Endogenous Institutions and Multiple Equilibria: The Role of Commitment

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Abstract

Current empirical work and discussions of the impact of institutions on economic performance either implicitly or explicitly assume that institutions persist and that the quality of such institutions impacts investment. We incorporate these empirical patterns into a dynamic model of institutional choice, wherein the government invests in the legal infrastructure in response to the need for the protection of output from appropriation by households. In modeling the government's choice of policy as a dynamic problem, we must consider whether the government is capable of committing to a sequence of investments in legal infrastructure. When the government is able to commit to policy over the infinite horizon, we find a unique equilibrium. However, discretionary policy permits a lower steady state to exist, under which the government over-responds to appropriation of output, succeeding in the reduction of property theft, but at the cost of lower consumption. These results would suggest that a measure of institutional quality must not only consider the extent to which current policies protect property rights, but also include the ability of the government to commit to reform in the long run.

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1 Introduction

The causes of the 'Great Divergence' of incomes between advanced and developing economies has been the source of much debate within the empirical growth literature. Most of the existing studies attempt simply to explain cross-sectional differences in income per capita, and attribute the income gap largely to differences in total factor productivity. Hall and Jones (1997) (**?**) provide empirical evidence that the levels of economic performance vary considerably across countries, with differences persistent over time. Despite controlling for accumulation of human and physical capital, the Solow residual still varies significantly across countries.

Beginning with the work of Alchian (**?**), Demsetz (7) and North (18) in the early 1970s, economic historians have pointed to economic conditions as causes of changing institutions and to the evolution of institutions as efficiency enhancing and thus improving economic outcomes. What are institutions in this context? North defines institutions as "the rules of the game in a society or, more formally, the humanly devised constraints that shape human interaction." Typically, institutions are divided into two categories - economic and political. Political institutions regulate the limits of political power, and determine how political power changes hands. Economic institutions determine the degree of property rights enforcement and govern contractual arrangements. These institutions then influence the structure of economic incentives and thus the efficiency of resource allocation, particularly to productive capital and labor. A lack of sufficient institutions will drive a wedge between private and social returns to activities in an economy, resulting in suboptimal allocations of resources.

Hall and Jones form a proxy for this wedge between private and social returns by combining two indices – an index of government antidiversion policies, and an index measuring openness to trade. They then find that the quality of "social infrastructure" or of institutions acts as a significant "deep variable" in determining the level of output per worker. This result would seem to be robust to instrumenting for the measure of institutions. Institutions and their role in economic development have been the focus of a flurry of empirical activity since Hall and Jones (1999). Acemoglu (**?**, 1) has played a significant role in continuing to emphasize the importance of institutions, and Acemoglu, Johnson and Robinson (2001) points to the persistence of institutions as the primal cause of long-run economic growth, a conclusion which is supported by a number of other cross-country studies. Jeffrey Sachs has written, along with co-authors, a series of empirical papers, pointing to the direct effects of geographical factors, including the role of disease burden in diminishing economic growth, on divergence of income between countries. Glaeser et al (2004) (8), following the argument of Lipset (1960) (17), provides evidence to cast doubt on the conclusions of Acemoglu and others, and proposes instead that both political and economic institutions may be the consequence of development, and especially of human capital accumulation. However, Glaeser does not presume to say that his study provides well-founded evidence in favor of Lipset's perspective, and one interpretation of his work may simply be that empirical specifications which rely exclusively on the identification of deep variables (such as institutional and geographical measures) to the exclusion of physical and human capital may be inadequate to accurately describe the political and economic processes of development. All authors admit an important correlation between institutions and economic development.

The existing theoretical literature is similarly lacking in a unified approach to the subject. The first definitive research efforts in this area, including Grossman and Kim (1996)(11) and Tornell(1997) (23), considered security of property rights without exclusively modeling institutions. Economies are considered in a kind of Hobbesian state of nature, where there is no explicit role of government, and where security is determined through competition between interest groups. Zak (2002) (26) introduced expropriation, and security expenditures by the government, into an overlapping-generations model, where the rate of expropriation was dependent on both criminal effort and on security expenditures. His model generates a poverty trap result, but property rights protection is modeled as a policy choice, rather than an outcome of institutional quality. Gradstein (2004) (9) as well as Hoff and Stiglitz (2002) (13) allow for discrete states of such institutions, requiring either sufficient political constituency or tax revenues to move from one to the other. The need for a threshold level of political institutions or tax revenues in order for adequate economic institutions to exist again generates a poverty trap.

Current empirical work, including the studies referred to above, frequently assume that institutions persist. Institutional transitions are sometimes implemented as the deliberate outcome of bargaining among a small number of elite groups, or because of changes in the balance of power between the elite and ordinary citizens. Following this line of thought, more recent work by Acemoglu and Robinson (2007)( ) considers a model in which economic institutions are determined by the combination of de facto political power and a stochastic process. Alternatively, institutional change may either take place either as a decentralized process by which changes in practice occur informally among large groups of agents, or as a formal process occurring through the government. This essay will focus on the latter channel of institutional change.

The approach taken in this chapter is most similar to Zak (2002), in that institutions are introduced to secure property, in a model where agents choose between productive honest labor and theft of output. The government must optimally choose the level of investment into institutions governing property rights in order to maximize the utility of the representative household. However, I build the model in an infinite-horizon setting, in which the level of property rights is a continuous stock variable, thus differing from both Gradstein and Zak. This introduces additional persistence in institutions, or property protection, and makes a key distinction between current government policies and institutions. While it can be useful in some contexts (such as the transition economies of Eastern Europe) to think of institutions as a purely discrete variable, allowing the measure of property rights institutions to be both persistent and continuous allows us to think about the process of gradual reform by which institutional improvements take place in most countries. In considering the dynamic optimization problem of the government in choosing investment in the legal infrastructure which protects property rights, I analyze both equilibria that are attainable when the government can commit to a sequence of policies, and those that occur when policy is set by discretion in each period.

When there is a threshold level of tax revenues necessary for the rule of law or the formation of economic institutions, then even under commitment, there may exist a poverty trap equilibrium, corresponding to low levels of income and of investment in the legal infrastructure and to high levels of appropriation of output. Specifying legal infrastructure as a continuous state variable, however, implies that under commitment there is a single feasible steady state. Multiple steady states only exist when discretionary policy is employed. In modeling discretionary policy, I consider the set of Markov Perfect equilibria, in which governments will maximize life-time utility of households, given the policy functions of future governments. Unlike the results of Ortigueira and Pereira (2007) and Azzimonti-Renzo, Sarte and Soares (2006), the steady state under commitment is not a Markov-Perfect equilibrium. Instead, the return to appropriation acts like a wedge between the Markov-Perfect equilibrium and the the solution to the Ramsey problem. This illustrates a key distinction between models of endogenous institutions and more traditional optimal fiscal policy models, in which no direct imperfection in the production technology is introduced.

The rest of the paper is organized as follows. Section 2 describes the model, in particular the optimization problems of households and firms. Section 3 characterizes the steady state equilibria of the model when government policies are taken as exogenous. Section 4 formalizes the Ramsey problem for the government in choosing the optimal level of investment into legal infrastructure. Section 5 discusses the steady state equilibria when the government is able to commit to policy. Section 6 characterizes the steady state equilibria when commitment is not possible. Section 7 compares the results under commitment and discretion. Section 8 concludes.

2 The Model

The economy consists of a large number of individuals and firms. Individuals inelastically supply labor and capital to firms, and both purchase and appropriate output from them. The proportion of working hours spent in appropriation is $(1 - \sigma_t)$. They face a positive probability of being caught, which is increasing in the time spent in appropriation, and in the extent of protection of property rights, or the quality of the legal/judicial system. I capture the extent of property rights protection by a variable P_t that I call legal infrastructure. Legal infrastructure refers to the system of laws, as well as the physical and human capital invested in the judicial and legal enforcement systems. P_t is a continuous stock variable, to capture small improvements in laws and the quality of courts and of legal enforcement over time, as well as persistence of practices within the legal system.

Effective labor is equal to the product of the size of the labor force and the proportion of time each worker spends in productive activities. A constant-returns-to-scale technology is available to transform effective labor $\sigma_t L_t$ and capital K_t into output via the production function $F(K_t, \sigma_t L_t)$. The output can be used either for private consumption C_t, new capital K_{t+1}, or for new legal infrastructure, P_{t+1}, which serves to improve protection of output produced by firms. Feasibility requires that

$$C_t + K_{t+1} + P_{t+1} = F(K_t, \sigma_t L_t) + (1 - \delta_k)K_t + (1 - \delta_p)P_t \tag{2.1}$$

where δ_k is the depreciation rate on capital, and δ_p is the depreciation rate on legal infrastructure. Investment in legal infrastructure is financed by taxes on wages and capital income and by bonds. Let ϕ_{lt} denote the tax rate on labor income, and ϕ_{kt} the tax rate on capital income. Let B_{t+1} denote the debt issued in period t and $R_{b,t+1}B_{t+1}$ denote the debt service payment in the subsequent period.

Constant returns to scale implies that I may consider the following transformation of the aggregate production function

$$f(\frac{k_t}{\sigma_t}) \equiv \frac{1}{\sigma_t L_t} F(K_t, \sigma_t L_t) \tag{2.2}$$

where $k_t = \frac{K_t}{L_t}$. If I assume a constant, exogenous population growth rate of n, then the feasibility constraint may be written in per capita terms

$$c_t + (1+n)k_{t+1} + (1+n)p_{t+1} = \sigma_t f(\frac{k_t}{\sigma_t}) + (1 - \delta_k)k_t + (1 - \delta_p)p_t \tag{2.3}$$

where, as previously stated, the lower case refers to per capita variables.

Now I consider the behavior of households and firms, and then proceed to specify the optimization problem of policymakers in the Ramsey problem.

