On the Role of Emotions in Games

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Abstract

Numerous experiments have shown that players cooperate in a prisoner's dilemma (PD) game. I contribute to this literature by showing that emotions might be the key to overcoming the coordination failure. I model emotions by generally distinguishing between two types of emotions: First, expected emotions play a central role. These are experienced not during the decision-making, but are rather ex post, i. e. after the outcomes are realized. In this regard, standard theory accounts for expected emotions by allowing for phenomenons such as fairness considerations, inequality aversion and temptation in iterated PD games. I describe how expected emotions - in particular satisfaction, anger, remorse and disappointment - may have an impact on the perceived payoff in a PD game. Second, immediate emotions seem to be equally important in my model. Immediate emotions, just as expected emotions, arise from considering the consequences of one's decision. The crucial difference, however, is that they are experienced during the decision-making. In introducing immediate emotions I apply the projection bias model developed by Loewenstein et al. (2003) and embed the perceived payoff into the projection bias model. I find that expected emotions might lead to the emergence of a new Pareto-efficient Nash equilibrium in which all players cooperate. However, immediate emotions might destroy this equilibrium instantaneously.

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1 Introduction

Rational decision making and emotional preference had long been seen as contradictions in economics. However, over the last 30 years a striking in-

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crease has been noted in research on the influence of emotions on judgment and choice; partly due to advancements in psychology amongst other social sciences (Lazarus (2006), Lerner & Keltner (2000), Lerner & Keltner (2001)). Indeed, emotions are now widely recognized as a critical part of the decision making process. Cosmides & Tooby (2000) describe emotions as "superordinate program" which coordinate human behavior. Without counting them in the equation, adverse consequences would ensue.
In addition, research from political science appears to be particularly well advanced in adopting emotions into the decision making process. Current research distinguishes emotions as two types (Renshon & Lerner (2010), Zimmerman & Lerner (2010)). The first class of emotions is composed of expected emotions. They arise from the well-known rational utility calculation and, as such, "are consistent with a rational choice framework (Renshon & Lerner 2010)". In this situation individuals anticipate the feeling they obtain from a specific activity and include it into their utility calculation.
In this situation individuals anticipate the feeling they obtain from a specific activity and include it into their utility calculation. The second, and probably more interesting category of emotions, are what Renshon & Lerner (2010) call immediate emotions: emotions from this group are experienced at the time of decision-making. A mathematical approach to the modeling of immediate emotions is presented in Loewenstein (2000). The basic idea is that people exhibit a projection bias in terms of overestimating the extent to which their current situation will affect their future. More precisely, when people find themselves in a situation with little emotional excitement, they underestimate the effect of future emotions. Contrariwise, they also struggle to assess how they will decide in a relatively calm moment, once they are under strong emotional influence. Loewenstein (2000) calls it the "hot-cold empathy states"[1].
In referring to expected emotions I describe a simple model of how emotions, in particular satisfaction, anger, remorse, and disappointment may have an impact on the perceived payoff in a symmetric n-person prisoner's dilemma game. I assume that, when individuals are asked to either cooperate or defect, they assume to"feel" emotions. These emotions are expected to occur once the outcome materializes and might, in turn, alter the decision to be undertaken by the agent. I assume that players do not only put into perspective their payoff compared to the payoff of their counter player. Equally important is the fact that agents qualify the payoff they receive in one particular situation with respect to the payoff they receive in another. Hence, players

[1] Hot state e.g craving, angry, jealous. Cold state e.g. not craving, not angry, not jealous.

