Hot Spot Policing: A Theoretical Study of Place-Based Strategies to Crime Prevention*

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Abstract

Hot spot policing is a place-based policing strategy which addresses crime by assigning limited police resources to areas where crimes are more highly concentrated. We evaluate the theoretical soundness of this strategy using a game theoretic approach. The main argument against focusing police resources on hot spots is that doing so would simply displace criminal activity from one area to another. Our results give new insights into the nature of the displacement effect as well as useful hints for the econometric analysis of crime-reduction effects of police reallocation. We also propose alternative place-based policies that display attractive properties regarding crime reduction.

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1 Introduction

Crime mapping is a powerful tool used by analysts in law enforcement agencies to visualize and study crime patterns. Such maps indicate that crimes are not evenly distributed across geographic locations. Instead, clusters of crimes occur in specific areas, or hot spots. Hot spot policing is a place-based strategy which attempts to reduce crime by assigning limited police resources to places where crimes are more highly concentrated. This approach to crime prevention is relatively new and many crime experts argue it is one of the main reasons why New York City has achieved a dramatic decrease in crimes during the past decade (see, e.g., Zimring (2011)). While hot spot policing has gained in popularity, little research has been done to evaluate its effectiveness in reducing crime. This paper thus contributes to our understanding by performing a theoretical analysis of this strategy to crime prevention. In addition to providing new insights on the potential drawbacks of hot spot policing, we evaluate other place-based strategies which display very attractive properties.[1] Since our results have simple testable implications, we believe they may prove useful to guide further empirical research.

To study the effectiveness of hot spot policing, we develop an approach that combines ideas from various crime theories. We believe each of these theories capture different relevant aspects of crime decisions, and are therefore needed to make predictions consistent with observed patterns of crimes. Specifically, the approach we propose is based on the rational choice model and uses game theory to incorporate into the analysis strategic interactions among potential offenders. We also borrow from the theory of environmental criminology which highlights the role of spatial factors in affecting crime location choice. More formally, we propose a two-stage game. In our model, we first divide our defined region into a finite number of areas which differ in terms of attractiveness for potential offenders. We capture crime attractiveness via two attributes, risk of apprehension and potential productivity. The riskiness of a place for a potential offender can be thought of as an index function that captures structural factors affecting the successful apprehension of offenders in that location, such as the presence of

[1] Place-based strategies are receiving renewed attention in economics. Glaeser and Gottlieb (2008) offer a theoretical and empirical evaluation of this type of policies.

illumination or video cameras. Our second attribute, potential productivity, relates to the expected gains from committing a crime there, such as the presence of a shopping mall or a bank. In the first period of the two-stage game, the enforcement agency decides how to allocate the limited police resources across alternative areas. In the second period, upon observing police allocation, people decide whether to commit a crime and, in case of doing so, where to perform the criminal act.

Using the standard backward induction principle we solve our game by first modeling the people choices for a given police assignment. Following the rational choice perspective, we assume people develop a cost-benefit analysis when considering whether to commit a crime (Becker (1968), Cornish and Clarke (1986), and Ehrlich (1973)).[2] The cost-side of each individual's analysis depends on both his perceived probability of being apprehended and the penalty he would have to pay if so.[3] To develop people payoffs, we also draw on the model presented in Sah (1991) where one individual's choice to become a criminal lowers the probability that any other individual ends up arrested. The motivation behind this model is as follows. Since one police officer cannot be at two different places at the same time, the probability of being apprehended in a given location is lower when its overall level of criminal activity is higher. By extension, we assume the probability of being caught decreases with the number of other people who decide to commit a crime in that location. Furthermore, we assume the probability of being caught in an area increases with both the level of police resources in the area and its natural apprehension risk level. On the benefit-side of the equation, we assume the expected payoff of the criminal act increases with the productivity of the area; by contrast, we assume it decreases with the number of offenders in the area, as the total potential productivity has to be shared among more people.

After identifying people's payoffs for the alternative choices, we conduct the second stage of

[2]Durlauf, Navarro, and Rivers (2010) provide a general description of criminal choices at the individual level to understand the implicit assumptions in aggregate crime regressions. They highlight the relevance of modeling the microfoundations of the empirical analysis of crimes.

[3]Durlauf and Nagin (2011) suggest that increasing the perceived risk of apprehension seems to have considerable deterrent effects on crimes.

our game, which is a game among potential offenders. Note that, for most of our analysis, the negative interaction effect among criminals dominates the positive one, inducing a congestion game with an outside option (not to commit a crime). The outcome of the second stage of our game is a vector of choices that explains crime concentration in terms of area attributes and police allocation. In particular, we find that, at the second-stage equilibrium, the criminal density of each area is inversely associated with its level of police, and positively related to its productivity-to-risk ratio. This conclusion is consistent with the conjecture that opportunity makes a thief (Felson and Clarke (1998)).

Our model allows us to study the effectiveness of hot spot policing more thoroughly. The first part of our work sheds light on one of the most controversial issues of this policing strategy, namely, the displacement effect. The main argument offered by those against re-directing police resources to hot spots is that doing so would simply displace criminal activity from one area to another.[4] Empirical research has shown increasing police resources in problematic areas reduces their criminal activity without increasing criminality in nearby locations (Braga (2008)). The model we provide features this empirical observation when the value of the outside option does not depend on the number of people who opt not to commit a crime. That is, under this assumption we also find that increasing police resources in a hot spot reduces its criminal activity and this policy in and of itself does not induce any initial displacement of criminality. However, in contrast to previous studies we find that displacement effects occur because police resources are limited. Therefore, in order to increase the amount of police in one area, the enforcement agency must reduce it in another one, making the latter more attractive to offenders due to reduced risk of apprehension. (We explain below how results change if the outside option displays congestion effects.) This simple mechanism helps us to characterize the most effective allocation of limited police resources regarding crime reductions.

Going back to the first period, we find that the optimal allocation of police resources does not necessarily induce an even distribution of crimes across areas. In other words, though areas that are *a priori* more attractive to offenders (i.e., display a larger productivity-to-risk ratio) receive indeed more police attention, the extra efforts in these areas do not fully offset the

[4] Reppeto (1976) offers an early discussion on different types of displacement effects in the criminal activity.

impact of their initial structural differences. Thus, in our model some hot spots remain under the optimal allocation strategy. This result is robust to all the extensions we consider for our initial model. Note that this finding should not be interpreted as a justification for sustaining differences in criminal densities across areas. Rather, we interpret our result as indicating that an egalitarian crime rate strategy will have the unintentional consequence of increasing the number of overall crimes relative to the optimal strategy.[5] Regarding the opportunity cost of hot spot policing in terms of overall crime levels, we find that it increases with differences in the productivity-to-risk ratios across locations.

The model we develop also provides insight into the issue of how to reduce crime on a broader level. This issue has been addressed by a number of crime theorists, including Braga and Wisburd (2010), who state that:

> "The attributes of a place are viewed as key factors in explaining clusters of criminal events... To reduce and better manage problems at crime hot spots, the police need to change the underlying conditions, situations, and dynamics that make them attractive to criminals and disorderly persons."

Having characterized the distribution of crimes at the optimal police allocation, we address a question that relates to the last passage. We modify the attributes in certain area in such a way that it becomes *a priori* less attractive to potential offenders. We find that this policy reduces the crime not only in the target area but also in the other locations. The mechanism by which it does so is quite interesting. The direct effect of the policy is to make the target area less attractive for potential offenders, thereby reducing its criminal activity. The indirect effect is due to subsequent police re-allocation from the improved area to the other ones, where the criminal activity diminishes as well. Glaeser and Gottlieb (2008) provide evidence of similar external effects in alternative place-based strategies.

