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Bargaining in Standing Committees

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July 31, 2012

Abstract

Committee voting has mostly been investigated from the perspective of the standard Baron-Ferejohn model of bargaining over the division of a pie, in which bargaining ends as soon as the committee reaches an agreement. In standing committees, however, existing agreements can be amended. This paper studies an extension of the Baron-Ferejohn framework to a model with an evolving default that reflects this important feature of policymaking in standing committees: In each of an infinite number of periods, the ongoing default can be amended to a new policy (which in turn determines the default for the next period). The model provides a number of quite different predictions. In particular: (i) Substantial shares of the pie are wasted each period and the size principle fails in some pure strategy Markov perfect equilibria of non-unanimity games with patient enough players; and (ii) All Markov perfect equilibria are Pareto inefficient when discount factors are heterogenous. However, there is a unique equilibrium outcome in unanimity standing committee games, which coincides with the unique equilibrium outcome of the corresponding Baron-Ferejohn framework.

JEL classification: C73, C78, D71, D72.

Keywords: Legislative bargaining, endogenous default, efficiency, pork barrel.

1 Introduction

In Baron and Ferejohn's (1989) closed rule model, equally patient risk neutral players bargain over division of a pie, earning nothing until agreement is reached. This renowned game has a unique stationary equilibrium outcome, in which a minimal winning coalition

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agree to fully share the pie; and this agreement is reached immediately, so every equilibrium is efficient. The closed rule game is naturally interpreted as a model of an ad hoc committee: the game ends as soon as the committee reaches an agreement, which can therefore not be renegotiated. This property differentiates ad hoc from standing committees, which can amend existing agreements. In the simplest formulation, agreements reached by standing committees remain in place until they are amended, yielding a bargaining game with an endogenous default in which a pie is available for division each period. We study such models in this paper: Each period begins with a default policy inherited from the previous period and a player is randomly drawn to make a proposal which is then voted up or down by the committee; if voted up, the proposal is implemented and becomes the new default; if voted down, the ongoing default is implemented and remains in place until the next period. This process continues ad infinitum. Costless renegotiation implicitly restricts the interpretation of "policy." This model applies naturally, for example, to Congressional bargaining over policies like protection or tax exemptions.[1]

Analysis of standing committees raises various interesting questions, such as: (1) When do (Markov perfect) equilibrium exist and, when they do, are equilibrium outcomes unique? (2) Must the pie be divided between a minimal winning majority — as predicted by the size principle — in every equilibrium? (3) Is the pie fully divided (that is, is the division of the pie statically efficient) each period in every equilibrium? And (4) Are equilibria Pareto efficient? Answers to these questions would also allow us to compare play in ad hoc and in standing committees.

The literature on standing committees has posed the first two questions, but not questions (3) and (4). Our contribution is to bypass technical issues which have stymied progress, and thereby to say much more about each of the four issues. Indeed, our model generalizes Baron and Ferejohn (1989) in various other respects: we allow players to have different discount factors, and any concave utility functions; and we consider any quota (including majority and unanimity rules).[2] Eraslan (2002) has already extended Baron and Ferejohn's (1989) results to closed rule, ad hoc committees with heterogeneous discount factors and any quota.[3]

[1] Nelson (1989) argues that Congress treated protection as a purely distributive issue until the 1934 Reciprocal Trade Agreements Act.

[2] We also allow players to be selected to propose with different probabilities.

[3] Efficiency and the size principle hold when players have strictly concave preferences, but uniqueness might fail.

We provide the following answers to the four questions above:

(1) Equilibrium existence and multiplicity of equilibrium outcomes. We construct pure strategy Markov perfect equilibria for any game with a non-unanimity quota and patient enough players, and prove (again using constructive arguments) that unanimity games possess pure strategy equilibria, irrespective of patience. By contrast, we have radically different results on multiplicity for games with and without a unanimity quota. We start with the latter case. Take *any* point in the policy space at which at least a minimal winning majority have a positive share of the pie. If players are sufficiently patient then we can construct a pure strategy Markov perfect equilibrium in which that policy is implemented in the first period and never amended: a property which we describe as *no-delay*. Now consider games with a unanimity quota: Any such game has a unique equilibrium outcome, which is no-delay, and in which the first offer is statically efficient. In addition, the policies proposed by each player also coincide with the unique equilibrium outcome in the equivalent model of an ad hoc committee.

The previous literature (which we will survey in the next section) has focused on existence of Markov perfect equilibria in bargaining games with an evolving default. Our results demonstrate that existence is not a problem when players are patient enough.

(2) The size principle. The size principle has been central to the study of legislatures since Riker (1962). The class of solutions which we construct for non-unanimity games contains equilibria in which the pie is shared amongst more than a minimal winning coalition. Our model therefore provides a new explanation for why majorities in legislatures are typically supraminimal.

(3) Waste. Our results on the division of the pie again differ, depending on the quota. We show that equilibrium agreements in games without a unanimity quota typically waste some of the pie when all players are patient enough. Specifically, for every $\varepsilon > 0$, we can construct an equilibrium in which a policy which wastes proportion $1 - \varepsilon$ of the pie is agreed to in the first period and never amended. By contrast, none of the pie is wasted in any equilibrium, irrespective of players' patience, in games with a unanimity quota.

More strongly, players can waste any proportion of the pie in equilibria which fail the size principle. Our model can therefore explain some common features of pork barrel politics (cf. Evans (2004)).

(4) Pareto inefficiency. If preferences are linear in share of the current pie (as in Baron and Ferejohn (1989)) then, in the generic case where all players have different discount factors, every equilibrium of a non-unanimity game is inefficient. On the other hand, unanimity committees immediately reach an agreement which is never amended in every equilibrium; and any equilibrium is inefficient if two or more players have different discount factors. The intuition is that an efficient policy sequence should yield the most patient [resp. impatient] players an increasing [resp. decreasing] share of the pie.

These results stand in sharp contrast to those obtained in the case of ad hoc committees (i.e. Baron and Ferejohn (1989) and Eraslan (2002)), where equilibrium payoffs are unique, only minimal winning coalitions form, none of the pie is wasted, and all equilibria are efficient.

We relate our model and results to the literature in the next section. We present our model in Section 3, and provide results on committees with a non-unanimity and a unanimity quota respectively in Sections 4 and 5. Section 6 concludes. Most of the proofs appear in the Appendix.

2 Related Literature

Baron and Ferejohn (1989) has spawned an enormous literature; we refer readers to Eraslan and McLennan (2011) for a recent list of contributions. The literature on bargaining in simple games with an endogenous default is much smaller, most likely for technical reasons: in equilibrium, the proposals which would be accepted may vary discontinuously with the default policy because of expectations about future play. The ensuing discontinuous transition probabilities preclude the use of conventional fixed point arguments to establish existence of even mixed strategy equilibria.

Kalandrakis (2004) studies majority rule games with three equally patient, risk neutral players and a statically efficient initial default; Kalandrakis (2009) extends the model to games with five or more players with concave preferences. These games have an equilibrium in which the default immediately reaches an ergodic distribution in which each proposer takes the entire pie; but players mix over extra-equilibrium proposals. By contrast, we follow Baron and Ferejohn (1989) by supposing that the initial default is statically inefficient, and allowing players to propose policies which waste some of the pie. (We show that this is possible in non-unanimity games.) In the equilibria which we construct, the default reaches a single policy (immediately).

Duggan and Kalandrakis (forthcoming) use a fixed point argument to establish existence of pure strategy equilibria for games in which preferences and the default are subject to stochastic shocks.[4] By contrast, we prove existence in unperturbed games (by and large) using constructive arguments.

Kalandrakis' (2004) and (2009) equilibria violate the size principle, in the sense that a subminimal winning coalition shares the pie. (Some of) our constructed equilibria violate the size principle, in the more conventional sense that a supraminimal winning coalition shares the pie, as in the equilibria constructed by Bowen and Zahran (2009) and Richter (2011):

Bowen and Zahran require preferences to be strictly concave and the initial default to be statically efficient, and show that the size principle is violated when discount factors take intermediate values and the initial default is not too inequitable. We also allow for (but do not require) strictly concave preferences;[5] but the size principle fails in our construction when all players are patient enough.

Richter (2011) constructs an egalitarian equilibrium by allowing offers to waste some of the pie. These offers are only made in order to deter deviations from equilibrium play, and are therefore never observed on the path. We also follow Baron and Ferejohn (1989) by allowing for such statically inefficient offers. However, in contrast to Richter, these offers are made on the equilibrium path in (some of) our constructions. In other words, we explain waste.

Baron (1991) argues that Congress often both wastes resources and splits the remainder among a supraminimal majority during distributive bargaining. Baron shows that closed and open rule models based on Baron and Ferejohn (1989) can explain waste (aka pork), but can only explain these violations of the size principle by appealing to a norm of universalism. By contrast, equilibria in our model exhibit both features.

The literature on bargaining in games with an evolving default started with Baron (1996), who provided a dynamic median voter theorem when the policy space is an interval. Other examples of bargaining in a simple game include Baron and Herron (2003), Gomes and Jehiel (2005), Bernheim et al (2006), Battaglini and Coate (2007), Anesi (2010), Diermeier and Fong (2011, 2012), Zápal (2011a,b), Acemoglu et al (2012), Anesi and

[4]Their results apply to a larger class of stage games (which includes pie division).

[5]As Battaglini and Palfrey (forthcoming) note, their experimental evidence on such games suggests that some subjects have strictly concave preferences.

Seidmann (2012), Battaglini and Palfrey (2012), Battaglini et al (2012), Nunnari (2012), and Bowen and Zahran (forthcoming).

Seidmann and Winter (1998) and Okada (2000), inter alia, study bargaining with an evolving default in superadditive characteristic function games.[6] Hyndman and Ray (2007) prove that all (including history-dependent) equilibria of characteristic function games are absorbing, and that they are asymptotically statically efficient if there is a finite number of feasible policies. They also show by example that these results do not carry over to games in partition function form. Now simple games are in characteristic function form if and only if the quota is unanimity. We exploit their first result when proving that every equilibrium of a unanimity game is no-delay; their second result also holds in our model (without requiring finiteness). Furthermore, statically inefficient equilibria exist both in our model with a non-unanimity quota and in Hyndman and Ray's model with a partition function form because each player's continuation function fails a monotonicity condition. However, Hyndman and Ray focus on asymptotic static efficiency, and assume a common discount factor; we consider Pareto efficiency and, crucially for associated results, allow discount factors to differ.

Finally, we turn to the no-delay property. Policy outcomes of our no-delay equilibria can be interpreted as a special case of Acemoglu et al's (2012) "dynamically stable states," which are defined as political states reached in a finite number of periods (and never changed) in pure strategy Markov perfect equilibria of bargaining games with evolving defaults and patient players. Hence, our results characterize and prove existence of a class of dynamically stable states in voting situations where, in contrast to those studied in Acemoglu et al (2012), the set of policies is infinite and policy preferences are not acyclic.

By definition, the default changes once in a no-delay equilibrium: policy is persistent. This prediction is consistent with a widespread claim that agencies are never terminated.[7] A related literature explains why statically inefficient policies may be persistent (so the policy sequence is inefficient). However, the mechanisms in this literature rely on privately incurred adjustment costs (Coate and Morris (1999)), incomplete information (e.g. Mitchell and Moro (2006)) or the growing power of incumbent factions (Persico et al (2011)). By contrast, no-delay equilibria are inefficient in our model because impatient players cannot

[6] Seidmann and Winter focus on equilibria in which the grand coalition forms after a number of steps. While we cannot exclude delay with a non-unanimity quota, our constructions all involve no-delay equilibria.

