Cheap Talk Equilibria - A Note On Two Senders

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Abstract

We study multisender cheap talk in a one-dimensional setting. We characterize partially-revealing equilibria of a two-sender game in which the receiver can extract more information than by consulting only one sender as in Crawford and Sobel (1982). While focusing on one-directional biases, we so close a lacuna that has remained open since the Krishna and Morgan (2001) who show that when senders act in sequence, a single sender at best transmits as much information as do multiple like-biased senders.

We compare our result with the fully-revealing equilibria under one-directional biases. Fully-revealing equilibria are not robust to small mistakes of at least one sender, once this sender induces a point that is strictly preferred by the receiver. In turn, the equilibrium characterized in this paper is robust against small mistakes, making the equilibrium concept studied here applicable to real-world situations.

JEL classification: C72, D72, D74, D80, D82.

Keywords: Strategic information transmission, multiple senders, expertise, fully revealing vs. partitional equilibria, robustness against small mistakes

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1 Introduction[1]

A distinct part of the large game-theory literature on expertise has researched the case of multiple senders. This strand, because of its early focus on bi-partisan signaling in Congress (Gilligan and Krehbiel, 1989; Austen-Smith, 1993; Epstein, 1998), has encompassed 2-sender cheap talk with opposing biases. In particular, Krishna and Morgan (2001a, KM hereafter) show that while fully revealing equilibria (FRE) typically do not exist, the decision maker will always derive some benefit from consulting two experts, as long as the experts have opposing biases.[2]

With Battaglini (2002), the focus of the literature has shifted toward a treatment of multidimensional disclosure. While studying the existence of FRE both in a unidimensional and multidimensional setting, papers by Ambrus and Takahashi (2008) as well as by Feldmann (2007) have provide conditions under which full revelation is possible.

Different from the case of opposing biases, two-sender cheap talk with one-directional biases has not received comparable attention. We argue that a detailed treatment of this case is overdue. And although political expertise is generally known as the textbook case for opposing biases, it as well covers cases with one-directional biases such as homogenous committees in which the chairperson prefers a lower budget than the two experts.

Outside political institutions, examples with like-biased experts abound. Think of an uninformed consumer who shops for e.g. a plasma TV in a consumer electronics outlet: would he prefer to consult only one salesperson before making a decision, namely the one closer to his bliss point (or his preferred budget), or would he benefit from the additional presence of a more biased salesperson?[3] Alternatively, think of a corporate CEO who calls two of his subordinates into his office to hear their advice on a project. Will he do better by including the more biased subordinate in the meeting, or should he only ask the less biased expert?

A detailed analysis of settings under one-directional biases seems important, for several reasons. First, to our best knowledge, equilibria with two senders have not been studied thoroughly under simultaneous disclosure.

[1]I thank Attila Ambrus, Oliver Board, Sidartha Gordon, Navin Kartik, Joel Sobel and Phil Stocken for comments on an earlier draft, as well as the participants of the Games 2008 conference for additional suggestions. All errors are mine.

[2]See Krehbiel (2001) for a critique of Krishna and Morgan (2001b).

[3]A related example can be found in Dziuda (2008).

Second, KM's findings under one-directional biases apply to sequential disclosure. They show that when senders act in sequence, a single sender at best transmits as much information as do multiple like-biased senders. This may be counterintuitive to real-world observations and it follows from their analysis of sequential disclosure. It is rather obvious that consulting multiple experts can *never be worse* than consulting a single expert in a CS setting when comparing "best" equilibria: there there is always an equilibrium in which one sender babbles, and the remaining sender and the receiver play the most-informative CS equilibrium of single-expert game.

Third, partially revealing equilibria with two senders have not been the object of study anywhere else in the literature, in our view because of the existence of FRE. Since Battaglini (2002) and Ambrus and Takahashi (2007) the literature has however shown that FRE exist only if both senders observe the state *perfectly.* Once at least one sender is believed by the receiver to make some mistake with a positive probability, FRE are no longer stable. This would suggest that in real life decision makers will rather develop disclosure protocols that develop mutual punishment strategies that, per se, are counterintuitive.[4] Specifically, this includes the case of partitional equilibria with two senders.

