

Intertemporal Choice and Legal Constraints

M. Keith Chen*

Yale School of Management

Alan Schwartz

Yale Law School and Yale School of Management

April 6, 2009[†]

Abstract

We study the effect of legal constraints in an environment in which agents face demand shocks they would like to smooth, but the agents also have weakness of will: their long and short run preferences are misaligned. Some agents are sophisticated – they know they will make inconsistent intertemporal choices – while other agents are naive. The consequent public policy problem is complex. The state apparently should facilitate consumer borrowing, to help agents cushion the effect of shocks, but also should facilitate pre-commitment, to help agents control excessive present-based preferences. We show that naive and sophisticated agents make similar consumption/savings choices, which simplifies the policy problem. We also show that all agents borrow when they experience consumption shocks, and that agents with relatively strong present-based preferences who face relatively mild consumption shocks will borrow to finance excessive current consumption. Other agents save appropriately. Legal constraints that severely restrict agents' access to credit thus would be over-inclusive. Offering agents access to both a liquid and an illiquid savings vehicle is welfare improving relative to allowing agents complete freedom to borrow or strongly restricting their access to the credit market.

*Comments are welcome at keith.chen@yale.edu or at alan.schwartz@yale.edu. This paper has benefitted from workshops at Boston University Law School, the NBER summer meetings, and the Swiss Institute of Technology, Zurich.

[†]Keywords: JEL Classifications:

1 Introduction

1.1 The Problem

Recent existing and proposed consumer regulation partly rests on the view that persons make inconsistent intertemporal choices. An agent may have a financial plan, but her present biased preferences may cause her to depart from it in favor of current consumption, much of which is financed with debt.¹ The recent bankruptcy law responds to this concern by increasing the complexity and cost of filing for bankruptcy and by making discharge more difficult; these changes raise borrowing costs and thus discourage borrowing. Proposed reforms would either hold lenders liable or bar lenders from collecting debts if a lender had reason to believe that the borrower could not repay.² The borrowers, in turn, take on too much debt because they have weakness of will.

This cognitive justification for regulation does not follow from formal analyses but rests on seemingly plausible extensions of psychology experiments to the consumer credit context. This paper, in contrast, formally models persons who make financial plans but who have present based preferences that may cause them to deviate from those plans. The model focuses on two factors that affect persons' intertemporal choices. First, an agent wants to smooth consumption over time; that is, to have her marginal utility of consumption be constant in every period. An agent may experience an exogenous shock in a particular period that raises her marginal utility. To ensure constant marginal utility, agents therefore must transfer resources from periods when their marginal utility is low to periods when their marginal utility is high. More particularly, the agent should consume less than her income in good times - she should save then - and consume more than her income in bad times - she should dis-save then. If an agent's income and savings - her endowment- are insufficient to cushion a shock fully, she will borrow against future earnings.

As suggested, we also focus on agents' excessively present based preferences. An agent with present-based preferences does not discount future utility exponentially; rather, viewed from time t_i , the agent acts as if her discount rate between any t_n and t_{n+1} is greater than her discount rate between t_i and t_{i+1} , for all $i > n$. This agent may spend more in any period than her plan specifies, not to cushion the effect of a shock, but just to consume.³

¹Pottow (2007) at 412-13 is a typical example of the legal literature: "Consumers of unsecured revolving credit are notoriously irrational.....The principal concern with credit borrowing is with the cognitive bias for risk underestimation and the irrational discounting (myopia) that makes 'seduction by plastic' so attractive." To the same effect, see, e.g., Bar-Gil (2004).

²Section 1229(a) of the Bankruptcy Abuse Prevention and Consumer Protection Act (2005), recites: "(1) certain lenders may offer credit to consumers indiscriminately, without taking steps to ensure that consumers are capable of repaying the resulting debt, and in a manner which may encourage certain consumers to incur additional debt; and

(2) resulting consumer debt may be a major contributing factor to consumer insolvency."

The statute directs the Federal Reserve Board to study the practice of predatory lending. A recent bill that passed the House would make debts uncollectible if credit was excessive.

³Agents have been shown to exhibit present-based preferences, but there is no settled explanation as to why. Among the theories, agents may be uncertain about whether they will

The literature commonly treats these factors separately. Analyses of how people smooth consumption suppose that agents make time consistent choices; analyses of weakness of will omit the possibility that agents suffer consumption shocks.⁴ Actual agents must cope with both factors, however. A person would like the flexibility to borrow in order to cushion the effect of an exogenous shock, but she also would like the ability to pre-commit not to borrow, in order to protect herself against weakness of will. The regulator should understand the intertemporal choices that this agent would make under various legally imposed constraints.

As an example, if Congress is concerned with intertemporal consumption smoothing, it should enact a tough bankruptcy law, that would restrict a borrower's right to discharge and limit the assets that she could keep after bankruptcy. Such a law maximizes creditors' bad state payoffs and thus minimizes interest rates; the lower the interest rate, in turn, the easier it is for people to borrow. In contrast, if Congress is concerned with weakness of will, it should enact a soft bankruptcy law, that would make discharge easy and shield a substantial fraction of a person's assets from collection. The resultant high interest rates would reduce peoples' ability to finance deviations from their financial plans with borrowed money.⁵ Which bankruptcy law is optimal should turn, at least partly, on which bankruptcy law would improve the unconstrained agent's trade off between flexibility and precommitment.

1.2 The Analysis Below

We develop a three period model to address issues such as this. In the initial version of the model, the agent earns income in periods one and two, after which the game ends. The agent may experience exogenous shocks in these periods that raise her marginal utility of consumption. Illustrative shocks include medical problems against which the agent had not fully insured, an unanticipated child or grandchild, or a divorce. Since the second (i.e., the last) period agent

receive a payoff (Azfar 1999) or when they will receive a payoff (Dasgupta and Maskin 2005); they may focus on attributes of the choice using a lexicographic semi-order rule, giving priority to delay (Roelofsma and Read 2000) or dominance and similarity (Rubenstein 2003); they may have non-linear probability weightings (Baucells, et al 2006); they may be risk averse with respect to length of life, which implies hyperbolic discounting (Bommier 2006); they may face costly self-control problems (Gull and Pesendorfer 2004); they may discount subadditively, which implies that discount rates increase as the intervals between periods get small (Read 2001); their decision problems reflect a game between a short run patient and a long run impulsive self (Fudenberg and Levine 2006); and they may discount semi-hyperbolically (Laibson 1997). The experimental evidence that persons discount hyperbolically is mixed (Sopher and Sheth 2005; Cairns and van der Pol 2000). Our analysis requires us only to assume that agents' financial plans may be affected by the knowledge that they have present-based preferences. This assumption seems consistent with the prevalent explanations.

⁴See, e.g., Imrohoroglu, et al (2003) and papers cited note 1. An interesting exception is Amador, et al (2006), who argue that a minimum savings rule is a good response when agents face self control problems but may gain information over time regarding their tastes.

⁵The US Bankruptcy Code once made discharge easy, but the 2005 amendments to the Code much increased the difficulty of discharge for middle class consumers. Congress did not appear to apprehend the consequences of moving from one corner solution to the other.

may be shocked, the period one agent has a motive to save. We ask how the agent trades off this precautionary saving motive against her desire to engage in present biased consumption.

A two period model is sufficient to exhibit this tradeoff but a three period model is necessary to distinguish between agents who are naive or sophisticated about their biases. Every agent in the model has present based preferences, but agents differ in their level of self-awareness. One agent type consistently behaves as if her desire for immediate consumption is unique to her current state (she is "naive"); the other agent type understands that her future self will yield to a present-based bias (she is "sophisticated"). To see how an agent's type may affect her behavior, we add a "period zero". The agent earns no income in this period and cannot be shocked, but she can create a plan to govern her future behavior. Both naive and sophisticated agent types behave identically in period one because they (are assumed to) have identical present based preferences. A three period model permits us ask how an agent's self awareness regarding her later tendency to over-consume influences the agent's period zero plan.

1.3 Results

We begin with the benchmark case in which agents have unrestricted access to credit markets, and we then compare agents' behavior there to behavior in the other polar case, in which agents are entirely constrained from borrowing.⁶ Which regime is best is parameter specific, but the data suggest that complete freedom is better than total restraint. Agents borrow for good and bad reasons. The good reason is that the agent has experienced a shock to her marginal utility that requires her to spend more than her current endowment to cushion. The bad reason is that the agent is financing excessive current consumption with borrowed money. A strong borrowing constraint can be desirable relative to no constraint, both because it restrains excessive consumption and because agents save more when they know that they cannot borrow to cushion later shocks. In particular, a non-shocked first period agent's marginal utility is lower than that of her second period self, who may be shocked. A strong borrowing constraint forces this lucky first period agent to save for the last period. On the other hand, a strong borrowing constraint impedes the shocked first period agent's ability to respond to difficulty.

The question which policy response is best turns on whether financial flexibility is more important to agents than restraint. When agents have relatively low betas – i.e., their present bias is relatively slight – and face the possibility of severe shocks, there is less need to force them to save and more need to permit them to borrow in difficult times; but the welfare of agents with high betas who face relatively modest shocks would be improved by a strong constraint. Real persons exhibit behavior consistent with betas of approximately 0.9 in the field and in experiments.⁷ This relatively low level of present based bias implies that

⁶Obstacles to credit will always exist and the state cannot completely block access to the credit market. The polar cases are heuristically valuable, however.

⁷A beta of one implies that the agent discounts exponentially (she is not present biased).

agents save against modest to severe shocks. Thus, in our model, an agent with an apparently typical beta saves appropriately when the largest shock she may face in any period would increase her marginal utility of consumption by seventeen percent or more. Agents facing smaller possible shocks over-consume. This analysis suggests that the ability to borrow may be more important to actual persons than the need to pre-commit not to spend excessively.

