Managerial Attention Allocation in Optimal Incentive Contracts*

Ricard Gil        Jordi Mondria
UC Santa Cruz     University of Toronto

May 6, 2008

Abstract

This paper presents the introduction of managerial attention allocation constraints in optimal incentive contracts. There is an agent who provides non-contractible effort in a number of tasks and a principal who designs a linear incentive contract, composed by a variable and a fixed factor, and monitors the effort of the agent. The framework in Holmstrom and Milgrom (1991) is extended to allow the principal to decide the amount of monitoring allocated in each task. More attention allocated to a given task improves the task contractibility due to a decrease in the uncertainty about the effort provided by the agent. The principal allocates the same level of attention and provides the same incentive contract across tasks under symmetric decreasing returns to scale in production and monitoring. However, when there are increasing returns to scale in the monitoring technology, the principal offers an unbalanced incentive contract and allocates asymmetric amounts of attention across tasks.

---

*Ricard Gil is an Assistant Professor in the Department of Economics at UC-Santa Cruz, and Jordi Mondria is an Assistant Professor in the Department of Economics at the University of Toronto. All errors are ours.
1 Introduction

To this day, there is an extensive literature in Economics that focuses on the study of contract theory and therefore the principal-agent problem. Most of the papers in this literature start off by assuming the presence of a principal that hires an agent to undertake a certain action which is not perfectly observable. For this reason, the agent may not take the action if she is concerned that she may not be rewarded due to the less-than-perfect action observability. The principal takes into account this factor and designs a contract that aligns the interests of the agent with those of their own. The optimal contract will provide stronger incentives the more observable the actions are and the lower the agent’s degree of risk aversion. These predictions were first formalized by Holmstrom (1979) and Shavell (1979) for the case of one-task jobs and by Holmstrom and Milgrom (1991) in the case of jobs compounded by more than one task. In these papers and most of the previous literature, the main role of the principal is to design an optimal incentive contract for the agent while taking the contractibility of actions as given.\footnote{Some papers have allowed the principal to take costly actions that enhance the profitability of the agent’s actions.} In this paper we extend the existing framework from Holmstrom and Milgrom (1991) and provide a new framework in which the contractibility of different tasks is endogenous. The goal of this paper is then to derive the optimal incentive contracts in such a setting where the principal simultaneously shapes the contracting environment and designs the incentive contract that she will offer to the agent.

For this purpose, in this paper we build upon the multi-tasking contracting scenario in Holmstrom and Milgrom (1991) by introducing an attention allocation constraint on the principal/manager side. The model presents a risk-neutral principal contracting with a risk-averse agent over effort in two different tasks. The agent’s effort is not perfectly observable and thus the moral hazard problem. We introduce to the classic contracting problem an attention constraint for the manager. The principal is endowed with a monitoring capacity that must be allocated among all tasks that define the job. More attention allocated to a given task increases the precision (decreases the uncertainty) of pay performance measures in the given task used by the principal in the incentive contract. Since the principal is constrained on the total level of attention that she can allocate among all tasks, she faces a trade-off on effort measurability across tasks. In other words, in this paper the principal’s monitoring decisions are endogenous and this affects the contractibility of the tasks that define the job. This differs from previous research in that typically the monitoring decisions of the principal are implicitly taken as given.
We solve the model through backward induction where the only difference with the regular model is that in the first period the principal chooses the allocation of her attention endowment across all tasks. Our solution generates two main results that we describe next. Our first result is consistent with findings in the previous literature. An increase in the attention allocated to one task leads to a decrease in the uncertainty of the performance pay measure and an increase in the contractibility of that task. As a consequence, the agent will increase the effort exerted in that task due to the decrease in the uncertainty of the performance pay measure.

Once established the link between managerial attention and the effort of the agent, our second set of results follows. Our model allows us to investigate how the manager allocates the monitoring capacity under different circumstances and how she optimally shapes incentives across tasks within a contract given the allocation of her attention endowment. For this purpose, we compare results under different technologies in production and monitoring. We find that a symmetric managerial attention allocation across tasks is optimal under symmetric decreasing returns to scale in production and monitoring. However, even in the case of symmetric technologies across tasks, there exists an optimal asymmetric attention allocation across tasks when there are increasing returns to scale to monitoring. Not surprisingly, we also obtain the same asymmetric result when introducing increasing returns to scale in production. Not surprisingly, the principal or manager will also choose an asymmetric allocation of attention when there is an asymmetry in the monitoring technology or in the contribution to profits of each task.

To the best of our knowledge, the framework and results in this paper are a contribution to this literature for a variety of reasons. On the content side of the paper, we believe it provides a more realistic view of the manager’s role in agency relationships by showing how managers interact and combine incentive contracts with monitoring. On the side of its place in the literature, this paper represents the first application of the ideas of the rational inattention literature started off by Mankiw and Reis (2002) and Sims (2003) and (2006) to optimal incentive contracts. Finally, on the empirical content of the implications of the model, we find that there is no direct evidence at the moment that confirm the testable implications derived in our model. Despite this, there are several papers out there that may provide indirect evidence consistent with our implications. Baker and Hubbard (2003) is an example of one of such papers in that they document the impact of an exogenous change in monitoring costs on the type of contracting used in the trucking industry. Lower monitoring costs may have increased the amount of monitoring optimally chosen by trucking companies and consequently the use of outsourcing as opposed to in-house contracting.
Additionally, our model is quite rich in detail and provides a number of other testable implications for which there is no evidence yet in the empirical contracting literature.

The paper is structured as follows. Section 2 relates this paper to the relevant preceding literature. Section 3 presents and solves the model under symmetric task technologies. This section shows how the introduction of managerial attention allocation shapes the design of optimal incentive contracts. In section 4, we consider the introduction of asymmetries between tasks. We extend our model in section 5 by adding complementarities in production across tasks and we obtain that results do not change qualitatively. Finally, section 6 comments on supporting evidence in the existing contracting empirical literature and section 7 concludes.

