Social Capital and Economic Growth

This Version: May 16th, 2012

Sumant Rai*

Abstract

This paper presents a new framework to analyze the dynamic relationship between social capital and economic growth. This relationship has been analyzed in a variant of quality-ladder growth model. We consider three institutional environments: first, perfect and costless institutions, second, social capital is the only form of institutions and third, both social capital and formal institutions determine the strength of institutions. We characterize an equilibrium in which a higher level of social capital increases growth but higher growth itself weakens social capital by increasing labor reallocation rate and by reducing socialization time. We show that in the absence of formal institutions, a higher rate of innovation lowers R&D investment by weakening existing informal institutions highlighting the need to improve formal institutions. A poor country lacking in resources or will to develop formal institutions will be caught up in poverty trap even if they transplant the technologies of rich countries. The model, therefore, provides another explanation of why poor countries do not catch up.

JEL Classification: E02, O43, Z13

Keywords: Social Capital, Institutions, Economic Growth

* PhD Candidate, Department of Economics, University of Washington, Seattle, 98195. Email: sumant@uw.edu
1 **Introduction**

After the pioneering work by Coleman (1988, 90) and Putnam (1993, 2000), research on social capital has received an enormous attention from economists. Social capital has been recognized as an important determinant of economic performance of a country.\(^1\) While a country with high stock of social capital tends to grow faster than a country with low stock of social capital, higher growth rate may itself be detrimental to social capital which, in turn, may hamper the growth performance. Putnam et al (1993) argued that a large part of the differences in per capita income between Northern and Southern Italy can be explained by their differences in the level of social capital, measured by membership in formal and informal groups and clubs. Routledge and von Amsberg (2003), and Miguel (2003) argued that a higher growth rate erodes social capital by increasing labor migration rate.\(^2\)

Additionally, Putnam (2000) documents that social capital in the US declined monotonically since 1960s, but there was no apparent adverse impact on the US economy. Particularly, during the 1990s US experienced rapid economic growth, a period when there was a sharp decline in social capital. He identifies some possible determinants of this decline as rising female participation in the labor market, increase in geographical mobility, replacement of small stores by supermarkets, individualization of leisure time etc. He, further, argued that during this period alternative (formal) sectors increased rapidly in response to decline in the strength of the informal sector (social capital). Putnam (2000) writes, “... during the 1980s both public and private spending on security rose rapidly as a share of GNP ... By 1995 America had 40% more

---


\(^2\) Social capital is person and place specific. See Glaeser and Redlick (2009)
police and guards and 150% more lawyers and judges than would have been projected in 1970, even given the growth of population and economy”.

What is `social capital'? According to Putnam et al (1993), “social capital... refers to features of social organizations, such as trust, norms, and [social] networks that can improve the efficiency of society ...”. Durlauf and Fafchamps (2004) identify three main underlying ideas behind social capital; first, it generates positive externalities in the society, second, these externalities are achieved through shared trust, and norms, and third, these shared trust and norms arise from informal forms of organizations. In a nutshell, any form of social organization or informal institution that facilitates cooperation and coordination, reduces transaction costs or improves market efficiency can be regarded as social capital. For example, since it is extremely difficult and prohibitively expensive to write complete and enforceable contracts in most cases, contracting parties, therefore, can lower these costs by writing a weaker incentive intensive contract. Social connections or social networks may also reduce the impact of moral hazard problem. Granovetter (1995) argued that social networks play a useful role in channeling information about jobs and job applicants in the labor market. In many cases, social capital is necessary in resolving conflicts among competing interests, reducing free riders problem and internalizing the externality in the provision of public good. Guiso et al (2004) have shown that social capital plays an important role in the degree of financial development across different parts in Italy. Recently, Akcomak and Weel (2009) investigated 102 European regions and concluded that social capital increases growth rate by fostering innovation.

---

3 Rob and Zemsky (2002) show that weaker incentive intensive contracts are desired when output strongly depends on partially observed cooperative efforts of workers.
4 Jackson and Schneider (2011) have shown that social connections significantly reduced the effects of moral hazard in New York City taxi industry.
This paper presents a model of dynamic relationship between social capital and economic growth where not only the positive impact of social capital on economic growth but also the detrimental impact of growth on social capital has been considered. This relationship has been analyzed in a variant of Aghion-Howitt (1992) Schumpeterian growth model where a representative consumer makes labor-socialization and consumption-saving decisions. Following Zak and Knack (2001) and Guiso et al (2004), we assume that consumers can invest their savings only through investment brokers. These brokers are opportunists in the sense that given the opportunity they would cheat and run away with the money. However, their ability to cheat (or the frequency of getting caught and money recovered) depends upon the strength of informal (social capital) and formal institutions. Therefore, a higher level of social capital increases investment and growth by reducing the broker's ability to cheat.\(^6\)

To capture the dynamics of social capital, we assume that the stock of social capital increases when people socialize and decreases with labor migration.\(^7\) We, further, assume that social capital is a by-product of individual’s rational decision where the reason for socialization is the pleasure derived from social interaction and labor migration is the result of technological shocks to the economy. The assumption that social capital is an externality is in line with the observations made by Arrow (2000). He writes, ``There is considerable consensus ... that much of the reward for social interactions is intrinsic - that is, the interaction is the reward - or at least that the motives for interaction are not economic ... The relations between the market and social interactions appear to be two-sided. On the one hand ... the market needs supplementation (for efficiency) by nonmarket relations [social capital]. On the other hand, labor or supplier

---

\(^6\) Alternatively, it can be argued that social capital raises the return from investment by reducing the cost of finding an honest broker or by reducing the cost of contracting because it may allow for writing a weaker contract.

\(^7\) Similar to time investment in Glaeser et al (2002).
turnover in response to price [changes] may destroy the willingness to offer trust or, more generally, to invest in the future of the relation".

We consider three different institutional environments in this paper. The benchmark case corresponds to the standard Schumpeterian growth model with a labor-socialization tradeoff where institutions are perfect and costless to maintain. The other two cases consider imperfect institutional environments. The second case includes only informal institutions (social capital) and the third incorporates both formal and informal institutions. Social capital is determined endogenously while the expenditure on formal institutions is optimally chosen.

Given the complexity of the model, most of the analysis is conducted numerically. Using plausible parameter values, we show that in the absence of formal institutions, a higher rate of innovation lowers R&D investment as it weakens existing informal institutions. Social capital declines through two sources, first, due to decline in socialization time and second, because of an increase in labor migration rate. As a result, increase in growth rate is much lower compared to the benchmark case.

It has been argued that the reason for the failure of poor countries to catch up is the lack of institutions.\(^8\) That is, in the absence of functional institutions, economic performance of poor countries may not improve significantly even if the technologies of the developed countries, which have been proven useful, are used.\(^9\) We show that formal institutions need to be developed in response to new technological breakthrough because of its detrimental impact on social capital. Although improvement in formal institutions increases growth, it reduces social capital.

\(^9\) See Francois and Zabojnik (2005).
even further.\textsuperscript{10} Therefore, improvement in formal institutions should not only take into account initial decline in social capital but also consider its own negative impact on social capital. A poor country, therefore, lacking in either resources or will to improve its institutions will be caught in a poverty trap.