Finally, we consider three extensions of our initial model. The outside option people face (not to commit a crime) can be interpreted as the possibility of getting a formal job. We

[5] The fact that re-allocating police resources to achieve equal crime rates across areas may not be the most effective way to reduce crime is also highlighted by Espejo et al. (2011).

introduce alternative specifications of the outside option and study the effects of improvements in the job market on the labor supply of the economy. By doing so, we confirm that increasing the minimum wage raises the opportunity cost of criminal activity and thus increases the number of people who opt not to commit a crime. This analysis implicitly assumes no congestion effects in the outside option. However, we can imagine a mechanism by which the opposite is true. For instance, when the number of people hoping to obtain a legal job increases, this may push salaries down or increase unemployment, thereby making this outside option less attractive. In the second extension, we allow for this possibility. While the characterization of equilibrium hot spots does not change, the displacement mechanism varies. Under this specification increasing police resources in areas with high criminality pushes criminal activity from this location to the others. This result raises a new simultaneity issue in the empirical studies that aim to measure the effect of increases in police on crime reductions by using cross-sectional data. Specifically, any such study should take into account that the crime rate in each area depends on the whole vector of police allocation (as opposite to just the police resources that were allocated in the corresponding area). In the third extension of our model, we explore how results change if the positive interaction effects dominate the negative ones, as in Freeman et al. (1996) and Sah (1991). Under this condition, our induced game among potential offenders displays strategic complementarities. This type of game, known as supermodular game in the economics literature, often displays multiple equilibria. In terms of policy making, interventions are a delicate matter, as they can easily affect equilibrium selection (see Blume (2006)). We explore this effect with a simple example in our study.

Our research contributes to work in both criminal studies and economics. In an early study, Becker (1968) examines individual decisions to commit crimes from an economic perspective.[6] His cost-benefit analysis is consistent with the rational choice approach used by Cornish and Clarke (1986), which we follow as well. Our study also relates to subsequent work on the importance of social interactions in motivating criminal behavior (Ballester et al. (2006), Chen and Shapiro (2007), Freeman et al. (1996), Glaeser et al. (1996), and Sah (1991)). Specifically, we assume that the decisions of others impact one's decision whether to commit a crime.

[6]See also Ehrlich (1973).

Furthermore, we draw on the work of Espejo et al. (2011), who provide an evaluation of hot spot policing by using a leader and follower model as we do in this investigation. However, our aim and approach differ from theirs. We want to describe crime displacement in a simple (and testable) way and to provide an alternative definition of hot spots. To this end we introduce three new features in our model. First, to make predictions consistent with the theory of environmental criminology we link people's payoffs with the relevant attributes of the areas. Second, we introduce a natural outside option (not to commit a crime) that plays a fundamental role in the analysis. Third, we also study the possibility of positive interactions among potential criminals. We model people's expected payoffs as in Hugie and Dill (1994), who study habitat selection by modeling the behavior of predators and prey. Finally, Braga and Wisburd (2010) provide a deep analysis of place-based policies.[7] In addition to new interesting insights about hot spots and crime prevention, they provide a thorough and updated overview of the theoretical and empirical research regarding this topic. Our analysis and discussions are inspired by their work and the literature therein.

The rest of the paper is organized as follows. Section 2 describes our model. Section 3 solves the equilibrium of the game and displays our main findings. Section 4 evaluates the decision of a hypothetical enforcement agency that aims to reduce crime by changing the attributes of a certain area. Section 5 discusses three extensions of our model. Section 6 concludes, and proofs are collected in the Appendix.

2 The Model

2.1 Main Variables

This sub-section describes the main variables of our model making a clear distinction between the features that we assume are exogenous to the incumbents (i.e., people and enforcement agency) and the features that are under their control. Sections 4 and 5 examine some

[7] See also Braga (2008), Eck et al. (2005), Felson and Clarke (1998), Sherman (1995), and Weisburd and Green. (1995).

extensions to this initial model structure.

Exogenous Variables We let N and M represent the size of the mass of people and police, respectively. There are K alternative areas where criminal activity can take place. With only a slight abuse of notation, K represents the set as well as the number of locations. These areas differ with respect to three attributes, namely, size of area, risk of apprehension, and productivity of criminal activity.

S_k refers to the geographic size of area k (e.g., in square feet). Riskiness R_k is a probability measure of the successful apprehension of offenders in the same area. Differences in riskiness across areas capture differences in location characteristics that reflect the level of police search activity or the ability of police to capture offenders. For example, better lighting may increase the risk of apprehension as offenders are more likely to be seen by someone who might call the police. Conversely, the presence of nearby highways may reduce this risk, as it becomes easier for criminals to escape. We use f to indicate the fee an offender must pay if apprehended. The fee (f) should capture, for instance, the opportunity cost of time spent in prison.

Productivity A_k captures the richness of the area in terms of expected benefits to criminals. For example, a rich area may be a neighborhood that is populated by high-income people whose houses contain high-value items. It may also be a location with stores or banks available as potential targets.

Endogenous Variables The incumbents in the model are the people and the law enforcement agency. Specifically, people decide whether to commit a crime and, if they do so, where to perform the criminal act. In our model, p_k represents the fraction of people in N who decide to commit a crime in location k. We indicate the density of offenders in that location by $d_k = p_k N/S_k$.

The enforcement agency decides how to assign the mass of police to different areas. We let q_k denote the fraction of police resources in M which is assigned to location k; consequently $e_k = q_k M/S_k$ is the corresponding police density. We assume $M/S_k \leq 1$, for all $k \in K$, so that the per capita apprehension rate (defined below) lies between 0 and 1.

2.2 Payoffs for People

We model encounters between police and offenders as a random process, such that the overall rate of apprehension in location k is given by:

$$\mathcal{A}\left(k, p_k, q_k\right) = d_k e_k R_k S_k,$$

where all variables are defined as in the last section. Furthermore, the per capita apprehension rate of an offender in location k is represented by:

$$\mathcal{P}\left(k, p_k, q_k\right) = \mathcal{A}\left(k, p_k, q_k\right) / d_k S_k = R_k e_k.$$

It follows from the last two expressions that the expected penalty for a person who commits a crime is:

$$\mathcal{P}\left(k, d_k, e_k\right) f.$$

On the other hand, the offender's expected benefit of committing a crime is:

$$\mathcal{Y}\left(k, p_k, q_k\right) = A_k / d_k.$$

Thus, the overall expected utility of an offender in location k is given by:

$$\mathcal{U}\left(k, p_k, q_k\right) = \mathcal{Y}\left(k, p_k, q_k\right) - \mathcal{P}\left(k, p_k, q_k\right) f = A_k / d_k - R_k e_k f.$$

Recall that our model allows people not to commit a crime. This outside option can be thought of as the possibility of working in a legal job. Under this interpretation, the number of people who opt not to commit a crime comprises the labor supply in the economy. To simplify the exposition, we initially assume the expected payoff of this outside option is 0. We relax this restriction in Sub-section 5.1 to evaluate the impact on crimes of public policies that affect the labor market in the economy. We refer to the outside option as $k = 0$, so that $\mathcal{U}(0) \equiv 0$ and the choice set of each person is $K_0 \equiv 0 \cup K$. The outcome of their decisions is a probability vector $\mathbf{p} \equiv \left(p_k\right)_{k \in K} \in \Delta^{K_0}$ where

$$\Delta^{K_0} \equiv \left\{ \mathbf{p} : p_k \geq 0 \text{ and } \textstyle\sum_{k=0}^{K} p_k = 1 \right\}.$$

Thus, $N p_0$ represents the number of people who decide not to commit a crime.