2 Literature Review

This paper builds on and contributes directly to two different economic literatures. These are the literature on optimal incentive contracts and its recent stream of papers departing from the standard rationality assumptions, and the literature on attention allocation that has been mainly developed in macroeconomics and only now recently applied to other fields in Economics.

2.1 Literature on Optimal Incentive Contracts

The literature on optimal incentive contracts is extensive and confronts many and very different types of information asymmetries. Here we review the literature on incentive contracts dealing with moral hazard issues and risk aversion. Among the first contributors to this literature are Holmstrom (1979) and Shavell (1979) who studied the case of one-dimensional effort problems, and Holmstrom and Milgrom (1991) who extended the literature by examining multi-dimensional effort problems. The former papers established the optimality of the negative relationship between uncertainty and incentive intensity while the latter emphasized the necessity of balancing incentives across tasks and the importance of job design. Following this literature, others have studied the distortion of performance measures in incentive contracts (Baker (1992)) or the role of subjective pay performance (Baker, Gibbons and Murphy (1994)) in the optimal design of incentive contracts. Yet all these studies take the contractibility of effort as given assuming an uncertainty variance-covariance matrix $\Sigma$.

In this paper, we introduce a managerial attention allocation constraint and relax the agent’s effort constraint by assuming a convex disutility of effort. This novelty allows us to endogeneize the
up-to-now exogenous variance-covariance matrix and therefore explore how the manager balances the use of incentives and attention across tasks. A direct consequence of this novelty of our approach is that managers may decide to mute incentives in one task and concentrate their attention into another. Other papers before ours have modelled the endogenous decision of leaving a task outside a formal contract and therefore choosing the degree of contractual completeness (see Hart and Moore (2004) or Wernerfelt (2007) among others). Our approach here differs from those in that our principal faces a trade-off of decreasing the contractibility of a task to increase the contractibility of another task and therefore we are very precise about the nature of the cost incurred by the principal.

This paper also contributes to a recent stream of papers that has introduced into the analysis of optimal contracting new elements that depart from traditional rationality assumptions. See recently Hart and Moore (2007) bringing entitlements into bargaining or Fehr and others\textsuperscript{2} for inequity aversion or reciprocity. Our paper differs from these in that managers in our framework are self-interested but instead of working around the contractibility shortcomings we allow for an endogenous solution to the degree of effort contractibility.

2.2 Literature on Attention Allocation

This paper is not the first to apply inattentiveness to other fields in Economics, but it is, to the best of our knowledge, among the first to examine the role of inattentiveness in contracting while endogeneizing the degree of effort contractibility within a standard moral hazard model. Gifford (2004) derives a model of make-or-buy decisions and endogenous transaction costs with attention allocation. Her paper follows the transaction cost economics approach to explaining make-or-buy decisions and therefore assumes that contractual incompleteness of tasks performed inside the firm are unimportant since all distortions can be taken care of within the firm. Our approach here differs from hers and examines the role of attention allocation in dealing with employment contracts within a firm.

This paper also relates to the recent literature on attention allocation and inattentiveness. Inattentive agents have been used in several fields. In macroeconomics, inattention has explained sticky prices in Mankiw and Reis (2002) and Mackowiak and Wiederholt (2007) and consumption dynamics in Gabaix and Laibson (2002), Reis (2006) and Luo (2007). In finance, attention al-

\textsuperscript{2}See Fehr and Schmidt (2003), Fehr, Klein and Schmidt (2004), and Englmaier and Wambach (2007) among others.
location decisions have been used to understand financial contagion across emerging economies in Mondria (2007) and portfolio under-diversification in Van Nieuwerburgh and Veldkamp (2007a). In international finance, inattentive investors help explain the forward discount puzzle in Bacchetta and Van Wincoop (2006) and the home bias puzzle in Van Nieuwerburgh and Veldkamp (2007b).

Despite the novelty of our approach, we recognize that previous research has characterized the main role of the principal as one of allocating resources across workers or tasks or even choosing the optimal number of workers being managed by one sole manager. See for example Lucas (1978) studying the division of persons into managers and employees. He shows that higher skill workers are more likely to become managers and are more likely to manage bigger firms. His result speaks about the distribution of firm sizes in the economy, but does not focus into the attention allocation constraint of the managers. Similarly, Rosen (1982) examines the allocation of talent within the hierarchy of a firm and across firms within the economy. He shows how more skilled managers should be solving more important problems and therefore located in higher up positions in the hierarchy of bigger firms. His result also focuses on the distribution of the size of firms and the distribution of earnings in the economy. For both these papers (and the literature that followed) higher skilled managers are allocated to more important problems to maximize revenues and therefore the same principle that drives the introduction of attention allocation is at use. Despite this, our approach differs from all these in that here attention allocation helps monitoring tasks and increases the contractibility of effort exerted on a given task by increasing the precision of the effort on that task.

3 The Model

In this section we present an extension to the framework in Holmstrom (1979) and Holmstrom and Milgrom (1991). This model presents a principal contracting over the non-contractible multidimensional effort of an agent. The agent provides effort in a number of tasks and the principal designs a linear contract, composed by a variable and a fixed factor, and monitors the effort of the agent. Since the effort in each task is not contractible, the principal writes an incentive contract contingent to some public (observable to a third party and contractible) signal non-perfectly correlated with effort. We present here the benchmark model with two tasks.\(^3\)

\(^3\)The case presented here is easily generalizable to the case of \(n\) tasks.
3.1 Benchmark Description

