Mergers, Managerial Incentives, and Efficiencies*

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Abstract

We analyze the effects of synergies from horizontal mergers on managerial incentives. In contrast to synergies, efficiency gains resulting from managerial effort are not merger specific, i.e., they may be realized by all firms before and after a merger. We show that synergies suppress managerial incentives within the non-merging firms, whereas the effect on the merged firm critically depends on the number of agents employed by its principal. An important implication for merger policy is that consumer surplus may be monotonically decreasing in the synergy level, which opposes the use of an efficiency defense in merger control.

JEL-Classification: D21, D86, L22, L41

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1 Introduction

The existing literature on mergers relies on the presumption that firms are entrepreneurial, and it thereby neglects the agency relationships inherent in firms. On top of that, this literature largely presumes that productive efficiency gains from mergers are exogenous.[1] We build on this observation and, instead, consider managerial firms that are characterized by an agency relationship with ex post asymmetric information and whose productive efficiency is a result of managerial effort. When firms merge, efficiency gains may be additionally generated through synergies in the form of marginal cost reductions. As is standard in the related literature, synergies are per se merger specific and thus contrast productive efficiency gains from managerial effort, which can also be realized without a merger.[2] However, we specify that synergies are not automatically generated following a merger. Rather, we explicitly require the merging firms to combine their *core hard-to-trade assets*, which refer to managerial skills in our framework, in order to realize synergies.[3] That is, we follow Farrell and Shapiro's (2001) view which defines synergies as those productive efficiency gains *"in which firms truly combine their core hard-to-trade assets in new ways that lead to lower costs or improved quality."*

This paper focuses on two questions. First, we ask to which extent synergies from a merger affect managerial incentives within firms to cut marginal costs. Since productive efficiency gains from managerial effort are costly, in contrast to synergies (given the merger), one would expect that the merged firm's principal would strictly have an incentive to substitute managerial

[1]Exceptions are Banal-Estanol et al. (2008) and Jovanovic and Wey (2012). The former specify efficiencies as stemming from relationship-specific investments by managers, while the latter consider a technology adoption game where the merging firms both with and without the merger decide to adopt a more efficient technology.

[2]For example, the US merger guidelines define merger-specific efficiencies as "those efficiencies likely to be accomplished with the proposed merger and unlikely to be accomplished in the absence of either the proposed merger or another means having comparable anticompetitive effects." Note that according to both the US and the EU merger guidelines efficiencies have to be classified as verifiable and beneficial to consumers (both requirements define so-called "cognizable" efficiencies), in addition to being merger specific. If these three criteria are cumulatively met, then a claimed efficiency will be accepted.

[3]This prerequisite is not new; it has not been explicitly considered, but rather implicitly presumed so far. For instance, Farrell and Shapiro (1990) specify synergies as requiring the recombination of the merging firms' assets *"to improve their joint production capabilities,"* but do not explicitly account for such a recombination of assets in their model.

effort with synergies. However, we will see that the answer to this question is rather ambiguous. Second, we ask how synergies from a merger affect consumer surplus in the presence of managerial firms. Recall that existing papers on horizontal mergers, starting with the seminal work by Farrell and Shapiro (1990), as well as the efficiency defense in merger control rely on the supposition that synergies monotonically increase consumer surplus.[4] However, in the case of managerial firms, we will see that there may exist a negative relationship between consumer surplus and synergies, which undermines the efficiency defense in current merger control.

We use a general Cournot oligopoly model in which firms consist of a principal-agent pair. Whereas principals design incentives for their respective agents and set the output in the product market, agents can exert effort to reduce marginal costs.[5] The game consists of two stages. In the contracting stage, principals first offer contracts and then agents decide on their optimal effort level which is not verifiable and thus cannot be contracted upon. Subsequently, in the market stage, uncertainty is resolved and principals learn their costs. Finally, they compete in quantities. We solve the game by backward induction.

When a merger occurs, which we specify as being an acquisition rather than a merger of equals, the acquiring firm's principal can either keep only one agent or employ multiple agents at the same time. Since we specify the combination of *core hard-to-trade assets* as the necessary prerequisite for synergies, we postulate that the merged firm's principal must employ several agents in order to generate synergies. That is, synergies stem from the coexistence of several agents within one firm. Internal knowledge transfer between the merged firm's agents serves as one example which can be interpreted as a source of synergies. Moreover, we need to assume that each firm's agent has specific knowledge of the firm's production capabilities, which makes it hard to replace him. In addition to synergies, having more than one agent gives the merged firm's principal the ex post flexibility to choose between several uncertain production capabilities. That is, the principal will choose the agent who is associated with minimal costs.[6]

[4]See also, e.g., Besanko and Spulber (1993) and, for more recent papers, Nocke and Whinston (2010) and Nocke and Whinston (2012). Notice that Williamson's (1968) seminal paper offers the first work which proposes the use of an efficiency defense in merger control.

[5]Notice that managerial effort, as specified in our model, shows parallels to process innovations as treated by, e.g., d'Aspremont and Jacquemin (1988), Kamien et al. (1992), and, even more generally, Vives (2008).

[6]In that respect, our model shows parallels to a tournament, since only the most efficient ("best") agent is

We show that a merger without synergies always leads to stronger managerial incentives and productive efficiency gains within all firms in the market. However, the effect of a merger creating synergies is rather ambiguous. While synergies always exert a negative externality on managerial incentives within the non-merging firms, they decrease managerial incentives within the merged firm only if the number of its agents is too large. The latter follows from synergies increasing the merged firm's wage costs so that the principal substitutes managerial effort with synergies. Otherwise, i.e., whenever the number of employed agents is sufficiently low, synergies will trigger the merged firm's principal to induce her agents to work harder.

Though synergies are generated within the merged firm, it is not necessarily true that it will exhibit a higher productive efficiency than its rivals. The reason is that managerial effort within the merged firm may not only be reduced by synergies, but the principal's ex post flexibility to choose between multiple agents additionally frustrates her managers' effort. Overall, we demonstrate that productive efficiency gains are not exclusively realized by the merged firm: if synergies are sufficiently large (small), then it is only the merged firm (non-merging firms) which operates (operate) more efficiently after a merger.

We also offer an important implication for merger policy by showing that the relationship between consumer surplus and synergies is not strictly positive as presumed by the efficiency defense in current merger control. The reason is that, in addition to the direct positive *synergy effect*, which was already emphasized by Farrell and Shapiro's (1990) Lemma, synergies may reduce managerial incentives overall and may thus create a countervailing *incentive effect*. The sum of these two effects appears to be ambiguous. Finally, a linear Cournot model is applied to our general setup. We demonstrate that consumer surplus is decreasing in the synergy level if and only if initial marginal cost levels in the industry are sufficiently low.

In addition to the works on horizontal mergers, our paper is closely related to the literature on the effects of competition on managerial incentives. In general, this literature builds on the works by Hart (1983), Hermalin (1992), and Schmidt (1997) who were among the first to formalize the relationship between managerial incentives and competitive pressure.[7] Based on

chosen for production (see, e.g., Olsen, 1993, and Levitt, 1995).

[7]Further papers in this spirit are, e.g., Scharfstein (1988), Martin (1993), Raith (2003), and Baggs and de Bettignies (2007). In contrast to previous works, Raith (2003) and Baggs and de Bettignies (2007) explicitly take strategic interactions between the firms into account. Finally, Vives (2007) provides a model which presents a

these papers a merger could simply be seen as a (marginal) reduction of competitive pressure which would equally affect all firms in the market. However, this is no longer true when an increase of market concentration stems from a merger creating synergies. In addition to the unilateral effect of market power, we take the impact of synergies into account and thus focus on the interplay between synergies and managerial incentives.

The remainder is organized as follows. We present the model in Section 2. Section 3 provides the pre- and post-merger equilibria and discusses the effects of a merger on managerial incentives. In Section 4, we present our findings on the relationship between synergies and consumer surplus. Section 5 concludes the paper.

2 The Model

Pre-merger case. Consider a homogeneous Cournot oligopoly where $n \geq 3$ firms compete in quantities q_i, with $i = 1, ..., n$. Each firm consists of a risk-neutral principal and a risk-neutral agent who is protected by limited liability.[8] We analyze the following game. In the contracting stage, principals offer their respective agents contracts to induce marginal cost reductions, and then agents choose effort. Subsequently, in the market stage, principals learn their own and their rivals' costs and, finally, compete in quantities.[9]

Firms face an inverse demand function $p(Q)$, with $Q \equiv \sum_i q_i$ denoting total output, and exhibit constant marginal costs, $c_i > 0$. We invoke the following standard assumptions: *i)* $p'(Q) < 0$, i.e., demand is downward sloping, *ii)* $Qp''(Q) + p'(Q) < 0$, i.e., quantities are strategic substitutes implying strict concavity of firms' profits, and *iii)* $\lim_{Q_i \to \infty} p(Q) = 0$, i.e., total output is bounded in equilibrium.[10] Principal i's profit is given by

$$\pi_i = p(Q)q_i - c_i q_i - w_i, \tag{1}$$

generalization of Raith (2003) and Baggs and de Bettignies (2007), but where managerial effort is treated rather as an innovative activity.

[8]Notice that presuming risk-neutral agents who are protected by limited liability is an economically feasible alternative to risk-aversion. For a more general discussion see, e.g., Laffont and Martimort (2002, Ch. 4).

[9]That is, each principal receives a signal about her own and her rivals' costs without any noise.

[10]These assumptions are standard for guaranteeing the existence and stability of a unique equilibrium in Cournot oligopoly models, see, e.g., Shapiro (1989).

where $c_i \equiv c - e_i - \epsilon_i$. That is, marginal costs comprise a constant cost parameter, c, agent i's effort level, e_i, and a uniformly and independently distributed cost shock, $\epsilon_i \in [\underline{\theta}, \theta]$, with $\underline{\theta} = -\theta < 0 < \theta$.[11] Let $f(\epsilon_i)$ and $F(\epsilon_i)$ denote the probability density function and the cumulative density function, respectively.

Principals use linear incentive schemes, $w_i = d_i + b_i\,(c - c_i)$, to reward their agents, which consist of a (fixed) salary, d_i, and a variable component, $b_i\,(c - c_i)$.[12] In contrast to managerial effort, efficiency gains are verifiable, and can thus be contracted upon. The limited liability model implies that $d_i = 0$, which reduces the wage function to $w_i = b_i\,(c - c_i)$.[13] The piece rate, b_i, represents the incentive which principal i gives her agent to cut marginal costs termed managerial incentive throughout the paper.

The agent can accept or reject the contract which is a take-it-or-leave-it offer. If agent i rejects the contract, then he realizes his reservation utility which is normalized to zero. In contrast, if agent i accepts the offer, then he receives w_i and incurs convex costs of exerting effort given by $k(e_i) = e_i^2/2$. Expected utility is thus strictly concave in e_i and given by

$$E(u_i) = \int_{\underline{\theta}}^{\theta} w_i dF(\epsilon_i) - k(e_i). \tag{2}$$

Post-merger case. Suppose now that $m \geq 2$ firms merge, with $m < n$.[14] We specify the merger as being an acquisition, in which one firm entirely acquires the remaining $m-1$ firms. Thereby, the acquirer gets full corporate control over the acquirees. It follows that M's principal has m available agents at hand. Let M and $N = 1, ..., n-m$ indicate the merged firm and the non-merging firms, respectively. We denote the number of employed agents within M by k, with $k \in (1, ..., m)$. Assume that a merger gives rise to synergies, s, which come as marginal

[11]To avoid a finite support, we could have assumed that cost shocks follow a normal distribution as in, e.g., Raith (2003). However, in that case we would have to restrict the variance and the confidence level, respectively, to be able to ignore too large random cost differences. Nevertheless, our results hold under both assumptions.