2.3 Payoffs of Police Allocation Strategies

The public authority decides how to assign police to different locations. Specifically, it chooses $\mathbf{q} \equiv (q_k)_{k \in K} \in \Delta^K$ where

$$\Delta^K \equiv \left\{ \mathbf{q} : q_k \geq 0 \text{ and } \textstyle\sum_{k=1}^{K} q_k = 1 \right\}.$$

If the purpose of the enforcement agency is to reduce the overall level of criminal activity, then its payoff function is represented by

$$\mathcal{V}(\mathbf{p}) = p_0 \geq 0,$$

In our subsequent analysis, we contrast the behavior of a public authority interested in reducing the overall crime rate to that of a public authority whose aim is to minimize criminality while working toward an even distribution of crimes across areas. In the first case, the enforcement agency prioritizes efficiency at the expense of equity. In the second case, it emphasizes equity at the expense of efficiency.

2.4 Structure of the Game

Following Espejo et al. (2011), we model interactions between incumbents by using a leader and follower game, with the public authority as the leader and people as the followers.

In this game, the public authority first decides how to assign police to different locations with the goal of reducing the overall crime rate. This problem can be specified as follows:

$$\max_{\mathbf{q}} \left\{ \mathcal{V}(\mathbf{p}) : \mathbf{q} \in \Delta^K \right\}.$$

Upon observing the distribution of police, each person, taking as given the decisions of the others, decides whether to commit a crime and, in the case of doing so, the location where the criminal activity will be performed. Thus, the problem faced by each person is:

$$\max_k \left\{ \mathcal{U}(k, p_k, q_k) : k \in K_0 \right\}.$$

In the next section we solve the game using the standard backward induction principle.

3 Equilibrium Analysis

3.1 People Choices

The second stage of the game is itself a game among people. We use Nash equilibrium as our solution concept. Given a strategy profile $\mathbf{p}$ we let $\mathrm{b}(\mathbf{p})$ indicate the best-response correspondence of an arbitrary person, that is,

$$\mathrm{b}(\mathbf{p}) \equiv \{k' \in K_0 : k' \in \arg\max_k \mathcal{U}(k, p_k, q_k)\}.$$

It follows that $\mathbf{p}(\mathbf{q}) \in \Delta^{K_0}$ is a Nash equilibrium if, for each $k' \in K_0$, we obtain:

$$p_{k'}(\mathbf{q}) > 0 \text{ if } k' \in \mathrm{b}(\mathbf{p}) \quad \text{and} \quad p_{k'}(\mathbf{q}) = 0 \text{ otherwise.}$$

Given an initial police assignment and some belief regarding crime location, all people face the same choice set and expected payoffs. Thus, any option that is selected with a strictly positive probability will be among the options with the highest expected value. Since people are indifferent across these possibilities, we can interpret $\mathbf{p}(\mathbf{q})$ as either an asymmetric equilibrium in pure strategies or a symmetric mixed strategy equilibrium (see Hugie and Dill (1994)).

To simplify notation, we define $\theta_k \equiv (S_k)^2 A_k/R_k MNf$, for all $k \in K$. We can now describe, for each police assignment $\mathbf{q}$, the distribution of criminal activity across areas.

Proposition 1 *Fix some $\mathbf{q} \in \Delta^K$. The proportion of the population which decides to commit a crime in location k, for each $k \in K$, is given by*

$$p_k(\mathbf{q}) = S_k A_k/N\left[u(\mathbf{q}) + R_k q_k Mf/S_k\right],$$

with $u(\mathbf{q}) \geq 0$. Moreover, $p_0(\mathbf{q}) > 0$ if and only if $\sum_{k\in K} \theta_k/q_k < 1$, in which case $u(\mathbf{q}) = 0$. The unique equilibrium is globally evolutionary stable.

Remark In Proposition 1, $u(\mathbf{q})$ captures the utility level obtained by each person at the second-stage equilibrium when the police assignment is $\mathbf{q}$. In addition, requiring $\sum_{k\in K} \theta_k/q_k < 1$ is the same as assuming $\sum_{k\in K} (S_k)^2 A_k/R_k Nfq_k < M$. This means that, in our model, some people will opt not to commit a crime ($p_0(\mathbf{q}) > 0$) only if the mass of police is large enough.

Essentially, the second stage of our game can be thought of as a congestion game with a continuum of players and it belongs to the class of population potential games. Sandholm (2001) defines such games as those that admit a continuously differentiable function whose gradient equals the payoff vector of the alternative choices. This function receives the name of potential function and, when it exists, is uniquely defined up to an additive constant. In our case, the potential function can be expressed as follows

$$\mathcal{W}(\mathbf{p},\mathbf{q}) = \int_0^{p_0} \mathcal{U}(0)\,dt + \sum_{k\in K} \int_{\varepsilon}^{p_k} \mathcal{U}(k,t,q_k)\,dt, \tag{1}$$

with $\varepsilon > 0$.[8] Characterizing the game via a potential function is useful as there is a one-to-one relationship between the local maxima of this function and the Nash equilibria of the underlying game. Specifically, in our model, (1) is strictly concave in $\mathbf{p}$ on Δ^{K_0}. Therefore, it admits a unique maximizer. It follows that $\mathbf{p}(\mathbf{q})$ is a Nash equilibrium if and only if:

$$\mathbf{p}(\mathbf{q}) = \left\{\mathbf{p}' \in \Delta^{K_0} : \mathbf{p}' = \arg\max_{\mathbf{p}} \mathcal{W}(\mathbf{p},\mathbf{q})\right\}.$$

Note that our expression for $\mathbf{p}(\mathbf{q})$ in Proposition 1 derives from the Kuhn-Tucker conditions related to the corresponding constrained optimization problem. As uniqueness, global stability follows by the strict concavity of the potential function $\mathcal{W}(\mathbf{p},\mathbf{q})$ in (1).[9]

Proposition 1 shows that the criminal activity in a certain location increases with the perceived productivity of the area and decreases with both its apprehension risk and the amount of police force. These results allow us to determine the patterns of displacement of criminal activity as the public authority changes the initial police allocation.

Displacement We let $Q \equiv \left\{\mathbf{q} \in \Delta^K : \sum_{k\in K} \theta_k/q_k < 1\right\}$ and assume this set is non-empty.[10] For all $\mathbf{q} \in Q$ and all $k \in K$, we get $\mathcal{U}(k, p_k(\mathbf{q}), q_k) = 0$ and $p_k(\mathbf{q}) = \theta_k/q_k$. That is, when the mass of police is large enough, the outside option not to commit a crime regulates the second-stage equilibrium payoffs for potential offenders and the level of criminal activity in

[8] ε is an arbitrarily small constant we added to the integral for it to be well-defined.

[9] A vector $\mathbf{p}$ is globally evolutionary stable if $(\mathbf{p}-\mathbf{s})\left(\mathcal{U}(0), (\mathcal{U}(k,p_k,q_k))_{k\in K}\right)' < 0$ for all $\mathbf{s} \neq \mathbf{p}$, $\mathbf{s} \in \Delta^{K_0}$ (see Sandholm (2010)).

[10] It can be easily shown that Q is a convex set.

each area depends only on the amount of police assigned to that specific area (rather than on the whole vector of police allocation). This means that if q_k increases then location k becomes less attractive to potential offenders. Consequently, some of them will opt not to commit a crime. Note that increasing q_k in and of itself does not induce any initial displacement of criminal activity from area k to other areas. However, we do observe an increase in crime in other areas due to the removal of police from the latter. In other words, the displacement effect occurs in our model as in order to increase the police force in area k, the law enforcement agency has to reduce it in other areas which then experience increased crime rates.

The remark below Proposition 1 explains that $p_0(\mathbf{q}) > 0$ if and only if the mass of police is large enough. As a thought experiment, let us assume the opposite holds. For small changes in $\mathbf{q}$, while police allocation would still affect the location of criminal activity, the overall level of criminality would remain at its highest possible value. This result resembles the Pigou-Downs-Knight paradox found in studies of traffic congestion.

