Production, Appropriation and the Dynamic Emergence of Property Rights*

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May 14, 2010

Abstract

This paper analyzes the dynamic emergence of property rights in a decentralized economy devoid of an exogenous enforcement mechanism. Imperfection in property rights enforcement gives rise to appropriative activities, which take away resources from productive activities, and thus, hampers the performance of the economy. Therefore, agents in the economy strategically invest in definition and enforcement of property rights to limit the detrimental appropriative competitions for the use of resources. Using a differential game framework, this paper obtains the open-loop and the Markov-perfect equilibrium level of property rights enforcement in an economy. The exact level depends on the economy's characteristics, such as fractionalization, value of affected assets, productivity of the tools employed to build the institution of property rights, future discount rate, as well as the economy's norms, culture and traditions.

*I would like to thank my supervisors Michelle Garfinkel and Stergios Skaperdas for their help, support and encouragement. I also appreciate generous research support provided by University of California, Irvine's Department of Economics, Center for Global Peace and Conflict Studies and Institute for Mathematical Behavioral Sciences.

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"The first principle of economics is that every agent is actuated only by self-interest. The workings of this principle may be viewed under two aspects, according as the agent acts without, or with, the consent of others affected by his actions. In wide senses, the first species of action may be called war; the second contract."

Edgeworth (1881)

"The efforts of men are utilized in two different ways: they are directed to the production or transformation of economic goods, or else to the appropriation of goods produced by others."

Vilfredo Pareto.[1]

"The fundamental purpose of property rights, and their fundamental accomplishment, is that they eliminate destructive competition for control of economic resources. Well-defined and well-protected property rights replace competition by violence with competition by peaceful means."

Alchian (2006)

1 Introduction

Economists have long recognized the role of property rights in determining the resource allocation and output distribution, and thus, in shaping the incentive structures for successful economic performance of the economy. Acemoglu et al. (2005) points to work by John Locke, Adam Smith, John Stuart Mill, among many others on the topic. However, when it comes to a systematic economic analysis, economists, barring a few, tend to preclude study on the subject by assuming the presence of perfectly enforced property right in models.

The notable exceptions are Coase (1960) and Demsetz (1967), who study the role of property rights in resource allocation. Grossman and Hart (1986) and Hart and Moore (1990) extend the previous studies, and provide an elaborate and precise model on the assignment of property rights in an economic organization. While assignment of property rights is one important aspect, property rights can also be defined on the basis of how output from the use of resources is distributed. Alchian (1965) has defined the property rights over an asset as the ability to enjoy the outcome from the

[1] As quoted in Hirshleifer (2001).

use of that asset. Barzel (1997) christened the way property rights are assigned as a 'legal property right regime' and the way output is distributed as a 'economic property right regime'. Thus, economic rights are the end which economic agents seek while legal rights are means to achieve that end. The present paper takes the assignment of property right as given, and deals solely with the role of property rights in resource allocation and output distribution.

Economic studies which explicitly model the institutions of property rights tend to assume fixed structures of property rights in the economy. And, when it is beneficial, economies jump from one regime to another regime, in a costless manner (Gradstein, 2004), or by incurring one time fixed cost (Tornell, 1997). In reality, a regime change is neither perfect nor fixed, but rather evolves continuously over time. Thus, another aspect of property rights that is left untouched by previous models, the costly transition and maintenance of the institution of property rights. A report by the Word Bank (1997), *the state in a changing world*, emphasizes not only the need to have secured property rights but also the need to pay attention to the evolving nature of property rights for the development of third world countries. Given the importance of the subject, this paper develops a model of evolution of property rights in a decentralized economy.

A decentralized economy is defined as an economy in which a state actor who can enforce the property right is absent. While the choice of such an economy for the model is stark, the description is very close to reality in many developing and underdeveloped countries, as well as in many informal and unorganized economic sectors. In places where the state actor can not and does not enforce property rights, individuals and groups in these economies attempt to establish institutions. In some cases such private institutions are able to enforce property rights. Ensminger (1992) describes one such case study of Orma tribe in Kenya. Greif (2006) presents another case study of Maghribi traders' coalition in medieval Europe who developed a private order to enforce property rights. Instead of taking an outside enforcer of property rights as given, the present model discusses the evolution of property rights in an homogeneous economy.

When do property rights evolve? Demsetz (1967), Cheung (1970), Pejovich (1972), among many others, have proposed theories of property right emergence. All of them agree that a well defined property right system evolves when the benefit of having secured property rights outweighs the cost of having it. Demsetz (1967), for example, suggests “that property rights arise when it becomes economic for those affected by externalities to internalize benefits and costs.” Demsetz and others support their theory from case studies. They neither provide an analytical model nor mention variables which affect the benefits and costs of institution of property rights. This paper explicitly describes variables affecting the benefit and costs and develops a functional relationship among them.

The paper tries to answer questions such as why, when and how the institution of property rights emerges. However, the introduction of a model and two neglected but important aspects of property rights: its role in resources allocation and output distribution, and its continuous evolution set this paper apart from the existing literature.

The framework in the paper comes from the “state of nature models” of conflict and appropriation (Skaperdas, 1992; Grossman and Kim, 1995; Garfinkel and Skaperdas, eds, 1996). Instead of assuming a perfect market, where property rights are perfectly secured, these models begin the analysis from anarchy. In anarchy[2] no property rights are automatically enforced. The absence of enforcement gives rise to various appropriation activities which hamper economic performance, as these activities take away resources from productive activities. The balance between productive activities leading to higher wealth, along with conflict to decide who gets the wealth, plays the central role in these models. An economic agent, who is truly self interested and rational, balances on the margin two alternative way of generating income: a peaceful production or a forceful appropriation of goods produced by others.

The imperfection in enforcement gives rise to appropriation activities which hamper economic performance of the economy as these activities take away resources from productive activities. Even in the absence of property rights enforcement, economic activity can not grind to a halt, otherwise too much potential of the economy would go unrealized. Individuals and groups in these economies develop private institutions to provide the needed mechanism for economic activities to take place

[2] Hirshleifer (1995) defines anarchy as a system in which economic agents can seize and defend resources without regulations from the above.

(Dixit, 2004). The contributions from economic agents improve the level of property right enforcement, which in turn, dampens the severity of appropriative activities in the economy. However, cooperation over such a collective good is not easy to achieve because of the free rider problem. In dynamic settings, the shadow of the future somewhat mitigates the effects of the free rider problem.