[12]Though restrictive, limiting our model to linear contracts is standard and motivated by their common use in practice.

[13]This simplification relies on the fact that neither the participation constraint nor the wealth constraint is binding in models with limited liability. For an application which also analyzes linear contracts (but different performance measures) see, e.g., Raith (2008).

[14]By specifying $n \geq 3$ together with $m < n$, we exclude mergers to monopoly which is in line with the prevailing antitrust law in e.g., the US and the EU.

cost reductions and contrast productive efficiency gains from managerial effort by being merger specific. We postulate that a merger is a necessary prerequisite, while employing more than one agent is sufficient for synergies to be realized, with $s > 0$ for all $k > 1$, and $s = 0$ otherwise.

By keeping $k > 1$ agents, M's principal obtains, in addition to s, the ex post flexibility to choose between k marginal cost realizations, i.e., $\bar{c}_M = \min\{\bar{c}_1, ..., \bar{c}_k\}$, where the upper bar indicates the post-merger case. Let r denote one of the merged firm's k agents, with $r = 1, ..., k$. Then agent r's probability of exhibiting the lowest realized marginal cost is given by $\alpha \equiv (1 - F(\bar{c}_r))^{k-1}\ \forall r$, with $k\alpha = 1$.[15] Hence, M's expected marginal cost is[16]

$$E(\bar{c}_M) = \begin{cases} \alpha \sum_r (c - s - \bar{e}_r) & \text{if } k > 1, \\ c - \bar{e}_M & \text{otherwise.} \end{cases}$$

Agent r receives an expected wage of $E(w_r) = b_r\,(c - E(c_M))$. That is, M's principal is only able to reward her agents based on actual costs, $\bar{c}_M$, rather than some (non-minimum) counterfactual cost level, which will not be chosen when costs are realized.[17] The merged firm's total wage cost is $E(w_M) \equiv \sum_r E(w_r)$. We do not endogenize the merged firm's decision on how many agents to employ. Instead we distinguish between mergers with synergies and mergers without synergies and analyze both cases.

For the sake of simplicity, we invoke the following assumption.[18]

Assumption 1. *Given $k > 1$, $\partial^2 E(\bar{\pi}_M)/\partial \bar{b}_r \partial k < 0$ holds.*

Assumption 1 ensures that managerial incentives within M are, all other things being equal, negatively affected by a higher ex post flexibility measured by k. Moreover, we postulate that s and θ are not too large so that, in equilibrium, every firm is active in the market. Finally, note

[15]Note that $(1 - F(\bar{c}_r))$ represents the probability that agent r has lower marginal costs than one of the remaining $k - 1$ agents.

[16]Notice that $E(\epsilon_r) = \int_{\underline{\theta}}^{\theta} \epsilon_r dF(\epsilon_r) = 0\ \forall r$.

[17]We, in fact, assume that counterfactual cost levels are not verifiable, and thus cannot be contracted upon. In this respect, our setting essentially corresponds to the principal-agent literature on which a single principal faces multiple agents, see, e.g., Holmstrom (1982), Mookherjee (1984), and Demski and Sappington (1984). However, we specify that only the most efficient ("best") agent is chosen for production as in, e.g., Olsen (1993) and Levitt (1995).

[18]A more detailed discussion is provided in the Proof of Proposition 2. Note that Assumption 1 is always met for the linear Cournot model.

that all assumptions with respect to inverse demand made in the pre-merger case continue to hold so that the existence of a unique equilibrium is guaranteed.

3 Horizontal Mergers and Managerial Incentives

3.1 Equilibrium Analysis

We begin by presenting the equilibria for the pre-merger case and the post-merger case. Thereby, we use an asterisk to indicate equilibrium values in both cases.

Pre-merger case. In the market stage, principals know both their own costs and their rivals' costs. Hence, the equilibrium output per firm is implicitly defined by

$$q_i^* = -\frac{p(Q^*) - c_i}{p'(Q^*)},$$

where $p'(Q) = dp(Q)/dQ$. For given incentives, agents simultaneously decide on their effort levels. Agent i's optimal effort choice is given by $e_i^* = b_i$. That is, there is a direct link between the agents' optimal effort choice and the managerial incentive. This is a standard result of moral hazard models where $e_i^* = b_i$ represents the principal's incentive compatibility constraint.[19]

When designing incentives, each principal faces the following optimization problem[20]

$$\begin{aligned} \underset{b_i \in \mathbb{R}^+}{Max} E(\pi_i) &= \int_{\underline{\theta}}^{\theta} \left[p(Q^*)q_i^* - c_i q_i^* - w_i \right] dF \\ s.t.\ e_i^* &= b_i,\ E(u_i) \geq 0, \text{ and } w_i \geq 0 \forall \epsilon_i. \end{aligned} \tag{3}$$

Using the incentive compatibility constraint and the envelope theorem, the first order condition of problem (3) is given by

$$\int_{\underline{\theta}}^{\theta} \left[\left(p'(Q^*) \frac{\partial Q_{-i}^*}{\partial c_i} \frac{dc_i}{db_i^*} - \frac{dc_i}{db_i^*} \right) q_i^* - \frac{dw_i}{db_i^*} \right] dF = 0. \tag{4}$$

Let $b_i(b_j)$ denote principal i's best response function solving (4), where $j \neq i$.[21]

Lemma 1. *Managerial incentives are strategic substitutes, i.e., $\partial b_i / \partial b_j < 0$ holds.*

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[19] See, e.g., Levitt (1995) for the case of risk-averse agents.

[20] For notational simplicity, we use $\int_{\underline{\theta}}^{\theta} [\cdot]\, dF$ instead of $\int_{\underline{\theta}}^{\theta} ... \int_{\underline{\theta}}^{\theta} [\cdot]\, dF(\epsilon_1) ... dF(\epsilon_n)$ to calculate expected values accounting for each of the n agents' idiosyncratic cost shock, ϵ_i, with $i = 1, ..., n$.

[21] All omitted proofs are offered in the Appendix.

Since managerial effort aims at reducing the firms' marginal costs, it increases, all other things being equal, its output. Recall that firms' quantities are strategic substitutes. Then, it follows that principals always have an incentive to induce their respective agents to work less hard in response to an increase in their rivals' managerial incentives. Moreover, the reader should note that $\partial b_i / \partial b_j \in (-1, 0)$ is implied by the above stated standard assumptions on inverse demand.

In equilibrium, managerial incentives are implicitly defined by (4). The following proposition presents the equilibrium incentives in the pre-merger case under symmetry, i.e., $E(c_i) = E(c)$ $\forall i$.

Proposition 1. *In the pre-merger case, managerial incentives satisfy* $b^* = E(q^*)/2$ *in the symmetric equilibrium, where* $\int_{\underline{\theta}}^{\theta} q_i^* dF = E(q_i^*) = E(q^*)$ $\forall i$. *Moreover,* $\partial b^* / \partial n < 0$ *always holds.*

We show that equilibrium incentives are shaped solely by the direct effect of b_i on expected marginal costs, $E(c_i)$, and expected wage, $E(w_i)$. The strategic effect on the rivals' quantity cancels out due to symmetry. It follows that equilibrium incentives, and thereby firms' productive efficiency are proportional to $E(q^*)$. Moreover, we find that managerial incentives decrease when the level of competition in the product market, measured by the number of firms, n, increases.[22] This comparative static result relies on a well-known property of Cournot models: $\partial E(q^*)/\partial n < 0$.

Post-merger case. If the merged firm's principal decides to employ only one of the available m agents ($k = 1$), then the equilibrium of the entire game is symmetric and identical with the pre-merger case, except that the number of merging firms is now given by $n - m + 1$. According to Proposition 1, it is straightforward to verify that $\overline{b}_M^*(k = 1) = \overline{b}_N^*(k = 1) = \overline{b}^* > b^*$, i.e., a merger equally increases managerial incentives within both the merged firm and the non-merging firms given $k = 1$. Hence, productive efficiency gains are created by all firms.

If, however, M's principal employs $k > 1$ agents, then the result changes significantly. The reason is that M's principal now realizes synergies, s, and may choose between several cost realizations each associated with one of the k agents at hand, i.e., $\overline{c}_M = \min\{\overline{c}_1, ..., \overline{c}_k\}$. However, the non-merging firms' are not able to choose between several agents and cost realizations,

[22]This result is also found by Raith (2003) and Vives (2008) for an exogenous market structure.

respectively. Each of their principals continues to face a single agent and thus derives efficiency gains exclusively through managerial effort.

Before principals learn their marginal costs, M's principal does not know which of her agents will exhibit the lowest cost realization. Instead she knows the expected cost, $E(\overline{c}_M)$, where each of the k agents can be the most efficient, and thus chosen for production with probability α. Taking this into account, M's agents simultaneously decide on their effort levels.

Lemma 2. *The merged firm's agents' optimal effort choice is given by* $\overline{e}_r^* = \alpha \overline{b}_r$.

Lemma 2 shows that each of M's agents explicitly accounts for the probability of being the most efficient. Since $\alpha < 1$, all other things being equal, the agents' effort to cut marginal costs decreases compared to the pre-merger case. Then, the crucial question becomes whether or not M's principal will provide her agents with stronger incentives to compensate for this reduction. The answer to this question depends on the following optimization problem[23]

$$\begin{aligned} \underset{\overline{b}_r \in \mathbb{R}^+}{Max} E\left(\overline{\pi}_M\right) &= \int_{\underline{\theta}}^{\theta} \left[p(\overline{Q}^*)\overline{q}_M^* - \overline{c}_M^0 \overline{q}_M^* - \overline{w}_M\right] dF \qquad (5) \\ s.t.\ \overline{e}_r^* &= \alpha \overline{b}_r,\ E(\overline{u}_r) \geq 0, \text{ and } \overline{w}_r \geq 0\ \forall r \text{ and } \forall \epsilon_r, \end{aligned}$$

which M's principal solves for each of her k agents simultaneously and where $\overline{c}_M^0 = \alpha \sum_r (c - s - \overline{e}_r - \epsilon_r)$. Note that M's cost of creating synergies, and thus having the ex post flexibility to choose between k agents is reflected by the ex ante need to trigger all agents to choose optimal effort. Solving (5), we get the first order condition which is composed of the strategic effect on rivals and the direct effects of $\overline{b}_r$ on $\overline{c}_M$ and $\overline{w}_M$, i.e.,

$$\int_{\underline{\theta}}^{\theta} \left[\left(p'(\overline{Q}^*) \frac{\partial \overline{Q}_{-M}^*}{\partial \overline{c}_M^0} \frac{\partial \overline{c}_M^0}{\partial \overline{b}_r^*} - \frac{\partial \overline{c}_M^0}{\partial \overline{b}_r^*} \right) \overline{q}_M^* - \frac{\partial \overline{w}_M}{\partial \overline{b}_r^*} \right] dF = 0, \qquad (6)$$

where $-M$ indicates all firms other than M. In this case, the principal's best response per agent r is not only shaped by the strategic interaction between her incentives and the rival firms' managerial incentives, but also by intra-firm strategic relations.[24] More specifically, denote with $\overline{b}_r(\overline{b}_{-r})$ the best response solving (5) for agent r, where $-r$ denotes all of M's agents other than r.