have different reference points (Koeszegi & Rabin 2006). In this spirit, I identify three different coefficients for emotions, namely λ for jealousy, κ for disappointment and μ for remorse. κ and μ together constitute anger. The coefficient for satisfaction is 1 so pursuantly I will not explicitly mention it. By alluding to immediate emotions I use the projection bias model by Loewenstein et al. (2003) and extend it with the above mentioned emotions model. More precisely, "people exaggerate the degree to which their future tastes will resemble their current tastes (Loewenstein et al. 2003)". For a clearer understanding consider the following example: A "relaxed" person might waste her time imagining what emotional distress would feel like. Her present state of relaxation will influence her imagination of emotional distress, hence her prospects might be too rosy. In other words, people underestimate the impact of their current emotional state on the judgment of their future state. Thus, agents are subject to a projection-bias. The reason for the players to correct their payoff is the emotions they experience during the decision-taking. Thus, during the moment of decision-taking every individual is subject to a certain set of feelings. To illustrate, consider the following example: there are two players facing a symmetric one shot PD game. Both players are fully informed about the possible outcomes and are asked simultaneously to choose an action. Firstly, the players consider the situation in which both players cooperate. This result provides our players with a feeling of satisfaction. Secondly, our players check the results from the Nash-equilibrium. This result disappoints our agents. Disappointment, in turn, leads our agents to further reduce the payoff they obtain in a disappointing situation. In the next situation, our agents consider how they would feel as suckers for the other's deception. This line of thinking causes them distress and makes them susceptible to anger, which reduces the subjectively felt utility even further than disappointment. Finally, our agents consider what it would be like to cheat the other agent. Undoubtedly, the payoff from the game is the highest in this situation. However, it might not necessarily be the highest payoff corrected for expected emotions. Exploiting someone else leaves our agents with a bad taste in their mouths. Fehr & Schmidt (1999) call it fairness.

After checking how they would experience the payoff under emotions, each agent picks the highest utility. The result might be a shift of the equilibrium from the Nash-equilibrium in a PD to an equilibrium in which both players cooperate. Adjusted for these expected emotions the new equilibrium turns out to be a Pareto-efficient Nash-equilibrium. However, the agent is also subject to immediate emotions, which might destroy the new equilibrium instantaneously.

The rest of the paper is organized as follows. A short literature overview is provided in Section 2. Section 3 describes the two-player PD and shows how a new Pareto-efficient Nash-equilibrium may arise if we allow expected emotions to be a controlling factor. Section 4 extends the two-player approach to n players and derives a unified payoff formula for the perceived outcome of a player in an n-player game. In Section 5 I describe immediate emotions. Finally, a conclusion is found in Section 6.

2 Literature Overview

Empirical research has revealed that behavior is not always consistent with rational choice theory and the axioms stated in Von Neumann & Morgenstern (1944)[2]. In the past 30 years new theoretical innovations have emerged which have aided the overcoming of these conflicts. For instance, although expected utility theory assumes that people base their decision on changes in overall welfare, Kahneman & Tversky (1979) show that the assessment of future results might be based on the changes in incremental gains and losses [3]. Equally important is the fact that an agent qualifies the payoff she receives in one particular situation in relation to the payoff she receives in another one; hence she adjusts outcomes relative to expectations. In other words, unrealized outcomes might influence the evaluation of realized outcomes (Koeszegi & Rabin 2006). Other violations of the rational choice theory might be explained by regret-theory (Loomes & Sugden (1982), Zeelenberg & Beattie (1997), Zeelenberg et al. (1996), Gilbert et al. (2004)).

Further, Ariely & Loewenstein (2006) provide a very unusual approach to research on immediate emotions. They study how arousal might lead people toward riskier behavior. By contrast, fear might be a trigger to give up on it (Bechara et al. 1997).

To make a long story short, research suggests that emotions influence risky decision making, intertemporal choice, and social preferences. For instance, in Rick & Loewenstein (2007) excellent article, they show that the idea that both categories of emotions have a crucial impact on decision making is supported by the behavioral economics literature.

[2]The axiom of transitivity states that if alternative A is preferred to B and B to C, then A is also preferred to C. Although this axiom is easy to understand, it is regularly violated in experiments.

[3]Assume that someone is offered a chance to enter a lottery. The person has a monetary endowment of 100. The lottery offers a gain of 7 with the probability of 0.5, and a loss of 3 with the complementary probability. From the EU perspective the lottery results in a gamble between a 50-50 chance for either 107 or 97, and a safe outcome of 100. However, most people do not take their initial endowment into account. Thus, they perceive the aforementioned lottery as a gamble between a 50-50 chance for either winning 7 or losing 3, and a safe outcome of 0.

3 The two-player prisoner's dilemma

3.1 A simple example

I concern myself with a symmetric two-player prisoner's dilemma. A classic example of this dilemma is presented as follows:
Two players have each to decide whether to cooperate (C) or to defect (D) in a simultaneous game. If both cooperate, they each receive the highest payoff of 3. However, if only one cooperates while the other decides to defect, the defector ends up receiving a payoff of 5, while the cooperator ends up with nothing at all. Finally, if both defect, the payoff for each player is a modest 1.