Following Demsetz (1967), the institution of property rights is treated as a public good that is produced by economic agents. It is useful to discuss two polar scenarios: a commiment scenario which presupposes a full commitment by agents at the beginning of the game to follow the agreed upon contribution towards the institution of property rights, and the noncooperative scenario, in which economic agents choose their contribution to further their own interest given other agents contributions and the level of property right in the economy. The Nash-equilibrium is found in both scenarios. It needs to be emphasized that the Nash equilibrium in the first scenario may not be sub-game perfect, though, the equilibrium in the second scenario is self-enforcing, which means that at any stage in the game, it is in the best strategy of each agent to follow the equilibrium strategy. The presence of equilibrium strategy doesn't rule out any pre-game play, but only makes the equilibrium strategies renegotiation proof. The second scenario appears more intuitive than the committed scenario. However, both scenarios are desribed in detail. The use of cooperative scenario in this paper can be, in part, justified on its possible application in some close knit economies and its use in providing benchmark for the second scenario.

2 The Model

Consider an economy populated by $n \geq 2$ infinitely lived identical agents. These agents can be individuals or groups. If groups, intra-group conflicts and the free-rider problem are assumed to be resolved and each group acts as a unitary actor. At time t, an agent $i \in N = \{1, \ldots, n\}$ allocates her endowment $R_i(t)$ among production (e_i), appropriation (a_i) to seize part of other's produce, and investment (g_i) to strengthen the institution of property rights in the economy. Her resource constraint is, then, given by the following:

$$e_i(t) + a_i(t) + g_i(t) = R_i \qquad R_i(t) = R, \ \forall i \in \{1, \ldots, n\} \ \& \ \forall t \geq 0. \tag{1}$$

The production technology, which transforms agent i's effort into consumable goods y_i, is linear and given in the following equation.

$$y_i(t) \quad = \quad A \cdot e_i(t) \tag{2}$$

where A is the total factor productivity in the economy.

After this allocation is made, production takes place. Subsequently, individuals try to seize a part of other agents' produce. The fraction of each agent's output that is subject of appropriation depends on the level of property-rights enforcement $\theta \in [0, 1]$ in the economy. The total amount of output, subject to appropriation, $(X(t))$ is given in the following equation:

$$X(t) \quad = \quad (1 - \theta) \sum_{j=1}^{n} A \cdot e_j(t). \tag{3}$$

Agent i's share of the contestable output from the common pool depends on her appropriative effort vis-a-vis the appropriative effort of all other agents. The share of agent i is given by following appropriation technology:[3]

$$P_i(t) \quad = \quad \begin{cases} \frac{a_i(t)}{\sum_{j=1}^{n} a_j(t)} & \sum_{j=1}^{n} a_j > 0 \\ \frac{1}{n} & \text{Otherwise.} \end{cases} \tag{4}$$

Agent i's total consumption at any time t, $c_i(t)$, depends on her current level of productive effort, appropriative efforts and the level of property right enforcement in the economy.

$$\begin{aligned} c_i(t) \quad &= \quad \theta(t) A \cdot e_i(t) + P_i(t)(1 - \theta(t)) \sum_{j=1}^{n} A \cdot e_j(t) \qquad (5) \\ &= \quad \theta(t) A \cdot (R - a_i(t) - g_i(t)) + P_i(t)(1 - \theta(t)) \sum_{j=1}^{n} A \cdot (R - a_j(t) - g_j(t)) \,. \end{aligned}$$

[3]The appropriation technology has an alternative interpretation in that it gives agent i's probability of winning all goods in the appropriative pool.

The first term in the expression above represents the part of agent i's production not subject to appropriation by others, while the second term gives the share appropriated by agent i from the contestable pool.

All agents share identical preferences over consumption goods, and they maximize their utility over an infinite horizon. Agent i's instantaneous utility at time t is given as:[4]

$$u_i(t) \quad = \quad \log(c_i(t)). \tag{6}$$

$u_i(\cdot)$ is concave and increasing in agent i's consumption. The preferences of the agent i for consumption over time are aggregated by integrating the discounted sum of instantaneous utilities:

$$U_i(t) \quad = \quad \int_t^{\infty} e^{-\rho(\tau-t)} u_i(\tau) d\tau. \tag{7}$$

The parameter ρ is the rate of time preference in the economy, and is assumed to be strictly positive.

Having imperfect property right enforcement has negative consequences as it induces fighting, though improved property rights help avoid this negative consequence. However, the evolution of property rights is not taken as exogenous: the degree of property right enforcement at any given time depends on previous actions of economic agents in the economy, as they invest in defining and improving property-rights enforcement. We assume a linear property right production function that is additively separable in different agents' efforts, $\{g_j\}_{j=1}^n$. It is also assumed that the institution of property rights depreciates at constant rate δ. The equation of motion for θ is given by the following equation:

$$\dot{\theta} \quad = \quad B \sum_{j=1}^{n} g_j(t) - \delta\theta, \quad B > 0. \tag{8}$$

where B is the factor productivity of agents to build the institution of property rights in the econ-

[4]The use of a logarithmic utility function in place of a more general CRRA function form is to keep exposition simple.

omy.

Each agent, i, in maximizing her life time utility chooses how many resources to devote to production, appropriation, and to strengthen property rights. The optimization problem for agent i at time t is:

$$\max_{\{e_i,a_i,g_i\}_t^\infty} \int_t^\infty e^{-\rho(\tau-t)} \log(c_i(\tau))d\tau \tag{9}$$
$$\text{Subject To} \ : \qquad \dot{\theta} = B\sum_{j=1}^{n} g_j(\tau) - \delta\theta,$$
$$\theta(t) = \theta_t, \qquad \theta(\tau) \in [0,1],$$
$$\& \quad e_i(t) + a_i(t) + g_i(t) = R$$

where $c_i(t)$ is given in equation (5).

Since at any moment of time, agent i's endowment of resources, R, is exogenous, only two choices need to be made. One of these choice variables is investment g_i to improve the level of property right enforcement in the future; and have no effect on the current level of property right enforcement. The other two choice variables are appropriation efforts, a_i, and productive effort, e_i which do not affect present or future level of property rights, thus one can consider the individual's allocation of resource in two parts; the static allocation problem which takes $\{g_j\}_1^n$ as given, and the dynamic problem to choose g_i, given the solution of the static problem.