Pigou-Downs-Knight Paradox This paradox reflects the observation that in traffic congestion expanding road capacity in a given route may have no impact on travel time. To illustrate this phenomenon let us consider a scenario with two possible routes to commute from city A to city B. Suppose that route 1, a bridge, takes 10 minutes with no traffic. Furthermore, travel time increases linearly with the ratio of traffic flow (F_1) to bridge capacity (C_1). By contrast, route 2 always takes 15 minutes (T_2), regardless of the traffic levels. These scenarios can be represented, respectively, by:

$$T_1 = 10 + 10\,(F_1/C_1) \quad \text{and} \quad T_2 = 15.$$

Next suppose there are 1,000 travelers faced with the choice of route 1 or 2. If the bridge capacity is defined at a level lower than 2,000 travelers, then the travel flow for route 1 adjusts at $(1/2)C_1$, so that the travel time for each route is always 15 minutes.[11] This means that a small increase in bridge capacity in route 1 (when $C_1 < 2,000$) will have no effect on that route's travel time.

[11] This follows from the implicit function theorem.

This paradox can be applied to our model of crimes. Instead of focusing on travel time, we are interested in the amount of people who choose not to commit a crime. In the traffic scenario, a small change in location attributes (when $C_1 > 2,000$) affects people's payoffs but not their choices. Similarly, in our model, a small change in the police force in a certain area (when $M < \sum_{k\in K} (S_k)^2 A_k/R_k N f q_k$) impacts utility levels but not the number of people who choose not to commit a crime.

As mentioned before, the literature on criminality defines a hot spot as an area with an above-average level of criminal activity relative to the entire space. Assuming $p_0(\mathbf{q}) > 0$, we get from Proposition 1 that area k is a so-called hot spot if and only if:

$$A_k/R_k e_k > (1/K) \sum_{k\in K} A_k/R_k e_k.$$

Thus, in our model, a hot spot depends on both the productivity-to-risk ratio of the area and its density of police. In the next section we solve the game by finding the optimal police allocation for reducing overall criminal activity. In particular, we are interested in learning whether a public authority with the goal of efficiency should sequentially target hot spots until they disappear. That is, we are interested in determining whether the following equation holds always at equilibrium:

$$A_k/R_k e_k^* = (1/K) \sum_{k\in K} A_k/R_k e_k^*.$$

By extension, if it does not hold, we would like to measure how much an egalitarian enforcement agency loses in terms of efficiency and to explore the determinants of this loss.

Example 1 illustrates our conclusions so far. We return to this example in later sections.

3.2 Efficient and Egalitarian Police Assignments

The last sub-section described the behavior of potential offenders given different police assignments. Using this result we now characterize the optimal distribution of police ($\mathbf{q}^* \equiv (q_k^*)_{k\in K}$) for a public authority whose goal is to reduce the overall rate of criminal activity. We first define $\underline{M} \equiv \left(\sum_{k\in K} S_k \sqrt{A_k/R_k}\right)^2 / Nf$.

Proposition 2 (Efficient Allocation) *Let $M > \underline{M}$. Then, for each $k \in K$,*

$$q_k^* = \sqrt{\theta_k} / \sum_{k \in K} \sqrt{\theta_k} \quad \textit{and} \quad p_k^* = \sqrt{\theta_k} \sum_{k \in K} \sqrt{\theta_k}.$$

Remark $\underline{M}$ can be defined as the minimum mass of police such that $p_0^* > 0$. We assume $M > \underline{M}$ as, otherwise, the problem of the enforcement agency is trivial (see the Pigou-Downs-Knight paradox in the last section).

Proposition 2 indicates that the optimal amount of police in each area depends on both the productivity-to-risk ratio and the size of the area. Its proof is as follows. When the mass of police is large enough, there is a (convex) set of alternative police assignments that force the utility derived from committing a crime to be zero for each potential offender. This guarantees that both $p_0(\mathbf{q}) > 0$ and, for each $k \in K$, $p_k(\mathbf{q}) = \theta_k / q_k$. We can see from the latter expression that the equilibrium strategies of the game among people induce a level of criminality in area k that depends on only the amount of police assigned to *that* location. The problem of the enforcement agency can thus be posed as follows:

$$\max_{\mathbf{q}} \left\{ 1 - \sum_{k \in K} \theta_k / q_k : \mathbf{q} \in \Delta^K \right\}.$$

In this scenario, slightly increasing q_k pushes some potential offenders to the outside option. Given that the overall mass of police is fixed, increasing q_k can only be done by decreasing the police force in other areas. Consequently, an allocation strategy is optimal when the effect of increasing the mass of police in a given area does not differ across locations. In this context, $\mathbf{q}^*$ can be easily obtained.

As mentioned, Proposition 2 states that the optimal amount of police increases with both the productivity-to-risk ratio of an area and its size. Upon a simple calculation, this proposition also implies that, at equilibrium, the ratio of criminal densities of Area k versus Area l is given by:

$$d_k^* / d_l^* = \sqrt{A_k / R_k} / \sqrt{A_l / R_l}.$$

That is, though the public authority makes a greater effort in areas that are *a priori* more attractive to offenders, this extra effort is not enough to eliminate the effects of their initial attribute differences. Therefore, areas that are *a priori* more attractive remain so with an efficient allocation and our model contains hot spots as an equilibrium outcome.

Hot Spots Let $M > \underline{M}$. Area k is a hot spot at the efficient equilibrium if and only if:

$$\sqrt{A_k/R_k} > (1/K)\sum_{k\in K}\sqrt{A_k/R_k}.$$

This specification indicates that, while overall crime levels will decrease in an efficient equilibrium, some hot spots will remain. By contrast, we next examine the case in which the goal of the enforcement agency is to obtain an even distribution of criminal activity across all areas (i.e., $d_k^{**} = d_l^{**}$ for all $k, l \in K$).

We first assume that the available mass of police is large enough to induce some people to opt for the outside option at the egalitarian allocation. This result holds for all $M > \underline{M}' \equiv \left(\sum_{k\in K} S_k\sqrt{A_k/R_k}\right)^2/Nf$. Under this assumption, we obtain the following specification, for each $k \in K$:

$$q_k^{**} = (\theta_k/S_k)\,/\sum_{k\in K}(\theta_k/S_k)\,.$$

Comparing this egalitarian policy with the efficient allocation strategy, we obtain:

$$e_k^{**}/e_l^{**} = (A_k/R_k)\,/\,(A_l/R_l) > \sqrt{(A_k/R_k)}/\sqrt{(A_l/R_l)} = e_k^*/e_l^*,$$