[7] See Kaufman (1976) for the conventional claim, and Lewis (2002) for a dissenting view.

commit to decreasing shares of the pie.

3 Notation and Definitions

3.1 The Standing Committee Game

In each of an infinite number of discrete periods, indexed $t = 1, 2, \ldots$, one unit of a divisible resource — the "pie" — has to be allocated among the members of a committee $N \equiv \{1, \ldots, n\}$, $n \geq 2$. Thus, the set of feasible policies is

$$X \equiv \left\{ (x_1, \ldots, x_n) \in [0, 1]^n : \sum_{i=1}^{n} x_i \leq 1 \right\} .$$

We denote the policy implemented in period t, and therefore the default at the beginning of period $t+1$, by $x^t = \left(x_1^t, \ldots, x_n^t\right)$. At the start of each period t, player i is selected with probability $p_i \in (0, 1)$ to propose a policy in X. We say that a player who proposes the existing default *passes*. All players then simultaneously vote to accept or to reject the chosen proposal. The voting rule used in every period t is a quota q which satisfies $n/2 < q \leq n$. Specifically, if at least q players accept proposal $y \in X$ then it is implemented as the committee decision in period t and becomes the default next period (i.e. $x^t = y$); and if y secures less than q votes then the previous default, x^{t-1}, is implemented again and becomes the default in period $t + 1$ (i.e. $x^t = x^{t-1}$). The default in period 1 is $x^0 = (0, \ldots, 0)$.[8] We will refer to $\left(x^t\right)_{t=1}^{\infty}$ such that every x^t is feasible as a *policy sequence.*

Once policy x^t has been implemented, every player i receives an instantaneous payoff $(1 - \delta_i)\, u_i \left(x_i^t\right)$, where u_i is a strictly increasing, continuously differentiable concave utility function, and $\delta_i \in (0, 1)$ is i's discount factor.[9] Thus, player i's payoff from a policy sequence $\left(x^t\right)_{t=1}^{\infty}$ is $(1 - \delta_i) \sum_{t=1}^{\infty} \delta_i^{t-1} u_i \left(x_i^t\right)$. We say that discount factors are *heterogeneous* if $\delta_i \neq \delta_j$ for some pair of players i and j; and that discount factors are *strictly heterogeneous* if $\delta_i \neq \delta_j$ for every pair of players i and j.

The assumptions above define a dynamic game, which we will refer to as a *standing committee* game. Our main purpose is to analyze the equilibria of this game. However, as noted in the Introduction, we want to assess the implications of an evolving default. Accordingly, we will refer below to play in a variant of the standing committee game:

[8]None of our results depend on which policy in X is the (exogenous) initial default.

[9]If players are myopic then the non-unanimity [resp.unanimity] version of our model has a unique equilibrium in which each [resp. the first] proposer successfully claims the entire pie.

the *ad hoc committee* game, in which the bargaining process stops as soon as a proposal is accepted and implemented. This game is equivalent to a generalization of Baron and Ferejohn's (1989) seminal model, allowing for any quota, heterogeneous discount factors and recognition probabilities, and concave preferences over the pie.

3.2 Equilibrium and Efficiency

Equilibrium concept. We follow the standard approach of concentrating on stage-undominated stationary Markov perfect equilibria, i.e., subgame perfect equilibria with the following two properties: (i) all players use stationary Markov strategies; and (ii) at any voting stage, no player uses a weakly dominated strategy. The first condition means that, in proposal stages, players' choices (of probability distributions over X) only depend on the ongoing default; in voting stages, players' choices (of probability distributions over $\{$accept , reject$\}$) only depend on the current default and the proposal just made. The second condition excludes strategy combinations in which players all vote one way, and are indifferent when $q < n$ because they are nonpivotal. Henceforth, we leave it as understood that any reference to "equilibria" is to stage-undominated stationary Markov perfect equilibria. We will be particularly interested in pure strategy equilibria, i.e. those with the property that every player's choice is deterministic after every history.

Absorbing points and no-delay strategies. Every stationary Markov strategy σ (in conjunction with recognition probabilities) generates a stationary transition function P^{σ}, where $P^{\sigma}(x, Y)$ is the probability (given σ) that the committee chooses a policy in Y in the next period, given that policy x is implemented in the current period. Thus, for all $i \in N$ and all $x \in X$, player i's continuation value from implementing x in the current period is given by

$$V_i^{\sigma}(x) = (1 - \delta_i)\, u_i(x_i) + \delta_i \int V_i^{\sigma}(y) P^{\sigma}(x, dy) \ .$$

We say that $x \in X$ is an *absorbing point* of σ if and only if $P^{\sigma}(x, \{x\}) = 1$, and denote by

$$A(\sigma) \equiv \{x \in X : P^{\sigma}(x, \{x\}) = 1\}$$

the set of absorbing points of σ. We will say that σ is *no-delay* if and only if: (i) $A(\sigma) \neq \emptyset$; and (ii) $P^{\sigma}(x, A(\sigma)) = 1$ for all $x \in X$. In words, a strategy profile is no-delay if the committee implements an absorbing point *at any default.*

(In)efficiency. It is instructive to distinguish between two notions of inefficiency. First, policy $x \in X$ is *statically inefficient* if $\sum_{i \in N} x_i < 1$; we will refer to $1 - \sum_{i \in N} x_i$ as *waste*.[10] Second, σ is *Pareto inefficient* if the vector $\left(V_1^{\sigma}\left(x^0\right), \ldots, V_n^{\sigma}\left(x^0\right)\right)$ is Pareto dominated by the infinite-horizon payoff arising from some (possibly stochastic) policy sequence in X. Evidently, every equilibrium inducing a policy sequence which contains a statically inefficient policy is Pareto inefficient, but the converse is false.

4 Nonunanimity Committees

Let $\mathcal{W}$ be the collection of winning coalitions: $\mathcal{W} \equiv \{C \subseteq N : |S| \geq q\}$. Throughout this section, we assume that $q < n$: agreement requires less than unanimous consent.

4.1 Simple Solutions

We will construct a class of pure strategy no-delay equilibria, in which each player $j \in N$ can only be offered two different shares of the pie: a "high" offer $x_j > 0$ and a "low" offer $y_j < x_j$. In every period and for any ongoing default, each proposer i (conditional on her being recognized to make an offer) implicitly selects a winning coalition $C_i \ni i$ by making high offers to the members of C_i and low offers to the members of $N \setminus C_i$. If each player receives a low offer from at least one proposer, then we refer to the set of such proposals (one for each player) as a *simple solution.* Formally:

Definition 1. *Let $\mathcal{C} \equiv \{C_i\}_{i \in N} \subseteq \mathcal{W}$ be a class of coalitions such that, for each $i \in N$, $i \in C_i$ and $i \notin C_j$ for some $j \in N \setminus \{i\}$. Let $x = (x_1, \ldots, x_n)$ and $y = (y_1, \ldots, y_n)$ be two vectors in $[0,1]^n$ satisfying $x_i > y_i$ and*

$$\sum_{j \in C_i} x_j + \sum_{j \notin C_i} y_j \leq 1 \ ,$$

for all $i \in N$. The simple solution *induced by $(\mathcal{C}, x, y)$ is the set of policies $S \equiv \left\{x^{C_i}\right\}_{i \in N}$, where*

$$x_j^{C_i} \equiv \begin{cases} x_j & \text{if } j \in C_i \ , \\ y_j & \text{if } j \notin C_i \ , \end{cases}$$

for all $i, j \in N$.

[10] Recall that $u_i(.)$ is strictly increasing in x_i.

Before we turn our attention to the construction of equilibria themselves, a few remarks are in order about simple solutions. First, a simple solution exists if and only if $q < n$: if $q < n$ then the main simple solution, in which the pie is divided equally among every minimal winning coalition, is a notable example of a simple solution (cf. Wilson (1971)); if $q = n$ then each player must be included in the unique winning coalition N and, therefore, there is no simple solution. If $q < n$ then any policy which assigns a positive share to at least q players is part of some simple solution. To see this, take an arbitrary policy $z \in X$ such that $|\{i : z_i > 0\}| \geq q$. For expositional convenience, we order the players in N in such a way that $z_i \geq z_{i+1}$ for each $i = 1, \ldots, n-1$ (thus ensuring that $z_i > 0$ for all $i \leq q$). Consider the simple solution induced by $(\mathcal{C}, x, y)$, where

$$x_i = \begin{cases} z_i & \text{if } i \leq q \\ z_i + \frac{\varepsilon}{n-q} & \text{if } i > q \end{cases}, \quad y_i = \begin{cases} z_i - \varepsilon & \text{if } i \leq q \\ z_i & \text{if } i > q \end{cases}, \quad \varepsilon > 0 \text{ arbitrarily small,}$$

and C_i is the coalition that includes i and the next $q - 1$ players following the order $1, 2, \ldots, n-1, n, 1, 2, \ldots, q-1$. It is readily checked that $(\mathcal{C}, x, y)$ satisfies all the conditions of Definition 1 (in particular $y_i \geq 0$ for all $i \in N$), and that $x^{C_1} = z$.

Second, the definition of the class $\mathcal{C}$ of coalitions does not require all of them to be distinct; but it is easy to confirm that $\mathcal{C}$ must contain at least $n/(n-q)$ distinct coalitions. Third, the policies in a simple solution may all assign a positive share to a supraminimal coalition, and might all involve waste. Fourth, policies which assign a positive share to fewer than a minimal winning coalition cannot be included in a simple solution. Such policies include the initial default and the vertices of the simplex.

4.2 Preliminary Intuitions

The following example illustrates Definition 1, and provides an intuitive presentation of some key mechanisms behind our equilibrium construction.

Example 1. Let $n = 3$, $q = 2$, $p_i = 1/3$, $\delta_i = \delta$ and $u_i(x) = x_i$ for all $i \in N$.[11] Take, for example, the simple solution $S = \{(1/3, 1/3, 1/6), (1/6, 1/3, 1/3), (1/3, 1/6, 1/3)\}$ — that is, $C_1 = \{1, 2\}$, $C_2 = \{2, 3\}$, $C_3 = \{1, 3\}$ and $x_j = 1/3$, $y_j = 1/6$ for every player $j = 1, 2, 3$. If $\delta \geq 12/13$ then the following strategy profile forms a pure strategy, no-delay equilibrium whose set of absorbing points is S:

[11] These are precisely the assumptions made by Kalandrakis (2004). In contrast to that paper, however, we require the initial default to be $(0, \ldots, 0)$, and allow for policies which do not exhaust the pie.

- Player i always offers $1/3$ to the players in C_i and $1/6$ to the player outside C_i if the ongoing default does not belong to S, and passes otherwise;
- Player i accepts proposal z when the ongoing default is w if and only if one of the following conditions holds: (i) $w \in S$ and $w_i = 1/6$; (ii) $w \notin S$, $z \in S$, and $z_i \geq (1-\delta)w_i + (5\delta/18)$; or (iii) $w, z \notin S$ and $z_i \geq w_i$.

A formal proof of this statement is obtained as a special case of Theorem 1. The intuition is as follows. It is readily checked that this (pure) strategy profile is no-delay and that S is the set of absorbing points: each policy x^{C_i} in S is proposed by player i with probability $1/3$, accepted by the two members of majority coalition C_i, and never amended.