The state need not choose between no or complete restraint, however, because a close to first best policy response exists. The state can provide agents with both a totally and a partly illiquid savings vehicle. If only a totally illiquid vehicle were to exist, the period zero agent may choose to save into it such that her shocked first period self is restricted to consuming her income. This plan would free the non-shocked first period agent to borrow against her last period income in order to consume excessively. In contrast, if only a savings vehicle that can be accessed in case of hardship were to exist, the period zero agent who saves into it loses control over the consumption of her shocked first period self, who can access the vehicle, so she instead saves enough in the vehicle to constrain the consumption of her non-shocked first period self. Each savings vehicle alone, that is, would permit the period zero agent to constrain only one of her possible first period selves. An agent with access to both vehicle types, however, can allocate her savings between them such as to restrict both of her later selves to consume exactly as much, when shocked and when not, as the period zero agent prefers. Partly illiquid savings vehicles exist today. For example, people can access funds in their IRAs only in the event of "hardship".⁸ Totally illiquid savings vehicles do not exist, but the state should offer them as well.

Sophisticated agents may plausibly use savings vehicles optimally, but the naive agent's lack of self awareness, it may be thought, would impede her ability to plan. We show, however, that both agent types are observationally equivalent. In the initial version of our model, the agent's consumption choices are entirely determined by her actual biases and the magnitude of the shocks she may face. An agent's type only influences her period zero decision how to allocate her future income between liquid and illiquid savings vehicles. The sophisticated agent would allocate optimally because she understands her situation. Naive agents inadvertently make similar allocation decisions because illiquid assets pay higher interest rates (to compensate lenders for their illiquidity). The naive period zero agent, that is, underestimates her present based bias, and thus she predicts that she will need less money in period one than she will turn out to want. As a consequence, she saves into illiquid vehicles as much as sophisticated agents do, but in her case, solely in order to take advantage of the higher interest

For an excellent review of the empirical and experimental evidence on time discounting, see Frederick, Loewenstein, & O'Donoghue 2002. Two recent studies found betas above .9. See Ahumada and Garegnani (2007) (low income Argentine consumers; beta of .97); Meier and Sprenger (2007) (experimental study of low income US consumers; beta of .94).

⁸Sections 403(b)(9) and 409A(2)(ii) of the Internal Revenue Code permit persons to access tax free retirement accounts in the case of hardship or emergency. Section 522(a)(3)(C) of the Bankruptcy Code exempts these tax free retirement accounts from the reach of creditors.

rates that these vehicles pay.

This observational equivalence result may seem to lack generality, however, because we permit agents to make a savings/consumption decision in only one period. Since both agent types behave similarly in this period – they act on their actual present based biases – the period zero agent’s plan is not based on the differential penchant of these agents to consume. In a more extended temporal framework, however, how much an agent saves in a particular period is partly determined by how much she expects to consume in later periods. Since sophisticated and naive agents have different expectations regarding later consumption, they may behave differently. To analyze this possibility, we add a third period to the model. The game ends in period three, and agents in periods one and two earn income and may be shocked. As before, the agent plans in period zero, but her period one saving/consumption choice now is affected by her belief as to the period two agent’s savings/consumption choice.

The observation equivalence result continues to hold in this more realistic setting. Both period one agent types want to save to cushion the effect of possible future shocks. A sophisticated agent knows that she faces a barrier to saving effectively, however. Money she saves out of her period one income, in the hope of increasing her final period three consumption, must be passed through her period two self. In period two, a present-biased agent will spend too much of her savings on current consumption. Saving for the far future, that is, is “taxed” by excessive consumption in periods in between. Agents who recognize the existence of the tax will moderate their saving behavior. A naive agent approaches the planning problem differently. She does not fully anticipate her future tendency to over-consume, and hence underestimates the resulting poverty of her far-future selves. She thus has a lesser incentive to save than the sophisticated agent has. On the other hand, the naive agent does not appreciate the extent to which savings for the far future will be taxed by the behavior of intermediate selves. This ignorance causes the naive agent to overestimate the efficacy of saving, and this induces her to save more than the sophisticated agent would. When persons exhibit log utility, as we assume, these two factors exactly offset; hence, the naive agent’s saving and consumption choices are identical to those of the sophisticated agent.⁹ And since both agent types make the same later period choices, both period zero agent types make the same plans.

We analyze two possible reasoning defects: agents are present biased and some agents underestimate the extent of their bias. Our observational equiv-

⁹More generally, these forces offset when agents display constant relative risk aversion (CRRA) with a coefficient (γ) near 1. There has been a long empirical literature estimating γ ; early studies from risky asset demand (Friend and Blume, 1975) and from property insurance prices (Szpiro 1986) find values of γ in excess of 1 (Szpiro estimates γ to lie between 1.2 and 1.8). A more recent estimate by Chetty aggregating 33 studies of labor-supply finds $\gamma \lesssim 1$, with a mean value for γ of 0.71. Additionally, a long literature in finance has demonstrated that in theoretic asset markets, only agents with $\gamma = 1$ hold any wealth asymptotically (Blume & Easley 1992). For simplicity and apparent realism we assume that agents have $\gamma = 1$; our results are qualitatively correct for values of γ near 1. Imrohroglu, et al, using a simulation derived from a different model, find that sophisticated and naive agents make similar consumption decisions when $\gamma = 2$.

alence result implies that policy need not respond to the second defect. Since welfare is commonly defined over consumption and consumer types are indistinguishable regarding consumption, the state should respond only to the existence of present based biases. Providing agents with access to both liquid and illiquid savings vehicles is a good response to this concern.¹⁰

An additional possible policy response to the offsetting factors we analyze is to open up the contracting space. Actual persons face shocks of varying probabilities and magnitudes, and suffer differentially from weakness of will. The state may respond with such policies as providing soft and tough bankruptcy and debt collection laws, and permitting persons to opt into the legal regime that best suits their situation¹¹ Space constraints preclude analyzing these possibilities. Part 2 below sets out the formal model; Part 3 derives results when credit markets are flexible; Part 4 analyzes borrowing constraints; Part 5 considers illiquid savings vehicles; Part 6 concludes.

2 The Model

Agents live for three periods. The initial period, labeled zero, is a "planning" period. The agent does not earn income in this period and cannot be shocked. Rather, the agent creates a plan to govern how she will consume, save, and react to potential demand shocks in future periods. In periods one and two, the agent earns income I . Since there is no period three, the period two agent consumes her income and any savings carried over from period one. In period one, the agent chooses how much to consume, and therefore how much to save. Also, in both periods one and two the agent may experience a demand shock that increases her marginal utility of consumption. In the model, with probability $1/2$ the agent's marginal utility increases by the factor α , where $1 \leq \alpha < \infty$. Agents have access to competitive credit markets, so the period one agent may borrow against her period two income, either to finance consumption or to cushion a shock to her marginal utility.

Agents with log utility exhibit constant relative risk aversion: that is, an agent's preferences between consumption and savings do not depend on her ab-

¹⁰Some commentators claim that persons borrow excessively because they are too optimistic regarding their earnings prospects or the future price level. See, e.g., Bar-Gill (2008). Making illiquid savings vehicles available does not respond to excessive optimism. Also, in a more general framework, the state could implement "macro" remedies. As examples, mandatory social security helps weak willed persons commit to a savings level – see Fehr, et al (2008) – and universal health care would help to cushion utility shocks. We focus here on credit markets, and our policy proposals respond to a subset of the cognitive errors that could affect persons in these markets. Thus, we claim only that implementing our views in that context would increase welfare.

¹¹Adler, et al (2000) suggest permitting consumers to make choices between bankruptcy regimes in a model that presupposed the consumer's ability to make time-consistent borrowing decisions. This recommendation may be robust to relaxing the time consistency assumption. Debt collection laws could vary in toughness depending, inter alia, on the fraction of the consumer's income the creditor is permitted to garnish.

solute level of wealth.¹² Agents also have excessively present based preferences. They discount future returns by the exponential discount factor δ times $\beta < 1$. We let $\delta = 1$ to focus on the time inconsistency concern. "Sophisticated" agents know that they discount future returns by β . "Naive" agents underestimate the extent to which they will over-consume. Formally, the naive agent believes that she discounts future returns by β_n where $\beta < \beta_n \leq 1$. Concluding the notation, the person consumes the amount c_n^α in period n . If she is not shocked in that period, $\alpha = 1$. Similarly, the person saves s_n^α in period n so she will have s_n^α plus her income I to spend in period $n + 1$. Summarizing this simple model, at time zero the agent utility function is:

$$\frac{1}{2}\beta(\ln[c_1^1] + a \ln[c_1^\alpha]) + \frac{1}{4}\beta(\ln[I + s_1^\alpha] + \alpha \ln[I + s_1^\alpha] + \ln[I + s_1^1] + \alpha \ln[I + s_1^1]) \quad (1)$$

The first terms are her expected utility from consumption in period one when she is not shocked and when she is, discounted by β ; and the last four terms represent her expected utility from consumption in each of the four possible period 3 states, again discounted by β .¹³

We solve the model backwards, beginning in period two. The agent consumes her entire endowment then $-I + s_1 -$, realizing utility of $\ln(I + s_1)$ if she is not shocked, and $\alpha \ln(I + s_1)$ if she is. The agent faces more difficult problems in periods one and zero, on which we focus. We consider three cases. In the first, analyzed in Part 3, the agent's ability to borrow is not legally constrained. This is the benchmark case for public policy. Part 4 initially identifies the circumstances in which a total borrowing constraint is preferable to an unconstrained market. Part 4 then goes on to consider an intermediate policy response, under which, again, borrowing is unconstrained but agents may access various illiquid savings vehicles.

