduce incentives for collective action, which would earn the community even more. Managing a forest to maximize NTFP value does not always protect other regionally or globally important ecosystem services like biodiversity or water storage capacity, depending on whether a forest’s ecosystem goods and services are more sensitive to the presence of activity (e.g., some types of fragile biodiversity, the presence of birds or large mammals, etc.) versus the intensity of activity (forest degradation, forest understory function and associated biodiversity, water storage capacity, etc.). Finally, I present empirical evidence from wild mushroom harvests in Yunnan, China, showing how spatial forest behavior is linked to individual and household characteristics and that the poor and vulnerable tend to travel farther to look for NTFPs.

In contrast to research on the ecological impacts of NTFP harvests, few studies explore the spatial aspects of populations’ decisions around harvesting wild forest products and potential impacts on ecosystem function. A relatively small literature on spatial economic models of
NTFP extraction exists (see Albers and Robinson, 2012 for a review). Related to the goals of this paper, Robinson et al (2002) explored labor and market interactions with a model that extracts NTFPs continually across space. Later, Robinson et al (2008) modeled the temporal dynamics and management of resource harvests, and welfare impacts and management institutions are explored for point-source harvesting by López-Feldman and Wilen (2008). In spirit, the effort here is somewhat similar to the last paper, but our motivation, modeling approach, and analysis are quite different. The focus here is to investigate management’s impact on the interaction between ecosystems and the community.

I begin the next section by setting up the harvest problem within a single forest “patch,” which is directly analogous to the non-spatial resource harvest model. The problem is expanded to include harvests over many patches, and is novel in how it deals with travel labor versus harvest labor and in making a patch’s carrying capacity of the resource an explicit function of space. In Section 3 I describe the numerical simulation and its implications. Datasets to test these models are limited, but I present some supporting empirical results from a unique dataset in Section 4. Section 5 summarizes the results. I conclude with implications for rural development policy and conservation of unique forest habitats.

2. Harvesting in a spatial forest

2.1. A single-patch forest: the standard bioeconomic framework

Consider a stylized community bordered by a forest with valuable non-timber forest products that will be consumed or sold at a market back in the community. Travel orthogonal to the village is costly, but travel tangent to a village is pure harvest labor since no additional costly travel labor is needed to return to the village.
Harvesting in a single patch of forest is denoted by $H_s(x_s, l_s)$ where $x_s$ is the biomass available and $l_s$ is total labor allocated in patch $s$. Total labor is an aggregate of all $n$ harvesters’ individual labor $l_s = \sum^n_{i=1} l_{i,s}$ and I assume a Schaefer-type (1954) production function, as is common in natural resource systems: $H_s(x_s, l_s) = q x_s l_s$. The parameter $q$ is a “catchability” coefficient that expresses how efficiently one can harvest. This could be a function of the density of resources in the forest or the time one needs to physically pick (“catch”) the resource in question. Resources grow according to a logistic growth function (Bluffstone, 1995; Gunatileke and Chakravorty, 2003) such that $x_{s,t+1} = x_{s,t} r \left(1 - \frac{x_{s,t}}{K_s}\right) - H_{s,t}$ where $r$ is the per-period growth rate of aggregate biomass and $K_s$ is the carrying capacity within patch $s$ (the stock of resources that would be present in a patch in the absence of any harvest).

To focus on the spatial aspects of the problem, we look at the system in equilibrium where harvests equal the resource’s growth rate $\tilde{H}_s = x_s r \left(1 - \frac{x_s}{K_s}\right)$, where ‘~’ denotes equilibrium. This implies resource stock has a deterministic relationship with labor $x_s = K_s \left(1 - q l_s / r\right)$ which results in the textbook inverted U-shaped relationship between harvests and equilibrium stock levels. Evaluating this equilibrium stock level in the harvest equation, equilibrium harvests as a function of labor are

$$\tilde{H}_s = q K_s l_s \left(1 - q l_s / r\right)$$

This single-patch framework is equivalent to the standard bioeconomic model (e.g. Clark, 1990; Conrad, 1999).
2.2. A spatially explicit forest

For a resource distributed evenly throughout space, denote its constant per-unit-area carrying capacity as $\bar{\rho}$. In a forest that extends radially out from a village, the total carrying capacity of a forest area with a radius $s$ from a village center is $\bar{\rho}\pi s^2$. Within a ring defined by the distance between $s$ and $s - \Delta$, where $\Delta$ is an interval width, resource availability is the per-unit-area density times the area of a ring of the interval width:

$$\bar{\rho} \left( \pi s^2 - \pi (s - \Delta)^2 \right) = \bar{\rho} \pi \left( 2s\Delta - \Delta^2 \right).$$

Thus, for a small $\Delta$, we can approximate the carrying capacity $K$ at distance $s$ as $K_s = \bar{\rho} \pi \left( 2s\Delta - \Delta^2 \right)$.\(^1\) Without loss of generality allow $s$ to increase at the unit interval and, since $\pi$ is a constant, let it be absorbed into our measure of carrying capacity so that $\rho = \bar{\rho} \pi$. Carrying capacity now simplifies to $K_s = \rho(2s - 1)$. Substituting $K_s$ into equation (1), harvests in steady state at radius $s$ are

$$\hat{H}_s = q\rho(2s - 1) \left( s - \frac{r}{l_s} l_s^2 \right). \tag{2}$$

Uniquely, this framework models a forest of any arc-width around a village: equivalently a wedge extending into space from a village center or a circular-forest surrounding a village, since both are proportional. A linear forest is assumed in all previous modeling efforts (although Albers and Robinson, 2011, use "rays" to model a quasi two dimensional forest), assuming carrying capacity is independent of space so that for all $s$, $K_s = \bar{\rho}$. Analytic solutions for the linear forest case are presented in the Appendix for comparison with previous work, but in the

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\(^1\) This relationship could be modeled equivalently as a continuous function with the same result. The infinitesimal resources available at distance $s$ are $x_s = f(s) = \bar{\rho} 2\pi s$ or, over an interval, $x_s = f(s) = \int_{r-\Delta}^{r} \bar{\rho} 2\pi s ds$. If we evaluate the integral and use $\rho = \bar{\rho} \pi$, the relationship becomes $x_s = f(s) = 2\rho \int_{r-\Delta}^{r} s ds = 2\rho \frac{s^2}{2}\bigg|_{r-\Delta}^{r} = 2\rho \left( \frac{r^2}{2} - \frac{(r-\Delta)^2}{2} \right) = \rho \left( r^2 - (r - \Delta)^2 \right) = \rho \left( 2r\Delta - \Delta^2 \right)$, and we have recovered the discrete space form.
text below we focus on solutions for the more novel radial case. In the simulation we test for differences in outcomes when one assumes a radial versus linear forest.

In addition to casting the carrying capacity as a function of distance, space enters the model in a second fundamental way. I assume travel from patch \( s \) to \( s + 1 \) (orthogonal movement away from the village) requires a fixed amount of round-trip labor \( \bar{t}_i \) per individual\(^2\) such that

\[
\bar{t} = \sum_{i}^{n} \bar{t}_i .
\]

Taxing only orthogonal travel \( (\bar{t}) \) is a conceptually simple way to separate costly travel labor from productive harvest labor. Also note this definition of carrying capacity \( (K_s) \) increases with \( s \), effectively increasing the area over which one can harvest.