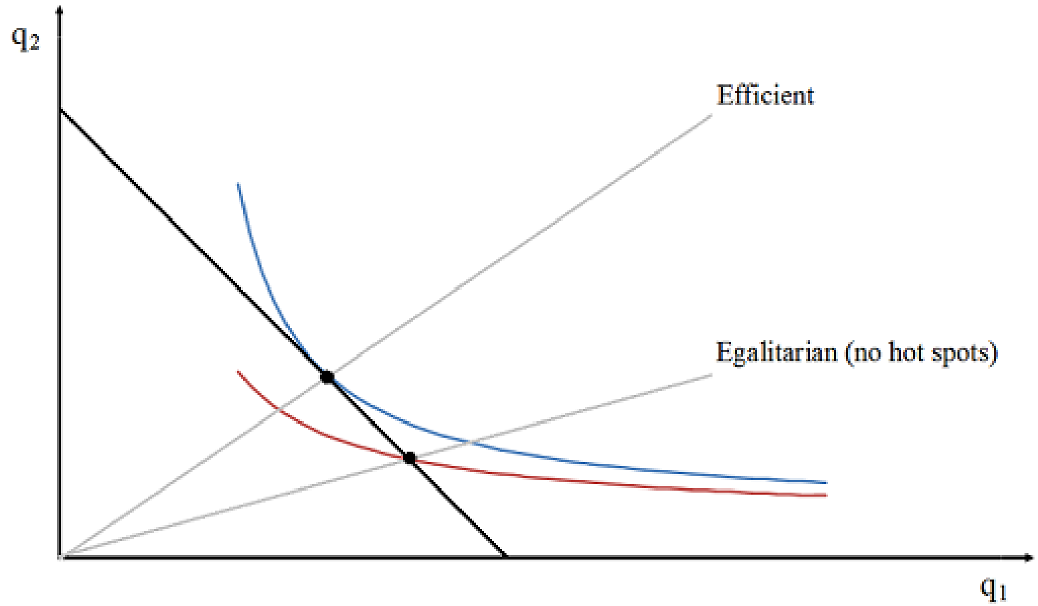
whenever $(A_k/R_k) > (A_l/R_l)$. That is, the egalitarian approach targets areas that are *a priori* more attractive to offenders more intensively than does a public authority who aims to reduce overall crime levels. This leads to our next proposition.

Proposition 3 (Opportunity Cost of the Egalitarian Allocation) *Let $M > \underline{M}'$. The opportunity cost of equity in terms of overall crime levels is given by*

$$p_0^* - p_0^{**} = (1/MNf)\sum_{k<l} S_k S_l\left(\sqrt{A_k/R_k} - \sqrt{A_l/R_l}\right)^2.$$

Proposition 3 indicates that the opportunity cost (in terms of criminal activity) of an equity oriented policy increases with the variability of productivity-to-risk ratios across locations.

Figure 1: Efficient versus Egalitarian Police Allocation Strategies



Example 1: Let $K = \{1, 2\}$, $A_1 = 8$, $A_2 = 4$, $R_1 = 1$, $R_2 = 2$, $S_1 = S_2 = 1$, $f = 1$ and $NM > 20$. Thus, Area 1 is both more productive and less risky than Area 2. Furthermore:

$$\mathcal{U}(1, p_1, q_1) = 8/Np_1 - Mq_1 \text{ and } \mathcal{U}(2, p_2, q_2) = 4/Np_2 - 2Mq_2.$$

Given that $MN > 20$, by Proposition 1, the efficient policy solves:

$$\max_{q_1,q_2} \{1 - (\theta_1/q_1 + \theta_2/q_2) : 0 \le q_1 \le 1, 0 \le q_2 \le 1, q_1 + q_2 = 1\} \tag{2}$$

with $\theta_1 = 8/MN$ and $\theta_2 = 2/MN$. Figure 1 exhibits a graphical representation of this result. In Figure 1, the constraint set is denoted by the bold line. Furthermore, the two curves can be thought of as indifference curves: each displays combinations of q_1 and q_2 that induce the same level of criminal activity and higher indifference curves are associated with lower crime levels.

We define an efficient allocation as one that occurs whenever the marginal efficacy of police resources is the same across locations. Upon a simple calculation, we obtain that this holds whenever

$$q_2 = \sqrt{\theta_1/\theta_2}q_1.$$

Using the fact that $q_1^* + q_2^* = 1$, we get $q_1^* = 2/3$ and $q_2^* = 1/3$. This police assignment corresponds to the upper-left intersection in Figure 1. Under this allocation, $p_1^* = 12/NM > 6/NM = p_2^*$ meaning that Area 1 is a hot spot.

In contrast to the efficient allocation strategy, the egalitarian policy satisfies the following condition:

$$q_2 = (\theta_1/\theta_2)\, q_1.$$

Using the constraint, we obtain $q_1^{**} = 4/5$ and $q_2^{**} = 1/5$. This police assignment corresponds to the lower-right intersection in Figure 1. Note that these two intersections coincide if only if $\theta_1 = \theta_2$. Under the egalitarian allocation, $p_1^{**} = p_2^{**} = 10/NM$; thus there are no remaining hot spots. However, as Figure 1 shows the egalitarian allocation is on a lower indifference curve. The opportunity cost of this policy in terms of crime level is $2/NM$.

Note that, if we increase either the penalty in case of being caught (f) or the amount of police (M), then θ_1 and θ_2 each decrease by the same percentage. Thus, while neither scenario change the police allocations, the indifference curves get re-leveled with the induced criminal activity curves shifting downward. Alternatively, if we reduce θ_1 by reducing productivity (A_1) and/or increasing riskiness (R_1), then criminal activity shifts downward by $1/q_1^*$ (this follows by applying the envelope theorem to expression (2)). This change flattens the indifference curves, so that now both the efficient and the egalitarian allocations entail a lower q_1 and a higher q_2. This means that structural changes in Area 1 have beneficial spillover effects on crime levels in Area 2 via subsequent police re-allocation. The discussion in the next section elaborates on this argument. $\square$

4 Modifying the Attributes of the Areas

This section extends our model to consider an enforcement agency that aims to change the characteristics of places that give rise to criminal opportunities. Specifically, we are interested in two questions: (i) What is the effect of changing the attributes of an area on the overall rate of criminal activity? and (ii) What is the impact on the criminal level of the areas not directly benefited by such a policy?

The question of how best to reduce criminal activity has received both academic and practical consideration. For example, Braga and Wisburd (2010) state that the aim of place-based policy strategies to fight crime should go beyond hot spot policing. As they state,

> "We should solve the conditions and situations that give rise to the criminal opportunities that sustain high-activity crime places."

Similarly, public authorities have enacted a number of area changes to increase apprehension risk or decrease productivity potential. These measures include improving the lighting in dark areas or inking store merchandise.

To represent such initiatives, recall that (assuming $M > \underline{M}$) the problem of the enforcement agency regarding the allocation of police resources is as follows:

$$\max_{\mathbf{q}} \left\{ p_0 = 1 - \sum\nolimits_{k \in K} \theta_k / q_k : \mathbf{q} \in \Delta^K \right\}. \tag{3}$$

Note that lowering the productivity-to-risk ratio in area k is similar to decreasing θ_k. By applying the envelope theorem on (3), we then obtain:

$$\partial p_0^* / \partial \left(-\theta_k\right) = 1/q_k^*.$$

Thus, reducing θ_k in any given area increases the number of people who opt not to commit a crime. We next elaborate on the mechanism by which this change happens.

Specifically, there are two forces behind the last result which reinforce each other. First, the target area becomes less attractive to potential offenders and thus its criminal activity naturally diminishes. Second, by using Proposition 2 we get that, for each $l \neq k$:

$$\partial q_l^* / \partial(-\theta_k) = (1/2) \left(q_l^*\right)^2 / \sqrt{\theta_l \theta_k} > 0 \quad \text{and} \quad \partial p_l^* / \partial(-\theta_k) = -(1/2)\, p_l^* / p_k^* < 0.$$

This means that the police force is optimally re-allocated from area k to the subsequent ones, thereby reducing the criminal activity in these areas as well. That is, structural changes in a certain area have beneficial spillover effects on all other locations via subsequent police re-allocations.

5 Extensions of the Model

5.1 Outside Option and the Labor Market

In our model, the expected payoff of the outside option (not to commit a crime) is assumed to be 0. This restriction simplifies our exposition without changing the two main implications, namely, the nature of the displacement of criminal activity and the characterization of hot spots. Nevertheless, it impacts both the effectiveness of the public authority in reducing the overall crime rate and the optimal police allocation. To formalize this effect, we extend Proposition 1 with $\mathcal{U}(0) \equiv c$, so that c measures the opportunity cost of committing a crime. This specification leads to the next result.