3 Equilibrium Analysis

Agent i's static optimization problem at time t given $\{g_j\}_{j\neq i}$ can be expressed as:[5]

$$\max_{e_i,a_i} \ \log(c_i), \qquad \text{Subject To} \ : \quad e_i + a_i = R - g_i. \tag{10}$$

log is an increasing function of its argument, so whatever values of a_i and e_i maximize c_i maximize

[5] Even though the model is dynamic, time subscripts are suppressed where possible to avoid notational cluttering.

$\log(c_i)$. Using the expression for c_i from equation (5), and taking $R - g_j, \forall j$, as given, one can give the maximization problem of agent i as:

$$\max_{a_i}\ \theta(t)A \cdot (R - a_i(t) - g_i(t)) + P_i(t)(1 - \theta(t)) \sum_{j=1}^{n} A \cdot (R - a_j(t) - g_j(t)) \tag{11}$$

Agents i takes other agents, choice of appropriation efforts, $\{a_j\}_{j \neq i}$ and chooses a_i to maximize her payoff. The appropriation technology shown in equation (4) ensures that all agents make a positive appropriation effort as long as $X(t) > 0$. As if all but one makes zero effort for appropriating the common pool, that one agent needs to make only an infinitesimally small effort to capture the entire common pool. Hence, $\sum a_j(t) > 0$. Therefore, in the homogeneous economy, all agents expend positive effort. Agent i's optimizing appropriation effort level satisfies the following first-order condition:

$$-\theta A - \frac{a_i}{\sum_{j=1}^{n} a_j(t)}(1-\theta)A + \frac{\sum_{j \neq i} a_i}{(\sum_{j=1}^{n} a_j(t))^2}(1-\theta)\sum_{j=1}^{n} A \cdot (R - g_i - a_i) \leq 0 (= 0 \text{ if } a_i > 0), \forall i \in N \tag{12}$$

At the margin, an increase in appropriation effort by individual a_i implies decreased production of good, y_i. This effect is reflected in first two terms. The first term represents the marginal decrease in the secured share of production; the second term represents the marginal decrease in agent i's share of appropriated goods from common pool $X(t)$ because her increased appropriation effort decreases the size of production, thus decreasing her contribution to the common pool. The third term represents the marginal increase in the fraction that agent i captures from the common pool due to increased appropriation effort.

The first order conditions in appropriation efforts for all agents of N, if satisfied with equality, yield the following result[6] :

Proposition 1. There exists a unique, symmetric pure-strategy Nash equilibrium in appropriation

[6]The economy is populated by identical agents having the same endowments. As such, satisfying each first order conditions simultaneously as a strict equality requires only that there is not too much variation among individuals' choice of $g_j s, \forall j$.

efforts provided agents' contribution towards improving property rights are sufficiently close. The equilibrium appropriation effort profile can be expresed as

$$a_i^* \quad = \quad a^* \ = (1-\theta)\frac{(n-1)}{n}(R - \sum_{j=1}^{n}\frac{g_j}{n}), \quad \forall i \in N. \tag{13}$$

Proof: See Appendix

Corollary 1.1. The maximized payoff expected by agent i, C_i in equation (10), in this symmetric equilibrium is given by:

$$C_i \quad = \quad A\left[\left(\frac{1}{n} + \frac{n-1}{n}\theta\right)R - \left(\theta + \frac{1-\theta}{n^2}\right)g_i - \left(\frac{1-\theta}{n^2}\right)\sum_{j\neq i} g_j\right]. \tag{14}$$

C_i is obtained using $a_j^* = a^*, \forall j \in N$, in equation (5).

As described earlier, efforts $g_i(t), \forall i$, work towards improving the level of property rights in the future. It follows immediately, from the above expression, that an increase in g_i induces two opposite effects. It increases θ, thus increasing the future value of total available consumable good for agent i, C_i; however, it also decreases the current C_i. As mentioned, an infinitely lived agent i will dynamically optimize the effort levels for production, appropriation, and strengthening future property rights.

3.1 The Dynamic Allocation Problem

Using the result from the static allocation problem, agent i's optimization problem at time t expressed in equation (9) can be restated as

$$\max_{\{g_i\}_t^\infty} \int_t^\infty e^{-\rho(\tau-t)} \log(C_i(\tau)) d\tau \quad \forall i \in N \tag{15}$$

$$\text{Subject To} \qquad \dot{\theta} = B \sum_{j=1}^{n} g_j(\tau) - \delta\theta,$$

$$\theta(t) = \theta_t, \quad \& \quad \theta(\tau) \in [0,1].$$

where $C_i(t)$ is obtained as a solution of the static allocation problem, and is expressed in equation (14).

In control-theoretic problems, the felicity equation and the state of motion is typically exogenously given. Here, however, both the felicity equation and the equation of motion for θ are determined endogenously. They depend on the strategies of all individuals in the economy. Such problems are modeled as differential games.[7] In the setting, agents are viewed making their choice of g_j in a noncooperative manner. In a differential game, the players interact repeatedly through time. However, the differential game is not a simple repetition of the original game. Instead, there is a state variable θ which continuously changes. Since, other agents' choice variable affects agent i's optimization problem, she must take into account the other agents' choice of control variable $\{g_j\}_{j \neq i}$ in choosing her control variable g_i. As this is true for all agents $j \in N$, each agent needs to choose her control variable so as to maximize her payoff for every possible choice of other player's control variable. All agents choose their control variable simultaneously. Accordingly, in order to make optimal choice, agents need to guess what others are doing and going to do in the future. After observing the real choices, some agents might like to revise their choice of control variable. When there exist no incentive for any agent to revise their choice of control variable, then the choices are said to be in a Nash equilibrium. If the expression J_i denotes agent i's objective function

$$J_i(\{g_1\}, \ldots, \{g_i\}, \ldots, \{g_n\}) \quad = \quad \int_t^\infty e^{-\rho(\tau-t)} \log(C_i(\tau)) d\tau \quad \forall i \in N,$$