Harvests receive a price of \( p \) per unit biomass and come at opportunity cost of time \( w \) so that the village’s total profits from the system are

\[
\Pi = \sum_{s=1}^{S} \Pi_s = \sum_{s=1}^{S} \left[ p \hat{H}_s - w (l_s + \bar{t}) \right]
\]  

(3)

where \( S \leq d \), so that the maximum distance traveled \( S \) is no greater than the extent of the forest boundary \( d \). We assume villages have exclusionary rights over the surrounding forest and thus an implicit labor constraint \( \sum_{s=1}^{S} (l_s + \bar{t}) \leq L \) where \( L \) is a village’s total available labor. The model flexibly deals with forest geometry (via carrying capacity), travel costs and necessary real-world constraints, providing a generalized spatial bioeconomic framework appropriate for a range of terrestrial harvest problems.

[Figure 1: Linear versus Radial Forests]

\(^2\) Alternatively, we could model this as a continuous cost as one moves orthogonally from the village as done by Robinson et al. (2002). However, assuming a fixed cost of travel between patches makes exposition of the model clearer and directly applies to the numerical simulation below.
Of interest is the spatial distribution of labor and profits in cooperative versus non-cooperative harvest settings. To model institutional arrangements, I employ two spatial extensions of standard economic first principles formulated in the following two propositions.

**Proposition 1.** Cooperative spatial management implies \( \frac{\partial \Pi}{\partial \tilde{l}_s} = \frac{\partial \Pi}{\partial \tilde{l}_{s+1}} \quad \forall s \in \{1,...,S\} \).

The marginal utility of harvests is equal across all harvested patches. Note that travel labor drops out of the cooperative equilibrium condition since it is a sunk cost.

**Proposition 2.** Non-cooperative spatial management implies \( \frac{n_s}{l_s} = \frac{n_{s+1}}{l_{s+1}} \quad \forall s \in \{1,...,S\} \).

The average utility of harvests is equal across all harvested patches. Comparing average utility between this patch and one more distant must take into account the cost of traveling to the next patch, so the average utility in patch \( s \) must be equal to the average utility from traveling to and harvesting in patch \( s + 1 \).

2.3. **Cooperative harvests**

Cooperative forest harvesters aim to maximize total social profits from harvests. Following Proposition 1, the village’s maximization problem is

\[
\max_{l_s} \quad \Pi = \sum_{s=1}^{S} \left[ pH_s - w(l_s + \bar{t}) \right] \\
\text{subject to:} \quad L - \sum_{s=1}^{S} (l_s + \bar{t}) \geq 0 \quad d - S \geq 0 .
\]

Profits are maximized subject to the labor available within the village and the area of the forest over which they have use rights. In practice, labor and distance constraints cannot

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3 The cooperative rule implies profit maximization while the non-cooperative rule implies rent-dissipation. These are the extreme institutional conditions a community may face, and are the common institutional settings analyzed in other spatial resource settings (Sanchirico and Wilen, 1999; Robinson et al., 2008). Game-theoretic models of non-cooperative behavior bridge these two extremes (Cheung, 1970; Dasgupta and Heal 1979). However, spatial extensions would be analytically complex, so we focus on these two bounding institutional cases.
simultaneously be slack. A village either “runs out” of harvest labor or they have more labor than is needed given the forest size (both constraints can be slack only if \( d = \infty \) and \( L = \infty \)). Thus there are three cases to examine (see Appendix 9.3 for proofs):

**Proposition C1.** *The distance constraint binds:* If there is more labor than needed for optimal harvests (\( L < \sum_{s=1}^{S^*} l^*_s + d \bar{\ell} \)) then harvesters forage to the forest edge (\( S^* = d \)). Optimal labor allocation is \( l^*_s = \frac{\bar{\ell}}{2q} \left(1 - \frac{w}{pq^{(2s-1)}} \right) \forall s \in \{1,\ldots,d\} \).

**Remark C1:** More labor is allocated further out in the forest to account for increases in the harvest area, but these increases are less than proportional to the increase in land area (since the change in labor with distance \( s \) is positive but decreasing: \( \frac{\partial l^*_s}{\partial s} = \frac{-rw}{pq^{s-1}(2s+1)} \)). In a linear forest of equal-area patches (\( K_s = \bar{D} \)), the first order conditions are independent of patch \( s \), so harvesting evenly from all areas \( l^*_s = \bar{l}^* \forall s \) maximizing the forests’ value (see Appendix).

**Proposition C2.** *The labor constraint binds:* If labor is constrained (\( L = \sum_{s=1}^{S^*} l^*_s + S^* \bar{\ell} \)) or “used up” before reaching the village’s forest boundary (\( S^* < d \)) then the optimal solution solves for optimal labor \( (l^*_s) \), stopping distance \( (S^*) \) and the shadow price of labor \( (\lambda^*) \) in the system of equations: \( l^*_s = \frac{\bar{\ell}}{2q} \left(1 - \frac{w}{pq^{(2s-1)}} \right) \forall s \in \{1,\ldots,S^*\}; \)

\[
\lambda^* = \frac{pq\left(2S^*-1\right)\left(1-\frac{2\bar{\ell}}{l^*_s+\bar{\ell}}\right)}{l^*_s+\bar{\ell}} - w; \text{ and } L = \sum_{s=1}^{S^*} l^*_s + d \bar{\ell} .
\]
Remark C2: If all labor is optimally allocated without reaching the forest boundary, then there is a positive value on the shadow price of labor since an additional unit of labor could secure more profits.

**Proposition C3.** The labor and distance constraints bind: In the special case where all a village’s labor is used ($L = \sum_{s=1}^{S^*} l_s^* + d \bar{\ell}$) and the forest edge is reached ($S^* = d$), optimal conditions are $S^* = d$, $\lambda^* = pq\rho \left(1 - 2 \frac{s}{d} l_s^*\right) - w \quad \forall s \in \{1, \ldots, d\}$ (the shadow price of labor), $\gamma^* = pq\rho \left(2d - 1\right) l_d^* \left(1 - \frac{s}{d} l_d^*\right) - \left(w + \lambda^*\right) \left(l_d^* + d \bar{\ell}\right)$ (the shadow price of forest area) and $L = \sum_{s=1}^{d} l_s^* + d \bar{\ell}$.

Remark C3: As opposed to Prop C2, we know the number of equations in the system ($d + 3$), but analytic solutions still require a defined forest boundary ($d$). Appendix 9.3 gives an example solution set for $d = 3$. In the linear forest case ($K_s = \bar{\rho}$), optimal conditions are simply $S^* = d$ and $l_s^* = \frac{d}{d} l_d^*$ since optimal labor is independent of spatial distance $s$.

Figure 2 presents a graphical illustration of cooperative harvests in a 1- and 3-patch forest (for a linear forest where $K_s = \bar{\rho}$ for simplicity). The 1-patch case represents the traditional bioeconomic harvest model. The 3-patch case shows this model “stacked” as one reaches new patches in space, and that profit maximizing labor allocation is equal across patches. Profits are maximized when a community exerts labor $l_1$ such that the marginal cost of harvesting ($w$) is equal to the marginal benefit (line tangent to the growth curve). The bold dashed line shows within-patch profits. The 3-patch forest extends the 1-patch model to make space explicit. Now the growth curves are “stacked”; harvesting begins after leaving the previous patch and
expending a fixed unit of labor \( \bar{\ell} \) to get there. We denote the opportunity cost as \( w' \) since it may include the shadow values for labor when the constraint binds. Also note that in this linear forest case \( l_1 = l_2 = l_3 \).