In related literature, Zak and Knack (2001), analyze the impact of trust on growth in a heterogeneous agent growth model. Consumers are randomly matched with investment brokers every period and decide how much time to spend in monitoring. Trust varies inversely with the level of monitoring. Routledge and von Amsberg (2003) analyze the impact of growth on social capital. They argue that technological innovation results in reallocation of labor which reduces social capital. Other approaches that include social capital into growth models use human capital, degree of marketization, and participation in social networks as determinants of social capital.\textsuperscript{11} This paper incorporates social capital dynamics in a variant of Aghion and Howitt (1992) Schumpeterian growth model by using optimal socialization time and labor migration as determinants of social capital. This paper also highlights the need to develop formal institutions in response to declining social capital in a growing economy.

The rest of the paper is organized as follows. Section 2 presents the model, Section 3 characterizes the equilibrium in a decentralized economy, Section 4 solves for social planner's problem, Section 5 calibrates the model and discusses short- and long-run effects of technological shocks, and Section 6 discusses the results and presents some possible extensions of the model.

\textsuperscript{10} A higher expenditure on formal institution increases investment income and growth by reducing cheating, however, because of crowding out and the resulting decline of socialization time together with an increase in labor migration rate reduces social capital which, turn, hampers growth.

\textsuperscript{11} See Bartolini and Bonatti (2009), Sequeira and Ferreira-Lopes (2011), and Beugelsdijk and Smulder (2009).
2 The Model

In order to analyze the dynamic relationship between social capital and growth, we use a variant of the quality-ladder growth model of Aghion and Howitt (1992). We extend the model by incorporating the labor-socialization trade off and by incorporating institutional factors that can affect the return from investment.

2.1 Production

In this economy, there is a final consumption good produced by competitive firms. It can be produced using an intermediate good and the best available technology in that intermediate good sector. There is a continuous mass one of intermediate good sectors.\(^\text{12}\) Intermediate goods are produced using only labor.\(^\text{13}\) Each unit of labor hour produces exactly one unit of intermediate good irrespective of the sector in which a worker works in. Therefore, we denote \(l_{it}\) as the output of intermediate good sector \(i\) at time \(t\), which employs \(l_{it}\) units of labor hour. Each sector produces only one type of intermediate good in which they have complete monopoly power. The contribution of intermediate good sector, \(i\), towards the final good, \(Y_{it}\), at time \(t\) is given by

\[
Y_{it} = A_{it} l_{it}^g
\]

where \(A_{it}\) is the state of the art technology in intermediate good sector \(i\). Aggregate final good is the sum of the contributions of all intermediate good sectors towards the final good.

\[
Y_t = \int_0^1 Y_{it} \, di = \int_0^1 A_{it} l_{it}^g \, di
\]

\(^{12}\) We assume that each intermediate good sector is located at different locations. This is to ensure that once a worker switches job he moves to a different location.

\(^{13}\) We abstract away from physical capital for simplicity.
2.2 Technology (R&D)

We assume that there is a different R&D sector for each intermediate good. Each R&D sector is competitive. The Poisson arrival rate of innovation in each sector is given by \( \lambda n_{it} \), where \( \lambda \) is an innovation parameter, \( n_{it} \left( \frac{N_{it}}{A_t} \right) \) is the productivity adjusted investment in the R&D sector \( i \), \( N_{it} \) is the investment into R&D sector and \( A_t \) is the state of the art technology in the economy at time \( t \). We assume that innovation is increasingly difficult, that is, the probability of innovation decreases when we go up in the ladder in the technological innovation for the same level of investment, \( N_{it} \). Each innovation at time \( t \) in any sector \( i \) permits the innovator to start producing in sector \( i \) using the leading edge technology, \( A_t \). Each innovation raises the technology parameter, \( A \), by a constant factor, \( \gamma \). Once an innovation occurs in sector \( i \), either the existing firm purchases the patent from the innovator or only the ownership of the firm changes. After the innovation, the technology in that sector jumps discontinuously from \( A_{it} \) to the state of the art technology, \( A_t \).

Although, technology grows discontinuously at the individual sector level, economy wide technology, \( A_t \), evolves gradually. We assume that this leading technology grows at a rate proportional to the aggregate flow of innovation, \( n_t \), per unit of time. The economy-wide growth rate of technology is given by

\[
\frac{\dot{A}_t}{A_t} = \lambda n_t \ln \gamma , \quad \gamma > 1
\]  

\(14\) Having R&D sector is more relevant for the developed countries. For other economies (emerging or poor) we can interpret R&D expenditure as the expenditure on buying technologies from the developed countries.

\(15\) This is to ensure that after the innovation the same (previously employed) workers along with some new workers are working in that firm and there is not a complete restructuring inside that firm.
where $n_t = \int_0^1 n_{it} \, du$ is the aggregate productivity adjusted investment into R&D sector. We define $a_{it} = \frac{A_{it}}{A_t}$ as the relative productivity of intermediate good sector $i$ with respect to the state of the art technology in the economy. We assume that the relative productivities are distributed across different intermediate good sector according to:

$$F(a) = a^{\frac{1}{\ln \gamma}} , \quad 0 \leq a \leq 1 \quad (4)$$

where $F(a)$ is the cumulative distribution function of the relative productivities, $a$. The probability distribution function of the relative productivity, $a$, therefore, is $f(a) = \frac{1}{\ln \gamma} a^{\frac{1}{\ln \gamma} - 1}$. At any time, the distribution of relative productivities stays the same but the relative position of firms change.

### 2.3 Consumers

We consider an economy populated with a continuous mass one of representative consumers. Each consumer is endowed with one unit flow of time which is allocated between working $l$ and socializing $(1 - l)$. At each time, a consumer makes the following decisions: first, how much to consume and how much to save and, second, how to allocate its time between working and socializing. A representative consumer's preference is given by the following intertemporal isoelastic utility function$^{16}$:

$$U = \int_0^\infty u(C_t, 1 - l_t) \, e^{-\beta t} \, dt = \int_0^\infty \frac{1}{\rho} \left[ C_t(1 - l_t)^\eta \right]^{-\rho} e^{-\beta t} \, dt \quad (5)$$

$$-\infty < \rho \leq 1; \quad \eta > 0$$

where $C_t$ is the consumption in period $t$, $\beta$ is the discount factor, and $\eta$ captures the impact of socialization on the welfare of consumers.

$^{16}$ This is a standard labor-leisure tradeoff utility function where we treat leisure as socialization.
As in Zak and Knack (2001) and Guiso et al (2004), we assume that consumers can invest their savings only through investment brokers. There is a continuum of risk-neutral investment brokers. These brokers invest consumer's savings into R&D firms and receive the return after the realization period. However, they are opportunists and can abscond with the money with probability, \((1 - \phi)\), where \(\phi\) captures the strength of existing institutions in the economy which protects the consumers from fraudulent behavior of these brokers. The strength of institutions, \(\phi\), in turn, depends on the strength of informal institutions, which we call social capital, and the formal institutions.\(^{17}\) A higher stock of social capital and a better formal institutional environment, by reducing the probability of cheating, increases the expected return from R&D investment. The representative consumer's budget constraint, therefore, is given by:

\[
V_t = W_t l_t + \phi_t r_t V_t - C_t - T_t \quad ; \quad 0 \leq \phi_t \leq 1
\]  

(6)

where \(V_t\) is the value of the all assets held by consumers, \(W_t\) is the current wage rate, and \(r_t\) is the market interest rate. The strength of institutions, \(\phi_t\), depends on the services provided by the stock of social capital, \(s_t\), and productivity-adjusted expenditure, \(g_t = \frac{G_t}{A_t}\), on formal institutions finance by lump-sum tax (or contributions) \(T_t\), where \(G_t\) is the current expenditure on formal institutions at time \(t\). Alternatively, \(s_t\) and \(g_t\) can also be interpreted as a measure of personalized and generalized trust, respectively, in the economy. We consider the following functional form for \(\phi\):

\[
\phi_t = 1 - e^{-\theta_s s_t - \theta_g g_t} \quad ; \quad \theta_s, \theta_g > 0
\]  

(7)

where, \(\theta_s\) and \(\theta_g\) capture the impact of social capital and formal institutions on the effectiveness of the institutions.