This model presents a principal contracting over the non-contractible effort provided by an agent in two tasks. The agent chooses a vector of efforts \( t = (t_1, t_2) \), which are not directly observed by the principal. The agent faces a personal cost \( C(t_i) = \frac{1}{2} t_i^2 \) for a level of effort \( t_i \) in each task \( i \). Since the principal cannot observe the effort provided in each task, \( t_i \), directly (effort in task is not contractible, she may be able to observe it but there is no third party that can), she writes an incentive contract contingent to some public (observable to a third party and contractible) signal \( x_i \) correlated with the effort \( t_i \) such that

\[
x_i = t_i + \epsilon_i \text{ for each task } i
\]

where \( \epsilon_i \) is normally distributed with mean 0 and variance \( \sigma_i^2 \), and \( \epsilon_1 \) is independent of \( \epsilon_2 \). The principal designs (assume) a linear contract composed by a variable and a fixed factor. The agent receives a total compensation of \( w(X) = \beta + \alpha^T X \) where \( \alpha = (\alpha_1, \alpha_2)' \) is the vector of incentive intensity for each task and \( X = (x_1, x_2)' \) is a vector of observable signals about the effort provided by each agent. The agent, with an absolute coefficient of risk aversion \( r \), has CARA preferences over the total compensation such that \( u(w) = e^{rw} \). On the other hand, the principal is risk neutral. The efforts \( t_i \) provided by the agent generate a private gross expected profit to the principal

\[
B(t_1, t_2) = t_1^\theta + t_2^\theta.
\]

This gross expected profit function is flexible enough to provide decreasing, constant and increasing returns to each task depending on the value of the parameter \( \theta \).

In this model, unlike the rest of the literature, the principal is able to decide how much monitoring she wants to do about the effort provided in each task. The principal would like to observe a signal that reduces all the uncertainty about the effort. However, the principal faces a technological constraint on monitoring, which is called attention allocation constraint. The principal is assumed to be endowed with \( \kappa \) units of monitoring capacity, which needs to allocate to both tasks such that

\[
\kappa = \kappa_1 + \kappa_2 \quad (1)
\]

where the monitoring technology for each task \( i \) is given by \( \sigma_i^2 = \frac{1}{\kappa_i^2} \). The more attention is allocated to one task, the less uncertainty about the effort provided by the agent about that particular task.
The monitoring technology is flexible enough to provide decreasing, constant and increasing returns to the attention allocated to a particular task depending on the value of the parameter \( \phi \). This constraint restricts the amount of information that the principal can process about the efforts that the agent is providing. This restriction could be interpreted as the principal having a limited amount of time to concentrate on monitoring the agent. The principal faces a trade-off between how much monitoring to allocate to each task. The principal cannot allocate negative attention to any task, which means \( \kappa_i \geq 0 \) for any \( i \). The benchmark assumes away complementarities between tasks (in both production and monitoring technologies).

### 3.2 Model Solution

The model is solved using backward induction. First, for a given wage contract \((\alpha, \beta)\) and managerial attention allocation, \((\kappa_1, \kappa_2)\), the agent chooses the effort, \( t \), she wants to provide in each task. Second, given the optimal effort of the agent, the principal chooses the wage contract for any managerial attention allocation. Third, given the optimal effort and the optimal wage contract, the principal chooses the optimal managerial attention allocation.

Following Holmstrom and Milgrom (1991), since the contract wage is normally distributed, the agent’s certainty equivalent can be written by

\[
CE = \alpha^T t + \beta - C(t) - \frac{1}{2} r \alpha^T \Sigma \alpha
\]

where \( \Sigma \) is the diagonal matrix of the vector of error terms in the private signal \((\epsilon_1, \epsilon_2)'\). For a given wage contract \((\alpha, \beta)\) and managerial monitoring technology \((\kappa_1, \kappa_2)\), which implies a given \( \Sigma \), the agent optimally chooses an effort in each task that is given as

\[
t_i = \alpha_i
\]

The principal expected profits are given by \( B(t) - \alpha^T \mu(t) - \beta \). Since the principal is risk neutral, she chooses the wage contract and the managerial attention allocation to maximize the following joint certainty equivalent of the principal and the agent (their joint surplus) for an optimal effort provided by the agent

\[
\max_{\{\alpha_i, \kappa_i\}_{i=1}^2} B(t) - C(t) - \frac{1}{2} r \alpha^T \Sigma \alpha \quad \text{subject to} \quad t_i = \alpha_i, \ k = \kappa_1 + \kappa_2, \ \kappa_1 \geq 0, \ \kappa_2 \geq 0
\]
As Holmstrom and Milgrom (1991) noted, the joint surplus is independent of the intercept $\beta$ that is used to distribute the joint certainty equivalent between both parties. Thus, the optimal incentive intensity provided by the principal is given by

$$\alpha_i = \left[ \frac{1}{\theta} \left( 1 + \frac{r}{\kappa_i^\phi} \right) \right]^{\frac{1}{\phi-2}}$$

as long as $\theta < 2$. If $\theta \geq 2$, there would be a corner solution with zero or infinite effort. This is due to the assumption of quadratic costs to effort by the agent and the attention allocation constraint of the manager. The more attention allocated to one task, the higher is the incentive intensity the principal offers and the higher the effort the agent provides on that task. This result shows that the principal has a complementarity between the attention allocated to a task and the incentive intensity of that task. Since incentive design and attention allocation are the two tools through which the principal maximizes profits, this complementarity conditions the decision in each task.

The objective function for the managerial attention allocation optimization problem given the optimal effort and the optimal wage contract is obtained by plugging the optimal incentive intensity, $\alpha_i$ in equation (3) provided by the principal and the optimal effort, $t$, provided by the agent in equation (2) into the joint certainty equivalent. Once this is done, the principal’s managerial attention allocation is obtained by maximizing the principal’s objective function in terms of $\kappa_i$ such that

$$\max_{\{\kappa_i\}_{i=1}^2} \sum_{i=1}^2 A \left( 1 + \frac{r}{\kappa_i^\phi} \right)^{\phi-2} \quad \text{subject to } \kappa = \kappa_1 + \kappa_2, \kappa_1 \geq 0, \kappa_2 \geq 0$$

where $A = \left[ (\frac{1}{\theta})^{\frac{\phi}{\phi-2}} - \frac{1}{2} (\frac{1}{\theta})^{\frac{2}{\phi-2}} \right]$, which is always a strictly positive function as long as $\theta < 2$.