[Figure 2: Cooperative Harvests in a Linear Forest]

2.4. Non-cooperative harvests

Proposition 2 suggests non-cooperative harvesters will continue foraging further into the forest as long as the average benefits of harvesting in and traveling to the next patch \( s + 1 \) are greater than the average benefits of continuing to harvest in the current patch \( s \). In equilibrium, harvesters are indifferent. The equivalence condition that denotes this non-cooperative spatial equilibrium is (see Appendix 9.4):

\[
\frac{\Pi_s}{l_s} = \frac{\Pi_{s+1}}{l_{s+1} + \bar{\ell}} \Rightarrow \frac{l_s - \frac{q}{\ell} l_s^2}{l_s} = \left( \frac{2s + 1}{2s - 1} \right) \left( \frac{l_{s+1} - \frac{q}{\ell} l_{s+1}^2}{l_{s+1} + \bar{\ell}} \right)
\]

(5)

On the right hand side of condition (5) there is a “distance term”: \((2s + 1)/(2s - 1)\). Equation 5 without this term is the condition for a linear forest (see Appendix 9.2) and \( l_s^{OA} > l_{s+1}^{OA} \ \forall s \). Per-patch open-access labor decreases with distance from the village center, since the presence of \( \bar{\ell} \) drives up the average cost of harvesting with distance. For small values of \( s \), this factor may be large enough to offset \( \bar{\ell} \)’s otherwise decreasing effect so that labor increases for some initial distance. However, as \( s \) increases, the distance term approaches unity. Thus under non-cooperative conditions, the model predicts that equilibrium labor initially increases and then decreases with distance.
In the solutions that follow, I focus on the profits conditions that result from different constraint sets. The typical characteristic of the non-cooperative solution is rent dissipation, so it is of interest to show here how this situation deviates from that. Indeed, the non-cooperative case with binding constraints is effectively a reduced-form model of common property systems that have been considered in game theoretic terms (Cheung, 1970; Dasgupta and Heal, 1979).

**Proposition NC1.** *The distance constraint binds:* If \( S^{NC} = d \) and \( L > \sum_{s=1}^{S} I_{s}^{NC} + \bar{f} \) then \( \Pi > 0 \),

where \( S^{NC} \) and \( I_{s}^{NC} \) are the distance traveled and labor allocated when non-cooperative condition (5) holds, respectively.

**Remark NC1.** When a community allocates labor up to the forest boundary and its labor constraint is slack, it is not necessary to dissipate all resource rents to achieve equilibrium across all patches. Therefore, when village is labor constrained, non-cooperative harvesters earn positive profits (for a game-theoretic treatment of this case, see Dasgupta 2008).

**Proposition NC2.** *The labor constraint binds:* If the labor constraint binds then \( \Pi = 0 \).

**Remark NC2.** The typical zero-rent characteristic associated with non-cooperative resource management does not hold within patches, but rather across all patches.\(^4\) Nearer patches are over-harvested (for negative profits) and distant patches are harvested for positive profit to make up for the deficit created in earlier patches.

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\(^4\) To illustrate this, assume we impose the zero-rent condition on harvests in the first patch. With condition (5), within-patch profits must then be positive for all further patches since non-cooperative labor decreases with distance in equilibrium and other households would have an incentive to enter the system. Therefore, in equilibrium we impose that rents over all space are driven to zero when the labor constraint is slack.
The non-cooperative model for a 1-patch and a 2-patch forest are depicted graphically in Figure 3 for a linear forest where \( K = P \) for simplicity. The 1-patch case for non-cooperative harvests shows \( l_1 \) is defined where the total costs of harvesting equal the total revenues, resulting in no economic rent. The 2-patch forest extends the 1-patch model, but now resources are “over harvested” in the first patch and “under harvested” in the second patch so that total profits are zero. Like the profit-maximizing case, the growth curves are “stacked” and harvesting begins after leaving the previous patch and expending the fixed unit of labor \( \overline{t} \). Here \( l_1 > l_2 \).

[Figure 3: Non-cooperative Harvests in a Linear Forest]

Several results arise from the linear model. First, under cooperative management labor is allocated equally in all used patches, while in non-cooperative systems harvest activity is more intense in nearby patches due to the “race” to extract resources. Thus, in labor-limited cases, village labor is “used up” more quickly in non-cooperative relative to cooperative management, which distributes labor less intensely but further into the forest. For fragile forests, this has the implication that profit-maximizing behavior may have negative ecological consequences since human activity is spread over more space. In forests where ecosystem goods and services degrade relative to the intensity of human use, profit-maximizing management should have positive ecological impacts.
3. Numerical application

To illustrate the above results more clearly, I construct a numerical application of the model. The numerical application evaluates cooperative and non-cooperative management and the effects of labor and distance constraints. Simulations were run with a number of parameter values for the resource’s growth rate \( r \), carrying capacity \( K \), catchability coefficient \( q \), the resource price \( p \), the cost of labor \( w \) and the labor needed to travel to the next patch \( T \). All reasonable parameter values produce qualitatively similar results to those presented below. Appendix 9.5 explains the simulation methods and gives a table of the parameter values used.

I present the results below in per-unit area terms so that the radial and linear models are easily comparable. I first show how the density of labor and profits change with distance from a village. Section 3.2 examines the effects of labor and distance constraints for various forest sizes, comparing stopping distance, total labor and profit within forests of a depth of 1 up to 10 patches for several levels community labor. I present results from both the linear and radial cases to show the differences and similarities between these cases.

3.1. Within-forest labor and profit distributions

Figure 4a shows the distribution of per-unit area labor and per-unit labor profits throughout a 10-patch forest from the linear model. The village center is at the origin. Each cluster of bars represents labor allocation or profit distribution from non-cooperative (NC) and cooperative (C) solutions. First, consider labor allocation. In the cooperative case, labor is equally allocated throughout the linear forest, although communities that are small relative to the forest size (where the labor constraint binds) do not travel all the way to the forest edge. Non-cooperatively managed communities concentrate all their effort closer to the village, and in the patches visited per-area labor is always greater than in the cooperative case. Therefore, in smaller
communities, non-cooperative conditions each patch “uses up” more labor than cooperative conditions where a smaller amount of labor is allocated per-patch and harvesters travel further into the forest (Figure 4a and b).

[Figure 4: Within-forest distribution of labor and profits per-area]

Figure 4a shows that per-patch profit (per unit labor) is always equal in the cooperative case. In non-cooperative settings, per-patch profits increase as harvesters travel farther from the village as labor (and thus competition) decrease with distance. Still, when communities are large relative to the forest size (labor is not constrained), non-cooperative conditions lead to zero total profits as we expect in rent-dissipating open access equilibria.

Figure 4b shows that non-cooperative per-area labor monotonically decreases with distance in the radial model. The term $\frac{2s+1}{2s-1}$ in equation (5) causes total labor within a ring to first increase and then decrease, but labor per-unit area always decreases to equate average benefits across space. In a cooperative setting with a small community relative to the forest size (the constrained labor case in the first and second columns in Figure 4b), harvesters choose to skip or under-allocate labor in nearer rings since that labor would be more profitably used in more resource-rich distant rings (eg, in Figure 4b column 1, the first three patches are skipped). In large communities (Figure 4 columns 3 and 4), the optimal allocation decreases with distance due to the non-linear relationship between labor, harvests and resource availability drives these results in the radial model (see equation 5). Cooperative labor does not increase at the same rate
as the increase in resource availability. Since the increase in area with each ring is proportional to the increase in resource availability, per-unit area labor allocation decreases with distance.\(^5\)

In the non-cooperative case for the radial model, positive per-unit labor profits are attained when the community is small, and profits dwindle as we increase total labor availability. With cooperative management, profit per unit labor increases with distance. Again, the increase in optimal labor at more distant patches is not proportional to the increase in area. Therefore, profits per unit labor increase when more resources are available and less competition.\(^6\)

Linking this back to forest ecology, the patterns of labor distribution shown here are indicative of the level and extent of forest disturbance induced by human harvesting. I will summarize these result below in Table 1. First, however, if we are primarily concerned with securing local livelihoods, then the aggregate outcomes from different institutional arrangements and assumptions over forest geography are of interest. We now look at the effects of labor and forest size on the maximum distance traveled, aggregate labor expenditures, and total profits.