All things being equal, all rational players prefer defecting to cooperating, since the latter is dominated. Thus, the only possible equilibrium in the classic form of this game is for all players to defect (Rapoport 2001).

		Player 2	
		C	D
Player 1	C	$3;3$	$0;5$
	D	$5;0$	$\underline{1};\underline{1}$

Figure 1: Example for a prisoner's dilemma game.

3.2 Emotions in the game

Now assume that individuals are subject to expected emotions at the moment they are asked to decide whether to cooperate or to defect. More precisely, agents compare the payoff they receive in one particular situation to the payoff they receive in another situation. Additionally, they concern themselves with the size of their payoff compared to the size of the payoffs of the other agents. This relativization changes the perceived outcome compared to the absolute outcome. Chronologically, it happens when players are asked to decide on one action, right before they actually make a final decision. An agent contemplates different outcomes (such as both agents cooperating, no one cooperating, etc.), which then give rise to the following feelings:

Anger: A situation in which our agent feels betrayed can make her experience anger. A player gets angry in a PD every time she chooses to cooperate, while the opposite player decides to capitalize on her decision, hence to defect. Not only does the angry agent mourn the potential profit she would have received (if the other player had not betrayed her) and adjusts it for the coefficient κ. She is also "jealous" from the unrighteous profit of the other player, which she corrects for the coefficient λ.

Remorse: This is the mirror-image situation of anger. The agent enjoys the high profit from defection but fairness considerations stifle her utility gains. This follows from the fact that the other player offered her cooperation and, at the time of picking the action, hoped for cooperation by the other agent as well. μ is the coefficient for remorse.

Disappointment: Ending up in the "former Nash-equilibrium" disappoints our agent. She compares her payoff with the payoff when both players cooperate. For sake of simplicity I assume that our agent uses κ in this situation too.

Satisfaction: The mutual cooperation outcome is a situation in which our agent feels satisfied. There is no need for her to correct the outcome in any fashion. Thus, without loss of generality, we assume the satisfaction coefficient to be 1.

Note that our agent still behaves rationally (Harsanyi & Selten 1988). All she is doing is applying a correction function on the outcome she might receive. The correction function on the other hand is a result of the emotions an agent faces during the moment of decision-taking. $\lambda \geq 0$ constitutes the jealousy coefficient. Disappointment is depicted by the $\kappa \geq 0$ coefficient and $\mu \geq 0$ describes the coefficient for remorse. κ and μ constitute anger. The coefficient is 1 if the outcome is mutual cooperation.
One might claim that agents should add rather than deduct utility if they end up in a Nash-equilibrium. After all, the other agent also defected, and if she had not defected, she would have ended up being the sucker. However, our agent already enjoys the higher utility from the Nash-equilibrium, compared to the utility she would have received if she had been the exploited victim. Thus, I am using the definition by Selten (1991) for rational behavior:

> Rational Economic Behavior is the maximization of subjectively expected utility.

Expected emotions constitute parts of an agent's set of information. For this reason we can apply a definition given by Aumann (2006) for rational behavior as well:

> A person's behavior is rational if it is in his best interests, given his information.

3.3 Example

Player 1 and Player 2 encounter the following two games:

	C	D
C	$\underline{10}; \underline{10}$	$0; 12$
D	$12; 0$	$9; 9$

Game 1

	C	D
C	$11; 11$	$0; 12$
D	$12; 0$	$\underline{10}; \underline{10}$

Game 2

Figure 2: Example for two games

Now assume both players decide to cooperate in the first game while they defect during the second. Since the payoff in both games is an identical 10 for every player, we might easily believe that they should be indifferent towards those two outcomes. After all, no one faces consequences in their payment structure.
That said, the situation changes entirely if we allow for the application of a correction function. Since the result in game 1 constitutes an equilibrium in which both players cooperate, it also provides our players with satisfaction. In that case they do not correct the obtained payment. By contrast, the result in game 2 makes our players suffer from disappointment. Both would have been better off if they had cooperated. Thus, I assume the perceived outcome is $10 - \kappa(11 - 10) < 10 \ \forall \ \kappa$[4] > 0. Consequently, rational players prefer the outcome in game 1 to the outcome in game 2.
But why did the players manage to achieve cooperation in game 1 and failed to do so during game 2? During their decision-taking players adjust for the outcome they would obtain if they defected while the other player cooperated, which might be thought of as a fairness consideration. Hence, not having a clear remorse comes at a cost: since the betrayed player offered cooperation to the betraying player, the latter knows the aim of the offer was to reach a situation in which both players cooperate. Accordingly, she can effortlessly estimate the value the betrayed player was striving for. I model that the betrayer trims the difference between what the betrayed expected to receive and what she finally obtained from the betrayer's payment. Additionally, she customizes it accordingly to her remorse coefficient μ. For that reason the betrayer experiences $12 - \mu_{game\ 1}(10 - 0)$ for game 1 and $12 - \mu_{game\ 2}(11 - 0)$ for game 2. In this example $12 - \mu_{game\ 1}(10 - 0) < 10 => \mu_{game\ 1} \geq \frac{1}{5}$ and $12 - \mu_{game\ 2}(11 - 0) > 10 => \mu_{game\ 2} < \frac{2}{11}$.