**Proposition 1** The symmetric managerial attention allocation $\kappa_1 = \kappa_2 = \frac{\kappa}{2}$ is a strict local maximum if and only if the following parameter constraint is satisfied

$$\left( 1 + \frac{1}{\phi} \right) \left( \frac{\kappa}{r} + 1 \right) > \frac{2}{2 - \theta}$$

**Proof.** If we introduce the monitoring attention allocation constraint from equation (1) into
the objective function in equation (4), we obtain the following maximization problem

$$\max_{\kappa_1} A \left[ \left( 1 + \frac{r}{\kappa_1^\theta} \right)^{\frac{\theta}{\theta-2}} + \left( 1 + \frac{r}{(\kappa - \kappa_1)^\theta} \right)^{\frac{\theta}{\theta-2}} \right]$$

(5)

The second order condition of this problem when there is a symmetric attention allocation such that $\kappa_1 = \kappa_2 = \frac{\kappa}{2}$ is given by

$$\frac{\partial^2}{\partial \kappa_1^2} \left( \frac{\kappa}{2} \right) = (\text{strictly negative constant}) \left[ - \left( \frac{2}{2-\theta} \right) + \left( 1 + \frac{1}{\phi} \right) \left( \left( \frac{\kappa}{2} \right)^\phi r + 1 \right) \right]$$

(6)

which is negative if and only if

$$- \left( \frac{2}{2-\theta} \right) + \left( 1 + \frac{1}{\phi} \right) \left( \left( \frac{\kappa}{2} \right)^\phi r + 1 \right) > 0$$

Note that through all the paper we are also assuming that $\theta < 2$. ■

The inequality condition in Proposition 1 implies that a symmetric managerial attention allocation will be optimal with higher probability when the total monitoring capacity $\kappa$ is large, the agent’s degree of risk aversion $r$ is low, the returns to scale in effort $\theta$ (in the gross expected profit function) are low and the returns to scale in monitoring $\phi$ are low (provided the monitoring capacity $\kappa$ is not too large). We explain the intuition behind each one of these results next.

If the principal is endowed with a large monitoring capacity $\kappa$, the attention allocation decision loses relevance when there are decreasing returns to scale in monitoring (low $\phi$) since the marginal benefit of concentrating attention into any given task will be low. On the other hand, the fact alone that large $\kappa$ implies that the principal is less attention constrained and therefore the problem at hand that we study here would be less of a concern. A second implication from Proposition 1 above is that a lower degree of risk aversion (low $r$) implies that the agent is less sensitive to uncertainty and therefore the principal will have less incentives to allocate the same attention to both tasks. Our results also predict that lower returns to scale in the expected profits (lower $\theta$) increases the likelihood of observing a symmetric attention allocation. This is so because the principal’s marginal benefit to accumulating attention in one task decreases sharply. For this reason, the principal equally motivates both tasks and therefore allocate the same attention to both tasks. Finally, if there are low returns to scale in the monitoring technology, low $\phi$, as long
as the monitoring capacity is not too large, the principal allocates attention to the effort provided by the agent in both tasks for exactly the same reason as when we have low returns to scale in production (low $\theta$).

**Corollary 1** The symmetric managerial monitoring attention allocation $\kappa_1 = \kappa_2 = \frac{\kappa}{2}$ is a unique global maximum if the following monitoring parameter constraint is satisfied

$$
\left(1 + \frac{1}{\phi}\right) > \frac{2}{2 - \theta}
$$

**Proof.** The first order condition to the maximization problem in equation (5) equals zero if

$$
\kappa_1^{1+\phi} \left(1 + \frac{r}{\kappa_1^{\phi}}\right) \frac{2-\sigma}{r} = (\kappa - \kappa_1)^{1+\phi} \left(1 + \frac{r}{(\kappa - \kappa_1)^{\phi}}\right) \frac{2-\sigma}{r} \tag{7}
$$

The left hand side (LHS) is a continuous and strictly increasing function of $\kappa_1$ for $\kappa_1 \geq 0$ and the right hand side (RHS) is a continuous and strictly decreasing function of $\kappa_1$ for $\kappa_1 \geq 0$ if the following parameter constraint is satisfied

$$
\left(1 + \frac{1}{\phi}\right) > \frac{2}{2 - \theta}
$$

Hence, the first order condition equals zero given in equation (7) has a unique solution $\kappa_1 = \frac{\kappa}{2}$.

The first order conditions are strictly positive for $\kappa_1 \in [0, \frac{\kappa}{2})$ and strictly negative for $\kappa_1 \in (\frac{\kappa}{2}, \kappa]$. The second order condition in equation (6) at $\kappa_1 = \frac{\kappa}{2}$ is always negative. Therefore, the symmetric managerial monitoring attention allocation $\kappa_1 = \kappa_2 = \frac{\kappa}{2}$ is a unique global maximum when the sufficient parameter constraint is satisfied. ■

The result of Corollary 1 states that a simple comparison between the returns to scale in production and monitoring is enough to determine if allocating the same amount of attention to both tasks, $\kappa_1 = \kappa_2 = \frac{\kappa}{2}$, is a global maximum. This condition establishes that the lower the returns to scale to production, $\theta$, and to monitoring, $\phi$, the higher the probability the principal allocates the same amount of resources to the monitoring of both tasks. The reason follows from our comments above on the condition in Proposition 1. If the returns to scale in production and monitoring are low enough, the marginal benefit to accumulating attention in any of the tasks and
consequently focus production in that task will also be low to the point that the principal will be better off reallocating some of the attention across tasks and benefit from an increase in the marginal benefit to attention in both tasks.

**Corollary 2** There exists asymmetric managerial monitoring attention allocation equilibria if the following parameter constraint is satisfied

\[
\left(1 + \frac{1}{\phi}\right) \left(\left(\frac{\xi}{2}\right)^{\phi} + 1\right) < \frac{2}{2 - \theta}
\]

**Proof.** If the constraint is satisfied, the symmetric managerial attention allocation \(\kappa_1 = \kappa_2 = \frac{\kappa}{2}\) is a strict local minimum. The objective function is a continuous function over a compact set \(\kappa_1 \in [0, \kappa]\), hence there exists a maximum and a minimum. Furthermore, since the objective function in equation (5) is a symmetric function around \(\kappa_1 = \frac{\kappa}{2}\), there exits at least two asymmetric equilibria where one type of equilibria is such that \(\kappa_1 > \frac{\kappa}{2} > \kappa_2\) and the other type of equilibria is such that \(\kappa_1 < \frac{\kappa}{2} < \kappa_2\).