\(^{17}\) The strength of formal institution is captured by productivity-adjusted expenditure. It may either be financed by the government or by private organizations.
Assuming that the brokers do not save, their per period consumption is given by $C^b_t = (1 - \phi_t) r_t V_t$, where $(1 - \phi_t)$ is the probability with which the brokers can cheat. Alternatively, $(1 - \phi_t)$ can also be interpreted as the transaction cost of searching an honest broker or the cost of writing a complete and enforceable contract with the new broker. Assuming that expenditure ($G_t$) on formal institutions is fully financed by total contributions ($T_t$) every period, the economy-wide budget constraint can be given by:

$$\dot{V}_t = W_t l_t + \phi(s_t, g_t) r_t V_t - C_t - G_t$$  \hspace{1cm} (8)

### 2.4 Social Capital

As argued earlier, since social capital increases with socialization and decreases with labor migration, we consider the following equation for the evolution of the stock of social capital:

$$s'_t = (1 - l_t) - m_t s_t, \quad 0 \leq m_t \leq 1; \quad 0 \leq s_t < \infty$$  \hspace{1cm} (9)

where, $m_t$ is the rate of labor migration across different sectors in the economy. If a proportion $m_t$ of workers switch jobs then the social capital is destroyed by a measure of $m_t s_t$.

Since the motive of socialization is not economic, the consumers do not consider the impact of their socializing decision on the social capital. This creates an additional source of externality into the model (social capital externality) where the formation of social capital is the side product of the individual rational decision of socialization.

We consider three different institutional environments in this paper. The first case is associated with perfect institutions ($\phi = 1$). The second case includes only informal institutions (social capital) and the third incorporates both formal and informal institutions where the strength of institutions is endogenously determined.
3. Decentralized Economy

We start our analysis with the last case which includes both social capital, $s$, and formal institutions, $g$, as determinants of institutions, $\phi$, where social capital is determined endogenously within the model while the expenditure on formal institutions, $G$, is optimally chosen. The other two cases are treated as special cases.

3.1 Equilibrium

An allocation in this economy consists of the time paths of consumption, and aggregate output, $[C_t, Y_t]_{t=0}^\infty$, time paths of R&D expenditure, state of the art technology, and net present value of the assets, $[N_t, A_t, V_t]_{t=0}^\infty$, time paths of interest rate, and wage rate, $[r_t, W_t]_{t=0}^\infty$ and time paths of labor supply, migration rate, social capital, government expenditure, and the strength of institutions $[l_t, m_t, s_t, G_t, \phi_t]_{t=0}^\infty$. An equilibrium is an allocation where the representative consumers maximize utility, intermediate good producers maximize profit, innovators maximize their net present discounted value and the labor market clears.

We start with the production sector. We assume that the final good sector is competitive while each of the intermediate good sectors is monopolized.\(^{18,19}\) For simplicity, we also assume that the monopolists use first-degree price discrimination to extract all surplus from the final good sector. It implies that the monopolist can charge $A_{lt}l_{lt}^g$ from the firms in the final good sector. The objective of a monopolist intermediate good firm is choose the optimal level of $l_{it}$ to maximize its profit, $\Pi_{it} = A_{lt}l_{lt}^g - W_{lt}l_{lt}$. The demand for labor, output and profit of an intermediate good firm $i$ is given by,

\(^{18}\) Price of the final good is normalized to one.
\(^{19}\) Quality gap is assumed to be sufficiently large between any two consecutive innovations in order to rule out limit pricing.
where $w_t = \frac{W_t}{A_t}$ is the productivity adjusted wage rate. The demand for labor increases when the technology in that sector improves and decreases if they do not innovate because of the rise in wage in response to innovation in other sectors. The output and the profit, therefore, would also increase with innovation and would fall otherwise. The aggregate flow of demand for labor, $l_t$, can be found by summing equation (10a) over $i$.

$$l_t = \frac{1}{\alpha^{1-\alpha} (1 + \frac{1}{1-\alpha} \ln \gamma)} \cdot \frac{1}{W_t^{1-\alpha}}$$

(11a)

We, then, get the following expressions for the productivity-adjusted aggregate output, $y_t = \frac{Y_t}{A_t}$, and profit, $\pi_t = \frac{\Pi_t}{A_t}$, in the economy by summing equation (10b) and (10c) respectively over $i$, by using (11a), and then diving through by $A_t$ as
In order to find the labor reallocation (migration) rate, we, first, use the expression of \( w_t \) from (11a) into (10a) and then differentiate it with respect to time. Noting that \( A_{lt} \) is constant for non-innovating firms, rate of change of demand for labor for these firms can be expressed as:

\[
\pi_t = (1 - \alpha) y_t = \frac{1 - \alpha}{1 + \frac{1}{1 - \alpha} \ln y} \lambda \ln y
\]  

(11c)

In order to find the labor reallocation (migration) rate, we, first, use the expression of \( w_t \) from (11a) into (10a) and then differentiate it with respect to time. Noting that \( A_{lt} \) is constant for non-innovating firms, rate of change of demand for labor for these firms can be expressed as:

\[
\frac{i_{lt}}{l_{lt}} = -\frac{1}{1 - \alpha} \frac{\dot{A}_t}{A_t} + \frac{i_t}{l_t}
\]  

(12a)

The first part captures the decline in demand as the workers move from non-innovating to innovating firms and the second is the change in labor hour each worker puts in when the economy experiences a technological shock. As our interest lie in the fraction of workers who change jobs, we consider only the first component. Since the number of non-innovating firms is \((1 - \lambda n_t)\) at any time \(t\), by using equation (3), we get the following expression for the labor migration rate in the economy:

\[
m_t = \frac{1}{1 - \alpha} \lambda n_t \ln y \left(1 - \lambda n_t\right)
\]  

(12b)

We next turn to the equilibrium in R&D sector. Because the expected payoff to an innovation is the same in every sector, the same equilibrium flow of investment, \(N_t\), will be used in each R&D sector. The value of an innovation, \(V_{lt}\), (or the value of a firm that innovates at time \(t\)) in sector \(i\) at time \(t\) is given by the net present value of all future profits:\(^\text{20}\)

---

\(^\text{20}\) Recall that once the innovation occurs in sector \(i\) at time \(t\), the technology in that sector jumps from \(A_{lt}\) to the state of the art technology, \(A_t\).
\[
V_{it}(A_t) = \int_{\tau=t}^{\infty} e^{-\int_{\tau}^{t} r_u \, du} \, e^{-\int_{\tau}^{t} \lambda n_u \, du} \, \Pi_{it}(A_t) \, d\tau
\]  
(13a)

where \( \Pi_{it}(A_t) \) is the profit of a firm at time \( \tau \) in which innovation occurred at time \( t \) and \( e^{-\int_{\tau}^{t} \lambda n_u \, du} \) is the probability that this firm is still producing using technology \( A_t \) at time \( \tau \geq t \).

