A Theoty of Dynastic Cycle*

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Abstract

This paper proposes a dynamic politico-economic theory on dynastic cycle. I characterize the Markov Perfect Equilibrium of the dynamic game and derive the analytical solution to the equilibrium. The main conclusion is that the demise of any dictatorial regime is inevitable if there are discontinuity of power caused by dictator's physical death and the delegation of the dictator's unbalanced power, which are two common properties shared by all dictatorial regimes. Consistent with historical evidence, the model shows the overall pattern of the evolution of dictatorial regime is increasing real burden on the citizen caused by increasing bureaucrats' tax surcharge due to weakering dictator, and the decreasing fiscal revenue of the dictator due to the decreasing of tax base, as will cause the demise of dictatorship in the long run.

1 Introduction

To be written.

2 The model

The model economy has a two-period OLG structure and in every period, there are four types of risk neutral agents: the citizens, the dictator, the dictator's successor candidates and the bureaucrats. The mass of each generation of citizens is unitary. Each of the citizens undertakes an investment when young,

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which costs $\frac{i^2}{2}$, and yields a return i in both periods of lives. The dictator is the ruler of the economy. He sets an age-independent tax rate to maximize the tax revenue from the investment returns of the young and the old citizens. No matter how strong a dictator is, he must face the following two problems about power: *(i)* The discontinuity of power caused by the physical death of the dictator; *(ii)* The delegation of power.

The dictator has a dilemma when solving the first problem. If the dictator does not designate anyone to be his successor when alive, there will be some chaos, in which δ of the citizens' investment will be destroyed, caused by the power struggle for the crown after the dictator's death. Such a bad state ex post will decrease the citizens' investment ex ante and thus decreases the dictator's tax base. Alternatively, the dictator can designate his successor when alive. Although this can preclude the possibility of chaos after the dictator's death and thus increases the dictator's tax base, such a method reduces the dictator's safety when alive, since the successor always has an incentive to take the place of the incumbent earlier to enjoy the dictator's rent. I assume $\delta = 1$, such that designating the successor when alive always dominates leaving no successor after death[1].

Assume some successor candidates with mass m ($m < 1$) are born in every period. These candidates are the only people in the economy that have the privilege to be the future dictator. Every incumbent dictator designates his successor from one of the successor candidates in the beginning of the incumbent's second period of life and transfers the power to the successor before death. Given the above assumptions, the timing of the power transfer, unless there is a coup against the incumbent dictator, is as follows: at the beginning of any period t, the incumbent dictator, who is in his second period of life and is designated as successor by the previous dictator, becomes the ruler and designates the successor from the successor candidates born at period t; at the end of period t, the incumbent dictator transfers the power to the successor.

[1] Herz(1952) provides a detailed discussion about this problem and shows designating a successor when alive dominates any other method.

The strength of the successor candidate has a uniform distribution in $[0, m]$, such that a candidate j can be marked by his strength $\alpha_j \in [0, m]$. The probability of the incumbent dictator α_i, who is among the successor candidates in the previous period and thus can also be marked by his strength α_i, to win the power struggle with his successor candidate α_j is

$$P(\alpha_i \text{ wins}) = \begin{cases} 1 & \text{if} \quad \frac{\alpha_i - \alpha_j}{\alpha_i} \geq d \\ \frac{1}{2}, & \text{if} \quad -d \leq \frac{\alpha_i - \alpha_j}{\alpha_i} < d \\ 0, & \text{if} \quad \frac{\alpha_j - \alpha_i}{\alpha_i} > d \end{cases}$$

The intuition of conflict technology is that if the incumbent is sufficiently stronger than the successor, the incumbent will win for sure; if the difference between the dictator's strength and the successor's strength is not big enough, the probability that each side wins is one half; if the dictator is sufficiently weaker than the successor, then the dictator will lose for sure. d can be seen as a measure of incumbent advantage in power struggle, with the larger the size of d, the lower the incumbent advantage.

In addition to the problem of power transfer, the dictator also has to delegate (some of) his power to the bureaucrats. Due to the nature of dictatorship, there can not be any source of independent check and balance of the bureaucrats' power since this means the erosion of the dictator's power[2]. Moreover, the asymmetric information between the dictator and bureaucrats create the opportunities for corruption. The unbalanced power plus the asymmetric information between the dictator and the bureaucrats makes corruption hard to be eradicated in dictatorship. In the model economy, bureaucratic corruption is reflected as the surcharge of tax by the bureaucrats. That is, a bureaucrat can say a citizen, who actually has paid the tax, has not paid; or a bureaucrat can say a citizen, who actually has not paid the tax, has paid. In equilibrium, the bureaucrats can charge more than the tax rate announced by the dictator. Since the bureaucrats' surcharge distorts the citizens' investment decision and

[2] See Yi(2007) for a detailed discussion.

decreases the tax base of the dictator[3], it is not in the interest of the dictator. The size of the surcharge depends on the strength of the dictator in regulating the bureaucrats[4]. Note that the ability for a dictator (successor) to fight in the power struggle with a successor (dictator) and the ability to regulate the bureaucrats are in fact the same thing or at least positively correlated, since these two abilities both reflects of the leader's political skills.

Technically, I assume if the tax rate announced by the dictator α_i is τ_t^d ex ante, the bureaucrats can surcharge $(n - \alpha_i)$ ex post on the citizens without any risk. This means for given τ_t^d and α_t^d, the real tax rate τ_t^r that the citizens face ex post, is

$$\tau_t^r = \tau_t^d + n - \alpha_t^d$$

with $n \geq m$. As can be seen from the above expression, for given τ_t^d, the stronger the dictator, the lower the tax burden on the citizens.

3 Political Equilibrium

The purpose of this paper is to explore the impact of interest conflict between the incumbent dictator and his successor on the strength of dictator generation after generation, which affects the extent of bureaucratic corruption over time and the evolution of dictatorship. More specifically, can a regime with continuous interest conflict between current and future ruler, which affects the distortion on investment caused by bureaucratic corruption, be sustainable in the long run? In order to answer this question, I start to solve an equilibrium without crown prince problem as a benchmark. which can help to characterize the equilibrium with crown prince problem.

[3] Mauro(1995) shows corruption is negatively related to growth and investment, and corruption affects growth through investment. See also Fisman and Svensson (2001) for a study about corruption and growth in the firm level.

[4] pp.153 of Feng(1985) documented the dramatic decrease of bureaucrats' surcharge soon after a strong dictator took power in China. In some provinces, for example, Henan and Shandong, the surcharge rate went down from 80% to 13% and 18%, respectively.