[4]For the sake of simplicity I do not distinguish κ for each player respectively but assume that $\kappa_1 = \kappa_2 = \kappa$.

3.4 The model

Suppose players adjust their realized payoffs for expected emotions by applying a correction function. Emotions arise during the decision-making and have a direct impact on the outcome. The action player i chooses is the maximized payout she can receive, given the correction function. I assume that the game is symmetric and that the players are identical. Hence, an optimal decision for player i is also an optimal decision for player j. $\lambda \geq 0$ constitutes the jealousy coefficient. Disappointment is depicted by the $\kappa \geq 0$ coefficient and $\mu \geq 0$ describes the coefficient for remorse. κ and μ constitute anger. The coefficient is 1 if the result is cooperation for both agents . Players are rational utility maximizers in the sense of Selten (1991).

3.5 Equilibrium analysis in the symmetric two-player prisoner's dilemma

A more generalized way to depict a symmetric PD game is:
Consider a game with $n = 2$ players (i, j). Each player can decide whether to pick action A (to cooperate) or whether to choose action B (to defect). The payoffs read as follows:
$\pi_{j,BA} = \pi_{i,AB} < \pi_{i,BB} = \pi_{j,BB} < \pi_{i,AA} = \pi_{j,AA} < \pi_{i,BA} = \pi_{j,AB}$

		Player j	
		A	B
Player i	A	$\pi_{i,AA}; \pi_{j,AA}$	$\pi_{i,AB}; \pi_{j,AB}$
	B	$\pi_{i,BA}; \pi_{j,BA}$	$\underline{\pi_{i,BB}}; \underline{\pi_{j,BB}}$

Figure 3: Generalized prisoner's dilemma game.

The adjusted payouts in a symmetric two-players PD read for player i as follows:

$$\begin{aligned}
\widetilde{\pi}_{i,AA} &= \pi_{i,AA} \\
\widetilde{\pi}_{i,AB} &= \pi_{i,AB} - \lambda_i(\pi_{j,AB} - \pi_{i,AB}) - \kappa_i(\pi_{i,AA} - \pi_{i,AB}) \\
\widetilde{\pi}_{i,BA} &= \pi_{i,BA} - \mu_i(\pi_{j,AA} - \pi_{j,BA}) \\
\widetilde{\pi}_{i,BB} &= \pi_{i,BB} - \kappa_i(\pi_{i,AA} - \pi_{1,BB})
\end{aligned} \tag{1}$$

and mirror inverted for player j.

Proposition 1. *For AA to become an equilibrium the following conditions must be met:*

$\mu_i \geq \frac{\pi_{i,AA}+\pi_{i,BA}}{\pi_{j,AA}-\pi_{j,BA}}$ and $\mu_j \geq \frac{\pi_{j,AB}+\pi_{j,AA}}{\pi_{i,AA}-\pi_{i,AB}}$

Proof. AA is an equilibrium iff

$$\widetilde{\pi}_{i,AA} \geq \widetilde{\pi}_{i,BA} \text{ and } \widetilde{\pi}_{j,AA} \geq \widetilde{\pi}_{j,AB}. \tag{2}$$

Therefore, $\pi_{i,AA} \geq \pi_{i,BA} - \mu_i(\pi_{j,AA} - \pi_{j,BA})$ and $\pi_{j,AA} \geq \pi_{j,AB} - \mu_j(\pi_{i,AA} - \pi_{i,AB})$. □