By using (3) and after some algebraic manipulations, we get the productivity-adjusted value of an innovation, \( v_{it} = \frac{V_{it}}{A_t} \), as:

\[
v_{it}(A_t) = (1 - \alpha) \left( 1 + \frac{1}{1 - \alpha} \ln \gamma \right)^{\alpha} \int_{\tau=t}^{\infty} l_t^g \, e^{-\int_{\tau}^{t} (r_u + \lambda n_u + \frac{\alpha}{1 - \alpha} \lambda n_t \ln \gamma) \, du} \, d\tau
\]  
(13b)

The amount of resources devoted to research is determined by the research arbitrage condition which equates expected marginal benefit to marginal cost. That is,

\[
\lambda n_t \, V_{it}(A_t) = N_t \quad \text{or} \quad v_{it}(A_t) = \frac{1}{\lambda}
\]  
(14a)

Differentiating (13b) and (14a) with respect to time and by equating them to each other, we get \( v_{it} \) as:

\[
v_{it}(A_t) = (1 - \alpha) \left( 1 + \frac{1}{1 - \alpha} \ln \gamma \right)^{\alpha} \frac{l_t^g}{r_t + \lambda n_t + \frac{\alpha}{1 - \alpha} \lambda n_t \ln \gamma}
\]  
(14b)

Using (14a) and (14b), we get the familiar research-arbitrage condition:

\[
\frac{\lambda (1 - \alpha) \left( 1 + \frac{1}{1 - \alpha} \ln \gamma \right)^{\alpha} l_t^g}{r_t + \lambda n_t + \frac{\alpha}{1 - \alpha} \lambda n_t \ln \gamma} = 1
\]  
(15)

Finally, the productivity adjusted value of all the firms is given by:\(^{21}\)

\[
v_t = \frac{1}{\lambda} \frac{1}{1 + \frac{1}{1 - \alpha} \ln \gamma}
\]  
(16)

\(^{21}\) \( v_t = \int_{t_0}^{t} v_{it} \, dt \), where \( v_{it}(A_{it}) = a_{it}^{\frac{1}{\alpha}} \, v_{it}(A_t) \), is the value of a firm with technology \( A_{it} \) at time \( t \) and \( v_{it}(A_t) = \frac{1}{\lambda} \).
A representative consumer chooses consumption and labor to maximize utility (Eq. 5) subject to the budget constraint (Eq. 8). The first order conditions at the optimum are

\[ u_c = C^{\rho - 1} (1 - l)^{\eta \rho} = \mu \]  \hspace{1cm} (17a)

\[ -u_l = \eta C^\rho (1 - l)^{\eta \rho - 1} = \mu W \]  \hspace{1cm} (17b)

\[ \phi r = \beta - \frac{\mu}{\mu} \]  \hspace{1cm} (17c)

where \( \mu \) is the private shadow value of wealth, together with the transversality condition \( \lim_{t \to \infty} \mu V_t e^{-\beta t} = 0 \). The interpretation of these equations are standard; (17a) equates the private marginal utility of consumption to the shadow value of wealth; (17b) equates the private marginal utility of socialization to its opportunity cost, the real wage valued at the shadow value of wealth, while (17c) equates the return on assets to the rate of return of consumption.

By solving equations (17a) and (17b), we get the familiar relationship between labor and consumption,

\[ 1 - l_t = \frac{\eta c_t}{W_t} = \frac{\eta c_t}{w_t} \]  \hspace{1cm} (18a)

where \( c_t = \frac{c_t}{\hat{c}_t} \) is the productivity adjusted consumption. The Euler equation is given by using equation (3) and time derivatives of (17a) and (18a) into equation (17c),

\[ \frac{\dot{c}_t}{c_t} = \Omega(l_t) \left[ \phi_t r_t - \beta + \Psi(l_t) \lambda n_t \ln \gamma \right] \]  \hspace{1cm} (18b)

where, \( \Omega(l_t) = -\frac{1 - \alpha(1 - l_t)}{\eta l_t + (\rho - 1)(1 - \alpha(1 - l_t))} > 0 \) and \( \Psi(l_t) = -\frac{\eta p l_t}{1 - \alpha(1 - l_t)} > 0 \).

Finally, the optimal expenditure, \( G \), on formal institution equates the additional return from investment due to strengthening of institutions to its cost.

\[ \theta_g r_t v_t (1 - \phi_t) = 1 \]  \hspace{1cm} (19)
We summarize the equilibrium conditions as follows:

**Definition 1** An equilibrium in this economy is given by the time paths of consumption, and aggregate output, \([C_t, Y_t]_{t=0}^\infty\) that satisfies (8), and (11b), time paths of R&D expenditure, state of the art technology, and net present value of the assets, \([N_t, A_t, V_t]_{t=0}^\infty\) given by (15), (3) and (16), time paths of interest rate, and wage rate, \([r_t, w_t]_{t=0}^\infty\) consistent with (18b) and (11a) and time paths of labor supply, migration rate, social capital, and formal institutions \([l_t, m_t, s_t, G_t, \phi_t]_{t=0}^\infty\) given by (18a), (12b), (9), (19) and (7).

**Case I (Benchmark): Perfect Institution \((\phi = 1)\)**

In this case, the evolution of social capital is no longer relevant. The production and R&D sectors will have the same optimality conditions. Since the institutions are perfect and costless, the relevant budget constraint now is:

\[
\dot{V} = Wl + rV - C \tag{20a}
\]

Consumer’s optimization gives us the same labor supply function as earlier (18a).

However, the Euler condition is given by:

\[
\frac{\dot{C}_t}{C_t} = \Omega(l_t) \left[ r_t - \beta + \Psi(l_t) \lambda n_t \ln \gamma \right] \tag{20b}
\]

Additionally, optimality conditions with respect to government expenditure on formal institutions, G, is now no longer relevant.

**Case II: Social Capital is the only determinant of Institutions \((\phi_t = 1 - e^{\theta_s s_t})\)**

The optimality conditions in production and R&D sectors are again the same as earlier. The agent’s optimality condition and the Euler equation are again given by (18a) and (18b).

Again, as in case I, there is no optimality condition for G.