To see why this is an equilibrium, observe first that each (patient) player $i = 1, 2, 3$ can only end up in two possible states in the long-run: a "good state" in which she receives $1/3$ in all periods, and a "bad state" in which she receives $1/6$ in all periods. Indeed, any ongoing default w is either an absorbing point itself or will lead immediately to some absorbing point $x^{C_j} \in S$, with $x_i^{C_j} \in \{1/6, 1/3\}$. In the former case, player i's expected payoff is $w_i = 1/3$ if $i \in C_j$, and $w_i = 1/6$ otherwise. In the latter case, i receives w_i in the current period and $2/3 \times 1/3 + 1/3 \times 1/6 = 5/18$ in the next period ($i \in C_j$ with probability $2/3$). Her expected payoff is therefore $(1-\delta)w_i + (5\delta/18)$, which is less than $1/3$ for all $w_i \in (0, 1)$ (recall that $\delta \geq 12/13$).

Thus, every player i seeks to maximize (resp. minimize) the probability of ending up in a good (resp. bad) state. In voting stages, this includes rejecting any proposal to change a default policy x^{C_j} with $C_j \ni i$ to another policy y (even if y Pareto dominates x^{C_i}): being in a good state, i would not run the risk of ending up in a bad state. It also includes accepting any proposal x^{C_j} with $C_j \ni i$ when the ongoing default is not already a good state for i. As the C_j's are winning coalitions, these observations imply that any attempt to change a default in S would be unsuccessful, and that any proposal to change a default outside S to a policy in S would be successful. In proposal stages, it is therefore optimal for player i to propose x^{C_i} if the ongoing default is not an absorbing point, and to pass otherwise.

$\square$

This example illustrates why our results are radically different from those obtained in the standard ad hoc committee game (Baron and Ferejohn (1989)). In particular, it explains why shares of the pie can be perpetually wasted in equilibrium: any deviation to proposing a Pareto-superior policy would be rejected, as policy would revert to one of the statically inefficient absorbing points. It also shows that the pie can be shared amongst more than a minimal winning coalition in equilibrium.

4.3 Results

Our first result generalizes the argument above to any quota, any concave utility functions, and any simple solution. We describe a pure strategy no-delay equilibrium in which each policy in a simple solution is proposed by some player, and no other policy is proposed as a *simple equilibrium.*

Theorem 1. *Suppose that $q < n$, and let S be a simple solution. There exists $\bar{\delta} \in (0,1)$ such that the following is true whenever $\min_{i \in N} \delta_i \geq \bar{\delta}$: There exists a pure-strategy no-delay equilibrium whose set of absorbing points is S.*

Theorem 1 has several interesting implications:

Multiplicity of equilibrium outcomes. We noted above that any policy (say, z) which assigns a positive share to q or more players is part of a simple solution. Theorem 1 therefore implies that z is an absorbing point of an equilibrium of any game with $q < n$ and patient enough players. In that equilibrium, player 1 proposes z which is accepted by all members of coalition $C_1 = \{1, \dots, q\} \in \mathcal{W}$, and never amended.

This argument does not apply to policies which assign a positive share to fewer than q players (including the initial default), and can therefore not be part of a simple solution. Policies which assign a zero share to some winning coalition cannot be absorbing points because every member of such a coalition could profitably deviate as a proposer.[12]

Minimal winning coalitions. The Baron-Ferejohn model predicts that only minimal winning coalitions share the dollar in equilibrium. Theorem 1 immediately implies that this property, often referred to as the *size principle,* may fail in our model with an evolving default: As mentioned earlier, policies in a simple solution may all assign a positive share to a supraminimal coalition.

Waste. Another important implication of Theorem 1 is that endogeneity of the default may create substantial (static) inefficiencies in equilibrium. For any $\varepsilon \in (0,1)$,

[12] As Kalandrakis (2004, 2010) demonstrates, such policies could nevertheless be part of an ergodic set.

let X_ε be the set of policies such that the committee "wastes" more than $1-\varepsilon$: $X_\varepsilon \equiv \{x \in X : \sum_{i\in N} x_i < \varepsilon\}$. It is easy to find simple solutions that are subsets of X_ε. For instance, take the simple solution induced by $(\mathcal{C}, x, y)$ where, for each $i \in N$, $x_i = \varepsilon/2q$, $y_i = 0$, and C_i is the coalition that includes i and the next $q-1$ players following the order $1, 2, \ldots, n-1, n, 1, 2, \ldots, q-1$. Theorem 1 implies that any non-unanimity game with patient enough players has a pure-strategy no-delay equilibrium whose absorbing points all belong to X_ε: *the committee wastes at least* $1-\varepsilon$ *in every period along the equilibrium path.* This again stands in sharp contrast to the stationary equilibria of the Baron-Ferejohn model, in which waste never occurs.

Agreements may in fact be even worse relative to the initial default than our presentation has hitherto suggested. Specifically, the proof of Theorem 1 does not rely on our supposition that $x^0 = (0, \ldots, 0)$; so we can construct simple equilibria in which every absorbing policy strictly Pareto-dominates the initial default (by appropriately selecting x^0).[13]

Pork barrel politics. We have noted that equilibrium agreements may waste some of the pie and that the size principle may fail. Theorem 1 says something stronger: that patient players may waste some of the pie (pork, in Baron's terms) and distribute the remainder among a supra-majority of players. According to Schattschneider (1935), this combination of properties characterized US trade policy before 1934. Indeed, Baron (1991) claims that legislation on distributive issues often exhibits this combination.[14] He also argues that models of ad hoc committees can explain pork, but not violations of the size principle. By contrast, Theorem 1 implies that equilibrium agreements in a standing committee may satisfy both properties without appealing to a norm of universalism.

We record the observations above as

Corollary 1. *Suppose that* $q < n$. *For each of the following statements, there exists* $\bar{\delta} \in (0,1)$ *such that this statement is true whenever* $\min_{i\in N} \delta_i \geq \bar{\delta}$*:*

(i) There exist multiple pure-strategy no-delay equilibria;

[13] This property is stronger than a related result in Bernheim et al's (2006) and Anesi and Seidmann's (2012) models of bargaining with an evolving default: that the equilibrium agreement is worse than x^0 for some winning coalition.

[14] Evans (2004) documents the failure of the size principle, and argues that Congress may often pass inefficient public good projects.

(ii) Any policy which assigns a positive share to q or more players is an absorbing point in some pure-strategy no-delay equilibrium;

(iii) There are equilibria which fail the size principle;

(iv) For every $\varepsilon \in (0,1)$, there is a pure-strategy equilibrium σ such that $P^{\sigma}(x, X_{\varepsilon}) = 1$ for all $x \in X$;

(v) There are no-delay equilibria in which the agreement wastes some of the pie and fails the size principle.

Corollary 1(ii) implies that some simple equilibria are statically efficient. However, wasting some of the pie is not the only possible kind of inefficiency in dynamic models. Our next result asserts that, if all players have linear preferences ($u_i(x_i) = x_i$) and discount factors are strictly heterogeneous then *all* equilibria of a game with a non-unanimity quota (including those which are not simple) are Pareto inefficient. (Players have linear preferences in Baron and Ferejohn (1989), and much of the ensuing literature.)

Theorem 2. *Let $q < n$. If $u_i(x_i) = x_i$ for all $i \in N$ and $\delta_i \neq \delta_j$ for all $i, j \in N$ then all equilibria are Pareto inefficient.*

The argument for Theorem 2 is easiest to see when the equilibria are no-delay (like simple equilibria). As discount factors are strictly heterogeneous, efficiency requires front loading the shares of less patient players, and eventually assigns the entire pie to the most patient player. This is impossible in equilibrium. We conjecture that Proposition 1 holds whenever preferences are concave. However, to the best of our knowledge, characterization of the set of efficient policy sequences in such cases remains an open question.

In contrast to Theorem 1, the premise of Theorem 2 does not require that players be patient enough. It only requires strict heterogeneity. It is easy to confirm that the argument works as long as enough players have different discount factors.

The no-delay property of simple equilibria provides a possible explanation for why policies, such as protection, are persistent (cf. Ray and Marvel (1984) and Baldwin (1985)).[15] Theorem 2 implies that a persistent policy is inefficient when discount factors are strictly heterogeneous, even if the policy is statically efficient.

[15]Indeed, Brainard and Verdier (1997) describe persistent protection as "one of the central stylized facts in trade" (p222).

5 Unanimity Committees

This section examines equilibria of standing committee games in which agreement requires unanimous consent: that is, $q = n$.

5.1 Preliminary Example

As in the previous section, we begin with a simple example that will provide some intuition for the general results that follow.

Example 1 Continued. Consider a variant on Example 1 (of Section 4.2) in which the default can only be changed if all three players accept a proposal: that is, $q = n = 3$. The other primitives of the example remain the same: $p_i = 1/3$, $\delta_i = \delta$ and $u_i(x) = x_i$ for all $i \in N$. We will construct a no-delay equilibrium σ in which, at any default $x \in X$, the selected proposer (say i) successfully offers the committee a policy $x + s^i(x) \in \Delta_{n-1}$. We can think of proposer i offering to share the amount of pie not distributed yet — i.e. $1 - (x_1 + x_2 + x_3)$ — with the other players, with $s^i_j(x)$ being the (extra) share offered by proposer i to player j.[16] In such a situation, proposer i's optimal offer to player j, $x_j + s^i_j(x)$, must leave the latter indifferent between accepting and rejecting. If j rejected i's offer, she would receive her payoff from the ongoing default in the current period, $(1-\delta)x_j$, and would then receive offer $x_j + s^k_j(x)$ from each proposer $k = 1, 2, 3$ with probability 1/3 in the next period. The following condition must therefore hold:

$$x_j + s^i_j(x) = (1-\delta)x_j + \delta \left[x_j + \frac{s^1_j(x) + s^2_j(x) + s^3_j(x)}{3} \right]$$

or, equivalently

$$s^i_j(x) = \frac{\delta}{3} \left[s^1_j(x) + s^2_j(x) + s^3_j(x) \right] \tag{1}$$

for each i and $j \neq i$. Given the shares of the pie offered to the other committee members, proposer i receives the residual:

$$x_i + s^i_i(x) = 1 - \sum_{j=1}^{3} \left[x_j + s^i_j(x) \right] \ . \tag{2}$$

[16]Hence, all proposers pass when the ongoing default is already in the unit simplex: $s^i(x) = (0, 0, 0)$ for all $i = 1, 2, 3$ whenever $x \in \Delta_2$.

Combining (1) and (2), we obtain the policy $x+s^i(x)$ (absorbing point) successfully offered by each player i at any default $x \in X$:

$$x_i + s^i_i(x) = x_i + \frac{3-2\delta}{3}\left(1 - \sum_{j=1}^{3} x_j\right) ,$$

$$x_j + s^i_j(x) = x_j + \frac{\delta}{3}\left(1 - \sum_{j=1}^{3} x_j\right) , \forall j \neq i .$$

In particular, each player expects to earn 1/3 in the game itself: $V_i^\sigma\left(x^0\right) = 1/3$.

□

Its simplicity notwithstanding, there are two noteworthy features of this example. First, the set of absorbing points of the no-delay equilibrium σ coincides with the unit simplex: $x_j + s^i_j(x) \in \Delta_2$ for all $x \in X$ and all $i, j \in N$. Second, the unique equilibrium payoff coincides with that of the analogous ad hoc committee game with a unanimity quota. As the rest of this section will demonstrate, these properties are not coincidental.

5.2 Positive Results

Our first result asserts existence of a pure strategy no-delay equilibrium in which resources are never wasted. Theorem 3 differs from Theorem 1 (our analogous result for $q < n$) in two main respects. First, we no longer require that players be patient enough. Second, Theorem 3 asserts that the policies reached from any default (including the initial default) are statically efficient.