3 Case 1: Full Credit Markets

3.1 Consumption

The zero period is irrelevant to this case because plans cannot constrain agents who may access the capital markets whenever they wish. The game ends in period two so every agent, regardless of her type, consumes her entire endowment. Thus, we focus on period one. Sophisticated and naive consumers behave

¹²Technically, the agent has a utility function of the form $u(c) = \frac{1-\gamma}{1-\gamma}$, where c is consumption and γ is the coefficient of relative risk aversion. For simplicity we let the coefficient of relative risk aversion (γ) go to one, in which case the decision maker's choices can be represented by a logarithmic utility function (see footnote 9 for a discussion of this simplifying assumption). Exogenous shocks in our model are multiplicative: a person's marginal utility may increase by 20% or 120%, etc. The agent thus faces gambles that are not expressed in dollars (one gains or loses \$100), but in percentages. Constant relative risk aversion seems a simple and relatively neutral assumption in these cases.

¹³There are four possibilities: the agent is shocked in period one and in period two; the agent is shocked in period one but not in period two; the agent is not shocked in period one but is shocked in period two; and the agent is not shocked in either period.

identically in this period because they have the same present based preferences (they have the same β s). Thus, we initially ask how the period one agent with present based preferences behaves in an unconstrained credit market.

The period one agent has an endowment of I , her income. She maximizes her intertemporal utility function (equation 1) with respect to consumption under two possible conditions: she is shocked in this period or she is not. This yields the two first-order conditions that govern her consumption and savings choices:

Condition for period one with a demand shock:

$$\frac{\partial}{\partial c_1^\alpha} \{a \ln[c_1^\alpha] + \frac{1}{2}[\beta \ln(I + s_1^\alpha) + \alpha\beta \ln(I + s_1^\alpha)]\} = 0 \quad (\text{Condition One})$$

The first term is the agent's utility from her period one consumption when she has been shocked. The expression inside the brackets represents the agent's expected utility in period two. With probability $1/2$, the agent is not shocked, with probability $1/2$, the agent experiences a period two shock. In either case she consumes exactly her period two income and any savings passed on from period one. The agent discounts her expected period two utility to period one by $\beta\delta$ (recall that we let $\delta = 1$).

The second first order condition is similar to the first except that the agent is not shocked in period one.

Condition for period one with no demand shock:

$$\frac{\partial}{\partial c_1^1} \{\ln[c_1^1] + \frac{1}{2}[\beta \ln(I + s_1^1) + \alpha\beta \ln(I + s_1^1)]\} = 0 \quad (\text{Condition Two})$$

The agent passes on to period two her period one income minus her period one consumption. This yields the simple budget constraint:

$$s_1^x = I - c_1^x \quad (\text{Condition Three})$$

for both $x = \alpha$ - a shock - and $x = 1$ - no shock.

The agent's period one consumption is a function of whether she has been shocked or not. We characterize her decisions by substituting Condition Three into Condition One and Condition Two. If the agent experiences a demand shock in period one, she consumes:

$$c_1^\alpha = \frac{4\alpha I}{\alpha(2 + \beta) + \beta} \quad (2)$$

and if she is not shocked, she consumes:

$$c_1^1 = \frac{4I}{2 + \beta + \alpha\beta} \quad (3)$$

Expressions (2) and (3) characterize the consumption and savings decisions of both agent types in the first period.

Regarding the intuition underlying these expressions, a demand shock increases the agent's marginal utility of consumption ($\alpha > 1$). Therefore, the

agent consumes more in period one when she is shocked than when she is not, and her consumption increases as α increases. Less intuitively, the non-shocked agent consumes less in period one as α increases. This agent knows that she may experience a shock in period two, and as the magnitude of that possible shock increases, the amount needed to cushion it also increases; hence, the non-shocked agent allocates a larger fraction of her period one income to precautionary saving. Finally, as volatility decreases (α falls) the agent's consumption varies less across states; and as the agent becomes more present-oriented (β falls) period one consumption increases in both states.

3.2 Savings

An unlucky – i.e., a shocked – period one agent always borrows against future income. That is, her consumption, c_1^α , always exceeds her income I . This is apparent in expression (2). There are two reasons for this result. First, all agents are present biased; hence, they prefer to consume more today than tomorrow. An agent will finance this consumption by borrowing against future income if she can. Second, the agent's expected period two marginal utility of income is lower than her period one marginal utility because the agent has been shocked in period one but she may not be shocked in period two. The agent thus prefers to shift resources from period two to period one, which she does by borrowing against her period two income.

In contrast, these two forces counteract each other when the first period agent is unshocked. This lucky agent is still present biased; hence, she would like to borrow. On the other hand, the agent's expected period two marginal utility exceeds her period one marginal utility because she has not been shocked in period one but she may be shocked in period two. The agent thus has a reason to save. The agent consumes exactly her period one income, I , when these two forces exactly cancel each other out. This occurs when:

$$\alpha = \frac{2 - \beta}{\beta} \quad (4)$$

Using this expression,¹⁴ we illustrate the behavior of sophisticated agents in Figure 1. Alpha is plotted on the vertical axis and beta is plotted on the horizontal axis; the “full consumption line” traces the set of $\alpha\beta$ combinations that satisfy expression 4. Agents with $\alpha\beta$ combinations that lie above this full consumption line save in the first period if they are not shocked. Agents whose $\alpha\beta$ combinations lie below the line borrow in the first period.

4 Case 2: Borrowing Restrictions

Agents with unrestricted access to the credit market save less and borrow more than they would were they not present biased. This analysis suggests that

¹⁴Recalling Expression 3, the unshocked agent exactly consumes her first period income when $\frac{4t}{2+e+re} = I$. The solution to this is Expression 4.

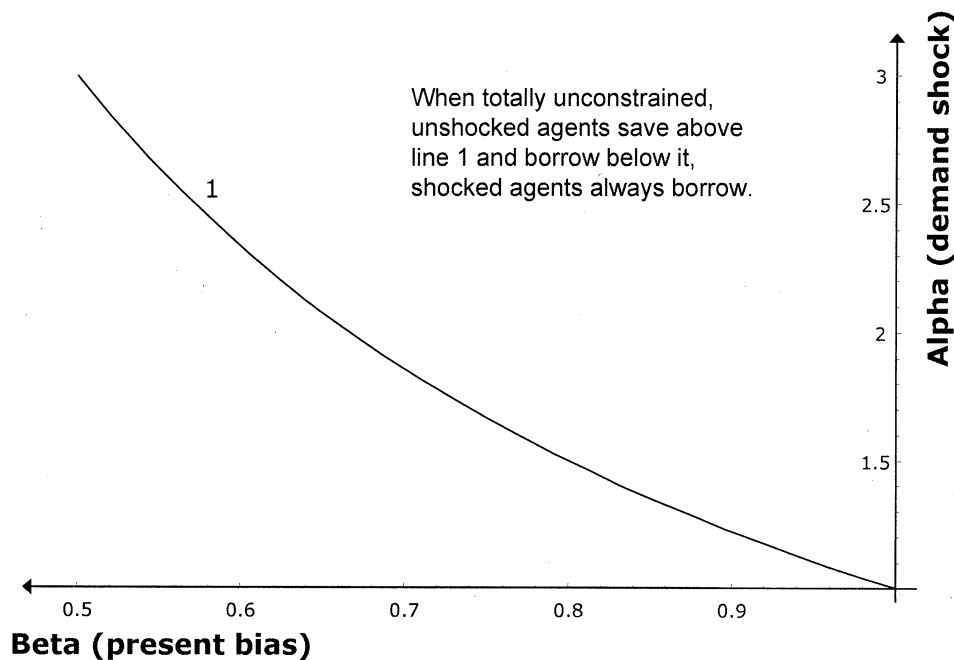


Figure 1:

constraining the agent's ability to borrow may improve welfare. A borrowing constraint, however, reduces the welfare of an agent who experiences a shock that increases her marginal utility of consumption to a level that requires more than her current endowment to cushion. Part 4 initially characterizes the behavior of agents who face a rigid borrowing constraint, and then evaluates the welfare effect of this restriction. When agents have relatively strong present based preferences, a state imposed ban on borrowing is welfare improving relative to permitting agents complete freedom. Part 4 goes on to show, however, that permitting agents to constrain themselves through illiquid savings vehicles is preferable on welfare grounds both to complete and to no freedom

4.1 A Total Borrowing Constraint

Borrowing constraints are irrelevant in period two because the game ends then. The period two agent cannot repay a period two loan with period three income, but rather exhausts her period two endowment. Turning to period one, a borrowing constraint does not affect the behavior of a non-shocked period one agent who prefers to save. Referring again to Figure 1, a borrowing constraint binds only those unshocked agents whose $\alpha\beta$ combinations put them below line one. Shocked period one agents always prefer to borrow against their period two

income. A borrowing constraint, however, restricts the consumption of every shocked agent to her period one income.