2.1 Households

My preference structure is standard. I assume that there are many identical consumers, each of whom maximizes the discounted expression for utility,

$$\sum_{t=0}^{\infty} \beta^t u(c_t) \tag{2.4}$$

In each period, while consumers supply labor and capital inelastically, they may choose both their level of consumption, c_t, and the proportion of their time they wish to devote to honest, productive activities. The remainder of their labor time they will spend in appropriating output from the firm. Their labor income consists of wage income, w_t, and appropriation income $\tau(1-\sigma_t)$, where σ_t is the proportion of their time allocated to productive activities, τ is the rate of return to appropriation. In each period, households face some probability that they will be caught by the government in the act of appropriation. If they are caught, all their labor income, including any appropriated output, will be forfeited to the firm. With probability $h(p_t, \sigma_t)$ they will retain all their income. Only "legal" income is taxed. The budget constraint for period t is thus given by

$$h(p_t, \sigma_t) \cdot ((1-\phi_{lt})w_t + \tau(1-\sigma_t)) + (1 + (1-\phi_{kt})(r_t - \delta_k))k_t + R_{b,t}b_t \\ \geq c_t + (1+n)k_{t+1} + (1+n)b_{t+1} \quad (2.5)$$

Here, I treat each household as a continuum of agents, so that each household receives the expected value of labor income from each agent. This avoids issues of risk which are tangential to the focus of this chapter. I make the following simplifying assumptions. First, the utility function is strictly concave and satisfies the Inada conditions. Second, the probability function $h(p, \sigma)$ satisfies the following:

Assumption 1. $\lim_{p\to 0} h(p, \sigma) = 1 \ \forall \sigma$ *and* $\lim_{p\to\infty} h(p, \sigma) = 0$ *for* $\sigma < 1$

Assumption 2. $\lim_{\sigma\to 1} h(p, \sigma) = 1 \ \forall p$ *and* $\lim_{\sigma\to 0} h(p, \sigma) = 0 \ \forall p$

Assumption 3. $h_\sigma \geq 0 \ \forall p, \sigma$ *and* $\lim_{\sigma_t \to 1} h_{\sigma,t} = 0$

Assumption 4. $h_p \leq 0$

Assumption 5. $\lim_{\sigma\to 0} h_{\sigma\sigma} > 0$

Assumptions 1 and 2 simply describe the limiting cases for the effects of legal infrastructure and labor productivity on the probability of retaining one's income. Assumption 3 captures decreasing returns to effort in the probability function. Assumption 4 simply implies that higher property rights will lower the probability of retaining income for a given level of effort. Finally, assumption 5 implies that there will be increasing returns to effort at low levels of σ. These conditions are sufficient to impose interior solutions on c_t and σ_t. I note for the reassurance of the reader that the set of functions satisfying the above assumptions is non-null. The household first-order conditions are then given by a static condition,

$$h_{\sigma,t}((1-\phi_{lt})w_t + \tau(1-\sigma_t)) = h(p_t,\sigma_t)\tau \tag{2.6}$$

the Euler equation for physical capital,

$$((1+n)u_{c,t} - \beta u_{c,t+1}(1+(1-\phi_{k,t+1})(r_{t+1}-\delta_k)))k_{t+1} = 0 \tag{2.7}$$

and the Euler equation for bonds

$$((1+n)u_{c,t} - \beta u_{c,t+1}R_{b,t+1})b_{t+1} = 0 \tag{2.8}$$

Equations (2.7) and (2.8) are conventional Euler equations with respect to capital and bonds respectively. In equation (2.6) the household is simply equating the marginal cost and benefit from an increase in the time spent productively, σ_t. $h_{\sigma,t}((1-\phi_{lt})w_t+\tau(1-\sigma_t))$ gives the marginal benefit of σ in the form of increased retained income. $h(p_t,\sigma_t)\tau$ gives the marginal cost, in the form of reduced income from appropriation.

2.2 Firms

Perfectly competitive markets imply that firms take rental rates of labor and capital, as well as the return on appropriation, as given. Since firms are not able to observe the appropriation of output by workers, the representative firm maximizes in each period the expected value of its profit

$$\max_{L_t} F(K_t,\bar{\sigma}_t L_t) - h(p_t,\sigma_t)(w_t L_t + \tau(1-\bar{\sigma}_t)L_t)) - r_t K_t \tag{2.9}$$

where $\bar{\sigma}_t$ is the firm's expectation of the proportion of time workers will devote to productive activities. In equilibrium this will be equal to σ_t.

The return to capital and labor thus equal their marginal products, namely

$$r_t = F_K(K_t,\sigma_t L_t) = f'(\frac{k_t}{\sigma_t}) \tag{2.10}$$

$$h(p_t,\sigma_t)(w_t+\tau(1-\sigma_t)) = F_L(K_t,\sigma_t L_t) = \sigma_t f(\frac{k_t}{\sigma_t}) - k_t f'(\frac{k_t}{\sigma_t}) \tag{2.11}$$

3 Competitive Equilibrium

As a first approach to the analysis of the equilibria of this model, I shall consider the case where the sequences of policies, $\{p_t, \phi_{l,t}, \phi_{k,t}\}_{t=0}^{\infty}$, are exogenously given. I assume that the government can borrow by issuing a one-period risk-free real bond. Per capita government debt evolves according to the law of motion

$$(1+n)b_{t+1} = R_{b,t}b_t + (1+n)p_{t+1} - (1-\delta_p)p_t - \phi_{lt}h(p_t,\sigma_t)w_t - \phi_{kt}(r_t-\delta_k)k_t \qquad (3.1)$$

where p_0, b_0, k_0 are given, as are $\phi_{k,0}$ and $R_{b,0}$. The economy will be fully characterized by the sequence of policies, the household budget constraint (2.5),the household first-order conditions (2.6-2.8), the rental rates (2.10), and the law of motion of government debt (3.1). The household budget constraint, the government budget constraint and the rental rates may be combined to obtain a resource constraint, which, following Chari et al, I will refer to as a feasibility constraint. Consider now the steady state equilibrium, given by c, σ, k, b and the policy variables p, ϕ_l, ϕ_k, R_b. This steady state must satisfy the feasibility constraint, the first-order condition for σ, and the Euler equation:

$$\sigma f\left(\frac{k}{\sigma}\right) = (n+\delta_k)k + (n+\delta_p)p + c \qquad (3.2)$$

$$h_\sigma\left((1-\phi_l)\frac{\sigma f\left(\frac{k}{\sigma}\right) - kf'\left(\frac{k}{\sigma}\right)}{h(p,\sigma)} + \phi_l\tau(1-\sigma)\right) = h(p,\sigma)\tau \qquad (3.3)$$

$$(1+n) = \beta\left(1 + (1-\phi_k)\left(f'\left(\frac{k}{\sigma}\right) - \delta_k\right)\right) \qquad (3.4)$$

The Euler equation for bonds implies that $b = 0$ or $R_b = (1+n)/\beta$. The government budget constraint then implies that the following constraint must hold:

$$\begin{aligned}(1+n)\left(\frac{\beta-1}{\beta}\right)b \quad &= \quad (n+\delta_p)p - \phi_l\sigma f\left(\frac{k}{\sigma}\right) + (\phi_l - \phi_k)kq1f'\left(\frac{k}{\sigma}\right) \\ &\quad + \phi_l h(p,\sigma)\tau(1-\sigma) + \phi_k\delta_k k \qquad (3.5)\end{aligned}$$

From the Euler equation for capital, equation (3.4), I find that k and σ are linearly related, or that $k = B\sigma$, where I define

$$B \equiv f'^{-1}\left(\frac{1+n-\beta(1-(1-\phi_k)\delta_k)}{\beta(1-\phi_k)}\right) \tag{3.6}$$

As a result, I may eliminate c and k from the steady state equations (3.2-3.4). We are left with one key equation in σ and the policy variables:

$$h_\sigma\left((1-\phi_l)\frac{\sigma f\left(B\right)-B\sigma f'\left(B\right)}{h(p,\sigma)}+\phi_l\tau(1-\sigma)\right)=(h(p,\sigma))^2\tau \tag{3.7}$$

One can show under optimal policy that $\phi_k = 0$, and that $R_b = (1+n)/\beta$. In this case, using equation (3.5), equation (3.7) may be rewritten as

$$h_\sigma\left(\sigma f(B)-B\sigma f'(B)-(n+\delta_p)p-\frac{(1+n)(1-\beta)}{\beta}b\right)=h(p,\sigma)\tau \tag{3.8}$$

This equation completely determines the steady state of the economy in the case of exogenous b, p where ϕ_l is chosen to satisfy the government budget constraint. We can now employ the assumptions made about the probability function $h(p, \sigma)$ to more fully characterize the steady state(s). These assumptions imply the following limiting behavior of the left-hand side of equation (3.8)

$$\lim_{\sigma\to 0} h_\sigma(\sigma f(B)-B\sigma f'(B)-(n+\delta_p)p-(R_b-(1+n))b)\leq 0 \tag{3.9}$$

$$\lim_{\sigma\to 1} h_\sigma(\sigma f(B)-B\sigma f'(B)-(n+\delta_p)p-(R_b-(1+n))b)=0 \tag{3.10}$$

However, $w = \sigma f(B) - B\sigma f'(B)$, and under optimal policy $\phi_k = 0$ so that investment in p is wholly funded through taxes on wages. Since I impose that $\phi_l \leq 1$ then $\sigma f(B) - B\sigma f'(B) - h(p,\sigma)\tau(1-\sigma) \geq (n+\delta_p)p + \frac{(1+n)(1-\beta)}{\beta}$. It may be shown that while equation (3.8) generally admits at most two solutions, the lower solution implies that $\phi_l > 1$, formalized in proposition 1 for which the proof is given in the appendix.

Proposition 1. *There exists at most a single steady state equilibrium associated with each value of p and τ, such that $\phi_l \in [0, 1]$.*

As $h(p, \sigma)$ is increasing in σ, the right-hand side of the equation will be increasing in σ for all values of σ. This would imply the possibility of two steady states of σ for a given

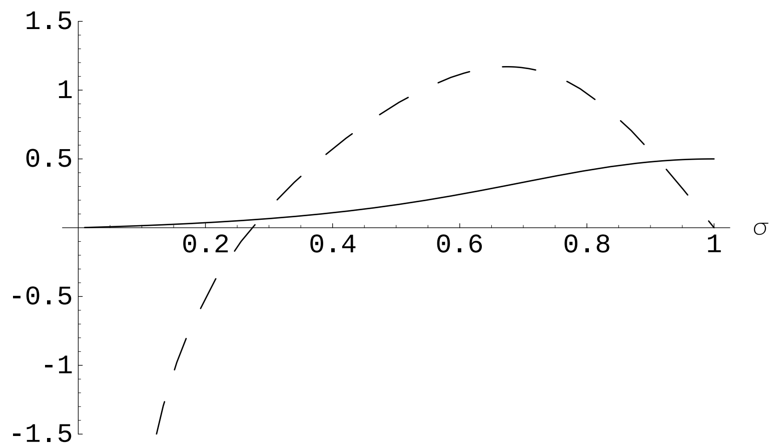


Figure 1: Possibility of multiple steady states for given value of p: Plot of values of left-hand/right-hand sides of equation(3.8)for $p = 2$. The dashed line indicates the value of the left-hand side as σ is varied, while the solid line indicates the value of the right-hand side.

value of p. In the case where $\lim_{\sigma\to 0} h_\sigma = 0$ the lower steady state would occur at $\sigma = 0$. Otherwise, as shown in proposition 1, there will generally be two steady states such that $\sigma > 0$ for a given value of p. The plot in figure (1) is generated for the case $b = 0$, assuming the functional forms

$$\begin{aligned} h(p,\sigma) &= \frac{1}{1 + \frac{\theta}{\sigma} p^\gamma (1-\sigma)^\gamma} \\ f\left(\frac{k}{\sigma}\right) &= \left(\frac{k}{\sigma}\right)^\alpha \end{aligned}$$

and the parameter values presented in table(1). We can observe in figure (1) the existence of two solutions to equation (3.8), but as argued above, only the upper steady state will satisfy the restriction that $\phi_l \leq 1$.