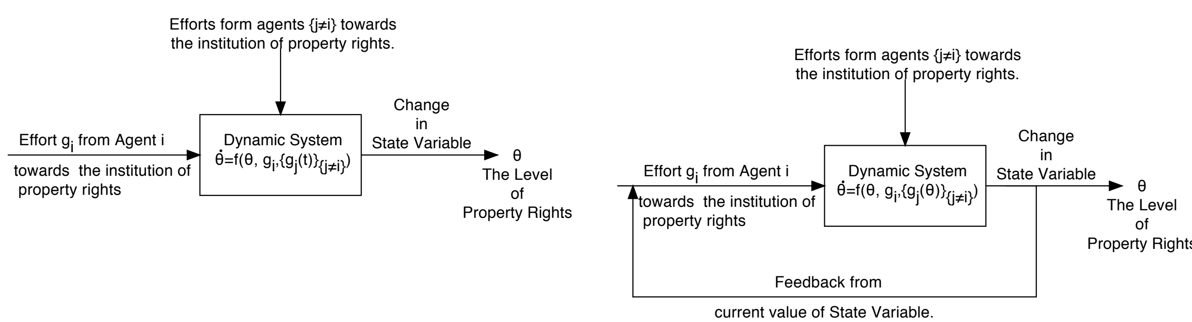
[7]For excellent introduction, see Kamien and Schwartz (1991) Sec.23, and for details see Dockner et al. (2000)

where $C_i(\tau)$ is a function of $g_j(\tau), \forall j \in N$, and $\{g_j\}$ in the above expression means $\{g_j\}_t^{\infty}$. Then, symbolically, the Nash equilibrium can be given as:

$$J_i(\{g_1^*\}, \ldots, \{g_i^*\}, \ldots, \{g_n^*\}) \quad \geq \quad J_i(\{g_1^*\}, \ldots, \{g_i\}, \ldots, \{g_n^*\}) \quad \forall i \in N \tag{16}$$

where superscript "*" denotes the equilibrium strategy.

As in any game-theoretic problem, the information structure available in the present problem plays a very important role in determining the equilibrium strategies of agents. The two most commonly employed assumptions regulating the information structures in differential games are: 1) each agent is aware of the initial condition of the state variable, 2) each agent observes the current state variable. The corresponding strategies are called "open-loop" and "feedback", respectively. Since open-loop strategies are conditioned on the initial value of the state variable, they imply that each player has committed to his entire course of action in the beginning of game and will not revise her strategy at any point of time (figure 1(a)). In the present problem, the open loop game corresponds to a cooperative scenario where agents are aware of the initial level of property rights enforcement; and based on this value either they commit to the life time stream of choice variable or they fail to observe the evolution of property rights. Both requirements, that agents either commit to entire sequence of actions through time in the beginning of game or do not observe the evolution of property rights, are relatively stringent and can be achieved only in few special conditions.



(a) Open Loop System

(b) Close Loop/ Feed Back System

Figure 1: Differential Games

Alternatively, we could assume all agents observe the current level of property rights enforcement, and then have option to revise their action throughout the game. This type of strategy is called feedback strategy, and is characterized by the requirement that the choice variable is a function of both time and the state of the system (figure 1(b)). Apart from considering the open-loop strategy for the benchmark, a special case of feed-back strategy is considered in the paper, that is stationary Markov feedback strategy. The term stationary indicates that the feedback strategy depends on time only through the state variable θ. The Markov perfection of the strategy implies that all the information about the state variable is captured into its current value. Feedback strategies have the property of being subgame perfect.

$$g_i \quad = \quad g_i(\theta(t)). \tag{17}$$

To grasp concepts, first open loop equilibrium is discussed in detail, and then Markov perfect feedback equilibrium is characterized.

3.1.1 The Open-Loop Equilibrium

When agents commit to an action plan at the beginning of the game and stick to that plan for ever, the open loop solution characterize the equilibrium behavior. Each agent takes the other agents' control variable as the function of time only. The current value Hamiltonian function for group i is:

$$H_i \quad = \quad \log\left(A\left[\left(\frac{1}{n}+\frac{n-1}{n}\theta\right)R-\left(\theta+\frac{1-\theta}{n^2}\right)g_i-\left(\frac{1-\theta}{n^2}\right)\sum_{j\neq i}g_j\right]\right)+\lambda_i\left[B\sum_{j=1}^{n}g_j-\delta\theta\right]$$

where λ_i is the shadow price that agent i sees associated to θ.[8] Necessary conditions for optimality include satisfaction of the equation of motion, constraints, and the following first order conditions.

[8] From the state of motion it is clear that $\theta \geq 0$ is always satisfied as θ falls to 0, it can not decrease thereafter. In order to restrict the equilibrium θ below 1, it is needed to introduce a multiplier with constraint $\theta \leq 1$ and form a Lagrangian, but to keep analysis simple, it is assumed that B is sufficiently small, so $\theta = 1$ is never achieved.

The first order condition with respect to g_i:

$$\frac{\partial H_i}{\partial g_i} = \frac{-\left(\theta + \frac{1-\theta}{n^2}\right)}{\left(\frac{1}{n} + \frac{n-1}{n}\theta\right) R - \left(\theta + \frac{1-\theta}{n^2}\right) g_i - \left(\frac{1-\theta}{n^2}\right) \sum_{j \neq i} g_j} + \lambda_i B \tag{18}$$

The first order condition with respect to state variable θ (The adjoint equation)

$$\frac{\partial H_i}{\partial \theta} = \frac{\frac{n-1}{n} R - \left(1 - \frac{1}{n^2}\right) g_i + \frac{1}{n^2} \sum_{j \neq i} g_j}{\left(\frac{1}{n} + \frac{n-1}{n}\theta\right) R - \left(\theta + \frac{1-\theta}{n^2}\right) g_i - \left(\frac{1-\theta}{n^2}\right) \sum_{j \neq i} g_j} - \lambda_i \delta = \rho \lambda_i - \dot{\lambda}_i \tag{19}$$

For an economy populated by identical agents, it can be shown that the denominator in the first order condition is always positive. Solving the first-order conditions with respect to g_i, for all $i = 1, 2...n$, for a symmetric result yields:

$$-\frac{\theta + \frac{1-\theta}{n^2}}{\left(\frac{1}{n} + \frac{n-1}{n}\theta\right)(R-g)} + \lambda B \begin{cases} > & 0 \quad \Rightarrow g = R \\ = & 0 \quad \Rightarrow g \in (0, R) \\ < & 0 \quad \Rightarrow g = 0 \end{cases} \qquad \lambda_i = \lambda, \ \& \ g_i = g \ \forall i \in N \tag{20}$$