4.1.1 Welfare under a total borrowing constraint

Agents subject to a borrowing constraint have different goals than society, however. Society, we suppose, does not respect present-based preferences. Recalling that the exponential discount factor δ is assumed to be one, the social goal is to maximize the following social welfare expression:¹⁵

$$\frac{1}{2}(\ln[c_1^1] + \alpha \ln[c_1^\alpha]) + \frac{1}{4}(\ln[I + s_1^\alpha] + \alpha \ln[I + s_1^\alpha] + \ln[I + s_1^1] + \alpha \ln[I + s_1^1]). \quad (5)$$

Agents do not act to maximize expression 5 because their preferences are present based; they add $\beta < 1$ to the second terms. We evaluate equation 5 when borrowing is constrained and when it is not. Society is indifferent between these possibilities when social welfare without a constraint is equal to social welfare with a constraint. This social indifference curve is plotted as line 2 in figure 2. A borrowing constraint reduces welfare for agents whose $\alpha\beta$ combinations are

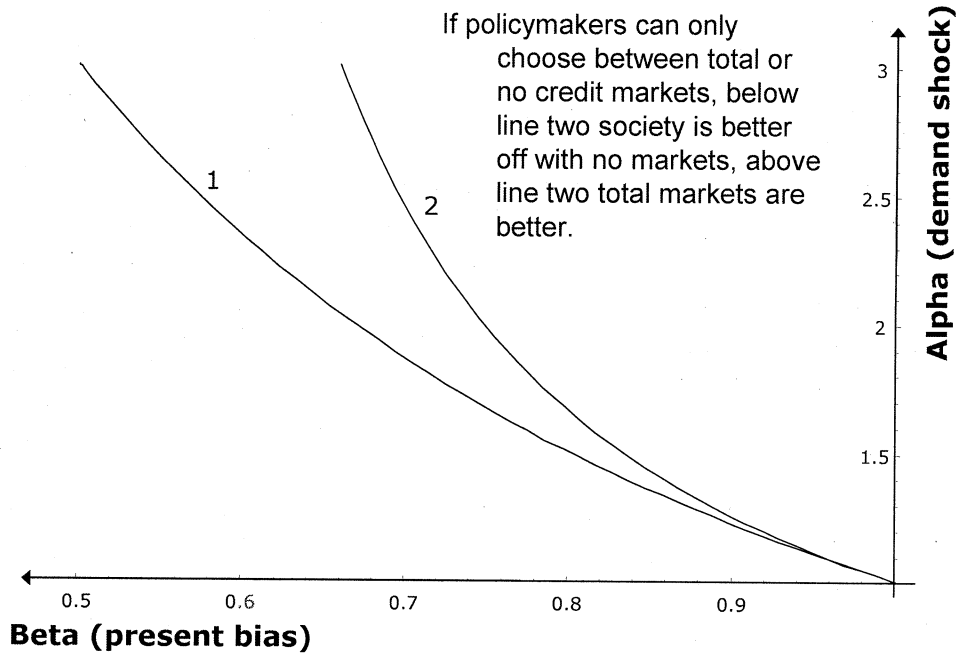


Figure 2:

¹⁵The first term in Expression 5 is the utility of consumption in the first period; the second expression characterizes the expected utility of consumption in the second period.

such as to put them above line 2 and increases welfare for agents whose $\alpha\beta$ combinations are such as to put them below line 2. To understand this welfare result intuitively, recall that the borrowing constraint reduces the ability of non-shocked first period agents with low betas to consume excessively. This is desirable because a non-shocked period one agent has a lower marginal utility of income than her expected second period self, who may be shocked. A borrowing constraint also reduces the ability of a shocked period one agent to access the credit market, however, and this can reduce welfare. When potential shocks are large relative to present bias, the ability to cushion shocks by borrowing wins out. When present bias is relatively strong relative to potential shocks, a constraint is welfare improving.

Remark 1 *Life-Cycle Income*

Agents in the model have the same income each period, which is plausible when the agent earns income in only two periods. Over real lives, however, the typical person's income rises for many periods and then declines. In this more realistic framework, a borrowing constraint is less desirable than the analysis here indicates. The constraint would apply to some agents who are on the increasing portion of their income profiles. These agents would be prevented from borrowing up to their (relatively) high next period's income to cushion shocks. On the other hand, borrowing constraints help little when agents are on the decreasing portion of their income profiles. These agents would be over-consuming largely out of savings. Thus, our welfare results hold in a more extended temporal framework.

Remark 2 *Creating Borrowing Constraints*

A total borrowing constraint is difficult to implement but the state could materially increase borrowing costs. As examples, interest on second homes or home equity loans could be made nondeductible and the amount of credit card debt could be restricted. Also, the amounts creditors could reach in case of default could be substantially reduced, which would increase interest rates. Since agents will not borrow at interest rates higher than the percentage increase in their marginal utility that a shock could cause, increasing interest rates increases the desirability of saving.

5 Case 3: Illiquid Savings Vehicles and Self-Control

The analysis to here has considered two extreme policy responses to present based preferences: permitting the agent either to have complete or to have no access to the credit market. We now consider an "intermediate" response: permitting the agent to access an illiquid savings vehicle. A totally illiquid vehicle could neither be cashed out nor borrowed against until the final period. A vehicle that the agent could access under restricted circumstances is denoted

"partly illiquid". An IRA is partly illiquid under this definition: there are heavy penalties for withdrawing funds prematurely, but the agent can access her IRA account in the event of verifiable hardship. The state today does not offer a totally illiquid vehicle.¹⁶

We have focused on the consumption/saving decision of the period one agent, but it is necessary also to consider the decisions of the agent in period zero when thinking about illiquid savings assets. This is because the period zero agent now has the power to precommit: she can partly constrain the behavior of her period one self by saving some of her future income into illiquid assets. In our model, the period zero agent may borrow against her future income and simultaneously save into the illiquid asset. This behavior is meant to capture cases in which people precommit to illiquid savings vehicles, such as automatic paycheck deductions into IRA savings accounts. These agents both save and incur debt. To here, the agents in the model acted identically, whether they were aware of the extent of their present bias or not. We now ask how an agent's degree of sophistication influences her choice of how much of her future income to allocate to an illiquid savings vehicle.

5.1 Solving for behavior

The initial step is to ask what the period zero agent prefers to consume in both possible first period states, and then to determine how an illiquid savings vehicle could help the agent to implement her preferences. The period zero agent trades off allowing her future self flexibility to deal with a shock but constraining that self from overconsuming. We show that offering agents access to both types of illiquid savings vehicles has better welfare properties than offering either vehicle alone. Further, if only one type of illiquid asset is possible, the asset that lacks a hardship exemption is preferable. These results hold for sophisticated and for naive agents, both of whom behave similarly under most conditions.

Both naive and sophisticated period zero agents have identical incentives. Neither agent can act on a present bias because agents have no wealth. Therefore, the period zero agent would like her first period self to consume just what the first period agent would want to consume if this agent were not present-biased. Taking the expressions for the first period agent's consumption preferences from equations 2 and 3, and substituting in a value of 1 for beta yields what we label with a hat the "first-period optimal" consumptions:

$$\begin{aligned} \widehat{c}_1^1 &= \frac{4I}{3+\alpha} \leq \frac{4I}{2+\beta+\alpha\beta} = \overline{c}_1^1 \\ \widehat{c}_1^\alpha &= \frac{4aI}{1+3\alpha} \leq \frac{4\alpha I}{\alpha(2+\beta)+\beta} = \overline{c}_1^\alpha \end{aligned} \quad (6)$$

Here \widehat{c}_1^1 and \widehat{c}_1^α are the zero period agent's preferred first period consumption for her non-shocked and shocked first period self, respectively. The \overline{c}_1^1 and \overline{c}_1^α expressions represent the period one agent's desired consumptions. The period

¹⁶Housing is not illiquid, according to our definition, because home equity loans traditionally have been easy to obtain. Proposed lending restrictions may decrease housing liquidity.

one agent will want to consume more than the period zero agent would want her to because the period one agent is present-biased. This reasoning results in the inequalities in expression 6.

A little algebra shows that the \hat{c} expressions are less than the \bar{c} expressions by the factor $(2 + \beta + \beta\alpha)/(3 + \alpha)$ for the unshocked agent, and the factor $(2\alpha + \beta + \beta\alpha)/(1 + 3\alpha)$ for the shocked agent.¹⁷ When β falls - the agent is more present based - and α rises - the possible shock increases - both of these factors shrink. This widens the gap between the period zero agent's preferences and her period one self's behavior. The period zero agent therefore would like to constrain the consumption of the first period agent by simultaneously borrowing against her future income and then saving into an asset that is accessible only in period 2. We turn to how such savings vehicles would work, and compare their performance to the other possible policy responses.

5.2 An Illiquid Asset with a Hardship Exemption

We initially consider a system such as that in the United States, in which state favored illiquid assets allow a hardship exemption; agents thus can open an illiquid vehicle to cushion a severe shock. We model this as an illiquid asset into which the period zero agent can save, and which the unlucky period one agent can access. There are two sub-cases: the period zero agent is sophisticated; or she is naive. We first analyze sophisticated agents.

5.2.1 Behavior of Sophisticated Agents

The period zero sophisticated agent understands that her future self will overconsume. She also realizes that she can partially prevent overconsumption by saving into the illiquid asset. The unshocked first period agent cannot access the asset but the shocked first period agent can because, we assume, a shocked agent has experienced a "hardship". The zero period agent thus uses the illiquid savings vehicle to constrain her first period unshocked self's consumption to exactly \hat{c}_1^1 , setting $c_1^1 = \frac{4I}{3+\alpha}$. The hardship exemption permits the shocked period one agent to consume the larger sum of $\bar{c}_1^\alpha = \frac{4\alpha I}{\alpha(2+\beta)+\beta}$.

5.2.2 Welfare

Social welfare is higher when a partially illiquid savings vehicle is available than welfare is under full credit markets. This is because, in one state of the world (the first period agent is unshocked), the zero period agent can compel her period one self to comply with her period zero plan. In the other state of the world (the first period agent is shocked), the period zero agent foresees that her period one self will behave as she would have under full credit markets: that is, the agent will borrow to cushion the shock and because she is present biased.¹⁸ When

¹⁷This factor is less than one as long as $\beta < 1$.