Last, adding a second sender can be regarded as a natural refinement of CS. This follows the line of thought illustrated in Chen, Kartik and Sobel (2008, CKS hereafter) as well as in Sobel (2008) who study partitional equilibria that build on CS. We so add an analysis of one-directional biases and provide some additional insight.

2 A 2-sender model with partial information revelation

We start with the canonical setting of CS but allow for two senders. Biases b are known to all players, with $b^R = 0$, $b^{S_1} < b^{S_2} < \frac{1}{4}$. Senders S_1 and S_2 observe the state of nature (=their type) t. The receiver, R, discloses a communication protocol in which he announces to play CS Best Equilibrium Strategies with S_1 and in addition permits S_2 to trigger a known default action $\tilde{a}$. Each sender discloses a message m^{S_j}. R observes the messages and chooses an action by best responding to his beliefs. We use j to label the senders, with $j \in \{1, 2\}$. When speaking of Sender j, the second sender is labeled S_{-j}. Otherwise, our notation follows closely the exposition in CKS.

[4] See Krehbiel, 2001.

2.1 Characterization

With two senders, $S_j, j \in \{1, 2\}$ and one-directional biases, with $0 < b_1 < b_2$, a pure-strategy PBE consists of

(i) a message strategy μ^{S_j}, with $\mu^{S_1} : [0, 1] \to M = \{m_1, ..., m_{N-1}\}$ and $\mu^{S_2} : [0, 1] \to \{\tilde{a}, \neg\tilde{a}\}$,

(ii) an action strategy $\alpha : M \times \{\tilde{a}, \neg\tilde{a}\} \to \mathbb{R}$ for the receiver, and

(iii) an updating rule $\beta(t|m^{S_1}, m^{S_2})$ such that

- for each $t \in [0, 1], \mu(t)$ solves $\max\limits_{m_i^{S_j}} U^{S_j}(\alpha(m_i^{S_j}, t),$

- for the message pair $(m_i^{S_1}, m^{S_2}), \alpha(m_i^{S_1}, m^{S_2})$ solves $\max\limits_{a} \int\limits_0^1 U^R(a, t)\beta(t|m_i^{S_1}, m^{S_2})$ dt,

where $(t|m_i^{S_1}, m^{S_2})$ is derived from μ and F from Bayes' Rule whenever possible.

Our two-sender equilibrium can be characterized by a partition of the set of types, $t(N) = (t_0(N), ..., t_N(N)$ with $0 = (t_0(N) < (t_1(N), ..., t_k(N) < a < t_{k+1}(N), ..., t_{N-1}(N) < (t_N(N) = 1, S_1$'s and messages $m_i^{S_1}, i = 1, ..., N$, and m^{S_2} such that for all $i = 1, ..., N-1$,

$$U^{S_1}(\bar{a}(t_i, t_{i+1}), t_i)) - U^{S_1}(\bar{a}(t_{i-1}, t_i), t_i)) = 0 \qquad \text{(S_1-types on the boundary are indifferent)}$$

$$\mu^{S_1}(t) = m_i^{S_1} \ \mathit{for}\ t \in (t_{i-1}, t_i]. \qquad \text{(S_1-types in a common element pool and send same message)}$$

$$U^{S_2}(\bar{a}(t_i, t_{i+1}), m_2 = \tilde{a})) > U^{S_2}(\bar{a}(t_i, t_{i+1}), m_2 = \neg\tilde{a})) \qquad \text{(Condition for S$_2$ to disclose $\tilde{a}$)}$$

while R's best response to the information of Sender 1 should Sender 2 disclose $m^{S_2} = \neg\tilde{a}$ is

$$\alpha(m^{S_j}) = \bar{a}^{S_1}(t_{i-1}, t_i),$$

or, in case of $m^{S_2} = \tilde{a}$ this response is either

$$\alpha(m^{S_j}) = \frac{\tilde{a} + \bar{a}(t_i^{S_1})}{2} \text{ } or \text{ } \frac{\tilde{a} + \bar{a}(t_{i+1}^{S_1})}{2}.$$