[23] Again, for notational simplicity, we use $\int_{\underline{\theta}}^{\theta} [\cdot]\, dF$ to calculate expected values accounting for each of M's agents' and the non-merging firms' agents' idiosyncratic cost shocks ϵ_r and ϵ_N, with $r = 1, ..., k$ and $N = 1, ..., n - m$, respectively.

[24] To see this, it is instructive to note that $E(c_M) = \alpha \sum_r (c - s - e_r) = \alpha\,(c - s - e_r) + \sum_{t \neq r} \alpha\,(c - s - e_t)$.

Lemma 3. *Managerial incentives within the merged firm are strategic substitutes, i.e.,* $\partial\overline{b}_r/\partial\overline{b}_{-r} < 0$*, holds.*

Lemma 3 states that agent r is induced to choose a lower effort level in response to an increase of managerial incentives for one or more of the merged firm's remaining agents. Intuitively, pushing one agent to work harder exerts a negative externality on all other agents' effort, because it increases M's wage payment by more than it creates gains from enhanced productive efficiency. Note that due to symmetry between M's agents, we can set $\overline{b}_r^* = \overline{b}_M^*$ $\forall r = 1, ..., k$.

The non-merging firms' agents' optimal effort choice continues to be determined according to Lemma 1, i.e., $\overline{e}_N^* = \overline{b}_N$. That is, their effort choice does not directly depend on α. It is straightforward to check that managerial incentives continue to be strategic substitutes on an inter-firm level. Thus, a merger implies that inter-firm strategic interactions relying on managerial incentives being strategic substitutes are partially replaced by intra-firm strategic relations where managerial incentives are strategic substitutes, too.

Since the non-merging firms do not have any flexibility to choose between several agents, their decision on the incentive scheme corresponds to (3). The first order condition of principal N is thus given by

$$\int_{\underline{\theta}}^{\theta} \left[p'(\overline{Q}^*) \left[\left(\frac{\partial \overline{q}_M^*}{\partial \overline{c}_N} \frac{d\overline{c}_N}{d\overline{b}_N^*} + \frac{\partial \overline{Q}_{-N}^*}{\partial \overline{c}_N} \frac{d\overline{c}_N}{d\overline{b}_N^*} \right) - \frac{d\overline{c}_N}{d\overline{b}_N^*} \right] \overline{q}_N^* - \frac{d\overline{w}_N}{d\overline{b}_N^*} \right] dF = 0, \tag{7}$$

where the interior brackets on the left-hand side of (7) represent the strategic effect on N's rivals, i.e., the merged firm, M, and the remaining non-merging firms, $-N$.[25] To simplify notation, define $\gamma_M \equiv 1 - p'(\overline{Q}^*)[\partial\overline{Q}_{-M}^*/\partial\overline{c}_M^0]$ and $\gamma_N \equiv 1 - p'(\overline{Q}^*)[\partial\overline{q}_M^*/\partial\overline{c}_N + \partial\overline{Q}_{-N}^*/\partial\overline{c}_N]$ as measures of M's rival firms' sensitivity toward changes in $\overline{c}_M^0$ and N's rival firms' sensitivity toward changes in $\overline{c}_N$, respectively. The following proposition presents the equilibrium incentives in the post-merger case and offers some comparative statics results.

Proposition 2. *In the post-merger equilibrium, the merged firm's and the non-merging firms' managerial incentives are defined by*

$$\overline{b}_M^* = \frac{\alpha}{2} \int_{\underline{\theta}}^{\theta} \overline{q}_M^* \gamma_M dF - \frac{s}{2\alpha}$$

[25]Note that due to symmetry the strategic effect on the remaining non-merging firms disappears.

and

$$\overline{b}_N^* = \frac{1}{2}\int_{\underline{\theta}}^{\theta} \overline{q}_N^* \gamma_N dF,$$

respectively, with $\partial \overline{b}_M^*/\partial k < 0$ *and* $\partial \overline{b}_N^*/\partial k > 0$*. Moreover,* $\partial \overline{b}_M^*/\partial s < 0$ *(*$\partial \overline{b}_M^*/\partial s > 0$*) holds if* $k > \widehat{k}$ *(*$k < \widehat{k}$*), while* $\partial \overline{b}_N^*/\partial s < 0$ *always holds. Finally, if* $k < \widehat{k}$*, then* $\overline{b}_M^* > \overline{b}_N^*$ *(*$\overline{b}_M^* < \overline{b}_N^*$*) holds whenever* $s > \widehat{s}$ *(*$s < \widehat{s}$*), with* $\partial \widehat{s}/\partial k > 0$*. Otherwise,* $\overline{b}_M^* < \overline{b}_N^*$ *always holds.*

A key implication of Proposition 2 is that a merger's externality on the non-merging firms' managerial incentives must be explicitly taken into account. More specifically, depending on the synergy level, s, and the number of employed agents, k, the non-merging firms may end up offering their agents stronger incentives than the merged firm. This is true whenever the merger involves too many agents within M or, otherwise, whenever it generates sufficiently small synergies, i.e., $s < \widehat{s}$ holds. The reason is that for a large number of agents, M's principal substitutes productive efficiencies from managerial effort with synergies and thus offers her agents weaker incentives. That is, synergies suppress managerial incentives because the merged firm thereby lowers its wage costs and uses synergies instead, which cause no additional costs. If, however, the merger involves a relatively small number of agents, i.e., $k < \widehat{k}$ holds, then synergies trigger M's principal to offer stronger incentives. It follows that managerial incentives within the non-merging firms are only larger than those within M if s is sufficiently small. Our results are illustrated by Figure 1, where the equilibrium incentives are depicted for both $k < \widehat{k}$ (dotted black lines) and $k > \widehat{k}$ (solid black lines).

Since the merged firm's productive efficiency is not only shaped by managerial effort, but also by synergies and the number of employed agents, a sole comparison of managerial incentives is not sufficient for gaining knowledge of the productive efficiency levels across firms. We need to compare M's expected marginal cost, $E(\overline{c}_M)$, with its rivals' expected marginal costs, $E(\overline{c}_N)$, in order to examine which firms will operate more efficiently after the merger. Corollary 1 summarizes our results.

Corollary 1. *If* $k > \widehat{k}$*, then* $E(\overline{c}_M) < E(\overline{c}_N)$ *(*$E(\overline{c}_M) > E(\overline{c}_N)$*) holds whenever* $s > \widetilde{s}$ *(*$s < \widetilde{s}$*). Otherwise, i.e.,* $k < \widehat{k}$*,* $E(\overline{c}_M) < E(\overline{c}_N)$ *(*$E(\overline{c}_M) > E(\overline{c}_N)$*) holds whenever* $s > \widetilde{s}'$ *(*$s < \widetilde{s}'$*), with* $\widetilde{s}' < \widetilde{s}$*.*

Corollary 1 shows that the merged firm exhibits a higher level of expected productive efficiency than its rivals whenever synergies are sufficiently large. Our results partially oppose the standard reasoning, in which managerial incentives are not taken into account, and which claims that the merged firm becomes more efficient than its rivals for any level of s.[26] This result is simply explained by a merger's externality on the non-merging firms' productive efficiency. More precisely, given $s = 0$, a merger involving $k > 1$ agents leads to $\overline{b}_M^* < \overline{b}_N^*$, since $\partial \overline{b}_M^* / \partial k < 0$ and $\partial \overline{b}_N^* / \partial k > 0$, and thus $E(\overline{c}_M) > E(\overline{c}_N)$ always holds. Hence, a merger must generate sufficiently large synergies to compensate for this efficiency loss compared to the non-merging firms. Furthermore, we demonstrate that the critical synergy levels, $\widetilde{s}'$ and $\widetilde{s}$, crucially depend on whether synergies suppress ($k > \widehat{k}$) or boost ($k < \widehat{k}$) managerial incentives within M. Since the direct effect of synergies on M's expected marginal costs is constant, irrespective of the indirect effect on $E(\overline{c}_M)$ via $\overline{b}_M^*$, it must be true that the critical synergy level in the former case, $\widetilde{s}$, is larger than in the latter case, $\widetilde{s}'$, where $\partial \overline{b}_M^* / \partial s > 0$ holds.

3.2 The Effects of a Merger on Managerial Incentives

Now we are in the position to analyze how a horizontal merger between m firms affects managerial incentives. For this purpose, we compare the equilibrium incentives in the pre-merger case with those in the post-merger case. It is instructive to note that the number of merging firms, m, is indicative of the unilateral effects of increased market power. That is, all other things being equal, the higher the number of merging firm, m, the higher the concentration in the post-merger case, and thus the higher a firm's market power.[27]

We know from Proposition 1 that managerial incentives increase when the number of firms decreases. In other words, a merger without synergies equally induces all firms to give their managers stronger incentives when the merged firm, M, employs only one agent, i.e., $k = 1$. As a consequence, productive efficiency gains are equally generated by both the merged firm and the non-merging firms. However, M's principal might choose to employ more than one agent after the merger so that she creates synergies and attains the ex post possibility to choose between

[26] Provided that firms are symmetric.

[27] It is well known that a firm's degree of market power can be measured based on the following identity $\frac{p(Q^*)-c_i}{p(Q^*)} = -\frac{s_i}{\varepsilon_i^p}$, where the term on the left-hand side is the Lerner index, s_i denotes firm i's market share, and ε_i^p is firm i's elasticity of demand. Then, it is straightforward to check that the Lerner index increases, and thereby a firm's market power when the number of firms in the market decreases.

k agents. We make use of this argument and thus focus on the effect of a merger leading to synergies.

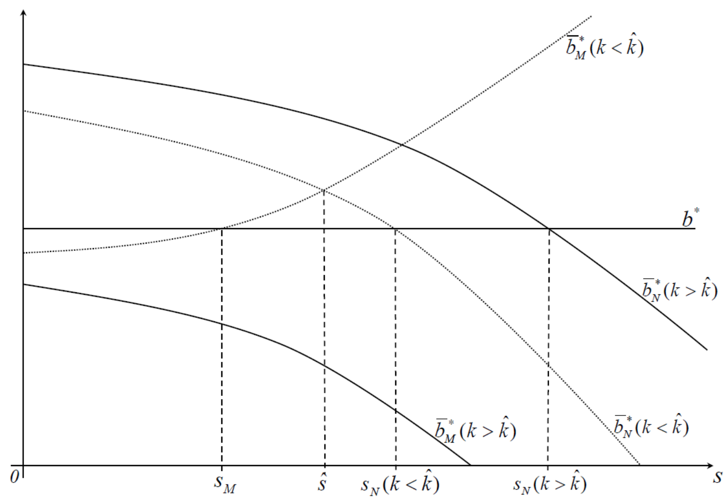

Figure 1: The Impact of Synergies on Managerial Incentives

We show that for sufficiently low synergies the non-merging firms push their agents to work harder than in the pre-merger case. This is even more true the more agents M's principal employs, since $\partial\overline{b}_N^*/\partial k > 0$ always holds. Recall that synergies always exert a negative externality on the non-merging firms. Then, it immediately follows that synergies must be relatively low for managerial incentives within the non-merging firms to increase due to a merger. In contrast, managerial incentives within the merged firm are larger than those in the pre-merger case if and only if k is sufficiently small, $k < \widehat{k}$, and, at the same time, s is sufficiently large. Note that increased ex post flexibility, i.e., a larger k, makes it harder for managerial incentives within M to surpass b^*, since $\partial\overline{b}_M^*/\partial k < 0$ holds. Otherwise, the wage costs are such that M's principal has no incentive to push her agents to exert more effort. Finally, note that there are cases in which both the non-merging firms' principals as well as M's principal are forced to give their agents weaker incentives to cut marginal costs. This is true whenever both the synergy level and the number of employed agents by M are sufficiently high. The reason is that in those cases synergies simply suppress managerial effort within both the merged firm and the non-merging firms. Proposition 3 summarizes our results.