We define a balanced growth path as an equilibrium path in which all variables grow at a constant rate except for labor allocation, interest rate, migration rate, social capital and the
strength of institutions, which are constant. Following our definition of balanced growth path, it is convenient to write the system in terms of stationary productivity adjusted variables. It is straightforward to express the dynamics of the decentralized economy in terms of \( c, l, \phi, m \) and \( n \) as

\[
\begin{align*}
\dot{c}_t &= c \Omega(l_t) [\phi_t r_t - \beta + (\rho - 1)\lambda n_t \ln \gamma] \\
\dot{s}_t &= (1 - l_t) - m_t s_t
\end{align*}
\]

(21a)

(21b)

along with labor market clearing conditions,

\[
\begin{align*}
l_t + \frac{\eta}{\alpha} \left( 1 + \frac{1}{1 - \alpha} \ln \gamma \right)^{1 - \alpha} c_t l_t^{1 - \alpha} &= 1 \\
\frac{\alpha^{1 - \alpha}}{1 - \alpha} \ln \gamma &+ \eta \frac{c_t}{w_t^{1 - \alpha}} = 1
\end{align*}
\]

(22a)

(22b)

research arbitrage condition (15), labor migration rate (12b), optimal expenditure on formal institutions (19), strength of institutions (7), and the economy-wide budget constraint\(^{22}\)

\[w_t l_t + (\phi_t r_t - \lambda n_t \ln \gamma) v_t - c_t - g_t = 0\]

(22c)

Imposing the steady-state conditions \( \dot{c} = \dot{s} = 0 \), we can solve for the steady-state values of productivity-adjusted variables, consumption (\( \tilde{c} \)), R&D investment (\( \tilde{n} \)), expenditure on formal institutions (\( \tilde{g} \)), and wage rate (\( \tilde{w} \)), and the other variables, interest rate (\( \tilde{r} \)), labor (\( \tilde{l} \)), migration rate (\( \tilde{m} \)), social capital (\( \tilde{s} \)), and the strength of institutions (\( \tilde{\phi} \)). Finally, productivity-adjusted output (\( \tilde{y} \)), profit (\( \tilde{\pi} \)) and value of assets (\( \tilde{v} \)) can be found by using (11b), (11c) and (16) respectively.

Linearizing (21a) and (21b) around the steady-state yields an approximation to the underlying dynamic system. This system forms the basis for our dynamic simulations. For all

\(^{22}\) We, first, write (8) in terms of \( \dot{\psi} \), and then use \( \dot{\psi} = 0 \) from (16).
plausible parameter values, the system has one positive (unstable) and one negative (stable) eigenvalues, leading us to conclude that it is saddle point stable.

4 Social Planner

Now we briefly discuss the Pareto optimal allocation. Decentralized equilibrium is Pareto suboptimal because of two sources of externalities. The first is the externality in the R&D sector where the monopolists do not internalize the loss to the earlier monopolist caused by new innovation (*business stealing effect*) resulting in too much innovation and they ignore the impact of their innovation on the next innovation (*intertemporal spillover effect*) leading to too little innovation in the decentralized economy. The second source of externality is the *social capital externality* where consumers do not take into account the impact of socialization on social capital as they take the stock of social capital as given at any point of time. Since the full benefit of socialization is not taken into account, consumers spend less time socializing in decentralized economy.

4.1 Equilibrium

Since there is no inefficiency in the production side, the equilibrium conditions are again given by equations (9), (10) and (11). The resource constraint can now be written as:

\[ N_t = (\alpha + \phi_t (1 - \alpha))Y_t - C_t - G_t \]  

(23)

The social planner chooses consumption, labor and R&D investment to maximize utility (5) subject to the technology growth (3), evolution of social capital (9), resource constraint (23) and labor migration rate (12b). The optimality conditions are:

\[
\frac{1}{\lambda \ln Y} (C^{\rho-1} (1 - l)^{\eta p} + \mu_2 m_N s) = \mu_1
\]  

(24a)

---

23 See Appendix A for derivation of resource constraint for the Social Planner.
\[ \frac{1}{\lambda \ln \gamma} \left( \eta C^\alpha (1 - l)^{\eta^\alpha - 1} + \mu_2 (1 + m_N N_t s) \right) = \mu_1 N_t \]  

(24b)

\[ \left( \lambda \ln \gamma - \frac{\mu_2}{\mu_1} m_N s \right) N_A - \frac{\mu_2}{\mu_1} m_A s = \beta - \frac{\mu_1}{\mu_1} \]  

(24c)

\[ \frac{\mu_1}{\mu_2} \lambda \ln \gamma - m_N s \right) N_s - m = \beta - \frac{\mu_2}{\mu_2} \]  

(24d)

where \( \mu_1 \) and \( \mu_2 \) denote the shadow value of technology and social capital respectively, together with the transversality conditions

\[ \lim_{t \to \infty} \mu_1 A e^{-\beta t} = \lim_{t \to \infty} \mu_2 s e^{-\beta t} = 0 \]  

(24e)

There are some key differences from the corresponding conditions for the decentralized economy. First, (24a) equates the utility of an additional unit of consumption, adjusted by its impact on social capital multiplied by the shadow value of social capital, to the shadow value of technology. Since additional consumption reduces the funds available for R&D investment and thereby increases social capital by reducing labor migration rate (12b), people would, therefore, consume less compared to decentralized economy. Second, (24b) equates the social marginal benefit of socialization (which includes its positive impact on social capital as well) to the real wage valued at the shadow value of technology. Third, (24c) and (24d) are the intertemporal efficiency conditions, where (24c) equates the rate of return of technology to the social return of consumption and (24d) equates the return of social capital to the rate of return of consumption evaluated in terms of the shadow value of social capital.

We can express the macrodynamic equilibrium of the centrally planned economy in terms of productivity adjusted variables as:\textsuperscript{24}

\[ \dot{c}_t = c[\Theta_1 (c, l, n, s, g, m, \phi) - \lambda n_t \ln \gamma] \]  

(24a)

\[ \dot{l}_t = \Theta_2 (c, l, n, s, g, m, \phi) \]  

(24b)

\[ s_t = (1 - l_t) - m_t s_t \]  

(24c)

\textsuperscript{24} see Appendix B.
5 Quantitative Results

Due to complexity of the model, we calibrate the system in order to obtain further insight. The baseline parameter values are given as follows: \( \alpha = 0.7; \beta = 0.02; \gamma = 2; \lambda = 0.3, 0.4; \eta = 0.1; \theta_s = 1; \theta_g = 100. \) Our choice of the preference parameters, \( \alpha \) and \( \beta \), are standard. The parameter \( \eta \) describes the degree of substitution between socialization/leisure and consumption. We chose \( \eta \) as 0.1 in order to ensure that people socialize 10-20% of the total available time out of working and socializing, however, the results are qualitatively similar for other values of \( \eta \). It is in contrast with the previous literature where the estimated work time is approximately 1/3 of the total available time. The reason for this difference is that we are not considering any other leisure activities and therefore our total time is approximately 10 hours a day, not 24 hours. The choice \( \gamma \), the size of innovation, and \( \lambda \), innovation probability parameter, are such that growth rate in the decentralized economy in the most general framework ranges between 2 to 7%. However, we can easily change these parameters to reflect varying growth experience of different countries. In this regard, it can be argued that the countries experiencing higher growth are able to either innovate more frequently or get the necessary resources (for example, foreign investment) in order to sustain higher growth. Once again, the qualitative results of the model are unchanged for other reasonable parameter values as well. As social capital is accumulated over time whereas expenditure on formal institutions is a flow variable, the effectiveness of formal institutions, which is captured by \( \theta_g \), should be sufficiently high for it to have any significant impact on the economy. One may also argue that formal institutions should have larger impact as it affects the whole economy in general, as compared to social capital which is more local in nature. It is because of these reasons, we have chosen a significantly high value for \( \theta_g \). Equation (19) also provides some idea about the magnitude of \( \theta_g \).
We know that, for $g$ to be positive, $\theta_g$ must be greater than $\frac{1}{r \cdot v \cdot (1 - \phi)}$. Using the values of $r$, $v$ and $\phi$ from case II of Table 1A, we get $\theta_g > 36.28$. Although some of the parameters are difficult to pin down, the calibration exercise still provides useful insights into the dynamics of social capital and economic growth.