¹⁸We assume that the agent can verify to a third party that she has experienced a hardship - she has been shocked - but the extent of the shock is private information. A partial

present-bias is sufficiently severe, the period zero agent's inability to constrain her first-period shocked self can be such as make totally shutting down credit markets the preferable policy response. The $\alpha\beta$ combinations that make this response preferable lie below line 3 in figure 3:

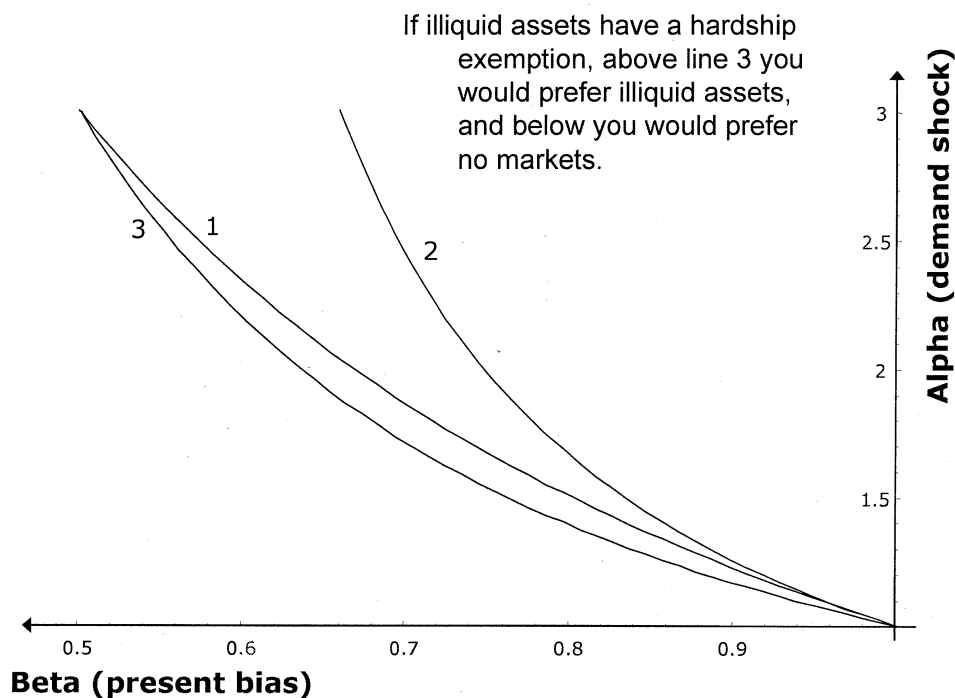


Figure 3:

5.2.3 Behavior of Naive Agents

The sophisticated period zero agent foresees two things. First, her future unshocked self may overconsume, a concern that she addresses with illiquid savings. Second, her future shocked self has the incentive and the ability to escape the constraint. The naive agent, in contrast, likely understands how a hardship exemption works but she does not fully apprehend the extent of her period one self's present bias.¹⁹ In our model, however, naive agents behave almost identically to sophisticated agents if the naive agent plans not to access her

consequence of the shock is an increase in the agent's marginal utility, which is entirely unobservable. Also, the administrator of an IRA for a number of persons cannot conveniently observe a plan participant's medical or other needs. Thus, while a shock is necessary to permit the agent to borrow, the agent also can exploit the shock to engage in present-biased consumption.

¹⁹Recall that this agent is plausibly assumed to discount the future using $\beta_m < \beta$.

savings until the final period. To see why, first note that the illiquid savings vehicle generates a higher return than liquid assets (to compensate investors for the illiquidity). More formally, we normalized the interest rate to zero above, but suppose instead that the illiquid savings vehicle (which the period zero agent knows cannot be accessed by her period one unshocked self) produces the slightly higher return of $\varepsilon > 0$.

The naive agent believes that her period one self will act as her period zero self prefers. She thinks, that is, that she later will want to consume \widehat{c}_1^1 and \widehat{c}_1^α , and because alpha is greater than one, $\widehat{c}_1^1 \leq \widehat{c}_1^\alpha$. The naive period zero agent therefore predicts that her period one self would prefer to save $I - \widehat{c}_1^1$ and $I - \widehat{c}_1^\alpha$ for period two consumption, depending on whether she is shocked in period one or not. This agent believes that she is actually compensated for satisfying these preferences because saving into the illiquid asset generates a slightly higher interest rate than saving into liquid assets. Hence, this agent inadvertently protects $I - \widehat{c}_1^1$ of her period one income. In the event of a shock, however, the naive period zero agent knows that she can access the funds. In sum, the naive period zero agent saves into the partly illiquid asset because she believes that is paid a premium to increase her period two wealth at no cost to her period one consumption. The unshocked naive period one agent thus is surprised to find that she cannot fully satisfy her period one preferences because a portion of her assets are locked up.

5.3 Illiquid Assets with no Exemption

A totally illiquid asset – an illiquid asset with no hardship exemption – permits the first period agent to constrain much of the consumption of her second period self, but this vehicle also reduces the flexibility with which the second period self can react to a shock. How the agent should use a total self-control vehicle turns on which of these consequences is more important. To solve for behavior, first note that the period zero agent maximizes the non-discounted sum of future consumption utilities, taking into account the probability of being shocked. This is equal to:

$$\frac{1}{2}\alpha \ln(c_1^\alpha) + \frac{1}{2}\ln(c_1^1) + \frac{1}{4}\ln(I+s_1^1) + \frac{1}{4}\alpha \ln(I+s_1^1) + \frac{1}{4}\ln(I+s_1^\alpha) + \frac{1}{4}\alpha \ln(I+s_1^\alpha) \quad (7)$$

If the first period agent saved amount S into the totally illiquid asset, she would leave $2I - S$ for a period two agent to consume. Denote the maximum amount that the first period agent could consume as \check{c} . This gives us the first-order condition that determines the period zero agent's choice:

$$\frac{\partial}{\partial \check{c}} \left[\frac{1}{2}\alpha \ln(c_1^\alpha) + \frac{1}{2}\ln(c_1^1) + \frac{1}{4}\ln(I+s_1^1) + \frac{1}{4}\alpha \ln(I+s_1^1) + \frac{1}{4}\ln(I+s_1^\alpha) + \frac{1}{4}\alpha \ln(I+s_1^\alpha) \right] = 0 \quad (8)$$

subject to the following constraints:

$$s_1^1 = I - c_1^1 \quad \text{and} \quad s_1^\alpha = I - c_1^\alpha \quad (9)$$

and

$$c_1^1 = \min\{\tilde{c}, \bar{c}_1^1\} = \min\left\{\tilde{c}, \frac{4I}{2+\beta+\alpha\beta}\right\} \quad (10)$$

and

$$c_1^\alpha = \min\{\tilde{c}, \bar{c}_1^\alpha\} = \min\left\{\tilde{c}, \frac{4\alpha I}{\alpha(2+\beta)+\beta}\right\} \quad (11)$$

The constraints in equation 9 are the intertemporal budget constraints; that is, the agent consumes in period two her income plus what she did not consume in period one. The more important constraints are expressed in equations 10 and 11. Since there is no hardship exemption, the agent must impose the same constraint \tilde{c} on both her shocked and her unshocked period one self (though this constraint need not always bind). The agent prefers to consume more when shocked than when not shocked, which yields the zero period agent three choices: to set the constraint such as to restrict both of her period one selves; to constrain only her shocked self; or to constrain neither of her future selves. Similarly, three first order conditions arise when we maximize equation 8 subject to these three constraints. Examining these three first order conditions:

First-order condition one: The first mathematically possible first-order condition is:

$$\frac{3\tilde{c}+\alpha\tilde{c}-4I}{4\tilde{c}^2-8\tilde{c}I} = 0 \quad (12)$$

which must hold when:

$$\tilde{c} \leq \frac{4I}{2+\beta+\alpha\beta} \quad \text{and} \quad \tilde{c} > \frac{4\alpha I}{\alpha(2+\beta)+\beta} \quad (13)$$

Though this is a mathematical possibility, it can be ignored because the constraint binds the unshocked first period self but not the shocked first period self. We assume, however, that the shock α is greater than one, as a consequence of which the shocked agent will want to consume more than her unshocked counterpart. Therefore the zero period agent would reject these constraints. The second and third first-order conditions, however, describe possible states of the world.

First-order condition two: The second mathematically possible first-order condition is:

$$\frac{\tilde{c}+3\alpha\tilde{c}-4\alpha I}{4\tilde{c}^2-8\tilde{c}I} = 0 \quad (14)$$

which must hold when

$$\tilde{c} > \frac{4I}{2+\beta+\alpha\beta} \quad \text{and} \quad \tilde{c} \leq \frac{4\alpha I}{\alpha(2+\beta)+\beta} \quad (15)$$

In this case, the period zero agent constrains her shocked first period self, but does not constrain her unshocked self. If the agent would find this optimal, then solving this first-order condition shows that she would find it best to set

$$\tilde{c} = \frac{4\alpha I}{1+3\alpha} = \widehat{c}_1^\alpha \quad (16)$$

That is, the agent would set the constraint \tilde{c} equal to what she would want her shocked period one self to consume. This self would use her available income

and ability to borrow to cushion the shock, but her unshocked self would be free to act on her present bias. The constraint thus is desirable when possible shocks are severe but the agent's present bias is moderate or, put mathematically, when:

$$\widehat{c}_1^1 < \overline{c}_1^1 \ll \widehat{c}_1^\alpha < \overline{c}_1^\alpha \quad (17)$$

Intuitively, setting the constraint marginally higher, to say $\tilde{c} + \varepsilon$, would not change the behavior of the unshocked agent (she would still not find the constraint binding), but it would allow the shocked agent to consume ε more than the zero period agent prefers. Similarly, lowering the constraint would not change the behavior of the unshocked agent, but would force the shocked agent to consume ε less than she should. Therefore, the zero period agent maximizes by constraining her shocked first period self perfectly (she cannot access the savings in the vehicle), while permitting her unshocked first period self to consume as she wishes.