Proposition 3. *Suppose that $\overline{b}_M^* < b^*$ at $k = \widehat{k}$. If $k < \widehat{k}$, then $\overline{b}_M^* > b^*$ ($\overline{b}_M^* < b^*$) holds whenever $s > s_M$ ($s < s_M$). Otherwise, i.e., $k > \widehat{k}$, we obtain that $\overline{b}_M^* < b^*$ always holds. Irrespective of the level of k, we find that $\overline{b}_N^* > b^*$ ($\overline{b}_N^* < b^*$) holds whenever $s < s_N$ ($s > s_N$). Moreover, $\partial s_M/\partial k > 0$ and $\partial s_N/\partial k > 0$ hold.*

For the sake of simplicity, we have assumed that $\overline{b}_M^* < b^*$ holds at $k = \widehat{k}$. In other words, we have postulated that the negative effect of ex post flexibility is large enough at $k = \widehat{k}$, so that managerial incentives within M are lower than those in the pre-merger case. In Figure 1, we illustrate our results, where the solid lines depict the case in which $k > \widehat{k}$ holds and the dotted lines reflect the case in which $k < \widehat{k}$ holds. The critical synergy levels, s_M, $\widehat{s}$, and s_N, are represented by the dashed lines.[28]

In a next step, we ask which implications our findings in Proposition 3 provide in terms of productive efficiency. More specifically, we present conditions under which a horizontal merger creates productive efficiency gains. Note again that we explicitly take the merger's externality on the non-merging firms' productive efficiency into account. We say that efficiency gains have been realized by M (N) if and only if $E(c_M) < E(c)$ ($E(c_N) < E(c)$) holds. Our results are presented in Corollary 2.

Corollary 2. *If $k < \widehat{k}$, then $E(\overline{c}_M) < E(c)$ ($E(\overline{c}_M) > E(c)$) holds whenever $s > \widetilde{s}'_M$ ($s < \widetilde{s}'_M$). Otherwise, i.e., $k > \widehat{k}$, we obtain that $E(\overline{c}_M) < E(c)$ ($E(\overline{c}_M) > E(c)$) holds whenever $s > \widetilde{s}_M$ ($s < \widetilde{s}_M$), with $\widetilde{s}'_M < \widetilde{s}_M$. Moreover, $\partial \widetilde{s}'_M/\partial k > 0$ and $\partial \widetilde{s}_M/\partial k > 0$ hold. Irrespective of the level of k, we find that $E(\overline{c}_N) < E(c)$ ($E(\overline{c}_N) > E(c)$) holds if $s < s_N$ ($s > s_N$).*

Corollary 2 basically mirrors our findings in Proposition 3, as a merger may create productive efficiency gains not only within the merged firm, but also within the non-merging firms. However, an important difference compared to our incentive comparison in Proposition 3 is that, in addition to their ambiguous effect on $\overline{b}_M^*$, synergies directly and positively affect M's productive efficiency. The total effect of synergies is positive, because the direct (negative) effect of s on $E(\overline{c}_M)$ is stronger than the indirect effect via $\overline{b}_M^*$ on $E(\overline{c}_M)$, so that $\partial E(\overline{c}_M)/\partial s < 0$ holds for any level of k. Overall, we stress that a merger may always create productive efficiency gains, but, due to its externality on the non-merging firms' productive efficiency, through different channels.

[28]In Figure 1, we assume that the ordering $s_M < \widehat{s} < s_N$ holds.

We illustrate the results of Corollary 2 in Figure 2, where the grey rectangle highlights all possible levels of $E(\bar{c}_M)$ and $E(\bar{c}_N)$, respectively, for which efficiency gains are created. Again, the dotted lines depict the case of $k < \widehat{k}$, while the solid lines reflect the case of $k > \widehat{k}$.[29]

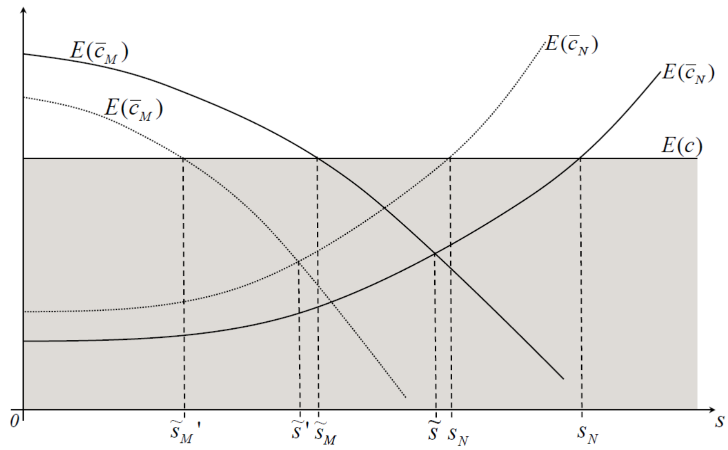

Figure 2: The Impact of Synergies on Productive Efficiency

4 Synergies and Consumer Surplus

In this section, we ask how synergies affect (expected) consumer surplus in the post-merger case. Focusing on consumer surplus is motivated by the supposition that many antitrust authorities seem to apply a consumer standard, rather than a welfare standard (see, e.g., Whinston, 2007).[30] Recent papers on horizontal mergers, such as, e.g., Nocke and Whinston (2010) and Nocke and Whinston (2012), also build on this view. In addition to using a consumer standard, many antitrust authorities allow for an efficiency defense which relies on the common belief that consumer surplus is monotonically increasing in the synergy level.[31] However, we will show that in the presence of endogenous efficiencies resulting from managerial effort the relationship

[29] In Figure 2, we assume that the ordering $\widetilde{s}'_M < \widetilde{s}' < s_N$ and $\widetilde{s}_M < \widetilde{s} < s_N$, respectively, holds.

[30] One reason for a consumer surplus standard could be, e.g., the firms' possibility to lobby efficiently (see Neven and Röller, 2005).

[31] Monotonicity is also used in empirical works to identify the competitive effects of mergers (see Duso et al., 2007, and Duso et al., 2011).

between (expected) consumer surplus, $E(\overline{CS}^*)$, and synergies, s, is ambiguous. We present our findings in the following proposition.

Proposition 4. *A marginal increase in s has the following two effects on $E(\overline{CS}^*)$: a (direct) synergy effect, which is positive, and an (indirect) incentive effect, which is ambiguous. The total effect is ambiguous.*

Our result in Proposition 4 can be presented as follows[32]

$$\frac{dE(\overline{Q}^*)}{ds} = \underbrace{\frac{\partial E(\overline{Q}^*)}{\partial s}}_{\oplus} + \underbrace{\textstyle\sum_N \frac{\partial E(\overline{Q}^*)}{\partial b_N^*}\frac{\partial b_N^*}{\partial s}}_{\ominus} + \underbrace{\frac{\partial E(\overline{Q}^*)}{\partial b_M^*}\frac{\partial b_M^*}{\partial s}}_{\ominus\ (\oplus)\text{ if } k>\widehat{k}\ (k<\widehat{k})} . \tag{8}$$

The (direct) *synergy effect* is reflected by the first term on the right-hand side of (8) and it mirrors the standard reasoning when endogenous efficiencies from managerial effort are absent. An increase in s raises, all other things being equal, M's output. The non-merging firms will respond by reducing their output, but by less, so that total output increases.[33] The second and the third term on the right-hand side of (8) represent the (indirect) *incentive effect* via *i)* managerial incentives within the non-merging firms and *ii)* managerial incentives within the merged firm. Recall that by Proposition 2 an increase in s always reduces managerial incentive within the non-merging firms, while the effect on managerial incentives within M depends on the number of employed agents, k.

[32]Notice that $sign(dE(\overline{CS}^*)/ds) = sign(dE(\overline{Q}^*)/ds)$.

[33]The reasoning is based on Farrell and Shapiro's (1990) Lemma.

Table 1. Managerial Incentives and Consumer Surplus in the Linear Cournot Example

Pre-merger case	Post-merger case with synergies
$b^{*}= \frac{n(1-c)}{1+n+n^2} \forall i = 1, ..., n$	$\overline{b}_M^{*}= \begin{cases} \frac{1}{2}\frac{4(1-c-7s)-2s\overline{n}^4+\gamma}{30+3\overline{n}^4+\delta} & \text{for } s < s_0 \\ 0 & \text{for } s \geq s_0 \end{cases}$ $\overline{b}_N^{*}= \begin{cases} \frac{1}{2}\frac{14(1-c)-8s+\lambda}{30+3\overline{n}^4+\delta} & \text{for } s < s_0 \\ \frac{1-c-s+(c-s)\overline{n}}{2+4\overline{n}c+\overline{n}^2} & \text{for } s \geq s_0 \end{cases}$
$E(CS^{*}) = \frac{n^2(1-c+b^{*})^2-\sigma^2(n-2)}{2(n+1)^2}$	$E(\overline{CS}^{*}) = \frac{\left[1-c+s+\left(1-c+b_N^{*}\right)\overline{n}+b_M^{*}/2\right]^2+\sigma^2(\overline{n}+1)^2}{2(\overline{n}+2)^2}$
$s_0=\frac{(\overline{n}+1)[\overline{n}(\overline{n}-1-2\overline{n}c+5c-4c^2)+2(1-c)]}{\overline{n}^4+(5+4c)\overline{n}^3+(7+24c)\overline{n}^2+(11+28c)\overline{n}+14}$, $\delta = (13 + 12c)\overline{n}^3+(19 + 56c)\overline{n}^2+(27 + 60c)\overline{n}> 0$	
$\lambda = (3c - 2s)\overline{n}^3+(2 + 12c - 10s)\overline{n}^2+(12 + 3c - 16s)\overline{n}$	
$\gamma = [2 - 10s - c(4 + 8s)]\overline{n}^3+[c(6 - 48s - 8c) + 14s]\overline{n}^2+[2 - 22s + c(6 - 56s - 8c)]\overline{n}$	

Linear example. We apply a linear Cournot oligopoly model to our general setup to illustrate a merger's effect on consumer surplus. For this purpose, we consider a merger between two firms, $m = 2$, and an inverse demand which is given by $p(Q) = 1 - Q$. Moreover, we assume that $c \in (0, 1/2)$ to ensure that all firms offer positive expected outputs. Equilibrium incentives and (expected) consumer surplus for both the pre-merger case and the post-merger case with synergies are summarized in Table 1, where, for simplicity, we define $\overline{n} \equiv n - m$.[34]

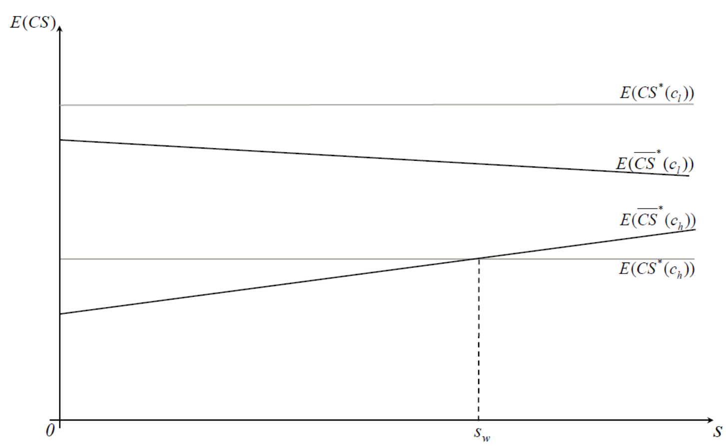

Figure 2: The Relationship between Synergies and Consumer Surplus

[34] A more detailed discussion of the derivation of the equilibrium values can be requested from the author.