First, we will compare the steady-state results in the decentralized economy (table 1A) and central planner’s (table 1B) case under the, abovementioned, three institutional environments. Thereafter, we analyze the comparative static results and finally, we examine the dynamic effect of increase in innovation parameter, $\lambda$, on the economy.

5.1 Steady-State

In the benchmark model (Case I), the steady state growth rate in a decentralized economy (Table 1A) is given by 2.75%, when we consider the innovation parameter, $\lambda$, to be 0.3. Consumption is 0.515, that is, people consume 79.53% of total output produced, spend 10.21% of the time socializing, and 8.8% of them change job. However, once we allow for endogenous institutions (Case II and Case III), growth performance deteriorates. In case II, in the absence of any formal institution, growth rate falls by 0.43 percent points. In this less than perfect institutional environment, $\phi = 0.74$, a lower effective rate of return shifts the supply of R&D investment left, thereby reducing R&D investment from 0.1325 to 0.1116 (that is, from 20.46 to 17.22% of total output) and increasing the market interest rate from 8.9% to 10.5%. As investment income falls from 0.09 ($r \cdot v$) to 0.08 ($\phi r v$), consumption declines, inducing people to work more from 0.898 to 0.899 and reducing wage rate from 0.5048 to 0.5046. Overall, wage income rises from 0.4433 to 0.4437 which, in turn, raises consumption. As overall income falls, consumption falls from 0.515 to 0.509. Once we include formal institutions into the model (Case

---

$25 r = 0.1054$, $v = 1.0069$, and $\phi = 0.7403$. 

21
III), growth rate rises by 0.3 percent point (to 2.6%) compared to case II and is closer to the benchmark case. An increase in expenditure on formal institutions crowds out current consumption. As argued earlier, decline in consumption raises labor hour and reduces wage rate. The resulting increasing in wage income raises consumption, however, the overall impact is negative. Improving formal institution raises the health of the institutions in the economy which increases the supply of R&D investment by raising the return from investment. In addition, increase in labor hour increases the demand for R&D investment because of its impact on profit and on value of an innovation. Overall supply side dominates resulting in an increase in R&D investment and a fall in interest rate. Increase in \( n \) raises economic growth directly but at the same time increases labor migration as well. Reduction in socialization time together with increased migration rate reduces social capital which adversely affects the economy. However as the direct impact of \( g \) dominates, while consumption and social capital declines, R&D investment and growth rate rises. In this case, 1.6% of the total output is spent on formal institutions.

The results in the case of social planner (Table 1B) are qualitatively similar except that socialization increases from 9% to 16% of total time when social capital is included in the model. This is because the social planner internalizes social capital externality and therefore considers the additional benefit of socialization on growth through an improvement in social capital.

---

\(^{26}\) If, however, indirect effect dominates then it is optimal to set \( g =0 \).

\(^{27}\) In the United States, on average approximately 1.9% of GDP was used on public order and safety in the last 15 years before the current recession during which the average growth rate was approximately 3%. Data source: Bureau of Economic Analysis.
5.2 A Permanent Increase in the Innovation Parameter (λ): Long Run Effects

We now introduce a permanent increase in the innovation parameter, $\lambda$, from 0.3 to 0.4. In the benchmark case, we observe that the growth performance improves by 1 percent point. Output, R&D investment and interest rate increases while consumption, socialization and wage rate declines. As the rate of innovation increases, causing an increase in the demand for R&D investment, investment rises along with the market interest rate. As mentioned earlier, as consumption is crowded out, it raises wage income which, in turn, raises consumption and leads to a further rise in R&D investment through increase in labor hour. Growth rate in the economy rises, therefore, not only due to the initial rise in $\lambda$, but also due to the resulting increase in R&D investment which, in turn, increases labor migration rate. Overall, consumption falls from 0.515 to 0.511 (from 79.54 to 78.95% of total output), and investment rises from 0.132 to 0.136.

Once social capital is included into the model, not only the rise in growth rate is lower (0.66 percent points), but, in fact, investment into R&D sector falls from 0.112 to 0.107 (from 17.22 to 16.55% of output) declines. The intuition is as follows: increase in $\lambda$ increases the demand for R&D investment, raising investment and market interest rate. Consequently, consumption falls while work hour rises. As social capital declines with a fall in socialization time and a rise in labor migration rate (because of higher growth rate), effective return from investment falls. R&D investment falls and market interest rate rises further as the supply of funds in R&D sector declines hampering growth rate. Overall R&D expenditure falls as institutional factors dominate its initial rise.

Therefore, the countries which are trying to grow faster in the absence of effective formal institutions may not be able to reap the full benefit of new technologies because of its detrimental impact on the existing informal institutions. In fact, contrary to the popular belief that new
technologies brings in more investment, we observe that investment falls in the absence of any formal institution. However, if a country chooses the strength of formal institutions optimally, in response to declining social capital, growth performance is far better (increase in growth rate is 0.98 percent point, only marginally lower than the benchmark case). In this case, growth rate (at 3.6%) itself is very close to the benchmark case (3.8%). Expenditure on formal institutions increases from 0.106 to 0.0135, which is an increase from 1.64% to 2.08% of total output. Therefore, if a country would like to improve its growth performance, it should change its formal institution in response to declining social capital in order to experience sustained higher economic growth.

The last column of the tables report the long run welfare change measured by the optimized utility of the representative agent where $C$ and $l$ are evaluated along the equilibrium path. These welfare changes are measures of equivalent variations, calculated as the percentage change in the flow of income necessary to maintain the level of welfare unchanged following the shock. As anticipated, the welfare gain is highest in case I (15.23%) and is lowest in case II (12.31%). Again, we get very similar qualitative results for the social planner's problem. Our simulation exercise shows that the optimal expenditure on formal institution should rise from 3.5% to 3.8% of total GDP, as the economy experiences technological breakthrough (as $\lambda$ goes up from 0.3 to 0.4). For an economy growing at 2.55%, 2.66% of GDP should be devoted to formal institutions.\footnote{This is the case when $\lambda$ is 0.2. This result is not reported in the table.}
5.3 Transient Dynamics

The transitional adjustment paths for case III following an increase in the probability of innovation are illustrated in Fig. 1. Fig. 1.1 illustrates the stable adjustment locus in c-s space, indicating how c and s both generally decrease together during the transition.

The short-run responses are reported in Table 2. As argued earlier, an increase in the innovation parameter immediately increases the demand for investment into R&D sector leading to an increase in the market interest rate, $r$, and overshooting R&D investment, $n$, while crowding out current consumption, $c$ and expenditure on formal institutions, $g$. This decline in c, in turn, induces people to work more and thereby increasing equilibrium labor hour and reducing the wage rate. The strength of institutions, $\phi$, deteriorates as $g$ falls. As institutions weakens, rate of return on investment falls which, in turn, reduces the supply of R&D funds. Through this channel, market interest rate rises while R&D investment falls. Also, increase in labor hour raises the demand for R&D investment, raising investment and interest rate. Overall, on impact, R&D investment rises from 0.125 to 0.130 which along with an increase in $\lambda$, raises economic growth from 2.6 to 3.6% and labor migration rate from 8.35 to 11.4%. Consumption falls from 0.503 to 0.499, labor hour jumps from 0.9004 to 0.901, and market interest rate rises from 9.5 to 12.19%. As a result of decline in $g$ from 0.0106 to 0.0103, the strength of institutions decline from 0.896 to 0.891.