In order to satisfy assumptions 1-5 regarding its behavior, $h(p,\sigma)$ must have the property that h_σ is initially increasing in σ for low values of σ, but that for high values of σ, h_σ is decreasing. In particular, there will exist a σ^* for which h_σ is maximized. The value σ^* will be increasing in p. The marginal effect of σ on retained income will thus attain a peak and then start to decline as σ approaches unity. As property rights improve, or p increases, the peak will be attained for a higher value of σ. (Figure 2) The behavior of the expected

Table 1: Baseline values of key parameters.

α	1/3
n	0.03
β	0.98
δ_k	0.1
δ_p	0.1
θ	1
γ	2
τ	0.5

return to appropriation, $h(p, \sigma)\tau$ is much simpler, as it is simply monotonically increasing in σ and decreasing in p. From the previous arguments with regard to the behavior of the marginal effect of σ on retained labor income, and the return to appropriation, I may conclude that the solution for σ will be increasing in p. From the household's perspective, taking wages as given, an increase in property rights will tend to reduce the relative return in appropriation, resulting in a lower fraction of time spent in appropriation. An increase in τ will increase the upper solution for σ, the only feasible solution. This can be seen by implicitly differentiating σ with respect to τ in equation (3.8)to obtain

$$\frac{d\sigma}{d\tau} = \frac{h(p, \sigma)}{h_{\sigma\sigma}(\sigma f(B) - Bf'(B)\sigma - (n + \delta_p)p - (R_b - (1 + n))b) + h_\sigma(f(B) - Bf'(B) - \tau)}$$

For high values of σ, $h_{\sigma\sigma} < 0$, which can result in $d\sigma/d\tau < 0$.

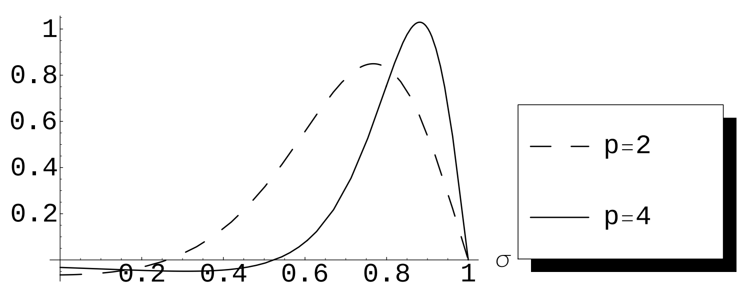


Figure 2: Marginal effect of increase in σ on retained labor income

Consumption in the steady state can be directly determined from the resource constraint, given p and the level of time spent in appropriation σ

$$c = \sigma f(B) - (n + \delta_k)B\sigma - (n + \delta_p)p$$

If social welfare is measured by the lifetime utility of the representative household in the steady-state, it will be given by

$$U = \frac{1}{1-\beta}u(c)$$

If the utility function is monotonically increasing in consumption, then an analysis of the variation of household welfare in the steady state will be equivalent to the variation of steady-state consumption. The effect of an improvement of legal infrastructure on steady-state consumption or

$$\frac{dc}{dp} = (f(B) - (n + \delta_k)B)\frac{d\sigma}{dp} - (n + \delta_p) \tag{3.11}$$

will depend on how σ responds to an increase in p. It will be highly responsive to increases in legal infrastructure when p is low, but not for relatively high values of p. This is due to the fact σ will rapidly approach the upper limit of $\sigma = 1$, where $h_\sigma = 0$, regardless of the value of p. As a result, in the steady state, consumption will be maximized for a relatively low level of p.

As the return to appropriation τ increases, households will choose to allocate more time to appropriation, reducing σ for a given level of p. Now address the responsiveness of p to σ. Differentiate (3.8) with respect to p to obtain:

$$\frac{d\sigma}{dp} = \frac{h_p\tau + (n+\delta_p)h_\sigma - h_{\sigma p}\left(\sigma f(B) - \sigma B f'(B) - (n+\delta_p)p - \frac{(1+n)(1-\beta)}{\beta}b\right)}{h_\sigma(f(B) - Bf'(B)) + h_{\sigma\sigma}\left(\sigma f(B) - Bf'(B)\sigma - (n+\delta_p)p - \frac{(1+n)(1-\beta)}{\beta}b\right)}$$

The denominator of the expression will be positive for all values of σ except when σ is sufficiently close to 0 or 1, so that

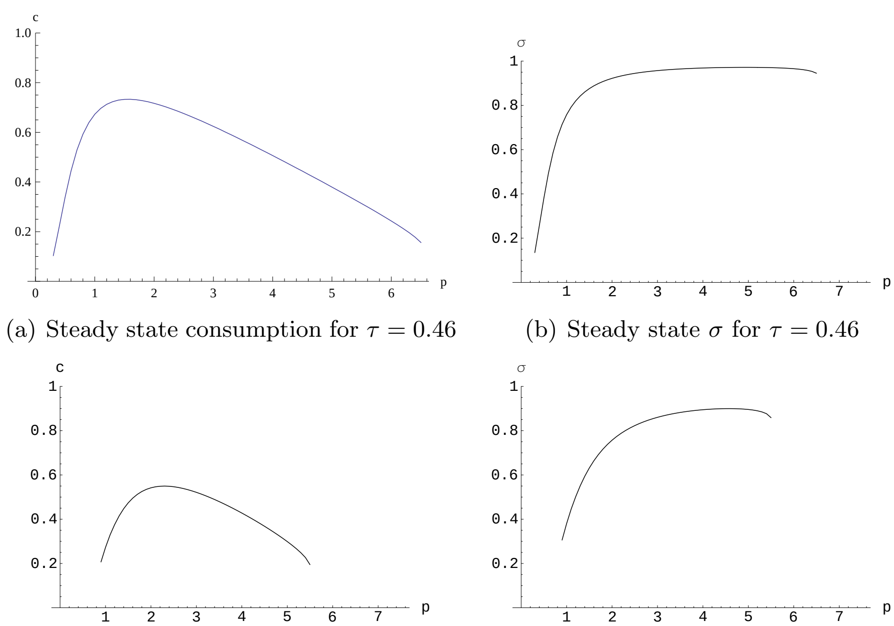
$$h_{\sigma\sigma}\left(\sigma f(B) - Bf'(B)\sigma - (n+\delta_p)p - \frac{(1+n)(1-\beta)}{\beta}b\right) < 0$$

As a result, except for the case where σ is either sufficiently small or large, $d\sigma/dp$ declines as τ increases. In the latter cases, an increase in the return to appropriation will increase $d\sigma/dp$. The increased sensitivity of σ to p at lower levels of legal infrastructure in the case of high return to appropriation will imply a higher level of p for which steady-state consumption is maximized, which one can observe in figure (3). Let us define p^* as the value of p for which c will be maximized for a given value of τ. Similarly, define $c^* = \max_p c(p,\tau)$.

$$\frac{dc^*}{d\tau} = c_p \frac{dp^*}{d\tau} + c_\tau$$

At c^*, $c_p = 0$, so that $dc^*/d\tau = dc(p,\tau)/d\tau$. The function $c(p,\tau)$ will be declining in the second argument, since σ declines as τ increases, causing income will be reduced. As a result, c^* will be decreasing in τ.

While the preceding analysis is certainly intuitive, and allows us to characterize the effect of policy on the steady state, it does not allow us to determine optimal policy, which is set in a dynamic framework. To find optimal steady state levels of p, I must solve the Ramsey problem with endogenous government spending.



(a) Steady state consumption for $\tau = 0.46$ (b) Steady state σ for $\tau = 0.46$

(c) Steady state consumption for $\tau = 1.46$ (d) Steady state σ for $\tau = 1.46$

Figure 3: Effect of τ on steady state values of c,σ for $b = 0$

4 The Ramsey Problem

Consider now the problem faced by the government in setting optimal policy. For simplicity, I assume that there is an institution through which the government can bind itself to a particular sequence of policies. Since policies need to account for consumer and firm responses to policies, allocations and rental rates on labor and capital will be given by sequences of functions that associate allocations and prices with policies.

Definition 1. A Ramsey equilibrium is a policy $\{p_{t+1}, b_{t+1}, \phi_{l,t}, \phi_{k,t}, R_{b,t}\}_{t=t_0}^{\infty}$, an allocation rule $\{c(\cdot), k(\cdot), \sigma(\cdot)\}$ and price rules $w(\cdot)$, and $r(\cdot)$ such that

i. The policy maximizes the discounted lifetime utility of the representative household (2.4),subject to the government budget constraint (3.1), where allocations and rental rates are given by the rules defined above.

ii. For every policy, the corresponding allocation and rental rates will maximize (2.4), subject to the household budget constraint (2.5).

iii. For every policy, the rental rates satisfy the profit-maximizing conditions given by (2.10) and (2.11).

Proposition 2. *The allocations of consumption, physical capital and time spent in appropriation in a Ramsey equilibrium solve the Ramsey allocation problem*

$$\max_{c_t, \sigma_t, k_{t+1}, p_{t+1}} \sum_t \beta^t u(c_t) \tag{4.1}$$

subject to the following constraints:

i. Resource constraint

$$c_t + (1+n)k_{t+1} + (1+n)p_{t+1} = \sigma_t f(\frac{k_t}{\sigma_t}) + (1-\delta_k)k_t + (1-\delta_p)p_t \tag{4.2}$$

ii. Implementation constraint

$$\sum_t \beta^t u_{c,t}(c_t - \frac{(h(p_t, \sigma_t))^2}{h_{\sigma,t}}\tau) = u_{c,0}\left(R_{b,0}b_0 + (1 + (1-\phi_{k0})(r_0 - \delta_k))k_0\right) \tag{4.3}$$

where $R_{b,0}$ and ϕ_{k0} are taken as given. [1]

Proof. See Appendix □

4.1 A Recursive Formulation of the Ramsey Problem

The formulation of the Ramsey problem, as presented so far, has the property that the policy problem is not recursive, and in particular, that the first-order conditions in the initial period differ from those in subsequent periods. Let us now consider an alternative approach, advocated by Benigno and Woodford (2006), which has the technical advantage of making the optimal policy stationary. To proceed I slightly alter the set of initial conditions. That is, rather than taking the initial tax rate on capital as given, I instead assume that there is a pre-existing commitment with regard to the value of initial household assets.

Definition 2. The value of consumption net of wage income over periods $T \geq t$ is given by

$$W_t \equiv \sum_{T=t}^{\infty} \beta^{T-t} u_{c,T} \left(c_T - \frac{(h(p_T, \sigma_T))^2}{h_{\sigma,T}} \tau \right) \tag{4.4}$$

$\overline{W}_t$ is defined to be the pre-existing commitment regarding the value of household assets in period t.