We summarize our finding about the existence of a 2-sender equilibrium in the following proposition:

Proposition 1 EXISTENCE OF 2-SENDER EQUILIBRIA. *For $\bar{a}(t_k) < \tilde{a} < \bar{a}(t_{k+1})$, there exist 2-sender partitional equilibria that are more informative than CS Best equilibria in which the Receiver best responds to his beliefs by implementing the actions $\frac{\tilde{a}+\bar{a}(t_i^{S_1})}{2}$ or $\frac{\tilde{a}+\bar{a}(t_{i+1}^{S_1})}{2}$, contingent on the messages in M.*

Note that the senders disclose simultaneously, and since S_j's response is independent of S_{-j}'s message strategy, it is sufficient to characterize the equilibrium in the way done above. More generally, Sender j's strategy is always fully described by the state of nature, and by the biases, once the communication protocol is revealed and the Receiver always best responds to his beliefs. It is a distinct feature concerning the stability of this equilibrium that each sender can rely on the opponent's sender disclosure.

2.2 A parametric example with $b_1 = \frac{1}{20}, b_2 = \frac{1}{15}, \tilde{a} = \frac{1}{2}$

• RECEIVER'S POSTERIOR BELIEFS

With $\mathcal{U}[x, y]$ denoting the uniform distribution over the interval $[x, y]$, R's posterior beliefs are: [5]

$$P(\cdot \mid m^{S_1}, m^{S_2}) = \begin{cases} \mathcal{U}[0, \frac{2}{15}] \text{ if } m^{S_1} \text{ suggests } \frac{1}{15} \\ \mathcal{U}[\frac{2}{15}, \frac{7}{15}] \text{ if } m^{S_1} \text{ suggests } \frac{9}{30} \\ \mathcal{U}[\frac{7}{15}, 1] \text{ if } m^{S_1} \text{ suggests } \frac{11}{15} \\ \mathcal{U}[\frac{1}{3}, \frac{7}{15}] \text{ if } m^{S_1} \text{ suggests } \frac{9}{30} \text{ and } m^{S_2} \text{ suggests } \tilde{a} \\ \mathcal{U}[\frac{7}{15}, \frac{38}{60}] \text{ if } m^{S_1} \text{ suggests } \frac{11}{15} \text{ and } m^{S_2} \text{ suggests } \tilde{a} \\ \mathcal{U}[0, 1] \text{ else.} \end{cases}$$

[5]To simplify the notation, a disclosure of $\mathrm{m}_2 = \neg a$ is omitted. This comes without loss of generality.

• Strategy Profile of Sender 1:

$$\mu^{S_1} = \begin{cases} \dfrac{1}{15} \text{ if } t \in [0, \frac{2}{15}) \\ \dfrac{9}{30} \text{ if } t \in [\frac{2}{15}, \frac{7}{15}) \\ \dfrac{11}{15} \text{ if } t \in [\frac{7}{15}, 1] \end{cases}$$

• Strategy Profile of Sender 2:

$$\mu^{S_2} = \begin{cases} \tilde{a} \text{ if } t \in [\frac{1}{3}, \frac{31}{60}] \\ \neg\tilde{a} \text{ otherwise.} \end{cases}$$

• Receiver's strategy profile:

$$a(m^{S_1}, m^{S_2}) = \begin{cases} \dfrac{1}{15} \text{ if } m^{S_1} \text{ suggests } \frac{1}{15} \\ \dfrac{9}{30} \text{ if } m^{S_1} \text{ suggests } \frac{9}{30} \\ \dfrac{11}{15} \text{ if } m^{S_1} \text{ suggests } \frac{11}{15} \\ \dfrac{2}{5} \text{ if } m^{S_1} \text{ suggests } \frac{9}{30} \text{ and } m^{S_2} \text{ suggests } \tilde{a} \\ \dfrac{37}{60} \text{ if } m^{S_1} \text{ suggests } \frac{11}{15} \text{ and } m^{S_2} \text{ suggests } \tilde{a} \end{cases}$$

Note that the action $\frac{1}{2}$ is never chosen *in equilibrium*, but it permits to implement an informationally superior 2-sender mechanism compared to CS.[6]

[6]Similarly, note that the receiver's posterior beliefs may stay uniformy distributed over [0,1]. In equilibrium, this never occures once Sender 1 discloses. A somewhat similar construction is given in a different setting in Krishna and Morgan (2001c).