3.1 Equilibrium without Crown Prince problem

In this case, I assume the successor's moral concerns always dominate his economic concerns. That is, the successor never tries to get the power one period earlier from the incumbent dictator. The timing of the game is as following:

1. At the beginning of period t, the old incumbent dictator chooses his successor, who gets the power in the end of period t when the incumbent dies;
2. The successor candidates other than the one chosen by the dictator as the successor are eradicated;
3. The incumbent sets the tax rate τ_t^d;
4. The young citizens born at period t make their investment i_t;
5. The bureaucrats surcharge and collect the tax for the old incumbent;
6. The incumbent transfers his power to the successor at the end of period t.

Given the assumption about the game, the indirect utility functions of the living agents are as follows

$$\begin{aligned} V^{oc} &= \left(1-\tau_t^r\right) i_{t-1} \\ V^{yc} &= \left(1-\tau_t^r\right) i_t + \beta\left(1-\tau_{t+1}^r\right) i_t - \frac{i_t^2}{2} \\ V^{od} &= \tau_t^d\left(i_{t-1}+i_t\right) - 2w_t, \end{aligned} \tag{1}$$

where V^{oc}, V^{yc}, V_t^{od} denote the objective of the old citizen, the young citizen, and the incumbent old dictator, respectively. τ_t^d, τ_t^r, α_t^d, i_t, w_t denote the tax rate imposed by the dictator, the real tax rate that the citizens face, the strength of the incumbent dictator, the investment made by young agent and the wage to the bureaucrats at period t, respectively. Simple maximization in (1) shows

that $w_t = 0$ and the solution to the optimal investment problem of the young citizen, given the real rates in his two periods of life, τ_t^r and τ_{t+1}^r, is

$$i_t^* = (1 - \tau_t^r) + \beta \left(1 - \tau_{t+1}^r\right) \tag{2}$$

Definition 1 *A (Markov Perfect) political equilibrium is defined as a triplet of functions* $\langle A, T, I \rangle$, *where* $A : [0, m] \times [0, 1 - \beta] \to [0, m]$ *is the dictator's decision rule on the strength of his successor,* $a_{t+1}^d = A\left(a_t^d, i_{t-1}\right)$, $T : [0, m] \times [0, 1 + \beta] \to \left[0, 1 - n + a_t^d\right]$ *is the dictator's policy decision rule on the tax rate,* $\tau_t^d = T\left(a_t^d, i_{t-1}\right)$ *and* $I : [0, m] \times [0, 1] \to [0, 1 + \beta]$ *is the young citizens' private investment decision rule* $i_t = I\left(\alpha_{t+1}^d, \tau_t^r\right)$, *such that the following functional equations hold:*

1. $\left\{A\left(a_t^d, i_{t-1}\right), T\left(a_t^d, i_{t-1}\right)\right\} = \arg\max_{a_{t+1}^d, \tau_t^d} V^{od}\left(\tau_t^d, \tau_{t+1}^d, \alpha_{t+1}^d, \alpha_t^d, i_{t-1}\right)$ *subject to* $\tau_{t+1}^d = T\left(A\left(a_t^d, i_{t-1}\right), I\left(\alpha_{t+1}^d, \tau_t^r\right)\right)$.
2. $I\left(\alpha_{t+1}^d, \tau_t^r\right) = 1 - \tau_t^r + \beta\left(1 - \left(T\left(a_{t+1}^d, I\left(\alpha_{t+1}^d, \tau_t^d + n - a_t^d\right)\right) + n - \alpha_{t+1}^d\right)\right)$
3. $V^{od}\left(\tau_t^d, \tau_{t+1}^d, \alpha_{t+1}^d, \alpha_t^d, i_{t-1}\right) = \tau_t^d\left(i_{t-1} + I\left(\alpha_{t+1}^d, \tau_t^r\right)\right)$.

According to Definition 1, the state of the model economy at period t is captured by two state variables, a_t^d and i_{t-1}. The first equilibrium condition requires that the incumbent old dictator chooses $a_{t+1}^d$and τ_t^d to maximize his indirect utility function, taking into account that future dictator's decisions about tax rate and the successor's strength depend on the current dictator's choice via the equilibrium decision rules. Furthermore, it requires $A\left(a_t^d, i_{t-1}\right)$ and $T\left(a_t^d, i_{t-1}\right)$ are both fixed points in the functional equation in part 1 of the definition. The second equilibrium condition implies that all young citizens choose their investment optimally, given a_t^d and τ_t^r, and that these agents hold rational expectations about how future tax rate and dictator's strength are determined. The third equilibrium condition means the old incumbent does not need to worry about his safety since by assumption, the successor never tries to seize the power one period earlier. The constraint that $\tau_t^d \in \left[0, 1 - n + a_t^d\right]$ is equivalent to $\tau_t^r \in [0, 1]$, which means the real tax rate that the citizens face can not be larger than one as there is no saving in the economy.

Proposition 2 *If $m \leq n \leq 1 - \frac{1}{2}m\beta$, in the equilibrium without crown prince problem, $\langle A, T, I \rangle$ is characterized as follows:*

$$A\left(a_t^d, i_{t-1}\right) = m$$

$$T\left(a_t^d, i_{t-1}\right) = \begin{cases} \frac{1}{2-\beta} i_{t-1} + \frac{1}{2} a_t^d + \frac{\frac{\beta m}{2} + 1 + \beta - n - n\beta}{2+\beta}, & if \quad i_{t-1} \in [0, \bar{\imath}_{t-1}] \\ 1 + n - a_t^d, & if \quad i_{t-1} \in (\bar{\imath}_{t-1}, 1+\beta] \end{cases}$$

$$I\left(\alpha_{t+1}^d, \tau_t^a\right) = \begin{cases} 1 - \tau_t^r, & if \quad \tau_t^r \in [0, \bar{\tau}_t^r] \\ -\frac{(2-\beta)\tau_t^a}{2} + \frac{\beta}{4} a_{t+1}^d + \frac{(2-\beta)\left(4\beta - 2n\beta - m\beta^2 + 4\right)}{4(\beta+2)}, & if \quad \tau_t^r \in (\bar{\tau}_t^r, 1] \end{cases}$$

for given a_0^d and all t,where

$$\bar{\imath}_t \equiv \frac{2-\beta}{\beta+2}\left(1 - n - \frac{1}{2}m\beta + a_{t+1}^d + \frac{1}{2}\beta a_{t+1}^d\right)$$

and

$$\bar{\tau}_t^r \equiv \frac{(\beta-2)\,\alpha_{t+1}^d}{(2+\beta\,(1-d))} + \frac{1}{\beta+2}\left(2n + 2\beta - n\beta\right)$$