At any interior optimum, the following holds:

$$\lambda B = \frac{\theta + \frac{1-\theta}{n^2}}{\left(\frac{1}{n} + \frac{n-1}{n}\theta\right)(R-g)} = \frac{\left(1 + (n^2-1)\theta\right)}{n\left(1 + (n-1)\theta\right)(R-g)} \tag{21}$$

The above relation gives an expression for the shadow price of property rights, λ, in terms of choice variable, $g_j = g, \forall j$. Using this expression the equation of motion can be expressed in terms of λ:

$$\dot{\theta} = BnR - \frac{n^2\theta + 1 - \theta}{1 + (n-1)\theta} \cdot \frac{1}{\lambda} - \delta\theta \tag{22}$$

The first order condition for θ implies:

$$\dot{\lambda} = (\rho + \delta)\lambda - \frac{n-1}{1 + (n-1)\theta} \tag{23}$$

The above two equations along with the initial condition, $\theta(t) = \theta_t$ describes the system completely.

Using these two equations, the initial condition and the transversality condition determines the time path of g^*, θ^* and λ^*. For an interior solution, these equations can be analyzed using the phase diagram in figure (2).

The schedule $\dot{\lambda} = 0$ shows the combination of the state variable θ and its value λ for which the value of the property rights remains momentarily unchanged. The negative slope of this schedule can be understood as follows. An increase in the value of property rights raises the rate of return from having the property rights θ. Then economy will experience zero instantaneous rate of change in the value of property rights, λ, only if the increase in rate of return is completely absorbed in corresponding increase in dividend received by the agent i for having the property right θ.[9]. The utility of agent i is concave in the level of property rights, so higher dividend implies lower level of property rights in the economy.

The curve describing the $\dot{\theta} = 0$ in the θ-λ plane is positively sloped. This can be explained with the help of the equation of motion in θ. As the level of property rights, θ, increases, the total disintegration in the institution of property right increases ($\delta > 0$). To maintain the same level of property rights, more efforts from agents are required. Agents put more effort only if the value of property right increases which gives positively sloped $\dot{\theta} = 0$ schedule.

Now one can use figure (2) to describe the equilibrium dynamics. Suppose that at the beginning of the game, t, the level of property right in the economy is given as θ_t. The figure shows three of infinitely many possible trajectories that begins from points with initial condition θ_t. Along all these trajectories all first order conditions are satisfied, however the initial value/shadow price of property right differ. Trajectory 1, which assigns the highest shadow price, leads in the figure, to a very high level of property rights with a very high shadow price. A high shadow price implies higher contribution towards improvement of property rights (equation 21), which chokes off flow of efforts towards production increasing the scarcity of consumption good in economy unnecessarily. As the initial valuation of property rights is too high to be consistent with perfect foresight, one

[9]Dividends are payments made by a firm to its shareholders. In a sense, dividend from improving property rights from θ to $\theta + d\theta$ can be defined as rate of utility gain of agent i at the property right θ multiplied by $d\theta$, which is $\frac{\partial u_i}{\partial \theta} \cdot d\theta$.

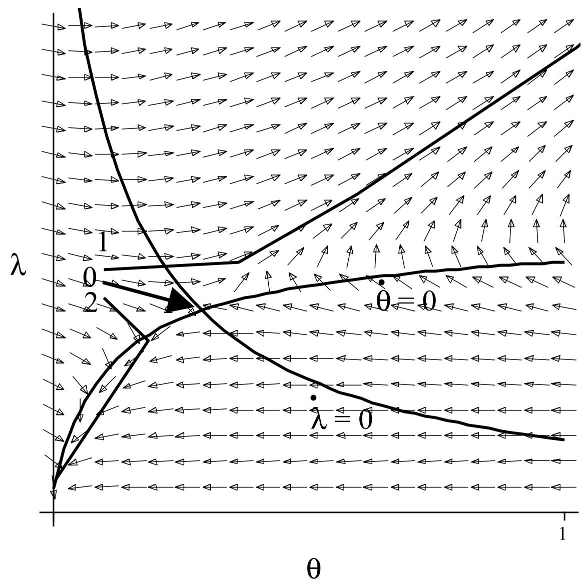


Figure 2: Open loop Equilibrium

can rule out this type of trajectories as candidates for equilibrium. Trajectory 2, which assigns the lowest shadow price with the figure, leads to 0 property right. These trajectories can also be ruled out based on the fact that with perfect foresight, agents would never contribute towards improving property rights to achieve 0 level of property rights. Only the saddle path, denoted by trajectory 0, reflects expectations that can be fulfilled everywhere. The saddle path describes the unique dynamic equilibrium. The equilibrium has a property that the current price of effort level to maintain this level of property rights is equivalent to the current value of future gain that accrue to agent i for having the property right as this level.

Proposition 2. There exists a unique and dynamically stable open-loop solution that results in a steady state level of property right enforcement given by:

$$\theta = \begin{cases} 0 & : \frac{B}{\rho+\delta} < \frac{1}{Rn(n-1)} \\ \frac{BnR - \frac{\rho+\delta}{n-1}}{(n+1)(\rho+\delta)+\delta} & : \frac{1}{Rn(n-1)} \leq \frac{B}{\rho+\delta} \leq \left[\frac{n}{(n-1)R} + \frac{\delta}{(\rho+\delta)nR}\right] \\ 1 & : \frac{B}{\rho+\delta} > \left[\frac{n}{(n-1)R} + \frac{\delta}{(\rho+\delta)nR}\right] \end{cases} \tag{24}$$

As it is discussed earlier, $\dot{\lambda} = 0$ curve is downward slopping and $\dot{\theta} = 0$ curve is upward slopping. There are two conditions under which these two curves don't intersect. Under one condition, the $\dot{\lambda} = 0$ lies completely below the $\dot{\theta} = 0$ curve. In this case, the economy ends up with no property rights ($\theta = 0$). This is because at all points of curve $\dot{\theta} = 0$, $\dot{\lambda}$ is positive, which renders the cost of property right very high, which, in turn, makes any contribution towards property right improvement very costly. The opposite is true when $\dot{\theta} = 0$ lies completely under the curve $\dot{\lambda} = 0$. At all points of $\dot{\theta} = 0$, $\dot{\lambda}$ is negative making the maintenance and improvement of property rights virtually costless.