3.2. Aggregate results

To show the effects of the interaction between the size of the community and the size of the forest on aggregate outcomes, Figure 5 presents four sets of graphs. Each column of graphs represents a fixed amount of community labor, as indicated at the top of the column. Each curve depicts results from a series of models run at the forest size as indicated by the x-axis, from \(d = 1\)

\(^5\) Formally, one can show that per unit-area labor \(\tilde{I}_s^* = \frac{p_r}{\sum \tilde{I}_s} \) decreases with distance from the village \(\partial \tilde{I}_s^*/\partial s < 0 \) in the unconstrained labor case.

\(^6\) López-Feldman et al A. López-Feldman and J. E. Wilen (2008), 'Poverty and spatial dimensions of non-timber forest extraction', Environment and Development Economics 13(5): 621-642. develop a model similar to the linear case above. Their results are quite different from our findings and reflect their assumption that harvests take place at one point in space, as opposed to this model of resources that are collected continually as one travels through a forest. In their model, cooperative and non-cooperative labor distributions are proportional over space. The distribution of revenue is constant for non-cooperative case (equal to the opportunity cost of time) and constantly decreasing for the cooperative case, equaling just the opportunity cost of time in last profitable patch. Our model finds quite different allocation of labor across space based on management type and forest geometry.
to \( d = 10 \). For example, consider a community with total labor of 65 units and a maximum forest depth of 5 (as seen within column 2 of Figure 5). In all models, labor is allocated the maximum distance (5) except in the linear non-cooperative case, which stops at a depth of 4. All cases allocate all 65 units of labor when \( d = 5 \) and radial non-cooperative profits come very close to those with optimally cooperative institutions. With the same labor availability and a maximum forest depth of \( d = 8 \), harvesters stop before the forest’s end in all cases except the cooperative radial case.

**Maximum distance.** When the community is small (the labor constraint binds), harvesters travel farther in radial than linear forests since resources increase with distance in the radial models and cooperative harvesters always travel further than non-cooperative ones. In larger communities, harvesters in all cases travel the extent of the forest.

**Labor.** Optimally cooperative harvesters never allocate more labor in a system than non-cooperative harvesters. When communities are large (Figure 5 column 4), in the non-cooperative case excess effort leads to the zero-profit condition that typifies open access systems. We also see that forest geometry does not affect aggregate labor allocation. This makes sense since total resource availability is the same in both the linear and radial models, and is actually a good check that our models are consistent and simulations perform correctly.

**Profit.** When communities are large, in both the linear and radial models, cooperative profits increase proportional to the size of the forest, and non-cooperative profits are zero. When labor is constrained, however, non-cooperative systems earn positive profits. For instance, Figure 5 column 2 shows labor is not constrained when \( d = 2 \), so open access profits are 0. But for \( d > 3 \), there is not enough labor in the non-cooperative system to dissipate all rents, so earnings are positive and in proportional terms quite close to the earnings in the cooperative case. In general,
non-cooperative profits are highest when the forest depth $d$ coincides with the patch where the labor constraint first fully binds.

Interestingly, when profits are positive in the non-cooperative case, the radial model always predicts profits greater than those in the linear model, even though aggregate labor allocation is the same. This is because the ratio of the benefits (potential harvest profit) to cost ($\bar{t}$) of moving to the next patch increases with distance in the radial model, but is constant in a linear framework. In the radial model, the cost of travel to a more distant patch relative to the per-labor benefits of harvesting gets cheaper.

[Figure 5: Total forest travel distance, aggregate labor and total profits]

3.3. Summary of modeling results

From a development perspective the most fundamental lesson from this model is that even under “open access” conditions – when harvesters compete uncooperatively – if the forest is large relative to the size of the community then harvesters still profit. Table 1a compares welfare outcomes from the two management regimes and two constraint conditions.\(^7\) To be sure, gains can be much greater when harvesters optimally cooperate. But if policymakers’ goal is to ensure livelihoods from wild forest resources are more profitable than available alternatives, forest dwellers may be capable of such earnings in the absence of collective action or policy intervention. That said, logistical constraints, such as the distance a harvester can physically travel in a day, are not explicitly modeled here and may play a role. Still, a forest without spatial restrictions on harvesting welfare-dominates the same forest with spatial restrictions.

\(^7\) I have shown the differences in linear and radial outcomes are mostly in magnitude and not outcome, we do not include them in this comparison. Although within-forest distribution of labor can be quite different between the two models, these aggregate qualitative results generalize to either context.
From an ecological perspective, we can distinguish between two types of ecosystem services (aside from the products harvested) forests may provide: ones sensitive to the intensity of human activity and others sensitive to any amount of human activity (e.g., some types of fragile biodiversity, the presence of birds or large mammals). The former may include, for example, regulating ecosystem services such as water absorption that may be compromised by intense soil compaction, or decimation of the understory altering habitat conditions. The latter include, for example, biodiversity provision in fragile ecosystems, habitat for endemic species that may be compromised from even light harvest activity, or the presence of humans that may alter foraging and habitat ranges of birds or large animals. In both cases, harvesting NTFPs can impose an externality on these other regional or global ecosystem services provided by forests.

With respect to ecosystem services that are sensitive to the intensity of harvest activity (Table 1b), non-cooperative conditions always result in higher per-area labor compared to cooperative management, yielding greater disturbance. Labor constraints will serve to lower disturbance and intensity even further in both the cooperative and non-cooperative cases.

In forest environments where ecosystem services are sensitive to any amount of anthropogenic disturbance, the magnitude of labor is not as important as the presence or absence of human activity. Table 1c compares model outcomes with the fragility of forest in mind where the distance traveled is the cause of negative outcomes. In smaller forests, harvesters forage throughout all the forest. In smaller communities, optimal management results in less intense activity, but requires further travel into the forest (Figure 5). Thus cooperative management is uniformly worse than the competitive outcome for particularly fragile ecosystem services. In this
case, a lack of local management may provide greater regional and global benefits since non-cooperative conditions keep a larger portion of the forest “pristine”.

4. Empirical support

Data to empirically test a spatial harvest model such as this presents challenges. First, it is difficult to know whether a village is labor or distance constrained in the aggregate, especially in the face of heterogeneous opportunity costs. Second, ideally the researcher would also know how resources and labor are distributed throughout the forest to accurately judge the profitability of any particular travel path as implied by the model. Both these challenges are difficult to overcome.

Here we test some simple hypotheses predicted by the model with a unique dataset, looking for household and individual level factors associated with harvesters’ estimates their travel distance into a forest. The data are from ten villages that collect wild matsutake mushrooms in northwest Yunnan, China. In each village 25-30 household interviews were conducted in which households reported on matsutake collection activity and typically how far each harvester travels into the forest in their search. Matsutake grow wild in these forests, and are sold to buyers at the village center. The sample used in this analysis consists of more than 600 individuals that also engage in farming or wage-earning activities between the ages of 12 and 65. More details regarding data collection and the setting are in Robinson et al (2013b).