Furthermore,

(1) With any $a_0^d \in [0, m]$ and $i^{-1} \in [0, 1+\beta]$, $\langle A, T, I \rangle$ converges to the following equilibrium in one period with

$$A\left(a_t^d, i_{t-1}\right) = m$$

$$T\left(a_t^d, i_{t-1}\right) = \begin{cases} \frac{1}{2-\beta} i_{t-1} + \frac{\beta+1}{\beta+2}\left(m - n + 1\right), & if \quad i_{t-1} \in [0, \bar{\imath}] \\ 1 + n - m, & if \quad i_{t-1} \in (\bar{\imath}, 1+\beta] \end{cases}$$

$$I\left(\alpha_{t+1}^d, \tau_t^r\right) = \begin{cases} 1 - \tau_t^r, & if \quad \tau_t^r \in [0, \bar{\tau}^r] \\ -\frac{(2-\beta)\tau_t^r}{2} + \frac{\beta(2-\beta)}{4} m + \frac{(2-\beta)\left(4\beta - 2n\beta - m\beta^2 + 4\right)}{4(\beta+2)}, & if \quad \tau_t^r \in (\bar{\tau}^r, 1] \end{cases}$$

where

$$\bar{\imath} \equiv \frac{2-\beta}{\beta+2}\left(1 - n + m\right)$$

and

$$\bar{\tau}^r \equiv \frac{\beta-2}{2} m + \frac{\left(4n + 4\beta + 2m\beta - 2n\beta - m\beta^2\right)}{2\left(\beta+2\right)}$$

(2) The equilibrium law of motion of τ_t^d is as follows

$$\tau_{t+1}^d = \begin{cases} 1 - n + m, & if \quad \tau_t^d \in \left[0, \bar{\tau}_t^d\right] \\ -\frac{\tau_t^d + n - a_t^d}{2} + \frac{\beta+2}{4} m + \frac{8\beta - 4n + 2m\beta - 6n\beta - m\beta^2 + 8}{4(\beta+2)}, & if \quad \tau_t^d \in \left(\bar{\tau}_t^d, 1 - n + a_t^d\right] \end{cases}$$

where

$$\bar{\tau}_t^d \equiv \bar{\tau}_t^r - n + a_t^d$$

(3) The equilibrium law of motion of τ_t^r *is*

$$\tau_{t+1}^r = \begin{cases} 1, & if \quad \tau_t^r \in [0, \bar{\tau}_t^r] \\ -\frac{\tau_t^r}{2} + \frac{\beta-2}{4}m + \frac{\left(4n+8\beta+2m\beta-2n\beta-m\beta^2+8\right)}{4(\beta+2)}, & if \quad \tau_t^r \in (\bar{\tau}_t^r, 1] \end{cases}$$

(4) Starting with any $a_0^d \in [0, m]$ *and* $i_{-1} \in [0, \bar{\imath}_{-1}]$*, then* $\tau_t^d \in \left(0, 1 - n + a_t^d\right)$ *and* $\tau_t^r \in (0, 1)$ *for all* $t \geq 0$*. Starting with any* $a_0^d \in [0, m]$ *and* $i_{-1} \in (\bar{\imath}_{-1}, 1 + \beta]$*, then* $\tau_t^d = 1 - n + a_0^d$*,* $\tau_0^r = 1$ *and* $\tau_t^d \in \left(0, 1 - n + a_t^d\right), \tau_t^r \in (0, 1)$ *for all* $t > 0$*. In either of the above two cases, the economy converges asymptotically with an oscillatory pattern to the following steady state with*

$$a^{ss} = m$$

$$\tau_{ss}^d = \frac{1}{3\beta + 6}\left(4m - 4n + 4\beta + 4m\beta - 4n\beta + 4\right)$$

$$\tau_{ss}^r = \frac{2n - 2m + 4\beta + m\beta - n\beta + 4}{3\left(\beta + 2\right)}$$

$$i^{ss} = \frac{\left(-\beta^2 + \beta + 2\right)\left(m - n + 1\right)}{3\left(\beta + 2\right)}$$

Figure 1 here

Figure 1 represents the equilibrium decision rules of the incumbent dictator and the citizens when there is no Crown Prince problem. Panel a shows that any incumbent will choose the strongest successor. Panel b shows that for given a_t^d, the equilibrium τ_t^d increases linearly with i_{t-1}, which is sunk at period t, before some threshold $\bar{\imath}_{t-1}$ and then achieves a corner solution with $\tau_t^d = 1 + n - a_t^d$ and a corresponding $\tau_t^r = 1$ henceforth. Panel c shows that for given a_{t+1}^d, the citizens' investment decreases with τ_t^r. The discontinuity at $\tau_t^r = \bar{\tau}_t^r$ reflects the fact that to the left of this point, the next period real tax rate, τ_{t+1}^r, will get a corner solution of one and the citizens' investment rule is different than that to the right. Intuitively, without Crown Prince problem, an incumbent with any strength will choose the strongest successor, who distorts least in i_t, since

the citizens' investment increases with a^d_{t+1}. Given the choice of the strongest successor, the incumbent chooses a τ^d_t that makes the tax income at the peak of the Laffer curve, taking into account that how the future dictator makes decisions about tax rate and successor's strength. Therefore, in this case, the tax base effect dominates the safety effect.

Figure 2 here

Figure 2 represents the equilibrium law of motion of tax rates. Panel a shows that if τ^d_t is lower than some threshold level $\bar{\tau}^d_t$, then τ^d_{t+1} will get a corner solution with $\tau^d_{t+1} = 1 + n - m$ and a corresponding $\tau^r_{t+1} = 1$, while if τ^d_t is higher than $\bar{\tau}^d_t$, then τ^d_{t+1} will decrease linearly with τ^d_t. The intuition is as following, other things given, a lower τ^d_t will lead to a higher i_t, which is sunk seen at period $t+1$. This increases the period $t+1$ incumbent dictator's tax base and will be taxed more heavily. This will generate an oscillatory pattern of equilibrium τ^d_t across time. Panel b shows the equilibrium law of motion of τ^r_t. The shape and the mechanism is similar as the equilibrium law of motion of τ^d_t.

Figure 3 here

Figure 3 represents the time series of the tax rates. Panel a and b show that if $i_{-1} \in (\bar{\imath}_{-1}, 1+\beta]$, then τ^d_t and τ^r_t get a corner solution only at $t = 0$. Panel c and d show that if $i_{-1} \in [0, \bar{\imath}_{-1}]$, then τ^d_t and τ^r_t never get corner solution. In both cases, τ^d_t and τ^r_t converge asymptotically with an oscillatory pattern and without any trend to their steady states, respectively.