One interesting case is to see what happens if political fragmentation, n, in economy increases. One can consider two scenarios. In the first scenario, more and more group share the resource available in economy T. In this case, resources available to an agent (a group) can be given as $R = \frac{T}{n}$. In the second scenario, one can analyze a situation where economy has more agents, however per capita resources in the economy remains unchanged.

Corollary 2.1. The level of property right in the economy with a fixed amount of resources either decreases or first increases and then decreases with increasing political fragmentation (n), and asymptotically reaches to zero.

Corollary 2.2. The level of property rights in the economy increases as more and more identical agents having the same amount of resources R joins in the economy, and asymptotically reaches to the level, θ_∞ given in the following expression:

$$\theta_\infty = \min\left[\frac{BR}{\rho + \delta}, 1\right] \tag{25}$$

The open-loop solution provides the level of property right enforcement that can be achieved if agents either do not observe the evolution of the state variable or they commit in the beginning of the game to ignore the effect of change in the state on their strategy. However, such conditions cannot be enforced as all agents have an incentive to free ride and deviate based on the observation of the state variable.

3.1.2 The Feedback Equilibrium

The concept of feedback strategy is more intuitive and appealing in the present problem as agents cannot gain by unilaterally deviating from his equilibrium strategy. Here, agents optimize their actions in all subgames. These subgames can be understood as a new game which starts after each agent's action have caused the level of property right to evolve from its initial state to a new state. The continuation of the game with a new level of property rights can be considered as a subgame of the original game. A feedback strategy permits agents to take best possible action in each subgame. A feedback strategy is, therefore, optimal not only in the beginning of the game but throughout the game. Although feedback strategies appear very appealing, they are very difficult to compute. In order to simplify the analysis, two assumptions are made:

1. All information emanating from the observation of the state variable θ is available through its current value. (Markov Perfect Property)

2. The feedback strategies depend on time only through state variable. (Stationary Property)

Feedback strategies are difficult to calculate because finding agent i's strategy requires that all other agent's optimal strategies be known which, in turn, requires player i's optimal strategies be known. In order to make optimal choice, as in the case of open loop strategies, agents need to guess what others are doing and going to do. However, in case of feedback strategies the agent i's guess of other agents' strategies are function of θ, the state variable, which leads to the presence of an interaction term in agent i's adjoint equation which, in turn, makes the computation of the shadow price difficult.

$$H_i = \log\left(A\left[\left(\frac{1}{n}+\frac{n-1}{n}\theta\right)R-\left(\theta+\frac{1-\theta}{n^2}\right)g_i-\left(\frac{1-\theta}{n^2}\right)\sum_{j\neq i}g_j(\theta)\right]\right)+\lambda_i\left[B\left(g_i+\sum_{j\neq i}g_j(\theta)\right)-\delta\theta\right]$$

where λ_i is the shadow price that agent i see associated to θ. Necessary conditions for optimality include satisfaction of the equation of motion, constraints, and following first order conditions.

The first order condition with respect to g_i:

$$\frac{\partial H_i}{\partial g_i} = \frac{-\left(\theta + \frac{1-\theta}{n^2}\right)}{\left(\frac{1}{n} + \frac{n-1}{n}\theta\right) R - \left(\theta + \frac{1-\theta}{n^2}\right) g_i - \left(\frac{1-\theta}{n^2}\right) \sum_{j \neq i} g_j(\theta)} + \lambda_i B \tag{26}$$

Implies (Same as in the case of Open Loop Strategies)

$$-\frac{\theta + \frac{1-\theta}{n^2}}{\left(\frac{1}{n} + \frac{n-1}{n}\theta\right)(R-g)} + \lambda B \begin{cases} > & 0 \quad \Rightarrow g = R \\ = & 0 \quad \Rightarrow g \in (0, R) \\ < & 0 \quad \Rightarrow g = 0 \end{cases} \tag{27}$$

Using the above optimization condition and looking at symmetric solutions we get

$$\lambda B = \frac{\theta + \frac{1-\theta}{n^2}}{\left(\frac{1}{n} + \frac{n-1}{n}\theta\right)(R-g)} = \frac{\left(1 + (n^2-1)\theta\right)}{n\left(1 + (n-1)\theta\right)(R-g)} \tag{28}$$

The above relation gives an expression for the shadow price of property rights, λ, in terms of choice variable, $g_j = g, \forall j$. Using this expression the equation of motion can be expressed in terms of λ: (Same as the open loop system)

$$\dot{\theta} = BnR - \frac{n^2\theta + 1 - \theta}{1 + (n-1)\theta} \cdot \frac{1}{\lambda} - \delta\theta \tag{29}$$

The first order condition with respect to state variable θ (The adjoint equation)

$$\frac{\partial H_i}{\partial \theta} = \frac{\frac{n-1}{n} R - \left(1 - \frac{1}{n^2}\right) g_i + \frac{1}{n^2} \sum_{j \neq i} g_j(\theta) - \left(\frac{1-\theta}{n^2}\right) \sum_{j \neq i} g_j'(\theta)}{\left(\frac{1}{n} + \frac{n-1}{n}\theta\right) R - \left(\theta + \frac{1-\theta}{n^2}\right) g_i - \left(\frac{1-\theta}{n^2}\right) \sum_{j \neq i} g_j} + \lambda_i \left(B \sum_{j \neq i} g_j'(\theta) - \delta\right) = \rho\lambda_i - \dot{\lambda}_i \tag{30}$$

It implies:

$$\dot{\lambda} = (\rho + \delta)\lambda - \frac{n-1}{1 + (n-1)\theta} - B(n-1)g'(\theta)\lambda + \frac{(1-\theta)(n-1)g'(\theta)}{n\left(1 + (n-1)\theta\right)[R - g(\theta)]} \tag{31}$$

One can rewrite the adjoint equation using first order condition with respect to the choice variable given in equation (28) as:

$$\dot{\lambda} \;=\; \underbrace{(\rho+\delta)\,\lambda - \frac{n-1}{1+(n-1)\theta}}_{\text{Same as Open loop Strategies}} \underbrace{-\frac{n(n-1)\theta}{(1+(n-1)\theta)} \cdot \frac{g'(\theta)}{[R-g(\theta)]}}_{\text{Interaction Term}} \tag{32}$$