Proposition 2. *AB is never an equilibrium.*

Proof. For AB to become an equilibrium the following conditions must be met:

$$\widetilde{\pi}_{i,AB} \geq \widetilde{\pi}_{i,BA} \text{ and } \widetilde{\pi}_{i,AA} \leq \widetilde{\pi}_{i,AB} \tag{3}$$

This results in $\pi_{i,AB} - \kappa_i(\pi_{i,AA} - \pi_{i,AB}) - \lambda_i(\pi_{j,AB} - \pi_{i,AB}) \geq \pi_{i,BB} - \kappa_i(\pi_{i,AA} - \pi_{i,BB})$. Hence, $\pi_{i,BB} - \pi_{i,AB} \leq -\kappa_i(\pi_{i,BB} - \pi_{i,AB}) - \lambda_i(\pi_{j,AB} - \pi_{i,AB})$ and consequently $(1+\kappa_i)(\pi_{i,BB} - \pi_{i,AB}) \leq -\lambda_i(\pi_{j,AB} - \pi_{i,AB})$. However, since $(1+\kappa_i)(\pi_{i,BB} - \pi_{i,AB}) > 0 > -\lambda_i(\pi_{j,AB} - \pi_{i,AB})$ this is never the case. Furthermore, $\pi_{i,AA} \leq \pi_{i,AB} - \kappa_i(\pi_{i,AA} - \pi_{i,AB}) - \lambda_i(\pi_{j,AB} - \pi_{i,AB})$ is never possible, due to the fact that $(1+\kappa_i)(\pi_{i,AA} - \pi_{i,AB}) \leq -\lambda_i(\pi_{j,AB} - \pi_{i,AB})$ ↯. □

Accordingly, the equilibrium BA is never possible either.

Proposition 3. *AA is at least as desirable as BB.*

Proof. First I show that BB remains an equilibrium:

$$\widetilde{\pi}_{i,AB} \leq \widetilde{\pi}_{i,BB} \text{ and } \widetilde{\pi}_{j,BA} \leq \widetilde{\pi}_{j,BB}. \tag{4}$$

Again, note that $(1+\kappa_i)(\pi_{i,BB} - \pi_{i,AB}) \geq -\lambda_i(\pi_{j,AB} - \pi_{i,AB})$ and $\pi_{j,BA} - \kappa_j(\pi_{j,AA} - \pi_{j,BA} - \lambda_j(\pi_{i,BA} - \pi_{j,BA}) \leq \pi_{j,BB} - \kappa_j(\pi_{j,AA} - \pi_{j,BB})$ which yields $(1+\kappa_j)(\pi_{j,BB} - \pi_{j,BA}) \geq -\lambda_j(\pi_{i,BA} - \pi_{j,BA})$ are always true. Now that we know that BB is still an equilibrium let us compare the two equilibrium outcomes AA and BB.
$\widetilde{\pi}_{i,AA} = \pi_{i,AA}$
$\widetilde{\pi}_{i,BB} = \pi_{i,BB} - \kappa_i(\pi_{i,AA} - \pi_{i,BB})$
$\widetilde{\pi}_{i,AA} \vee \widetilde{\pi}_{i,BB}$
$\pi_{i,AA} \vee \pi_{i,BB} - \kappa_i(\pi_{i,AA} - \pi_{i,BB}) => \pi_{i,AA} - \pi_{i,BB} \vee -\kappa_i(\pi_{i,AA} - \pi_{i,BB})$
$(1+\kappa_i)(\pi_{i,AA} - \pi_{i,BB}) > 0$
The previous Nash-equilibrium remains in place, but it is dominated by a new equilibrium, the Pareto-efficient Nash-equilibrium. □

4 The n-player game

4.1 Structure

Consider a game with $n \geq 3$ players, in which each player can decide whether to pick action A (to cooperate) or whether to choose action B (to defect).
If everyone decides to cooperate the payoff for each player is n, thus the payoff for the entire group is n^2.
If everyone defects the payoff for each player falls to 1, which results in n for all players together.
Assuming there are $0 < k < n$ defectors in the game, I define the payoff for the non-defectors: the players that decide on picking strategy A, as $n-k-1$. By contrast, the defectors which represent the players choosing strategy B receive $2n-k$. Note that the sum of all payoffs in a game with $0 < k < n$ defectors is $n^2 - (n-k)$. This sum is smaller than n^2 for $k \in [1, n-1]$.
Likewise, $n^2 - (n-k) \geq n^2 - (n-1) \geq n => n^2 - 2n + 1 = (n-1)^2 \geq 0$.
These are the necessary conditions for a prisoner's dilemma.