Proposition 4 *Let $M > \underline{M} / \left(1 + c \sum_{k \in K} \gamma_k\right)$ with $\gamma_k = S_k / R_k M f$. Then, for each $k \in K$,*

$$q_k^{***} = q_k^* \left(1 + c \textstyle\sum_{k \in K} \gamma_k\right) - c\gamma_k \quad \textit{and} \quad p_k^{***} = p_k^* / \left(1 + c \textstyle\sum_{k \in K} \gamma_k\right)$$

where $\underline{M}$, q_k^* and p_k^* are defined as in Proposition 2.

Proposition 4 shows that a higher opportunity cost of committing a crime facilitates the condition under which $p_0(\mathbf{q}) > 0$ and reduces the level of criminality in all locations.

As mentioned, the outside option could be thought of as the possibility to work in a legal activity. Under this interpretation, the number of people who opt not to commit a crime comprises the labor supply in the economy. Thus, an increase in c could correspond to either a decrease in the unemployment rate or an increase in the minimum wage. We can then use Proposition 4 to suggest that criminal activity decreases when labor market conditions improve. Alternatively, improvements in the attributes of areas would also have a positive impact on the labor supply of the economy.

5.2 Congestion Effects in the Outside Option

Our previous analysis rules out the possibility of congestion effects in the outside option. However, we can imagine a simple mechanism by which the opposite is true. For instance, when the number of people hoping to obtain a legal job increases, this may push salaries down or increase unemployment, thereby making this outside option less attractive. In this section, we incorporate congestion effects by assuming $\mathcal{U}(0, p_0) \equiv A_0/d_0$ and address two interesting implications of this modeling assumption.

Under this specification the model does not have a closed form solution neither for the people choices conditional on police assignments nor for the optimal police allocation. Nevertheless, it still delivers relevant information regarding both the displacement mechanism and the characterization of hot spots at the optimal allocation of police resources. We start by describing the implication of congestion effects on the displacement mechanism.

Proposition 5 *Let* $\mathcal{U}(0, p_0) \equiv A_0/d_0$ *and* $Q \equiv \left\{\mathbf{q} \in \Delta^K : q_k > 0, k \in K\right\}$*. For all* $\mathbf{q} \in Q$*, all* $k \in K$ *and all* $m \in K$ *with* $m \neq k$*, we get* $\partial p_k(\mathbf{q}) / \partial q_k \leq 0$ *and* $\partial p_m(\mathbf{q}) / \partial q_k \geq 0$.

Proposition 5 states that more police resources in area k reduce its criminal activity, but they increase the criminal level in all other locations. The reason is as follows: When q_k increases, location k becomes less attractive to potential offenders, which pushes some criminals to the outside option. When the value of this outside option is independent of the number of people who decide not to commit a crime, there are no further consequences. However, when the outside option displays congestion effects, the value of not to commit a crime decreases, incentivizing people to commit crimes in other locations. This has the unfortunate effect of shifting $p_m(\mathbf{q})$ up in all other areas.

We next describe an econometric challenge raised by Proposition 5.

Estimates of Crime-Reducing Effect of Police Academics have long studied the relationship between the scale of policing and the level of criminal activity by using panel data. The first few studies on this issue did not find evidence of a strong causal effect of police on crimes. As Levitt and Miles (2007) explain, one of the reasons behind such disappointing result is that early studies did not take into account a simultaneity bias. Namely, jurisdictions with higher crime rates react by hiring more police, and this response induces a positive cross-sectional correlation between police and crimes. Marvell and Moody (1996) and Levitt (1997) address this difficulty by using an approach based on Granger causality, and Lazzati (2012) proposes a partial identification approach that relies on the use of police resources as a monotone instrumental variable.[12] Proposition 5 poses a new identification challenge. Under congestion effects in the outside option, the crime rate in each area depends not only on the police resources assigned to that location but on the whole vector of police allocation. That is, any study that uses cross-sectional data to evaluate the effect of police on crimes should be based on a simultaneous equations approach.

The next result shows that some hot spots remain at the optimal police allocation. It also states that whether an area is a hot spot depends on its productivity-to-risk ratio in the same way as when $U(0, p_0) \equiv 0$.

Proposition 6 *Let $\mathcal{U}(0, p_0) \equiv A_0/d_0$. Area k is a hot spot at the efficient equilibrium if and only if:*

$$\sqrt{A_k/R_k} > (1/K) \textstyle\sum_{k \in K} \sqrt{A_k/R_k}.$$

This proposition shows that equilibrium hot spots are a robust feature of our model.

5.3 Complementarities in Criminal Activity

In the previous analysis, the game induced in the second stage displays negative interactions among potential offenders. We next evaluate the consequences of an alternative specification.

The overall expected utility of an offender in location k is given by:

$$\mathcal{U}(k, p_k, q_k) = \mathcal{Y}(k, p_k, q_k) - \mathcal{P}(k, p_k, q_k) f.$$

Differentiating this expression with respect to p_k, we obtain:

$$\partial \mathcal{U}(k, p_k, q_k) / \partial p_k = \partial \mathcal{Y}(k, p_k, q_k) / \partial p_k - (\partial \mathcal{P}(k, p_k, q_k) / \partial p_k) f. \qquad (4)$$

We expect both derivatives on the right-hand-side of (4) to be (weakly) negative. Specifically, congestion effects in the rewards are expected as the higher the number of criminals in an area,

[12] See also MacCray and Chalfin (2012).

the lower the piece of the pie for each offender. Congestion effects in costs are also expected as the police cannot be in two different places at the same time. Therefore, the higher the number of criminals in a given area, the lower the probability that any one of them is apprehended (Freeman et al. (1996) and Sah (1991)). The sign of the total effect then depends on the relative size of these two forces. That is, for each $k \in K$:

$$\partial \mathcal{U}(k, p_k, q_k) / \partial p_k \geq (\leq) \, 0 \text{ if } |\partial \mathcal{Y}(k, p_k, q_k) / \partial p_k| \leq (\geq) \, |\partial \mathcal{P}(k, p_k, q_k) / \partial p_k| \, f.$$

In our previous analysis, the second term dominated the first one thereby inducing a congestion game among potential offenders. When the opposite holds, the second-stage game is a game of strategic complements. Such supermodular games often display multiple equilibria and involve coordination problems. In our case, people may coordinate in the same option and police allocation choices can easily affect the one selected. The possibility that policy interventions may affect equilibrium selection is well-described by Blume (2006) for a discrimination model. The next example applies this phenomenon to our model of crimes.

Example 2: Let $K = \{1, 2\}$, $N = 1$, and $M = 1$. We further assume that $\mathcal{U}(0) = 0$. In addition,

$$\mathcal{U}(1, p_1, q_1) = 1/p_1 - (q_1 + 1/2) / (p_1)^2 \text{ and } \mathcal{U}(2, p_2, q_2) = 1/p_2 - 4(q_2 + 1/2) / (p_2)^2.$$

In this example, Area 2 shows greater apprehension risk than Area 1 and the second stage of the game displays strategic complementarities.

Specifically, when $q_1 = q_2 = 1/2$ then the second-stage game has two Nash equilibria: $\mathbf{p}(1/2, 1/2) \in \{(1, 0, 0), (0, 1, 0)\}$. While these two equilibria imply the same level of utility for people—zero—the last one is far riskier for potential offenders. The reason is that the first equilibrium guarantees each person a payoff of zero independently of what other people choose. Alternatively, the second equilibrium gives each offender a payoff of zero if and only if all other people select to commit a crime in Area 1. Otherwise, the payoff is negative. Thus, when $q_1 = q_2 = 1/2$, choosing not to commit a crime is a dominant strategy and it is therefore reasonable to predict that everyone will choose this option.