Theorem 3. *If $q = n$ then: (i) every equilibrium σ is a pure strategy no-delay equilibrium with $A(\sigma) = \Delta_{n-1}$; and (ii) such an equilibrium exists.*

Thus, under unanimity rule, a standing committee selects an absorbing point in the simplex immediately *at any ongoing default*. In contrast to non-unanimity committees, therefore, waste never occurs in an equilibrium of unanimity committee games.

Our construction (using simple solutions) in the proof of Theorem 1 relied on the possibility that deviation could be punished by changing the default (with positive probability) without the deviator's assent. By contrast, such punishments could only be implemented with the deviator's assent in unanimity games. Indeed, stage payoffs must satisfy a monotonicity condition in such games: in any equilibrium σ, any player i's continuation value

$V_i^\sigma(x)$ from a given default x must be at least the net present value $u_i(x)$ of always implementing her stage payoff at that default. This monotonicity condition allows us to exploit Hyndman and Ray (2007) Proposition 1, which implies (in our model) that the equilibrium default converges almost surely.

We prove Theorem 3(ii) using a construction which generalizes that employed in Example 1 above: A fixed point point argument is used to show that there are proposals for each player which move the default into the simplex and make every respondent indifferent between accepting and rejecting, given that defaults in the simplex would not be amended; and that no player can profitably deviate from proposing such policies or accepting such an offer.

Theorem 3 establishes that the unanimity game has and only has no-delay, statically efficient pure strategy equilibria. These properties also hold for an ad hoc, unanimity game (Banks and Duggan (2000, 2006)). Our next result strengthens the analog between equilibrium play in ad hoc and standing committees with a unanimity quota.

Theorem 4. *If $q = n$ then there is a unique equilibrium payoff, which coincides with the (stationary subgame perfect) equilibrium payoff of the ad hoc committee game.*

The proof of this theorem establishes that pure-strategy equilibrium outcomes in the two games coincide. The result then follows from Merlo and Wilson (1995) Theorem 2, which shows that the ad hoc committee game has a unique equilibrium payoff when $q = n$.

In the Introduction, we asked how play in ad hoc and standing committees differs. Our results in the last section entail a significant contrast across equilibrium outcomes in the two games when $q < n$. Theorem 4 implies that this contrast does not carry over to games with a unanimity quota.

5.3 Pareto efficiency

Theorem 2 states that every equilibrium of a non-unanimity game with linear preferences is inefficient if discount factors are strictly heterogeneous. Pareto efficiency then requires that the most patient player eventually gets the entire pie: which is impossible in equilibrium. In addition, Corollary 1(ii) states that there are no-delay, statically inefficient equilibria. If $q = n$ then waste is impossible in equilibrium (by Theorem 3). However, the inefficiency result carries over:

Theorem 5. *If $q = n$ and $\delta_i \neq \delta_j$, for some $i, j \in N$, then any equilibrium is Pareto inefficient.*

In contrast to Theorem 2, the premise of Theorem 5 does not require linear preferences, and weakens strict heterogeneity to heterogeneity. We obtain this stronger result because the equilibrium of a unanimity game is no-delay (Theorem 3). This extra structure allows us to prove inefficiency by constructing a Pareto-improving policy sequence.

6 Concluding Remarks

This paper has identified a class of pure strategy (stationary Markov perfect) equilibria for pie-division bargaining games with an evolving default, which supplements existing constructions. This has allowed us to provide a number of predictions about decision making in standing committees, which differ from those of ad hoc committee games (in the tradition of Baron and Ferejohn (1989)) exactly when the quota is less than unanimity. Some notable differences are:

(i) Substantial shares of the pie can be indefinitely wasted and the size principle may fail in non-unanimity standing committees, whereas waste never occurs and only minimal winning coalitions form in ad hoc committees. Thus, while models of ad hoc committees can explain pork but not violations of the size principle, agreements in a standing committee may possess both properties.

(ii) Equilibrium play in the standing committee game is Pareto inefficient when discount factors are heterogeneous; whereas it is well known that equilibrium play is efficient in the Baron-Ferejohn (1989) model, where play ends as soon as the committee agrees. So far, we have interpreted the latter framework as a model of an ad hoc committee, in which the pie is divided once. In light of the efficiency results, it is instructive to reinterpret it as a model in which a proposal represents a policy sequence: that is, how the pie is to be divided thereafter. On this interpretation, the key difference from our model is that a Baron-Ferejohn committee cannot renegotiate an agreement. Given this commitment capacity, we would expect players to propose Pareto efficient policy sequences in any equilibrium. Viewed in this light, our model demonstrates that equilibrium play in a standing committee is generically inefficient because players cannot commit not to renegotiate the existing agreement.

In addition, the identified equilibria to the standing committee game have a no-delay property: the first policy proposal is accepted and remains in place in all future periods. Our results thus contribute to the political-economy literature on the persistence of inefficient policies.

Banks and Duggan (2000, 2006) have generalized the standard model of bargaining in ad hoc committees to include any convex set of policies as well as purely distributional policies, and established existence of a (mixed-strategy) stationary subgame perfect equilibrium. Before concluding, a similar extension of our model of bargaining in standing committees to more general policy spaces is worth discussing.

Our positive results for non-unanimity games relied on the existence of simple solutions. Though the definition of a simple solution needs to be extended to this more general setting, the logic behind this extension remains the same as for Definition 1. Each player i can be in two possible states: a "good state," in which she has a high utility u_i, or a "bad state," in which she has a low utility v_i. Each proposer i selects a policy x^{C_i} which gives all members of winning coalition C_i their high utility, and gives the other players their low utility. Put differently, each proposer i selects the coalition C_i of players who will be in a good state.

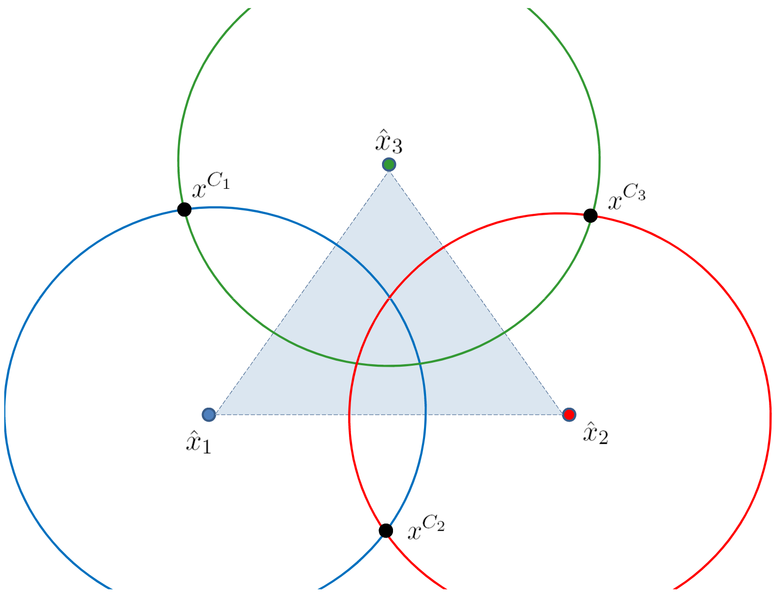

Figure 1: Simple Solution in the Spatial Model

Figure 1 provides an example of a simple solution in the standard spatial model: $n = 3$; $q = 2$; X is a nonempty, compact and convex subset of $\mathbb{R}^2$; and $u_i(x) = -\|x - \hat{x}_i\|$ for all $x \in X$ and all $i \in N$, where $\hat{x}_i \in X$ stands for the ideal policy of player i. The set of policies $S = \{x^{C_1}, x^{C_2}, x^{C_3}\}$ constitutes a simple solution and, therefore, the set of absorbing points of some pure-strategy no-delay equilibrium whenever players are sufficiently patient. (The arguments used to prove Theorem 1 still apply.) This equilibrium is both statically and Pareto inefficient: all the policies in S lie outside the static Pareto set (the grey triangle in Figure 1) and all players would be strictly better off if the expected policy $\sum_i p_i x^{C_i}$ were agreed immediately and never amended. This is in accord with our findings for the distributive setting: The non-unanimity standing committee game (extended to a nonempty, compact policy set in $\mathbb{R}^k$) possesses a pure-strategy no-delay equilibrium whenever a simple solution exists and players are patient enough. This equilibrium may be statically and/or Pareto inefficient.

Simple solutions can again only exist if $q < n$. Pie-splitting problems possess a main simple solution; but this is only known to exist for strong simple symmetric games in characteristic function form with transferable utility, and remains an open question for more general simple games.[17] Indeed, no simple solution can exist when X is a compact interval on the real line, as the median voter cannot be excluded from any winning coalition. If players are patient enough then both ad hoc and standing committees reach policies close to the median voter's ideal policy in no-delay equilibria (cf. Baron (1996) and Banks and Duggan (2006))).

We now turn to unanimity games. We showed in the last section that, in distributive settings, the sets of policies which can be implemented in ad hoc and standing committees coincide. Indeed, a brief inspection of the proof of Theorem 4 reveals that it does not rely on the restriction to pie-division problems. Hence, when $q = n$, the equilibrium outcomes of the extended standing committee game are also (stationary subgame perfect) equilibrium outcomes of the related ad hoc committee game. Whether the converse is also true, however, remains an open question. Indeed, the monotonicity condition used to establish Theorem 4 (cf. Lemma 1(i) in the Appendix) still holds but is insufficient to prove that optimal proposals in the standing committee game are also optimal in the ad hoc committee game (given the same voting behavior).

Appendix

Theorem 1. *Suppose that $q < n$, and let S be a simple solution. There exists $\bar{\delta} \in (0,1)$ such that the following is true whenever $\min_{i \in N} \delta_i \geq \bar{\delta}$: There exists a pure-strategy no-delay equilibrium whose set of absorbing points is S.*

[17] We refer the reader to Ordeshook (1986, Chapter 9) for an in-depth discussion.

Proof: Let $\{C_1, \ldots, C_n\} \subseteq \mathcal{W}$ and $(x_1, \ldots, x_n), (y_1, \ldots, y_n) \in [0,1]^n$ satisfy the conditions in Definition 1, and let $S \equiv \left\{x^{C_i}\right\}$ be the simple solution induced by $\left(\{C_i\}_{i \in N}, x, y\right)$. Our goal is to construct a pure-strategy stationary Markov strategy σ such that $A(\sigma) = S$, and then to show that σ is a no-delay subgame perfect equilibrium when $\min_{i \in N} \delta_i$ exceeds some threshold $\bar{\delta} \in (0,1)$.

We begin by defining $\bar{\delta}$. Let $p^{\min}$ be the minimal probability of recognition among the members of the committee: $p^{\min} \equiv \min_{i \in N} p_i$. For each player $i \in N$, define the threshold $\bar{\delta}_i$ as

$$\bar{\delta}_i \equiv \max \left\{ \frac{u_i(1) - u_i(x_i)}{u_i(1) - p^{\min} u_i(y_i) - (1 - p^{\min}) u_i(x_i)}, \frac{u_i(y_i) - u_i(0)}{p^{\min} u_i(x_i) + (1 - p^{\min}) u_i(y_i) - u_i(0)} \right\} \in (0,1) \ .$$

The threshold $\bar{\delta}$ is defined as $\bar{\delta} \equiv \max_{i \in N} \bar{\delta}_i$.

We henceforth assume that $\min_{i\ inN} \delta_i \geq \bar{\delta}$.

We now turn to the construction of strategy profile $\sigma = (\sigma_1, \ldots, \sigma_n)$. For each $i \in N$, define the function $\phi^i : X \to S$ as follows: (1) if $w \in S$ then $\phi^i(w) = w$; (2) if $x \notin S$ then $\phi^i(w) \equiv \left(\phi^i_1(w), \ldots, \phi^i_n(w)\right)$ where $\phi^i_j(w) = x^{C_i}_j$ for all $j \in N$.