First-order condition three: The final first-order condition is:

$$\frac{(1+\alpha)(\tilde{c}-I)}{\tilde{c}(\tilde{c}-2I)} = 0 \quad (18)$$

which must hold when:

$$\tilde{c} \leq \frac{4I}{2+\beta+\alpha\beta} \quad \text{and} \quad \tilde{c} \leq \frac{4\alpha I}{\alpha(2+\beta)+\beta} \quad (19)$$

In this case, the constraint \tilde{c} is set so low as constrain both the shocked and the unshocked period one agent. The first-order condition from equation 18 holds when

$$\tilde{c} = I \quad (20)$$

That is, a zero period agent who finds it optimal to constrain both of her period one selves will force these selves to consume exactly their income. This case holds when both period one selves are extremely present-biased. These selves, the zero period agent foresees, will consume exactly the same amount: everything they are allowed to consume. Since the period zero agent thus cannot allow her future selves to adjust their consumption to the presence or absence of a shock, she chooses instead to smooth her consumption between periods one and two by forcing her period one self to consume exactly her income. An agent who foresees that her future self is very present-biased thus will shut down her access to the credit market, which she does by preventing her period one self from borrowing against future income.

5.4 The Welfare Effects of Illiquid Savings Vehicles

Society is indifferent between shutting down credit markets and providing an illiquid asset with no hardship exemption because, in the latter case, the agent could shut down her own access to credit markets by constraining her future selves to consuming their income. To see when society would strictly prefer to

offer an illiquid asset to shutting down the market, we let equation 7 equal 0, the social utility realized when all agents consume exactly their income in every period. The set of points for which this is exactly true is line 4 in figure 4. An illiquid asset is better than a total borrowing constraint when (eq 18) holds. In this event, the first period agent sets her second period consumption to either \bar{c}_1^I and \widehat{c}_1^α (when there is no hardship exemption), or \widehat{c}_1^I and \bar{c}_1^α (when there is a hardship exemption).

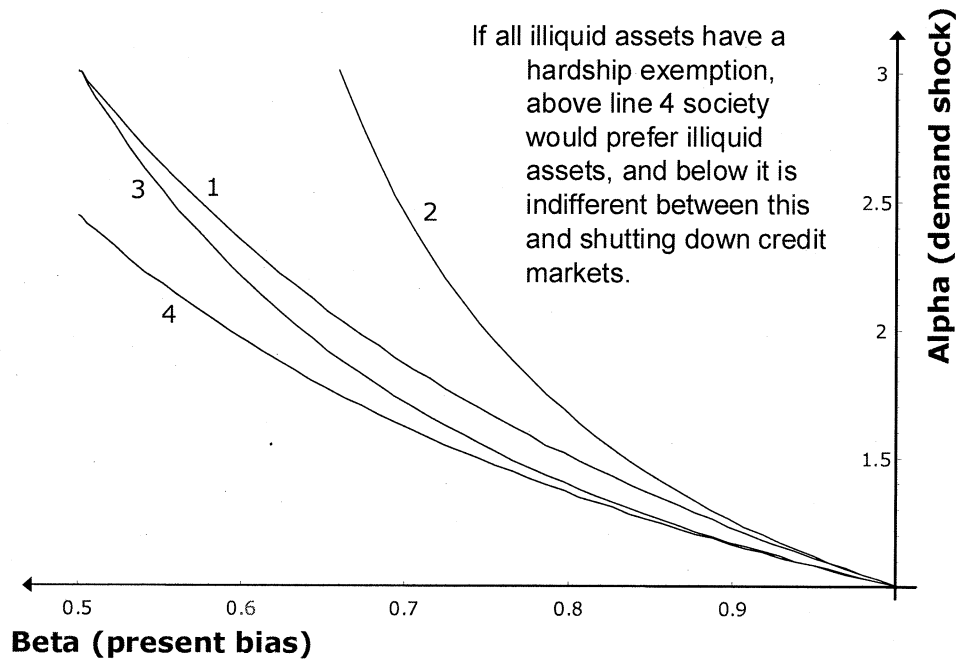


Figure 4:

To summarize, when the illiquid savings vehicle cannot be accessed, the period one agent overconsumes when she is not shocked, but consumes optimally when she is. When the illiquid savings vehicle is accessible in case of hardship, the period one agent consumes optimally when she is not shocked but overconsumes when she is. The period one agent thus can use either of the illiquid savings vehicles to constrain one of her second period selves to act optimally (despite this self's present bias), but must permit her other second period self to overconsume. In our simple model, social utility is identical under both possibilities. Finally, society weakly prefers agents to have access to an illiquid asset with no hardship exemption to prohibiting borrowing.

5.4.1 Naive agents and Observational Equivalence:

These welfare implications hold for the most part if agents are naive about their present biases. Recall that, when the illiquid asset had a hardship exemption, but illiquid assets paid a slightly higher interest rate than liquid assets, the results above were independent of the agent's degree of self knowledge. Now let the illiquid asset have no hardship exemption. Above line four in figure 4 the sophisticated agent saves enough to constrain her future shocked self to consume exactly the right amount, $\widehat{c}_1^\alpha = \frac{4aI}{1+3\alpha}$. The naive agent does the same, not because she thinks that she needs self-control, but because she thinks that saving into the illiquid asset maximizes her return. She does not save more than this, however, because in the absence of a hardship exemption, she does not want to distort her shocked future self's consumption. Therefore, the observational equivalence between sophisticated and naive agents that we observed under illiquid assets with a hardship exemption also holds when agents find themselves above line four in figure 4. This equivalence disappears when agents fall below line four. In these cases, present bias is so strong that the sophisticated agent restricts both of her future selves to consume exactly their income I . Put another way, the sophisticated agent uses the totally illiquid asset to block her future selves from any access to the credit market. A naive agent underestimates how much her future self would benefit from the self control that an illiquid asset provides. Thus, she continues to save into the illiquid asset just enough to permit her to consume $\widehat{c}_1^\alpha = \frac{4aI}{1+3\alpha}$, if she is shocked in period one. Below line four, then, the naive agent will not constrain her future unshocked self as much as she would have had she anticipated her future self control problems. In this case, both sophisticated and naive agents save into illiquid assets but the naive agent saves less than the sophisticated agent.

5.5 Behavior and Welfare with both Types of Illiquid Asset

Suppose now that society provides agents with both types of illiquid asset. In this world, a sophisticated period zero agent can independently target the consumption of her future shocked and her future unshocked selves. She does this by accessing the capital markets and borrowing $2I$, her total future income. Recall that the period zero agent wants her shocked period one self to consume no more than $\widehat{c}_1^\alpha = \frac{4aI}{1+3\alpha}$. She thus saves $2I - \frac{4aI}{1+3\alpha}$ into the illiquid asset that lacks a hardship exemption. Since $\widehat{c}_1^\alpha \geq \widehat{c}_1^\alpha$ (the first period agent will always want to consume more than her period zero self wanted her to consume), the shocked first period agent is compelled to consume exactly the right amount.

The period zero agent also can control her unshocked first period self's consumption by saving into the illiquid asset with a hardship exemption the difference between her preferred shocked and her preferred unshocked selves' consumption levels. Precisely, the zero period agent saves $\widehat{c}_1^\alpha - \widehat{c}_1^1$ into the illiquid asset with a hardship exemption. This will not change how much her shocked first period self can consume (because in the event of a shock, this asset can

be accessed early). Her unshocked self, however, cannot access this vehicle and hence is constrained to consume $\widehat{c}_1^\alpha - (\widehat{c}_1^\alpha - \widehat{c}_1^1)$, which is \widehat{c}_1^1 , exactly the right amount. In sum, a sophisticated agent with access to both illiquid asset types can achieve the first best consumption path for herself in every state of the world. Therefore, the optimal policy response is to offer agents access to both types of savings vehicle.

5.5.1 Behavior of Naive agents

Naive agents also achieve the first best when they have access to both types of illiquid assets if the three forms of savings (liquid savings and illiquid assets without and with a hardship exemption) are attractive to naive agents in ascending order of illiquidity. When interest rates are identical across these three types of savings, naive agents are indifferent between the savings behavior that sophisticated agents engage in (which achieves the first best) and, for example, saving entirely in liquid savings (which yields sub-optimal consumption). If this indifference is broken in favor of the restrictive assets, however, the naive agent will (unwittingly) bind herself in exactly the same way as would a sophisticated agent. To be precise, recall that we have normalized the real interest rate of liquid savings to 0. Assume that, as compensation for their illiquidity, the illiquid assets without and with a hardship exemption pay interest rates ε and δ respectively, with $\varepsilon > \delta > 0$. This pattern seems likely because illiquid assets typically pay higher interest rates in competitive capital markets to compensate for their illiquidity. Then for relatively small ε and δ , naive agents behave identically to sophisticated agents, saving as much as they foresee not needing in period one into the relevant illiquid assets.

5.5.2 General Observational Equivalence

Sophisticated and naive agents behave in identical ways, in the model so far, because they make similar period zero plans and similar illiquid asset allocations, not because they make similar consumption decisions. This observational equivalence result may be thought to depend on our permitting agents to make a saving/consumption choice in only one period. We show here, in a multi-period extension, that sophisticated and naive agents are observationally equivalent more generally; they make similar consumption choices as well as similar illiquid asset choices.

To investigate the consumption decisions of agents who are more or less aware of the extent of their present-bias, we need a period in which the agent bases her consumption choices partly on her expectation as to how she will consume in the future. Naive and sophisticated agents have different expectations, and this raises the possibility that they will behave differently. To investigate this possibility, we now assume that agents live for four periods. As before, period zero is the planning period, and now period three is the final period, in which the agent consumes her entire endowment. In periods one, two and three, the agent earns income I and, with probability of $1/2$, experiences a demand shock

that increases her marginal utility of income by the factor α . The agent must make consumption/savings choices in periods one and two. Competitive credit markets permit agents to save for (and thus to borrow against) the future.