Proposition 3. *Given $\left\{k_{t_0}, p_{t_0}, b_{t_0}, \overline{W}_{t_0}\right\}$, consider the sequential optimization problem in which $\{x_t\} = \{c_t, \sigma_t, k_{t+1}, p_{t+1}, W_{t+1}\}$ are chosen for each period $t \geq t_0$ to maximize the function $J(x_t, \overline{W}_{t+1})$, subject to the feasibility constraint (4.2) and*

$$\overline{W}_t = u_{c,t} \left(c_t - \frac{(h(p_t, \sigma_t))^2}{h_{\sigma,t}} \tau \right) + \beta \overline{W}_{t+1} \tag{4.5}$$

given the values for $\left\{k_t, p_t, \overline{W}_t\right\}$ determined in the previous period. The function $J(\cdot)$ is defined by:

$$J(x_t, \overline{W}_{t+1}) \equiv u(c_t) + \beta V(k_{t+1}, p_{t+1}, \overline{W}_{t+1}) \tag{4.6}$$

[1] This assumption is to avoid the typical result of the Ramsey problem that the government will have the incentive to set the initial tax rate on capital as large as possible.

where $V(k_{t+1}, p_{t+1}, \overline{W}_{t+1})$ denotes the maximum attainable value of

$$U_t = \sum_{T=t}^{\infty} \beta^{T-t} u(c_T) \tag{4.7}$$

subject to the feasibility and implementation constraints mentioned above. The allocation chosen in this manner will maximize the lifetime utility of households subject to the feasibility constraint (4.2) and the implementation constraint $W_{t_0} = \overline{W}_{t_0}$.

Proof. See Appendix. □

A similar approach may be used to show that the policy problem as originally defined (with a constraint on the capital tax rate, rather than on the value of initial assets) is also equivalent to a two-stage problem. In the first stage, $(c_{t_0}, \sigma_{t_0}, k_{t_0+1}, p_{t_0+1})$ and $\overline{W}_{t_0+1}$ are chosen, given the initial capital tax rate and the initial levels of capital and bonds. For $t \geq t_0$, policy is chosen to maximize U_{t_0+1} subject to the feasibility constraint and $W_{t_0+1} = \overline{W}_{t_0+1}$. Proposition (3) implies that the second stage of this problem is characterized by stationary optimal policies, for which I can define a steady state equilibrium.

5 Steady State under Optimal Policy with Commitment

Finding the steady state will correspond to finding, for an initial level of debt b, the initial commitment $\overline{W}$ and initial levels of physical capital and legal infrastructure (k, p) such that the optimal sequence of allocations is a constant allocation and set of policies $\{c, \sigma, k, p, b, \phi_k, \phi_l, \overline{W}\}$, where $b, k, \overline{W}$ are identical to the initial conditions.

Rewriting the optimization problem as a Lagrangian, I find

$$\mathscr{L}_{t_0} = \sum_{T=t_0}^{\infty} \Bigg(u(c_T) + \psi_T \left(\sigma_T f\left(\frac{k_T}{\sigma_T}\right) + (1-\delta_k)k_T + (1-\delta_p)p_T - (1+n)k_{T+1} - (1+n)p_{T+1} - c_T \right) - \lambda u_{c,T} \left(c_T - \frac{(h(p_T, \sigma_T))^2}{h_{\sigma,T}} \tau \right) \Bigg) + \lambda \overline{W}_{t_0} \tag{5.1}$$

The first-order conditions with respect to c_t, σ_t are then given by

$$(1-\lambda)u_{c,t} - \tilde{\psi}_t - \lambda u_{cc,t}\left(c_t - \frac{(h(p_t,\sigma_t))^2}{h_{\sigma,t}}\tau\right) = 0 \tag{5.2}$$

$$\tilde{\psi}_t\left(f\left(\frac{k_t}{\sigma_t}\right) - \frac{k_t}{\sigma_t}f'\left(\frac{k_t}{\sigma_t}\right)\right) + \tau\lambda u_{c,t}\left(2h(p_t,\sigma_t) - \left(\frac{h(p_t,\sigma_t)}{h_{\sigma,t}}\right)^2 h_{\sigma\sigma,t}\right) = 0 \tag{5.3}$$

These are accompanied by two Euler-like equations, for k_{t+1} and p_{t+1} respectively:

$$-(1+n)\tilde{\psi}_t + \beta\tilde{\psi}_{t+1}\left(1-\delta_k + f'\left(\frac{k_{t+1}}{\sigma_{t+1}}\right)\right) = 0 \tag{5.4}$$

$$\beta\tau\lambda u_{c,t+1}\left(2h(p_{t+1},\sigma_{t+1})\frac{h_{p,t+1}}{h_{\sigma,t+1}} - \left(\frac{h(p_{t+1},\sigma_{t+1})}{h_{\sigma,t+1}}\right)^2 h_{\sigma p,t+1}\right) - (1+n)\tilde{\psi}_t + \beta\tilde{\psi}_{t+1}(1-\delta_p) = 0 \tag{5.5}$$

In a steady-state solution, these conditions reduce to the following system of equations:

$$(1-\lambda)u_c - \psi - \lambda u_{cc}\left(c - \frac{(h(p,\sigma))^2}{h_\sigma}\tau\right) = 0 \tag{5.6}$$

$$\tilde{\psi}\left(f\left(\frac{k}{\sigma}\right) - \frac{k}{\sigma}f'\left(\frac{k}{\sigma}\right)\right) + \lambda\tau u_c\left(2h(p,\sigma) - \left(\frac{h(p,\sigma)}{h_\sigma}\right)^2 h_{\sigma\sigma}\right) = 0 \tag{5.7}$$

and

$$1+n = \beta\left(1-\delta_k + f'\left(\frac{k}{\sigma}\right)\right) \tag{5.8}$$

$$\tilde{\psi}(1+n-\beta(1-\delta_p)) = \beta\tau\lambda u_c\left(2h(p,\sigma)\frac{h_p}{h_\sigma} - \left(\frac{(h(p,\sigma))}{h_\sigma}\right)^2 h_{\sigma p}\right) \tag{5.9}$$

Equation (5.8) and the household first order condition for capital (3.4) imply together that the tax rate on capital will be equal to zero in the steady state. Equations (5.6)-(5.9) together with the steady versions of the feasibility and implementation constraint and a constraint on $\overline{W}$ to make the commitment feasible

$$\sigma f\left(\frac{k}{\sigma}\right) = c + (n+\delta_k)k + (n+\delta_p)p \tag{5.10}$$

$$(1-\beta)\overline{W} = u_c\left(c - \frac{(h(p,\sigma))^2}{h_\sigma}\tau\right) \tag{5.11}$$

$$\overline{W} = u_c\left(R_b b + \frac{1+n}{\beta}k\right) \tag{5.12}$$

form a system of equations which determine the steady state. The steady state tax rate on wages is determined by combining the household first order condition for σ with the market wage rate to obtain

$$(1-\phi_l)\left(\sigma f\left(\frac{k}{\sigma}\right) - kf'\left(\frac{k}{\sigma}\right)\right) + \phi_l h(p,\sigma)\tau(1-\sigma) = \frac{(h(p,\sigma))^2}{h_\sigma}\tau \tag{5.13}$$

The following budget constraint gives the relationship between ϕ_l and b

$$(1+n-R_b)b = (n+\delta_p)p - \phi_l\left(\sigma f\left(\frac{k}{\sigma}\right) - kf'\left(\frac{k}{\sigma}\right)\right) + \phi_l h(p,\sigma)\tau(1-\sigma) \tag{5.14}$$

The law of motion for government debt, equation (3.1), implies that b will determined by the history of the economy prior to convergence to the steady state. The steady state allocation, c, σ, k, p will depend on b, and thus is history-dependent. In order to obtain numerical results for the steady state under full commitment, I consider the case of log utility. In my initial analysis, I assign to the parameters $\alpha, n, \beta, \delta_k, \delta_p, \theta, \gamma$ the values listed in Table 1.

In section 3, I considered the effect of p, taken at that point to be exogenous, on steady-state consumption and time spent in appropriation, σ. To more clearly compare the results under exogenous policy to steady state consumption in the Ramsey equilibrium, I define $c(p)$ as the steady state consumption as a function of legal infrastructure p, where $\phi_k = 0$ and $b = 0$. Furthermore, let $c^* = \max_p c(p)$, and let c_r denote the steady state value of consumption in the Ramsey equilibrium. Suppose that $u(c_r) > u(c^*)$. By the definition, the steady state allocation of the Ramsey equilibrium satisfies the household FOCs given policies p, ϕ_l, ϕ_k. Therefore, c^* violates its own definition. We can therefore state that $u(c^*) \geq u(c_r)$. Furthermore, c^*, and corresponding σ^*, p^*, k^* must satisfy the steady-state versions of the implementation and feasibility constraint, since these are derived from the household first order conditions. As a result, I can use the solution to the following maximization problem to find c^*

$$\max_{c,\sigma,p} u(c) + \psi\left(\sigma f\left(\frac{k}{\sigma}\right) - (n+\delta_k)k - (n+\delta_p)p - c\right) \\ - \lambda u_c\left(c - \tau\frac{(h(p,\sigma))^2}{h_\sigma}\right) + \lambda\overline{W} \quad (5.15)$$

given the steady state Euler equation, which implies $k = B\sigma$. The first order conditions of this problem with respect to c and σ are identical to those obtained for the steady state of the Ramsey equilibrium. The FOC with respect to p is given by

$$\psi(n+\delta_p) = \lambda u_c \tau\left(2h(p,\sigma)\frac{h_p}{h_\sigma} - \left(\frac{h(p,\sigma)}{h_\sigma}\right)^2 h_{\sigma p}\right) \quad (5.16)$$

which is identical in form to equation (5.9), and is identical to equation (5.9) for $\beta = 1$. Thus, as β approaches 1, c_r approaches c^*.