Equipped with functions $\left(\phi^i\right)_{i \in N}$, we are now in a position to define σ. For each $i \in N$, σ_i prescribes the following behavior to player i:

(a) In the proposal stage of any period t with ongoing default $x^{t-1} = w$, i's proposal (conditional on i being selected to make a proposal) is $\phi^i(w)$;[18]

(b) In the voting stage of any period t with ongoing default $x^{t-1} = w$, player i accepts proposal $z \in X \setminus \{w\}$ if and only if: either (a) $w \in S$ and $w_i = y_i$; or (b) $w \notin S$ and

$$(1 - \delta_i) u_i(z_i) + \delta_i \sum_{j \in N} p_j u_i \left(\phi^j_i(z)\right) \geq (1 - \delta_i) u_i(w_i) + \delta_i \sum_{j \in N} p_j u_i \left(\phi^j_i(w)\right) \ .$$

Observe that σ is a pure strategy stationary Markov strategy profile. We will now prove that σ is a no-delay, stage-undominated subgame perfect equilibrium in a number of easy-to-prove steps.

Claim 1: The collection of functions $\left(\phi^i\right)_{i \in N}$ satisfies the following inequality for all $i \in N$ and $w \notin S$:

$$(1 - \delta_i) u_i(w_i) + \delta_i \sum_{j \in N} p_j u_i \left(\phi^j_i(w)\right) \leq u_i(x_i) \ .$$

[18] Recall that proposing the default w is interpreted as passing.

Proof: Consider any player $i \in N$ and any policy $x \notin S$. By definition of the ϕ^j's, we have

$$(1-\delta_i)\,u_i\,(w_i)+\delta_i\sum_{j\in N}p_j u_i\left(\phi_i^j(w)\right)=(1-\delta_i)\,u_i\,(w_i)+\delta_i\left[u_i\,(x_i)\sum_{j:i\in C_j}p_j+u_i\,(y_i)\sum_{j:i\notin C_j}p_j\right]$$

$$\leq(1-\delta_i)\,u_i(1)+\delta_i\left[\left(1-p^{\min}\right)u_i\,(x_i)+p^{\min}u_i\,(y_i)\right]$$

$$\leq u_i\,(x_i)$$

where the last inequality follows from $\delta_i \geq \bar{\delta} \geq \bar{\delta}_i$.

Claim 2: σ is no-delay with $A(\sigma)=S$; and, for all $w \in X$ and $i \in N$,

$$V_i^{\sigma}(w)=(1-\delta_i)\,u_i\,(w_i)+\delta_i\sum_{j\in N}p_j u_i\left(\phi_i^j(w)\right)\ , \tag{3}$$

where $V_i^{\sigma}(w)$ is i's continuation value, which is implicitly defined in Section 3.2.

Proof: When $w \notin S$ is implemented, each player $i \in N$ receives $(1-\delta_i)\,u_i\,(w_i)$. Then, in the next period, player j is selected to make a proposal with probability p_j. From the definition of proposal strategies, she proposes $z=\phi^j(w)$. As $z \in S$, proposal strategies prescribe all proposers to pass when the default is z: $\phi^j(z)=z$ for all $j \in N$. This implies that z would be implemented in all future periods if it were voted up in the next period. From part (b) in the definition of voting strategies, this implies that j's proposal is voted up (and never amended): each member i of $C_j \in \mathcal{W}$ is offered $z_i = x_i$ and

$$(1-\delta_i)\,u_i\,(x_i)+\delta_i\sum_{j\in N}p_j u_i\left(\phi_i^j(z)\right)=u_i\,(x_i)\geq(1-\delta_i)\,u_i\,(w_i)+\delta_i\sum_{j\in N}p_j u_i\left(\phi_i^j(w)\right)$$

(where the inequality is obtained from Claim 1). Hence, player i's expected continuation value from the next period on is $\sum_{j\in N}p_j u_i\left(\phi_i^j(w)\right)$. This proves that (3) holds and $P^{\sigma}(w,S)=1$ for all $w \notin S$.

Now suppose that $w \in S$ is implemented. From the definition of proposal strategies, all proposers pass in future periods — i.e. $w_i=\phi_i^j(w)$ for all $i,j \in N$ — so that i's continuation value is $u_i\,(w_i)$. This implies that

$$V_i^{\sigma}(w)=u_i\,(w_i)=(1-\delta_i)\,u_i\,(w_i)+\delta_i\sum_{j\in N}p_j u_i\left(\phi_i^j(w)\right)$$

and $P^{\sigma}(w,S)=1$ for all $w \in S$. $A(\sigma)=S$ then follows from $P^{\sigma}(w,S)=1$ for all $w \in X$.

Claim 3: Given default w and proposal z, each voter $i \in N$ accepts only if $V_i^\sigma(z) \geq V_i^\sigma(w)$, and rejects only if $V_i^\sigma(w) \geq V_i^\sigma(z)$.

Proof: If $w \notin S$ then this claim is an immediate consequence of Claim 2 and the definition of voting strategies (part (b)).

Suppose that $w \in S$ — so $V_i^\sigma(w) = u_i(w_i)$. We must prove that part (a) in the definition of voting strategies prescribes i to accept only if $V_i^\sigma(z) \geq V_i^\sigma(w)$, and to reject only if $V_i^\sigma(w) \geq V_i^\sigma(z)$. To do so, we distinguish between two different cases:

• Case 1: $z \in S$ — so $V_i^\sigma(z) = u_i(z_i) \in \{u_i(x_i), u_i(y_i)\}$. In this case, if i accepts then $w_i = y_i$. Hence, $V_i^\sigma(w) = u_i(y_i) = \min\{u_i(x_i), u_i(y_i)\} \leq V_i^\sigma(z)$. If i rejects then $w_i = x_i$ and $V_i^\sigma(w) = u_i(x_i) = \max\{u_i(x_i), u_i(y_i)\} \geq V_i^\sigma(z)$.

• Case 2: $z \notin S$. In this case, if i accepts then $w_i = y_i$. As $\delta_i \geq \bar{\delta} \geq \bar{\delta}_i$,

$$\begin{aligned} V_i^\sigma(w) = u_i(y_i) &\leq (1-\delta_i)\, u_i(0) + \delta_i \left[p^{\min} u_i(x_i) + \left(1-p^{\min}\right) u_i(y_i)\right] \\ &\leq (1-\delta_i)\, u_i(z_i) + \delta_i \sum_{j \in N} p_j u_i\left(\phi_i^j(w)\right) = V_i^\sigma(z) \ . \end{aligned}$$

If i votes rejects then $w_i = x_i$. Claim 1 then implies that

$$V_i^\sigma(w) = u_i(x_i) \geq (1-\delta_i)\, u_i(z_i) + \delta_i \sum_{j \in N} p_j u_i\left(\phi_i^j(z)\right) = V_i^\sigma(z) \ .$$

Claim 4: There is no profitable one-shot deviation from σ in the proposal stage of any period.

Proof: Suppose, first, that the current default w is an element of S. Passing is evidently an optimal action for the selected proposer, for part (a) in the definition of voting strategies implies that members of some winning coalition — i.e. those voters j who receive $w_j = x_j$ — would reject any proposal in X.

Now suppose that $w \notin S$. If proposer i followed the prescription of σ_i then her proposal would be accepted (Claim 2) and her payoff would be $u_i(x_i)$ (which is the highest payoff she can obtain by making a proposal in S). She must therefore propose a policy $z \notin S$ if she is to profitably deviate from σ_i. By Claim 1, however, we have

$$V_i^\sigma(z) = (1-\delta_i)\, u_i(z_i) + \delta_i \sum_{j \in N} p_j u_i\left(\phi_i^j(z)\right) \leq u_i(x_i) \ ,$$

for all $z \notin S$. This proves that no proposer has a profitable one-shot deviation from σ.

Combining Claims 1-4, we obtain Theorem 1.

□

Theorem 2. *Let $q < n$. If $u_i(x_i) = x_i$ for all $i \in N$ and $\delta_i \neq \delta_j$ for all $i, j \in N$ then all equilibria are Pareto inefficient.*

Proof: We assume without loss of generality that $\delta_i < \delta_{i+1}$ for each $i = 1, \ldots, n-1$. Now suppose, contrary to the statement of Theorem 2, that an efficient equilibrium σ exists.

Let $\mathbf{u}_i^t$ denote player i's expected period-t payoff in this equilibrium. To obtain the desired contradiction, we first need to establish the following result for every $i \in N$: If $\mathbf{u}_i^t > 0$ for some $t \in \mathbb{N}$ then $\mathbf{u}_j^\tau = 0$ for all $j < i$ and all $\tau > t$. To see this, suppose instead that $\mathbf{u}_i^t > 0$ and that $\mathbf{u}_j^\tau > 0$ for some $j < i$ and some $\tau > t$. This implies that there is a feasible marginal utility transfer $d\mathbf{u}_j^t$ from player i to player j in period t, and a feasible marginal utility transfer $d\mathbf{u}_j^\tau$ from player j to player i in period τ. In particular, consider transfers that would leave player j indifferent; that is: $d\mathbf{u}_j^t - \delta_j^{\tau-t} d\mathbf{u}_j^\tau = 0$. As σ cannot be Pareto improved, this implies that $\delta_i^{\tau-t} d\mathbf{u}_i^\tau \leq d\mathbf{u}_i^t = \delta_j^{\tau-t} d\mathbf{u}_j^\tau$, which is impossible since $\delta_j < \delta_i$.

The result above implies that, in equilibrium σ, if $\mathbf{u}_n^t > 0$ in some period t then $\mathbf{u}_j^\tau = 0$ for all $j \neq n$ and all $\tau > t$. This in turn implies that, in equilibrium, $\mathbf{u}_n^t = 0$ in all periods $t \in \mathbb{N}$. However, this is impossible because player n proposes with positive probability; and $\max_{i \in N} \delta_i < 1$ implies that a winning coalition would accept an offer which yields n a positive share.

□

Theorem 3. *If $q = n$ then: (i) every equilibrium σ is a pure strategy no-delay equilibrium with $A(\sigma) = \Delta_{n-1}$; and (ii) such an equilibrium exists.*

Proof: Part (i)

The proof of Theorem 3(i) hinges on the following lemmata.

Lemma 1. *Suppose that $q = n$, and let σ be an equilibrium. For all $i \in N$ and all $x \in X$:*

(i) $V_i^\sigma(x) \geq u_i(x_i)$; and

(ii) $V_i^\sigma(y) \geq u_i(x_i)$ for all policies y that are accepted with positive probability when the default is x.

Proof: (i) Let $x \in X$. By definition,

$$V_i^\sigma(x) = (1 - \delta_i)\, u_i(x_i) + \delta_i \int V_i^\sigma(y) P^\sigma(x, dy)\ .$$

When offered policy y at default x, player i accepts only if $V_i^\sigma(y) \geq V_i^\sigma(x)$. Hence,

$$V_i^\sigma(x) \geq (1-\delta_i)\, u_i\,(x_i) + \delta_i V_i^\sigma(x)$$

or, equivalently, $V_i^\sigma(x) \geq u_i\,(x_i)$.

(ii) As $q = n$, sequential rationality implies that player i only accepts policy y with positive probability when the default is x if $V_i^\sigma(y) \geq V_i^\sigma(x)$ for all $i \in N$. By (i), this implies that $V_i^\sigma(y) \geq u_i\,(x_i)$ for all $i \in N$.