In Corollary 3, we present the condition under which (expected) consumer surplus is decreasing in the synergy level.

Corollary 3. *Synergies decrease consumer surplus in the post-merger case if and only if* $c < \widehat{c}$ *holds, with* $\widehat{c} \equiv (\overline{n} - 2)/4\overline{n}$ *and* $d\widehat{c}/d\overline{n} > 0$.

Our finding in Corollary 3 stresses that in the presence of endogenous efficiencies resulting from managerial effort an efficiency defense in merger control may be misleading. That is, there exist mergers which reduce consumer surplus even more when synergies get larger so that an efficiency defense would necessarily fail to benefit consumers. It follows that for those mergers there is no critical synergy level above which a merger increases consumer surplus. The reason is that for sufficiently low initial marginal cost levels in the industry, i.e., $c < \widehat{c}$, the *incentive effect* becomes negative and cannot be offset by the positive *synergy effect* so that $dE(\overline{CS}^*)/ds < 0$ holds. We illustrate this result in Figure 3, where expected consumer surplus in the pre-merger case (post-merger case) is depicted by the grey lines (black lines), with $c_l < \widehat{c} < c_h$, and $s_w = \{s \in \mathbb{R}_+ : E(\overline{CS}^*) = E(CS^*)\}$.

Corollary 3 also demonstrates that the relevant threshold level, $\widehat{c}$, is increasing in the number of firms in the post-merger case. In other words, a lower concentration level in the post-merger case is accompanied by a higher chance that synergies will negatively affect expected consumer surplus. This result contradicts the intuition of the standard practice in merger control which relies on the use of concentration measures in the first phase of its competitive appraisal.

5 Conclusion

We have analyzed the effects of synergies from horizontal mergers on *i)* managerial incentives within firms to cut marginal costs and *ii)* consumer surplus. First, we have shown that synergies differently impact managerial incentives within the non-merging firms when compared with the merged firm. Whereas synergies always suppress managerial incentives within the non-merging firms, the effect on managerial incentives within the merged firm is ambiguous. More precisely, the merged firm's principal induces her agents to work harder only if she does not employ too many agents. In that case, synergies boost managerial incentives within the merged firm. Otherwise, the merged firm's principal faces wage costs which are so high that she substitutes managerial effort with synergies. That is, the agents are paid less than in the pre-merger case and thus choose lower effort levels. In terms of productive efficiency, we have demonstrated that it is not only the merged firm which can realize efficiency gains following a merger. Rather, the non-merging firms may offer their respective agents stronger incentives, too. This is true whenever the synergy level is sufficiently low. It follows that the non-merging firms may also realize efficiency gains, although they do not enjoy synergies.

Second, we have shown that in the presence of managerial firms the relationship between consumer surplus and synergies is not necessarily positive. We have identified two effects of an increase in synergies on consumer surplus: a (direct) *synergy effect* and an (indirect) *incentive effect*. The former is strictly positive and was already emphasized by Farrell and Shapiro's (1990) Lemma. However, the latter appears to be ambiguous and depends on the effects of synergies on managerial incentives within the firms (see Proposition 2). In addition, we have used a linear Cournot model to illustrate our results and derive conditions under which consumer surplus is reduced when synergies get larger. It has been shown that for sufficiently low initial marginal cost levels in the industry, the *incentive effect* becomes negative and cannot be offset by the positive *synergy effect*. An important implication for merger policy is that the use of an efficiency defense could be misleading, as consumer surplus may be monotonically decreasing in the synergy level. Hence, synergies, if accepted by the respective antitrust authority, do not always represent a countervailing factor toward increased unilateral market power. They may rather lead to an additional reduction in consumer surplus and may thus make a merger's unilateral market power effects even more harmful to consumers.

Appendix

In this Appendix we provide the omitted proofs.

Proof of Lemma 1. Decompose $\partial b_i/\partial b_j$ into

$$\frac{\frac{\partial E(q_i^*)}{\partial E(q_j^*)}\frac{\partial E(q_j^*)}{\partial E(c_j)}\frac{\partial E(c_j)}{\partial b_j}}{\frac{\partial E(q_i^*)}{\partial E(c_i)}\frac{\partial E(c_i)}{\partial b_i}}. \tag{9}$$

It is easily checked that the denominator of (9) is positive, whereas the numerator is negative. In addition, notice that $\partial b_i/\partial b_j = \partial E(q_i^*)/\partial E(q_j^*)$ must hold true due to symmetry, i.e., $\partial E(c_i)/\partial b_i = \partial E(c_j)/\partial b_j$ and $\partial E(q_i^*)/\partial E(c_i) = \partial E(q_j^*)/\partial E(c_j)$. ■

Proof of Proposition 1. Using $dw_i/db_i^* = 2b_i^* + \epsilon_i$ and $dc_i/db_i^* = -1$, we can rewrite the first order condition presented in (4) as $\int_{\underline{\theta}}^{\theta}[-p'(Q^*)(\partial Q_{-i}^*/\partial c_i)q_i^* + q_i^* - 2b_i^* - \epsilon_i]dF = 0$. Recall that $\int_{\underline{\theta}}^{\theta}\epsilon_i dF\left(\epsilon_i\right) = 0$. Due to symmetry the (positive) strategic effect on the non-merging firms' output is canceled out so that (4) further becomes

$$\int_{-\infty}^{+\infty}\left[q_i^* - 2b_i^*\right]dF = 0. \tag{10}$$

Rearranging (10), we obtain the implicitly defined equilibrium expression in Proposition 1, where $\int_{\underline{\theta}}^{\theta}2b_i^* dF = 2b_i^*$ and $\int_{\underline{\theta}}^{\theta}q_i^* dF = E(q_i^*) = E(q^*)\ \forall i$ due to symmetry. Moreover, we know that $\partial b^*/\partial n < 0$ must be true, since $\partial E(q_i^*)/\partial n < 0$ always holds under Cournot competition and $\partial b_i^*/\partial n = \left(\partial E(q_i^*)/\partial n\right)/\left(\partial E(q_i^*)/\partial b_i^*\right)$, with $\partial E(q_i^*)/\partial b_i^* > 0$. ■

Proof of Lemma 2. Each agent, r, maximizes $E(\overline{u}_r) = \overline{b}_r\left(c - E(\overline{c}_M)\right) - \overline{e}_r^2/2$, with $E(\overline{c}_M) = \alpha\sum_r\left(c - s - \overline{e}_r\right) = \alpha(c - s - \overline{e}_r) + \sum_{t\neq r}\alpha(c - s - \overline{e}_t)$, where t denotes all of M's agents except r. The first order condition is given by $\alpha\overline{b}_r - \overline{e}_r^* = 0$. Solving for $\overline{e}_r^*$ yields the optimal effort level presented in Lemma 2. ■

Proof of Lemma 3. To evaluate the sign of the slope of $\overline{b}_r(\overline{b}_{-r})$, which is given by

$$\frac{\partial\overline{b}_r}{\partial\overline{b}_{-r}} = -\frac{\partial^2 E\left(\overline{\pi}_M\right)/\partial\overline{b}_r\partial\overline{b}_{-r}}{\partial^2 E\left(\overline{\pi}_M\right)/\partial\overline{b}_r^2},$$

we only need to sign $\partial^2 E\left(\overline{\pi}_M\right)/\partial\overline{b}_r\partial\overline{b}_{-r}$, as $\partial^2 E\left(\overline{\pi}_M\right)/\partial\overline{b}_r^2 < 0$ holds due to strict concavity.

Differentiating $E(\overline{\pi}_M)$ with respect to $\overline{b}_r$ and then with respect to $\overline{b}_{-r}$ gives

$$\frac{\partial^2 E(\overline{\pi}_M)}{\partial \overline{b}_r \partial \overline{b}_{-r}} = \int_{\underline{\theta}}^{\theta} \left[p''(\overline{Q}^*) \frac{\partial^2 \overline{Q}^*_{-M}}{\partial (\overline{c}^0_M)^2} \frac{\partial^2 \overline{c}^0_M}{\partial \overline{b}_r \partial \overline{b}_{-r}} \overline{q}^*_M + p'(\overline{Q}^*) \frac{\partial \overline{Q}^*_{-M}}{\partial \overline{c}^0_M} \frac{\partial \overline{c}^0_M}{\partial \overline{b}_r} \frac{\partial \overline{q}^*_M}{\partial \overline{c}^0_M} \frac{\partial \overline{c}^0_M}{\partial \overline{b}_{-r}} - \frac{\partial^2 \overline{c}^0_M}{\partial \overline{b}_r \partial \overline{b}_{-r}} \overline{q}^*_M - \frac{\partial \overline{c}^0_M}{\partial \overline{b}_r} \frac{\partial \overline{q}^*_M}{\partial \overline{c}^0_M} \frac{\partial \overline{c}^0_M}{\partial \overline{b}_{-r}} - \frac{\partial^2 \overline{w}_M}{\partial \overline{b}_r \partial \overline{b}_{-r}} \right] dF. \tag{11}$$

Note that $\partial^2 \overline{c}^0_M / \partial \overline{b}_r \partial \overline{b}_{-r} = 0$. Using Lemma 2, $\overline{c}^0_M$ can be presented as $\overline{c}^0_M = \alpha(c - s - \alpha \overline{b}_r - \epsilon_r) + \sum_{t \neq r} \alpha(c - s - \alpha \overline{b}_t - \epsilon_t)$. Correspondingly, the merged firm's wage cost, $\overline{w}_M$, is written as $\overline{w}_M = \overline{b}_r(c - \overline{c}^0_M) + \sum_{t \neq r} \overline{b}_t(c - \overline{c}^0_M) = \left[\overline{b}_r + (k-1)\overline{b}_{-r}\right] [\alpha^2(\overline{b}_r + (k-1)\overline{b}_{-r}) + c - \alpha(k(c - s - \epsilon_{-r}) - \epsilon_r + \epsilon_{-r})]$. Notice that $\partial \overline{c}^0_M / \partial \overline{b}_r = -\alpha^2$, $\partial \overline{w}_M / \partial \overline{b}_r = 2\alpha^2 \left[\overline{b}_r + (k-1)\overline{b}_{-r}\right] + c - \alpha \left[k(c - s - \epsilon_{-r}) - \epsilon_r + \epsilon_{-r}\right]$, $\partial \overline{c}^0_M / \partial \overline{b}_{-r} = -(k-1)\alpha^2$ and $\partial^2 \overline{w}_M / \partial \overline{b}_r \partial \overline{b}_{-r} = 2\alpha^2(k-1)$, so that (11) becomes

$$\frac{\partial^2 E(\overline{\pi}_M)}{\partial \overline{b}_r \partial \overline{b}_{-r}} = \int_{\underline{\theta}}^{\theta} \left[p'(\overline{Q}^*) \frac{\partial \overline{Q}^*_{-M}}{\partial \overline{c}^0_M} \alpha^2 \frac{\partial \overline{q}^*_M}{\partial \overline{c}^0_M} (k-1)\alpha^2 - \alpha^2 \frac{\partial \overline{q}^*_M}{\partial \overline{c}^0_M} (k-1)\alpha^2 - 2\alpha^2(k-1) \right] dF. \tag{12}$$