3.2 Equilibrium with Crown Prince problem

This equilibrium can be analyzed in three steps. Firstly, I solve the Markov Perfect Equilibrium where all the incumbent dictators chooses a sufficiently weak successor $\left(\frac{\alpha^d_t - \alpha^d_{t+1}}{\alpha^d_t} \geq d\right)$, and derive the indirect utility of the old incumbent

dictator as a function of α_{t+1}^d for given i_{t-1} and α_t^d. Secondly, I analyze the case in which the old incumbent dictator at period t chooses a non-sufficiently weak successor $\left(-d \leq \frac{\alpha_i - \alpha_j}{\alpha_i} < d\right)$, given that all the past and future dictators choose a sufficiently weak successor, and derive the indirect utility of the old incumbent dictator as a function of α_{t+1}^d for given i_{t-1} and α_t^d. Thirdly, I derive the condition under which the indirect utility of the old incumbent in the first case is always higher than that in the second case for any i_{t-1} and α_t^d. If this condition holds, then by one-stage deviation principle, the Markov Perfect Equilibrium where all the incumbent dictators chooses a sufficiently weak successor is a Subgame Perfect Nash Equilibrium without retrictions on the successor's strength.

3.2.1 Equilibrium without threat from the successor

In this case, the safety effect still dominates the tax base effect. This means $\frac{\alpha_t^d - \alpha_{t+1}^d}{\alpha_t^d} \geq d$ for all t. The timings of the game and the indirect utility functions of living agents at period t are the same as in the equilibrium without Crown Price problem as there is no threat from the successor.

Definition 3 *A (Markov Perfect) political equilibrium is defined as a triplet of functions* $\langle A, T, I \rangle$, *where* $A : [0, m] \times [0, 1 - \beta] \to [0, m]$ *is the dictator's decision rule on the strength of his successor,* $a_{t+1}^d = A\left(a_t^d, i_{t-1}\right)$, $T : [0, m] \times [0, 1 + \beta] \to \left[0, 1 - n + a_t^d\right]$ *is the dictator's policy decision rule on the tax rate,* $\tau_t^d = T\left(a_t^d, i_{t-1}\right)$ *and* $I : [0, m] \times [0, 1] \to [0, 1 + \beta]$ *is the young citizens' private investment decision rule* $i_t = I\left(\alpha_{t+1}^d, \tau_t^r\right)$, *such that the following functional equations hold:*

1. $\left\{A\left(a_t^d, i_{t-1}\right), T\left(a_t^d, i_{t-1}\right)\right\} = \arg\max_{a_{t+1}^d, \tau_t^d} V^{od}\left(\tau_t^d, \tau_{t+1}^d, \alpha_{t+1}^d, \alpha_t^d, i_{t-1}\right)$ *subject to* $\tau_{t+1}^d = T\left(A\left(a_t^d, i_{t-1}\right), I\left(\alpha_{t+1}^d, \tau_t^r\right)\right)$ *and* $\frac{\alpha_t^d - \alpha_{t+1}^d}{\alpha_t^d} \geq d$.
2. $I\left(\alpha_{t+1}^d, \tau_t^r\right) = 1 - \tau_t^r + \beta\left(1 - \left(T\left(a_{t+1}^d, I\left(\alpha_{t+1}^d, \tau_t^r\right)\right) + n - \alpha_{t+1}^d\right)\right)$
3. $V^{od}\left(\tau_t^d, \tau_{t+1}^d, \alpha_{t+1}^d, \alpha_t^d, i_{t-1}\right) = \tau_t^d\left(i_{t-1} + I\left(\alpha_{t+1}^d, \tau_t^r\right)\right)$.

According to Definition 3, the state of the model economy at period t is captured by two state variables, a_t^d and i_{t-1}. The first equilibrium condition requires that a_{t+1}^d and τ_t^d maximize the indirect utility function of the old incumbent dictator, taking into account that future dictators' decisions about tax rate and the successor's strength depend on the current dictator's choice via the equilibrium decision rules. Also, it requires $A\left(a_t^d, i_{t-1}\right)$ and $T\left(a_t^d, i_{t-1}\right)$ are both fixed points in the functional equation in part 1 of the definition. Furthermore, the constraint $\frac{\alpha_t^d - \alpha_{t+1}^d}{\alpha_t^d} \geq d$ needs to be satisfied as all the dictators secure their power by choosing a sufficiently weak successor. The second equilibrium condition implies that all young citizens choose their investment optimally, given a_t^d and τ_t^r, and that these agents hold rational expectations about how future tax rate and dictator's strength are determined. The third equilibrium condition means the old incumbent does not need to worry about his safety since in this case, the sufficiently weak successor has no chance to win the power struggle.

Proposition 4 *If $m \leq n \leq 1$ and $0 < d < \frac{1-\beta}{2-\beta}$, in the equilibrium with crown prince problem but without threat from the successor, $\langle A, T, I \rangle$ is characterized as follows:*

$$A\left(a_t^d, i_{t-1}\right) = (1-d)\, a_t^d$$

$$T\left(a_t^d, i_{t-1}\right) = \begin{cases} \frac{1}{2-\beta} i_{t-1} + \frac{1+\beta(1-d)}{2+\beta(1-d)} a_t^d + \frac{1+\beta-n-n\beta}{2+\beta}, & if \quad i_{t-1} \in [0, \bar{i}_{t-1}] \\ 1+n-a_t^d, & if \quad i_{t-1} \in (\bar{i}_{t-1}, 1+\beta] \end{cases}$$

$$I\left(\alpha_{t+1}^d, \tau_t^r\right) = \begin{cases} 1-\tau_t^r, & if \quad \tau_t^r \in [0, \bar{\tau}_t^r] \\ -\frac{(2-\beta)\tau_t^r}{2} + \frac{\beta(2-\beta)\alpha_{t+1}^d}{2(2+\beta(1-d))} + \frac{(2-\beta)(2\beta-n\beta+2)}{2(\beta+2)}, & if \quad \tau_t^r \in (\bar{\tau}_t^r, 1] \end{cases}$$

for given a_0^d and all t, where

$$\bar{i}_{t-1} \equiv (2-\beta)\left(\frac{(1-n)}{2+\beta} + \frac{a_t^d}{2+\beta\,(1-d)}\right)$$

$$\bar{\tau}_t^r \equiv \frac{(\beta-2)\,\alpha_{t+1}^d}{(2+\beta\,(1-d))} + \frac{1}{\beta+2}\,(2n+2\beta-n\beta)$$

Furthermore,

1. The equilibrium law of motion of τ_t^d is as follows

$$\tau_{t+1}^d = \begin{cases} 1-n+a_t^d, & if \quad \tau_t^d \in \left[0, \bar{\tau}_t^d\right] \\ -\frac{\tau_t^d+n-a_t^d}{2} + \left[\frac{2+2\beta(1-d)+\beta}{2(2+\beta(1-d))}\right]\alpha_{t+1}^d + \frac{1}{2(\beta+2)}\left(4\beta-2n-3n\beta+4\right), & if \quad \tau_t^d \in \left(\bar{\tau}_t^d, 1-n+a_t^d\right] \end{cases}$$

where

$$\bar{\tau}_t^d \equiv \bar{\tau}_t^r - n + a_t^d$$

2. The equilibrium law of motion of τ_t^r, is as follows

$$\tau_{t+1}^r = \begin{cases} 1, & if \quad \tau_t^r \in [0, \bar{\tau}_t^r] \\ -\frac{\tau_t^r}{2} - \left[\frac{2-\beta}{2(2+\beta(1-d))}\right]\alpha_{t+1}^d + \frac{1}{2(\beta+2)}\left(2n+4\beta-n\beta+4\right), & if \quad \tau_t^r \in (\bar{\tau}_t^r, 1] \end{cases}$$