Over time, social capital starts declining as a result of a reduction in socialization time and an increase in labor migration rate. Since the marginal benefit of improving formal institutions exceeds its cost, $g$ rises over time. Increase in $g$ however crowds out current consumption, consumption continues to declines, labor hour rises and wage rate declines during the transition period. Although, the strength of institutions improves during the transition period.
after its initial fall as improvement in formal institutions dominates declining social capital, it remains below its steady state level. Also, it is not enough to fully compensate initial decline in \( \phi \) and therefore, the strength of institutions declines in the long run. As a result, R&D investment falls after during the transition while interest rate rises. Growth rate and labor migration rate, therefore, also decline during the transitional path.

The reduction in initial consumption and leisure results in short-run welfare loss of 0.86% but as the consumption grows overtime, welfare rises (Fig 1.12) and the overall intertemporal welfare gain is 14.71%.

6 Conclusions

We have developed a new framework to analyze the endogenous relationship between social capital and economic growth. Our model is based on the argument that a higher social capital is beneficial for economic performance of a country but higher growth itself destroys it. We show that a higher growth prospect reduces social capital by reducing socialization time and by increasing labor reallocation rate. In the absence of any formal institutions, an increase in the rate of innovation reduces the investment into R&D sector. The reason behind this decline in R&D expenditure is the erosion of the strength of existing informal institutions. It is generally argued that technological advancements require the strengthening of existing formal institutions and in some cases new institutions need to be set up. This paper argues that in addition to that, formal institutions needs to be improved further to fill the void created by the erosion of existing informal institution, a result of technological advancements.

This model also provides an alternative explanation to the growth convergence conundrum. Even if a poor country may acquire better technologies of rich countries, catch up rate might still be low if the decline in social capital is not compensated by improving formal
institutions. Therefore, a poor country lacking in resources or lacking in will to develop formal institutions may, therefore, will remain poor.

Although we have abstained from capital accumulation for simplicity, it is straightforward to include it in the model. Also, a more complete model should incorporate some form of heterogeneity among the economic agents. Improvements in communication technology such as telephone, internet, or online social networks by lowering the cost of maintaining contacts with friends and family increases the size and strength of social networks and therefore the impact of labor reallocation on social capital may not be too strong. Incorporating these features into a model of social capital and economic growth will help in better understanding of this inter-relationship and hence have strong policy implications. In this model, we have assumed that the formal institution is financed by lump-sum contributions. It would be interesting to characterize the optimal tax policy for these institutions. As very little is known about the absolute and relative effectiveness of various types of institutions, new research, both at theoretical as well as at empirical level is desired. Lastly, social capital has many dimensions such as trust, norms, social network to name a few. We need to look into the dynamics of each of them separately and their relationship with growth in order to have more precise predictions.
Table 1: Comparative Statics of Steady-State Results

Case I: Benchmark Case ($\phi = 1$)

Case II: Social Capital is the only determinant of institutions

Case III: Both social capital and formal institutions determine the strength of institutions

Baseline Parameters: $\alpha = 0.7, \beta = 0.02, \gamma = 2, \eta = 0.1, a = 1, b = 100$

Table 1A: Decentralized Economy

$\lambda = 0.3$

<table>
<thead>
<tr>
<th></th>
<th>$c$</th>
<th>$l$</th>
<th>$w$</th>
<th>$r$</th>
<th>$n$</th>
<th>$y$</th>
<th>$m$</th>
<th>$s$</th>
<th>$g$</th>
<th>$\phi$</th>
<th>Growth</th>
<th>$\Delta$Welfare</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case I</td>
<td>0.5151</td>
<td>0.8979</td>
<td>0.5048</td>
<td>0.0889</td>
<td>0.1325</td>
<td>0.6476</td>
<td>0.0882</td>
<td>1.1567</td>
<td>-</td>
<td>1.0000</td>
<td>0.0275</td>
<td>-</td>
</tr>
<tr>
<td>Case II</td>
<td>0.5089</td>
<td>0.8991</td>
<td>0.5046</td>
<td>0.1054</td>
<td>0.1116</td>
<td>0.6482</td>
<td>0.0748</td>
<td>1.3481</td>
<td>-</td>
<td>0.7403</td>
<td>0.0232</td>
<td>-</td>
</tr>
<tr>
<td>Case III</td>
<td>0.5030</td>
<td>0.9003</td>
<td>0.5044</td>
<td>0.0950</td>
<td>0.1252</td>
<td>0.6487</td>
<td>0.0835</td>
<td>1.1939</td>
<td>0.0106</td>
<td>0.8955</td>
<td>0.0260</td>
<td>-</td>
</tr>
</tbody>
</table>

$\lambda = 0.4$

<table>
<thead>
<tr>
<th></th>
<th>$c$</th>
<th>$l$</th>
<th>$w$</th>
<th>$r$</th>
<th>$n$</th>
<th>$y$</th>
<th>$m$</th>
<th>$s$</th>
<th>$g$</th>
<th>$\phi$</th>
<th>Growth</th>
<th>$\Delta$Welfare</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case I</td>
<td>0.5115</td>
<td>0.8987</td>
<td>0.5047</td>
<td>0.1146</td>
<td>0.1364</td>
<td>0.6479</td>
<td>0.1192</td>
<td>0.8502</td>
<td>-</td>
<td>1.0000</td>
<td>0.0378</td>
<td>15.23%</td>
</tr>
<tr>
<td>Case II</td>
<td>0.5029</td>
<td>0.9003</td>
<td>0.5044</td>
<td>0.1453</td>
<td>0.1074</td>
<td>0.6488</td>
<td>0.0949</td>
<td>1.0497</td>
<td>-</td>
<td>0.6499</td>
<td>0.0298</td>
<td>12.31%</td>
</tr>
<tr>
<td>Case III</td>
<td>0.4967</td>
<td>0.9015</td>
<td>0.5042</td>
<td>0.1228</td>
<td>0.1291</td>
<td>0.6494</td>
<td>0.1132</td>
<td>0.8702</td>
<td>0.0135</td>
<td>0.8921</td>
<td>0.0358</td>
<td>14.71%</td>
</tr>
</tbody>
</table>

Table 1B: Social Planner

$\lambda = 0.3$

<table>
<thead>
<tr>
<th></th>
<th>$c$</th>
<th>$l$</th>
<th>$w$</th>
<th>$r$</th>
<th>$n$</th>
<th>$y$</th>
<th>$m$</th>
<th>$s$</th>
<th>$g$</th>
<th>$\phi$</th>
<th>Growth</th>
<th>$\Delta$Welfare</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case I</td>
<td>0.4319</td>
<td>0.9140</td>
<td>0.5022</td>
<td>0.1363</td>
<td>0.2238</td>
<td>0.6557</td>
<td>0.1447</td>
<td>0.5943</td>
<td>-</td>
<td>1.0000</td>
<td>0.0465</td>
<td>-</td>
</tr>
<tr>
<td>Case II</td>
<td>0.4135</td>
<td>0.8372</td>
<td>0.5156</td>
<td>0.1615</td>
<td>0.1625</td>
<td>0.6166</td>
<td>0.1072</td>
<td>1.5190</td>
<td>-</td>
<td>0.7811</td>
<td>0.0338</td>
<td>-</td>
</tr>
<tr>
<td>Case III</td>
<td>0.4139</td>
<td>0.9086</td>
<td>0.5030</td>
<td>0.1399</td>
<td>0.2061</td>
<td>0.6529</td>
<td>0.1340</td>
<td>0.6818</td>
<td>0.0229</td>
<td>0.9489</td>
<td>0.0428</td>
<td>-</td>
</tr>
</tbody>
</table>