It is immediately checked that (12) is always negative, since

$$2k^2 > -\int_{\underline{\theta}}^{\theta} \left[\frac{\partial \overline{q}^*_M}{\partial \overline{c}^0_M} \gamma_M \right] dF$$

is implied by strict concavity of $E(\overline{\pi}_M)$ in $\overline{b}_r$, where $\alpha^2 = 1/k^2$ and $\gamma_M \equiv 1 - p'(\overline{Q}^*)[\partial \overline{Q}^*_{-M} / \partial \overline{c}^0_M] > 0$. ■

Proof of Proposition 2. First, we present the sufficient conditions for a unique equilibrium to exist, i.e., $\partial^2 E(\overline{\pi}_M) / \partial \overline{b}^2_r < 0$ and $\partial^2 E(\overline{\pi}_N) / \partial \overline{b}^2_N < 0$ hold. It can be immediately verified that the second order conditions are

$$\frac{\partial^2 E(\overline{\pi}_M)}{\partial \overline{b}^2_r} = \int_{\underline{\theta}}^{\theta} \left[p''(\overline{Q}^*) \frac{\partial^2 \overline{Q}^*_{-M}}{\partial (\overline{c}^0_M)^2} \frac{\partial^2 \overline{c}^0_M}{\partial \overline{b}^2_r} \overline{q}^*_M + p'(\overline{Q}^*) \frac{\partial \overline{Q}^*_{-M}}{\partial \overline{c}^0_M} \frac{\partial \overline{c}^0_M}{\partial \overline{b}_r} \frac{\partial \overline{q}^*_M}{\partial \overline{c}^0_M} \frac{\partial \overline{c}^0_M}{\partial \overline{b}_r} - \frac{\partial^2 \overline{c}^0_M}{\partial \overline{b}^2_r} \overline{q}^*_M - \frac{\partial \overline{c}^0_M}{\partial \overline{b}_r} \frac{\partial \overline{q}^*_M}{\partial \overline{c}_M} \frac{\partial \overline{c}^0_M}{\partial \overline{b}_r} - \frac{\partial^2 \overline{w}_M}{\partial \overline{b}^2_r} \right] dF \tag{13}$$

and

$$\frac{\partial^2 E(\overline{\pi}_N)}{\partial \overline{b}_N^2} = \int_{\underline{\theta}}^{\theta} \left[p''(\overline{Q}^*) \frac{\partial^2 \overline{q}_M^*}{\partial \overline{c}_N{}^2} \frac{\partial^2 \overline{c}_N}{\partial \overline{b}_N^2} \overline{q}_N^* + p''(\overline{Q}^*) \frac{\partial^2 \overline{Q}_{-N}^*}{\partial (\overline{c}_N)^2} \frac{\partial^2 \overline{c}_N}{\partial \overline{b}_N^2} \overline{q}_N^* + \right. \tag{14}$$

$$p'(\overline{Q}^*) \frac{\partial \overline{q}_M^*}{\partial \overline{c}_N} \frac{\partial \overline{c}_N}{\partial \overline{b}_N} \frac{\partial \overline{q}_N^*}{\partial \overline{c}_N} \frac{\partial \overline{c}_N}{\partial \overline{b}_N} + p'(\overline{Q}^*) \frac{\partial \overline{Q}_{-N}^*}{\partial \overline{c}_N} \frac{\partial \overline{c}_N}{\partial \overline{b}_N} \frac{\partial \overline{q}_N^*}{\partial \overline{c}_N} \frac{\partial \overline{c}_N}{\partial \overline{b}_N} -$$

$$\left. \frac{\partial^2 \overline{c}_N}{\partial \overline{b}_N^2} \overline{q}_N^* - \frac{\partial \overline{c}_N}{\partial \overline{b}_N} \frac{\partial \overline{q}_N^*}{\partial \overline{c}_N} \frac{\partial \overline{c}_N}{\partial \overline{b}_N} - \frac{\partial^2 \overline{w}_N}{\partial \overline{b}_N^2} \right] dF.$$

Recall that $\partial \overline{c}_M^0 / \partial \overline{b}_r = -\alpha^2$, $\partial \overline{w}_M / \partial \overline{b}_r = 2\alpha^2 [\overline{b}_r + (k-1)\overline{b}_{-r}] + c - \alpha[k(c - s - \epsilon_{-r}) - \epsilon_r + \epsilon_{-r}]$, $\partial^2 \overline{c}_M^0 / \partial \overline{b}_r^2 = 0$, and $\partial^2 \overline{w}_M / \partial \overline{b}_r^2 = 2\alpha^2$. We can thus rewrite (13) as

$$\frac{\partial^2 E(\overline{\pi}_M)}{\partial \overline{b}_r^2} = \int_{\underline{\theta}}^{\theta} \left[p'(\overline{Q}^*) \frac{\partial \overline{Q}_{-M}^*}{\partial \overline{c}_M^0} \frac{\partial \overline{q}_M^*}{\partial \overline{c}_M^0} \alpha^4 - \frac{\partial \overline{q}_M^*}{\partial \overline{c}_M^0} \alpha^4 - 2\alpha^2 \right] dF,$$

which yields the following sufficient condition

$$2k^2 > -\int_{\underline{\theta}}^{\theta} \left[\frac{\partial \overline{q}_M^*}{\partial \overline{c}_M^0} \gamma_M \right] dF$$

for a unique maximum to exist, where $\gamma_M \equiv 1 - p'(\overline{Q}^*)[\partial \overline{Q}_{-M}^* / \partial \overline{c}_M^0]$ and $\alpha = 1/k$. Turning to the non-merging firms, where $d\overline{c}_N / d\overline{b}_N = -1$, $d^2 \overline{c}_N / d\overline{b}_N^2 = 0$, $d\overline{w}_N / \overline{b}_N = 2\overline{b}_N + \epsilon_N$, and $d^2 w_N / d\overline{b}_N^2 = 2$, (14) can be rewritten as

$$\frac{\partial^2 E(\overline{\pi}_N)}{\partial \overline{b}_N^2} = \int_{\underline{\theta}}^{\theta} \left[p'(\overline{Q}^*) \frac{\partial \overline{q}_M^*}{\partial \overline{c}_N} \frac{\partial \overline{q}_N^*}{\partial \overline{c}_N} + p'(\overline{Q}^*) \frac{\partial \overline{Q}_{-N}^*}{\partial \overline{c}_N} \frac{\partial \overline{q}_N^*}{\partial \overline{c}_N} - \frac{\partial \overline{q}_N^*}{\partial \overline{c}_N} - 2 \right] dF$$

which gives the following sufficient condition

$$2 > -\int_{\underline{\theta}}^{\theta} \left[\frac{\partial \overline{q}_N^*}{\partial \overline{c}_N} \gamma_N \right] dF$$

for a unique maximum to exist, where $\gamma_N \equiv 1 - p'(\overline{Q}^*)[\partial \overline{q}_M^* / \partial \overline{c}_N + \partial \overline{Q}_{-N}^* / \partial \overline{c}_N]$.

Second, we derive the implicit equilibrium expressions in Proposition 2. Using Lemma 2, $\overline{c}_M$ and $\overline{w}_M$ can be presented as $\overline{c}_M^0 = \alpha(c - s - \alpha \overline{b}_r - \epsilon_r) + \sum_{t \neq r} \alpha(c - s - \alpha \overline{b}_t - \epsilon_t)$ and $\overline{w}_M = \overline{b}_r(c - \overline{c}_M^0) + \sum_{t \neq r} \overline{b}_t(c - \overline{c}_M^0) = [\overline{b}_r + (k-1)\overline{b}_{-r}][\alpha^2(\overline{b}_r + (k-1)\overline{b}_{-r}) + c - \alpha(k(c - s - \epsilon_{-r}) - \epsilon_r + \epsilon_{-r})]$, respectively, with $\partial \overline{c}_M^0 / \partial \overline{b}_r = -\alpha^2$ and $\partial \overline{w}_M / \partial \overline{b}_r = 2\alpha^2[\overline{b}_r + (k-1)\overline{b}_{-r}] + c - \alpha[k(c - s - \epsilon_{-r}) - \epsilon_r + \epsilon_{-r}]$. Furthermore, we can set $b_M^* = b_r^* \forall r = 1, .., k$, due to symmetry and eliminate the cost shocks ($\int_{\underline{\theta}}^{\theta} \epsilon_r dF(\epsilon_r) = 0 \ \forall r$) so that (6) becomes

$$\int_{\underline{\theta}}^{\theta} \left[-p'(\overline{Q}^*) \frac{\partial \overline{Q}_{-M}^*}{\partial \overline{c}_M^0} \alpha^2 \overline{q}_M^* + \alpha^2 \overline{q}_M^* - 2\alpha^2 \overline{b}_M^* k + \alpha k (c - s) - c \right] dF = 0. \tag{15}$$

Notice that $\alpha ck - c = 0$ and $\alpha ks = s$. Rearranging (15) yields the implicitly defined equilibrium expression presented in Proposition 2.

The same procedure is performed with respect to the non-merging firms. Note that $d\overline{c}_N/d\overline{b}_N = -1$ and $d\overline{w}_N/d\overline{b}_N = 2\overline{b}_N + \epsilon_N$. The first order condition in (7) can be presented as follows

$$\int_{\underline{\theta}}^{\theta} \left[-p'(\overline{Q}^*)\frac{\partial \overline{q}_M^*}{\partial \overline{c}_N}\overline{q}_N^* - p'(\overline{Q}^*)\frac{\partial \overline{Q}_{-N}^*}{\partial \overline{c}_N}\overline{q}_N^* + \overline{q}_N^* - 2\overline{b}_N^* - \epsilon_N \right] dF = 0. \tag{16}$$

Isolating (16) for $\overline{b}_N^*$ and noticing that $p'(\overline{Q}^*)[\partial \overline{Q}_{-N}^*/\partial \overline{c}_N]$ cancels out due to symmetry, we obtain the implicitly defined equilibrium expression in Proposition 2.

Third, we analyze the impact of k on the equilibrium incentives. The marginal effect of k on $\overline{b}_M^*$ is given by

$$\frac{\partial \overline{b}_M^*}{\partial k} = -\frac{\partial^2 E\left(\overline{\pi}_M\right)/\partial \overline{b}_r \partial k\big|_{\overline{b}_r = \overline{b}_M \forall r}}{\partial^2 E\left(\overline{\pi}_M\right)/\partial \overline{b}_r^2\big|_{\overline{b}_r = \overline{b}_M \forall r}}.$$

Recall that $\partial^2 E\left(\overline{\pi}_M\right)/\partial \overline{b}_r^2\big|_{\overline{b}_r = \overline{b}_M \forall r} < 0$ always holds due to strict concavity so that $sign(\partial \overline{b}_M^*/\partial k)$ $= sign\left(\partial^2 E\left(\overline{\pi}_M\right)/\partial \overline{b}_r \partial k\big|_{\overline{b}_r = \overline{b}_M \forall r}\right)$. By Assumption 1, we know that $\partial \overline{b}_M^*/\partial k < 0$ always holds. Moreover, $\partial \overline{b}_M^*/\partial k < 0$ implies that $\partial \overline{b}_N^*/\partial k > 0$ which is simply derived from $\overline{b}_M^*$ and $\overline{b}_N^*$ being strategic substitutes (see Lemma 1).