Finally, the second and last corollary shows that even in our very simple setting where the tasks enter symmetrically in the principal and agent’s problem, asymmetric attention allocation among tasks can result in equilibrium. Mainly, under strong enough increasing returns to scale in production, \(\theta\) close to 2, or monitoring (as long as the total amount of monitoring capacity \(\kappa\) is not too large), \(\phi > 1\), the principal optimally concentrates all her monitoring capacity in one task. This, in turn, strengthens the incentives for that task by increasing \(\alpha\) and increasing the precision in which \(t\) is measured. Under our functional assumptions on the monitoring technology, when the principal decides to allocate no attention to one of the tasks, the precision of that task decreases radically and therefore the principal is forced to set \(\alpha = 0\) for the task that is not being monitored. This result would change if we were to allow for a finite lower bound (similar to the Holmstrom and Milgrom (1991) framework where \(\Sigma\) matrix is taken as given). In that case, the optimal incentive contract would depict \(\alpha > 0\) for all tasks where the value of some \(\alpha\)'s would be higher than others.

This result shows that in scenarios where jobs are defined by a number of differentiated tasks principals may still find optimal to provide unbalanced incentive contracts even if all tasks that define the job enter the principal and the agent objective functions symmetrically. Traditional
explanations emphasized differences in the contractibility or the returns to scale across tasks as the main reason for observing asymmetries of incentive provision in these contracts. Here we show that even under total symmetry among tasks, principals may find optimal to mute incentives in a task to strengthen incentives in the other task. This result is driven by the main contribution in this paper, which is to allow the manager to optimally choose how to allocate her monitoring activities across tasks. These monitoring activities have a positive impact in the productivity of individuals since they increase the precision at which effort on a given task is measured. Symmetric increasing returns to scale in production or monitoring increases the likelihood of observing an optimal asymmetric attention allocation across tasks and the consequent unbalanced provision of incentives.

4 Introducing asymmetries between tasks

The benchmark case above assumes symmetry across tasks in the gross expected profit function and the monitoring technology. In this section, we relax this symmetry assumption and introduce asymmetries first in the gross expected profit function and then in the monitoring technology. We aim to understand how sensitive our previous results are to the symmetry assumption and at the same time examine how our results compare to traditional results in the incentive contract literature when asymmetries across tasks exist.

4.1 Asymmetry in the principal’s expected profits

The benchmark model assumes that both tasks provided by the agent generate the same gross expected profit to the principal. In this section, we show the optimal monitoring attention allocation when the tasks generate asymmetric gross expected profits to the principal. Assume that for the same amount of effort in both tasks, the principal receives a higher expected profit from the second task such that the gross expected profits of the principal are given by

\[ B(t_1, t_2) = t_1^\theta + \tau t_2^\theta, \text{ where } \tau > 1 \]

The optimal effort decision by the agent is not distorted and is still given by equation (2) from the benchmark model. However, the incentive intensity chosen by the principal is affected since her

---

4 This result would generalize easily to the case of more than two tasks.
gross expected profits have changed. The principal chooses the wage contract and the managerial monitoring technology to maximize the following joint certainty equivalent of the principal and the agent (their joint surplus) for an optimal effort provided by the agent

\[
\max_{\{\alpha_i, \kappa_i\}_{i=1}^2} B(t) - C(t) - \frac{1}{2} r \alpha^T \Sigma \alpha \text{ subject to } t_i = \alpha_i, \ k = \kappa_1 + \kappa_2, \ \kappa_1 \geq 0, \ \kappa_2 \geq 0
\]

The optimal incentive intensity provided by the principal is given by

\[
\alpha_1 = \left[ \frac{1}{\theta} \left( 1 + \frac{r}{\kappa_1^\phi} \right) \right]^{\frac{1}{2-\phi}}, \ \ \ \alpha_2 = \left[ \frac{1}{\theta} \left( 1 + \frac{r}{\kappa_2^\phi} \right) \right]^{\frac{1}{2-\phi}}
\]
as long as \( \theta < 2 \). The optimal managerial monitoring attention allocation given the optimal effort and the optimal wage contract by the principal is obtained by plugging the optimal incentive intensity, \((\alpha_1, \alpha_2)\), provided by the principal and the optimal effort, \(t\), provided by the agent into the joint certainty equivalent

\[
\max_{\{\kappa_i\}_{i=1}^2} A \left( 1 + \frac{r}{\kappa_1^\phi} \right)^{\frac{1}{2-\phi}} + \tau \frac{2}{\theta} A \left( 1 + \frac{r}{\kappa_2^\phi} \right)^{\frac{1}{2-\phi}} \text{ subject to } \kappa = \kappa_1 + \kappa_2, \ \kappa_1 \geq 0, \ \kappa_2 \geq 0 \quad (8)
\]

where \( A = \left[ \left( \frac{1}{\theta} \right)^{\frac{1}{2-\phi}} - \frac{1}{2} \left( \frac{1}{\theta} \right)^{\frac{2}{2-\phi}} \right] \), which is always a strictly positive function as long as \( \theta < 2 \).