The expression for the interaction term in the terms of θ, n, and λ can be obtained from differentiating the first order condition in choice variable. Using equation of motion, one can obtain: (See appendix for the derivation of the equation)

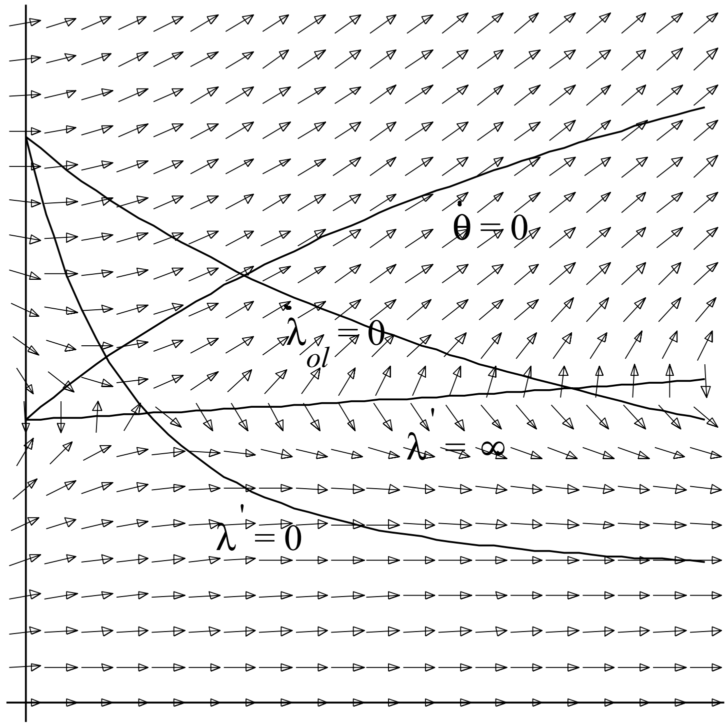
$$\lambda'(\theta) \;=\; \frac{(\rho+\delta)\,\lambda - F(n,\theta)}{\left[BnR - \delta\theta - \frac{1}{\lambda(\theta)}\right]} \tag{33}$$

where $F(n,\theta) = \frac{n-1}{1+(n-1)\theta} - \frac{n(n-1)\theta}{(1+(n-1)\theta)}\left[\frac{(n^2-1)}{(1+(n^2-1)\theta)} - \frac{(n-1)}{(1+(n-1)\theta)}\right]$

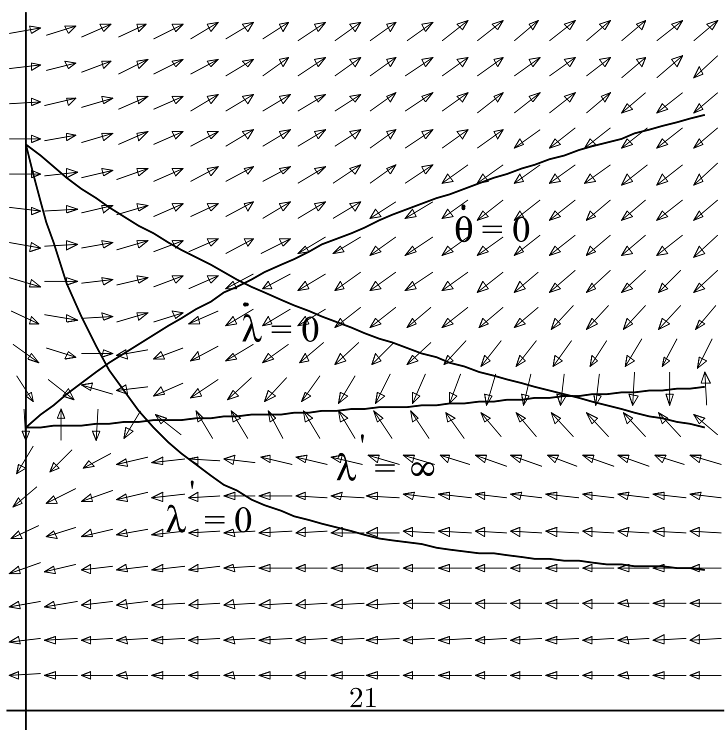
The above equation along with the equation of motion for θ given in equation (29) and the initial condition, $\theta(t) = \theta_t$ describes the system completely. It is important to note that for the feed back system, we don't give the canonical equations for the system in terms of $\dot{\lambda}$ and $\dot{\theta}$. We do so because the expression for $\dot{\lambda}$ of this system can only be expressed in terms of $\dot{\theta}$, as $\dot{\lambda} = \lambda'(\theta)\dot{\theta}$, as a consequence the expression in $\dot{\lambda}$ and $\dot{\theta}$ no longer remain independent of each other.

By definition, in equilibrium, the shadow price λ and the level of property right θ in the economy do not change. So, $\dot{\lambda}$ and $\dot{\theta}$, both, should be equal to zero in equilibrium. As we discussed, $\dot{\lambda}$ is always zero whenever $\dot{\theta}$ is zero. So all strategies which cross the $\dot{\theta}$ curve provide equilibria. However, not all of these equilibria are stable. Since, in this feedback system, we possibly encounter multiple equilibria (see figure (3)), we use stability condition to characterize the equilibria rather than use phase diagrams to analyze the system.

Stability criterion for the system can be obtained from the equation of motion expressed in terms of the shadow price λ given in equation (29). $\dot{\theta}$ is positive above the curve $\dot{\theta} = 0$, shown in figure (3), and negative below. If we assume $\lambda(\theta)$ be a feedback price corresponding to a feedback strategy