3. Starting with any $a_0^d \in [0,m]$ and $i_{-1} \in [0, \bar{\imath}_{-1}]$, then $0 < \tau_t^r < 1$ for all $t \geq 0$. Starting with any $a_0^d \in [0,m]$ and $i_{-1} \in [\bar{\imath}, 1+\beta]$, then $\tau_t^r = 1$ and $0 < \tau_t^r < 1$ for all $t > 0$, where

$$\bar{\imath}_{-1} = (2-\beta)\left(\frac{(1-n)}{2+\beta} + \frac{a_0^d}{2+\beta(1-d)}\right)$$

In either of the above two cases, the economy converges asymptotically with an oscillatory pattern to the following steady state with

$$a^{ss} = 0$$

$$\tau_{ss}^d = \frac{4(\beta+1)(1-n)}{3(\beta+2)}$$

$$\tau_{ss}^r = \frac{1}{3(\beta+2)}\left(2n+4\beta-n\beta+4\right)$$

$$i_{ss} = \frac{1}{3(\beta+2)}\left(-\beta^2+\beta+2\right)(1-n)$$

Figure 4 here

Figure 4 represents the equilibrium decision rules of the incumbent dictator and the citizens when there is no Crown Prince problem. Panel a shows that the successor's strength increases linearly with the incumbent's strength. Panel b shows that for given a_t^d, the equilibrium τ_t^d increases linearly with i_{t-1}, which is

sunk at period t, before some threshold $\bar{\imath}_{t-1}$ and then achieves a corner solution with $\tau_t^d = 1 + n - a_t^d$ and a corresponding $\tau_t^r = 1$ henceforth. Panel c shows that for given a_{t+1}^d, the citizens' investment decreases with τ_t^r. The kink at $\tau_t^r = \bar{\tau}_t^r$ reflects the fact that to the left of this point, the next period real tax rate, τ_{t+1}^r, will get a corner solution of one and the citizens' investment rule is different than that to the right. Intuitively, when there is Crown Prince problem, the dictator's choice of a_{t+1}^d and τ_t^d can be separate, given the model's assumption about agents' preferences and how the winner of the power struggle being determined. That is, firstly, to ensure his safety, an incumbent with any strength will choose a successor as strong as possible to keep the distortions on investment as low as possible, given the constraint $\frac{\alpha_t^d - \alpha_{t+1}^d}{\alpha_t^d} \geq d$ is satisfied. Secondly, given the choice of the successor, the incumbent chooses a τ_t^d that makes the total taxation on the peak of the Laffer curve, taking into account that how the future dictator makes decisions about tax rate and successor's strength.

Figure 5 here

Figure 5 represents the equilibrium law of motion of tax rates. Panel a shows that if τ_t^d is lower than some threshold level $\bar{\tau}_t^d$, then τ_{t+1}^d will get a corner solution with $\tau_{t+1}^d = 1 + n - a_t^d$ and a corresponding $\tau_{t+1}^r = 1$, while if τ_t^d is higher than $\bar{\tau}_t^d$, then τ_{t+1}^d will decrease linearly with τ_t^d. The intuition is as following, other things given, a lower τ_t^d will lead to a higher i_t, which is sunk seen at period $t+1$. This increases the period $t+1$ incumbent dictator's tax base and will be taxed more heavily. This will generate an oscillatory pattern of τ_t^d. Panel b shows the equilibrium law of motion of τ_t^r. The pattern is similar as the equilibrium law of motion of τ_t^d. The oscillatory pattern of τ_t^d and τ_t^r has three important implications: *(i)* Growth-enhancing economic reforms in dictatorial regime will probably to be reversed with the change of the ruler, if there is no institutional reform that balances the power of the ruler, because without institutional reform, the power to set the policies stays on

the dictator, and as the tax base becomes larger due to the growth-enhancing economic reforms, the new dictator will tax heavily on the sunk investment. This will reverse the growth-enhancing economic reform; *(ii)* Bureaucratic corruption and economic growth can be positively correlated in dictatorial regime. The intuition is as following. When the tax base is low due to less sunk investment, the dictator has an incentive to lower the tax rate, which is growth-enhancing to increase the tax base. However, the lower tax rate itself can not put any constraint on bureaucratic corruption. On the contrary, this increases the rent base of the bureaucrats to get corrupt income. Thus, bureaucratic corruption and growth can be positively correlated. This explains the high corruption and high growth puzzle in east Asia after Second World War after which not much capital is left. *(iii)* As the oscillatory tax rates between generations can be seen as the variations of economic policies that are growth-enhancing or growth-retarding and can be controlled by dictators, it is wrong to use variables that reflect economic institutions as an indicator of political institutions in empirical analysis. This supports the view of Gleaser et al.(2004).

Figure 6 here

Figure 6 represents the time series of the tax rates. Panel a and b show that if $i_{-1} \in [0, \bar{\imath}_{-1}]$, then τ_t^d and τ_t^r get a corner solution only at $t = 0$. Panel c and d show that if $i_{-1} \in (\bar{\imath}_{-1}, 1+\beta]$, then τ_t^d and τ_t^r never get corner solution. In both cases, τ_t^d converges asymptotically with an oscillatory pattern and a *downward* trend to the steady steady state. The *downward* trend is reflected in the term $\left[\frac{2+2\beta(1-d)+\beta}{2(2+\beta(1-d))}\right]\alpha_{t+1}^d$ in the equilibrium law of motion of τ_t^d as this term is decreasing period by period due to decreasing α_{t+1}^d. Also in both cases, τ_t^r converges asymptotically with an oscillatory pattern and an *upward* trend to the steady steady state. The *upward* trend is reflected in the term $-\left[\frac{2-\beta}{2(2+\beta(1-d))}\right]\alpha_{t+1}^d$ in the equilibrium law of motion of τ_t^r as this term is increasing period by period due to decreasing α_{t+1}^d. The mechanism to generate the trends is as follows. Other things given, the weaker the dictator, the worse

is he in controlling his bureaucrats and the higher the bureaucrats' surcharge will be. This will increase real tax rate that the citizens face and shift Laffer curve to the left, which means tax rate set by the dictator will be lower. As the dictator becomes weaker and weaker within one dictatorial dynasty, the real tax rate faced by the citizens tends to increase and the tax rate charged by the dictator will be lower and lower. This means dictatorial government's revenue will be lower and lower because on the one hand, the increasing real tax burden will reduce the citizens' investment, which decrease the dictator's tax base and on the other hand, the dictator's share of the pie becomes lower and lower.