We saw in our analysis of the steady state with exogenous policy, that while there were in general two potential solutions to the first order condition for σ, the lower value of σ implied a value of ϕ_l which was not feasible, i.e. $\phi_l \geq 1$. In my numerical analysis, I found that for a given value of τ there was a single value of p that maximized steady-state consumption. One could use these results to argue for the existence of a unique steady state under policy with commitment. Alternatively, I may reduce the equations (5.6 - 5.9) to a two-dimensional system in σ and p, given by

$$(f(B) - Bf'(B))\Bigg((1-\lambda(p,\sigma))u_c(p,\sigma) - \lambda(p,\sigma)u_{cc}(p,\sigma)\left(c(p,\sigma) - \tau\frac{(h(p,\sigma))^2}{h_\sigma}\right)\Bigg) \\ + \lambda(p,\sigma)\tau u_c(p,\sigma)\left(2h(p,\sigma) - \left(\frac{h(p,\sigma)}{h_\sigma}\right)^2 h_{\sigma\sigma}\right) = 0 \quad (5.17a)$$

and

$$(1-\beta)\frac{1+n}{\beta}(b + B\sigma) = c(p,\sigma) - \tau\frac{(h(p,\sigma))^2}{h_\sigma} \quad (5.17b)$$

First, consider the behavior of equation (5.17b), which is simply the implementation constraint in the steady state. On the right-hand side, consumption will be linear in both σ and p. Income from labor, given by $\tau(h(p,\sigma))^2/h_\sigma$, is increasing in σ, decreasing in p and has the following limiting behavior

$$\lim_{\sigma\to 0}\frac{(h(p,\sigma))^2}{h_\sigma} = 0 \tag{5.18}$$

$$\lim_{\sigma\to 1}\frac{(h(p,\sigma))^2}{h_\sigma} = \infty \tag{5.19}$$

As a result, there is a unique value of σ for which $c - \tau(h(p,\sigma))^2/h_\sigma$ is maximized. Since for σ close to both 0 and 1, consumption net of labor income will be negative. There will thus be at most 2 solutions to equation (5.17b) with the case of no solutions occurring for p sufficiently low or high. For the moment, let us refer to these two solutions as $\sigma_l(p,b)$ and $\sigma_h(p,b)$. An increase in p, i.e. in the capital invested in legal infrastructure, will lower steady steady-state consumption for every value of σ, and will similarly lower labor income. The value of the right-hand side of equation (5.17b) will decline for every value of σ. Consequently, both σ_l and σ_h will be increasing in p.

The behavior of equation (5.17a) is more complicated, and the reasoning is considerably less straightforward. However, numerical analysis shows that for a range of parameter values, and in particular, for a range of values of τ, there are at most 2 solutions in σ to the equation, where the upper solution is decreasing in p and the lower solution is increasing in p. In particular, when the capital invested in the legal infrastructure approaches 0, the lower solution will approach 0 and the upper solution will approach 1. This implies that there will be at most two solutions to the set of equations, which one can see in figure 4. We refer to these two solutions as $\{\sigma_{lr}, p_{lr}\}$ and $\{\sigma_{hr}, p_{hr}\}$. In general, however, the solution $\{\sigma_{lr}, p_{lr}\}$ will in general be infeasible, requiring that $\phi_l > 1$.

In section (3), I showed that c^* was decreasing in τ. The dependence of the steady state on τ is entirely driven by its influence on the household's choice of σ_t. In particular, an increase in τ increases the return on appropriation relative to wage income. The reduction in σ would imply a decline in steady state capital stock and consumption. The government could in theory increase the tax rate and improve the legal infrastructure in order to restore σ to its original level. However, an increase in the tax rate would also increase the relative return from appropriation, reducing the net effect of the increase in the tax rate and in p

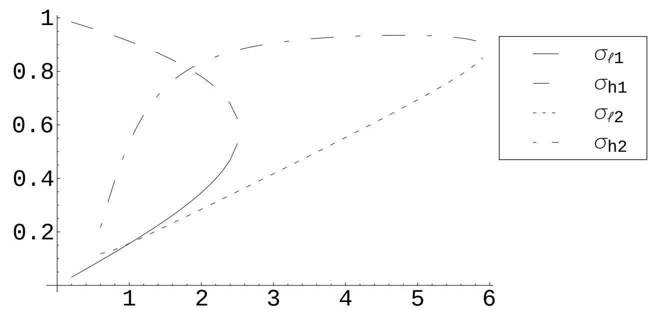


Figure 4: Possibility of multiple steady states under commitment: Solutions to the equations (5.17a-5.17b) for $b = 0$ and $\tau = 1$. σ_{l1} and σ_{h1} are the solutions to (5.17a), while σ_{l2} and σ_{h2} indicate the solutions to (5.17b).

on σ. In addition, maintaining a higher level of p would reduce steady state consumption. An increase in legal infrastructure will thus be optimal but σ will still decline.

6 Discretionary Policy

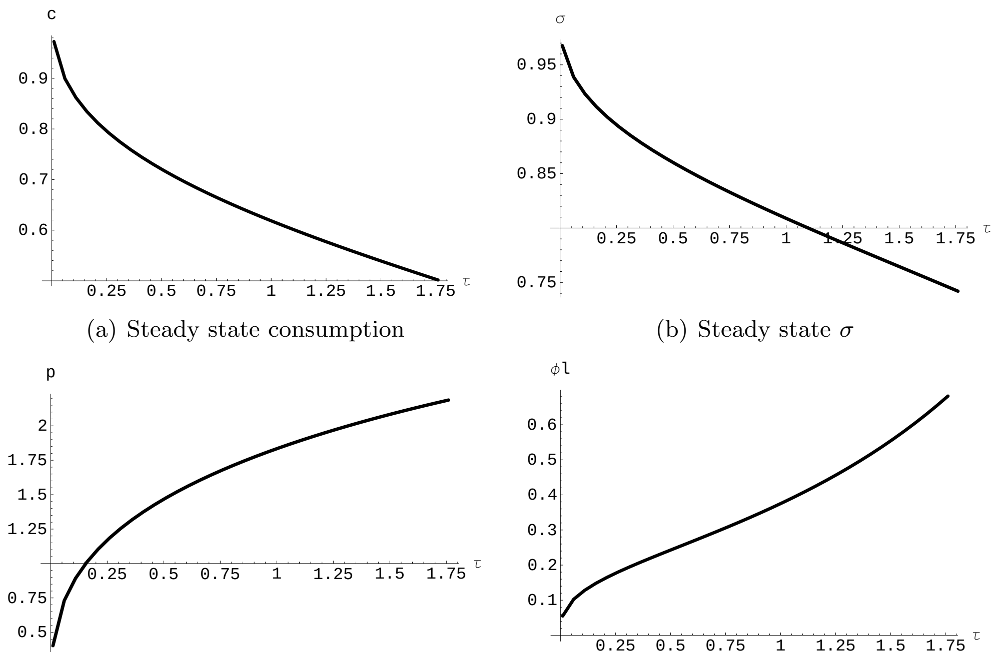
6.1 Discretion as the first period of Ramsey problem

Now consider the case where the government is unable to commit to a sequence of policies for all time. Instead, they are only able to commit one period in advance. Specifically, they choose the capital tax rate and the level of investment in the legal infrastructure for the subsequent period, and then set current labor tax rates in order to satisfy the government budget constraint. In this initial approach to the optimal policy problem under discretion, they assume that they will be able to commit to optimal policy in subsequent periods.

Rather than using an implementation constraint such as equation (4.3) based on initial endowments in capital and bonds, the government uses the following constraint in each period, to assure that the allocation found as a solution to the discretionary policy problem is in fact a solution to the household's optimization problem:

$$\sum_{T=t}^{\infty} \beta^{T-t} u_{c,T} \left(c_T - \frac{(h(p_T, \sigma_T))^2}{h_{\sigma,T}} \tau \right) = u_{c,t}(R_{b,t} b_t + (1 + (1 - \phi_{k,t})(r_t - \delta_k))k_t) \tag{6.1}$$

The derivation of this constraint is isomorphic to that for equation (4.3), with the exception



(a) Steady state consumption

(b) Steady state σ

(c) Steady state legal infrastructure

(d) Steady state tax rate

Figure 5: Effect of τ on steady state of Ramsey equilibrium

that rather than summing over all time, I am simply considering the household's first-order conditions and budget constraint for $T \geq t$. The constraint simply represents the fact that present and future consumption in excess of period labor income must be funded by income from assets with which the representative household begins period t. The Lagrangian for the discretionary policy problem may thus be written

$$\begin{aligned}\mathscr{L}_t = \sum_{T=t}^{\infty} \beta^{T-t} \Bigg(u(c_T) + \tilde{\psi}_T \Bigg(\sigma_T f\left(\frac{k_T}{\sigma_T}\right) + (1-\delta_k)k_T + (1-\delta_p)p_T \\ - (1+n)(k_{T+1} + p_{T+1}) - c_T \Bigg) - \lambda u_{c,T} \left(c_T - \frac{(h(p_T, \sigma_T))^2}{h_{\sigma,T}} \tau \right) \Bigg) \\ + \lambda u_{c,t} \left(R_{b,t} b_t + \left(1 + (1-\phi_{k,t}) \left(f'\left(\frac{k_t}{\sigma_t}\right) - \delta_k \right) \right) k_t \right) \quad (6.2)\end{aligned}$$

The first order condition with respect to c_t takes the form

$$u_{c,t} - \tilde{\psi}_t - \lambda \left(u_{c,t} + u_{cc,t} \left(c_t - \frac{(h(p_t, \sigma_t))^2}{h_{\sigma,t}} \tau \right) \right) \\ + \lambda u_{cc,t} \left(R_{b,t} b_t + \left(1 + (1 - \phi_{k,t}) \left(f' \left(\frac{k_t}{\sigma_t} \right) - \delta_k \right) \right) k_t \right) = 0 \quad (6.3)$$

The first-order condition with respect to σ_t can similarly be written as

$$\tilde{\psi}_t \left(f \left(\frac{k_t}{\sigma_t} \right) - \frac{k_t}{\sigma_t} f' \left(\frac{k_t}{\sigma_t} \right) \right) + \lambda u_{c,t} \tau \left(2h(p_t, \sigma_t) - \left(\frac{h(p_t, \sigma_t)}{h_{\sigma,t}} \right)^2 h_{\sigma\sigma,t} \right) \\ - \lambda u_{c,t} (1 - \phi_{k,t}) \left(\frac{k_t}{\sigma_t} \right)^2 f'' \left(\frac{k_t}{\sigma_t} \right) = 0 \quad (6.4)$$

Finally, I obtain two Euler-like equations, the first order conditions with respect to k_{t+1} and p_{t+1}

$$-(1+n)\tilde{\psi}_t + \beta \tilde{\psi}_{t+1} \left(1 - \delta_k + f' \left(\frac{k_{t+1}}{\sigma_{t+1}} \right) \right) = 0 \quad (6.5)$$

$$\beta \lambda \tau u_{c,t+1} \left(2h(p_{t+1}, \sigma_{t+1}) \frac{h_{p,t+1}}{h_{\sigma,t+1}} - \left(\frac{h(p_{t+1}, \sigma_{t+1})}{h_{\sigma,t+1}} \right)^2 h_{\sigma p,t+1} \right) \\ -(1+n)\tilde{\psi}_t + \beta \tilde{\psi}_{t+1} (1 - \delta_p) = 0 \quad (6.6)$$