The data contain weekly time budgets from each individual within the household during matsutake harvesting season. Most in our sample are farmers (n=512, with 358 report harvesting). The surveys collected a detailed account of households’ agricultural production and activity, with which I construct an agricultural production function to estimate harvester’s opportunity
cost of time (Jacoby, 1993; Skoufias, 1994). For wage-earners (n=105, 26 of which harvest) I use the wage from their off-farm employment activity as a measure of opportunity cost.

From the full sample (n=602, 15 report being both a worker and a farmer), 64% of the individuals harvest matsutake. When choosing how far to travel in the forest, each villager \( i \) in household \( j \) solves the maximization problem \( s_{ij} = \max(0, s_{ij}^*) \), where \( s_{ij}^* \) is a continuous latent variable. This choice may depend on one’s opportunity cost of time \( w_{ij} \), or other household \( Y_j \) or individual-level factors \( X_{ij} \). We estimate these impacts through a truncated regression (Tobit) model \( s_{ij} = \alpha + \beta w_{ij} + \delta Y_j + \eta X_{ij} + \varepsilon_{ij} \) where \( s_{ij} = 0 \) if \( s_{ij}^* \) is negative. The results from several specifications of this model are presented in Table 2. I present two specifications for the dependent variable (natural log of the distance traveled, and the natural log of distance normalized by the village’s maximum harvest distance) and two measures of opportunity cost (the full sample, and only farmers).

[Table 2: Marginal effects from Tobit estimations of factors associated with travel distance]

Table 2 presents the marginal effects from Tobit models of factors correlated with harvesters’ reported travel distance. We see mixed results for the relationship between the travel distance and a harvester’s opportunity cost of time. In the full sample, the relationship between opportunity cost and travel distance is not precisely estimated. Farmers’ opportunity cost, calculated as the marginal product of labor from an agricultural production function, is positive and significant, indicating those most reliant on farming also engage heavily in NTFP collection. Further, farmers with more education are associated with further travel distances.
Across both sets of dependent variable specifications, the factors that are consistently correlated with a harvester’s distance traveled are their household’s asset index (as a measure of wealth; see Filmer and Pritchett, 2001), the number of years engaged in harvesting, and the harvester’s age. A household’s dependency ratio, as the number of non-productive to productive household members, does not have a precise impact on individuals’ travel. The sign and relative magnitude of the models that use natural log of the normalized distance as the dependent variable are quite similar to those that use the log of the raw distance, but interpretation of the coefficients with a normalized dependent variable is more challenging, so we focus on columns one and two of Table 2. At our sample mean, a 10% increase in a harvester’s asset index is related to about a 26-27% decrease in the distance traveled, a 10% increase in years of experience relates to roughly 28-31% increase in distance, and another year in age is associated with about 4-5% increase in distance traveled.

5. Institutions for harvesting wild forest products

The analytical model above categorizes institutional settings in a relatively simplistic way. The canonical example of cooperative management is privatized rights to the forest, where individuals, households or even groups of households would hold user rights to a specific forest plot. However, collective management strategies can approximate this condition (Ostrom, 1990; Poteete et al., 2010), and governance over wild forest products is has been documented for a variety of resources and can take varied forms (Laird et al., 2010). In places where forest tenure is generally less secure, traditional forms of customary rule often dictate wise-use policies (Wynberg and Laird, 2007). Even in the context of our empirical results, various governance methods have been devised around mushroom harvesting including auctions for rights to harvests, small groups defined to harvest in rotating extraction areas, and limiting the number of
harvest days per week (Yeh, 2000; Yang et al., 2009; Menzies and Li, 2010; Robinson et al., 2013b). We have all these types of management methods in mind when discussing “cooperative” management and take the two institutional settings modeled above as bounds on most real-world situations.

Our analytical results suggest management institutions should lead to livelihood improvements. But at the same time we must recognize that institutions are not costless interventions, and can come at an expense to communities even with external aid. Communities must often take on management duties, especially long-run responsibilities such as monitoring and enforcement, which alone may too high to be internally desirable. Developing and enforcing such institutions can also be contentious and detrimental a community’s social capital. Moreover, institutions are not perfect; the relative gains from implementing management are not likely as high as the theoretical profit-maximizing solution.

As opposed to managing harvester labor, forest managers may consider restricting the areas of the forest where villagers can go to harvest. One option is to restrict the depth to which a community can harvest in order to protect promote a “pristine” outer forest and establish an exclusionary boundary. In non-cooperative situations where most of the community participates in harvesting (i.e., the labor constraint binds), this may limit the extent of forest disturbance, but will drive down community profits. In cooperative settings, profits are also likely to decrease when distance becomes more constrained due to fewer available resources. The empirical results suggest these kind of distance-restricting policy actions likely disproportionately affect those with lower socioeconomic status and those who are older.

Instead of a hard exclusionary boundary, another option is to create an interior buffer where harvest activity is restricted close to the village, but allow for harvests farther away. This
increases the overall upfront cost of engaging in harvesting and thus may not be a pareto-improvement, but should better preserve the welfare of the poor and vulnerable who have lower opportunity costs of harvesting.

6. Conclusions

This paper proposes a general model for the spatial harvest of non-timber forest products and provides some empirical results for factors that are correlated with harvesters’ travel distances. The theoretical model contributes to our understanding of the spatial nature of resource harvest in several ways. First, the institutional context is explicit, and in particular I develop the economic intuition and methods for modeling non-cooperative behavior over space. Second, I make the geometry of forest conditions explicit and highlight differences in assuming a linear versus a circular forest. Third, we make clear that either labor or distance constraints (or both) must bind for a model to be realistic (and tractable), and analyze the effect of each constraint on the outcomes of harvesting from a forest ecosystem.

The numerical simulation helps highlight how forest disturbance differs when assuming a linear forest versus a radial forest model. Namely, a linear model predicts within-forest labor distribution consistent with a constant degradation of forests over space under optimal cooperative management. In the radial model, per unit area labor allocation drops off considerably in both the cooperative and non-cooperative cases since harvesters can forage in any direction. However, aggregate labor outcomes do not differ appreciably between the two forest geometries. Yet the radial model predicts greater non-cooperative profits from the forest when labor is limited.
In general, when the forest is small relative to its labor pool, welfare can improve only if the forest is managed cooperatively. Without cooperation, excess effort will lead to the typical non-profit open access result. When the forest is larger than the community can exploit, non-cooperative foragers will still profit. Whether or not cooperation is actual welfare-improvement will depend on the size of potential gains relative to the costs of implementing a cooperative solution, and possibly who bears the burden of those costs.

When looking to manage the forest to preserve other ecological benefits, ecosystems services that are particularly fragile (i.e., they are impacted by the presence or absence of human activity) may actually be hurt when trying to institute cooperative NTFP management since foraging activity is spread further throughout space. Alternatively, ecosystem services from forests that are more sensitive to the intensity of human use are likely to benefit from cooperative management since non-cooperative conditions always result in greater harvest intensities. Our empirical results also show that imposing distance-based restrictions on foraging are likely to disproportionately affect poor and vulnerable populations. Older, less-wealthy households are the ones that tend to rely on harvests further from the village.
7. Figures