Fourth, we sign the marginal effects of s on firms' equilibrium incentives. We only need to evaluate $\partial^2 E\left(\overline{\pi}_M\right)/\partial \overline{b}_r \partial s\big|_{\overline{b}_r = \overline{b}_M \forall r}$, which is given by

$$\begin{aligned} \int_{\underline{\theta}}^{\theta} \quad & \left[p''(\overline{Q}^*)\frac{\partial^2 \overline{Q}_{-M}^*}{\partial (\overline{c}_M^0)^2}\frac{\partial^2 \overline{c}_M^0}{\partial \overline{b}_r \partial s}\overline{q}_M^* + \right. \\ & p'(\overline{Q}^*)\frac{\partial \overline{Q}_{-M}^*}{\partial \overline{c}_M^0}\frac{\partial \overline{c}_M^0}{\partial \overline{b}_r}\frac{\partial \overline{q}_M^*}{\partial \overline{c}_M^0}\frac{\partial \overline{c}_M^0}{\partial s} - \\ & \left. \frac{\partial^2 \overline{c}_M^0}{\partial \overline{b}_r \partial s}\overline{q}_M^* - \frac{\partial \overline{c}_M^0}{\partial \overline{b}_r}\frac{\partial \overline{q}_M^*}{\partial \overline{c}_M^0}\frac{\partial \overline{c}_M^0}{\partial s} - \frac{\partial^2 \overline{w}_M}{\partial \overline{b}_r \partial s} \right] dF, \end{aligned} \tag{17}$$

since

$$\frac{\partial \overline{b}_M^*}{\partial s} = -\frac{\partial^2 E\left(\overline{\pi}_M\right)/\partial \overline{b}_r \partial s\big|_{\overline{b}_r = \overline{b}_M \forall r}}{\partial^2 E\left(\overline{\pi}_M\right)/\partial \overline{b}_r^2\big|_{\overline{b}_r = \overline{b}_M \forall r}}$$

and and $\partial^2 E\left(\overline{\pi}_M\right)/\partial \overline{b}_r^2\big|_{\overline{b}_r = \overline{b}_M \forall r} < 0$. Note that $\partial \overline{c}_M^0/\partial s = -1, = \partial^2 \overline{c}_M^0/\partial \overline{b}_r \partial s = 0$, $\partial^2 \overline{w}_M/\partial \overline{b}_r \partial s$ $= \alpha k = 1$ so that (17) becomes

$$\int_{\underline{\theta}}^{\theta} \left[p'(\overline{Q}^*)\frac{\partial \overline{Q}_{-M}^*}{\partial \overline{c}_M^0}\frac{1}{k^2}\frac{\partial \overline{q}_M^*}{\partial \overline{c}_M^0} - \frac{1}{k^2}\frac{\partial \overline{q}_M^*}{\partial \overline{c}_M^0} - 1 \right] dF,$$

which can be rearranged to

$$k^2 = -\int_{\underline{\theta}}^{\theta} \left[\frac{\partial \overline{q}_M^*}{\partial \overline{c}_M^0} \gamma_M \right] dF.$$

Note that the right-hand side is always positive, since $\gamma_M > 0$ and $\partial \overline{q}_M^* / \partial \overline{c}_M^0 < 0$. Hence, we can calculate the following implicit threshold value

$$\widehat{k} = \sqrt{-\int_{\underline{\theta}}^{\theta} \left[\frac{\partial \overline{q}_M^*}{\partial \overline{c}_M^0} \gamma_M \right] dF},$$

where $\partial \overline{b}_M^* / \partial s < 0$ ($\partial \overline{b}_M^* / \partial s > 0$) holds, whenever $k > \widehat{k}$ ($k < \widehat{k}$). Since $\int_{\underline{\theta}}^{\theta} [(\partial \overline{c}_M^0 / \partial \overline{b}_M^*)(\partial \overline{b}_M^* / \partial s)] dF > \int_{\underline{\theta}}^{\theta} (\partial \overline{c}_M^0 / \partial s) dF$ and $-\alpha^2 (\partial \overline{b}_M^* / \partial s) > -1$, respectively, i.e., the indirect effect of a marginal increase in s is lower than the direct effect of s on $\overline{c}_M^0$, it follows that $\partial \overline{b}_N^* / \partial s < 0$ must always hold.

Finally, we compare $\overline{b}_M^*$ and $\overline{b}_N^*$. Since $\partial \overline{b}_M^* / \partial k < 0$ and $\partial \overline{b}_N^* / \partial k > 0$ always hold, it must be that $\overline{b}_M^* < \overline{b}_N^*$ for $s = 0$. Hence, given $k < \widehat{k}$ and thus $\partial \overline{b}_M^* / \partial s > 0$, there exists a threshold synergy level $\widehat{s} \in \{s \in \mathbb{R}_+ : \overline{b}_M^* = \overline{b}_N^*\}$ such that $\overline{b}_M^* > \overline{b}_N^*$ ($\overline{b}_M^* < \overline{b}_N^*$) whenever $s > \widehat{s}$ ($s < \widehat{s}$). Moreover, $\partial \widehat{s} / \partial k > 0$ is implied by $\partial \overline{b}_M^* / \partial k < 0$ and $\partial \overline{b}_N^* / \partial k > 0$. Our results in Proposition 2 follow immediately. ■

Proof of Corollary 1. The merged firm's and the non-merging firms' expected marginal costs in equilibrium are given by $E(\overline{c}_M) = c - s - \overline{b}_M^* / k$ and $E(\overline{c}_N) = c - \overline{b}_N^*$, respectively. Depending on whether $\overline{b}_M^*$ increases or decreases in s, i.e., $k < \widehat{k}$ or $k > \widehat{k}$ holds, we can distinguish two critical synergy levels. If $k > \widehat{k}$ and thus $\partial \overline{b}_M^* / \partial s < 0$ holds, then there exists a synergy $\widetilde{s} \in \{s \in \mathbb{R}_+ : E(\overline{c}_M) = E(\overline{c}_N) \wedge k > \widehat{k}\}$ for which $E(\overline{c}_M) > E(\overline{c}_N)$ ($E(\overline{c}_M) < E(\overline{c}_N)$) whenever $s < \widetilde{s}$ ($s > \widetilde{s}$). If, however, $k < \widehat{k}$ and thus $\partial \overline{b}_M^* / \partial s > 0$, then there exists a synergy $\widetilde{s}' \in \{s \in \mathbb{R}_+ : E(\overline{c}_M) = E(\overline{c}_N) \wedge k < \widehat{k}\}$ for which $E(\overline{c}_M) > E(\overline{c}_N)$ ($E(\overline{c}_M) < E(\overline{c}_N)$) whenever $s < \widetilde{s}'$ ($s > \widetilde{s}'$). It is straightforward that $\widetilde{s}' < \widetilde{s}$ must hold. Our results in Corollary 1 follow immediately. ■

Proof of Proposition 3. First, we compare b^* with $\overline{b}_M^*$. Assume that $\overline{b}_M^* < b^*$ at $k = \widehat{k}$. Since $\partial \overline{b}_M^* / \partial s < 0$ holds if $k > \widehat{k}$, we can infer that $\overline{b}_M^* < b^*$ holds for all feasible s given $k > \widehat{k}$. If, however, $k < \widehat{k}$ holds and thus $\partial \overline{b}_M^* / \partial s > 0$, then there exists a threshold synergy defined by $s_M \in \{s \in \mathbb{R}_+ : \overline{b}_M^* = b^* \wedge k < \widehat{k}\}$ such that $\overline{b}_M^* > b^*$ ($\overline{b}_M^* < b^*$) holds whenever $s > s_M$ ($s < s_M$). Note that $\partial s_M / \partial k > 0$ must be true, since $\partial \overline{b}_M^* / \partial k < 0$ holds by Assumption 1.

Second, comparing $b^*$and $\overline{b}_N^*$, we know from $\partial\overline{b}_N^*/\partial s < 0$ and $\partial\overline{b}_N^*/\partial k > 0$ that there must exist a threshold synergy denoted by $s_N \in \{s \in \mathbb{R}_+ : \overline{b}_N^* = b^*\}$ such that $\overline{b}_N^* > b^*$ ($\overline{b}_N^* < b^*$) if $s < s_N$ ($s > s_N$). Due to $\partial\overline{b}_N^*/\partial k > 0$, it must be true that $\partial s_N/\partial k > 0$. ■

Proof of Corollary 2. First, we compare $E(c)$ with $E(\overline{c}_M)$. Recall that $\partial\overline{b}_M^*/\partial s < 0$ ($\partial\overline{b}_M^*/\partial s > 0$) holds whenever $k > \widehat{k}$ ($k < \widehat{k}$) (see Proposition 2). It follows that $E(\overline{c}_M)$ will fall below $E(c)$ at a smaller synergy if $k < \widehat{k}$ than in the case of $k > \widehat{k}$. As for the former, we obtain the critical synergy $\widetilde{s}'_M \in \{s \in \mathbb{R}_+ : E(\overline{c}_M) = E(c) \wedge k < \widehat{k}\}$, for which $E(c) < E(\overline{c}_M)$ ($E(c) > E(\overline{c}_M)$) holds whenever $s < \widetilde{s}'_M$ ($s > \widetilde{s}'_M$). As for the latter, there exists another critical synergy given by $\widetilde{s}_M \in \{s \in \mathbb{R}_+ : E(\overline{c}_M) = E(c) \wedge k > \widehat{k}\}$ such that $E(c) < E(\overline{c}_M)$ ($E(c) > E(\overline{c}_M)$) holds whenever $s < \widetilde{s}_M$ ($s > \widetilde{s}_M$), with $\widetilde{s}'_M < \widetilde{s}_M$.

Second, we compare $E(c)$ and $E(\overline{c}_N)$. Since the non-merging firms' post-merger marginal costs do not involve any synergies, the comparison essentially equals the incentive comparison in the Proof of Proposition 3. We conclude that the relevant threshold value must equal s_N. ■

Proof of Corollary 3. We provide the omitted Proof of Corollary 3 and discuss the equilibria for all three cases in more detail. We start with the pre-merger case.

Pre-merger case. Each firm's equilibrium output is given by

$$q_i^* = \frac{1 - nc_i + \sum_{j \neq i} c_j}{n+1}. \tag{18}$$

The agents' optimal effort levels are given by $e_i^* = b_i$ (see Lemma 1). Solving the optimization problem presented in (3) and making use of symmetry, i.e., $b_i^* = b^* \forall i$, equilibrium incentives are

$$b^* = \frac{n(1-c)}{1+n+n^2},$$

where $\partial b^*/\partial n < 0$ obviously holds.

Knowing the equilibrium values, we are able to compute the expected consumer surplus which is presented by

$$E(CS^*) = \frac{n^2(1-c+b^*)^2 - \sigma^2(n-2)}{2(n+1)^2}.$$

$E(CS^*)$ is strictly increasing in n, although managerial incentives are reduced when n increases:

$$\frac{\partial E(CS^*)}{\partial n} = \underbrace{\frac{\partial E(CS^*)}{\partial n}}_{>0} + \underbrace{\sum\nolimits_n \frac{\partial E(CS^*)}{\partial b^*}}_{>0} \underbrace{\frac{\partial b^*}{\partial n}}_{<0} > 0.$$

Post-merger case. In the post-merger case, the merged firm, M, competes with $\overline{n} = n - 2$ non-merging firms denoted by N. The total number of firms is given by $\overline{n} + 1$. The number of agents employed within M is indicated by k, with $k \leq m$. Note that we cannot make use of symmetry as we did in the pre-merger case if the merged firm employs $k > 1$ agents. Nevertheless, it is instructive to note that M's agents are symmetric and the non-merging firms are symmetric. We distinguish two cases in the post-merger scenario. First, we consider a merger which does not create any synergies within M, i.e., $k = 1$. Second, we analyze the post-merger case in which synergies are realized within M, i.e., $k = 2$ and $s \geq 0$.