**Proposition 2** There is a unique global maximum managerial monitoring attention allocation with \( \kappa_1^* < \frac{\kappa}{2} < \kappa_2^* < \kappa \) when the tasks provide asymmetric gross expected profits (due \( \tau > 1 \)) if the following parameter constraint is satisfied

\[
\left( 1 + \frac{1}{\phi} \right) > \frac{2}{2 - \theta}
\]

_Proof_. When we introduce the monitoring attention allocation constraint from equation (1) into the objective function in equation (8), the first order condition equals zero when

\[
\tau \frac{2}{\theta} \kappa_1^{1+\phi} \left( 1 + \frac{r}{\kappa_1^\phi} \right)^{\frac{2}{2-\phi}} = (\kappa - \kappa_1)^{1+\phi} \left( 1 + \frac{r}{(\kappa - \kappa_1)^\phi} \right)^{\frac{2}{2-\phi}}
\]

The left hand side (LHS) is a strictly increasing function of \( \kappa_1 \) for \( \kappa_1 \geq 0 \) and the right hand side (RHS) is a strictly decreasing function of \( \kappa_1 \) for \( \kappa_1 \geq 0 \) if the following parameter constraint is
satisfied \( (1 + \frac{1}{\theta}) > \frac{2}{\tau - \theta} \). If \( \kappa_1 = \frac{\tau}{2} \), the LHS > RHS. If \( \kappa_1 = 0 \), LHS < RHS. Therefore, there exists a unique solution \((\kappa_1^*, \kappa_2^*)\) that makes RHS=LHS and \(\kappa_1^* < \frac{\tau}{2} < \kappa_2^* < \kappa\). The first order conditions are strictly positive for \(\kappa_1 \in [0, \kappa_1^*]\) and strictly negative for \(\kappa_1 \in (\kappa_1^*, \kappa]\). The second order condition at \(\kappa_1^*\) is always negative. Therefore, the asymmetric managerial monitoring attention allocation \((\kappa_1^*, \kappa_2^*)\) is a unique global maximum when the parameter constraints \( (1 + \frac{1}{\theta}) > \frac{2}{\tau - \theta} \) and \( \tau > 1 \) are satisfied. \(\blacksquare\)

In this case, the inequality condition in Proposition 2 shows that, due to the asymmetry in the gross expected profit function, the principal optimally allocates a bigger share of her monitoring capacity to the task with higher returns as long as both tasks exhibit the same degree of returns to scale. The incentive strength \(\alpha\) placed to different tasks differs and the principal places \(\alpha_2 > \alpha_1\) provided that \(\tau > 1\). This result is consistent with the literature in that incentive contracts in multi-tasking settings optimally place stronger incentives on tasks that are more profitable to the principal.

Similarly to the results in the previous section, it is easy to see that in situations where the gross expected profit function or the monitoring technology exhibit increasing returns to scale (high \(\theta\) or \(\phi\) respectively), the principal chooses to allocate all the monitoring capacity in task 2 \((\kappa_2 = \kappa)\) and place none in task 1 \((\kappa_1 = 0)\). Therefore, this extreme uneven allocation of attention would mute incentives in task 1 \((\alpha_1 = 0)\) and maximize incentives in task 2 \((\alpha_2 > 0)\). Again, we find that incentive contracts in a multi-tasking setting that mute incentives for one task may not come from differences in returns to effort but be a consequence of the optimal combination of attention allocation across tasks and the optimal provision of incentives. The principal understands that there is a complementarity between the provision of incentives and the allocation of attention in each task. In the presence of increasing returns to scale to effort in the gross expected profit function, the marginal benefit of accumulating attention on a given task is greater than the marginal benefit of allocating attention to another task. In this case, the principal chooses to allocate all her monitoring capacity to a given task and write an optimal incentive contract that provides incentives to effort on one task only.

4.2 Asymmetry in monitoring technology

Similarly to the case presented above, the benchmark model assumes that monitoring both tasks cost the same. In this section, we show the optimal monitoring attention allocation when the
tasks have different monitoring costs. To proceed with this analysis, we assume that the second task requires more time of monitoring to reduce the same amount of uncertainty about the non-observable effort of the agent such that the attention allocation constraint is given by

\[
\kappa = \kappa_1 + \tau \kappa_2, \text{ where } \tau > 1
\]  

(9)

This constraint does not affect the optimal effort chosen by the agent in the benchmark model, which is given in equation (2), and it does not affect the optimal incentive intensity provided by the principal and given by equation (3) either. However the optimal monitoring attention allocation is distorted and this leads to our next proposition.

**Proposition 3** There is a unique global maximum managerial monitoring attention allocation with \( \kappa > \kappa_1^* > \frac{\kappa}{2} > \kappa_2^* \) when the tasks have different costs of monitoring \((\tau > 1)\) if the following parameter constraint is satisfied

\[
\left(1 + \frac{1}{\phi}\right) > \frac{2}{2 - \theta}
\]

**Proof.** When we introduce the monitoring attention allocation constraint from equation (9) into the objective function in equation (4), the first order condition equals zero when

\[
\kappa_1^{1+\phi} \left(1 + \frac{r}{\kappa_1^\phi}\right)^{\frac{2}{\phi-2}} = \tau \kappa_2^{1+\phi} \left(1 + \frac{r}{\kappa_2^\phi}\right)^{\frac{2}{\phi-2}}
\]

where \(\kappa_2 = \frac{(\kappa - \kappa_1)}{\tau}\). The left hand side (LHS) is a strictly increasing function of \(\kappa_1\) for \(\kappa_1 \geq 0\) and the right hand side (RHS) is a strictly decreasing function of \(\kappa_1\) for \(\kappa_1 \geq 0\) if the following parameter constraint is satisfied \((1 + \frac{1}{\phi}) > \frac{2}{2 - \theta}\). If \(\kappa_1 = \kappa_2\), the LHS<RHS. If \(\kappa_1 = \kappa\), LHS>RHS. Therefore, there exists a unique solution \((\kappa_1^*, \kappa_2^*)\) that makes RHS=LHS and \(\kappa > \kappa_1^* > \frac{\kappa}{2} > \kappa_2^*\). The first order conditions are strictly positive for \(\kappa_1 \in [0, \kappa_1^*]\) and strictly negative for \(\kappa_1 \in (\kappa_1^*, \kappa]\). The second order condition at \(\kappa_1^*\) is always negative. Therefore, the asymmetric managerial monitoring attention allocation \((\kappa_1^*, \kappa_2^*)\) is a unique global maximum when the parameter constraints \((1 + \frac{1}{\phi}) > \frac{2}{2 - \theta}\) and \(\tau > 1\) are satisfied. ■