As we can see, in presence of the crown prince problem, if all the dictator wants to be safe, the evolution of dictatorship can be summarized as following:

1. The dictator will become weaker and weaker period by period.
2. Bureaucratic corruption, which is measured by the fraction of tax income that goes to the bureaucrats, will become higher and higher.
3. The real tax rate that the citizens face, τ_t^r, will become higher and higher, which makes the tax base to be smaller and smaller.
4. The fraction of tax income that goes to the dictator, τ_t^d, will become lower and lower.
5. Dictatorial can hardly survive in the long run due to the decreasing fiscal revenue.

3.2.2 Equilibrium with threat from the successor

Now I explore the following question: given all the past and future dictators choose a sufficiently weak successor, is it optimal for the incumbent dictator at period t to deviate for one period from choosing a sufficiently weak successor or equivalently, to choose an insufficient weak successor ($-d \leq \frac{\alpha_t^d - \alpha_{t+1}^d}{\alpha_t^d} \leq d$)? If the answer is no, then by the one-stage deviation principle, the Markov Perfect

Equilibrium where all the incumbent dictators choose sufficiently weak successors is a Subgame Perfect Nash Equilibrium without retrictions on the successor's strength

As there is now threat from the successor and the result of the political struggle is probablistic, the timing of the game at period t is modified as following:

1. At the beginning of period t, the old incumbent dictator chooses his successor with strength α_{t+1}^d;
2. The successor candidates other than the one chosen by the dictator as successor and the one with strength $\alpha_{t+1}^d + \varepsilon$,with $\varepsilon \to 0$, are eradicated[5];
3. The old incumbent sets the tax rate τ_t^d;
4. The young citizens born at period t make their investment i_t;
5. The bureaucrats surcharge $n - \alpha_t^d$ and collect the tax;
6. The power struggle between the incumbent and the successor takes place;
7. If the old incumbent wins, the successor is replaced with the candidate with strength $\alpha_{t+1}^d + \varepsilon$ at the end of period t.
8. If the successor wins, he gets the tax income at period t and also rules in period $t+1$. In this case, the utility of the old incumbent is Ψ.

Giving the timing of the game, the indirect utility function of the old incumbent at period t is

$$V_{nsw}^{od} = \frac{1}{2}\tau_t^d \left(i_{t-1} + i_t\right) + \frac{1}{2}\Psi$$

[5] If there is a power struggle between the incumbent and the successor at period t, then a potential question is, who will be the ruler in period $t+1$ if the successor loses in the power struggle at period t. For simplicity, I assume the dictator keeps a candidate with almost the same strength as the successor and if the successor loses in the power struggle, then the incumbent transfers his power to the candidate with strength $\alpha_t^s + \varepsilon$ at the end of period t. With this assumption, the equilibrium tax rate and the young citizens' investment will not be affected by the result of the political struggle.

This indirect utility function consists of two terms: with probability $\frac{1}{2}$, the old incumbent can maintain his power and get the tax at period t; and with probability $\frac{1}{2}$, he loses the power and the utility of being removed is Ψ. Furthermore, as the power struggle at period t takes place after the strength of successor (or equivalently, the strength of period $t+1$ dictator), the tax rate τ_t^d and the real tax rate τ_t^r are determined, no matter who wins the power struggle at period t, the citizens' investment decision rule will be the same as in the case when all the incumbents choose sufficiently weak successors, given that all the future successors choose sufficiently weak successors.

Proposition 5 *If*

$$\Psi < \min \left\{ \begin{array}{c} (2-\beta)\left(\frac{2(1-n)^2(1+\beta)^2-(\beta+2)^2\left(\frac{3+\beta(1-d)}{4+2(1-d)}m+\frac{(1-n)(1+\beta)}{\beta+2}\right)^2}{2(\beta+2)^2} \right), \\ \frac{2(2-\beta)(1-n)^2(1+\beta)^2-(\beta+2)^2(2+\beta-n+m)^2}{2(\beta+2)^2} \end{array} \right\}$$

then all the dictators will choose a sufficiently weak successor and the Markov Perfect Political Equilibrium defined in Definition 3 is a Subgame Perfect Nash Equilibrium without the constraint $\frac{\alpha_t^d-\alpha_{t+1}^d}{\alpha_t^d} \geq d$.

The intuition of Proposition 5 is that, if the utility Ψ of the old incumbent from being replaced by the successor is sufficiently low, then any dictator will concern more about his own safety than his rent. Therefore, all the dictators will choose a sufficiently weak successor. Figure 7 illustrates the relationship between the incumbent's utility and a_{t+1}^d for given a_t^d and i_{t-1}. In panel a, V^{od} increases with a_{t+1}^d for all $a_{t+1}^d \in \left((1-d)\,a_t^d, \min\left\{(1+d)\,a_t^d, m\right\}\right]$, and if Ψ is sufficiently low, the incumbent's indirect utility of choosing a sufficiently weak successor(V_{sw}^{od}) is higher than that of choosing a non-sufficiently weak successor$\left(V_{nsw}^{od}\right)$. In panel b, V^{od} increases with a_{t+1}^d for all $a_{t+1}^d \in \left((1-d)\,a_t^d, \bar{a}_{t+1}^d\right]$ and gets a corner solution henceforth because τ_{t+1}^r will get a corner solution of one for $a_{t+1}^d \in \left(\bar{a}_{t+1}^d, \min\left\{(1+d)\,a_t^d, m\right\}\right]$. In this case, sufficiently low Ψ also ensures the incumbent's indirect utility of choosing a sufficiently weak successor(V_{sw}^{od}) is higher than that of choosing a non-sufficiently weak successor$\left(V_{nsw}^{od}\right)$.