$\Diamond$

Lemma 2. *Suppose that $q = n$. If σ is an equilibrium then $\emptyset \neq A(\sigma) \subseteq \Delta_{n-1}$.*

Proof: It is easy to see that $A(\sigma) \neq \emptyset$ (for instance, take policy $(1, 0, \ldots, 0) \in \Delta_{n-1}$).

Let σ be an equilibrium and suppose, contrary to the statement of the result, that there exists $x \in A(\sigma) \setminus \Delta_{n-1}$. As $x \notin \Delta_{n-1}$, there is $y \in \Delta_{n-1}$ such that $u_i\,(y_i) > u_i\,(x_i)$ for each $i \in N$. By definition of $A(\sigma)$, the following must be true for each $i \in N$:

$$V_i^\sigma(x) = u_i\,(x_i) < u_i\,(y_i) \leq V_i^\sigma(y)\ ,$$

where the last inequality follows from Lemma 1(i). This implies that, at default x, any proposer could profitably deviate by (successfully) proposing to amend x to y.

$\Diamond$

At this point, we need some notation. Any stationary Markov strategy profile $\sigma = (\sigma_i)_{i\in N}$ induces a stochastic process $\left(\tilde{x}^t\right)$ on the policy space, where the random variable $\tilde{x}^t$ stands for the policy implemented in period t.[19] Let $x \in X$ and $m \in \mathbb{N}$. As σ is stationary Markov, we can define a random variable $\tilde{x}^m(x)$, which describes the policy implemented in period $t + m$ given that x is the policy implemented in period t. Put differently, $\tilde{x}^m(x)$ is the random variable obtained from the distribution of $\tilde{x}^{t+m}$ conditional on the event "$\tilde{x}^t = x$." Thus, for every $m \in \mathbb{N}$, $\tilde{x}^m\left(x^0\right) = \tilde{x}^m$ and $\mathbb{E}\left[\tilde{x}^m(x)\right] = \mathbb{E}\left[\tilde{x}^{t+m}|\tilde{x}^t = x\right]$, where $\mathbb{E}\left[\cdot\right]$ is the expectation operator with respect to the stochastic process engendered by σ.

Part (i) of Theorem 3 is obtained from the following lemmata: Lemmata 3 and 4 show that $P^\sigma\left(x, \Delta_{n-1}\right) = 1$ for all $x \in X$; Lemma 5 shows that $A(\sigma) = \Delta_{n-1}$.

[19] In what follows, we use "~" to indicate random variables.

Lemma 3. *If σ is an equilibrium then the following statements are true for all $x \in X$:*

(i) $(\tilde{x}^m(x))$ converges almost surely to a limit $\tilde{x}(x)$;

(ii) For every $i \in N$,

$$u_i(x_i) \leq \mathbb{E}\left[V_i^\sigma\left(\tilde{x}^1(x)\right)\right] \leq \mathbb{E}\left[V_i^\sigma\left(\tilde{x}^2(x)\right)\right] \leq \ldots \leq \mathbb{E}\left[u_i\left(\tilde{x}_i(x)\right)\right] . \tag{4}$$

Proof: Take an arbitrary $x \in X$.

(i) By Proposition 1 in Hyndman and Ray (2007), the stochastic sequence $\left(u_i\left(\tilde{x}_i^m(x)\right)\right)_{i\in N}$ converges almost surely to a limit.[20] As the u_i's are strictly increasing functions, the stochastic sequence of policies $(\tilde{x}^m(x))$ converges along any sample path for which $\left(u_i\left(\tilde{x}_i^m(x)\right)\right)_{i\in N}$ converges. Hence, $(\tilde{x}^m(x))$ converges almost surely to a limit $\tilde{x}(x)$.

(ii) Lemma 1 implies that any realization of the random variable $\tilde{x}^1(x)$, say x^1, must satisfy $u_i(x_i) \leq V_i^\sigma\left(x^1\right)$ for every $i \in N$. Hence, $u_i(x_i) \leq \mathbb{E}\left[V_i^\sigma\left(\tilde{x}^1(x)\right)\right]$ for each $i \in N$.

Take an arbitrary $m \in \mathbb{N}$, and let x^m be some realization of $\tilde{x}^m(x)$. As $q = n$, sequential rationality implies that a proposal $x^{m+1} \in X$ is only voted up if $V_i^\sigma(x^m) \leq V_i^\sigma\left(x^{m+1}\right)$ for every $i \in N$. Consequently, $V_i^\sigma(x^m) \leq \mathbb{E}\left[V_i^\sigma\left(\tilde{x}^{m+1}(x)\right)|\tilde{x}^m(x) = x^m\right]$. This in turn implies that

$$\mathbb{E}\left[V_i^\sigma\left(\tilde{x}^m(x)\right)\right] \leq \mathbb{E}\left[\mathbb{E}\left[V_i^\sigma\left(\tilde{x}^{m+1}(x)\right)|\tilde{x}^m(x) = x^m\right]\right] = \mathbb{E}\left[V_i^\sigma\left(\tilde{x}^{m+1}(x)\right)\right] . \tag{5}$$

To complete the proof of the lemma, therefore, it remains to establish that

$$\mathbb{E}\left[V_i^\sigma\left(\tilde{x}^m(x)\right)\right] \leq \mathbb{E}\left[u_i\left(\tilde{x}_i(x)\right)\right]$$

for all $m \in \mathbb{N}$ and all $i \in N$. To do so, observe first that for any $\varepsilon > 0$, there exists $M_\varepsilon > 0$ such that $\mathbb{E}\left[V_i^\sigma\left(\tilde{x}^m(x)\right)\right] \leq \mathbb{E}\left[u_i\left(\tilde{x}_i(x)\right)\right] + \varepsilon$ for all $m > M_\varepsilon$. Indeed,

$$\mathbb{E}\left[V_i^\sigma\left(\tilde{x}^m(x)\right)\right] = (1-\delta_i)\,\mathbb{E}\left[\sum_{\tau=0}^{\infty} \delta_i^\tau u_i\left(\tilde{x}^{m+\tau}(x)\right)\right] = (1-\delta_i)\sum_{\tau=0}^{\infty}\delta_i^\tau \mathbb{E}\left[u_i\left(\tilde{x}^{m+\tau}(x)\right)\right] .$$

As $(\tilde{x}^m(x))$ converges almost surely to a limit $\tilde{x}(x)$, Lebesgue's Dominated Convergence Theorem implies that $\mathbb{E}\left[u_i\left(\tilde{x}^m(x)\right)\right] \to \mathbb{E}\left[u_i\left(\tilde{x}(x)\right)\right]$. This in turn implies that, for any $\varepsilon > 0$, there exists $M_\varepsilon > 0$ such that $\mathbb{E}\left[u_i\left(\tilde{x}^m(x)\right)\right] \leq \mathbb{E}\left[u_i\left(\tilde{x}_i(x)\right)\right] + \varepsilon$ and, consequently,

$$\mathbb{E}\left[V_i^\sigma\left(\tilde{x}^m(x)\right)\right] \leq \mathbb{E}\left[u_i\left(\tilde{x}_i(x)\right)\right] + \varepsilon \tag{6}$$

for all $m > M_\varepsilon$.

[20] Note that Hyndman and Ray's result applies to a more general class of coalitional games — in which unanimous voting is only a special case — and does not require Markov stationarity.

Now, for each $m \in \mathbb{N}$ and $\varepsilon > 0$, define $M(m, \varepsilon) \equiv \max\{M_\varepsilon, m+1\}$. Combining (5) and (6), we obtain that

$$\mathbb{E}\left[V_i^\sigma\left(\tilde{x}^m(x)\right)\right] \leq \mathbb{E}\left[V_i^\sigma\left(\tilde{x}^{M(m,\varepsilon)}(x)\right)\right] \leq \mathbb{E}\left[u_i\left(\tilde{x}_i(x)\right)\right] + \varepsilon$$

for any $\varepsilon > 0$. This proves that $\mathbb{E}\left[V_i^\sigma\left(\tilde{x}^m(x)\right)\right] \leq \mathbb{E}\left[u_i\left(\tilde{x}_i(x)\right)\right]$ for all $m \in \mathbb{N}$ and all $i \in N$; completing the proof of Lemma 3.

$\Diamond$

Lemma 4. *If σ is an equilibrium then $P^\sigma\left(x, \Delta_{n-1}\right) = 1$ for all $x \in X$.*

Proof: For any $w \in X$, define policy $y(w)$ as $y(w) \equiv \mathbb{E}\left[\tilde{x}(w)\right]$. As the u_i's are concave, Jensen's inequality implies that

$$\mathbb{E}\left[u_i\left(\tilde{x}_i(w)\right)\right] \leq u_i\left(y_i(w)\right) \tag{7}$$

for all $i \in N$.

Now suppose that, contrary to the statement of the lemma, there is some $x \in X$ such that $P^\sigma\left(x, \Delta_{n-1}\right) < 1$. This implies that some policy $x' \notin \Delta_{n-1}$ is successfully proposed when the default is x. Using (7) and Lemma 3(ii), we obtain

$$u_i\left(x_i'\right) \leq \mathbb{E}\left[V_i^\sigma\left(\tilde{x}^1\left(x'\right)\right)\right] \leq \mathbb{E}\left[u_i\left(\tilde{x}_i(x')\right)\right] \leq u_i\left(y_i(x')\right) \ , \tag{8}$$

which implies that $y_i(x') \geq x_i'$ for all $i \in N$. Consequently, there must be some $y \in \Delta_{n-1}$ such that $y_i \geq x_i'$ for all $i \in N$. Moreover, as $x' \notin \Delta_{n-1}$ and $y \in \Delta_{n-1}$, there must be a nonempty subset of players, say W, such that $y_i > x_i'$ for all $i \in W$.

If $W = N$ then every player who proposes x' could profitably deviate by proposing y instead, as (8) implies that

$$V_i^\sigma\left(x_i'\right) = \left(1 - \delta_i\right) u_i\left(x_i'\right) + \delta_i \mathbb{E}\left[V_i^\sigma\left(\tilde{x}^1\left(x'\right)\right)\right] < u_i\left(y_i\right) \leq V_i^\sigma\left(y_i\right)$$

for all $i \in N$ (where the last inequality follows from Lemma 1(i)).

If $W \subset N$ then the (strict) inequality above still holds for the members of W: $V_i^\sigma\left(x_i'\right) < u_i\left(y_i\right)$ for every $i \in W$. Let $z(\varepsilon) \in X$ be the policy defined as

$$z_i(\varepsilon) \equiv y_i - \frac{n - |W|}{|W|}\varepsilon \ , \forall i \in W \ ,$$

$$z_i(\varepsilon) \equiv y_i + \varepsilon \ , \forall i \in N \setminus W \ .$$

As the u_i's are continuous and strictly increasing, there exists a sufficiently small $\varepsilon > 0$ such that $V_i^\sigma\left(x_i'\right) < u_i\left(z_i(\varepsilon)\right) \leq V_i^\sigma\left(z_i(\varepsilon)\right)$. By the same argument as above, every player who proposes x' could profitably deviate by proposing $z(\varepsilon)$ instead.

$\diamondsuit$

We already know from Lemma 2 that $A(\sigma) \subseteq \Delta_{n-1}$. To complete the proof of the Theorem, we must show that every point in the unit simplex is absorbing.