4.2 The payoff structure in a symmetric n-person game

Different situations can occur:
S_I is a multi-index with $I = i_1 \dots i_k$ indicating the amount of defectors: $S_0 = AA\dots A$ describes a situation in which no one defects. All players choose action A.
$S_1 = BA\dots A$ describes a situation in which Player 1 defects, everyone else remains a cooperator.
$S_2 = AB\dots A$ describes a situation in which Player 2 defects, everyone else remains a cooperator.
$S_{12} = BB\dots A$ describes a situation in which Player 1 and Player 2 defect, everyone else remains a cooperator.

The payoff for player l in the event of k defectors playing the game is $\pi_{l,S_I} = \pi_{l,S_{i_1 \dots i_k}}$.
Four different scenarios may arise:

Scenario 1: There are no defectors in the game. Everyone obtains:

$$\widetilde{\pi}_{l,S_0} = \pi_{l,AA\dots A}. \tag{5}$$

Scenario 2: Player l is a defector: Player $l = i_m$ for $1 \leq m \leq k$:

$$\widetilde{\pi}_{l,S_I} = \pi_{l,S_0} - \mu_l \sum_{j \neq i_1,\dots,i_k} (\pi_{j,S_0} - \pi_{j,S_I}). \tag{6}$$

Scenario 3: There are defectors in the game. However, player l cooperates:

$$\widetilde{\pi}_{l,S_I} = \pi_{l,S_0} - \kappa_l(\pi_{l,S_0} - \pi_{l,S_I}) - \lambda_l \sum_{m=1}^{k} (\pi_{j_m,S_I} - \pi_{l,S_I}). \tag{7}$$

Scenario 4: Everyone defects:

$$\widetilde{\pi}_{l,S_I} = \pi_{l,S_0} - \kappa_l(\pi_{l,S_0} - \pi_{l,S_I}). \tag{8}$$

These four scenarios can be summarized as follows:

$$\begin{aligned} \widetilde{\pi}_{l,S_I} = \pi_{l,S_I} - \kappa_l \frac{sgn(\pi_{l,S_0} - \pi_{l,S_I}) + 1}{2} \cdot (\pi_{l,S_0} - \pi_{l,S_I}) \\ - \lambda_l \sum_{m=1}^{k} (\pi_{j_m,S_I} - \pi_{l,S_I}) \\ - \mu_l \sum_{j=1}^{n} \left\lfloor \frac{sgn(\pi_{l,S_I} - \pi_{j,S_I}) + 1}{2} \right\rfloor \cdot (\pi_{j,S_0} - \pi_{j,S_I}). \end{aligned} \tag{9}$$

Equation (9) shows the corrected payoff for player l. To be more precise, this is the perceived payoff with all information, and particularly emotions, included. Usually $\widetilde{\pi}_{l,S_I} \neq \pi_{l,S_I}$, even though rational choice theory assumes exactly the opposite, namely $\widetilde{\pi}_{l,S_I} = \pi_{l,S_I}$. Hence decision-makers in the real world might behave differently from what a model predicts. This gives rise to seemingly bizarre results, such as cooperation in one shot PD games.

5 Immediate emotions

Loewenstein et al. (2003) propose the following model: $u(c_t, z_t)$ is a person's utility in period t, given state z and consumption level c. A person currently facing state z' is predicting a future utility $\widehat{u}(c, z|z')$, whenever she tries to foretell $u(c, z)$. Broadly speaking, this means that a person's predicted utility is in between her true future utility and her utility given her current state. Thus, a decision-maker underestimates the effect of her instantaneous utility on the predicted utility.