By comparing the above equation with the household Euler condition, one may find once again that the steady capital tax rate will be equal to zero. The latter conditions may be combined with the resource constraint and equation (6.1) to obtain the following system of equations characterizing the steady state under discretionary policy

$$\psi = (1 - \lambda) u_c - \lambda u_{cc} \left(c - \frac{(h(p, \sigma))^2}{h_\sigma} \tau \right) \\ + \lambda u_{cc} \left(R_b b + \left(1 + f' \left(\frac{k}{\sigma} \right) - \delta_k \right) k \right) \quad (6.7a)$$

$$\psi\left(f\left(\frac{k}{\sigma}\right)-\frac{k}{\sigma}f'\left(\frac{k}{\sigma}\right)\right)+\lambda\tau u_c\left(2h(p,\sigma)-\left(\frac{h(p,\sigma)}{h_\sigma}\right)^2 h_{\sigma\sigma}\right)$$
$$-\lambda u_c\left(\frac{k}{\sigma}\right)^2 f''\left(\frac{k}{\sigma}\right)=0 \quad (6.7b)$$

$$\psi(1+n-\beta(1-\delta_p))=\beta\lambda\tau u_c\left(2h(p,\sigma)\frac{h_p}{h_\sigma}-\left(\frac{h(p,\sigma)}{h_\sigma}\right)^2 h_{\sigma p}\right) \quad (6.7c)$$

$$(1+n)=\beta\left(1-\delta_k+f'\left(\frac{k}{\sigma}\right)\right) \quad (6.7d)$$

$$c+(n+\delta)k+(n+\delta_p)p=\sigma f\left(\frac{k}{\sigma}\right) \quad (6.7e)$$

$$\frac{1}{1-\beta}\left(c-\frac{(h(p,\sigma))^2}{h_\sigma}\tau\right)=R_b b+\left(1-\delta_k+f'\left(\frac{k}{\sigma}\right)\right)k \quad (6.7f)$$

As in the case of commitment, I may reduce the equations characterizing the steady state to a two-dimensional system:

$$(f(B)-Bf'(B))\Bigg((1-\lambda(b,p,\sigma))u_c(p,\sigma)-\lambda(b,p,\sigma)u_{cc}(p,\sigma)\Bigg(c(p,\sigma)-\tau\frac{(h(p,\sigma))^2}{h_\sigma}$$
$$-\frac{1+n}{\beta}(b+B\sigma)\Bigg)\Bigg)+\lambda(p,\sigma)\tau u_c(p,\sigma)\left(2h(p,\sigma)-\left(\frac{h(p,\sigma)}{h_\sigma}\right)^2 h_{\sigma\sigma}\right)$$
$$-\lambda u_c(p,\sigma)B^2 f''(B)=0 \quad (6.8a)$$

and

$$(1-\beta)\frac{1+n}{\beta}(b+B\sigma)=c(p,\sigma)-\tau\frac{(h(p,\sigma))^2}{h_\sigma} \quad (6.8b)$$

The behavior of the two solutions to (6.8b) is identical to that of (5.17b). The key difference in the Ramsey problem first-order conditions under discretion and commitment is that in the case of discretion policymakers consider the effect of policy on the value of assets in every period. In determining the sequence of allocations $\{c_T,\sigma_T,k_{T+1},p_{T+1}\}_{T=t}^{\infty}$, the policymakers must balance dual objectives of maximizing lifetime utility and maximizing the current value of assets in order to relax the implementation constraint. The second objective causes the introduction of additional terms in the FOCs for c_t and $\sigma)_t$. To see the intuition behind this, I observe that an increase in c_t, while increasing current-period

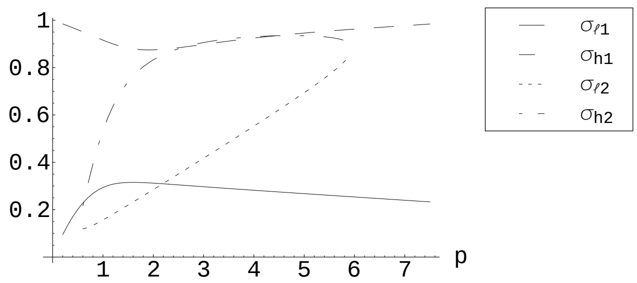


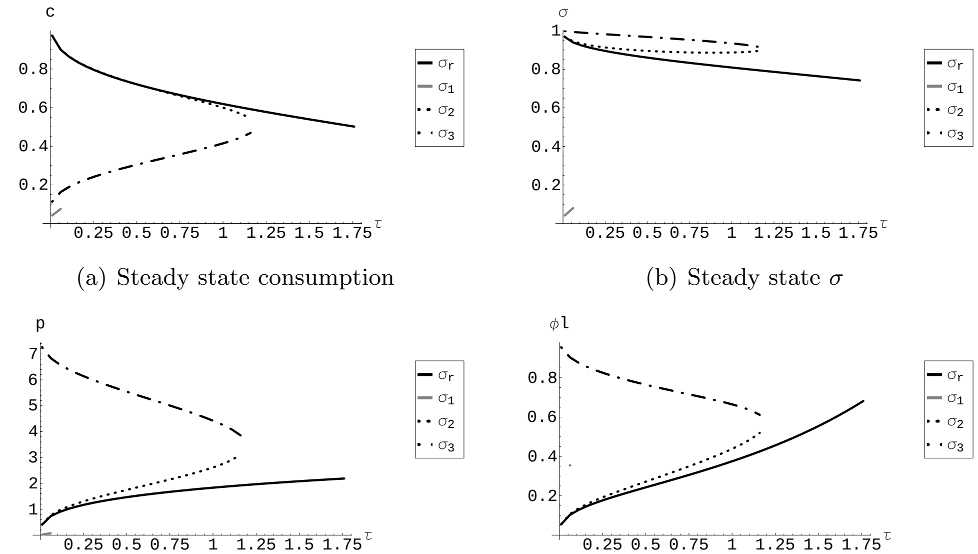
Figure 6: Possibility of multiple steady states under discretion: Solutions to the equations (6.8a-6.8b) for $b = 0$ and $\tau = 1$. σ_{l1} and σ_{h1} are the solutions to (6.8a), while σ_{l2} and σ_{h2} indicate the solutions to (6.8b).

utility, will also lower the marginal utility of consumption. If the current value of assets is given by

$$u_{c,t}(R_{b,t}b_t + \left(1 + (1 - \phi_{k,t}) \left(f' \left(\frac{k_t}{\sigma_t}\right) - \delta_k\right) k_t\right)$$

then the increase in c_t will reduce the value of these assets. Similarly, an increase in σ_t both increases wage income, and by increasing the interest rate, increases the current value of household assets. Under discretion, policy-makers will have an incentive to decrease current consumption and increase σ_t, relative to their values under commitment. In the steady state, for relatively low values of p, numerical analysis shows that σ_{h1} will initially decrease as p increases, and in fact, will not differ significantly from its value under commitment. However, σ_{h1} will be increasing in p once property rights exceed a certain threshold (Figure (6)). This behavior may be justified by the fact that when property rights are high, wage income is relatively more important than income from appropriation, and the tax base will thus be higher. It becomes less costly to set investment in legal infrastructure and tax rates so that households spend less time in appropriation. Under discretion, the government will have an added incentive to do so.

The above characterization of the loci of solutions to equations (6.8a-6.8b) implies the possibility of four steady states, two with relatively low values of σ and two with relatively high values. The lower two steady states will not be feasible, except for very low values of τ, as they would require $\phi_l > 1$. However, the upper two steady states are indeed feasible for a broad range of values of τ, the return to appropriation.



(a) Steady state consumption

(b) Steady state σ

(c) Steady state legal infrastructure

(d) Steady state tax rate

Figure 7: Effect of τ on steady state equilibrium, under discretionary policy. σ_r corresponds to the equilibria under commitment, while σ_1, σ_2 and σ_3 correspond to the 3 possible steady states under discretionary policy

6.2 Markov Perfect Equilibria

The approach to discretionary policy outlined in the previous section has the disadvantage that it assumes that despite having perfect foresight, governments and households will continue to assume throughout time that the government is always in the first period of a new commitment to policy. The framework of Markov Perfect Equilibria, as used by Klein et al (2003) and Ortigueira and Pereira (2007), assumes that governments are conscious of their inability to commit. Instead, fiscal policy will be chosen to maximize the lifetime utility of the representative household, given the policy functions of the sequence of future governments. Equilibria will correspond to the identical policy function being used by current and future governments.

The optimization problem for the government may be formalized as follows[2]. The Euler equation and the static first order condition with respect to σ, give us expressions

[2] A slight change in notation is employed in this section. B, P, K all will refer to aggregate values of variables b, p, k, while k refers to the choice of physical capital accumulation by the household

for the choice variables c, σ in terms of state variables and policy variables, namely $\sigma = \sigma(K, P, \phi_l)$ and $c = c(K, B, P, \phi_l, \phi_k)$. Given these functions, the government's problem may be formulated in two stages. In the second stage, given tax rates and investment in the legal infrastructure, the government will choose the level of public debt for the subsequent period. This will give public debt as a function of current bond holdings, the aggregate stock of physical capital and legal infrastructure, along with tax rates. In the first step of the government's problem, tax rates and legal infrastructure are chosen subject to the capital accumulation equation, the government's budget constraint, with public debt and household choice variables being included as functions of state variables, as described above.

The value function for the government then becomes:

$$W(K, B, P) = \max_{\phi_l, \phi_k, P'} \left\{ U(C(K, B, P, \phi_l, \phi_k)) + \beta W(K', B'(K, B, P, \phi_l, \phi_k), P'(K, B, P)) \right\} \tag{6.9}$$

subject to

$$(1+n)K' = \sigma(K, P, \phi_l) f\left(\frac{K}{\sigma(K, P, \phi_l)}\right) + (1-\delta_k)K + (1-\delta_p)P - (1+n)P' - C(K, B, P, \phi_l, \phi_k) \tag{6.10}$$

and

$$(1+n)P' = (1-\delta_p)P + \phi_l h(P, \sigma(K, P, \phi_l)) * w\left(\frac{K}{\sigma(K, P, \phi_l)}\right) + \phi_k f'\left(\frac{K}{\sigma(K, P, \phi_l)}\right) + (1+n)B'(K, B, P, \phi_l, \phi_k) - R_b B \tag{6.11}$$

The next step is to obtain what I shall refer to as generalized Euler equations, by combining the first order conditions for the government's maximization problem with appropriate envelope conditions.[3] The first order condition with respect to legal infrastructure implies that $W'_K = W'_{P'}$, while that with respect to bonds implies $W'_K P'_{B'} = W'_{B'}$. The

[3] Subscripts indicate a derivative with respect to the variable in the subscript. Primes in the subscript indicate that the derivative is being taken with respect to the value in the next period, while primes in a variable indicate next period values for that variable.

first order condition with respect to tax rates implies that:

$$U_c c_{\phi_l} + \beta W'_{K'} \left(\sigma_{\phi_l} \left(f\left(\frac{K}{\sigma}\right) - \frac{K}{\sigma} f'\left(\frac{K}{\sigma}\right) \right) - (1+n)P'_{\phi_l} - (1+n)P'_{B'}B'_{\phi_l} \right) \frac{1}{1+n} - \beta W'_{K'} \left(\frac{c_{\phi_l}}{1+n} \right) + \beta W'_{B'} B'_{\phi_l} + \beta W'_{P'} P'_{\phi_l} = 0 \quad (6.12)$$

A similar first order condition is found for capital taxes. Combining the above first conditions I obtain the following condition, with a similar one to be satisfied once again for capital taxes:

$$(1+n)u_c c_{\phi_l} + \beta W'_{K'} \left(\sigma_{\phi_l} \left(f\left(\frac{K}{\sigma}\right) - \frac{K}{\sigma} f'\left(\frac{K}{\sigma}\right) \right) - c_{\phi_l} \right) = 0 \quad (6.13)$$

In the previous section, for the case of commitment, I found that in a steady state that capital taxes would be equal to zero, and that the value of bonds in the steady state would be equal to their value prior to entering the steady state, so that legal infrastructure would be primarily funded by labor taxes. Ortigueira and Pereira (2007) find that when Markov Perfect Equilibria are applied to case of unbalanced budgets, that one of the multiple steady states existing under discretionary policy will correspond to that obtained as a solution to the Ramsey problem. However, in the case of the model examined in this chapter, this result does not hold.