**Figure 1: Linear versus Radial Forests**

a. Linear examples

b. Radial examples

**Figure 2: Cooperative Harvests in a Linear Forest**
Figure 3: Non-cooperative Harvesst in a Linear Forest

1 Patch Forest (non-spatial)

2 Patch Forest

U-shaped revenue curve in equilibrium

Labor, Cost $
Figure 4: Within-forest distribution of labor and profits per-area

<table>
<thead>
<tr>
<th>L =</th>
<th>60</th>
<th>110</th>
<th>200</th>
<th>250</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>A: Linear model</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Labor unit area</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>NC</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>NC</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Profit unit labor</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>NC</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>NC</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Smax</td>
<td>3</td>
<td>6</td>
<td>8</td>
<td>10</td>
</tr>
<tr>
<td>Profits</td>
<td>424</td>
<td>2976</td>
<td>1642</td>
<td>5210</td>
</tr>
</tbody>
</table>

| **B: Radial model** |
| Labor unit area |
| C | NC |
| C | NC |
| Profit unit labor |
| C | NC |
| C | NC |
| Smax | 4 | 8 | 10 | 10 |
| Profits | 549 | 2250 | 2096 | 5459 |

Notes: In the linear model, non-cooperative per-unit area labor decreases with distance while labor under cooperative management is always constant. For the radial model, per-unit area labor monotonically decreases at a decreasing rate with distance into the forest. In both models, non-cooperative labor is always higher than cooperative labor when unconstrained. When constrained, cooperative labor is allocated further into the forest. Profits under cooperative management always dominate non-cooperative earnings, but when labor is constrained, non-cooperative cases also earn positive profits.
Figure 5: Total forest travel distance, aggregate labor and total profits

<table>
<thead>
<tr>
<th>$L$</th>
<th>40</th>
<th>65</th>
<th>120</th>
<th>250</th>
</tr>
</thead>
<tbody>
<tr>
<td>Max dist traveled</td>
<td><img src="image1.png" alt="Graphs" /></td>
<td><img src="image2.png" alt="Graphs" /></td>
<td><img src="image3.png" alt="Graphs" /></td>
<td><img src="image4.png" alt="Graphs" /></td>
</tr>
<tr>
<td>total Labor</td>
<td><img src="image5.png" alt="Graphs" /></td>
<td><img src="image6.png" alt="Graphs" /></td>
<td><img src="image7.png" alt="Graphs" /></td>
<td><img src="image8.png" alt="Graphs" /></td>
</tr>
<tr>
<td>total Profit</td>
<td><img src="image9.png" alt="Graphs" /></td>
<td><img src="image10.png" alt="Graphs" /></td>
<td><img src="image11.png" alt="Graphs" /></td>
<td><img src="image12.png" alt="Graphs" /></td>
</tr>
</tbody>
</table>

Notes: Each figure depicts cooperative and non-cooperative outcomes for total community labor ($L$) at the top of each figure over a range of potential forest sizes (1 to 10). Therefore, each point on a plot represents a solution for a particular set of constraints. Figure 4 looks at within-forest outcomes for $d=10$ over a range of labor conditions.
8. Tables

Table 1: Model comparison for welfare and ecology

**a. Community profits**

<table>
<thead>
<tr>
<th>Welfare</th>
<th>Limited labor</th>
<th>Limited distance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cooperative</td>
<td></td>
<td>highest</td>
</tr>
<tr>
<td>Non-cooperative</td>
<td>low</td>
<td>lowest (zero)</td>
</tr>
</tbody>
</table>

**b. Forest disturbance, labor per unit area matters**

<table>
<thead>
<tr>
<th>Intensity</th>
<th>Limited labor</th>
<th>Limited distance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cooperative</td>
<td>lowest</td>
<td>low</td>
</tr>
<tr>
<td>Non-cooperative</td>
<td>high</td>
<td>highest</td>
</tr>
</tbody>
</table>

**c. Forest disturbance, distance matters most**

<table>
<thead>
<tr>
<th>Fragility</th>
<th>Limited labor</th>
<th>Limited distance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cooperative</td>
<td>far</td>
<td>up to max distance</td>
</tr>
<tr>
<td>Non-cooperative</td>
<td>short</td>
<td></td>
</tr>
</tbody>
</table>

Each cell represents one modeling case. Shaded cells show the outcome relative to other cells. Positive outcomes are shaded lightly, negative outcomes are dark.
Table 2: Marginal effects from Tobit estimations of factors associated with travel distance

<table>
<thead>
<tr>
<th>Dep var:</th>
<th>ln(distance)</th>
<th>ln(normalized distance)</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Full sample</td>
<td>Farmer workers</td>
<td>Full sample</td>
</tr>
<tr>
<td>ln(combine opp. cost)</td>
<td>0.07 (0.07)</td>
<td>0.18 (0.08)**</td>
<td>0.03 (0.03)</td>
</tr>
<tr>
<td>ln(farm opp. cost)</td>
<td>0.05 (0.18)</td>
<td>0.15 (0.18)</td>
<td>0.02 (0.07)</td>
</tr>
<tr>
<td>ln(dependency ratio)</td>
<td>-0.27 (0.13)**</td>
<td>-0.26 (0.13)**</td>
<td>-0.11 (0.05)**</td>
</tr>
<tr>
<td>ln(asset index)</td>
<td>0.28 (0.07)***</td>
<td>0.31 (0.06)***</td>
<td>0.11 (0.02)***</td>
</tr>
<tr>
<td>gender (M=1)</td>
<td>0.07 (0.06)</td>
<td>0.13 (0.07) *</td>
<td>-0.02 (0.02)</td>
</tr>
<tr>
<td>yrs education</td>
<td>0.01 (0.01)</td>
<td>0.03 (0.01)***</td>
<td>0.01 (0.00)</td>
</tr>
<tr>
<td>age</td>
<td>0.04 (0.01)***</td>
<td>0.05 (0.02)***</td>
<td>0.01 (0.01)***</td>
</tr>
<tr>
<td>age^2</td>
<td>-0.00 (0.00)***</td>
<td>-0.00 (0.00)***</td>
<td>-0.00 (0.00)***</td>
</tr>
<tr>
<td>Private village</td>
<td>-0.54 (0.10)***</td>
<td>-0.67 (0.11)***</td>
<td>-0.15 (0.05)***</td>
</tr>
</tbody>
</table>

|          |                  |                  | N  |  | # censored |  | # uncensored |  |
|          |                  |                  | 602| 512 | 602  | 512 | 370 | 358 |
|          |                  |                  | 232| 154 | 232  | 154 | 370 | 358 |
|          | pseudo R^2       |                  | 0.13| 0.21 | 0.18  | 0.33 |
|          | log pseudolikelihood |              | -791 | -618 | -435  | -280 |

Notes: ***=p<0.01; **=p<0.05; *=p<0.10

All estimations included fixed effects at the village level (not shown). All Tobit estimates represent $E[s\mid \theta > 0]/\partial \theta$, where $\theta = [w, X, Y]$. This represents the marginal effect conditional on one choosing to harvest.
9. Appendix

9.1. Linear optimal conditions

The Lagrangian for the problem is:

\[ G = S^* \left[ pql_s K (1 - ql_s / r) - w (l_s + \bar{\ell}) \right] + \lambda \left( L - \sum_i^S (l_i + \bar{\ell}) \right) + \gamma (d - S). \]

The optimal solution finds profit maximizing values for \( S \) number of labor choice variables, the value of \( S^* \) and the multipliers on the two constraints, giving \( S+3 \) equations with \( S+3 \) unknowns. The complete set of first order conditions are:

\[
\begin{align*}
\frac{\partial G}{\partial l_s} &= \left( pqK - 2 \frac{q}{r} \right) pql_s^* - w - \lambda^* = 0 \quad \forall s = \{1, 2, ..., S\} \\
\frac{\partial G}{\partial S} &= pql_s^* \left( 1 - \frac{q}{r} l_s^* \right) - (w + \lambda^*) (l_s^* + \bar{\ell}) - \gamma^* = 0 \\
\frac{\partial G}{\partial \lambda} &= (L - S^* (l_s + \bar{\ell})) = 0 \\
\frac{\partial G}{\partial \gamma} &= (d - S^*) = 0
\end{align*}
\]