Merger without synergies. It is easily verified that if $k = 1$, then equilibrium output per firm, managerial incentives, and expected consumer surplus equal those in the pre-merger case except that the number of firms is now given by $\overline{n} + 1 < n$. Thus, it is straightforward that $\overline{\Delta}_{CS} = E(\overline{CS}^*) - E(CS^*) < 0$ must hold, since $\partial E(CS^*)/\partial n > 0$.

Merger with synergies. If $k = 2$, then equilibrium quantities are given by

$$\overline{q}_M^* = \frac{1 - (\overline{n}+1)\overline{c}_M + \sum_N \overline{c}_N}{\overline{n} + 2} \tag{19}$$

and

$$\overline{q}_N^* = \frac{1 - (\overline{n}+1)\overline{c}_N + \overline{c}_M + \sum_{j \neq N,M} \overline{c}_j}{\overline{n} + 2}. \tag{20}$$

According to Lemma 3, each of M's agents chooses $\overline{e}_r^* = \alpha \overline{b}_r$, whereas the non-merging firms' agents' optimal effort choice is $\overline{e}_N^* = \overline{b}_N$. In equilibrium, managerial incentives within M and within the non-merging firms, respectively, are given by

$$\overline{b}_M^* = \begin{cases} \frac{1}{2}\frac{4(1-c-7s)-2s\overline{n}^4+\gamma}{30+3\overline{n}^4+\delta} & \text{for } s < s_0 \\ 0 & \text{for } s \geq s_0 \end{cases}$$

and

$$\overline{b}_N^* = \begin{cases} \frac{1}{2}\frac{14(1-c)-8s+\lambda}{30+3\overline{n}^4+\delta} & \text{for } s < s_0 \\ \frac{1-c-s+(c-s)\overline{n}}{2+4\overline{n}c+\overline{n}^2} & \text{for } s \geq s_0 \end{cases},$$

where $\gamma = [2 - 10s - c(4 + 8s)]\overline{n}^3 + [c(6 - 48s - 8c) + 14s]\overline{n}^2 + [2 - 22s + c(6 - 56s - 8c)]\,\overline{n}$, $\delta = (13 + 12c)\overline{n}^3 + (19 + 56c)\overline{n}^2 + (27 + 60c)\overline{n} > 0$, and $\lambda = (3c - 2s)\overline{n}^3 + (2 + 12c - 10s)\overline{n}^2 + (12 + 3c - 16s)\overline{n}$. The critical synergy level is given by

$$s_0 = \frac{(\overline{n}+1)[\overline{n}(\overline{n} - 1 - 2\overline{n}c + 5c - 4c^2) + 2(1 - c)]}{\overline{n}^4 + (5 + 4c)\overline{n}^3 + (7 + 24c)\overline{n}^2 + (11 + 28c)\overline{n} + 14}.$$

It is immediately checked that equilibrium incentives always fulfill

$$\left.\frac{\partial \overline{b}_M^*}{\partial s}\right|_{s<s_0} = -\frac{28+\overline{n}^4+(10+8c)\overline{n}^3+(14+48c)\overline{n}^2+(22+56c)\overline{n}}{30+3\overline{n}^4+\delta} < 0,$$

$$\left.\frac{\partial \overline{b}_N^*}{\partial s}\right|_{s<s_0} = -\frac{8+2\overline{n}^3+10\overline{n}^2+16\overline{n}}{30+3\overline{n}^4+\delta} < 0,$$

and

$$\left.\frac{\partial \overline{b}_N^*}{\partial s}\right|_{s\geq s_0} = -\frac{1+\overline{n}}{2+4c\overline{n}+\overline{n}^2} < 0,$$

with $\left|\partial \overline{b}_M^*/\partial s\right| > \left|\partial \overline{b}_N^*/\partial s\right|$ given $s < s_0$. That is, an increase of s directly lowers M's (expected) marginal costs, but, at the same time, it increases M's wage payment. The latter (negative) effect turns out to dominate the former (positive) effect, so that M's principal is induced to offer weaker managerial incentives when s increases. Though $\overline{b}_M^*$ is reduced, M's (expected) marginal costs decrease, so that M's quantity increases when s gets larger. Comparing $\overline{b}_M^*$ and $\overline{b}_N^*$ for $s \in [0, s_0)$ yields the ordering $\overline{b}_N^* > \overline{b}_M^*$, since $\overline{b}_N^* > \overline{b}_M^*$ holds at $s = 0$ and $\left|\partial \overline{b}_M^*/\partial s\right| > \left|\partial \overline{b}_N^*/\partial s\right|$. Thus, $\overline{b}_N^* > \overline{b}_M^*$ holds for all feasible s.

Evaluating $\overline{b}_M^* - b^*$, we obtain that $\overline{b}_M^* < b^*$ holds for all feasible s. Turning to the non-merging firms, we find that $\overline{b}_N^* > b^*$ ($\overline{b}_N^* < b^*$) holds if $s < s_N$ ($s > s_N$) for $s \in [0, s_0)$, where

$$s_N = \frac{38(1-c)-\kappa}{2(7+5\overline{n}+\overline{n}^2)(\overline{n}+1)(\overline{n}+2)^2},$$

with $\kappa = [(3-6c)\overline{n}^5 + (17-34c-12c^2)\overline{n}^4 + (23-49c-80c^2)\overline{n}^3 + (22c-23-172c^2)\overline{n}^2 + (85c-70-120c^2)\overline{n}]$. If, however, $s \geq s_0$, then $\overline{b}_N^* > b^*$ ($\overline{b}_N^* < b^*$) holds, whenever $s < s_N'$ ($s > s_N'$), where

$$s_N' = \frac{3\,(1-c)-\nu}{(7+5\overline{n}+\overline{n}^2)(\overline{n}+1)},$$

with $\nu = [(1-2c)\overline{n}^3 + (1-2c-4c^2)\overline{n}^2 + (4c-3-8c^2)\overline{n}]$. Note that $E(\overline{c}_M) > E(\overline{c}_N)$ ($E(\overline{c}_M) < E(\overline{c}_N)$) holds for $s \in [0, s_0)$ if $s < \widetilde{s}$ ($s > \widetilde{s}$), where

$$\widetilde{s} = \frac{12(1-c)+(5c-1)\overline{n}^3+(2+9c+4c^2)\overline{n}^2+(11+4c^2)\overline{n}}{(3+\overline{n}+4c\overline{n}+\overline{n}^2)(\overline{n}+2)^2}.$$

For $s \geq s_0$, however, we obtain that $E(\overline{c}_M) > E(\overline{c}_N)$ ($E(\overline{c}_M) < E(\overline{c}_N)$) holds for $s < \widetilde{s}_0$ ($s > \widetilde{s}_0$), where

$$\widetilde{s}_0 = \frac{1-c+c\overline{n}}{3+\overline{n}+4c\overline{n}+\overline{n}^2}.$$

In a next step, we ask whether or not a merger with synergies, s, creates productive efficiency gains within the merged firm and/or within the rival firms when compared with the pre-merger case. Therefore, we compare $E(\bar{c}_M)$ with $E(c)$ and $E(\bar{c}_N)$ with $E(c)$, respectively, where $E(\bar{c}_M) = c - s - \bar{b}^*_M/k$, $E(\bar{c}_N) = c - \bar{b}^*_N$, and $E(c) = c - b^*$. Inspection of $E(\bar{c}_M)$ and $E(c)$ reveals that $E(\bar{c}_M) > E(c)$ always holds for $s \in [0, s_0)$. If, however, $s \geq s_0$, then $E(\bar{c}_M) > E(c)$ ($E(\bar{c}_M) < E(c)$) holds whenever $s < \widetilde{s}_M$ ($s > \widetilde{s}_M$), where

$$\widetilde{s}_M = \frac{(1-c)(\bar{n}+2)}{7+5\bar{n}+\bar{n}^2}.$$

Turning to $E(\bar{c}_N)$ and $E(c)$, we can immediately infer that the analysis entirely corresponds to the comparison of $\bar{b}^*_N$ and b^*.

Expected consumer surplus is calculated as

$$E(\overline{CS}^*) = \frac{[1-c+s+(1-c+b^*_N)\,\bar{n}+b^*_M/2]^2+\sigma^2(\bar{n}+1)^2}{2(\bar{n}+2)^2},$$

with

$$\begin{aligned}\left.\frac{dE(\overline{CS}^*)}{ds}\right|_{s<s_0} &= \underbrace{\frac{\partial E(\overline{CS}^*)}{\partial s}}_{>0} + \underbrace{\frac{\partial E(\overline{CS}^*)}{\partial b^*_N}\frac{\partial b^*_N}{\partial s}}_{<0} + \underbrace{\frac{\partial E(\overline{CS}^*)}{\partial b^*_M}\frac{\partial b^*_M}{\partial s}}_{<0} \\ &= \frac{24[\frac{4}{3}(1-c+s/2)+\psi][2+(4c-1)](\bar{n}+2)}{(30+3\bar{n}^4+\delta)^2}\end{aligned}$$

and

$$\begin{aligned}\left.\frac{dE(\overline{CS}^*)}{ds}\right|_{s\geq s_0} &= \underbrace{\frac{\partial E(\overline{CS}^*)}{\partial s}}_{>0} + \underbrace{\frac{\partial E(\overline{CS}^*)}{\partial b^*_N}\frac{\partial b^*_N}{\partial s}}_{<0} \\ &= \frac{(1-c+s+(1-c+b^*_N)\,\bar{n})\left(1-\frac{(\bar{n}+1)\bar{n}}{2+4c\bar{n}+\bar{n}^2}\right)}{(\bar{n}+2)^2},\end{aligned}$$

where $\psi = [(1-c)\bar{n}^4/4 + ((10+5c)/12 - c^2)\bar{n}^3 + ((15-2s-44c^2)/12 + c(3+2s/3))\bar{n}^2 + ((7-8c^2)/3 + c(1+4s)/3)\bar{n}]$. Inspecting the marginal effects of an increase in s, we obtain ambiguous results for all feasible s: $dE(\overline{CS}^*)/ds > 0$ ($dE(\overline{CS}^*)/ds < 0$) holds whenever $c > \widehat{c}$ ($c < \widehat{c}$), with $\widehat{c} \equiv (\bar{n}-2)/4\bar{n}$ and $d\widehat{c}/d\bar{n} = 1/2\bar{n}^2 > 0$.

Finally, we compare $E(\overline{CS}^*)$ with the (expected) consumer surplus in the pre-merger case, $E(CS^*)$. We find that $E(\overline{CS}^*) < E(CS^*)$ holds for any $s \in [0, s_0)$. If, however, $s \geq s_0$, then $E(\overline{CS}^*) > E(CS^*)$ ($E(\overline{CS}^*) < E(CS^*)$) holds whenever $s > s_w$ ($s < s_w$), where

$$s_w = \frac{10(1-c)+\phi}{\left(7+5\bar{n}+\bar{n}^2\right)[2+4\bar{n}(c-1/4)]},$$

with $\phi = (1-2c)\overline{n}^4 + (3-4c-4c^2)\overline{n}^3 + (2+7c-16c^2)\overline{n}^2 + (1+19c-20c^2)\overline{n}$. Obviously, $s_w > s_0$ holds, and $s_w > 0$ if and only if $c > \widehat{c}$. $\blacksquare$

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