If every agent type had access to a "complete" set of illiquid assets (assets with every possible expiry date), sophisticated and naive agents would again behave identically, so long as interest rates increase in the illiquidity of a savings vehicle. In that case, in period zero the sophisticated and the naive agent will save into these assets in identical ways. The set of illiquid assets may not be complete in this manner, however. For example, agents may only have access to illiquid assets that are tied to their retirement (in our model, assets that can only be accessed in period three). In this case, regardless of how much the period zero agent saves into illiquid assets for the far future, the agent is free to allocate her consumption across periods one and two.

For simplicity, we initially focus on the consumption and savings decisions of the non-shocked first period agent, who may be either sophisticated or naive. Analysis of the shocked first period agent turns out to be identical to that of the unshocked agent. Regarding the logic of the analysis, if sophisticated and naive agents behave identically when making later-period decisions, then sophisticated and naive period zero agents will also behave identically when deciding how to constrain their later selves. Therefore, if we show that sophisticated and naive agents do make the same period one and two choices, then observational equivalence holds more generally. We begin our analysis with period two, in which both agent types behave identically, and we then work backwards to period one.

Behavior when both Illiquid Assets are Available Agents behave the same in period two of the four period model as they behaved in period one of the three period model. Just as before, the agent must choose two consumption levels: if she is shocked or not shocked. The agent maximizes her period two utility with respect to consumption, which yields two first-order conditions; one condition for period two with a demand shock:

$$\frac{\partial}{\partial c_2^\alpha} \{a \ln[c_2^\alpha] + \frac{1}{2}[\beta \ln(I + s_2^\alpha) + \alpha\beta \ln(I + s_2^\alpha)]\} = 0 \quad (21)$$

and a second first order condition without a demand shock:

$$\frac{\partial}{\partial c_2^1} \{\ln[c_2^1] + \frac{1}{2}[\beta \ln(I + s_2^1) + \alpha\beta \ln(I + s_2^1)]\} = 0 \quad (22)$$

The agent can pass on to period three her period two income plus her savings from period one minus her consumption in period two. This yields:

$$s_2^x = I + s_1 - c_2^x \quad (23)$$

for both $x = \alpha$ - a shock - and $x = 1$ - no shock. Substituting 23 into 21 and 22, if the consumer experiences a demand shock in period two, she consumes:

$$c_2^\alpha = \frac{2\alpha(2I + s_1)}{\alpha(2 + \beta) + \beta} \quad (24)$$

and if she is not shocked, she consumes:

$$c_2^1 = \frac{2(2I + s_1)}{2 + \beta + \alpha\beta} \quad (25)$$

5.5.3 Sophisticated Agents

The sophisticated agent decides how much to spend and how much to save by maximizing her period one utility from consumption and her expected utility from consumption in periods two and three. She discounts expected utility to present value using the true beta. Looking forward from period one, the agent faces two period two possibilities – she is shocked or she is not – and four period three possibilities: she is shocked in period three and had been shocked in period two; she is shocked in period three but had not been shocked in period two; she is not shocked in period three but had been shocked in period two; she is not shocked in period three and had not been shocked in period two. The period one agent thus maximizes:

$$\ln[c_1] + \frac{1}{2}\beta(\ln[c_2^1] + a \ln[c_2^\alpha]) + \frac{1}{4}\beta(\ln[I + s_2^\alpha] + \alpha \ln[I + s_2^\alpha] + \ln[I + s_2^1] + \alpha \ln[I + s_2^1]) \quad (26)$$

with respect to consumption. The first term in this expression is the person's utility from period one consumption; the next two terms are her expected utility from consumption in period two when she is not shocked and when she is, discounted by β ; and the last four terms represent her expected utility from consumption in each of the four possible period 3 states, again discounted by β .

We have solved for the agent's consumption and savings decisions in periods two and three. Substituting for c_2^α , c_2^1 , s_2^α , s_2^1 , and s_1 and solving the agent's first period maximization problem yields the first order condition:

$$\frac{c_1(\beta + \alpha\beta + 1) - 3I}{c_1(c_1 - 3I)} = 0 \quad (\text{Condition Four})$$

Using Condition Four, we can characterize the sophisticated agent's period one consumption and savings decisions. The agent consumes:

$$c_1 = \frac{3}{1 + \beta + \alpha\beta} I \quad (27)$$

which permits her to save:

$$s_1 = \left(1 - \frac{3}{1 + \beta + \alpha\beta}\right) I \quad (28)$$

Remark 3 *Countervailing Forces*

The first-period unshocked agent balances two forces when choosing consumption. First, the agent knows that she may later experience exogenous shocks that will increase her marginal utility of consumption. More precisely,

the agent has a lower marginal utility of consumption in period one, in which she is not shocked, than she expects to have in periods two and three, in which she may be shocked. The recognition of this pushes the sophisticated agent to save, setting $c_1 < I$.²⁰ This agent knows, however, that her savings will be reduced by the tendency of the second period present-biased agent to over-consume. Expressions (24) and (25) show that the period two agent consumes $(\frac{2x}{x(2+\beta)+\beta})$ of the period one agent's savings, where $x \in \{\alpha, 1\}$. For example, if $\alpha = 3$ and $\beta = 2/3$, the shocked period two agent consumes 70% of her period one savings. A non-shocked agent consumes 60%. The period one agent believes that these "taxes" are too high, and she responds by reducing her precautionary saving for period three.

5.5.4 Naive Agents

The naive agent's incomplete awareness of her present-biased preferences may matter because she incorrectly supposes her discount rate between periods two and three to be β_n where $\beta < \beta_n \leq 1$. The agent thus predicts that if she is shocked in period two, she will consume:

$$c_{2,n}^\alpha = \frac{2\alpha(2I + s_1)}{\alpha(2 + \beta_n) + \beta_n} \quad (29)$$

and if she is not shocked she will consume:

$$c_{2,n}^1 = \frac{2(2I + s_1)}{2 + \beta_n + \alpha\beta_n} \quad (30)$$

The naive period one agent realizes that she may be shocked in either future period, and this again generates two possible period two cases and four period three cases. The naive agent maximizes:

$$\ln[c_1] + \frac{1}{2}\beta(\ln[c_{2,n}^1] + \alpha \ln[c_{2,n}^\alpha]) + \frac{1}{4}\beta(\ln[I + s_{2,n}^\alpha] + \alpha \ln[I + s_{2,n}^\alpha] + \ln[I + s_{2,n}^1] + \alpha \ln[I + s_{2,n}^1]) \quad (31)$$

with respect to period one consumption. Since we have solved for all of the variables in this maximization problem, we can substitute them to derive the first order condition:

$$\frac{c_1(\beta + \alpha\beta + 1) - 3I}{c_1(c_1 - 3I)} = 0 \quad (\text{Condition Five})$$

We use Condition Five to solve for the naive first period agent's consumption and savings decisions. This yields:

$$c_1 = \frac{3}{1 + \beta + \alpha\beta} I \quad (32)$$

²⁰The agent's marginal utility from consumption in period one is: $\frac{1}{3I}(\beta + \alpha\beta + 1)$. The agent's marginal utility of consumption in period 3 is: $\frac{1}{3I}(\beta + \alpha\beta + 1)[\frac{1}{2e + 2re}(\beta + 1) + \frac{1}{4re + 4r^2e}(2\alpha + \beta + \alpha\beta)]$. The first term in this expression is identical to the agent's first period marginal utility from consumption. The term in brackets exceeds β , which means that the present - i.e., period one - value of the agent's period three utility of consumption is higher than the agent's period one utility of consumption.

which leaves as savings:

$$s_1 = \left(1 - \frac{3}{1 + \beta + \alpha\beta}\right)I \quad (33)$$

Expressions 32 and 33 are identical to Expressions 27 and 28. This leads to the following proposition:

Proposition 1 *Sophisticated and naive persons make the same savings and consumption decisions in all periods.*

Further to clarify this Proposition, recall the expressions for the utility of the first period agent as given by equation 26 (if the agent is sophisticated) and equation 31 (if the agent is naive). In each of the four possible third period states, the consumption of the third period agent enters the utility of the first period agent as a multiple of

$$\ln[3I - c_1 - c'_2] \quad (34)$$

where c'_2 represents the naive first period agent's prediction of her second period consumption. The agent knows that her second period budget will be $2I + s$. Both the naive and the sophisticated period two agent consume a fraction of this budget. The first period agent knows that this fraction is a function only of her discount rate (β_n for the naive agent and β for the sophisticated agent) if she is not shocked, and is a function of her discount rate and the magnitude of the shock (α) if she is shocked. In this model, an agent's discount rate and her view as to the values α can take do not change; hence, the fraction, denoted χ , that determines the naive agent's second period consumption is a constant. She believes that she will consume $c'_2 = \chi(2I + s_1)$, where χ takes on a higher value if the agent is shocked than if she isn't (more generally, an agent with constant relative risk aversion consumes a constant share of her remaining wealth in every period). Recalling that $c_1 = I - s_1$, and substituting c_1 and c'_2 into equation 34, the naive first period agent's expected utility from third period consumption is given by

$$\ln[2I - s_1 - \chi(2I + s_1)] = \ln[(1 - \chi)(2I + s_1)] \quad (35)$$

The agent will maximize this expression with respect to s_1 to choose how much to save in period one. The solution to this maximization problem is

$$\frac{\partial}{\partial s_1} \ln[(1 - \chi)(2I + s_1)] = \frac{1 - \chi}{(1 - \chi)(2I + s_1)} = \frac{1}{(2I + s_1)} \quad (36)$$

The χ value drops out – it does not enter the first-order condition of the naive first-period agent – because it is a constant. Since the sophisticated first period agent also forecasts second period consumption by applying a constant fraction to her period two endowment (her fraction differs from χ only by the discount rate), both sophisticated and naive agents save the same amount in period one. Sophisticated and naive agents behave identically in period two as well, so

we have shown that sophisticated and naive agents behave identically in every period.