Proposition 4. *There does not exist a steady state Markov Perfect equilibrium that is identical to the steady state obtained as a solution to the Ramsey problem*

Proof. In a steady state, the envelope theorem implies that the derivative of the value function with respect to physical capital must satisfy:

$$(1+n)W_K - \beta W_K \left(\sigma_K \left(f\left(\frac{K}{\sigma}\right) - \frac{K}{\sigma} f'\left(\frac{K}{\sigma}\right) \right) - (1+n)P'_K - C_K \right) - (1+n)u_c C_K = \beta W_K (1 - \delta_k + f'\left(\frac{K}{\sigma}\right)$$

If it were the case that there existed a steady state Markov Perfect equilibrium identical to that obtained from the Ramsey problem, then the above first order condition would reduce to the Euler equation for the household under no capital taxes. This would imply that:

$$(1+n)W_K - \beta W_K\left(\sigma_K\left(f\left(\frac{K}{\sigma}\right) - \frac{K}{\sigma}f'\left(\frac{K}{\sigma}\right)\right) - (1+n)P'_K - C_K\right) \\ - (1+n)u_c C_K = (1+n)W_K$$

This would be satisfied if both $(1+n)U_C = \beta W_K$ and

$$\sigma_K\left(f\left(\frac{K}{\sigma}\right) - \frac{K}{\sigma}f'\left(\frac{K}{\sigma}\right)\right) - (1+n)P'_K = 0 \tag{6.14}$$

were satisfied. However, the first condition will in general only be consistent with equation (6.13) if $\sigma_{\phi_l} = 0$. This would be inconsistent with the static first order condition of the household which must be satisfied by $\sigma_{\phi_l} < 0$, except in the extreme case where $\tau = 0$. □

This result would imply a gap between the steady state under the Ramsey problem and that found under discretionary policy, which is consistent with my findings in the previous subsection in the simpler approach to discretionary policy as simply the first period of optimal policy under commitment. A third possible approach to discretionary policy would be to adapt the Lagrangian setup employed in the section under commitment. That is, one may think of the problem under discretion as simply maximizing the lifetime utility of the household, subject to the implementation constraint and resource constraint, but with an additional sequence constraints which give the household first order conditions for labor and capital in terms of the policy functions of future government. The government's problem under discretion may thus be thought of as simply a more highly constrained version of the problem under commitment. In particular, governments will no longer be able to take advantage of time inconsistencies in optimal policy, which was the source of zero capital taxes in the steady state under commitment. This justifies the gap between the steady states under commitment and under discretion found under both the previous approaches to discretionary policy.

7 Discussion

Three relevant strands of the literature in the literature on the differences between allocations under discretionary policy and policy under commitment can be identified. First, a series of studies on optimal fiscal policy argue that under commitment, capital taxes will be set higher in the first period, and lower in subsequent periods. Under discretion, government behavior is identical to that of the government under commitment during the initial period. As a result, capital taxes will typically be higher under discretion than under commitment. Second, Kydland and Prescott argue that commitment by the central bank to a low average rate of inflation may permit lower inflation than would result from discretionary monetary policy, with little loss of output. The third strand consists of a dynamic reformulation of the Kydland Prescott argument. Under traditional optimal control setting of the policy problem, the policymaker's optimal action will depend only on the economy's state in the current period. Under discretionary policy, its target variables will depend both on its current actions and on private-sector expectations. Commitment to earlier policy promises will imply that the central bank's behavior will depend on both current and past conditions, and thus introduces inertia into optimal policy rules.

All three strands of the literature share the characteristic that if policy is committed to at the initial period, then this will result in private-sector expectations being derived from certain knowledge of future policy actions in each possible state of the world. Policy may thus be chosen to affect both current allocations and private-sector expectations, which allows for a superior outcome to that attainable under discretionary policy.

While the model of economic growth and property rights determination presented here is deterministic, government policy under commitment affects capital accumulation just as it does in more traditional models of optimal fiscal policy. An increase in legal infrastructure for the subsequent period, p_{t+1}, will decrease the time spent in appropriation in that period, and thus increase returns to both capital and labor. The increase in the interest rate will affect the growth rate of consumption, and thus the accumulation of capital. As in prior work on the difference between discretionary policy and policy under commitment, it may be argued that discretionary policy is suboptimal with respect to policy under commitment.

Consider now the differences in the policymaker's problem under commitment and discretion. Under commitment, the sequence of policies are set so as to satisfy the implementation constraint in the initial period, thereby ensuring that the household's first order condition will be satisfied throughout subsequent periods. Under discretion, since the government cannot commit to a sequence of policies through time, it must assure in each period that policies are set to satisfy the implementation constraint from that period onwards. An increase in consumption will reduce $u_{c,t}$ and thus decrease the value of initial assets. Discretionary policy thus imposes an additional cost to increasing consumption in the current period. Similarly, decreasing time spent in appropriation will increase current interest rates, or increase the value of initial assets. The benefit of increasing σ_t is thus higher. This implies that a lower steady state exists, with a lower level of consumption, and a higher level of σ, induced by higher levels of legal infrastructure and of taxes on wage income. The policy in this lower steady state is effective, in the sense of discouraging appropriation and allowing for a higher steady state level of capital to be maintained. We assumed that $\lim_{\sigma \to 1} h_p = 0$, so that investments in the legal infrastructure are less effective as σ approaches 1. Thus, setting taxes and investment in legal infrastructure so that $\sigma > \sigma_r$, as occurs in the lower steady state under discretion, is highly costly, and results in lowered consumption.

While the upper steady state level of σ, σ_3, is close to its steady state level under commitment, σ_r when τ is close to zero, as the return to appropriation increases, σ_r responds more quickly than σ_3, given the added incentive under discretion to maintain low levels of appropriation. This causes consumption in the upper steady state under discretion to be lower than it is under commitment. In addition, I note that the upper and lower steady states under discretion approach each other as τ until the return to appropriation approaches a threshold level, above which no steady state under discretion exists.

8 Conclusion

I have developed a model in which legal infrastructure, or the level of protection of property rights, affects the probability of being caught if agents engage in appropriation from their employer. This model has the characteristic that under exogenous policy, there is a single steady state. I embed this in a model of optimal fiscal policy, characterize the steady states under commitment and under discretionary policy. Under commitment, I find that there is a unique feasible steady state, and that welfare, as measured by the utility of the representative household, will be decreasing in the level of return to appropriation. An intuitive result is that taxes and legal infrastructure will also be increasing in τ. When policy is discretionary, that is, when the government is unable to commit to a particular sequence of policies, then there will generally exist at least two steady states. The upper steady state will not generally be significantly different from the steady state under commitment. The lower steady state displays in fact a higher level of legal infrastructure, and less time devoted to appropriation, but this is so costly that consumption is lower than in the upper steady state.

This model differs from the existing theoretical literature on the connection between economic growth and institutions in two key respects. First, legal infrastructure or the level of protection of property rights is explicitly modeled as accumulating over time, and then as explicitly entering into the household's optimization problem via a probability function. Secondly, and perhaps most importantly, I look at both discretion and commitment. The result of multiple steady states does not occur, as it does in other models, because of strategic complementarities in the optimization problems of households and firms, but rather because of the inability of the government to commit to policy.

This model not only argues in favor of growing protection of property rights during the process of economic development, but also demonstrates the need for commitment to policy over the long-run. In the set-up of the model, this would imply that a sequence of policies, specifically investment in legal infrastructure, be committed to in some initial period of the economy. Alternatively, one may use Woodford's "timeless perspective" approach, and argue instead that the government simply needs to set policy according to the pattern of behavior to which it would have wished to commit itself at a date far in the past. This implies a history-dependent approach to the reform and improvement of a country's legal infrastructure.

One striking aspect aspect of this model is that through characterizing property rights as a continuous state variable, I do not obtain the poverty trap result previously obtained in the literature, i.e. that for sufficiently low initial levels of property rights the economy would converge towards a steady state with low levels of property rights and income. The lower steady state obtained under discretionary policy in my model instead corresponds to over-investment in legal infrastructure. However, this does not imply that under-development will not lead to poor quality of institutions. In the case that lack of development implies higher returns to appropriation, then such countries will display higher rates of appropriation of output. This would suggest that greater attention should be paid to endogenizing return to appropriation, rather than focusing exclusively on the failure of government to adequately invest in legal infrastructure.

The model presented in this paper serves as a benchmark for the relationship between economic growth and institutions. Future work could include first the characterization of the dynamics of the model, both in the case of commitment and discretionary policy, including transition dynamics between the two steady states found under discretion. As alluded to earlier, our assumption of an exogenous return to appropriation neglects the effect of development on return to appropriation. A possible extension would thus be to make the return to appropriation proportional to the level of per capita output. This would imply that both interest rates and wages would depend on the level of protection of property rights, and legal infrastructure would enter directly into the Euler equation. The level of steady state physical capital would thus depend both on σ and on p, introducing an additional interaction between property rights and time spent in appropriation. This will undoubtedly lead to richer dynamics and possibly more steady states, but my analysis suggests that the distinction between results under commitment and discretion will be preserved, arguing for the need of long-term perspective on the part of the governments of developing countries alongside any attempt to improve protection of property rights.

A Proof of Proposition 1

Proof. This proof will proceed in three parts. First, I will show that if we ignore the government budget constraint, at most two steady state equilibria will exist. Then, I will show that at $\tau = 0$, that $\phi_l = 1$ at the lower equilibrium. Finally, I will show that $\tau > 0$ implies that $\phi_l > 1$ at the lower equilibrium. For simplicity of notation, I define $g(p, \sigma) = (h(p, \sigma))^2/h_\sigma$. Equation (3.8) then implies that expected labor income, net of taxes, is equal to $g(p, \sigma)$.