Finally, we assume the public authority assigns all police force to the riskier area, so that $q_1 = 0$ and $q_2 = 1$. Though the equilibrium set does not change, the two predictions differ regarding expected payoffs. While the payoff of coordinating not to commit a crime is zero, the payoff of coordinating to commit a crime in Area 1 is $1/2$ for each offender. Therefore, it may now be more reasonable to predict that people will coordinate in the second equilibrium. $\square$

Given the previous analysis, whether crime decisions are substitute or complement is ultimately an empirical questions with relevant policy implications. De Paula and Tang (2012) and Aradillas-Lopez and Gandhi (2012) provide theoretical results on identification of signs of interaction effects in games. We believe their work could be very useful in addressing the previous question. This would provide fundamental insights for developing further theoretical and empirical research in the area of crimes.

6 Conclusion

Crimes rates fell sharply in US during the 1990s, including both violent and property crimes. In New York City the fall was so sharp that the media often refers to the phenomenon as the New York "miracle." The drop in crime rates has triggered crime experts to investigate its roots and reasons. There were so many changes and policies implemented during this time period that it is hard to sort out how successful each policy was at lowering the crime rate. Levitt (2004) evaluates frequently cited reasons for the crime decline in articles in major newspapers over the 1990s. He presents a list of six factors which has innovative policing strategies at the top and increased number of police as the least cited factor among the six. While he finds innovative policing strategies do not appear to have played an important role in the drop in crime, he suggests increased number of police may have been an important determinant. In our study, we go one step back and evaluate the theoretical soundness of hot spot policing as an effective policing strategy to crime prevention.

The main argument offered by those against re-directing police resources to hot spots is that doing so would simply displace criminal activity from one area to another without reducing overall crime. The empirical evidence for hot spot policing shows that increasing police resources in areas with high crime levels reduces their criminal activity without increasing criminality in nearby locations. While these results show that police resources are actually effective at fighting crime in hot spots, these analyses do not provide evidence that hot spot policing is a sensitive strategy to re-allocate the limited police resources. In evaluating hot spot policing the question is not whether police is effective at reducing crimes in hot spots, but rather whether limited police resources are more effective at fighting crime in hot spots as compared to doing so in alternative locations. We find that while police resources are initially more effective in areas that are *a priori* more attractive to offenders (i.e., display a larger productivity-to-risk ratio), their relative effectiveness is reversed before an even distribution of crimes across areas is reached. In other words, in our study some hot spots remain at the optimal police allocation. This result has direct policy implications, suggesting that further hot spot policing implementations should be carefully considered in terms of its ultimate objectives.

Regarding the displacement effect we find that it crucially depends on whether there are congestion effects in the outside option. This result is particularly important for the econometric studies that aim to measure the effect of police on crime rates using panel data. Specifically, under the presence of congestion effects in the outside option, then any of such studies should take into account that the level of crime in each area depends on the whole vector of police allocation.

Along the study, we view the distribution of criminal activity across areas as an endogenous outcome which depends on both the attributes of the respective locations and the initial police allocation assignments. Indeed, one drawback of the current practice of characterizing hot spots as a guide for police assignments is that this characterization is affected by the same policy that it is trying to guide. In short, a designated hot spot may actually be a location police avoids, as illustrated below:

"Dark Place was a "hot spot" of crime. It was so hot that the police said they stayed away from it as much as possible, unless they got a call." (Sherman (1995))

To better characterize differences in locations, we propose an index function based on structural factors that affect the productivity-to-risk ratios. In addition, we believe that interactions among choices of potential offenders should be taken into account to correctly asses counterfactual predictions of police re-allocations.

Last but not least, we explore the impact of area attribute changes on crime levels, as suggested by Braga and Wisburd (2010). Specifically, they propose that public authorities should consider

"Alternative prevention strategies such as razing abandoned buildings, controlling access to venues, target hardening, and protecting repeat victims..."

We find that improvements in an area attributes not only reduce its crime rate but also have positive spillovers on all other locations via subsequent police reallocation.

Overall, the main results in our paper provide useful insights that could guide both new policy strategies and further empirical research on the issues of crimes and crime reductions in modern societies.

7 Appendix: Proofs

Proof of Proposition 1 By Sandholm (2001), $\mathbf{p}(\mathbf{q})$ is a Nash equilibrium in the second stage of the game if it satisfies the Kuhn-Tucker conditions for the Lagrangian

$$\mathcal{L}(\mathbf{p},\mathbf{q}) = \int_0^{p_0} \mathcal{U}(0)\, dt + \sum_{k\in K} \int_\varepsilon^{p_k} \mathcal{U}(k,t,q_k)\, dt + \sum_{k\in K_0} \varphi_k p_k + \lambda\left(1 - \sum_{k\in K_0} p_k\right).$$

That is, if $(\mathbf{p}(\mathbf{q}), \boldsymbol{\varphi}(\mathbf{q}), \lambda(\mathbf{q}))$ satisfies the following conditions:

$$\begin{aligned}
&A_k S_k/N p_k(\mathbf{q}) - R_k M f q_k/S_k = -\varphi_k(\mathbf{q}) + \lambda(\mathbf{q}), \text{ for all } k \in K,\\
&0 = -\varphi_0(\mathbf{q}) + \lambda(\mathbf{q})\\
&\varphi_k(\mathbf{q}) \geq 0, p_k(\mathbf{q}) \geq 0 \text{ and } \varphi_k(\mathbf{q})\, p_k(\mathbf{q}) = 0, \text{ for all } k \in K_0,\\
&\lambda(\mathbf{q}) \geq 0 \text{ and } \left(1 - \sum_{k\in K_0} p_k(\mathbf{q})\right) = 0.
\end{aligned}$$

It is readily verified that the non-negativity constraints are non-binding, i.e., $\varphi_k^* = 0$, for all $k \in K$. Thus, the previous conditions reduce to:

$$\begin{aligned}
&A_k S_k/N p_k(\mathbf{q}) - R_k M f q_k/S_k = \lambda(\mathbf{q}) \text{ for all } k \in K\\
&\varphi_0(\mathbf{q}) = \lambda(\mathbf{q})\\
&\varphi_0(\mathbf{q}) \geq 0, p_0(\mathbf{q}) \geq 0 \text{ and } \varphi_0(\mathbf{q})\, p_0(\mathbf{q}) = 0\\
&p_k(\mathbf{q}) \geq 0 \text{ for all } k \in K\\
&\lambda(\mathbf{q}) \geq 0 \text{ and } \sum_{k\in K_0} p_k(\mathbf{q}) = 1.
\end{aligned}$$

As a consequence, we need to consider only two cases, namely, $\varphi_0(\mathbf{q}) > 0$ and $\varphi_0(\mathbf{q}) = 0$.

We first suppose $\varphi_0(\mathbf{q}) > 0$. Then $p_0(\mathbf{q}) = 0$ and $p_k(\mathbf{q}) = S_k A_k / N(\lambda(\mathbf{q}) + R_k q_k M f/S_k)$, for all $k \in K$. However, this is possible if and only if there exists $\lambda(\mathbf{q}) > 0$, such that $\sum_{k\in K} p_k(\mathbf{q}) = 1$. Note that$\sum_{k\in K} p_k(\mathbf{q})$ is decreasing in $\lambda(\mathbf{q})$ and $\sum_{k\in K} p_k(\mathbf{q}) \to 0$ as $\lambda(\mathbf{q}) \to \infty$. Thus, by the intermediate value theorem, this condition holds if and only if $\sum_{k\in K} \theta_k/q_k \geq 1$.