Regarding the intuition, the naive first period agent's delusion causes her to underestimate her period two consumption. As a consequence, she believes that more of the dollars she saves in period one will be passed on to her third period self. Since her expected third period utility is increasing in period one savings, the deluded agent is induced to save more than the sophisticated agent would save. On the other hand, an agent who underestimates her second period consumption necessarily overestimates her third period endowment. This mistake induces her to save less in period one than the sophisticated agent would save. When the agent is constrained to consume a constant fraction of her endowment in every period, these mistakes must offset. Hence, the naive period one agent saves as much as the sophisticated period one agent.

6 Conclusion

6.1 Results:

Persons may experience exogenous shocks that increase their marginal utility of consumption, and persons also have present-biased preferences. The prospect of a shock provides the agent with a motive to save while the present biased preference provides the agent with a motive to spend excessively. Some agents are aware of their tendency to yield to weakness of will but others are not. We analyze the tradeoff between saving and spending from the individual perspective – what do naive and sophisticated agents do? – and from the social perspective – what should society do? We develop the following results:

- Agents who are not shocked save if their present-biased preferences are relatively mild and they face relatively serious shocks. Conversely, unshocked agents borrow to finance current consumption if their present-biased preferences are relatively strong and they face relatively mild shocks.
- Every agent, if unconstrained, borrows to cushion the effect of an exogenous shock.
- A strong borrowing constraint is desirable when its principal effect is to cause agents to transfer wealth from states in which their marginal utility is low to states in which their marginal utility is high. The constraint is undesirable when agents in general save when they should, so that the principal effect of the strong constraint is to prevent an agent from fully cushioning a shock to her marginal utility. Data suggest that typical agents do not have strong present biased preferences (agents exhibit betas of .9 or above). This suggests caution in imposing strong constraints.
- Providing agents with access to both a completely and a partly illiquid savings vehicle is welfare improving relative to the more extreme policies of complete or no borrowing freedom. The agent can allocate her savings

between these vehicles such that all of her future selves consume and save as her initial self wishes.²¹

- Sophisticated and naive agents make similar savings and consumption decisions. The reform of making liquid and illiquid saving vehicles available to individual borrowers thus can be implemented though the regulator does not know the proportion of naive agents in any borrower population.

6.2 Remarks:

We assume that the agent correctly predicts α , the magnitude of the shock she may experience. An agent may underestimate α either rationally, which may be occurring today, or because she is excessively optimistic. Underestimating α has two, partly offsetting effects. First, the agent may save too little. Second, the agent will save more in the illiquid savings vehicle because she underestimates her need for the money. The former effect reduces welfare. The latter effect increases welfare for unshocked agents, whose present bias will be further curbed.

There is a debate whether a present biased agent has a reasoning defect or is acting on her preferences. We adopt a common solution to this concern by measuring welfare according to the preferences of the period zero, unbiased, agent. A less common solution is to measure welfare by the retrospective preferences of the last period unbiased agent. Final period agents put less weight on earlier periods than agents living in those periods, and thus final period agents prefer the early period agents to have saved more than they did. We use the period zero agent as a benchmark for two reasons. First, this is the more common measure. Second, implementing the preferences of the last period agent requires coercion: earlier agents must be made to save. We choose freedom in the absence of a compelling justification for making last period preferences the measure.

Finally, we assume that agents can create consumption/savings plans. The seemingly sparse data suggest that some agents plan to the "planning horizon" while others do not.²² Our analysis may be usefully extended by letting agents differ in their ability to plan.

References

- [1] Adler, Barry, Ben Polak and Alan Schwartz (2000) "Regulating Consumer Bankruptcy: A Theoretical Inquiry", 29 J. Legal Studies 585.

²¹Amador, et al argue that a minimum savings rule would be part of an optimal precommitment strategy. This is a thoughtful idea but it is unclear how the rule could be implemented.

²²Hey and Knoll (2007) test persons' ability to plan. In their experiment, some 57.8% of subjects "plan ahead to the planning horizon (as they should according to economic theory)" while some 41.4% of subjects do not plan ahead.

- [2] Ahumada, Hildegart A. & Maria Lorena Garegnani (2007), "Testing hyperbolic discounting in consumer decisions: Evidence for Argentina", 95 *Economics Letters* 146-150.
- [3] Amador, Manuel, Ivan Werning and George-Marios Angeletos (2006) "Commitment vs. Flexibility", 74 *Econometrica* 365-96.
- [4] Azfar, Omar (1999), "Rationalizing hyperbolic discounting", 38 *J. Econ. Behavior & Org.* 245-252.
- [5] Bar-Gill, Oren (2004), "Seduction by Plastic", 98 *Northwestern Law Review* 1373
- [6] _____ (2008), "The Law, Economics and Psychology of Subprime Mortgage Contracts", NYU L&E Working Paper No. 08-59.
- [7] Baucells, Manel, Franz H. Heukamp and Antonio Villasis (2006), "Risk and Time Preference Integrated", Working Paper.
- [8] Blume, L. and D. Easley (1992), "Evolution and market behavior", *Journal of Economic Theory* 58, pp. 9-40
- [9] Bommier. Antoine (2006), "Uncertain Lifetime and Intertemporal Choice: Risk Aversion as a Rationale for Time Discounting", 47 *Int. Econ. Rev.* 1223-1246.
- [10] Cairns, John and Marjon van der Pol (1999), "Valuing future private and social benefits: The discounted utility model versus hyperbolic discounting models", 21 *J. Econ. Psychology* 191-205.
- [11] Chatterjee, Satyajit, Dean Korbae, Makoto Nakajima and Jse-victor Rios-Rull (2007), "A Quantitative Theory of Unsecured Consumer Credit with Risk of Default", *Econometrica* 75(5) 1525-1589.
- [12] Chetty, Raj, (2006), "A New Method of Estimating Risk Aversion", *American Economic Review* 96(5): 1821-1834
- [13] Dasgupta, Partha and Eric Maskin (2005), "Uncertainty and Hyperbolic Discounting", 95 *Amer. Econ. Rev.* 1290-1299.
- [14] Deaton, Angus (1992), *Understanding Consumption*.
- [15] Eliehausen, Gregory, E. Christopher Lundquist and Michael E. Staten (2007), "The Impact of Credit Counseling on Subsequent Borrower Behavior", 41 *The Journal of Consumer Affairs* 1-28.
- [16] Fehr, Hans, Christian Habermann & Fabian Kindermann (2008), "Social security with rational and hyperbolic consumers", 11 *Rev. Econ. Dynamics* 884-903.

- [17] Frederick, Shane, George Loewenstein & Ted O'Donoghue (2002), "Time Discounting and Time Preference: A Critical Review," *Journal of Economic Literature*, vol. 40(2), pages 351-401.
- [18] Friend, Irwin; Marshall E. Blume (1975), "The Demand for Risky Assets", *The American Economic Review*, Vol. 65, No. 5, pp. 900-922
- [19] Fudenberg, Drew and David K. Levine (2006), "A Dual-Self Model of Impulse Control", 96 *AER* 1449-1475.
- [20] Imrohoroglu, Ayse, Selahattin Imrohoroglu and Douglas H. Jones (2003), "Time Inconsistent Preferences and Social Security", *Quarterly J. of Econ.* (May 2003) 745-784.
- [21] Gul, Faruk and Wolfgang Pesendorfer (2004), "Self-control, revealed preference and consumption choice", 7 *Rev. Econ. Dynamics* 243-64.
- [22] Hey, John D. and Julia A. Knoll (2007), "How far ahead do people plan?", 96 *Economics Letters* 8-13.
- [23] Kanheman, Daniel and Richard Thaler (2006), "Utility Maximization and Expected Utility", 20 *J. Econ. Persp.* 221-2-
- [24] Laibson, David (1997), "Golden Eggs and Hyperbolic Discounting", 112 *Q. J. Econ.* 443-77.
- [25] Livshits, Igor, James MacGee & Michele Tertilt (2007), "Consumer Bankruptcy: A Fresh Start", 97 *American Economic Review* 402-18.
- [26] Meier, Stephen & Charles Sprenger (2007), "Impatience and Credit Behavior: Evidence from a Field Experiment", Federal Reserve Bank of Boston, Working Paper No. 07-3.
- [27] Pottow, John A.E. (2007) "Private Liability for Reckless Consumer Lending", 2007 *U. Illinois Law Review* 405-465.
- [28] Read, Daniel (2001), "Is Time-Discounting Hyperbolic or Subadditive?", 23 *J. Risk and Uncertainty* 5-32.
- [29] Roelofsma, Peter H.M.P. and Daniel Read (2000), "Intransitive Intertemporal Choice", 13 *J. Behavioral Decision Making* 161-77.
- [30] Rubenstein, Ariel (2003), "Economics and Psychology'? The Case of Hyperbolic Discounting", 44 *Int. Econ. J.* 1207-16.
- [31] Sopher, Barry and Arnav Sheth (2005), "A Deeper Look at Hyperbolic Discounting", 60 *Theory and Decision* 219-55.
- [32] Szpiro, George G. (1986), "Measuring Risk Aversion: An Alternative Approach", *The Review of Economics and Statistics*, Vol. 68, No. 1, pp. 156-159