$\lambda = 0.4$

<table>
<thead>
<tr>
<th></th>
<th>$c$</th>
<th>$l$</th>
<th>$w$</th>
<th>$r$</th>
<th>$n$</th>
<th>$y$</th>
<th>$m$</th>
<th>$s$</th>
<th>$g$</th>
<th>$\phi$</th>
<th>Growth</th>
<th>$\Delta$Welfare</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case I</td>
<td>0.4228</td>
<td>0.9157</td>
<td>0.5019</td>
<td>0.1820</td>
<td>0.2338</td>
<td>0.6566</td>
<td>0.1958</td>
<td>0.4301</td>
<td>-</td>
<td>1</td>
<td>0.0648</td>
<td>16.67%</td>
</tr>
<tr>
<td>Case II</td>
<td>0.3927</td>
<td>0.8295</td>
<td>0.5170</td>
<td>0.2438</td>
<td>0.1646</td>
<td>0.6126</td>
<td>0.1421</td>
<td>1.2005</td>
<td>-</td>
<td>0.6989</td>
<td>0.0456</td>
<td>12.64%</td>
</tr>
<tr>
<td>Case III</td>
<td>0.4033</td>
<td>0.9132</td>
<td>0.5023</td>
<td>0.1868</td>
<td>0.2169</td>
<td>0.6552</td>
<td>0.1831</td>
<td>0.4744</td>
<td>0.0250</td>
<td>0.9491</td>
<td>0.0601</td>
<td>16.14%</td>
</tr>
</tbody>
</table>
Table 2: Short-Run Effects of increase in $\lambda$ (Case III)

<table>
<thead>
<tr>
<th></th>
<th>$c(0)$</th>
<th>$l(0)$</th>
<th>$w(0)$</th>
<th>$r(0)$ %</th>
<th>$n(0)$</th>
<th>$y(0)$</th>
<th>$m(0)$ %</th>
<th>$g(0)$</th>
<th>$\phi(0)$</th>
<th>$\text{Growth}(0)$ %</th>
<th>$\text{Welfare}(0)$ %</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Case III</strong></td>
<td>0.4990</td>
<td>0.9010</td>
<td>0.5043</td>
<td>0.1219</td>
<td>0.1299</td>
<td>0.6492</td>
<td>0.1138</td>
<td>0.0102</td>
<td>0.8913</td>
<td>0.0360</td>
<td>-0.86</td>
</tr>
</tbody>
</table>

Figure 1: Dynamic Responses to Innovation Shock

1.1 Phase Diagram

1.2 Social Capital

1.3 Productivity Adjusted Consumption

1.4 Labor
1.11 Growth Rate

1.12 Welfare Path

After Shock

Before Shock
Appendix

Appendix A: Resource constraint of the Social Planner

Since the social planner takes into account only the incremental profit when considering the value of an innovation, the profit generated by any innovation continues forever. Therefore, value of all the assets held by consumers is the present discounted value of all future profit at the current level of innovation.

That is, \( V = \frac{\Pi}{r} \Rightarrow rV = \Pi \) \hspace{1cm} (A1)

Resource constraint is given by:

\[ Y = C + N + G - (1 - \phi)rV \] \hspace{1cm} (A2)

Using \( rV = \Pi = (1 - \alpha)Y \), we get

\[ N_t = (\alpha + \phi (1 - \alpha))Y_t - C_t - G_t \] \hspace{1cm} (A3)

Appendix B: Dynamic Equations for Social Planner

\[ (u_{Cl}^2 + \Gamma_1 \mu_2 m_{NN} s + \Gamma_2 \mu_2 m_{NN} N_t s - u_{CC} \Gamma_3) \dot{C} = \]

\[ - \Gamma_1 \lambda \ln \gamma \ \dot{\mu}_1 + (\Gamma_1 m_N s + \Gamma_6)\dot{\mu}_1 + (\Gamma_4 \mu_2 - \Gamma_6 u_c N_t s)\dot{s} + (\Gamma_5 \mu_2 s - \Gamma_6 u_c N_{lA})\dot{A} \] \hspace{1cm} (B1)

and

\[ (u_{Cl}^2 + \Gamma_1 \mu_2 m_{NN} s + \Gamma_2 \mu_2 m_{NN} N_t s - u_{CC} \Gamma_3) \dot{l} = \]

\[ \Gamma_2 \lambda \ln \gamma \ \dot{\mu}_2 - (\Gamma_2 m_N s + \Gamma_7)\dot{\mu}_2 - (\Gamma_5 \mu_2 s - \Gamma_7 u_c N_t s)\dot{s} - (\Gamma_5 \mu_2 s - \Gamma_7 u_c N_{lA})\dot{A} \] \hspace{1cm} (B2)

where,

\[ \Gamma_1 = u_{Cl} + u_c N_{lA} + U_{Cl} N_l \]

\[ \Gamma_2 = u_{Cl} + u_c N_l \]

\[ \Gamma_3 = u_{Cl} + u_c N_{lA} \]

\[ \Gamma_4 = m_N + m_{NN} N_s s \]

\[ \Gamma_5 = m_{NN} N_A + M_{NA} \]

\[ \Gamma_6 = U_{Cl} + \mu_2 m_{NN} N_l s \]
\[ \Gamma_7 = u_{cc} - \mu_2 m_N n \]

\[ U_{cc} = \frac{1}{A} \frac{(\rho - 1)}{c} u_c, \quad U_{ll} = \frac{\eta(\eta \rho - 1) c}{(1 - l)^2} u_c, \quad U_{cl} = -\frac{\eta \rho}{1 - l} u_c, \]

\[ N_A = y - \phi g (1 - \alpha)y, \quad N_i = \frac{\alpha Y}{l}, N_s = \phi_s (1 - \alpha)y A, N_lA = \frac{\alpha (\alpha - 1) y}{l^2} A \]

\[ m_N = \frac{1}{1 - \alpha} \frac{\lambda \ln y}{A} (1 - 2 \lambda n), m_A = -\frac{1}{1 - \alpha} \frac{\lambda \ln y}{A} n(1 - 2 \lambda n) \]

\[ m_{NN} = -\frac{1}{1 - \alpha} \frac{\lambda \ln y}{A} \frac{2 \lambda}{A}, \quad m_{NA} = -\frac{1}{1 - \alpha} \frac{\lambda \ln y}{A} \frac{1}{A} (1 - 4 \lambda n) \]

\[ \mu_1 = \frac{1}{\lambda \ln y} \left( 1 - \left( \frac{\eta c A}{1 - l} - N_l \right) m_N n \right) u_c, \quad \mu_2 = -\left( \eta c A \right) \frac{1}{1 - l} u_c \]

\[ \mu_1 = \left( \beta - \left( \lambda \ln y - \frac{\mu_2}{\mu_1} m_N n \right) N_A - \frac{\mu_2}{\mu_1} m_A n \right) \mu_1, \quad \mu_2 = \left( \beta - \left( \frac{\mu_1}{\mu_2} \lambda \ln y - m_N n \right) N_s - m \right) \mu_2 \]

\[ \dot{s} = (1 - l) - m s \quad , \dot{A} = \lambda \ln y \quad A \]
References