Plus two complimentary slackness conditions for the inequality constraints:

\[
\lambda (L - \sum_i^S (l_i + \bar{\ell})) = 0 \quad \text{and} \quad \gamma (d - S) = 0.
\]

There are three cases to examine for which we need to determine optimal labor \( l_s^* \) and the optimal stopping distance \( S^* \):

i.) Only the labor constraint binds

\[
\begin{align*}
\frac{\partial G}{\partial l_s} &= pqK - 2 \frac{q}{r} \left( pql_s^* \right) - w = \lambda^* \\
\frac{\partial G}{\partial S} &= pql_s^* \left( 1 - \frac{q}{r} l_s^* \right) - (w + \lambda^*) (l_s^* + \bar{\ell}) = 0 \\
\frac{\partial G}{\partial \lambda} &= l_s^* = \frac{L}{S} - \bar{\ell}
\end{align*}
\]

To solve the system of equations, first collect terms in \( \frac{\partial G}{\partial S} \) to give \(- \frac{pqK}{r} l_s^{*2} + \left( pqK - (w + \lambda^*) \right) l_s^* - \bar{\ell} (w + \lambda^*) = 0 \). Then insert \( \lambda^* \) and \( l_s^* \) from \( \frac{\partial G}{\partial l_s} \) and \( \frac{\partial G}{\partial \lambda} \), respectively. Collecting and canceling terms from this aggregate equation and solving for \( S^* \) yields \( S^* = L/ \sqrt{\bar{\ell}^2 + \frac{z}{q} \bar{\ell}} \). Using this back in our expression from \( \frac{\partial G}{\partial \lambda} \), optimal per-patch labor is \( l_s^* = \sqrt{\bar{\ell}^2 + \frac{z}{q} \bar{\ell}} - \bar{\ell} \). Technically, the first order condition with respect to \( S \) contains only labor in the final patch \( S \), but since optimal labor is independent of one’s patch in space, \( l_s^* = l_s^* \quad \forall s \) and the first order condition from \( \frac{\partial G}{\partial S} \) applies at any point in space, not just the final patch.
ii.) Only the distance constraint binds
\[
\frac{\partial G}{\partial l_s} \Rightarrow pqK - 2 \frac{a}{r} pq Kl_s^* - w = 0
\]
\[
\frac{\partial G}{\partial S} \Rightarrow pq Kl_s^* \left(1 - \frac{a}{r} l_s^* \right) - w(l_s^* + \bar{r}) = \gamma^*
\]
\[
\frac{\partial G}{\partial \gamma} \Rightarrow S^* = d
\]
From \( \frac{\partial G}{\partial \gamma} \) we can solve directly for optimal labor \( l_s^* = \frac{r}{2q} \left(1 - \frac{w}{pqK} \right) \). The distance constraint binds so \( S^* = d \).

iii.) Both the labor and distance constrains bind
If both constraints bind, calculation of the optimal stopping distance and optimal labor is just the combination of the constraints themselves: \( S^* = d \) and \( l_s^* = \frac{r}{d} - \bar{r} \).

9.2. Linear non-cooperative conditions
For the average benefit of harvests in patch \( s \) to be equal to the average benefit of harvests in patch \( s+1 \):

\[
\frac{pH_s}{l_s} = \frac{pH_{s+1}}{(l_{s+1} + \bar{r})}
\]
\[
\frac{ql_l K(1 - q l_l / r)}{l_s} = \frac{ql_{l+1} K(1 - q l_{l+1} / r)}{(l_{s+1} + \bar{r})}
\]
\[
\frac{qK(l_s - q l_s^2 / r)}{l_s} = \frac{qK(l_{s+1} - q l_{s+1}^2 / r)}{(l_{s+1} + \bar{r})}
\]
\[
\frac{l_s - q l_s^2 / r}{l_s} = \frac{l_{s+1} - q l_{s+1}^2 / r}{(l_{s+1} + \bar{r})}
\]
\[
\left(1 - \frac{q}{r} l_s \right) = \frac{l_{s+1}}{(l_{s+1} + \bar{r})} \left(1 - \frac{q}{r} l_{s+1} \right)
\]
\[
\frac{1 - \frac{q}{r} l_s}{1 - \frac{q}{r} l_{s+1}} = \frac{l_{s+1}}{(l_{s+1} + \bar{r})}
\]

On the RHS, the denominator is greater than the numerator. This must also apply to the LHS, so
\[
1 - \frac{q}{r} l_s < 1 - \frac{q}{r} l_{s+1} \Rightarrow \frac{q}{r} l_{s+1} < \frac{q}{r} l_s \Rightarrow l_s > l_{s+1} \quad \forall s
\]
The non-cooperative labor allocation as a function of the next patch follows from above:
\[
l_s^{OA} = \frac{r}{q} \left(1 - \frac{l_{s+1} - (l_{s+1}^2 / r)}{(l_{s+1} + \bar{r})} \right)
\]
9.3. Radial optimal conditions

The radial Lagrangian is \( G^C = \sum_{i=1}^{S} \left[ pq \rho \left( 2s - 1 \right) l_s \left( 1 - \frac{w}{l_s} \right) - w(l_s + \bar{\ell}) \right] + \lambda \left( L - \sum_{i=1}^{S} \left( l_s + \bar{\ell} \right) \right) + \gamma(d - S) \). The complete set of first order conditions are:

\[
\begin{align*}
\frac{\partial G^C}{\partial l_s} &= pq \rho \left( 2s - 1 \right) \left( 1 - 2 \frac{w}{l_s} \right) - w - \lambda^* = 0 \quad \forall s \in \{1, \ldots, S^*\} \\
\frac{\partial G^C}{\partial S} &= pq \rho \left( 2S^* - 1 \right) \left( 1 - 2 \frac{w}{l_s} \right) - \left( w + \lambda^* \right) (l_s^* + \bar{\ell}) - \gamma^* = 0 \\
\frac{\partial G^C}{\partial \lambda} &= L - \sum_{i=1}^{S} \left( l_s + \bar{\ell} \right) = 0 \\
\frac{\partial G^C}{\partial \gamma} &= d - S = 0
\end{align*}
\]

Plus the two complimentary slackness conditions for the inequality constraints:

\[
\lambda \left( L - \sum_{i=1}^{S} \left( l_s + \bar{\ell} \right) \right) = 0 \quad \text{and} \quad \gamma(d - S) = 0.
\]

Similar to the linear case above, we have \( S + 3 \) equations and \( S + 3 \) unknowns, but deriving solutions is more complicated. We have \( S^* \) number of within-ring first order conditions \( \frac{\partial G^C}{\partial l_s} \) to solve – the number of first order conditions is determined by a choice variable in the first order conditions. Technically this is also the case for the linear problem, but there \( l_s^* = l_{s-s}^* \) so the solution simplifies to a single patch \( s \) held for all patches irrespective of stopping distance \( S^* \). In the radial case \( l_s \neq l_{s-s} \), so we must solve all conditions for \( l_1, l_2, \ldots, l_{s-s} \) separately, but also while jointly determining \( S^* \). The three constraint combinations to examine are:

i) Only the labor constraint binds (\( \gamma = 0 \)).