(a) phase Portrait of $\lambda'(\theta)$ in Feed-Back System



(b) Feed-Back Equilibrium

Figure 3: Feed-Back System

$g(\theta)$, and λ^*, θ^* be an equilibrium such that $\lambda^* = \lambda^*(\theta)$. This equilibrium is stable if

$$\frac{d}{d\theta} f(\lambda^*, \theta^*) < 0 \tag{34}$$

where $f(\cdot)$, the equation of motion, is equal to $BnR - \frac{n^2\theta+1-\theta}{1+(n-1)\theta} \cdot \frac{1}{\lambda} - \delta\theta$. The stability condition holds when

$$\frac{1}{\lambda^*(\theta) \cdot (1+(n-1)\theta^*)} \left[\frac{\lambda'(\theta)}{\lambda(\theta)}|_{\dot{\theta}=0} \left(1+(n^2-1)\theta^*\right) - \frac{n^2-n}{1+(n-1)\theta^*)} \right] < \delta \tag{35}$$

is satisfied. Intuitively, at any stable equilibrium point, the slope of shadow price $(\lambda'(\theta))$ actuated by the equilibrium strategy should be less than the slope of the curve $\dot{\theta} = 0$. From the figure 3(a), it is apparent that points on $\dot{\theta} = 0$ close to the origin are stable, while points closer to $\theta = 1$ are unstable. There is a point on the curve $\dot{\theta} = 0$ where equilibria change from stable to unstable. This point is unique and lies at the position where $\lambda'(\theta)$ is tangent to $\dot{\theta} = 0$. This point of inflection, called $\bar{\theta}$, can be obtained analytically, and is given in the following equation (see derivation in appendix):

$$\bar{\theta} \quad = \quad \frac{Bn(n-1)R - (\rho+\delta)}{2(n-1)\delta + (n^2-1)\rho} \tag{36}$$

Proposition 3. There exist multiple equilibria in the feedback system. Any $\theta \in [0, \bar{\theta}]$ is feasible as the equilibrium level of property rights.

For increasing political fragmentation in the economy, we can derive corollaries analogous to that derived for open loop system.

Corollary 3.1. The property rights enforcement in the economy with fixed amount of resources either decrease or first increase and then decreases with increasing political fragmentation (n), and asymptotically becomes zero.

Corollary 3.2. The feasible range of property right in the economy expands as more and more identical agents having the same amount of resources R joins in the economy, and asymptotically becomes $[0, \bar{\theta}_\infty]$, where $\bar{\theta}_\infty$ is given in the following expression:

$$\bar{\theta}_\infty \;=\; \min\left[\frac{BR}{\rho}, 1\right] \tag{37}$$

4 Conclusion

The main contribution of this paper is the analytical model that explains the evolution of property rights in a decentralized economy. The model captures the basic elements affecting the benefit and cost associated with the institution of property rights and describes the strategic interaction among agents involved in productive and appropriative activities. The paper uses dynamic-game framework and characterizes an open loop and a stationary Markov feedback strategies in the game of evolution of property rights without relying on the guessing method. For the open loop strategy, we show an unique and stable equilibrium which depends on the economy's characteristics such as productivity factor in institution building, institutional rate of depreciation, and also on individual characteristics such as individuals' endowment of resources, discount factor for future. For stationary Markov feedback strategies, we show that a range of equilibrium level of property rights are feasible in an economy.

A APPENDIX

A.1 Proof of Proposition 1

For an internal solution, equation (12) gives:

$$-\theta A - \frac{a_i}{\sum_{j=1}^{n} a_j(t)}(1-\theta)A + \frac{\sum_{j\neq i} a_i}{(\sum_{j=1}^{n} a_j(t))^2}(1-\theta)\sum_{j=1}^{n} A\cdot(R-g_i-a_i) = 0 \quad \forall i \in N \tag{A-1}$$

Taking the first order condition for agents i, and j, and dividing one by another, one can obtain:

$$\frac{\theta + \frac{a_i}{\sum_{k=1}^{n} a_k(t)}(1-\theta)}{\theta + \frac{a_j}{\sum_{k=1}^{n} a_k(t)}(1-\theta)} \;=\; \frac{\sum_{k\neq i} a_k}{\sum_{k\neq j} a_k} \tag{A-2}$$

The above equation is true for any i, j pair belonging to N. This is possible only when $a_i^* = a_j^*, \forall i, j \in N$. Using this fact in the first order condition for agent i, one can obtain the expression for a_i^* given in Proposition 1.

A.2 Derivation of equation (33)

Taking log and differentiating the first order condition with respect to choice variable g_i gives:

$$\frac{\lambda'(\theta)}{\lambda(\theta)} = \frac{(n^2-1)}{(1+(n^2-1)\theta)} - \frac{(n-1)}{(1+(n-1)\theta)} + \frac{g'(\theta)}{R-g(\theta)}, \tag{A-3}$$

which can be rearranged to obtain an expression for the interaction term:

$$\frac{g'(\theta)}{R-g(\theta)} = \frac{\lambda'(\theta)}{\lambda(\theta)} - \left[\frac{(n^2-1)}{(1+(n^2-1)\theta)} - \frac{(n-1)}{(1+(n-1)\theta)}\right]$$

Using the expression obtained for the interaction term and identity $\dot{\lambda} = \lambda'(\theta)\dot{\theta}$, one can rewrite the adjoint equation as

$$\lambda'(\theta)\left[\dot{\theta} + \frac{n(n-1)\theta}{(1+(n-1)\theta)}\frac{1}{\lambda(\theta)}\right] = (\rho+\delta)\,\lambda - \frac{n-1}{1+(n-1)\theta} + \frac{n(n-1)\theta}{(1+(n-1)\theta)}\left[\frac{(n^2-1)}{(1+(n^2-1)\theta)} - \frac{(n-1)}{(1+(n-1)\theta)}\right]$$

Using the expression for $\dot{\theta}$ in terms of λ in LHS of the above equation, one can obtain:

$$\lambda'(\theta)\left[BnR - \delta\theta - \frac{1}{\lambda(\theta)}\right] = (\rho+\delta)\,\lambda - \frac{n-1}{1+(n-1)\theta} + \frac{n^2(n-1)^2\theta}{(1+(n^2-1)\theta)\,(1+(n-1)\theta)^2} \tag{A-4}$$

A.3 Derivation of equation (36)

Using the expression for $\lambda'(\theta)$ given in equation (33) in the stability condition given in equation (35), we obtain:

$$\frac{1}{\lambda(\theta)}\frac{\left[(\rho+\delta)\lambda^*(\theta) - \frac{n-1}{1+(n-1)\theta^*}\right]}{(n^2-n)\theta^*} < \delta \tag{A-5}$$

Since λ^* lies on the curve $\dot{\theta} = 0$, we can obtain an expression for λ^* in terms of θ^* and other system parameters by equating the equation of motion to 0. Using this expression for $\lambda^*(\theta)$ in the above equation, we obtain the $\bar{\theta}$ that we give in equation (36).

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