When we consider the case that task 2 requires more units of monitoring capacity to increase precision of effort measurement by the same amount \((\tau > 1)\), we find that the principal optimally
allocates more attention to task 1 than to task 2 ($\kappa_1 > \kappa_2$). This asymmetric allocation of attention comes from the fact that to achieve equal precision across tasks the principal must allocate more units of attention to task 2 than to task 1. This means that at the margin the opportunity cost of the last unit of attention allocated to task 2 in terms of gains in precision of task 1 is higher than the increase in precision obtained in the measurement of effort exerted in task 2. This unequal trade-off induces the principal to allocate more units of attention to task 1 up to the point at which the marginal gain in precision are equal across tasks and $\kappa_1 > \kappa_2$. At this point, and given the existing complementarity between attention allocated and incentive provision to a task, the principal optimally chooses to provide stronger incentives to task 1 than to task 2 ($\alpha_1 > \alpha_2$).

This finding is indeed consistent to the main finding in Holmstrom (1979). Optimal incentive contracts mediating a risk-neutral principal and a risk-averse agent should provide stronger incentives for those tasks that are less costly to monitor. The novelty here is that the degree of monitorability is endogenous to the principal and she is able to combine that with the optimal incentive provision scheme.

Finally, and similarly to the previous section, under strong enough increasing returns to scale in the production function or in the monitoring technology (high $\theta$ or $\phi$ respectively), the principal chooses to allocate all her attention to the task where attention is less costly (task 1 such that $\kappa_1 = \kappa$) and allocate no attention to task 2 ($\kappa_2 = 0$). In this case, the principal would offer a contract that offers no incentives for task 2 ($\alpha_2 = 0$) and only provides incentives to task 1 ($\alpha_1 > 0$). We want to point out at this point that this corner result hinges on the specific functional assumption on the monitoring technology. In other words, there would be a positive provision of incentives to task 2, $\alpha_2 > 0$, if we were to allow for a lower bound of precision.

5 Complementarity between tasks

In the general framework above and the particular cases in the following sections, we assumed away any complementarity between tasks in production and monitoring. Next, we examine the case where the complementarity between tasks exists in the expected gross profit function, but still assuming away complementarities in the monitoring technology.
5.1 Complementarity in the Production Function

The benchmark model assumes that both tasks provided by the agent are independent of each other in generating gross expected profit to the principal. In this section, we show the optimal monitoring attention allocation when the tasks have strong complementarities. Assume that the expected gross profits by the principal are given by

\[ B(t_1, t_2) = t_1^\theta t_2^\phi \]

This expected profit function introduces a strong complementarity between tasks because if one of the tasks is not provided by the agent, then the principal receives zero profits. The optimal effort decision by the agent is not distorted and is still given by equation (2) from the benchmark model. However, the incentive intensity chosen by the principal is affected since her gross expected profits have changed. The principal chooses the wage contract and the managerial monitoring technology to maximize the following joint certainty equivalent of the principal and the agent (their joint surplus) for an optimal effort provided by the agent

\[
\max_{\{\alpha_i, \kappa_i\}_{i=1}^2} t_1^\theta t_2^\phi - \frac{1}{2} \sum_{i=1}^2 t_i^2 \left( 1 + \frac{r}{\kappa_i^\phi} \right) \quad \text{subject to} \quad t_i = \alpha_i, \quad \kappa = \kappa_1 + \kappa_2, \quad \kappa_1 \geq 0, \quad \kappa_2 \geq 0
\]

The optimal incentive intensity provided by the principal is given by

\[
\alpha_1 = \left[ \frac{1}{\theta B} \left( 1 + \frac{r}{\kappa_1^\phi} \right) \right]^{\frac{\phi}{2}} \quad \alpha_2 = \left[ \frac{1}{\theta B} \left( 1 + \frac{r}{\kappa_2^\phi} \right) \right]^{\frac{\phi}{2}}
\]

where \( B = \left( 1 + \frac{r}{\kappa_1^\phi} \right)^{\frac{1}{2}} \left( 1 + \frac{r}{\kappa_2^\phi} \right)^{-\frac{1}{2}} \)

as long as \( \theta < 1 \). If \( \theta > 1 \), the principal could give the agent enough incentives to choose an infinite effort in both tasks. The optimal managerial monitoring attention allocation given the optimal effort and the optimal wage contract by the principal is obtained by plugging the optimal incentive intensity, \((\alpha_1, \alpha_2)\), provided by the principal and the optimal effort, \(t\), provided by the agent into the joint certainty equivalent

\[
\max_{\{\kappa_i\}_{i=1}^2} A^c \left( 1 + \frac{r}{\kappa_1^\phi} \right)^{\frac{\phi}{2 \theta - 2}} \left( 1 + \frac{r}{\kappa_2^\phi} \right)^{\frac{\phi}{2 \theta - 2}} \quad \text{subject to} \quad \kappa = \kappa_1 + \kappa_2, \quad \kappa_1 \geq 0, \quad \kappa_2 \geq 0 \quad (10)
\]
where \( A^c = \left[ \theta^{\frac{\alpha}{1-\sigma}} - \theta^{\frac{1}{1-\sigma}} \right] \), which is always a strictly positive function as long as \( \theta < 1 \).

**Proposition 4** There is a unique global maximum managerial monitoring attention allocation with \( \kappa_1 = \kappa_2 = \frac{\kappa}{2} \).