This is the most complicated case since we need to solve explicitly for \( S^* \) and the value of \( S^* \) also determines our number of first order labor conditions. The most general solution we can present here is to state \( l_s^* = l_{s-s}^* \) so the system of equations:

\[
\begin{align*}
l_s^* &= \frac{r}{2q} \left( 1 - \frac{(w + \lambda^*)}{pq \rho \left( 2s - 1 \right)} \right) \quad \forall S \in \{1, \ldots, S^*\} \\
\alpha^* &= \frac{pq \rho \left( 2s - 1 \right) l_s^* \left( 1 - \frac{w}{l_s^*} \right)}{(l_s^* + \bar{\ell})} - w
\end{align*}
\]

and \( L = \sum_{l=1}^{S^*} l_s + S^* \ell \). We develop methods for a numerical solution in following sections.

ii) The distance constraint binds (\( \lambda = 0 \)).

\[
l_s^* = \frac{r}{2q} \left( 1 - \frac{w}{pq \rho \left( 2s - 1 \right)} \right) \quad \forall S \in \{1, \ldots, d\}
\]

\( S^* = d \)

In this case we need not make use of the first order condition with respect to \( S^* \) since distance is determined by \( d \). The optimal labor condition comes directly from the first order condition with respect to \( l_s \).

iii) The labor and distance constraints bind (\( \gamma = 0, \lambda = 0 \)).
Optimal conditions when both constraints bind are $S^* = d$; $L = \sum_{s=1}^{d} l_s^* + d \bar{\ell}$;

$l_s^* = \frac{r}{2q} \left( 1 - \frac{(w+ \bar{\chi})}{pq_p} (2s-1) \right) \forall s \in \{1, \ldots, d\}$; and $pq \rho \left( 2d - 1 \right) l_d^* \left( 1 - \frac{q}{r} l_d^* \right) - \left( w + \bar{\chi} \right) \left( l_d^* + \bar{\ell} \right) = \gamma^*$. The number of equations in the system is determined $(d + 3)$, but analytic solutions still require a defined value of $d$.

The set of equations for $S^* = d = 3$ is:

\begin{align*}
    l_1^* &= \frac{r}{2q} \left( 1 - \frac{(w+ \bar{\chi})}{pq_p} \right) \\
    l_2^* &= \frac{r}{2q} \left( 1 - \frac{(w+ \bar{\chi})}{3pq_p} \right) \\
    l_3^* &= \frac{r}{2q} \left( 1 - \frac{(w+ \bar{\chi})}{5pq_p} \right) \\
    L &= l_1^* + l_2^* + l_3^* + d \bar{\ell} \\
    \gamma^* &= 5 pq \rho l_3^* \left( 1 - \frac{q}{r} l_3^* \right) - \left( w + \bar{\chi} \right) \left( l_3^* + \bar{\ell} \right)
\end{align*}

We are only really concerned with the first four equations since they fully describe optimal labor allocation over this three-patch system (the value of $\gamma^*$ per se is not of interested and does not affect labor allocation). Solving these equations gives:

\begin{align*}
    l_1^* &= \frac{1}{23} \left( 15L - 11 \frac{q}{r} - 45 \bar{\ell} \right) \\
    l_2^* &= \frac{1}{23} \left( 4 \frac{L}{q} + 5L - 15 \bar{\ell} \right) \\
    l_3^* &= \frac{1}{23} \left( 7 \frac{L}{q} + 3L - 9 \bar{\ell} \right) \\
    \gamma^* &= \frac{1}{23} pq \rho \left( 45 - 30 \frac{q}{r} L + 90 \frac{q}{r} \bar{\ell} - 23 \frac{w}{pq_p} \right)
\end{align*}

9.4. Radial non-cooperative conditions

The equivalence condition for the average benefit of harvests in patch $s$ to be equal to the average benefit of harvests in patch $s+1$ is:

\begin{align*}
    &\frac{pq \rho \left( 2s - 1 \right) l_s - \frac{q}{r} l_s^2}{l_s} = \frac{pq \rho \left( 2(s+1) - 1 \right) \left( l_{s+1} - \frac{q}{r} l_{s+1}^2 \right)}{l_{s+1} + \bar{\ell}} \\
    &\frac{(2s-1)l_s - \frac{q}{r} l_s^2}{l_s} = \frac{(2s+1)\left( l_{s+1} - \frac{q}{r} l_{s+1}^2 \right)}{l_{s+1} + \bar{\ell}} \\
    &\frac{\left( l_s - \frac{q}{r} l_s^2 \right)}{l_s} = \frac{(2s+1)\left( l_{s+1} - \frac{q}{r} l_{s+1}^2 \right)}{(2s-1)\left( l_{s+1} + \bar{\ell} \right)}
\end{align*}

Labor in patch $s$ is an explicit function of labor in patch $s+1$:

\begin{align*}
    l_s &= \frac{r}{q} \left( 1 - \left( \frac{2s+1}{2s-1} \frac{\left( l_{s+1} - \frac{q}{r} l_{s+1}^2 \right)}{l_{s+1} + \bar{\ell}} \right) \right)
\end{align*}

(A.1)
9.5. Simulation methods and parameters

Numerical simulations were developed in MatLab®. For a cooperative forest, we first calculate optimal unconstrained labor \( (l_s^*) \), and then check whether enough community labor \( (L) \) exists for this optimal allocation given forest depth of \( (d) \). If so, Proposition C1 holds (only distance is constrained). If not, the labor constraint binds and we use the conditions given under Proposition C2. It is not possible to solve for these conditions analytically since the number of first order conditions is endogenous to the solution, so I calculate all possible optimal solutions \( \hat{l}^*_s \) and \( \hat{S}^* \) for forest depth \( s = \{1, \ldots, 10\} \), and evaluate which solution set \( (\hat{l}^*_s, \hat{S}^*) \) gives the greatest profit. This determines the optimal solution set \( l^*_s, S^* \). If \( S^* \) is greater than the exogenously given forest depth \( (d) \), then both constraints bind and Proposition C3 applies.

To calculate non-cooperative labor allocation I first give an initial (very high) guess for labor in the last patch at distance \( d \). I then use equation (A.1) in the Appendix section above to calculate the value of labor in patch \( d - 1 \) and similarly iterate backward over space calculating labor in all prior patches. I then check whether total profits are negative or if total labor allocation is greater than \( L \). If either is true, we reduce labor in the last patch by some small amount and repeat the backward iteration process. If we reduce labor in the last patch to zero and still no non-negative profit condition or admissible labor allocation are found, then labor equals zero for that patch and the next closest patch is the new maximum distance traveled. We stop when all labor is allocated and profits are positive, or when profits are (within a tolerance of) zero with an admissible (non-negative) labor solution.

Table A: Parameter values used in numerical simulations

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Resource growth rate, ( r )</td>
<td>0.5</td>
</tr>
<tr>
<td>Carrying capacity per patch (1-d), ( K )</td>
<td>500</td>
</tr>
<tr>
<td>Parameter</td>
<td>Value</td>
</tr>
<tr>
<td>-----------</td>
<td>-------</td>
</tr>
<tr>
<td>Catchability coefficient, $q$</td>
<td>0.03</td>
</tr>
<tr>
<td>Resource per-unit price, $p$</td>
<td>10</td>
</tr>
<tr>
<td>Value of a unit of labor, $w$</td>
<td>5</td>
</tr>
<tr>
<td>Units of labor needed to travel to the next patch, $\ell$</td>
<td>5</td>
</tr>
</tbody>
</table>
10. References


C. W. Clark (1990), Mathematical bioeconomics : the optimal management of renewable resources, New York: J. Wiley.


W. D. Sunderlin, J. Hatcher and M. Liddle (2008), 'From exclusion to ownership? Challenges and opportunities in advancing forest tenure reform', in, Washington D.C.: Rights and Resources.