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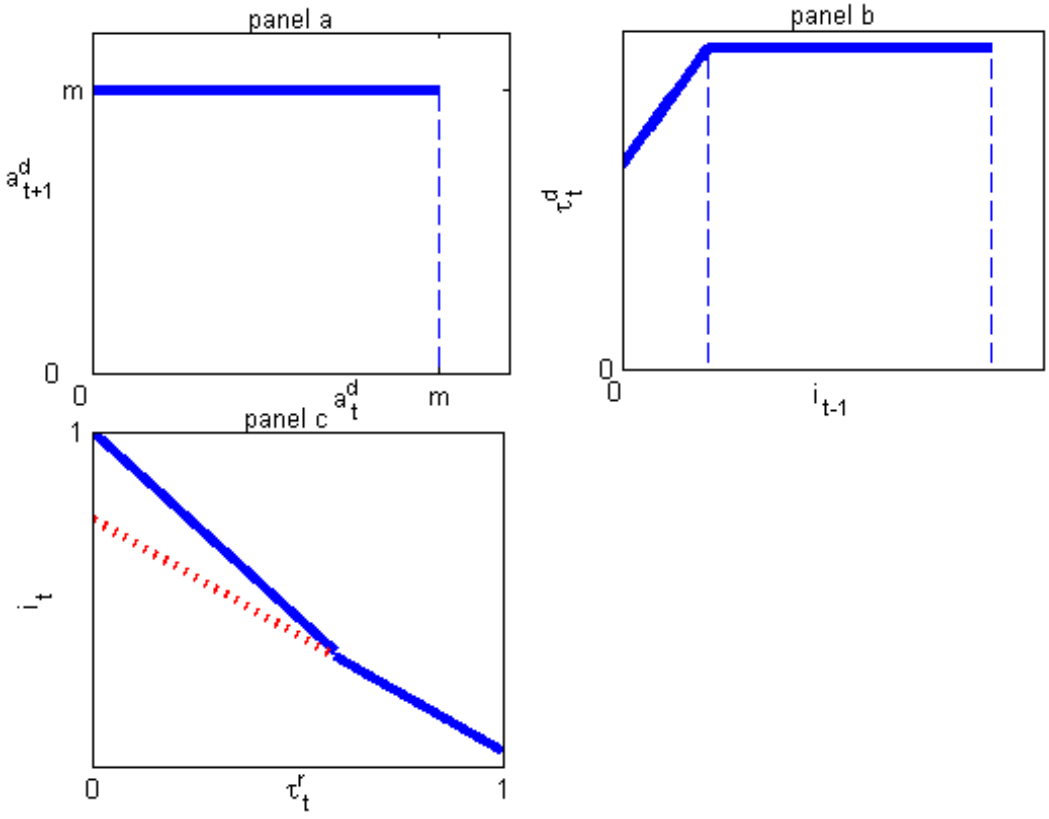

Figure 1: Decision rules when there is no Crown Prince problem.