An illustration of this parametric example is given in *Fig. 1* below.

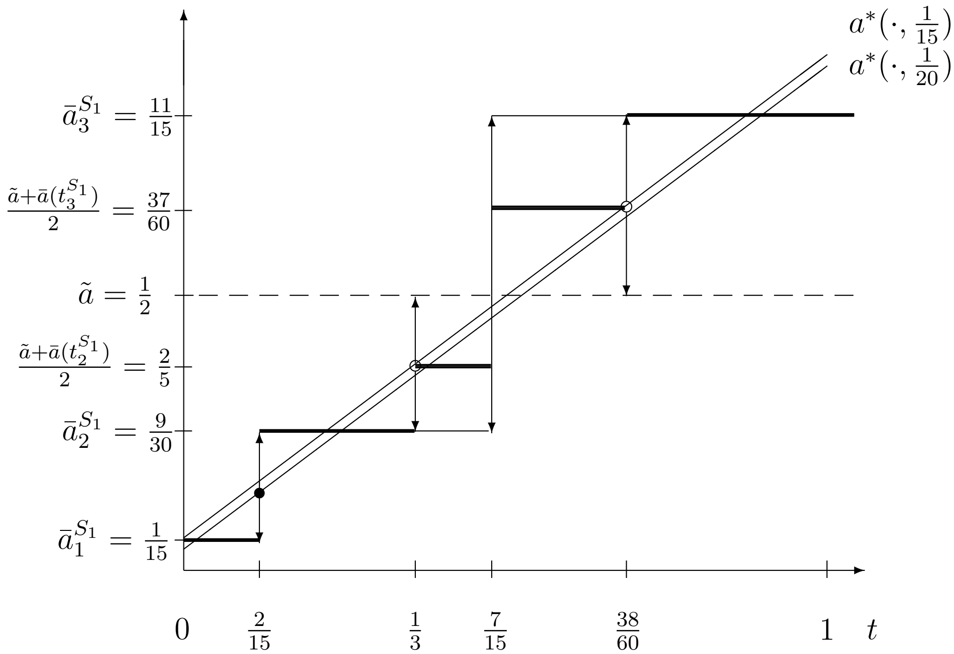

Fig. 1: Parametric example with $b_1 = \frac{1}{20}, b_2 = \frac{1}{15}, \tilde{a} = \frac{1}{2}$

Proposition 2. For $b_1 = \frac{1}{20}$ and $b_2 = \frac{1}{15}$, the two-sender PBE is more informative than the CS equilibrium with the less biased sender.

Proof.

$$-\left[\int_0^{\frac{2}{15}} \left(\frac{\frac{2}{15}}{2}\right)^2 + \int_{\frac{2}{15}}^{\frac{1}{3}} \left(\frac{\frac{1}{3}-\frac{2}{15}}{2}\right)^2 + \int_{\frac{1}{3}}^{\frac{7}{15}} \left(\frac{\frac{7}{15}-\frac{1}{3}}{2}\right)^2 + \int_{\frac{7}{15}}^{\frac{38}{60}} \left(\frac{\frac{38}{60}-\frac{7}{15}}{2}\right)^2 + \int_{\frac{38}{60}}^{1} \left(\frac{1-\frac{38}{60}}{2}\right)^2\right] = -0.00575 = -\frac{23}{4000}.$$

In turn, the CS equilibrium with $b_1 = \frac{1}{20}$ yields approximately -0.00637.