Lemma 5. *If σ is an equilibrium then $A(\sigma) = \Delta_{n-1}$.*

Proof: Let $x \in \Delta_{n-1}$ and suppose, contrary to the Lemma, that there is some $Y \subseteq X \setminus \{x\}$ such that $P^{\sigma}(x, Y) > 0$. Take an arbitrary policy x' in $Y \cap \Delta_{n-1}$ (which from the previous lemma is nonempty). By (8), we have $u_i(x'_i) \leq u_i(y_i(x'))$ for all $i \in N$. But, as $x' \in \Delta_{n-1}$, this implies that $x' = y(x')$, and therefore, that $u_i(x'_i) = u_i(y_i(x'))$ for all $i \in N$. Using (8) again, this in turn implies that $u_i(x'_i) = \mathbb{E}\left[V_i^{\sigma}\left(\tilde{x}^1(x')\right)\right]$ for every $i \in N$. Hence,

$$u_i\left(x'_i\right) = (1-\delta_i)\, u_i\left(x'_i\right) + \delta_i \mathbb{E}\left[V_i^{\sigma}\left(\tilde{x}^1\left(x'\right)\right)\right] = V_i^{\sigma}\left(x'\right) \geq u_i\left(x_i\right)$$

for all $i \in N$ (where the inequality follows from Lemma 1(ii)). As x is by assumption an element of the simplex, the inequality above implies that $x' = x$ (which contradicts $x' \in Y \subseteq X \setminus \{x\}$).

$\diamondsuit$

Finally, σ must be a pure strategy profile. Indeed, the no-delay property implies that, at any default, each proposer makes a proposal that is accepted by all players. By sequential rationality, the proposer must give the other players the minimum shares that they are willing to accept.

Part (ii)

To prove Theorem 3(ii), we will construct an equilibrium σ in which, at any default $x \in X$, the selected proposer — say i — offers the committee a policy $x + s^i(x) \in \Delta_{n-1}$, which is accepted by all players and then never amended. We can think of proposer i offering to share the amount of money not distributed yet — i.e. $1 - \sum_{j \in N} x_j$ — with the the other players, with $s^i_j(x)$ being the share offered by proposer i to player j.

Our first step is to define these transfers. For each $x \in X$, let

$$T_x \equiv \left\{ s \in [0,1]^n : \sum_{j \in N} x_j + s_j = 1 \right\} .$$

Thus, any element of the n-fold product of T_x, T_x^n, can be thought of as a vector of shares of the budgetary surplus $s = \left(s^i\right)_{i \in N}$, where $s^i \in T_x$ stands for the shares offered by proposer

i. Next, let $\phi(x)(\cdot) = \left(\phi^1(x)(\cdot), \ldots, \phi^n(x)(\cdot)\right)$ be a self-map on T_x^n defined as follows: for all $i \in N$ and all $s = \left(s^k\right)_{k \in N} \in T_x^n$,

$$u_j\left(x_j + \phi_j^i(x)(s)\right) \equiv (1-\delta_j)\, u_j\left(x_j\right) + \delta_j \sum_{k \in N} p_k u_j\left(x_j + s_j^k\right) \ , \forall j \neq i \ ,$$

$$\phi_i^i(x)(s) \equiv 1 - x_i - \sum_{j \neq i}\left[x_j + \phi_j^i(x)(s)\right] \ .$$

As all the u_i's are by assumption continuous, $\phi(x)(\cdot)$ is a continuous function from T_x^n (which is convex and compact in $\mathbb{R}^{n^2}$) into itself. Brouwer's Fixed Point Theorem then implies that there is $s(x) = \left(s_j^i(x)\right)_{i,j \in N} \in T_x^n$ such that $\phi(x)(s(x)) = s(x)$; that is

$$u_j\left(x_j + s_j^i(x)\right) = (1-\delta_j)\, u_j\left(x_j\right) + \delta_j \sum_{k \in N} p_k u_j\left(x_j + s_j^k(x)\right) \ , \forall j \neq i \ , \tag{9}$$

$$x_i + s_i^i(x) = 1 - \sum_{j \neq i}\left[x_j + s_j^i(x)\right] \ , \tag{10}$$

for all $i \in N$. Observe that, by construction, $x + s^i(x) \in \Delta_{n-1}$ for all $i \in N$ and all $x \in X$. Moreover, if $x \in \Delta_{n-1}$ then $T_x = \{(0, \ldots, 0)\}$ and, therefore, $s^i(x) = (0, \ldots, 0)$ for every $i \in N$.

We are now in a position to define the strategy profile $\sigma = (\sigma_1, \ldots, \sigma_n)$:

• In the proposal stage of any period t with ongoing default $x^{t-1} = x$, i's proposal (conditional on i being chosen to make a proposal) is $x + s^i(x)$;

• In the voting stage of any period t with ongoing default $x^{t-1} = x$, following any proposal $y \in X \setminus \{x\}$, player i accepts if and only if

$$(1-\delta_i)\, u_i\left(y_i\right) + \delta_i \sum_{j \in N} p_j u_i\left(y_i + s_i^j(y)\right) \geq (1-\delta_i)\, u_i\left(x_i\right) + \delta_i \sum_{j \in N} p_j u_i\left(x_i + s_i^j(x)\right) \ .$$

Observe that σ is a pure strategy stationary Markov strategy profile. To complete the proof of Theorem 3, therefore, it remains to show that σ is a no-delay, stage-undominated subgame perfect equilibrium. As in the proof of the previous theorem, we proceed in several steps.

Claim 1: σ is no delay with $A(\sigma) = \Delta_{n-1}$ and, for all $i \in N$ and all $x \in X$:

$$V_i^\sigma(x) = (1-\delta_i)\, u_i\left(x_i\right) + \delta_i \sum_{j \in N} p_j u_i\left(x_i + s_i^j(x)\right) \ .$$

Proof: If $x \in \Delta_{n-1}$ then σ prescribes all proposers to pass in all periods. This implies that $x \in A(\sigma)$ — thus establishing that $\Delta_{n-1} \subseteq A(\sigma)$ — and, for each $i \in N$,

$$V_i^\sigma(x) = u_i(x_i) = (1-\delta_i)\, u_i(x_i) + \delta_i \sum_{j\in N} p_j u_i\left(x_i + s_i^j(x)\right)$$

(since $x \in \Delta_{n-1}$ implies that $s_i^j(x) = 0$ for all $i, j \in N$).

If $x \notin \Delta_{n-1}$ then, in the next period, σ prescribes each proposer j to propose policy $x + s^j(x)$. As $x + s^j(x) \in \Delta_{n-1}$, we have $s_i^k\left(x + s^j(x)\right) = 0$ for all $i, k \in N$, so that

$$(1-\delta_i)\, u_i\left(x_i + s_i^j(x)\right) + \delta_i \sum_{k\in N} p_k u_i\left(x_i + s_i^j(x) + s_i^k\left(x + s^j(x)\right)\right) = u_i\left(x_i + s_i^j(x)\right) \ , \tag{11}$$

for all $i \in N$. From the definition of voting strategies, therefore, player i accepts if and only if

$$u_i\left(x_i + s_i^j(x)\right) \geq (1-\delta_i)\, u_i(x_i) + \delta_i \sum_{k\in N} p_k u_i\left(x_i + s_k^i(x)\right) \ ,$$

which by equation (9) holds for all $i \neq j$. To prove that j's proposal is voted up, therefore, it remains to check that she accepts her own proposal. By concavity of the u_i's, equation (9) implies that

$$\begin{aligned} u_i\left(x_i + s_i^j(x)\right) &= (1-\delta_i)\, u_i(x_i) + \delta_i \sum_{k\in N} p_k u_i\left(x_i + s_i^k(x)\right) \\ &\leq u_i\left((1-\delta_i)\, x_i + \delta_i \sum_{k\in N} p_k\left(x_i + s_i^k(x)\right)\right) = u_i\left(x_i + \delta_i \sum_{k\in N} p_k s_i^k(x)\right) \ , \end{aligned}$$

for all $i \neq j$, which in turn implies that $s_i^j(x) \leq \sum_{k\in N} p_k s_i^k(x)$ for all $i \neq j$ (recall that $\delta_i \in (0,1)$). Using this inequality and the concavity of u_j, we obtain

$$\begin{aligned} &(1-\delta_j)\, u_j(x_j) + \delta_j \sum_{k\in N} p_k u_j\left(x_j + s_j^k(x)\right) \leq u_j\left(x_j + \delta_j \sum_{k\in N} p_k s_j^k(x)\right) \\ =&u_j\left(x_j + \delta_j \sum_{k\in N} p_k\left[1 - \sum_{l\in N} x_l - \sum_{l\neq j} s_l^k(x)\right]\right) = u_j\left(x_j + \delta_j\left[\sum_{i\in N} s_i^j(x) - \sum_{i\neq j}\sum_{k\in N} p_k s_i^k(x)\right]\right) \\ \leq&u_j\left(x_j + \delta_j\left[\sum_{i\in N} s_i^j(x) - \sum_{i\neq j} s_i^j(x)\right]\right) = u_j\left(x_j + \delta_j s_j^j(x)\right) \leq u_j\left(x_j + s_j^j(x)\right) \\ =&(1-\delta_j)\, u_j\left(x_j + s_j^j(x)\right) + \delta_j \sum_{k\in N} p_k u_j\left(x_j + s_j^j(x) + s_j^k\left(x + s^j(x)\right)\right) \ , \end{aligned} \tag{12}$$

where the last equality comes from (11). Thus, σ_j prescribes player j to accept as well, and $x_j + s^j(x)$ is therefore voted up. This proves that policies outside the simplex cannot be absorbing points of σ — i.e. $(X \setminus \Delta_{n-1}) \cap A(\sigma) = \emptyset$ — and, therefore, that $A(\sigma) = \Delta_{n-1}$. This also proves that $P^\sigma(x, A(\sigma)) = P^\sigma(x, \Delta_{n-1}) = 1$ for all $x \in X$; that is, σ is no-delay.

Moreover, as $x_j + s^j(x) \in \Delta_{n-1}$, σ prescribes all proposers to pass in all future periods. This implies that, for all $i \in N$ and $x \notin X$,

$$V_i^\sigma(x) = (1-\delta_i)\, u_i(x_i) + \delta_i \sum_{j\in N} p_j u_i\left(x_i + s_i^j(x)\right) ,$$

thus completing the proof of the claim.

For future reference (see Claim 3 below), observe that (12) implies that $V_i^\sigma\left(x_i + s_i^i(x)\right) \geq V_i^\sigma(x_i)$ for any player $i \in N$.

Claim 2: Given default x and proposal y, each voter $i \in N$ accepts if and only if $V_i^\sigma(y) \geq V_i^\sigma(x)$, and rejects only if $V_i^\sigma(x) \geq V_i^\sigma(y)$.

Proof: This is an immediate consequence of Claim 1 and the definition of voting strategies.

Claim 3: There is no profitable one-shot deviation from σ in the proposal stage of any period.

Proof: Let $x^{t-1} = x$, and suppose that player i is recognized to make a proposal in period t. If she plays according to σ_i then she proposes $x + s^i(x)$ (or, equivalently, passes when $x \in \Delta_{n-1}$). As σ is no-delay (Claim 1), this offer is accepted and player i's payoff is $u_i\left(x_i + s_i^i(x)\right)$.

In the proof of Claim 1, we showed that $V_i^\sigma(x) \leq V_i^\sigma\left(x_i + s_i^i(x)\right)$. Hence, player i cannot profitably deviate by passing or by making a proposal that is voted down.