We now suppose $\varphi_0(\mathbf{q}) = 0$. This yields the following equation:

$$p_k(\mathbf{q}) = (S_k)^2 A_k/N R_k q_k M f = \theta_k/q_k,$$

for all $k \in K$. Since $p_0(\mathbf{q}) \geq 0$, then $\sum_{k\in K} \theta_k/q_k \leq 1$ with strict inequality if $p_0(\mathbf{q}) > 0$.

Uniqueness follows as the potential function is strictly concave in $\mathbf{p}$ on Δ^{K_0}. Sandholm (2010) shows that global evolutionary stability follows by the same condition. $\square$

Proof of Proposition 2 For M large enough, any efficient police allocation satisfies the following condition:

$$\mathcal{U}\left(k, p_k\left(\mathbf{q}\right), q_k\right) = 0.$$

Thus, for all $k \in K$,

$$p_k\left(\mathbf{q}\right) = \theta_k / q_k.$$

The problem of the public authority can then be posed as:

$$\max_{\mathbf{q}} \left\{1 - \sum_{k\in K} \theta_k / q_k : \mathbf{q} \in \Delta^K\right\}. \tag{5}$$

In equation (5), the objective function is differentiable and strictly concave for all $\mathbf{q}$ in the interior of Δ^K. Thus, the solution to (5) exists and is unique. Moreover, $\mathbf{q}^*$ is an equilibrium if it satisfies the Kuhn-Tucker conditions for the Lagrangian:

$$\mathcal{L}\left(\mathbf{q}\right) = 1 - \sum_{k\in K} \theta_k / q_k - \sum_{k\in K} \varphi_k q_k - \lambda\left(\sum_{k\in K} q_k - 1\right).$$

In this case, $(\mathbf{q}^*, \boldsymbol{\varphi}^*, \lambda^*)$ satisfies the following conditions:

$$\begin{aligned}
&\theta_k / \left(q_k^*\right)^2 - \varphi_k^* = \lambda^*, \text{ for all } k \in K,\\
&\varphi_k^* \geq 0, q_k^* \geq 0 \text{ and } \varphi_k^* q_k^* = 0, \text{ for all } k \in K,\\
&\lambda^* \geq 0 \text{ and } \left(1 - \sum_{k\in K} q_k^*\right) = 0.
\end{aligned}$$

It is readily verified that the non-negativity constraints are non-binding, i.e., $\varphi_k^* = 0$ for all $k \in K$. The characterization of $\mathbf{q}^*$ follows through a simple calculation.

To find $\underline{M}$, note that $\sum_{k\in K} \theta_k / q_k^* < 1$ needs to hold for $p_0^* > 0$ to be true. That is,

$$\left(\sum_{k\in K} S_k \sqrt{A_k / R_k}\right)^2 / NMf < 1.$$

It follows that $\underline{M} = \left(\sum_{k\in K} S_k \sqrt{A_k / R_k}\right)^2 / Nf$. $\square$

Proof of Proposition 3 From the previous analysis we know that:

$$p_0^* = 1 - \left(\sum_{k \in K} \sqrt{\theta_k}\right)^2 \quad \text{and} \quad p_0^{**} = 1 - \sum_{k \in K} S_k \sum_{k \in K} \theta_k / S_k.$$

Then,

$$p_0^{**} - p_0^* = \left(\sum_{k \in K} \sqrt{\theta_k}\right)^2 - \sum_{k \in K} S_k \sum_{k \in K} \theta_k / S_k.$$

By applying the Multinomial Theorem to the first term in the right hand side and expanding the second term, the last expression takes the form of:

$$\begin{aligned} p_0^{**} - p_0^* &= \sum_{k \in K} \theta_k + \sum_{k,l \in K, k \neq l} \sqrt{\theta_k}\sqrt{\theta_l} - \sum_{k \in K} \theta_k - \sum_{k,l \in K, k \neq l} \left(S_l / S_k\right) \theta_k \\ &= \sum_{k,l \in K, k \neq l} \sqrt{\theta_k}\sqrt{\theta_l} - \sum_{k,l \in K, k \neq l} \left(S_l / S_k\right) \theta_k. \end{aligned}$$

Since $\theta_k \equiv (S_k)^2 A_k / R_k M N f$, then:

$$\begin{aligned} p_0^{**} - p_0^* &= (1/MNf) \left[\sum_{k,l \in K, k \neq l} S_k S_l \sqrt{A_k / R_k}\sqrt{A_l / R_l} - \sum_{k,l \in K, k \neq l} S_k S_l \left(A_k / R_k\right)\right] \\ &= (1/MNf) \sum_{k,l \in K, k < l} S_k S_l \left(\sqrt{A_k / R_k} - \sqrt{A_l / R_l}\right)^2 \end{aligned}$$

which completes the proof. □

Proof of Proposition 4 The proof of this result is very similar to the proofs of Propositions 1 and 2, thus we omit it. □

Proof of Proposition 5 Under this specification, for all $\mathbf{q} \in Q$, people will re-distribute across options till the utility obtained in each of them is the same. Therefore, $p_0 = S_0 A_0 / N u(\mathbf{q})$ and, for each $k \in K$, we have:

$$p_k(\mathbf{q}) = S_k A_k / N \left[u(\mathbf{q}) + R_k q_k f M / S_k\right] \tag{6}$$

where $u(\mathbf{q})$ is the constant that solves $\sum_{k \in K_0} p_k(\mathbf{q}) = 1$. Differentiating (6) $p_k(\mathbf{q})$ and $p_m(\mathbf{q})$ with respect to q_k we get:

$$\begin{aligned} \partial p_k(\mathbf{q}) / \partial q_k &= -\left\{S_k A_k / N \left[u(\mathbf{q}) + R_k q_k f M / S_k\right]^2\right\} \left[\partial u(\mathbf{q}) / \partial q_k + R_k f M / S_k\right] \\ \partial p_m(\mathbf{q}) / \partial q_k &= -\left\{S_k A_k / N \left[u(\mathbf{q}) + R_k q_k f M / S_k\right]^2\right\} \partial u(\mathbf{q}) / \partial q_k. \end{aligned}$$

By the Implicit Function Theorem applied to $\sum_{k \in K_0} p_k(\mathbf{q}) = 1$ we get:

$$\partial u(\mathbf{q}) / \partial q_k = -\frac{\left\{S_k A_k / N \left[u(\mathbf{q}) + R_k q_k f M / S_k\right]^2\right\} R_k f M / S_k}{\sum_{k \in K} \left\{S_k A_k / N \left[u(\mathbf{q}) + R_k q_k f M / S_k\right]^2\right\} + S_0 A_0 / N u(\mathbf{q})^2} \leq 0.$$

Substituting the last expression in the previous two, we get $\partial p_k(\mathbf{q}) / \partial q_k \leq 0$ and $\partial p_m(\mathbf{q}) / \partial q_k \geq 0$. $\square$

Proof of Proposition 6 At the second stage equilibrium, $p_0 = S_0 A_0 / N u(\mathbf{q})$. Thus, maximizing p_0 is the same as selecting the vector $\mathbf{q}$ that minimizes $u(\mathbf{q})$. It follows that any optimal $\mathbf{q}^*$ must satisfy, for all $k, m \in K$:

$$\partial u(\mathbf{q}) / \partial q_k = \partial u(\mathbf{q}) / \partial q_k.$$

Using intermediate results from the proof of Proposition 5 we get that, for all $k, m \in K$:

$$\sqrt{A_k R_k} / \left[u(\mathbf{q}) + R_k q_k f M / S_k\right] = \sqrt{A_m R_m} / \left[u(\mathbf{q}) + R_m q_m f M / S_m\right] = H.$$

Thus, for each $k \in K$, we have:

$$N p_k(\mathbf{q}) / S_k = d_k = \sqrt{A_m / R_m} H \tag{7}$$

and the result follows immediately. $\square$

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