Definition 1. A decision-maker l faces expected and immediate emotions if there exists $\alpha \in [0, 1]$ such that for all $\widetilde{\pi}$,z, and z',

$$\widehat{u}(\widetilde{\pi}, z|z') = (1 - \alpha)u(\widetilde{\pi}, z) + \alpha u(\widetilde{\pi}, z'). \tag{10}$$

Note that both types of emotions enter the equation. First of all, expected emotions impact the utility of a player; however, the emotional state and thus immediate emotions also have an effect of the individual's utility level. Let the functional form of the utility be:

$$u(\widetilde{\pi}, z) = v(\widetilde{\pi} - \alpha\psi(z)). \tag{11}$$

First the marginal utility from $\widetilde{\pi}$ is increasing in the emotional state ($\partial \left[\partial u/\partial\widetilde{\pi}\right] \partial z > 0$). Second, the level of utility is declining in the emotional state ($\partial u/\partial z < 0$). Note that for $\alpha = 0 : u(\widetilde{\pi}, z) = v(\widetilde{\pi})$ and for $\alpha = 1 : u(\widetilde{\pi}, z) = v(\widetilde{\pi} - \psi(z))$.

5.1 Example for $\psi(z)$

5.1.1 The affective person

Let us consider the following C^1 function for $A, B, C, D > 0$:

$$\psi(z) = \begin{cases} Ae^{-B(z-z_0)^2}, & z < z_0 \\ A - Ce^{-\frac{D}{(z-z_0)^2}}, & z > z_0 \end{cases}$$

with $Y_{max} = Y(z_0), Y'(z) < 0$ for $z > z_0$ and $Y'(z) > 0$ for $z < z_0$:

$$\psi'(z) = \begin{cases} Ae^{-B(z-z_0)^2}(-2B(z-z_0)), & z < z_0 \\ -Ce^{-\frac{D}{(z-z_0^2)}} \frac{2D}{(z-z_0)^3}, & z > z_0 \end{cases}$$

The left part of the function ($z < z_0$) is a classical bell shape with $\psi(0) = Ae^{-Bz_0^2} > 0$. The right part of the function ($z > z_0$) is also a bell function. The magnitude of the bell depends on C and D. $\psi(z) < 0$ once a certain z is surpassed. Thus, little changes in z might cause extreme shifts in ψ.

5.1.2 The ln person; $\psi(z) = ln(z)$

(10) leads to:

$$\begin{aligned} \widehat{u}(\widetilde{\pi}, z|z') =& (1-\alpha)u(\widetilde{\pi}, z) + \alpha u(\widetilde{\pi}, z') \\ =& (1-\alpha)v(\widetilde{\pi} - \alpha ln(z)) + \alpha v(\widetilde{\pi} - \alpha ln(z')). \end{aligned} \tag{12}$$

Jensen's inequality yields:

$$\begin{aligned} &(1-\alpha)v(\widetilde{\pi} - \alpha ln(z)) + \alpha v(\widetilde{\pi} - \alpha ln(z')) \leq \\ &v((1-\alpha)(\widetilde{\pi} - \alpha ln(z)) + \alpha(\widetilde{\pi} - \alpha ln(z'))). \end{aligned}$$

Therefore,

$$\widehat{u}(\widetilde{\pi}, z|z') \leq v((1-\alpha)(\widetilde{\pi} - \alpha ln(z)) + \alpha(\widetilde{\pi} - \alpha ln(z'))).$$

Note that

$$v((1-\alpha)(\widetilde{\pi}-\alpha ln(z))+\alpha(\widetilde{\pi}-\alpha ln(z'))) =$$
$$v(\widetilde{\pi}-\alpha\left[(1-\alpha)ln(z)+\alpha ln(z')\right].$$

Because $(1-\alpha)ln(z)+\alpha ln(z') = ln(z^{1-\alpha}z'^{\alpha})$, $v(\widetilde{\pi}-\alpha\left[(1-\alpha)ln(z)+\alpha ln(z')\right] = v(\widetilde{\pi}-\alpha ln(z^{1-\alpha}z'^{\alpha}) = u(\widetilde{\pi}, z^{1-\alpha}z'^{\alpha})$. With $\widehat{z} = z^{1-\alpha}z'^{\alpha}$,

$$\widehat{u}(\widetilde{\pi}, z|z') \leq u(\widetilde{\pi}, \widehat{z}) = u(\widetilde{\pi}, z^{1-\alpha}z'^{\alpha}). \tag{13}$$

(13) provides a better upper bound for $\widehat{u}$ than $max\left[u(\widetilde{\pi}, z); u(\widetilde{\pi}, z')\right]$. Since $u(\widetilde{\pi}, \widehat{z}) \geq (1-\alpha)u(\widetilde{\pi}, z) + \alpha u(\widetilde{\pi}, z') = u(\widetilde{\pi}, z) + \alpha(u(\widetilde{\pi}, z') - u(\widetilde{\pi}, z))$, three cases can be distinguished:

Case 1: The player overestimates her utility: $u(\widetilde{\pi}, z') > u(\widetilde{\pi}, \widehat{z}) > u(\widetilde{\pi}, z)$

$$0 \leq \alpha \leq \frac{u(\widetilde{\pi}, \widehat{z}) - u(\widetilde{\pi}, z)}{u(\widetilde{\pi}, z') - u(\widetilde{\pi}, z)} \tag{14}$$

Case 2: The player estimates her utility properly.