3 Stability against small mistakes and FRE

So far, we have shown that there exists a partitional equilibrium that is more informative than Crawford and Sobel. What remains to be shown is that the equilibrium characterized here is robust against sender errors, while the existing fully revealing equilibrium is not.[7]

Consider an example with one-directional biases.[8] An FRE exists as long as the receiver can mutually punish both senders by implementing an extreme punishment action that makes both senders worse off than fully revealing the true state of nature.[9]

Now assume that one sender makes a mistake with probability ε. That is, with probability $1-\varepsilon$ he is still a expert and observes the state of nature perfectly, but with probability ε he observes a random state, that is a state between [0,1]. It can be shown that as soon as one sender makes a mistake with a positive probability, the FRE fails to exist since the discontinuity in the beliefs would be large enough to make the FRE collapse.[10]

We now use the same construction with sender errors to check whether our equilibrium is stable against small mistakes. In our equilibrium, Sender 1 plays the CS Best Equilibrium with the Receiver, while Sender 2 compares this result with a given action $\tilde{a}$. In what follows, we show that the higher-partition profile of Sender 1 is stable against small sender errors.

This construction is straightforward. Sender 1 now observes the true state with probability $1-\varepsilon$, and with probability ε he observes a state $t \in [0,1]$. Since each disclosure follows a sender error with probability ε, the receiver will now best respond to a disclosure by implementing action

$$\bar{a}_i = (1-\varepsilon)(\frac{t_{i-1}+t_1}{2}) + \varepsilon\frac{1}{2}.$$

Reducing the observation to this interval, the sender's expected utility when disclosing $m^{S_1} \in [t_{i-1}, t_i]$ can be defined

[7] I thank Attila Ambrus and Oliver Board for helpful suggestions on this case. See also Blume et al. (2007).

[8] See Battaglini (2002), Sobel (2007).

[9] For one-directional biases it would suffice to threaten the senders by e. g. implementing an action $\frac{1}{2}$ of the lowest disclosure.

[10] See e.g. Battaglini (2002)

$$E[U^{S_1}(t)] = -(1-\varepsilon)(t+b_1-\bar{a}_i)^2 - \varepsilon \int\limits_0^1 (t'+b_1-\bar{a}_i)^2 \ \ dt'$$

Because of the CS "no-arbitrage" condition, types t are indifferent between inducing action $_i and \bar{a}_{i-1} at any breakpo$

$$U^{S_1}(t, \bar{a}_i) = U^{S_1}(t, \bar{a}_{i-1})$$

leading to

$$t_{i+1} - t_i = t_i - t_{i-1} + \frac{4b}{1-\varepsilon},$$

which reveals that the last interval must be by $\frac{4b}{1-\varepsilon}$ longer than the first. This shows that sender errors in our equilibrium as well as in CS entail a loss of information for all players, reducing the utility. Under sender errors, the partitions are less evenly spaced as under perfect observation. However, the equilibria are generally robust against sender errors, while the FRE is not.

4 Concluding remarks

As Sobel (2008) and CKS have already argued, to add a second sender is a natural way to refine the information structure under cheap talk. We have closed a lacuna in this field and characterized a partially-revealing equilibrium with two senders and one receiver under simultaneous disclosure and with one-directional biases. In addition, we have discussed the stability of these equilibria vis-a-vis fully-revealing equilibria. Our results are of some value and apply for real-world expertise with two senders and one-directional biases. Typically, expertise – even with two senders – does not lead to truth telling. However, it reveals additional information, making it worthwhile for the receiver to add a second, more biased sender.

Parametric analysis furthermore reveals that the mechanism is indeed more informative for all small biases, and even for large bias differences.[11]

While not part of this note, a next extension of the literature could be to study default actions $\tilde{a}$ that are different from the case studied here, namely $\frac{1}{2}$. This would make the equilibrium concept broadly

[11] A detailed parametric analysis is available from the author on request.

applicable to series of real-world institutions. In homogeneous committees, chairpersons may chose to remain uninformed and thus trigger a previous and known default action when refusing to act. This and similar applications are left for future research.

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