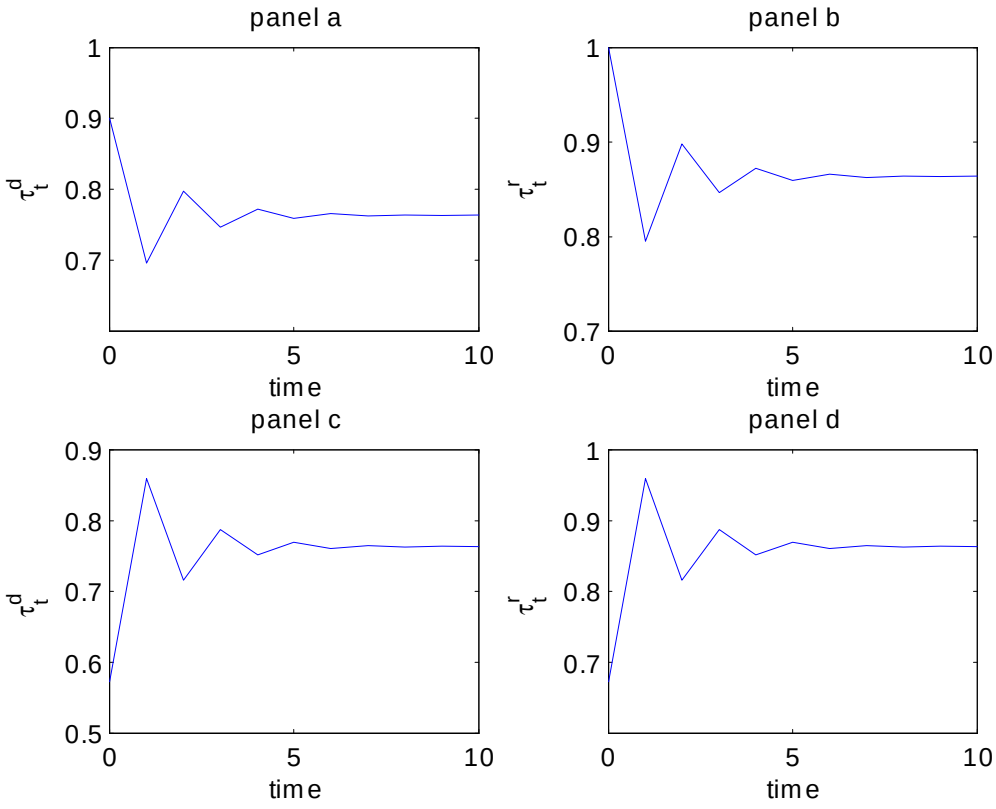

Figure 2: Time series of tax rates when there is no Crown Prince problem

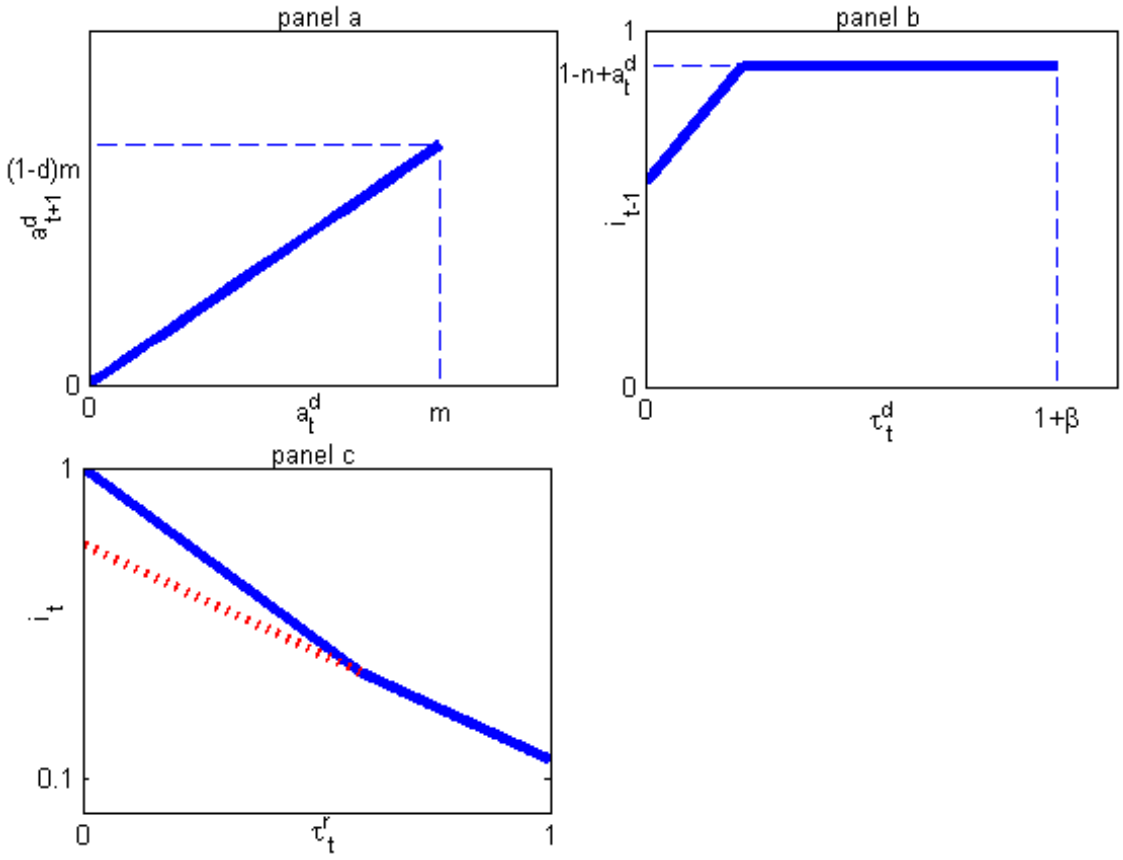

Figure 3: Decision rules when the incumbent chooses a sufficient weak successor

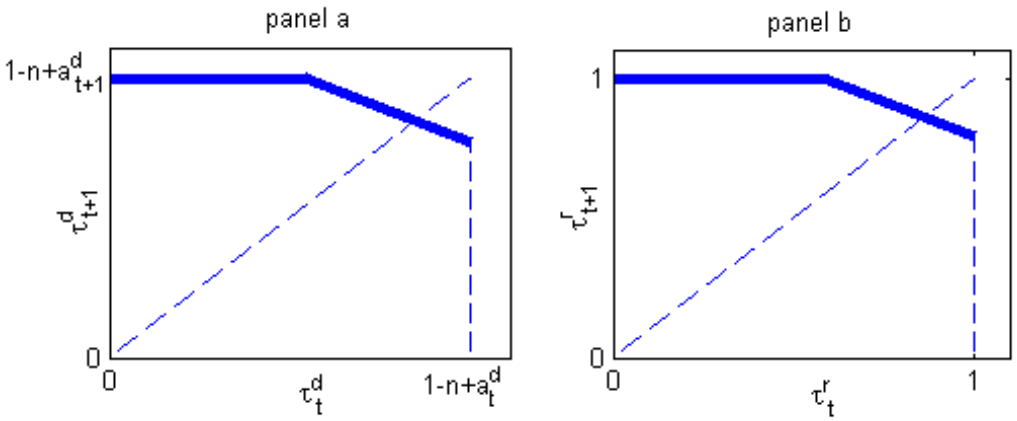

Figure 4: Equilibrium law of motion of tax rates when the incumbent chooses a sufficient weak successor

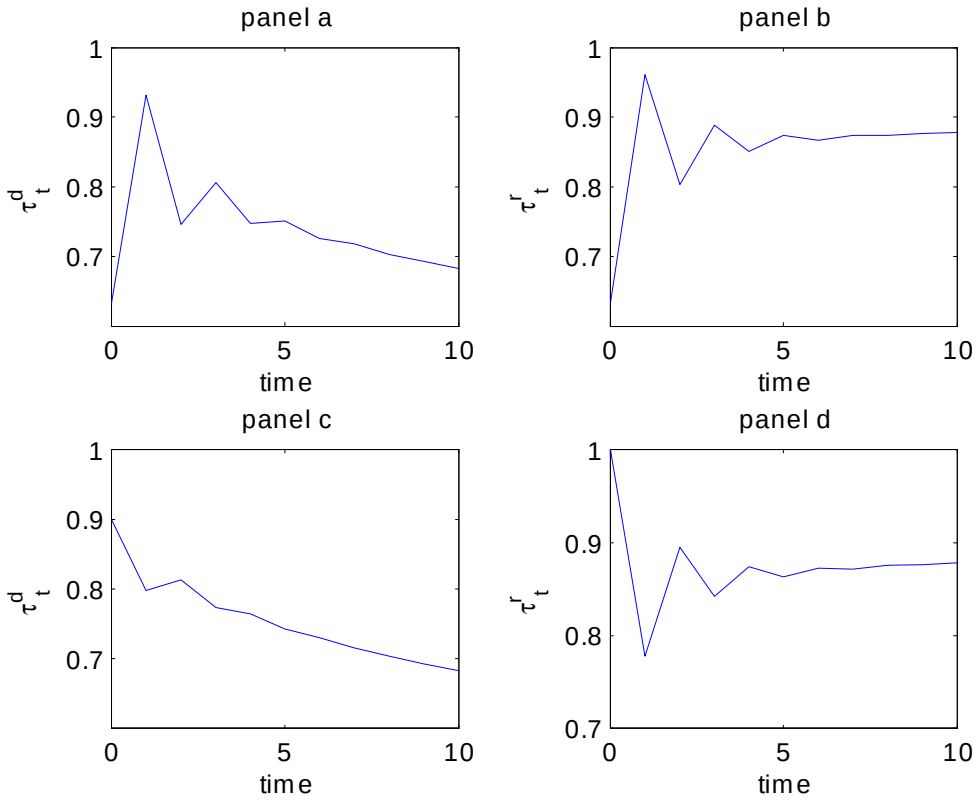

Figure 5: Time series of tax rates when the incumbent chooses a sufficient weak successor

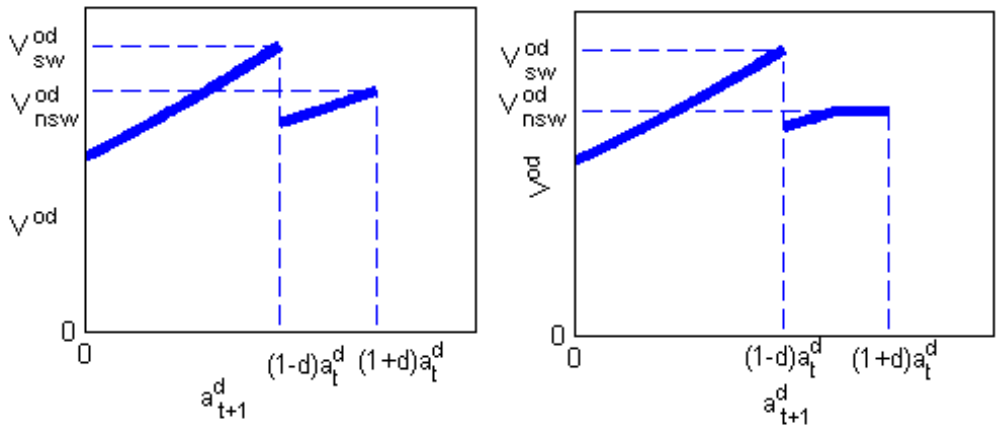

Figure 6: The indirect utility function of the old incumbent dictator