Case 3: The player underestimates her utility: $u(\widetilde{\pi}, z') < u(\widetilde{\pi}, \widehat{z}) < u(\widetilde{\pi}, z)$

$$1 \geq \alpha \geq \frac{u(\widetilde{\pi}, z) - u(\widetilde{\pi}, \widehat{z})}{u(\widetilde{\pi}, z) - u(\widetilde{\pi}, z')} \tag{15}$$

We have already seen that expected emotions might lead to cooperation in PD games [5]. If expected emotions were the only type of emotions that influence the decision of the player, she would never defect in this particular situation. However, the agent is also subject to immediate emotions and calculates her utility according to (11). Immediate emotions enter her utility calculation through the term $-\alpha\psi(z)$ which in turn depends on the emotional state z. This term might eventually prevent the agent from cooperation.

Although I base my article on the PD game there are plenty of other examples: Imagine a player has the possibility to buy a lottery ticket for \$

[5] Recall 3.1 and equation (1) which results in the agents adjusting the payoffs as follows:

$$\widetilde{\pi}_{1,AA} = \pi_{1,AA} = 3$$
$$\widetilde{\pi}_{1,AB} = \pi_{1,AB} - \lambda_1(\pi_{2,AB} - \pi_{1,AB}) - \kappa_1(\pi_{1,AA} - \pi_{1,AB}) = 0 - \lambda_1(5-0) - \kappa_1(3-0)$$
$$\widetilde{\pi}_{1,BA} = \pi_{1,BA} - \mu_1(\pi_{2,AA} - \pi_{2,BA}) = 5 - \mu_1(3-0)$$
$$\widetilde{\pi}_{1,BB} = \pi_{1,BB} - \kappa_1(\pi_{1,AA} - \pi_{1,BB}) = 1 - \kappa_1(3-1)$$

For the sake of simplicity I assume that both players are identical. If $\kappa = \lambda = 0.5$ and $\mu = 0.8$ than $\widetilde{\pi}_{AA} = 3, \widetilde{\pi}_{AB} = -4, \widetilde{\pi}_{BA} = 2.6$ and $\widetilde{\pi}_{BB} = 0$, hence both players cooperate.

1.00 which yields $ 1.000.000 with the probability of 0.0000001. Hence, the expected value of the lottery is a minuscule $ 0.1. Let us further assume that expected emotions make the player derive a higher utility from the lottery, namely the utility equivalent to $ 1.10. Clotfelter & Cook (1991) describe how hope might be the reason for people to overvalue lottery tickets. "Good feeling days" may enforce the gambler's willingness to pay for the lottery, since her positive feelings might translate into a more optimistic conception of her future if she hits the jackpot. Contrary, a pessimistic agent might view the likelihood of becoming a lotto millionaire as less likely and therefore be less willing to purchase the lotto ticket. Immediate emotions might be a trigger for the person's decision as to whether or not to buy the lottery ticket.

6 Conclusion

Agents compare their payoffs to the payoffs of their counter players. Agents also compare the payoff they receive in one particular situation to the payoff they receive in another situation. This relativization allows us to model expected emotions a decision taker might expect to feel once the outcome of a decision materializes. The decision maker might also be subject to immediate emotions during the actual instant of deciding. Emotions are most certainly effortlessly modeled, but one can be certain that they have a compelling impact on our decision-making. Economists have not necessarily embraced the idea of adding emotions to the utility calculus; however, continuous development is definitely happening. In this paper, I have attempted to show that emotions may alter the outcome of traditional PD games dramatically, and to model it accordingly. As shown earlier, emotions can make way to a new Pareto-efficient Nash-equilibrium. There are arbitrarily many real world situations and problems in economics that should be modeled as games under emotions. Thus, it is only essential to accommodate emotions into the bigger equation of real world behavior analysis.

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