1. First consider the limiting values of σ, or $0, 1$. When no time is spent in honest effort, expected labor income is negative, while $g(p, 0) = 0$ for $p > 0$. When all available time is spent in honest effort, then $\lim_{\sigma \to 1} g(p, \sigma) = \infty$, while expected labor income will be finite in value. The assumptions regarding $h(p, \sigma)$ implies that the derivative of $g(p, \sigma)$ with respect to σ will vary continuously from 0 at $\sigma = 0$ to ∞ at $\sigma = 1$.

   Therefore, for a given level of debt b, one may solve equation (3.8) and the following condition

$$f(B) - Bf'(B) = \tau \left(2h(p,\sigma) - \left(\frac{h(p,\sigma)}{h_\sigma} \right)^2 h_{\sigma\sigma} \right) \tag{A.1}$$

for the pair of values p, σ. This corresponds to the level of legal infrastructure in the steady state, such that the curve representing expected labor income is just tangent to the $g(p,\sigma)$ curve, holding p constant and varying σ. Let $p^*(b)$ denote this threshold value of legal infrastructure.

For small values of p, $g(p,\sigma) > \sigma(f(B) - Bf'(B)) - (n+\delta_p)p - \beta^{-1}(1+n)(1-\beta)b$, as h_σ quickly approaches zero. Thus, no equilibrium exists. Similarly, for large values of p, $g(p,\sigma)$ exceeds expected labor income. Thus, there will typically be two possible values of $p^*(b)$, which I now denote by $p_l^*(b)$ and $p_h^*(b)$. When p lies in the interval $(p_l^*(b), p_h^*(b)$, then there will exist two equilibria in σ, given p and b, still ignoring the government budget constraint. When p lies outside this interval, no equilibrium will exist.

2. Consider now the limiting case where $\tau = 0$. The government budget constraint, and the assumption that $\phi_k = 0$ imply that

$$\phi_l = \frac{(1+n)(1-\beta)\beta^{-1}b + (n+\delta_p)p}{\sigma f(B) - B\sigma f'(B) - h(p,\sigma)(1-\sigma)\tau} \tag{A.2}$$

There are two solutions to equation (3.8) at $\tau = 0$: $\sigma = 1$, and

$$\sigma = \frac{(n+\delta_p)p + (1+n)(1-\beta)\beta^{-1}b}{f(B) - Bf'(B)}$$

The latter solution I will refer to subsequently as the lower equilibrium. The lower equilibrium implies that $\phi_l = 1$.

3. Now consider the labor tax rate at the lower equilibrium when $\tau > 0$. *Note: still need to write up the remainder of this proof*

□

B Proof of proposition 2

Proof. In section (3), I showed that by combining the household budget constraint, the government budget constraint, and the competitive rental rates of labor and capital, I obtained the feasibility condition (4.2). Multiplying the household budget constraint (2.5) by ψ_t, summing over t

$$\sum_t \beta^t \psi_t [h(p_t, \sigma_t)((1-\phi_{lt})w_t + \tau(1-\sigma_t)) + (1 + (1-\phi_{kt})(r_t - \delta_k))k_t + R_{b,t}b_t$$
$$- c_t - (1+n)k_{t+1} - (1+n)b_{t+1}] = 0 \quad \text{(B.1)}$$

The Euler equations for capital and bands

$$(1+n)\psi_t k_{t+1} = \beta\psi_{t+1}(1 + (1-\phi_{k,t+1})(r_{t+1} - \delta_k))k_{t+1} \quad \text{(B.2)}$$
$$(1+n)\psi_t b_{t+1} = \beta\psi_{t+1}R_{b,t+1}b_{t+1} \quad \text{(B.3)}$$

and the corresponding transversality conditions may be used to eliminate to eliminate capital and bonds, with the exception of $t = 0$. Combining equations (B.1-B.3), and slightly rearranging, I may obtain

$$\sum_t \beta^t \psi_t (c_t - h(p_t, \sigma_t)((1-\phi_{lt})w_t + \tau(1-\sigma_t)))$$
$$= \psi_0(R_{b,0}b_0 + (1 + (1-\phi_{k0})(r_0 - \delta_k))k_0) \quad \text{(B.4)}$$

The first-order condition for consumption implies that $u_{c,t} = \psi_t$. The first order condition with respect to σ_t may then be used to obtain w_t in terms of σ_t and p_t. Equation (B.4) may thus be transformed into the implementation constraint (4.3). Thus, equations (4.2) and (4.3) are necessary conditions that any Ramsey equilibrium must satisfy.

Now, given any allocation that satisfies (4.2) and (4.3), I may construct sequences of bond holdings and policies such that these allocations satisfy the first-order conditions of the household's optimization problem. An allocation $\{c_t, \sigma_t, k_{t+1}, p_{t+1}\}_{t=0}^{\infty}$ will determine the rental rates of labor and capital, according to:

$$r_t = f'(\frac{k_t}{\sigma_t}) \quad \text{(B.5)}$$
$$h(p_t, \sigma_t)(w_t + \tau(1-\sigma_t)) = (\sigma_t f(\frac{k_t}{\sigma_t}) - k_t f'(\frac{k_t}{\sigma_t})) \quad \text{(B.6)}$$

To construct the bond allocations, multiply the household budget constraint (2.5) by ψ_t and sum over all periods following r. Slightly rearranging, I obtain

$$\sum_{t=r+1}^{\infty} \beta^t \psi_t [(1 + (1 - \phi_{kt})(r_t - \delta_k))k_t - (1+n)k_{t+1} + R_{b,t}b_t - (1+n)b_{t+1}]$$
$$= \sum_{t=r+1}^{\infty} \beta^t \psi_t [c_t - h(p_t, \sigma_t)((1 - \phi_{lt})w_t + \tau(1 - \sigma_t))] \quad \text{(B.7)}$$

Following the same steps as I used in deriving the implementation constraint, in particular, employing the Euler equations for capital and bonds, equation (B.7) may be transformed to

$$\beta^{r+1}\psi_{r+1}[(1 + (1 - \phi_{k,r+1})(r_{r+1} - \delta_k))k_{r+1} + R_{b,r+1}b_{r+1}] = \sum_{t=r+1}^{\infty} \beta^t \psi_t \left[c_t - \tau \frac{(h(p_t, \sigma_t))^2}{h_{\sigma,t}} \right] \quad \text{(B.8)}$$

But the household first order condition with respect to capital and bonds implies that

$$(1+n)\beta^r \psi_r (b_{r+1} + k_{r+1}) = \beta^{r+1}\psi_{r+1}[R_{b,r+1}b_{r+1} + (1 + (1 - \phi_{k,r+1})(r_{r+1} - \delta_k))k_{r+1}] \quad \text{(B.9)}$$

As a result, I obtain

$$(1+n)\beta^r \psi_r (b_{r+1} + k_{r+1}) = \sum_{t=r+1}^{\infty} \beta^t \psi_t \left[c_t - \tau \frac{(h(p_t, \sigma_t))^2}{h_{\sigma,t}} \right] \quad \text{(B.10)}$$

Or bonds issued in period r are given by

$$b_{r+1} = \sum_{t=r+1}^{\infty} \beta^{t-r} \frac{u_{c,t}\left(c_t - \frac{(h(p_t,\sigma_t))^2}{h_{\sigma,t}}\tau \right)}{(1+n)u_{c,r}} - k_{r+1} \quad \text{(B.11)}$$

The method of construction of the bond allocations implies that the household first-order conditions and budget constraint will be satisfied, provided that the return to bonds is given by

$$R_{b,t} = 1 + (1 - \phi_{kt})(r_t - \delta_k) \quad \text{(B.12)}$$

for periods in which $b_t > 0$. □

C Proof of proposition 3

Proof. Consider the Ramsey problem:

$$\max \sum_{t=t_0}^{\infty} \beta^{t-t_0} u(c_t) \tag{C.1}$$

subject to (4.2) and $W_{t_0} = \overline{W}_{t_0}$, given $\{k_{t_0}, p_{t_0}\}$. We now proceed by comparing the allocations chosen as the solution to the above problem to the outcome of a 2-stage problem. In the first stage of this problem, choose $\left\{x_{t_0}, \overline{W}_{t_0+1}\right\}$ to maximize $J(x_{t_0}, \overline{W}_{t_0+1})$ such that the feasibility constraint holds and

$$\overline{W}_{t_0} = u_{c,t_0}\left(c_{t_0} - \frac{(h(p_{t_0}, \sigma_{t_0}))^2}{h_{\sigma,t_0}}\tau\right) + \beta\overline{W}_{t_0+1} \tag{C.2}$$

In the second stage, $\{x_t\}$ is chosen to maximize U_{t_0+1} given the feasibility constraint and $W_{t_0+1} = \overline{W}_{t_0+1}$. We note that the solution of this two-stage problem is feasible as a Ramsey equilibrium, in that it satisfies the feasibility and implementation constraints. Clearly, both the first and second stage satisfy feasibility. Then, I can substitute $W_{t_0+1} = \overline{W}_{t_0+1}$ into equation (C.2), to obtain $W_{t_0} = \overline{W}_{t_0}$. It then remains only to show that there cannot be any other sequence of allocations $\{\tilde{x}_t\}$ that satisfies the constraints of the Ramsey problem and attains a higher level of utility U_{t_0}. Suppose that there does exist such a $\{\tilde{x}_t\}$, and let $\tilde{W}_{t_0+1}$ be the implied value for W_{t_0+1}, $\tilde{U}_{t_0+1}$ the implied forward-looking utility in period $t_0 + 1$ from the sequence of allocations, and $\tilde{U}_{t_0}$ the implied value of U_{t_0}. By hypothesis $\tilde{U}_{t_0} > U_{t_0}$. We note that $\{x_t\}$ satisfies the constraints of the first-stage problem. Because the allocation is feasible and is consistent by construction with the precommitment $\tilde{W}_{t_0+1}$, I must have

$$V(\tilde{k}_{t_0+1}, \tilde{p}_{t_0+1}, \tilde{W}_{t_0+1}) \geq \tilde{U}_{t_0+1} \tag{C.3}$$

Since the allocation also satisfies the constraints of the first-stage problem, then

$$J(\tilde{x}_{t_0}, \tilde{W}_{t_0+1}) \geq \tilde{U}_{t_0} \tag{C.4}$$

We must then conclude that

$$J(\tilde{x}_{t_0}, \tilde{W}_{t_0+1}) > U_{t_0} \tag{C.5}$$

which contradicts the assumption that $\{x_t\}$ solved the first-stage optimization. This contradiction implies that $\{x_t\}$ represents a Ramsey equilibrium. Furthermore, the second-stage problem as described here is of the same form as the Ramsey problem The same proof can be used to show that it is equivalent to a similar two-stage problem. By induction, I may establish that $(x_t, \overline{W}_{t+1})$ solve a similar "first-stage" problem to that described in this proof. The proposition follows. □

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