Now consider a deviation to a proposal $y \neq x + s^i$, which is accepted. According to the definition of voting strategies, y must satisfy

$$(1-\delta_j)\, u_j(y_j) + \delta_j \sum_{k\in N} p_k u_j\left(y_j + s_j^k(y)\right) \geq (1-\delta_j)\, u_j(x_j) + \delta_j \sum_{k\in N} p_k u_j\left(x_j + s_j^k(x)\right) \tag{13}$$

for all $j \in N$. We distinguish between two different cases:

- Case 1: $y \in \Delta_{n-1}$. In this case, inequality (13) becomes

$$u_j(y_j) \geq (1-\delta_j)\, u_j(x_j) + \delta_j \sum_{k\in N} p_k u_j\left(x_j + s_j^k(x)\right) = u_j\left(x_j + s_j^i(x)\right)$$

for all $j \neq i$ (the equality is obtained from (9)). As u_j is increasing, this implies that $y_j \geq x_j + s_j^i(x)$ for all $j \neq i$ and, consequently,

$$x_i + s_i^i(x) = 1 - \sum_{j \neq i} \left(x_j + s_j^i(x)\right) \geq 1 - \sum_{j \neq i} y_j = y_i \ .$$

This in turn implies that $V_i^\sigma\left(x_i + s_i^i(x)\right) = u_i\left(x_i + s_i^i(x)\right) \geq u_i\left(y_i\right) = V_i^\sigma\left(y\right)$. Hence, proposing $y \in \Delta_{n-1}$ is not a profitable (one-shot) deviation for player i.

- Case 2: $y \notin \Delta_{n-1}$. In this case, equations (9) and (13) imply that

$$\begin{aligned} u_j\left(y_j + \delta_j \sum_{k \in N} p_k s_j^k(y)\right) &\geq \left(1-\delta_j\right) u_j\left(y_j\right) + \delta_j \sum_{k \in N} p_k u_j\left(y_j + s_j^k(y)\right) \\ &\geq \left(1-\delta_j\right) u_j\left(x_j\right) + \delta_j \sum_{k \in N} p_k u_j\left(x_j + s_j^k(x)\right) \\ &= u_j\left(x_j + s_j^i(x)\right) \ , \end{aligned}$$

so that $y_j + \sum_{k \in N} p_k s_j^k(y) \geq x_j + s_j^i(x)$ for all $j \neq i$ (recall that $\delta_j \in (0,1)$ and $s_j^k(y) \geq 0$ for all $j, k \in N$). Consequently,

$$\begin{aligned} x_i + s_i^i(x) &= 1 - \sum_{j \neq i}\left[x_j + s_j^i(x)\right] \\ &\geq 1 - \sum_{j \neq i}\left[y_j + \sum_{k \in N} p_k s_j^k(y)\right] = 1 - \sum_{j \neq i} y_j - \sum_{k \in N}\left[p_k \sum_{j \neq i} s_j^k(y)\right] \ . \end{aligned} \tag{14}$$

Moreover, by equation (10),

$$\sum_{j \neq i} s_j^k(y) = 1 - \sum_{l \in N} y_l - s_i^k(y) \ . \tag{15}$$

Combining (14) and (15), we obtain

$$x_i + s_i^i(x) \geq 1 - \sum_{j \neq i} y_j - \sum_{k \in N}\left[p_k\left(1 - \sum_{l \in N} y_l - s_i^k(y)\right)\right] = y_i + \sum_{k \in N} p_k s_i^k(y) \ .$$

Hence:

$$\begin{aligned} V_i^\sigma\left(x_i + s_i^i(x)\right) = u_i\left(x_i + s_i^i(x)\right) &\geq u_i\left(y_i + \sum_{k \in N} p_k s_i^k(y)\right) \geq u_i\left(\left(1-\delta_i\right) y_i + \delta_i \sum_{k \in N} p_k\left[y_i + s_i^k(x)\right]\right) \\ &\geq \left(1-\delta_i\right) u_i\left(y_i\right) + \delta_i \sum_{k \in N} p_k u_i\left(y_i + s_i^k(x)\right) = V_i^\sigma(y) \ . \end{aligned}$$

This shows that proposing $y \notin \Delta_{n-1}$ is not a profitable deviation for player i, and completes the proof of Claim 3.

Combining Claims 1-3, we obtain Theorem 3.

□

Theorem 4. *If $q = n$ then there is a unique equilibrium payoff, which coincides with the (stationary subgame perfect) equilibrium payoff of the ad hoc committee game.*

Proof: Denote our game with an evolving default by Γ^e, and the game with a constant default by Γ^c.

Let $\sigma = (\sigma_i)_{i\in N}$ be an equilibrium of Γ^e, and let $\pi^i(x) \in X$ be the proposal made by player i when the ongoing default is x in this equilibrium (recall that, by Theorem 3, σ is a pure strategy profile). Hence, player i's expected payoff as evaluated after rejection of a proposal in the first period is given by:

$$V_i^\sigma\left(x^0\right) = (1-\delta_i)\, u_i\left(x^0\right) + \delta_i \sum_{j\in N} p_j u_i\left(\pi_i^j\left(x^0\right)\right)$$

(recall that, by Theorem 3, σ must be no-delay).

Now define the stationary strategy profile $\sigma^c = (\sigma_i^c)_{i\in N}$ in game Γ^c as follows. At the proposal stage of every period t, each player $i \in N$ makes proposal $\pi^i\left(x^0\right)$. At the voting stage of each period, player i accepts the proposal just made, say y, if and only if $u_i(y) \geq V_i^\sigma\left(x^0\right)$.

As σ is no delay, proposal $\pi^i\left(x^0\right)$, $i \in N$, must be accepted with probability 1 in Γ^e. By sequential rationality and unanimity rule, this implies that $V_j^\sigma\left(\pi^i\left(x^0\right)\right) = u_j\left(\pi_j^i\left(x^0\right)\right) \geq V_j^\sigma\left(x^0\right)$ for all $j \in N$, which in turn implies that proposal $\pi^i\left(x^0\right)$ is also accepted with probability 1 in any period of Γ^c. Two immediate consequences of this observation are that: (i) player i's expected payoff as evaluated after rejection of a proposal in the first period of Γ^c is $V_i^\sigma\left(x^0\right)$; and (ii) player $i \in N$ has no profitable deviation from the voting behavior prescribed by σ_i^c.

To complete the proof of the result, therefore, it remains to show that no player $i \in N$ can profitably deviate from σ^c in a proposal stage of Γ^c. Take an arbitrary player i. As σ is an equilibrium of Γ^e, player i cannot profitably deviate by proposing another policy

$y \in X \setminus \left\{\pi^i\left(x^0\right)\right\}$ instead of x or by making an unsuccessful proposal. Hence,

$$V_i^\sigma\left(\pi^i\left(x^0\right)\right) = u_i\left(\pi_i^0\right) \geq \max\left\{V_i^\sigma\left(x^0\right), \left(1-\delta_i\right)u_i(y) + \delta_i \sum_{j\in N} p_j u_i\left(\pi_i^j(y)\right)\right\} \geq u_i(y)$$

where the second inequality follows from Lemma 1(ii). Now consider a deviation from $\pi^i\left(x^0\right)$ in Γ^c. If i proposed some policy y then her expected payoff would be $u_i(y)$ if her proposal were successful, and $V_i^\sigma\left(x^0\right)$ otherwise. Hence, the inequality above implies that i cannot improve upon proposing $\pi^i\left(x^0\right)$ and, therefore, cannot profitably deviate from σ_i^c in proposal stages.

□

Theorem 5. *If $q = n$ and $\delta_i \neq \delta_j$, for some $i, j \in N$, then any equilibrium is Pareto inefficient.*

Proof: We start the proof of Theorem 5 with the following lemma:

Lemma 6. *Let $q = n$. If $\left(\tilde{x}^t\right)$ is the stochastic sequence of policies on some equilibrium path then, for any realization $\left(x^t\right)$ of $\left(\tilde{x}^t\right)$, $x_i^t \in (0,1)$ for all $i \in N$ and all $t \in \mathbb{N}$.*

Proof: Let $\left(x^t\right)$ be an arbitrary realization of the sequence $\left(\tilde{x}^t\right)$ engendered by some equilibrium σ. Suppose that, contrary to the statement above, $x_j^\tau = 0$ for some $j \in N$ and some $\tau \in \mathbb{N}$. Theorem 3 then implies that in period 1 (with default x^0) player j accepted a proposal x such that $x_j = x_j^t = 0$ for all $t \in \mathbb{N}$. By sequential rationality, this implies that $V_j^\sigma\left(x^0\right) = u_j(0)$; otherwise she would be strictly better off rejecting any such proposal.

By Theorem 3, $W \equiv \left\{i \in N : V_i^\sigma\left(x^0\right) > u_i(0)\right\}$ is nonempty and, for each $i \in W$,

$$V_i\left(x^0\right) = \left(1-\delta_i\right)u_i(0) + \delta_i \sum_{l\in N} p_l u_i\left(x_i^l\right) < \sum_{l\in N} p_l u_i\left(x_i^l\right) \leq u_i\left(\sum_{l\in N} p_l x_i^l\right) ,$$

where x^l denotes player l's successful proposal when the default is x^0 (and the second inequality follows from Jensen's inequality). By continuity of the u_i's, therefore, there exists a sufficiently small $\varepsilon > 0$ such that

$$V_i\left(x^0\right) < u_i\left(\sum_{l\in N} p_l x_i^l - \varepsilon\right) , \forall i \in W .$$

Let $y = (y_i)_{i \in N} \in X$ be defined as follows:

$$y_i \equiv \sum_{l \in N} p_l x_i^l - \varepsilon \text{ , for all } i \in W \text{ , and } y_i \equiv \frac{|W|}{n - |W|} \varepsilon \text{ for all } i \in N \setminus W \text{ .}$$

It is readily checked that $u_i(y_i) > V_i^\sigma(x^0)$ and then, by Lemma 1(i), $V_i^\sigma(y) > V_i^\sigma(x^0)$ for all $i \in N$.

Consider player i's proposal when she is recognized to make a proposal in period 1. As $V_i^\sigma(y) > V_i^\sigma(x^0)$ for all $i \in N$, she could successfully propose y and thus get a payoff of $V_j^\sigma(y) > V_j^\sigma(x^0) = u_j(0)$. As $p_j > 0$ and $q = n$ (and $u_j(0)$ is obviously the minimum payoff she can get), she must therefore reject any proposal x such that $x_j = 0$ in equilibrium.

◊

Suppose that there are $i, j \in N$ such that $\delta_i > \delta_j$. Now suppose that, contrary to the Theorem, there exists a Pareto efficient equilibrium σ. Theorem 3 implies that this equilibrium can be described by a policy vector $(x^1, \ldots, x^n) \in \Delta_{n-1}^n$, such that $x^t = x^k$ for all $t \in \mathbb{N}$ with probability p_k. By concavity of the u_k's, the policy $\bar{x} \equiv \sum_{k \in N} p_k x^k$ is weakly preferred by all players to the lottery engendered by σ. To obtain the desired contradiction, therefore, it suffices to show that the indefinite implementation of $\bar{x}$ can be Pareto improved.

From Lemma 6, $\bar{x}_i, \bar{x}_j \in (0, 1)$. Consequently, there is a feasible marginal transfer dx_j^1 from player i to player j in period 1, and a marginal transfer dx_j^2 from j to i in period 2, such that player 1's discounted payoff remains unchanged. If we assume by contradiction that the repeated implementation of policy $\bar{x}$ is Pareto efficient then the changes in players i and j's payoffs must satisfy:

$$-u_i'(\bar{x}_i)\, dx_j^1 + \delta_i u_i'(\bar{x}_i)\, dx_j^2 = 0 \text{ , and } u_j'(\bar{x}_j)\, dx_j^1 - \delta_j u_j'(\bar{x}_j)\, dx_j^2 \leq 0 \text{ .}$$

Combining these two conditions, we obtain $\delta_i = dx_j^1 / dx_j^2 \leq \delta_j$, which contradicts our initial assumption that $\delta_i > \delta_j$.

□

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