**Proof.** When we introduce the monitoring attention allocation constraint into the objective function in equation (10), the first order condition equals zero when

\[
\kappa_1^{1+\phi} \left( 1 + \frac{r}{\kappa_1^\phi} \right) = (\kappa - \kappa_1)^{1+\phi} \left( 1 + \frac{r}{(\kappa - \kappa_1)^\phi} \right)
\]

The right hand side (RHS) is a continuous and strictly increasing function of \( \kappa_1 \) for \( \kappa_1 \geq 0 \) and the left hand side (LHS) is a continuous and strictly decreasing function of \( \kappa_1 \) for \( \kappa_1 \geq 0 \). Hence, the first order condition equals zero has a unique solution \( \kappa_1 = \frac{\kappa}{2} \). The first order conditions are strictly positive for \( \kappa_1 \in [0, \frac{\kappa}{2}] \) and strictly negative for \( \kappa_1 \in (\frac{\kappa}{2}, \kappa] \). The second order condition is always negative. Therefore, the symmetric managerial monitoring attention allocation \( \kappa_1 = \kappa_2 = \frac{\kappa}{2} \) is a unique global maximum. ■

Proposition 4 establishes the result that in the presence of strong enough complementarities it will always be optimal to spread attention allocation across tasks since the introduction of a strong complementarity in the gross expected profit function of the principal eliminates the potential benefits of concentrating the principal’s attention in any given task. In the particular scenario that we examine here, the principal optimally allocates equal attention to each task if each task exhibits decreasing returns to scale. This particular case has an odd but otherwise senseful characteristic provided the assumptions of the model. If the task exhibit increasing returns to scale such that \( \theta > 1 \), since we have assumed at the beginning of the paper that the agent’s internal cost of effort is quadratic, there will be no finite solution due to the complementarity among tasks because the principal benefits from increasing the agent’s effort more than linearly even after compensating the agent above the cost of her effort.

### 6 Supporting Evidence and Empirical Content of the Model

The findings in our model have several empirical implications that are consistent with stylized facts in the existing empirical contracting literature. The main empirical implication is that managers choose to provide higher incentives in those tasks that they choose to monitor more closely. This is
so because of the complementarity that emerges endogenously out of the model between incentives and monitoring through the attention allocation constraint of the principal. To the best of our knowledge, there are no papers in the empirical literature that provide evidence on how managers allocate their attention and monitoring capabilities and how they combine these with the provision of incentives across tasks. Therefore there is no direct evidence of this testable implication. Despite the lack of direct evidence, many papers document the relation between contractibility and incentive provision in agency relationships and therefore provide evidence on contracting episodes that are consistent with some of our results.

Prior to the literature on optimal contracts in multi-tasking settings, most papers studied contracts that muted incentives for some of the tasks defining the job at hand (see Chiappori and Salanie (2003) for a list of a few examples). Our results show that under certain circumstances, the unbalanced provision of incentives in multi-tasking may be optimal from the point of view of the principal if she can allocate her attention to different tasks and endogenously change the contractibility of some tasks and not others. Therefore to test the implications of our model we should observe monitoring activities across tasks before and after an employer decides to mute incentives in some tasks and increase them in others. A positive correlation between incentives provided in a contract for a given task and monitoring activities would be consistent with the testable implications in our model. Similarly, our model only predicts that all tasks will move in the same direction (regarding incentives and monitoring) as long as the total amount of attention increases. Otherwise, our model assumes that incentives and monitoring across tasks within a contract/job should be negatively correlated.

Other papers have documented cases when the cost of monitoring has gone down due to an exogenous factor such as a change in technology. For example, Baker and Hubbard (2003) show that with the appearance of OBC (on-board computers) the costs of monitoring decreased and presumably the amount of monitoring increased helping firms in the trucking industry to transition from the use of in-house divisions to more outsourcing with high-powered incentive contracts. In another paper, Lerner and Malmendier (2007) study the relation between contractibility and the design of contracts in biotechnology research. In this scenario, financing firms worry that research firms use their funding to pursue side projects. They find that when actions are not contractible an option contract becomes optimal since the threat of termination strengthens the incentives of the

\[5\] In this same paper, the authors show that firm ownership may be a way to balance drivers’s incentives across tasks.
research firm. Finally, we find more supporting evidence in Slade (1996) who empirically examines contracts between private, integrated oil companies and their service stations in Vancouver. She shows that variation in characteristics of a task optimally changes compensation scheme for another tasks.

All of these three papers offer evidence that indirectly support our results in that a quasi-exogenous shock in the cost of monitoring (a change in the monitoring technology) affects the contracting scenario and changes the strength of incentives optimally offered. Despite this, there is no evidence of the first step suggested by our model, that is, an increase in monitoring. Therefore the direct test of the implication of simultaneous changes in monitoring and incentives positively correlated within tasks and negatively correlated across tasks is left untested by the current empirical literature in contracting.

7 Concluding Remarks

In this paper, we introduce managerial attention allocation in optimal incentive contracts. In our model, managers are constrained in the total amount of monitoring capacity that they can allocate across tasks. The allocation of attention across tasks becomes a managerial problem with clearly defined trade-offs. Specifically speaking, more attention allocated to one task implies less monitoring in other tasks. When managers allocate more attention to a given task, the worker’s effort on that task becomes more contractible and therefore the manager optimally provides stronger incentives on the realization of that same task.

We find that managers allocate the same level of attention and provide the same incentive contracts for both tasks when production and monitoring technologies exhibit a low degree of returns to scale. When relaxing these initial conditions and allowing for the presence of increasing returns for both tasks in either technology, the symmetry of the results disappears and we find that managers optimally concentrate all their attention and provide incentives only for one of the tasks. After that, we introduce asymmetries in the way different tasks enter the production and monitoring technologies. As a result of this asymmetry, we find that managers optimally allocate different amounts of attention to different tasks and provide different incentives to different tasks.

These findings provide an alternative and complementary explanation for the use of simple unidimensional contracts in multi-tasking settings. In cases when managers combine their attention allocation decision and incentive contracts, they may choose to concentrate all their attention
and incentives in a few tasks and leave some others unmentioned in the contracts at use. This explanation of ours is consistent with the fact that most jobs are multidimensional and yet managers and principals use simple contracts that concentrate in only a few of the tasks that compose the job.

A straightforward extension of the model at hand would be to generalize the case of two tasks presented here into $n$ tasks. We believe though that our main results will remain qualitatively, and we foresee that the analysis and testable implications presented in this paper are easily generalizable and do not change when the number of tasks defining the job increases. Future lines of research are to include complementarities between tasks in monitoring and study the implications of managerial attention allocation for other incentive problems and vertical integration decisions.
References


